The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2036-14

International Workshop: Quantum Chromodynamics from Colliders to Super-High Energy Cosmic Rays

25 - 29 May 2009

QCD evolution at small x

Anna Stasto

Penn State University & RIKEN Center BNL
University Park
USA

QCD evolution at small x

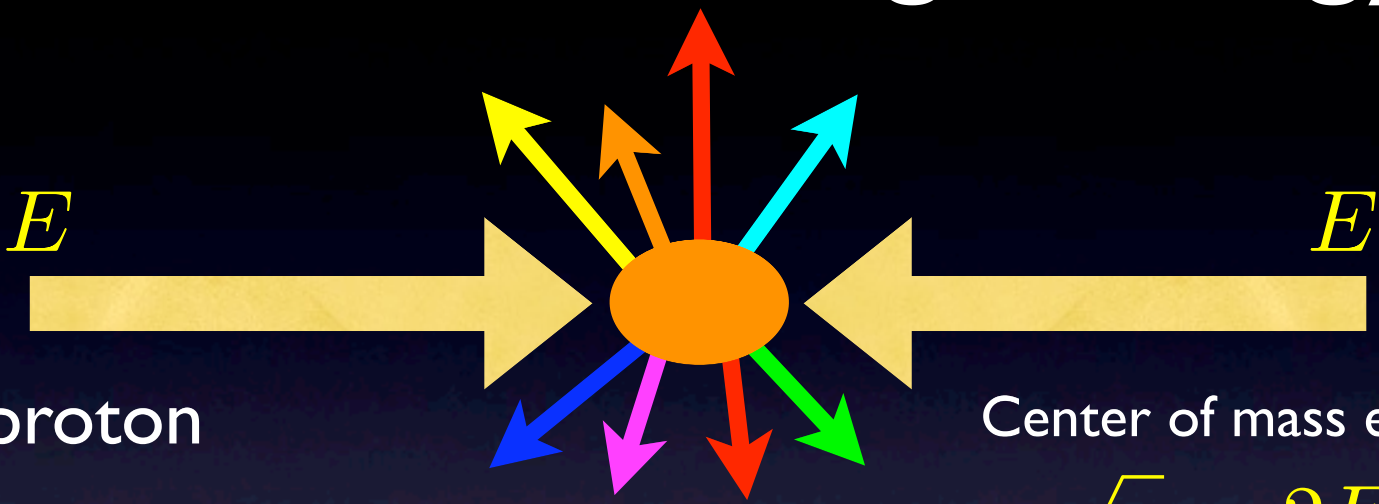
Anna Staśto
Penn State University & RIKEN Center BNL

ICTP, Trieste, May 25th, 2009

Outline

- Introduction and motivation:
  - Proton structure at high energy (small x).
  - Gluon radiation.
- Resummation in the evolution at small x.
- Unitarity and parton saturation.

Collisions at high energy



ex. proton

Center of mass energy

$$\sqrt{s} = 2E \gg m_h$$

center-of-mass energy:

- RHIC (pp,AA): 200 GeV
- HERA (ep): 300 GeV
- Tevatron (pp): 1.96 TeV
- LHC (pp,AA): 14 TeV, 5.5 TeV
- Cosmic Rays (pA,AA): 100 TeV

How well do we know theory of strong interactions - QCD - at these high energies?

Quantum Chromodynamics

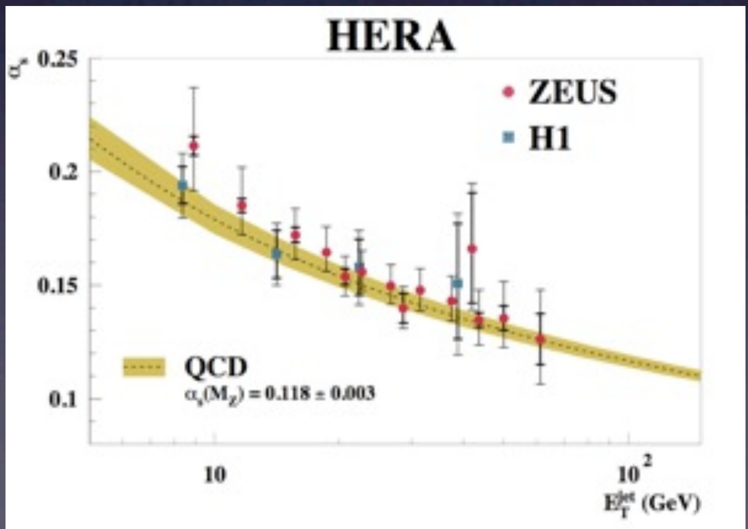
QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a} + \sum_{\mathrm{flavours}} \bar{q}_a (i D^{\mu} \gamma_{\mu} - m_f)_{ab} q_b$$

Field strength:

$$F^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\mu} A^{a}_{\nu} - g f^{abc} A^{b}_{\mu} A^{c}_{\nu}$$

Strong coupling as a function of energy



Small scales

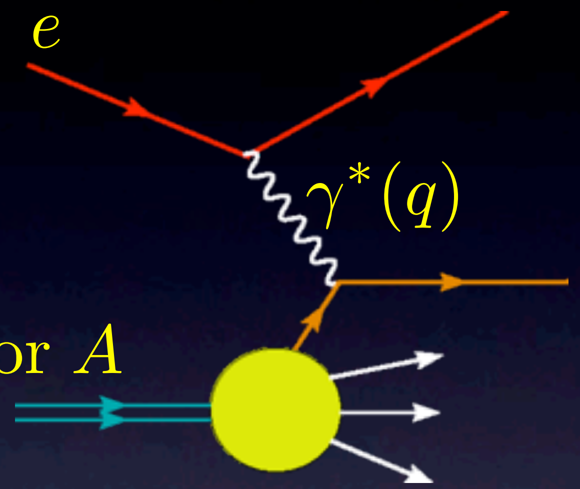
Large scales

- Rich and very complicated structure due to non-linear interactions of gluons.
- Emergent phenomena: confinement, Regge trajectories, hadron spectrum.
- Complex dynamics at high energies or at small Bjorken x.

How do we know that gluons play such an important role at high energies?

Deep Inelastic Scattering

Scattering of electron off a hadron(proton):



Photon virtuality:

$$Q^2 = -q^2 > 0$$

Total energy of the photon-proton system

$$s = (p+q)^2$$

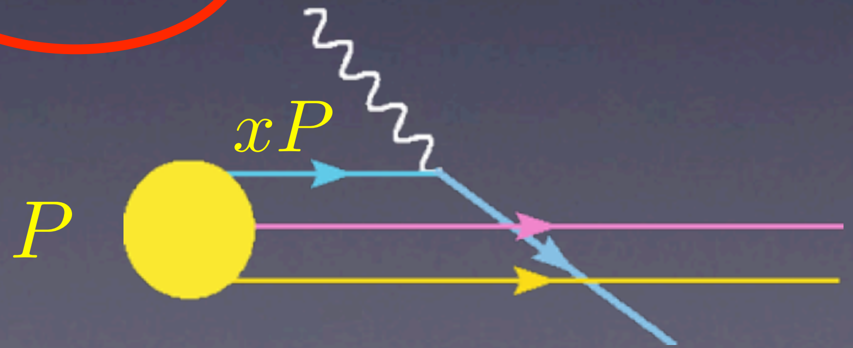
Bjorken x

$$x = \frac{Q^2}{s+Q^2} \simeq \frac{Q^2}{s}$$

cross section   parton density

$$\sigma^{\gamma^* p} \sim \frac{1}{Q^2} x f(x, Q^2)$$

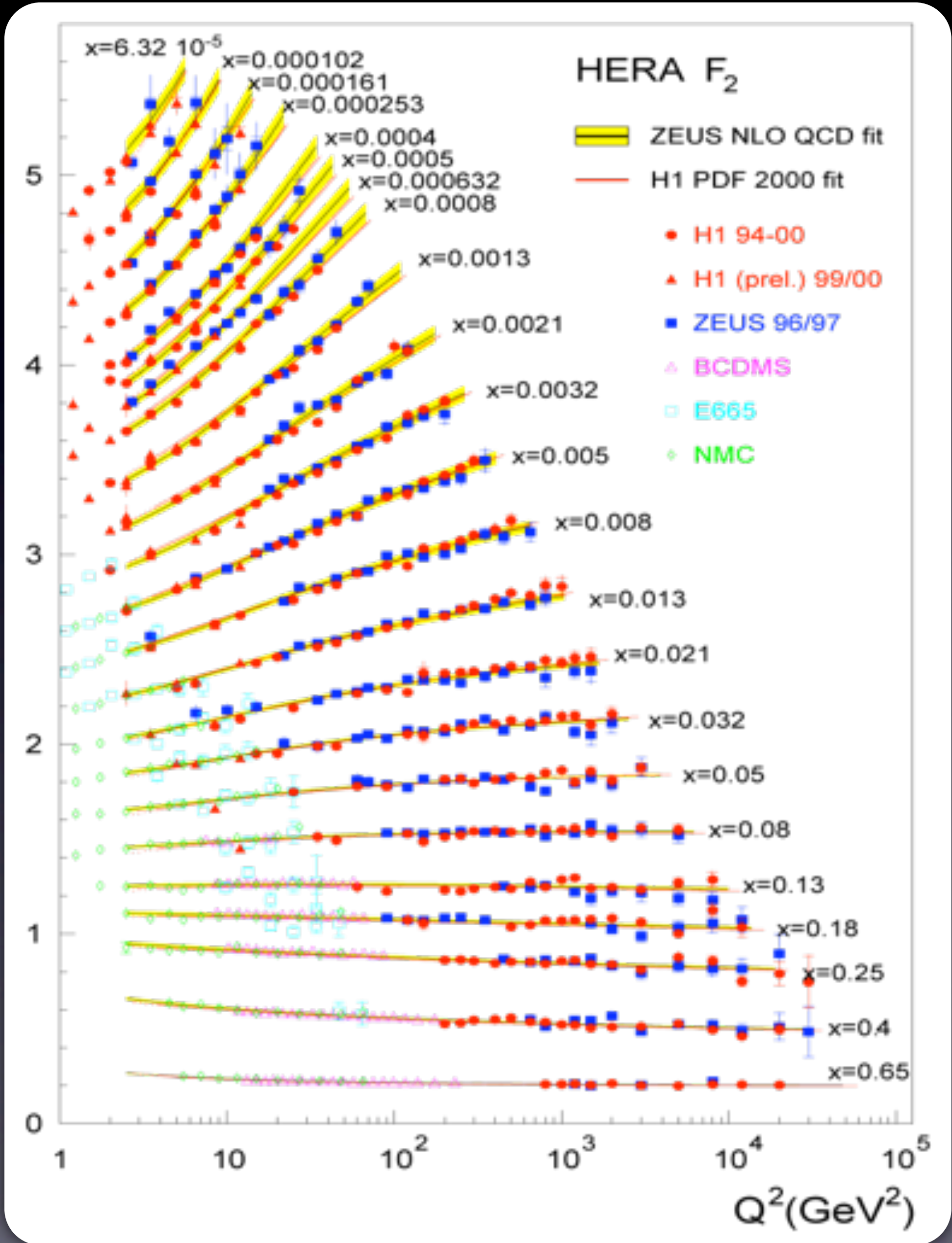
$$0 \leq x \leq 1$$



x fraction of the longitudinal momentum of the proton carried by the quark

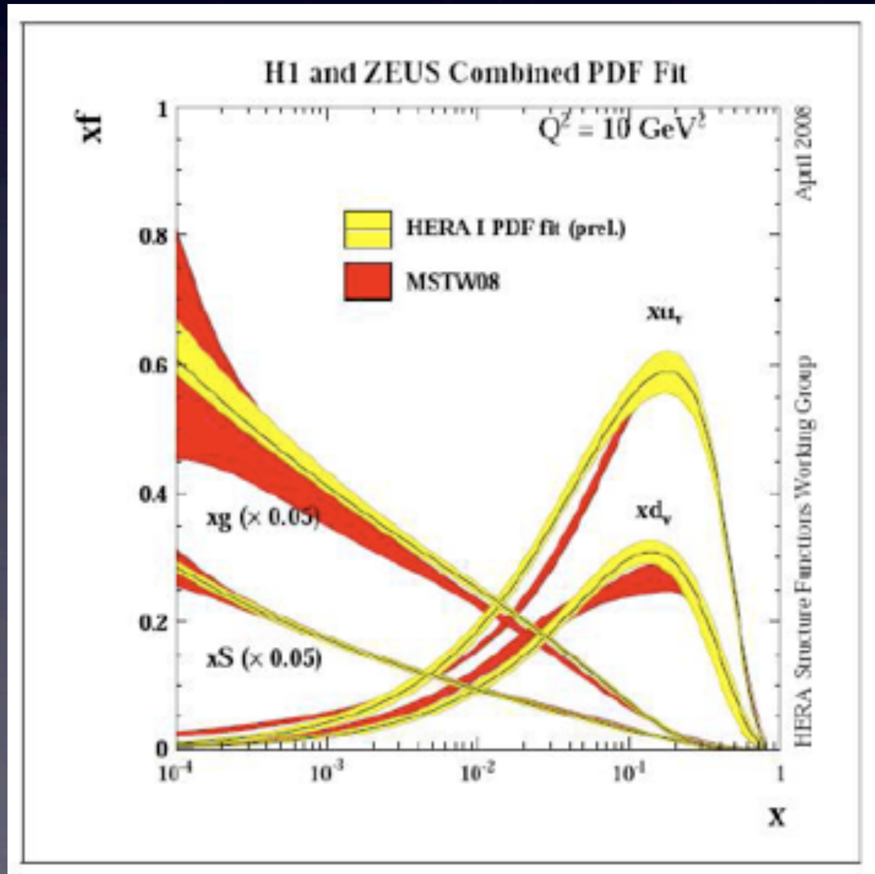
DIS

$$\frac{d^2\sigma^{ep\to eX}}{dxdQ^2} = \frac{4\pi\alpha_{\rm em}^2}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2)-\frac{y^2}{2}F_L(x,Q^2)\right]$$

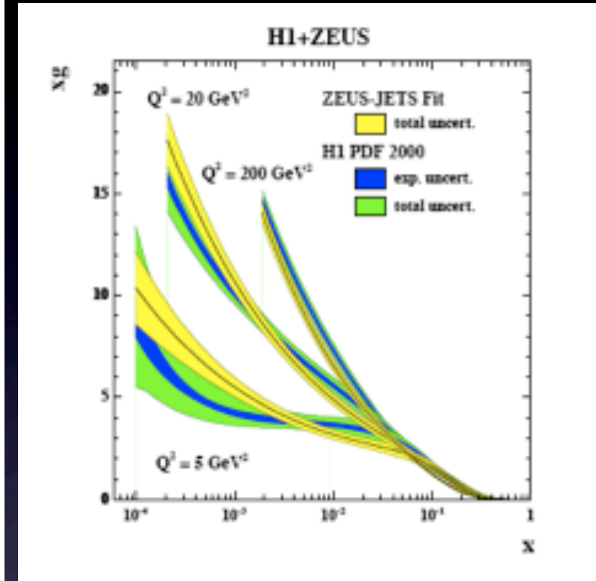


Observation of large scaling violations.

Gluon density dominates at small x!



Estimates of gluon density

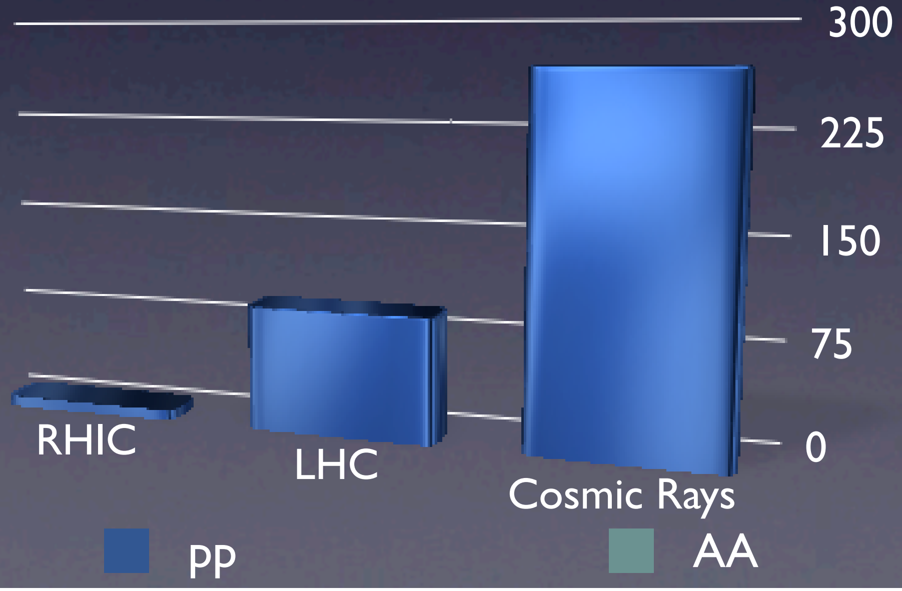


Extrapolation of gluon density
to various energies

Scale $Q^2 = 5\ \mathrm{GeV}^2$

$$x \sim \frac{Q^2}{s}$$

gluon density



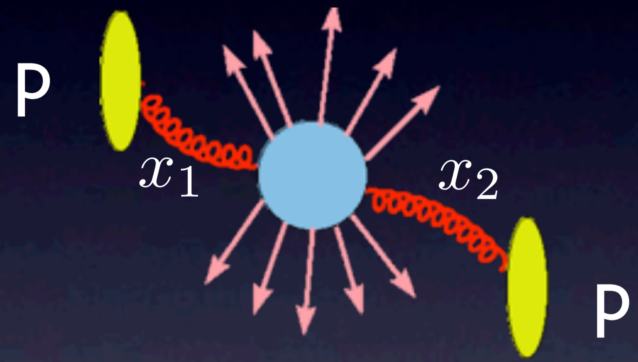
- LHC is a gluon-gluon collider.
- Extremely small x probed at Cosmic Ray energies.
- Large gluon densities.

In hadron-hadron collisions

LHC

pp 14 TeV

PbPb 5.5TeV



Typical values of x

Scale $2\ \mathrm{GeV}$

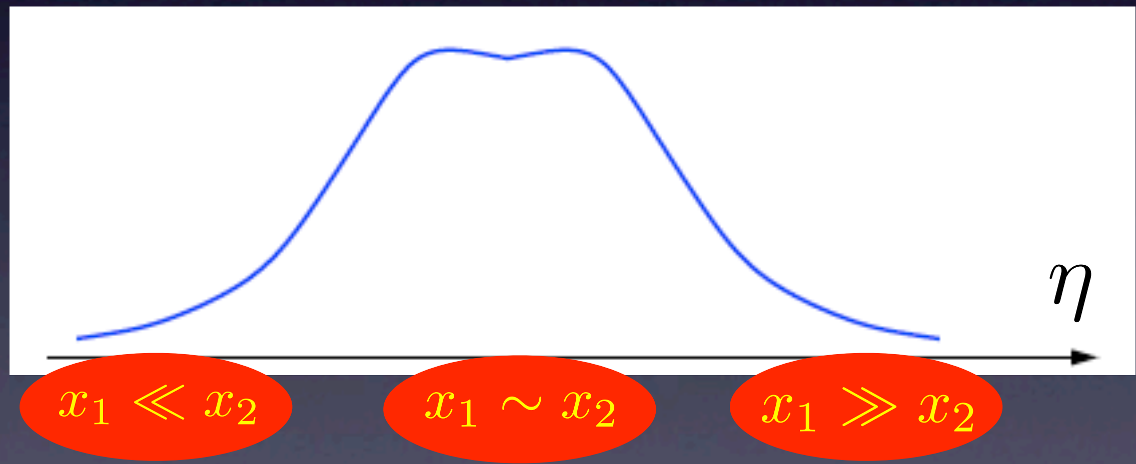
η	0	4
RHIC	0.01	0.0001
LHC	0.0001	0.000001

Cosmic Rays

pp or AA about 100 TeV

$x_1\ x_2$ fraction of longitudinal momenta of protons carried by the gluons

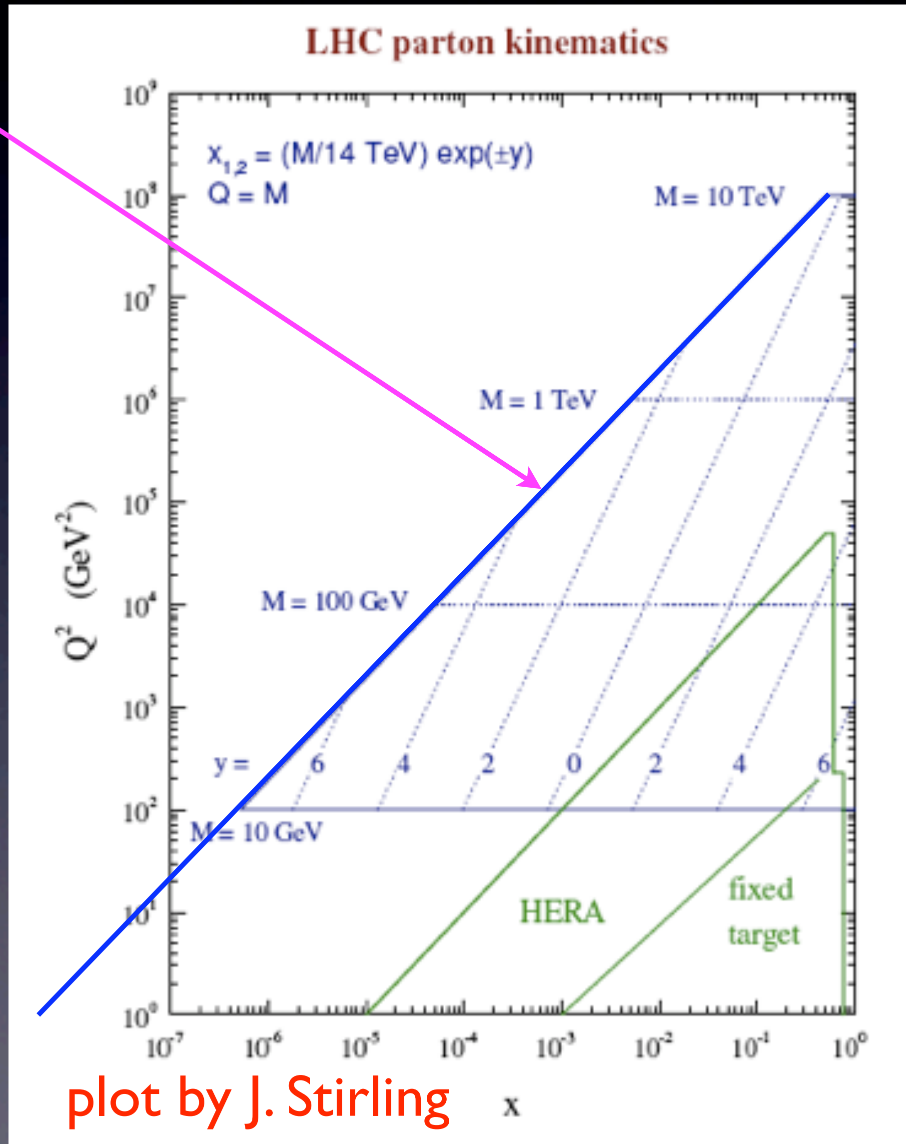
Pseudorapidity distribution of the produced particles



Pseudorapidity $\eta = -\ln(\tan\frac{\theta}{2})$

LHC and Auger kinematics

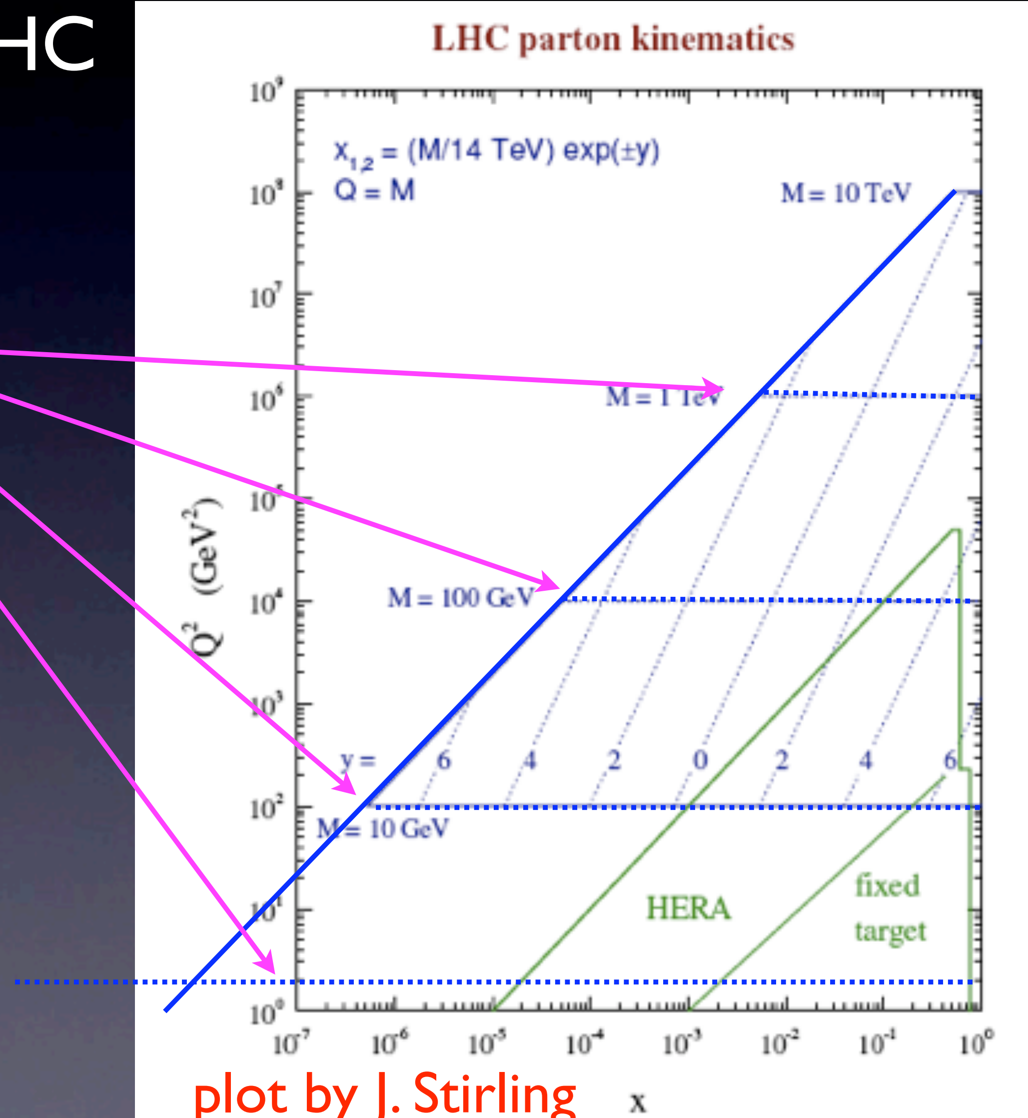
Kinematic boundary for LHC



LHC and Auger kinematics

Kinematic boundary for LHC

Different scales

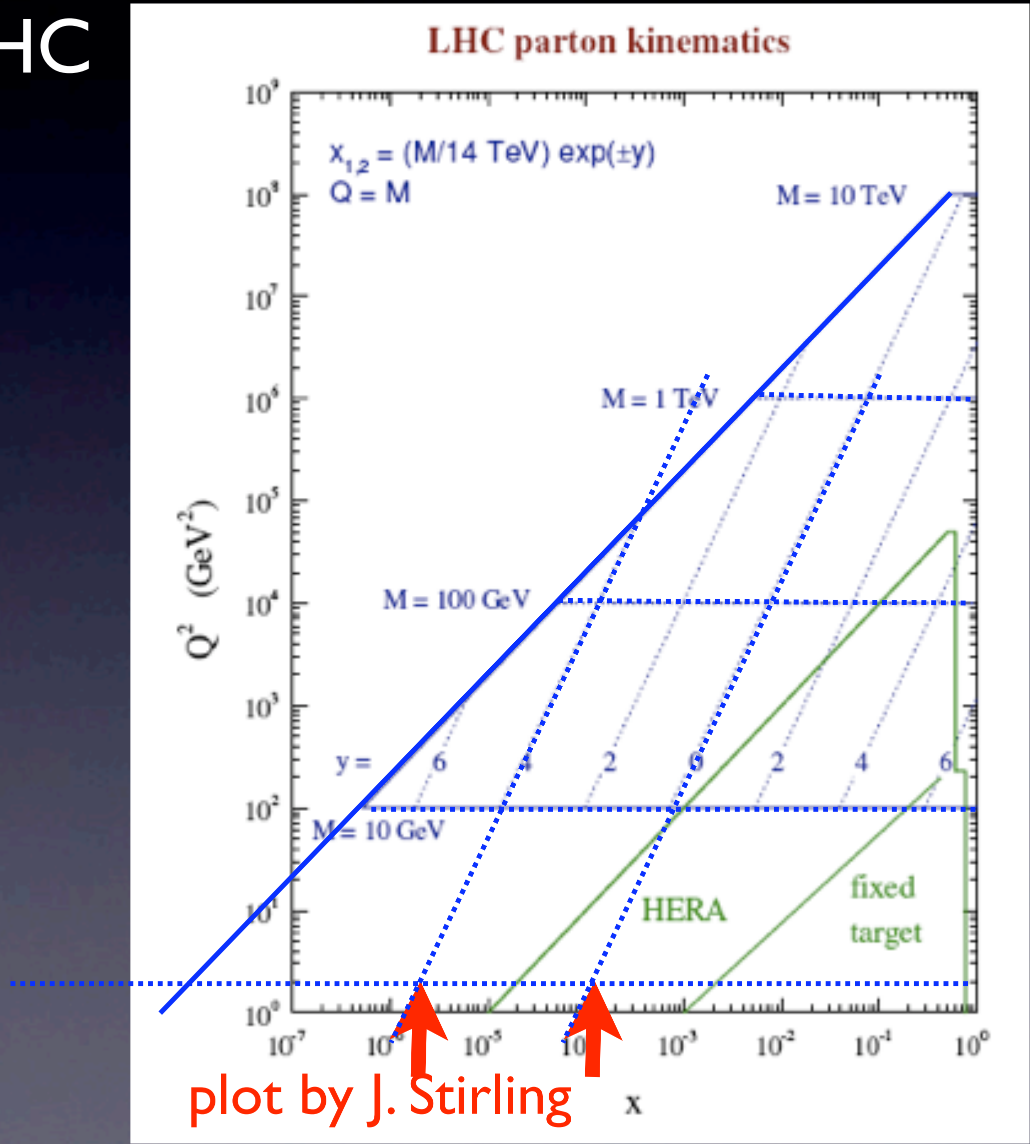


plot by J. Stirling

LHC and Auger kinematics

Kinematic boundary for LHC

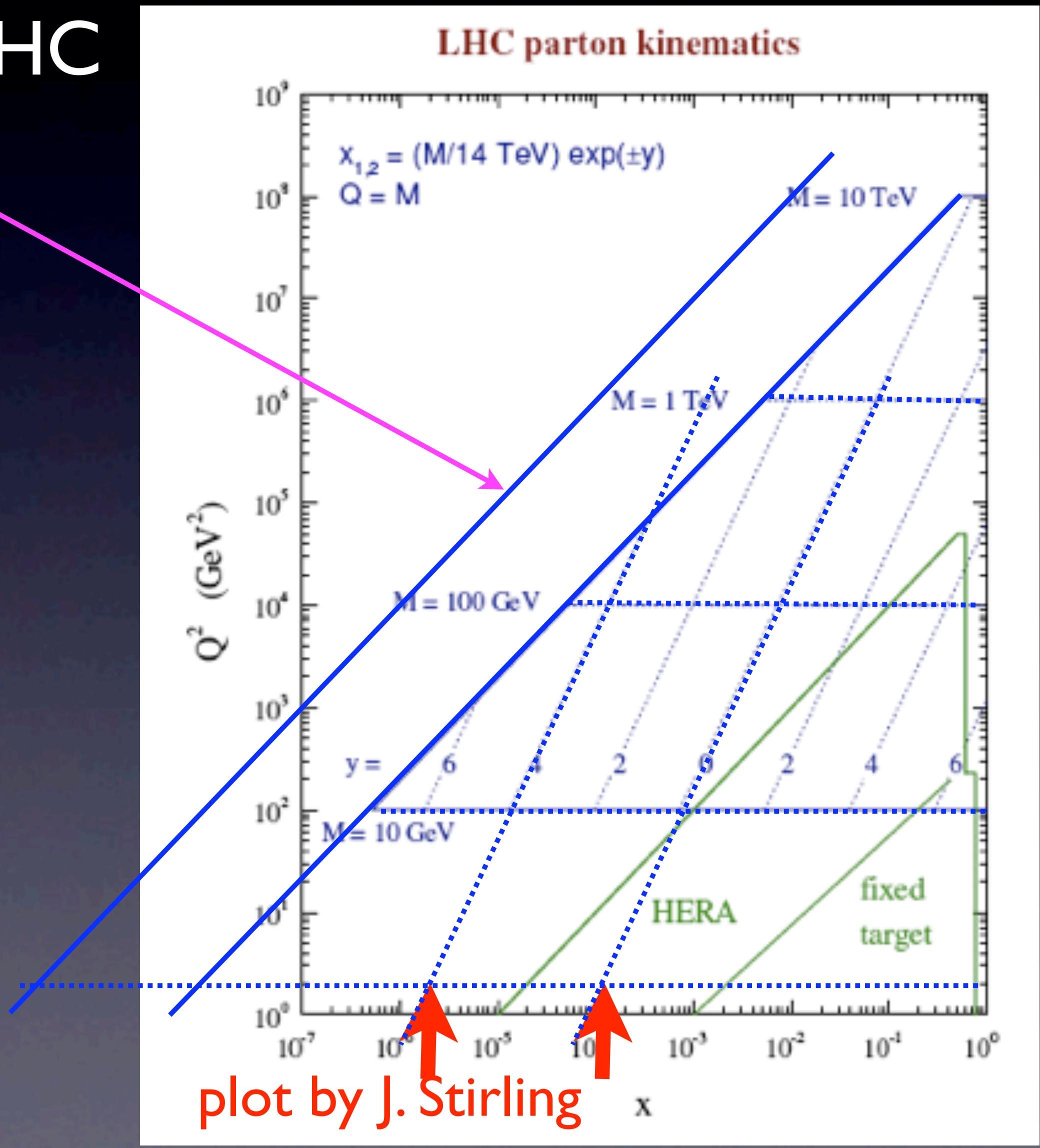
Different rapidities



LHC and Auger kinematics

Kinematic boundary for LHC

Kinematic boundary
for Auger: 100 TeV c.m.s.



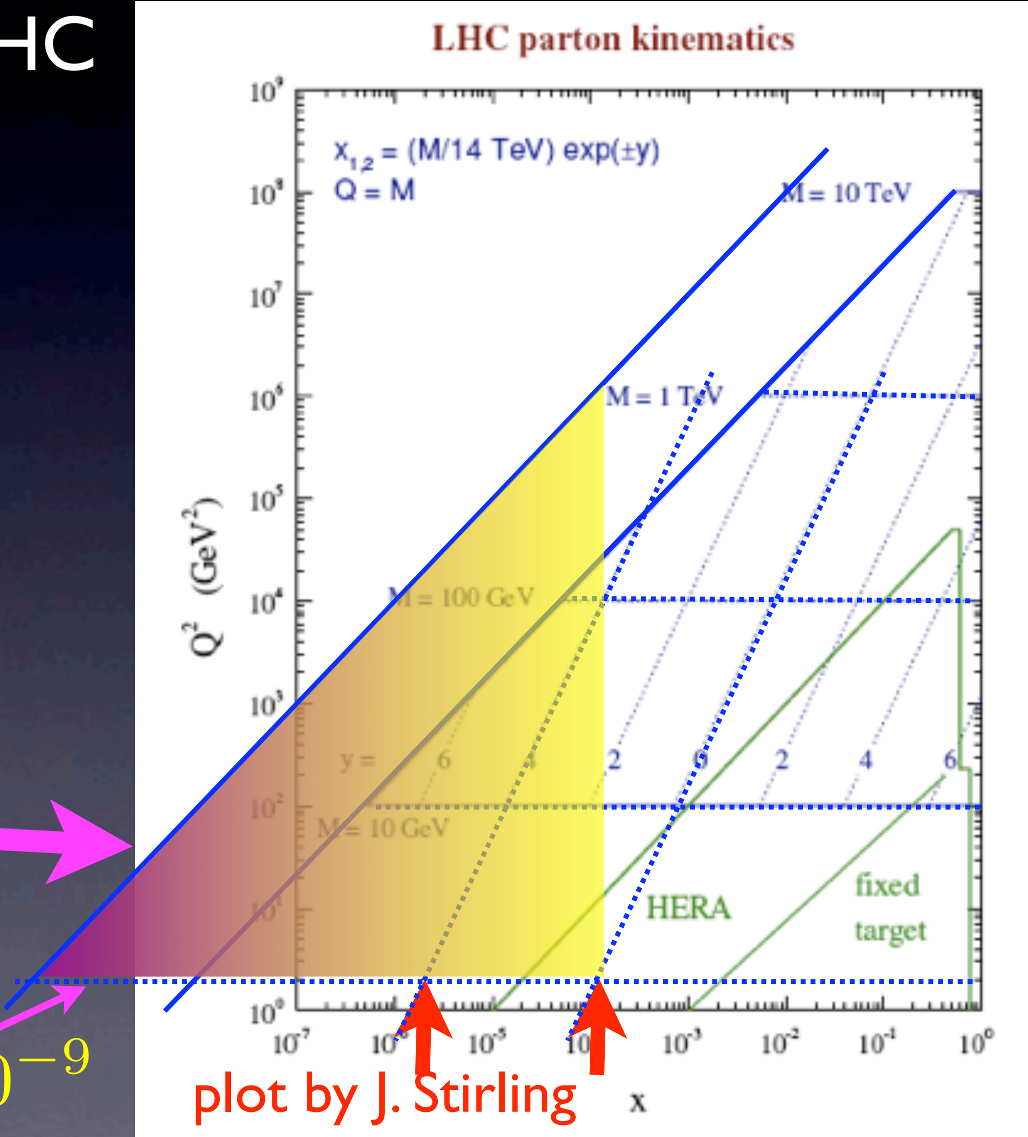
LHC and Auger kinematics

Kinematic boundary for LHC

Kinematic boundary for Auger: 100 TeV c.m.s.

Very large region with very small x!

$x \sim 10^{-9}$



plot by J. Stirling

- Increasing role of the gluon emissions and their interactions at high energy: at small x.
- LHC is really a ‘Large Gluon Collider’.
- Detailed knowledge of the gluon density important for the search of the new phenomena (etc. Higgs, SUSY).
- New kinematic regime for high - density QCD opens at LHC and in the cosmic ray interactions.

Questions and problems:

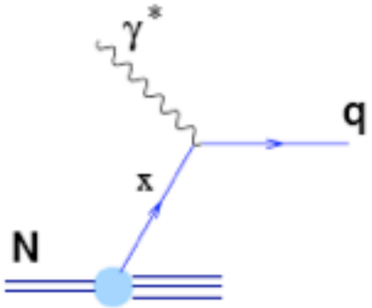
- What are the computational methods suitable at high energy/density limit ?
- What are the predictions for the phenomenology ?

Computing parton density in QCD: DGLAP evolution

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

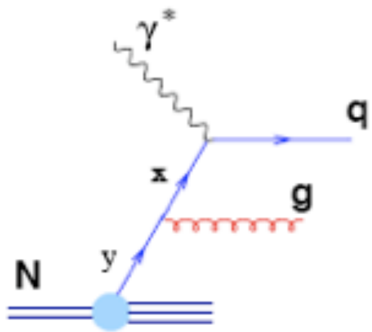
Scattering of photon γ with virtuality Q^2 off a hadron, at c.m.s energy $\sqrt{s}$.
Q^2 defines the resolution with which one probes partonic structure:

$t = \ln Q^2/Q_0^2$



$q(x,t)$

$t + \delta t$



$q(x,t) + \delta q(x,t)$

quarks with fraction $x = \frac{Q^2}{s}$ of target momentum

Evolution equation:

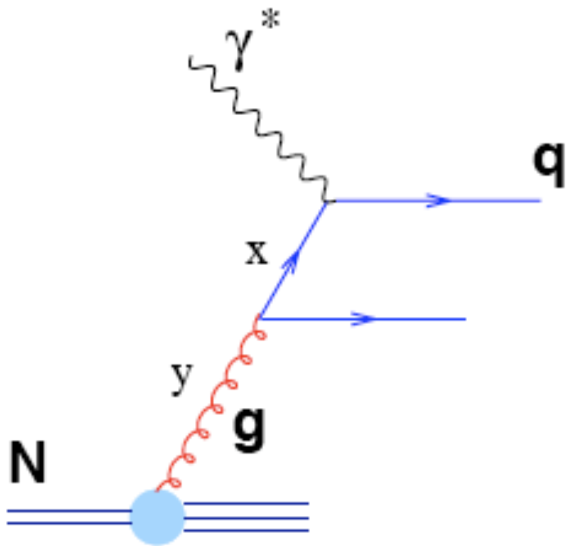
$$\frac{dq(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}(x/y)\, q(y,t)$$

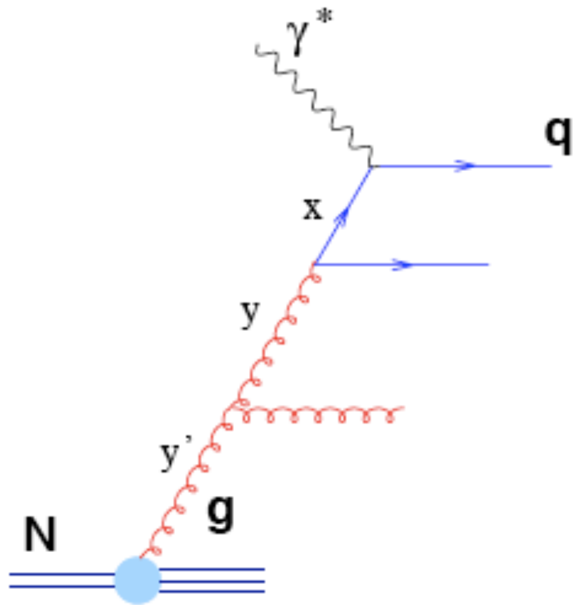
$P_{qq}(z)$ is the probablity of finding quark in the quark with $z(= x/y)$ fraction of its momentum

DGLAP evolution

Apart from $q(x,t)$ one has to include $g(x,t)$

t $\qquad\qquad$ $t+\delta t$





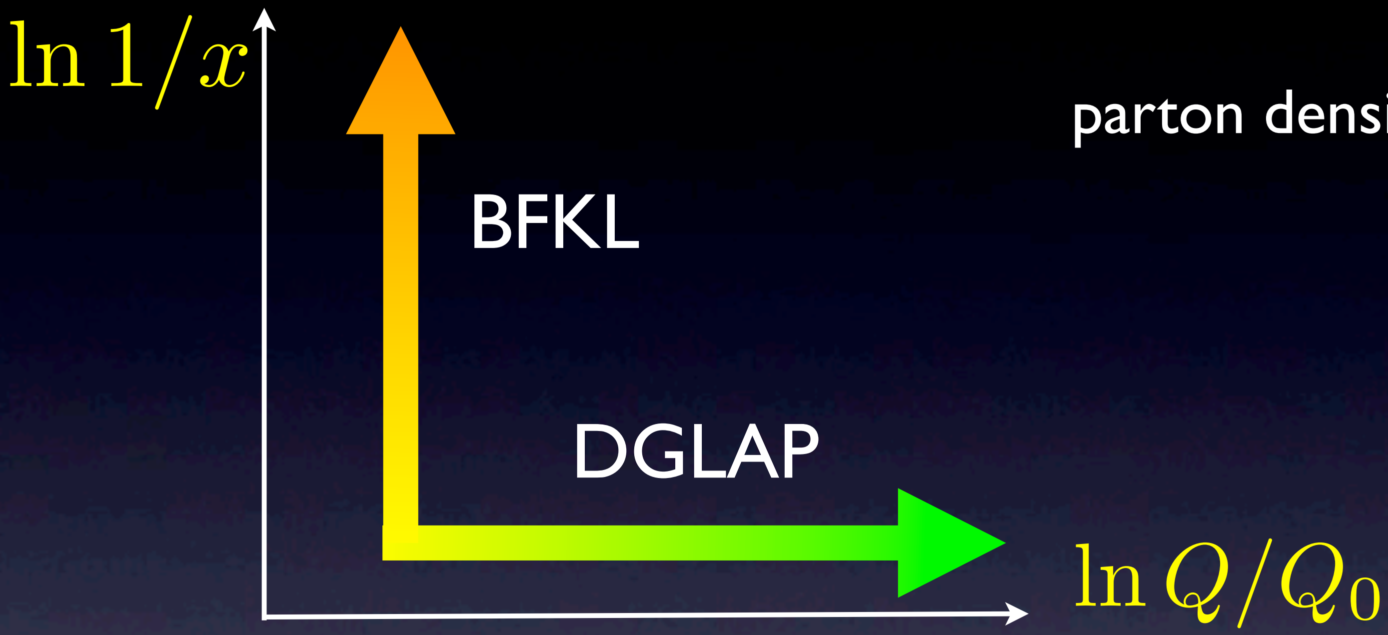
Set of evolution equations (flavour singlet case):

$$\frac{\partial}{\partial t}\begin{bmatrix}\Sigma(x,t)\\ g(x,t)\end{bmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{bmatrix}P_{qq} & 2N_f P_{qg}\\ P_{gq} & P_{gg}\end{bmatrix}\overset{x}{\otimes}\begin{bmatrix}\Sigma(x,t)\\ g(x,t)\end{bmatrix}$$

where $\Sigma(x,t) = \sum_i [q_i(x,t) + \bar{q}_i(x,t)]$

Parameter space for evolution

parton density $f(x, Q^2)$



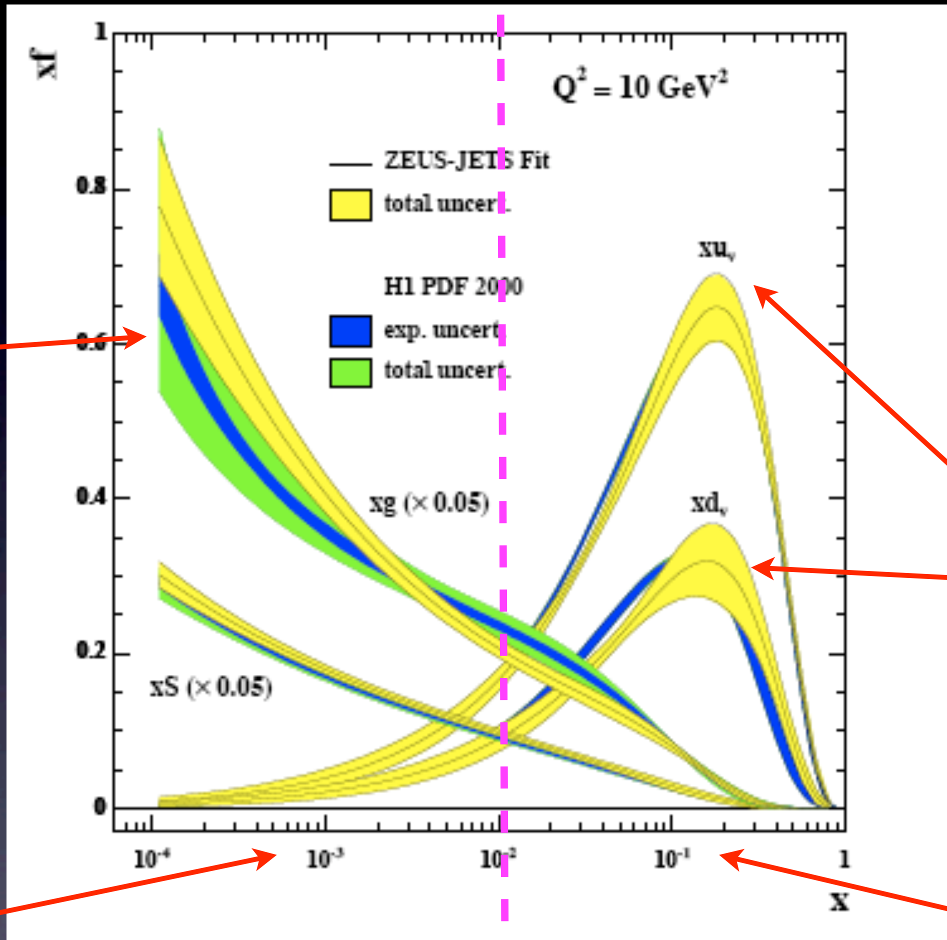
DGLAP: evolution of the parton densities with the hard scale (Q in DIS). Operator product expansion + renormalization group. Anomalous dimensions.

Important when $\alpha_s \ln Q/Q_0$ large.

BFKL: evolution with the energy **s**. Regge limit.
Important when $\alpha_s \ln 1/x$ large

Electron-Hadron scattering at very high energy

Sea and gluons: (soft) slow fields



Valence quarks: fast fields

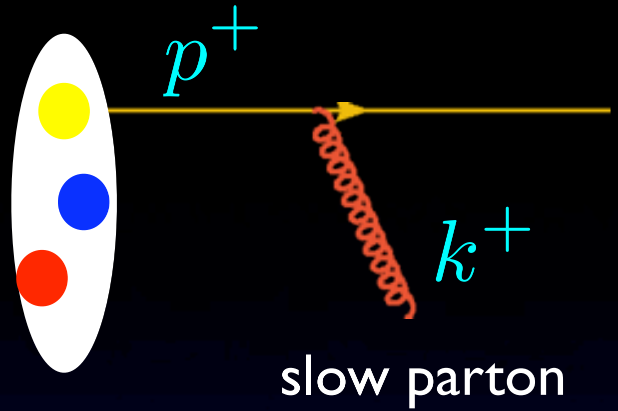
small x

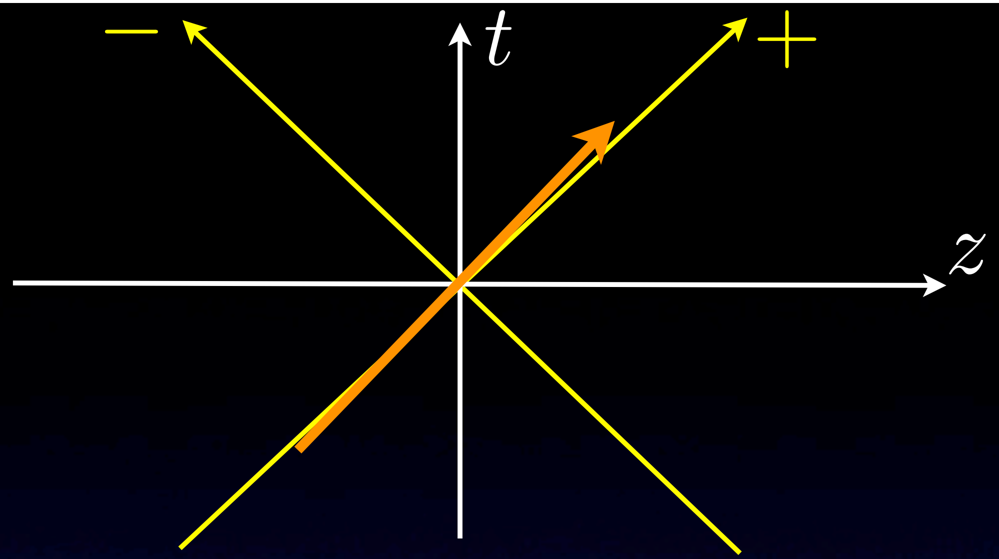
large x

separation scale

Separation between fast and slow fields

Very fast hadron(nucleus)





Gribov's spacetime picture: fast parton exists for a long time, slow parton is a short fluctuation.

Lifetime on the lightcone $\Delta x^+(p) \sim \dfrac{1}{p^-} \sim \dfrac{p^+}{m_T^2}$

$$p^+ \gg k^+$$
$$p^+ \to \infty$$

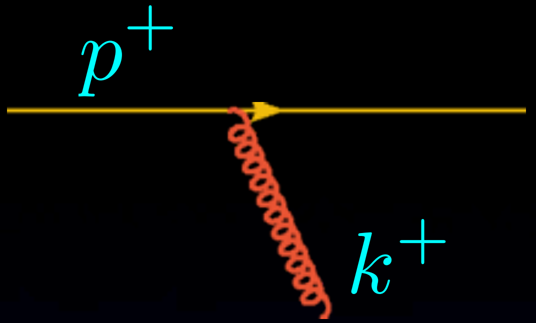
LC momentum:

$$p^+ = \frac{1}{\sqrt{2}}(E + p_z)$$

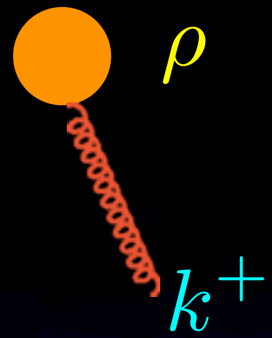
LC energy:

$$p^- = \frac{1}{\sqrt{2}}(E - p_z)$$

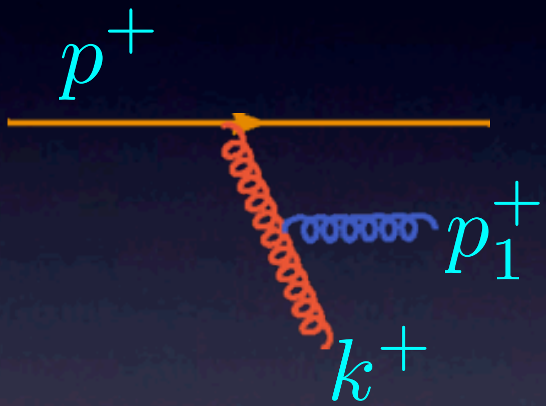
- Slow partons can only see the total charge of the fast partons.



$p^+ \gg k^+$



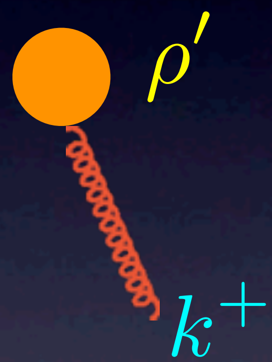
charge



One gluon emission

$p^+ \gg p_1^+ \gg k^+$

Separation of scales



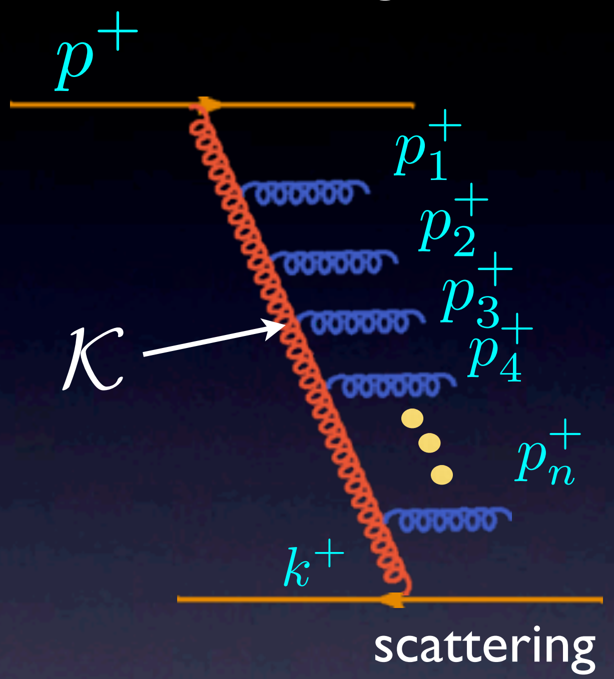
Radiation of gluons:
Bremsstrahlung

Renormalized charge

The effect of the additional gluon emission is to renormalize the effective color charge.

Generalization to many soft gluon emissions

Cascade of the n soft gluons



Strong ordering (in longitudinal momenta)

$$p^+ \gg p_1^+ \gg p_2^+ \gg \cdots \gg p_n^+ \gg k^+$$

Note: transverse momenta are not ordered

$$k^+ = xp^+$$

$$\frac{\alpha_s N_c}{\pi} \int_{k^+}^{p^+} \frac{dp_1^+}{p_1^+} = \frac{\alpha_s N_c}{\pi} \ln \frac{1}{x}$$

Large logarithm

Nested logarithmic integrals

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x} \right)^n$$

Resummation of the gluon emissions performed by the Bethe-Salpeter type equation

$$\frac{df_g(x, k_T^2)}{d \ln 1/x} = \frac{\alpha_s N_c}{\pi} \int d^2 k'_T \mathcal{K}(k_T, k'_T) \; f_g(x, k'_T)$$

I.Balitsky, V.Fadin, E.Kuraev, L.Lipatov

integral over transverse momenta

kernel describing branching of gluons

unintegrated gluon density at small x

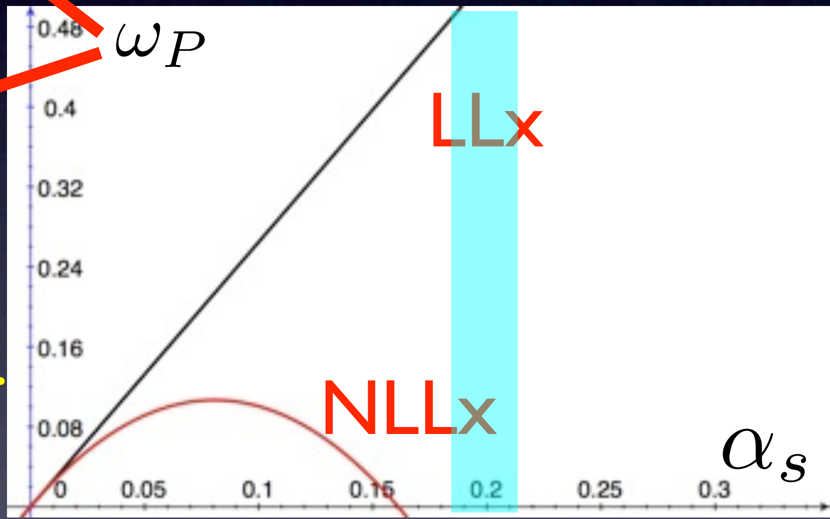
Evolution equation in longitudinal momenta

$$\frac{df_g(x,k_T^2)}{d\ln 1/x} = \frac{\alpha_s N_c}{\pi}\int d^2k'_T \mathcal{K}(k_T,k'_T)\, f_g(x,k'_T)$$

Solution:

$$f_g(x,k_T) \sim x^{-\omega_P} \qquad \omega_P = j-1 = \frac{\alpha_s N_c}{\pi} 4\ln 2$$

Leading exponent(spin)



Rise too strong for the data!

$$\sigma^{DIS}_{\gamma^* p} \sim s^{\omega_P}$$

Take higher order corrections.

$$\alpha_s \mathcal{K}_0 + \alpha_s^2 \mathcal{K}_1 + \ldots$$

V.Fadin,L.Lipatov, G.Camici,M.Ciafaloni

$$\omega_P \simeq \bar{\alpha}_s 4\ln 2(1-6.5\bar{\alpha}_s)$$

relevant values of α_s

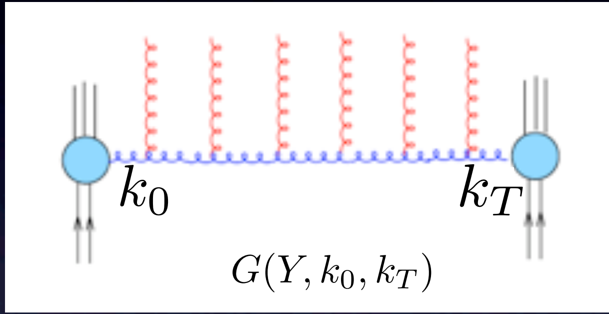
Leading logarithmic approximation, not compatible with the experimental data.
Very large next-to-leading correction!

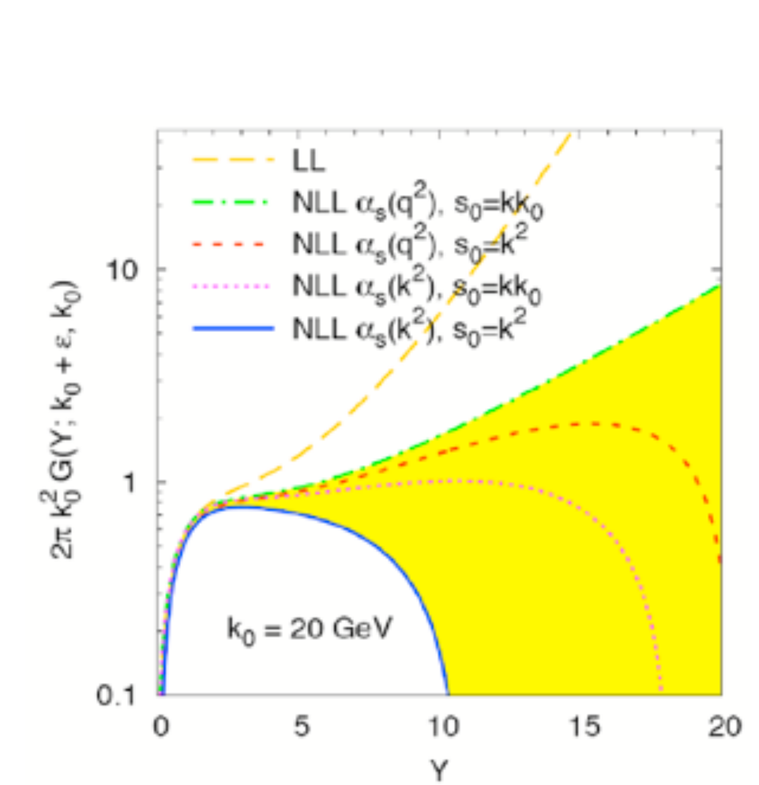
Solution at NLLx

$$\frac{dG(Y,k_0,k_T)}{dY} = \frac{\alpha_s N_c}{\pi} \int d^2k'_T \mathcal{K}(k_T,k'_T)\, G(Y,k_0,k'_T)$$

Rapidity

$Y = \ln 1/x$





Ciafaloni, Colferai, Salam, A.S.; Andersen, Sabio-Vera

- Numerical solution to the equation at NLLx accuracy.
- Sensitivity to the choice of the scale
- Sensitivity to the way coupling is run in the equation.
- Differences are of the NLLx order.
- Poor perturbative convergence.

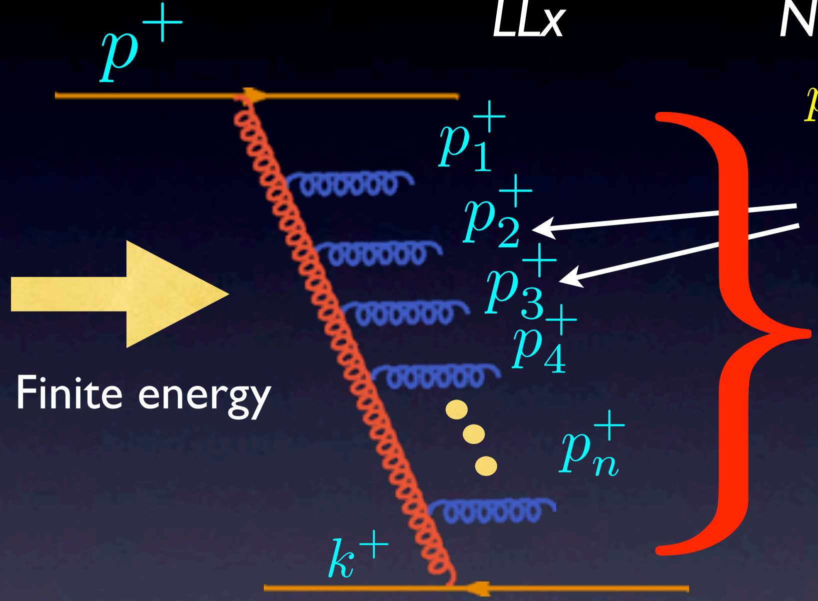
Resummation

J.Kwiecinski, A.Martin, A.Stasto
M. Ciafaloni, D. Colferai, G. Salam, A.Stasto

expansion of the kernel $\alpha_s \mathcal{K}_0 + \alpha_s^2 \mathcal{K}_1 + \ldots \sum_{i=3}^{\infty} \alpha_s^i \mathcal{K}_{i-1}$

LLx *NLLx*

$p^+ \gg p_1^+ \gg p_2^+ \gg \cdots \gg p_n^+ \gg k^+$



Treat phase space accurately (reduce the amount of gluons than can be produced)

Global energy momentum constraint

Take into account logarithms of the scale

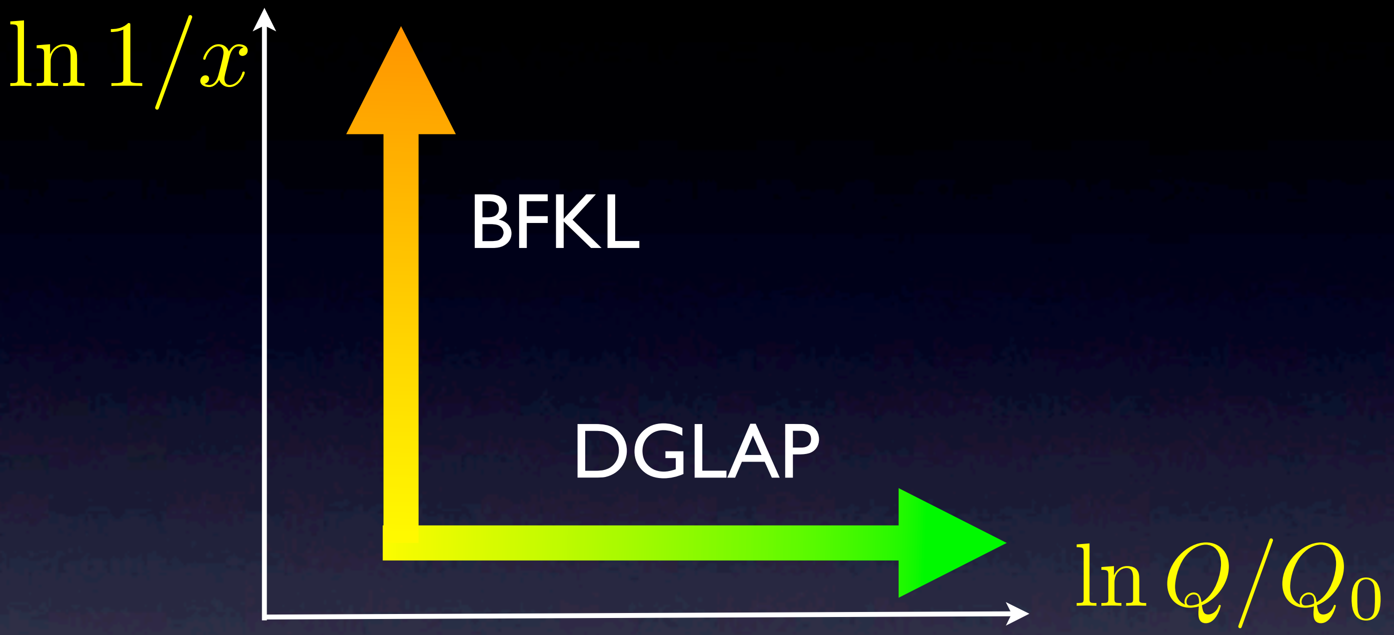
Running coupling

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x}\right)^n$$ total energy

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{Q}{Q_0}\right)^n$$ scale (related to transverse momentum)

Problem with two large parameters

Resummation at small x



Use DGLAP information to constrain the BFKL expansion. Very strong constraint!

Take care of both type of logarithms

$\alpha_s \ln 1/x$

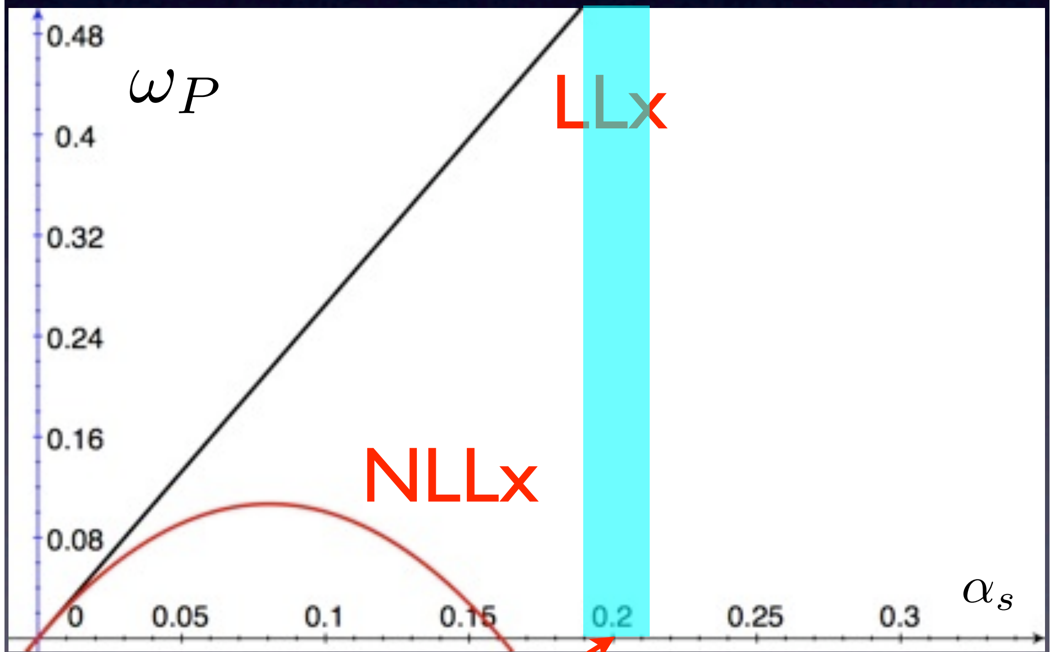
$\alpha_s \ln Q/Q_0$

Resummation ctd.

$$f_g(x, k_T) \sim x^{-\omega_P}$$

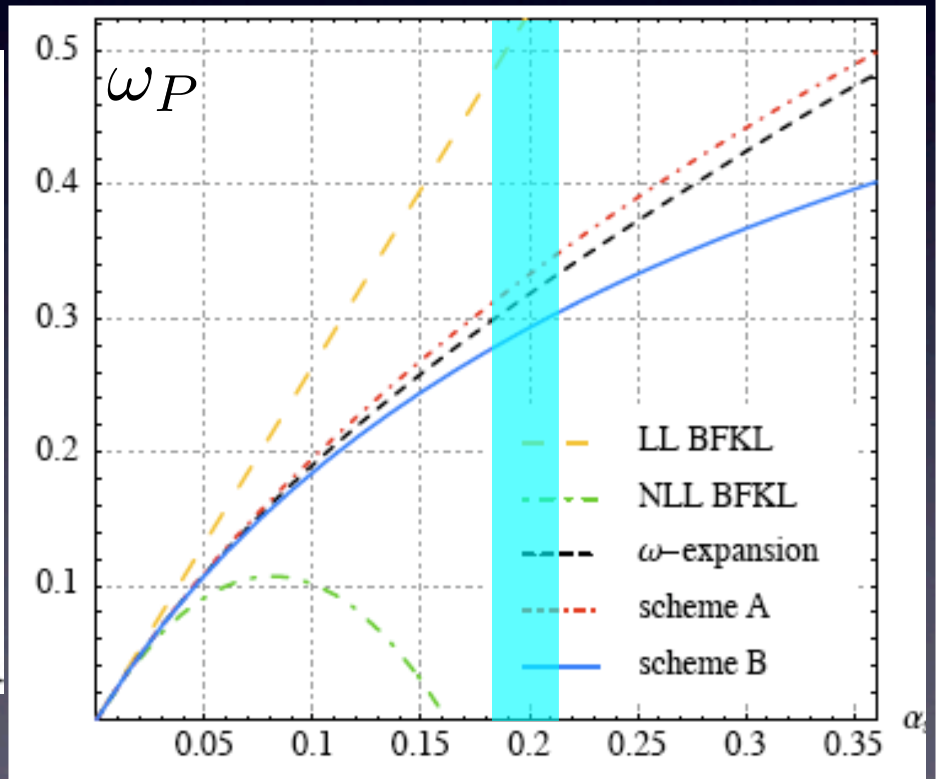
M. Ciafaloni, D. Colferai, G. Salam, A.Stasto

Before resummation



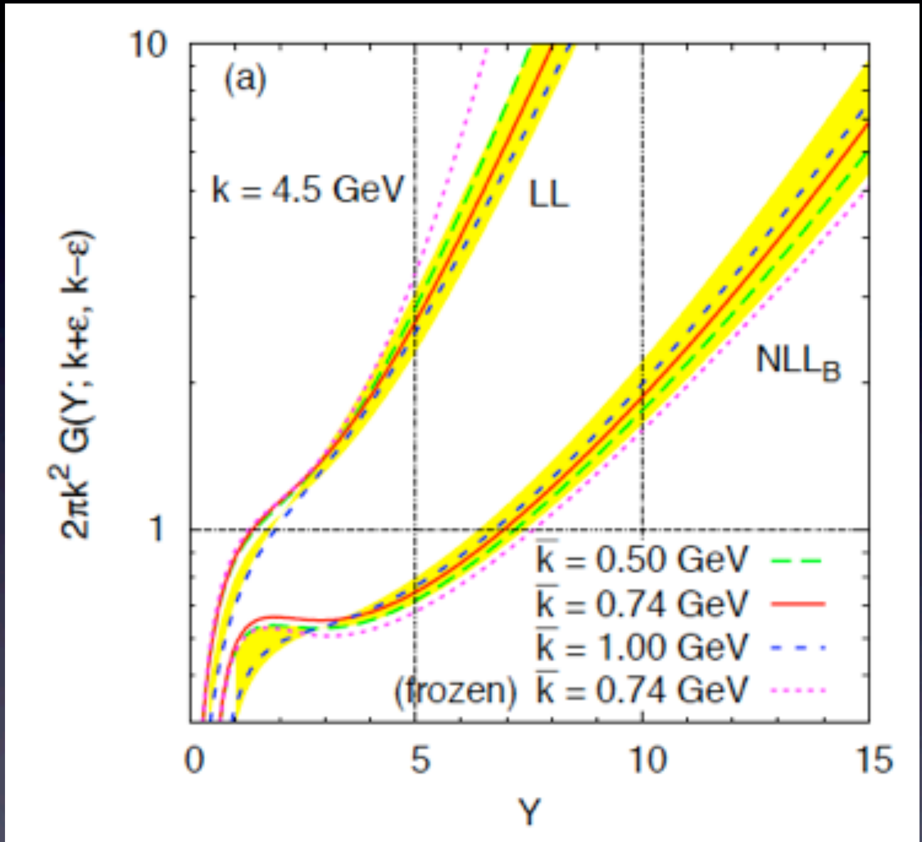
typical values of α_s

After...

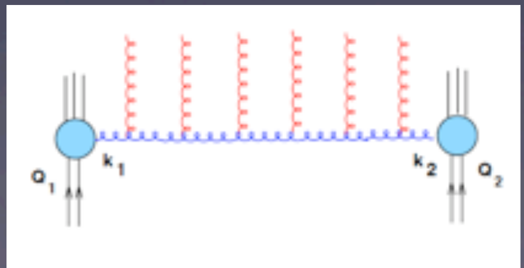


Gluon Green's function

Resummed prediction compared to the leading logarithmic (with running coupling)



- Renormalization scale variation
- Change of the infrared cutoff in the running coupling
- Suppressed for Y<5
- Growth beyond Y>5
- For full phenomenology need impact factors



Green function

Green function is perturbatively calculable.
Good for processes with two large scales.
Virtual photon-photon scattering.
Mueller-Navelet jets.

Proton structure

In deep inelastic scattering we have scales very different:
Q is perturbative, much larger than Λ_{QCD}

In general we can only predict Q evolution via DGLAP equations with anomalous dimensions.

Anomalous dimensions also get enhancements from low x.

Resummed splitting functions at small x

Construct the integrated gluon density from the Green's function

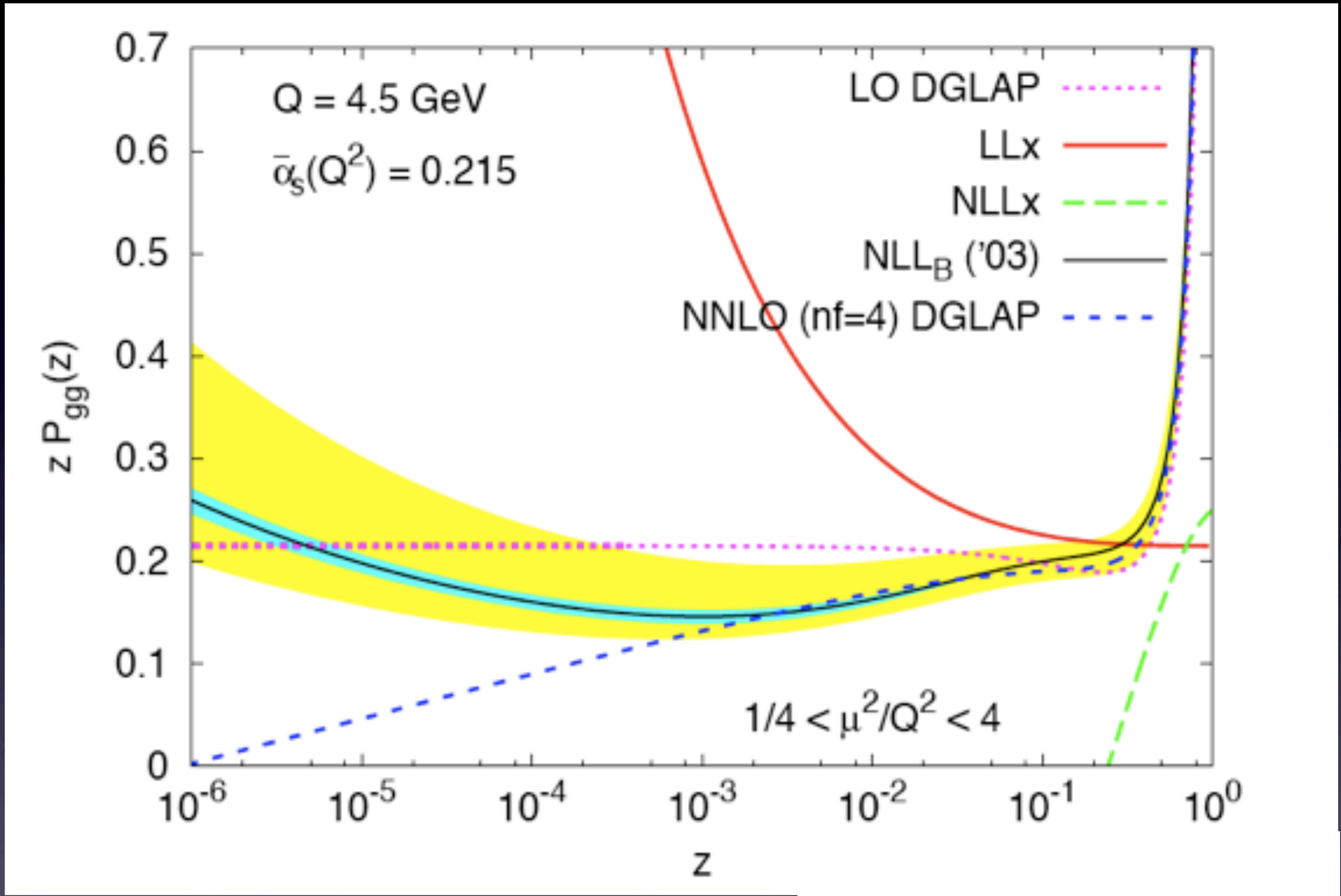
$$xg(x,Q^2) = \int_0^{Q^2} dk^2\, G(x;k,k_0)$$

Solve numerically for the effective splitting function that satisfies the DGLAP equation

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \int_x^1 \frac{dz}{z} P_{\rm eff}(z,Q^2) g(\frac{x}{z},Q^2)$$

The splitting function should be independent of the regularization of the coupling and the choice of the k_0 provided $Q \gg k_0$

Resummed gluon-gluon splitting function

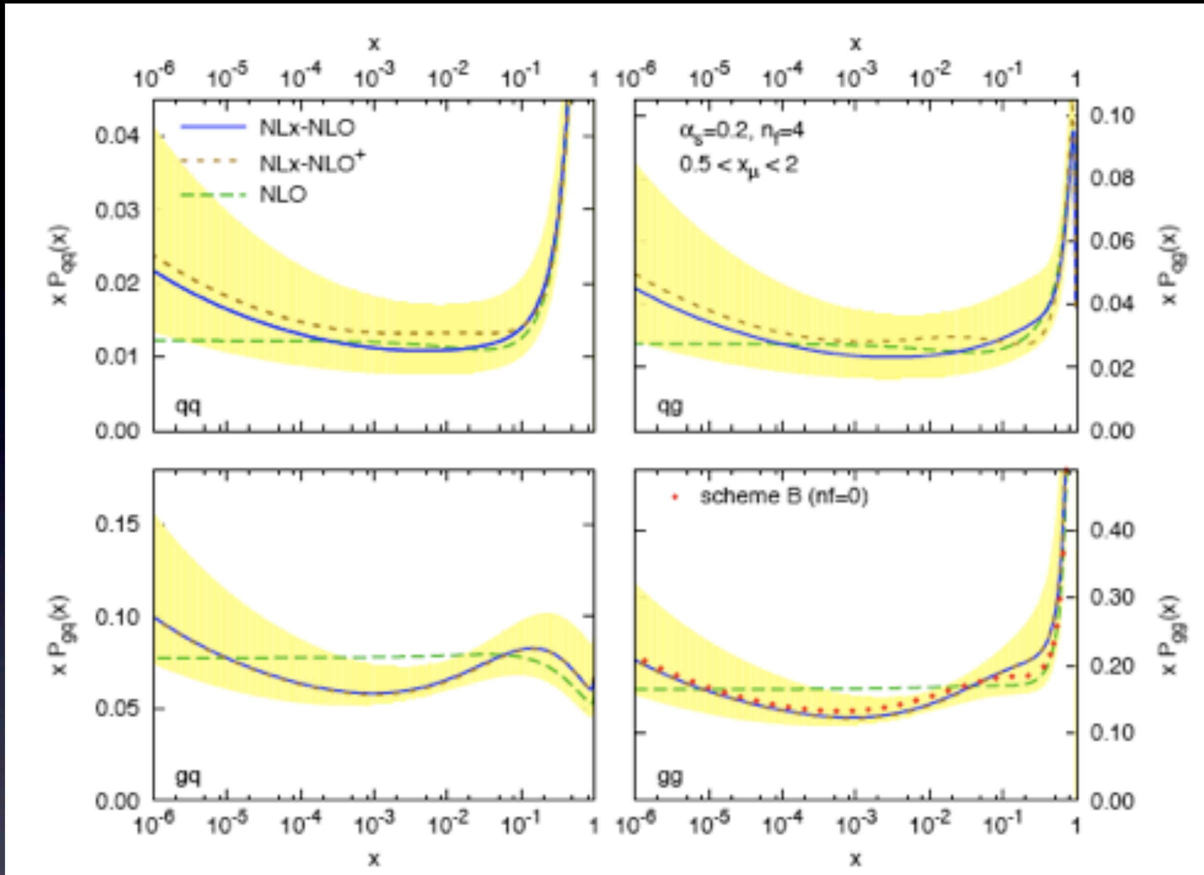


Characteristic 'dip' in the splitting function. The small x growth is delayed until $x < 10^{-3}$

Extension to quarks

- So far we discussed only single gluon channel.
- Extend the resummation procedure to include quarks.
- Matrix approach: collinear factorization scheme with resummed anomalous dimensions.
- Energy momentum sum rules satisfied.
- Requirement of the collinear and anticollinear symmetry.

Matrix of the resummed splitting functions

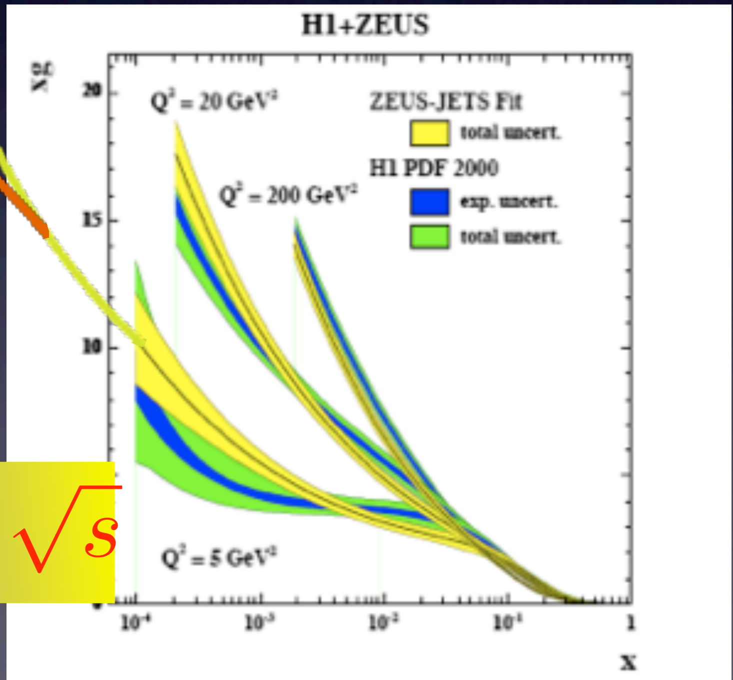


- Again, shallow dip present.
- Small x rise delayed in qq and qg channel down to x < 0.0001.
- Infrared cutoff independence, ensures matrix factorization.

Parton Saturation

Can the gluon density rise forever?
How fast?

very large density of gluons

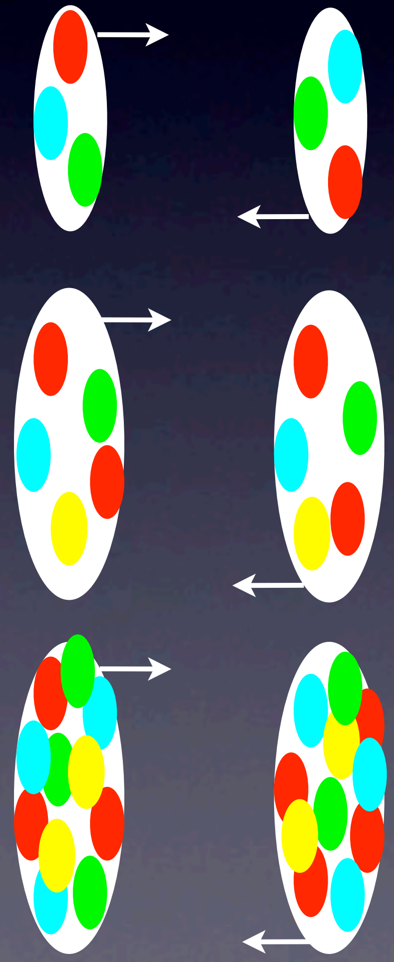


Increase energy $\sqrt{s}$

Can the gluon density rise forever?

Froissart bound is a limit on energy behavior of the total cross section:

$$\sigma_{hh}^{\mathrm{TOT}} \leq \frac{\pi}{m_\pi^2}(\ln s)^2$$



Origin of growth of the cross section:

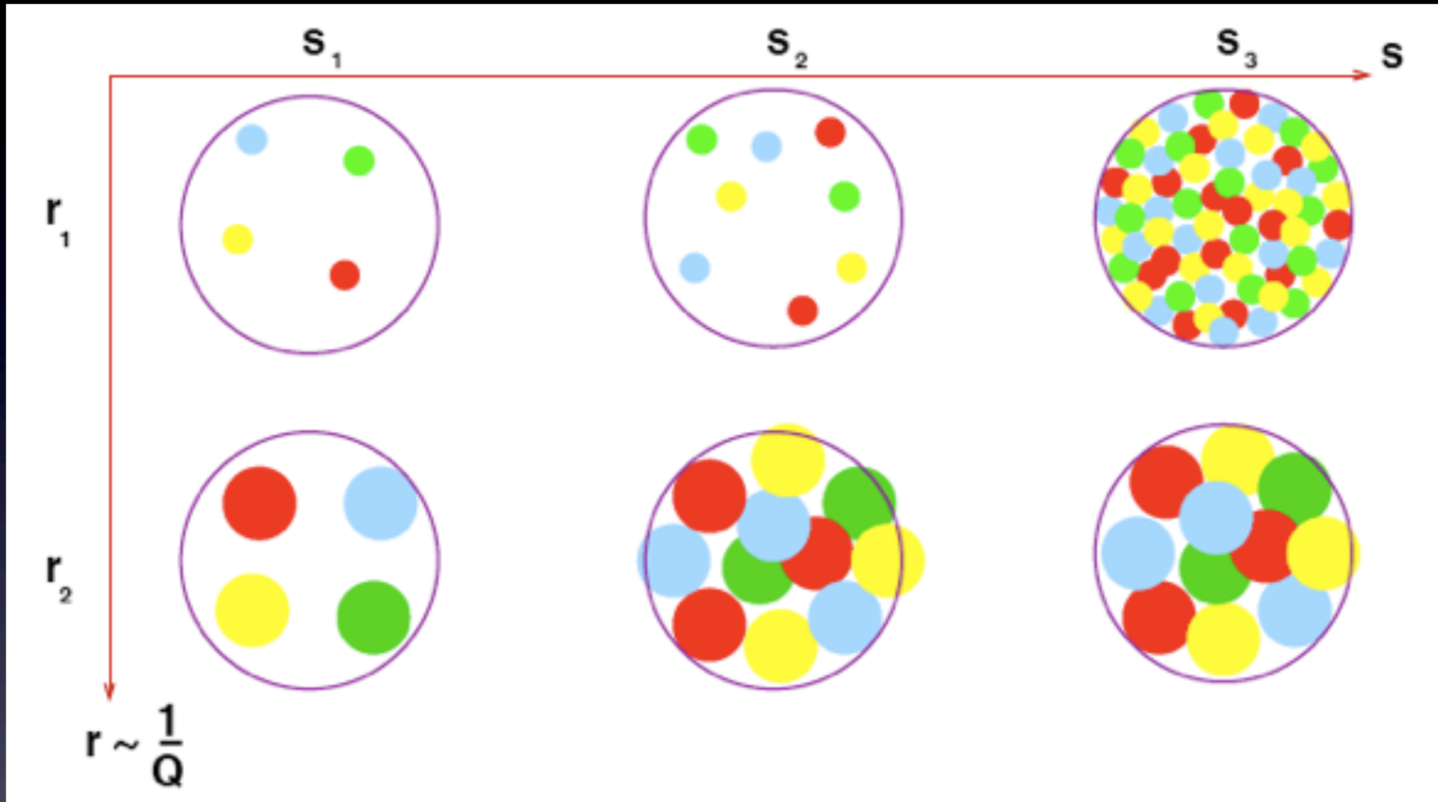
- Growth in the interaction area.
- Limited by the confinement.
- Strong force is short range: $R \to 1/m_\pi$

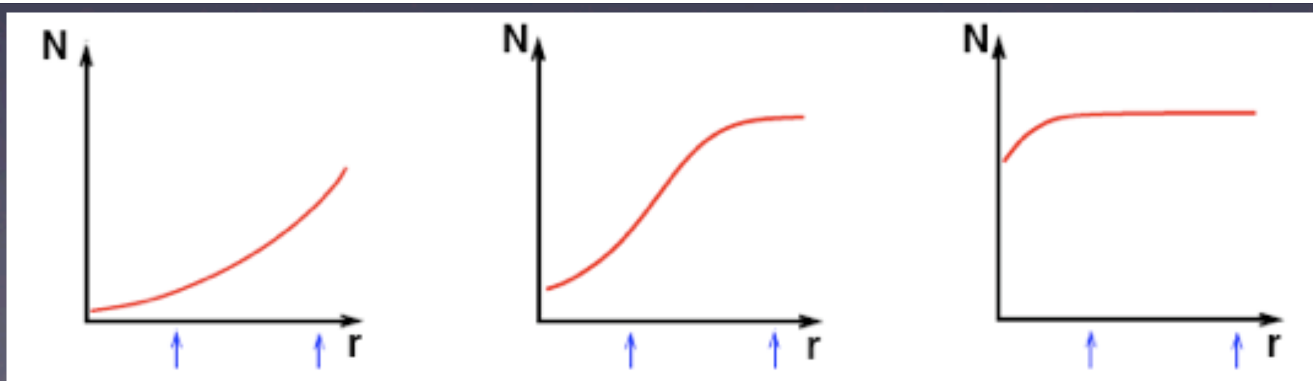
- Increase in the density of the projectile.
- It is limited by the unitarity at fixed impact parameter. Probability of the interaction: $N(b) \leq 1$

Parton saturation

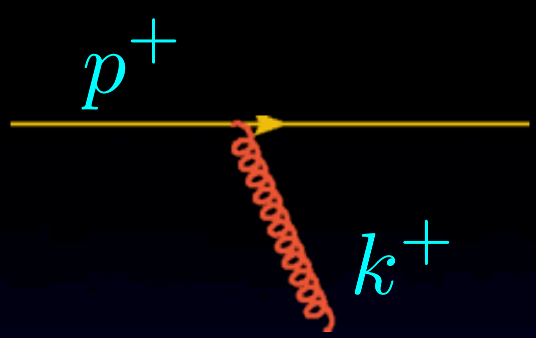
energy of the interaction

size of the probe

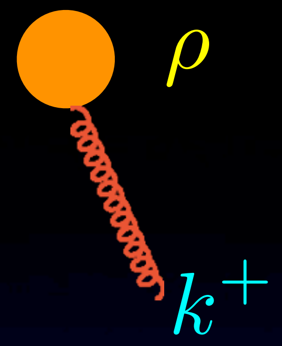




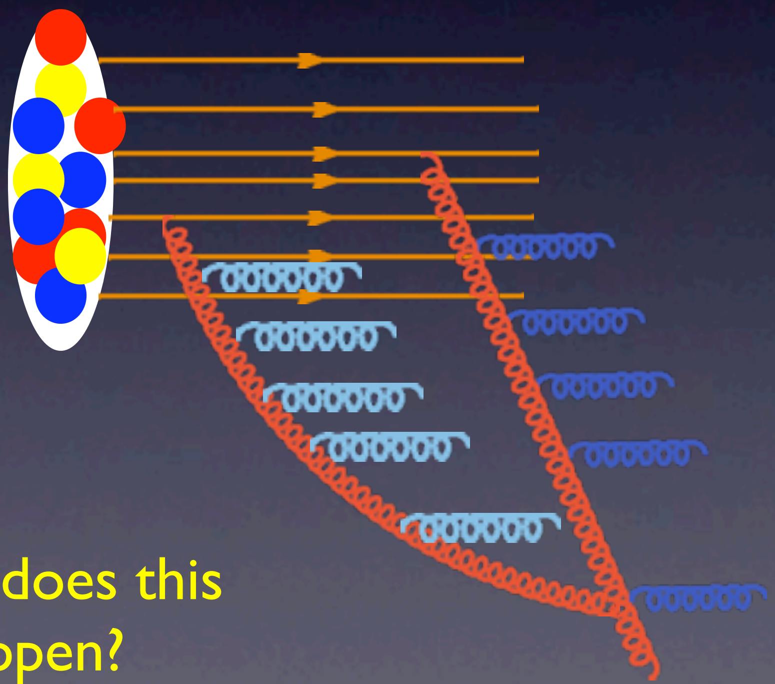
Microscopic mechanism:



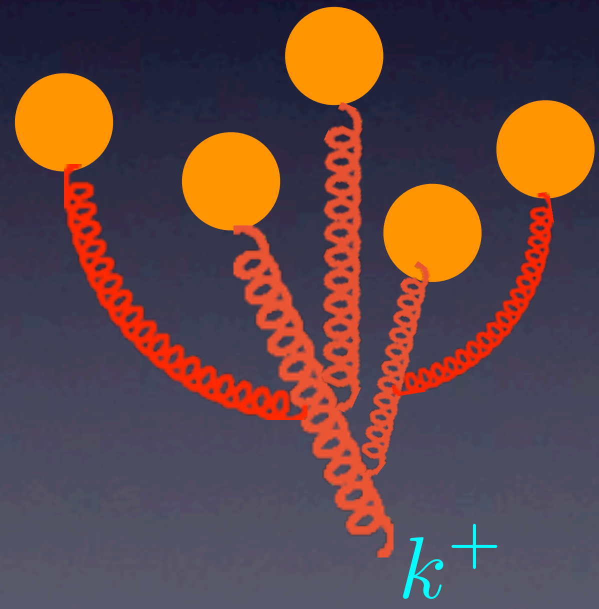
Single charge (source)



Fast hadron or a nucleus



Many charges (sources)



When does this happen?

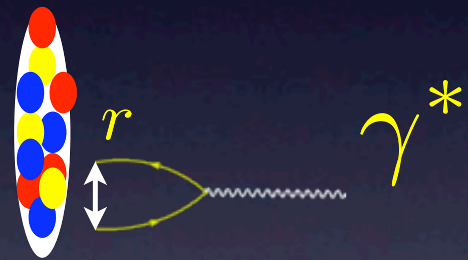
Unitarity

Parton saturation: important mechanism for restoration of unitarity in QCD

$f(x, k_T)$ $\longleftrightarrow$ $N(r, x)$ $r \sim 1/k_T$

gluon density

probability to interact



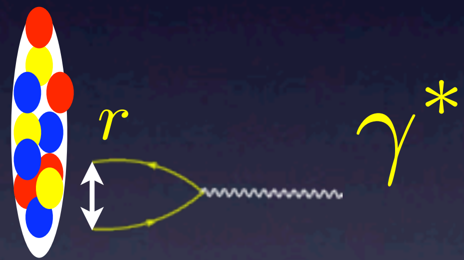
Unitarity

Parton saturation: important mechanism for restoration of unitarity in QCD

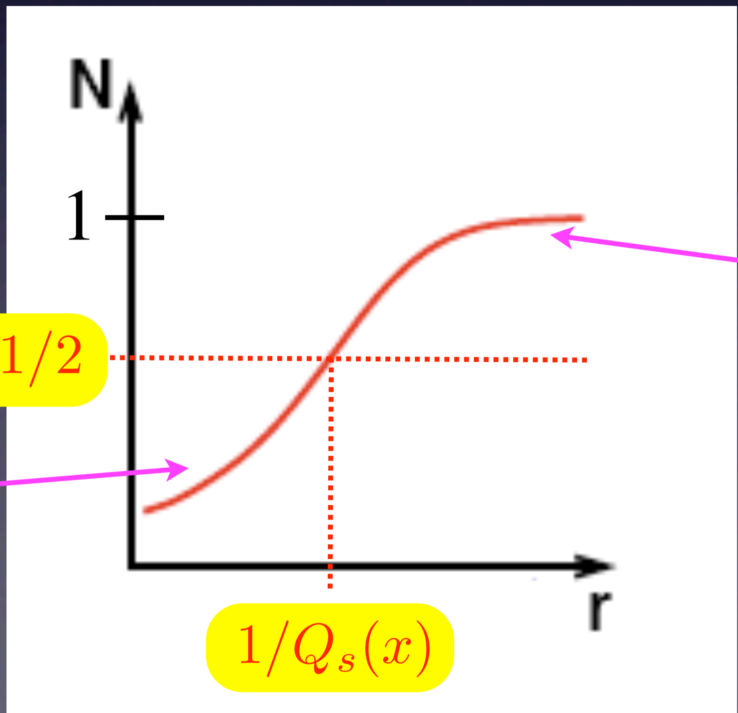
$f(x, k_T)$ $\longleftrightarrow$ $N(r, x)$ $\quad r \sim 1/k_T$

gluon density

dipole scattering amplitude



$N(r, x)$



color transparency

unitarity bound

dipole size

The evolution equation becomes nonlinear in density

$$\frac{df_g(x,k_T^2)}{d\ln 1/x} = \frac{\alpha_s N_c}{\pi} \int d^2k'_T \mathcal{K}(k_T,k'_T)\ f_g(x,k'_T) \quad \boxed{-\frac{\alpha_s N_c}{\pi}(f_g(x,k_T^2))^2}$$

L.V.Gribov, E. Levin, M. Ryskin;
I.Balitsky,Y.Kovchegov; J.Jalilian-Marian,
E.Iancu,L.McLerran,H.Weigert, Leonidov

Note: Compare with Verhulst logistic equation for the population dynamics.

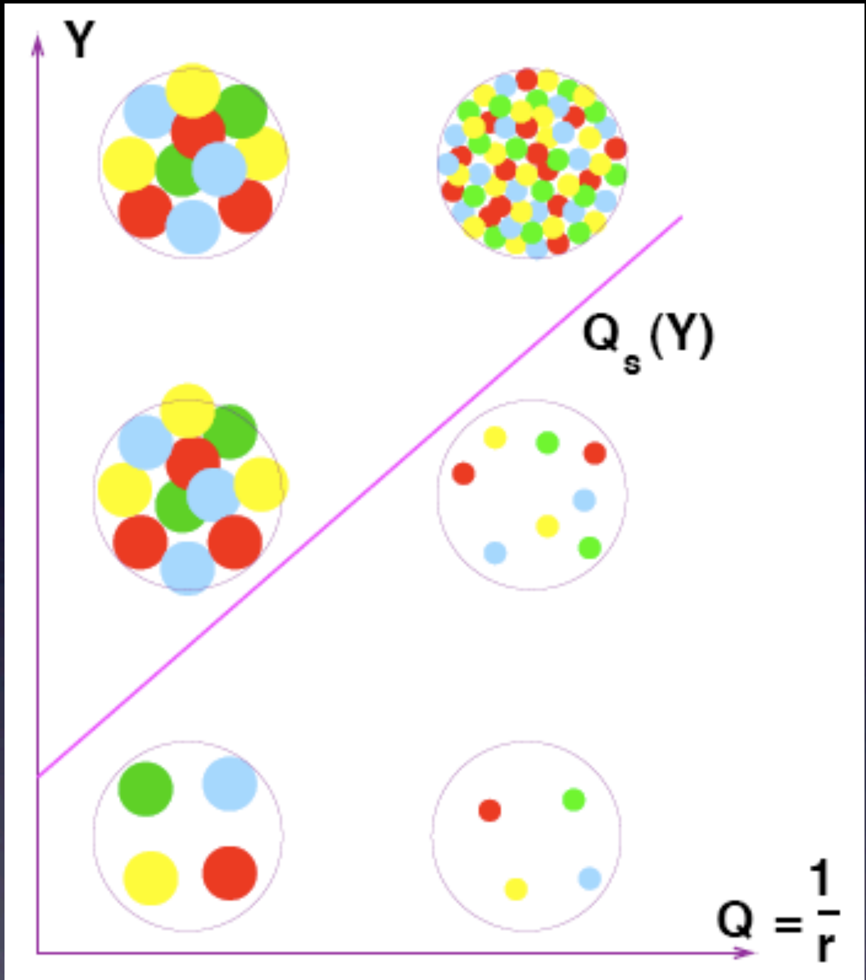
Linear term: gluon splitting, increase of the density

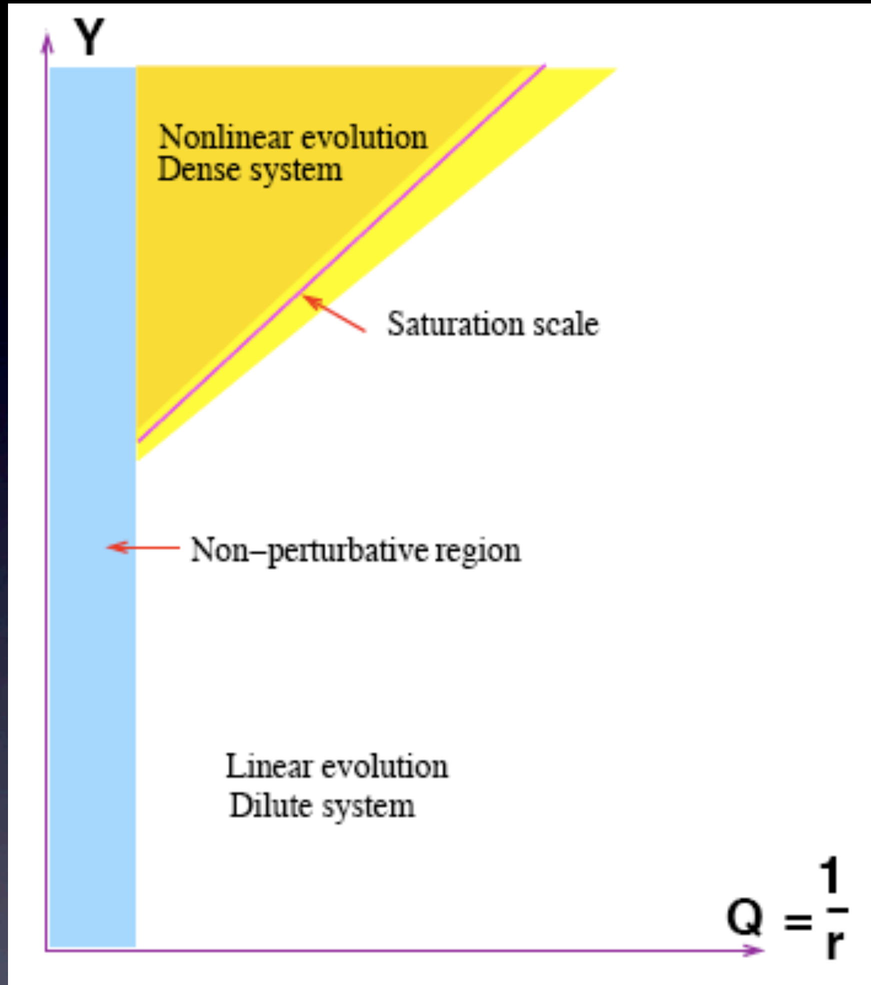
Nonlinear term: gluon merging, slow down the growth of the density with the energy

Equilibrium: nonlinear compensates the linear term.

Gluon density saturates: ***parton saturation***

Saturation scale





saturation scale

$$Q_s(x) \sim x^{-\lambda}$$

Dynamically generated

Parton saturation

$f(x, k_T)$ $\longleftrightarrow$ $N(r, x)$ $\quad r \sim 1/k_T$

gluon density

probability to interact

Parton saturation

$$f(x, k_T) \longleftrightarrow N(r, x) \quad r \sim 1/k_T$$

gluon density

dipole scattering amplitude

nonlinear equation for dipole amplitude

$$\frac{dN(x,r)}{d\ln 1/x} = \frac{\alpha_s N_c}{\pi} \bar{\mathcal{K}}(r,r') \otimes [N(x,r') - N(x,r')N(x,r-r')]$$

Solution to this nonlinear equation shows scaling:

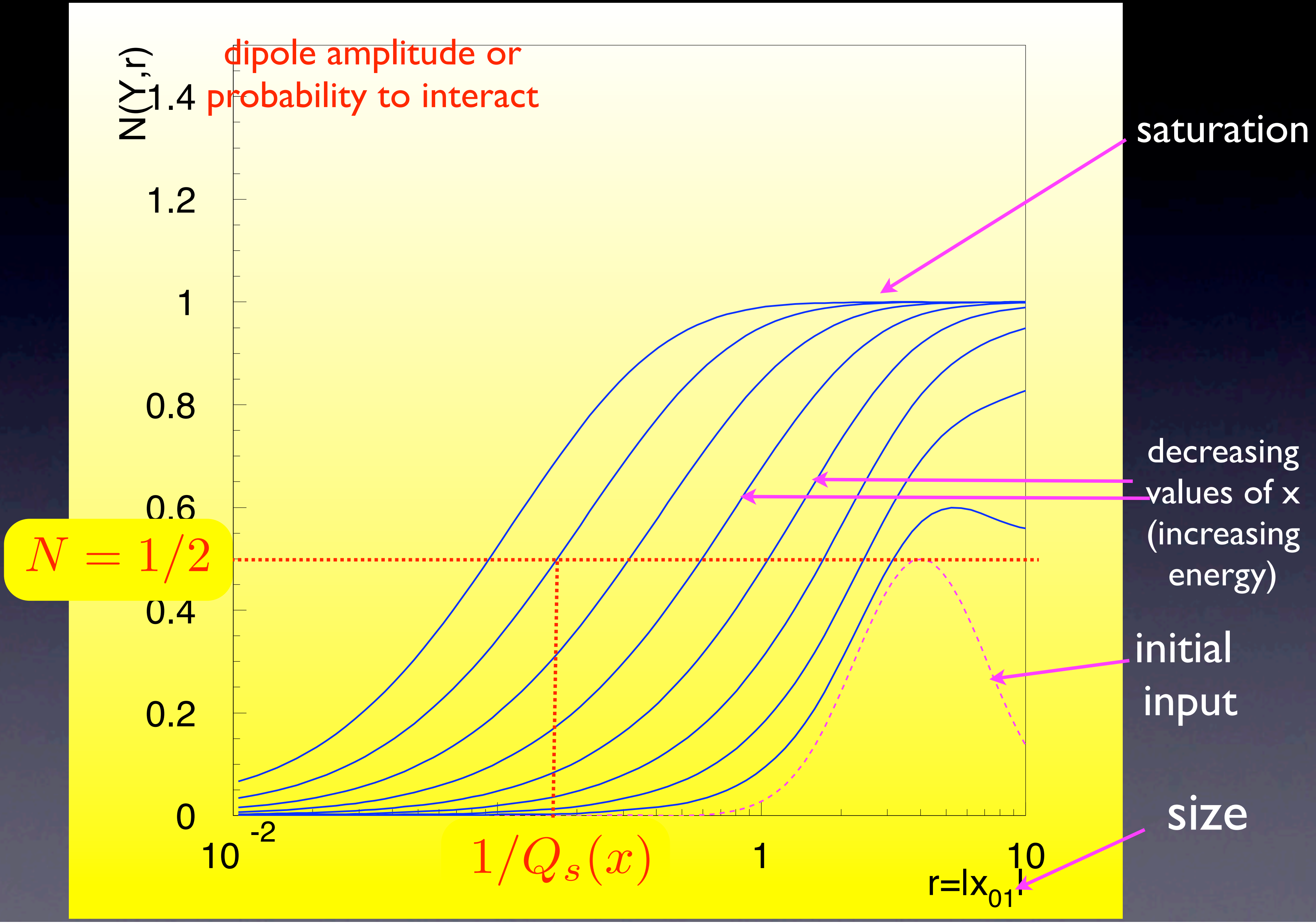
$$N(r,x) \;= N(\underbrace{r^2 Q_s(x)^2}_{\text{scaling}})$$

K.Golec-Biernat, J.Kwiecinski, A.Stasto

K.Golec-Biernat, L. Motyka, A.Stasto

if $\quad r \geq 1/Q_s(x) \quad$ in the dense regime

Saturation and the scattering amplitude

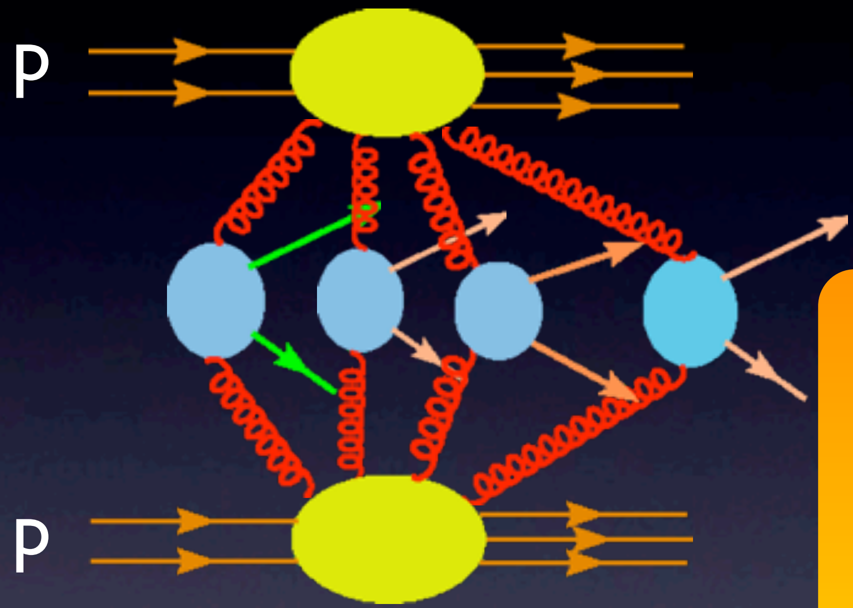


Summary

- In the high energy regime the gluon fields are strong: rapid increase of the gluon density.
- High energy approximation results in the very strong increase of the gluon density. The result however is not compatible with experimental data.
- By taking into account phase space effects and additional logarithms of the scale, one can resum effectively higher orders. To do: applications to the phenomenology.
- Large gluon density. Unitarity bound for the scattering amplitude. Need to take into account recombination, rescattering processes. New phenomena: parton saturation, saturation scale.

Backup Slides

Multiple interactions at LHC

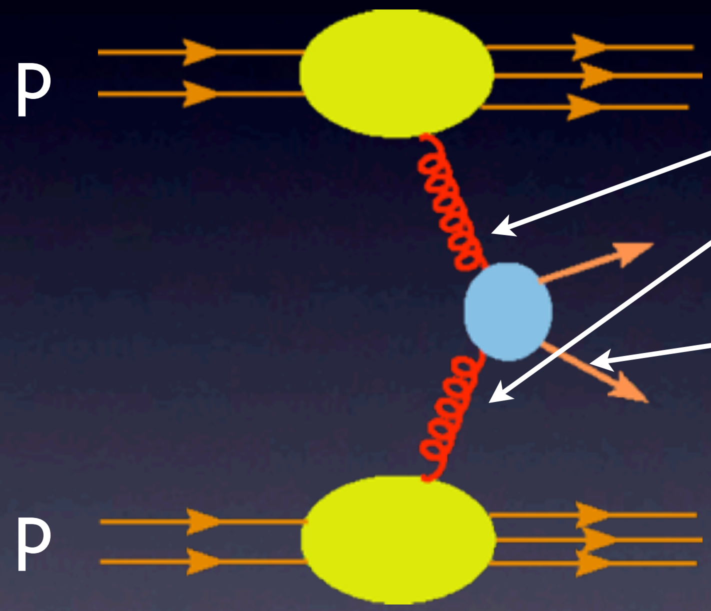


Multiple scatterings:
- Gluon level (saturation)
- Hadron level
- Correlations

- Important for search of new phenomena (SUSY, Higgs...).
- But also important for understanding QCD: unitarity.

- Combine saturation + rescattering + resummation (phase space and kinematics plays very important role).
- Take into account detailed geometry.
- What is the role of the saturation scale in the growth of the total cross section and of the multiplicity with energy?
- How does it affect the spectrum in rapidity?

Exclusive final states



The resummed gluon density includes information about the transverse momenta

Can calculate the transverse momentum distribution of the outgoing particles

The role of the exact kinematics in the parton densities

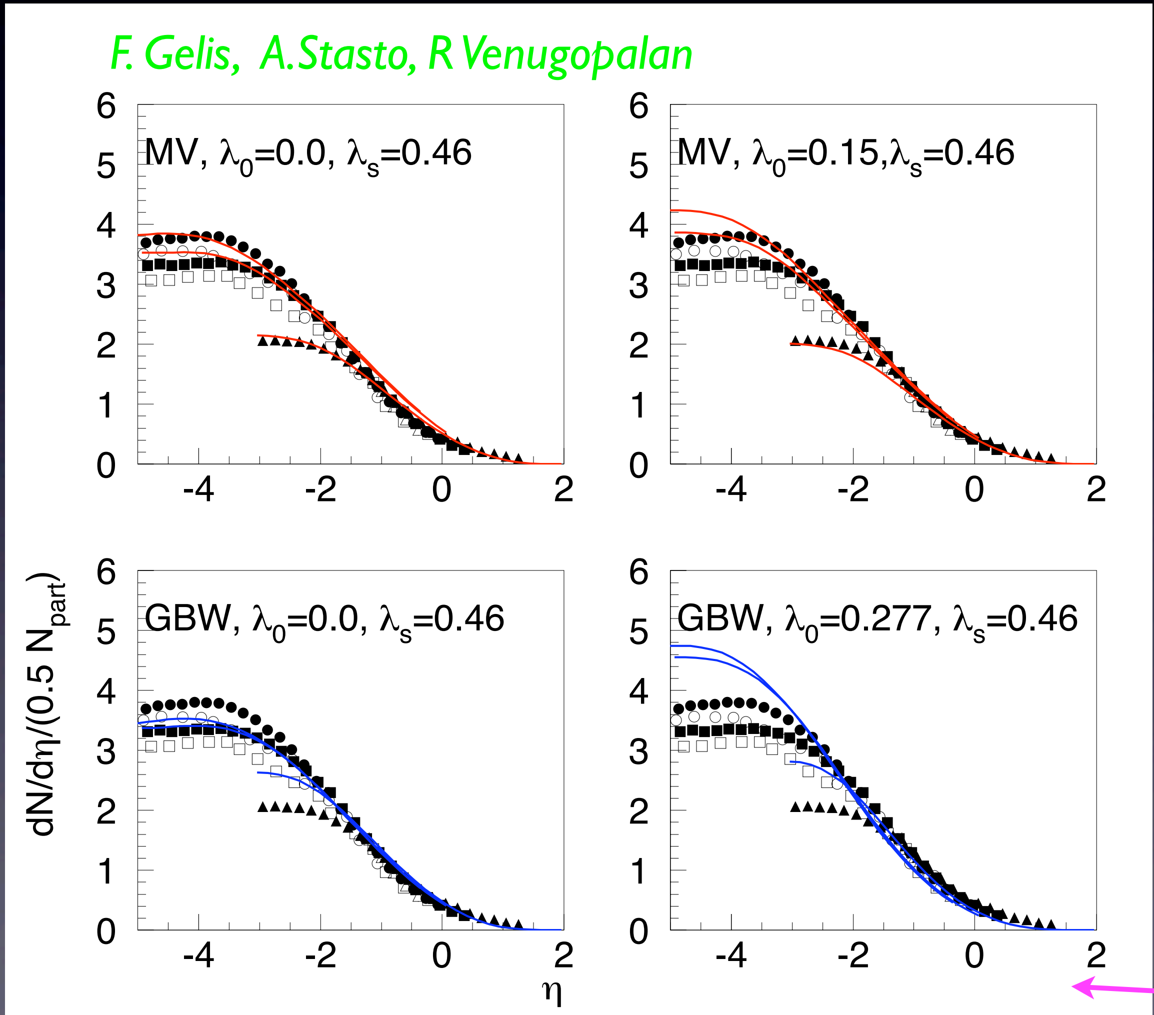
J. Collins, T.Rogers, A.Stasto

Examples:

- p_T distributions of hadrons; Higgs
- p_T distributions of heavy quarks
- Azimuthal decorrelations

Multiplicities in pp, AA

Multiplicity distributions in pseudorapidity in pp collisions.



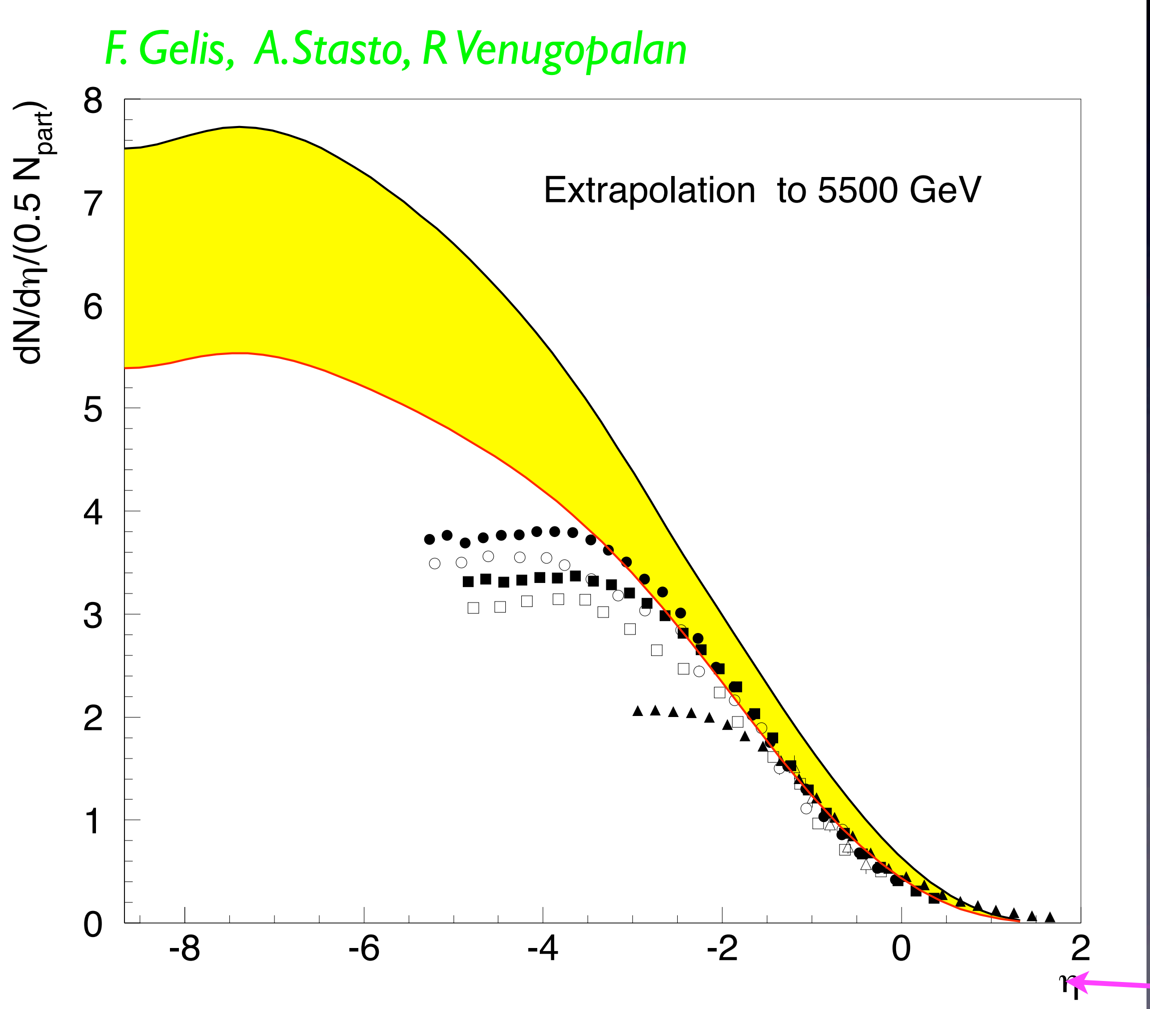
Estimates of multiplicities in the model with saturation. Energy: 19.6, 130, 200 GeV

rescaled pseudorapidity

$\eta - Y_{\mathrm{beam}}$

Multiplicities in pp, AA

Multiplicity distributions in pseudorapidity in pp collisions.



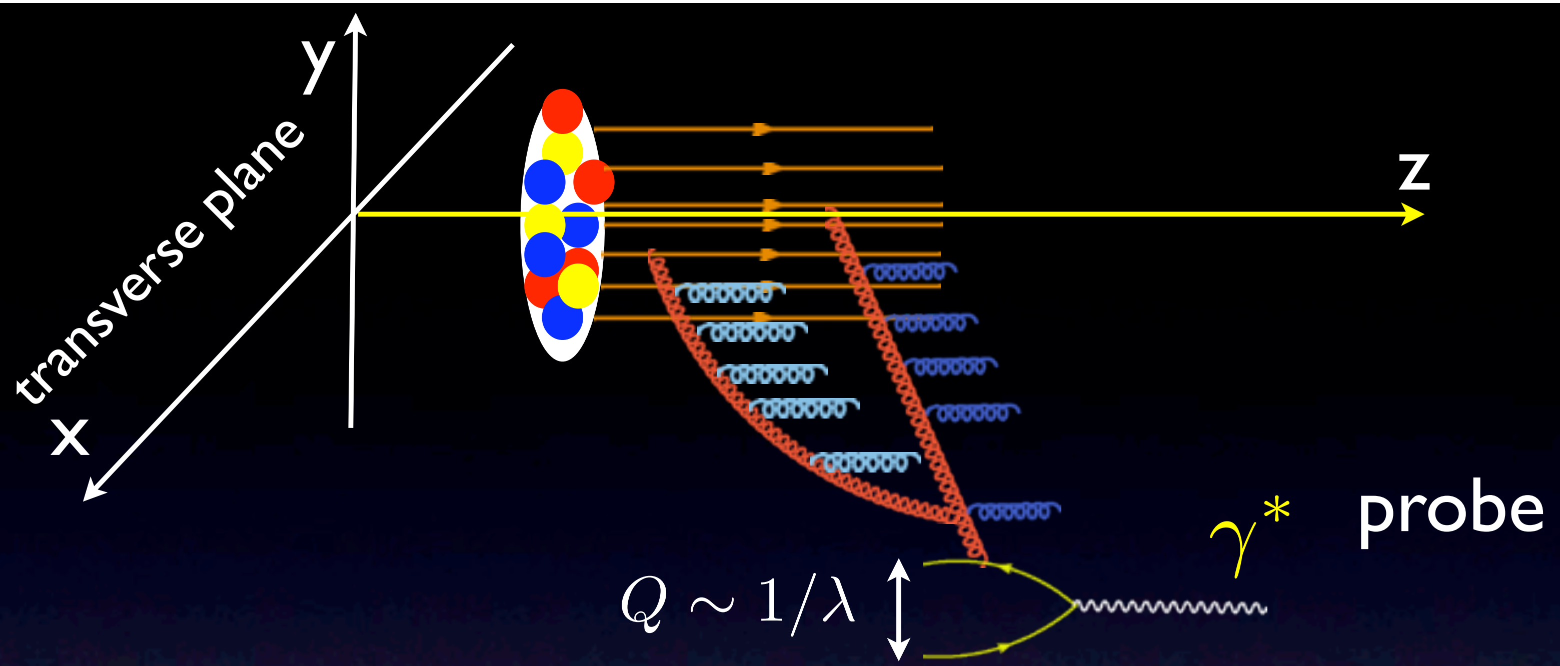
Extrapolations to LHC

- Better treatment of geometry.
- Inclusion of the kinematic (phase space effects).
- Does limiting fragmentation still hold?

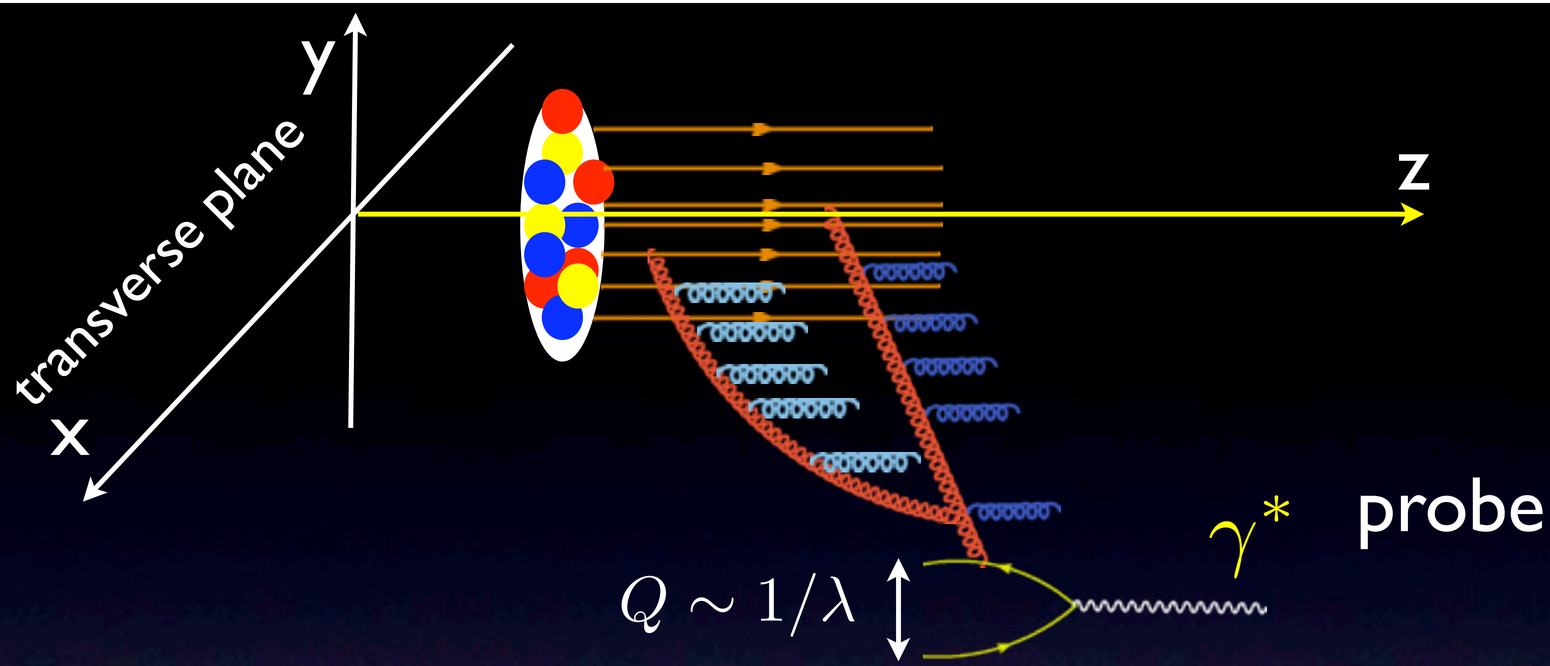
Estimates of multiplicities in the model with saturation.
Energy: 19.6, 130, 200 GeV

rescaled pseudorapidity

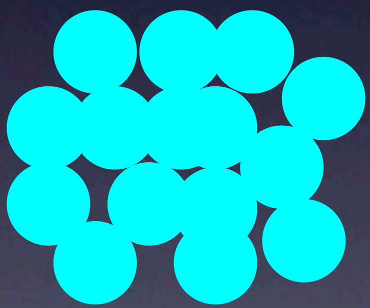
$\eta - Y_{\rm beam}$



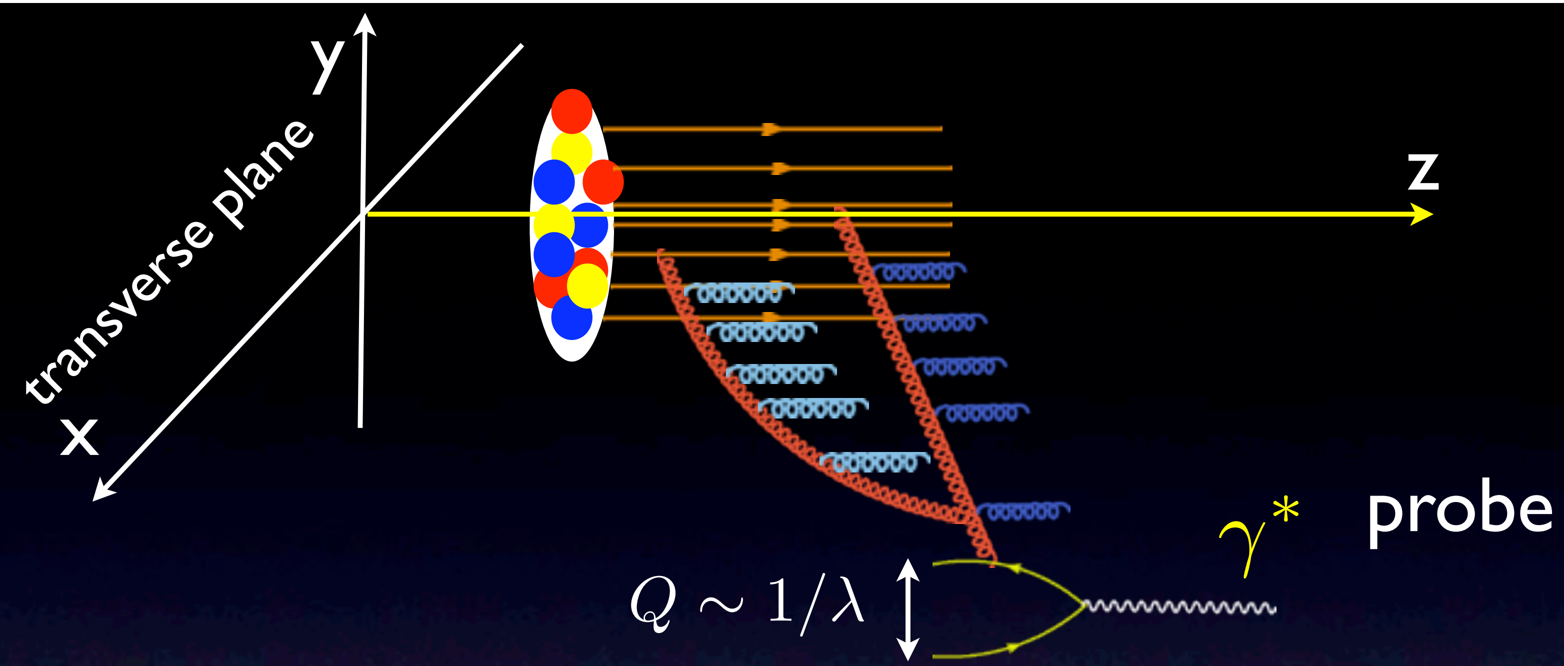
In the transverse plane



In the transverse plane

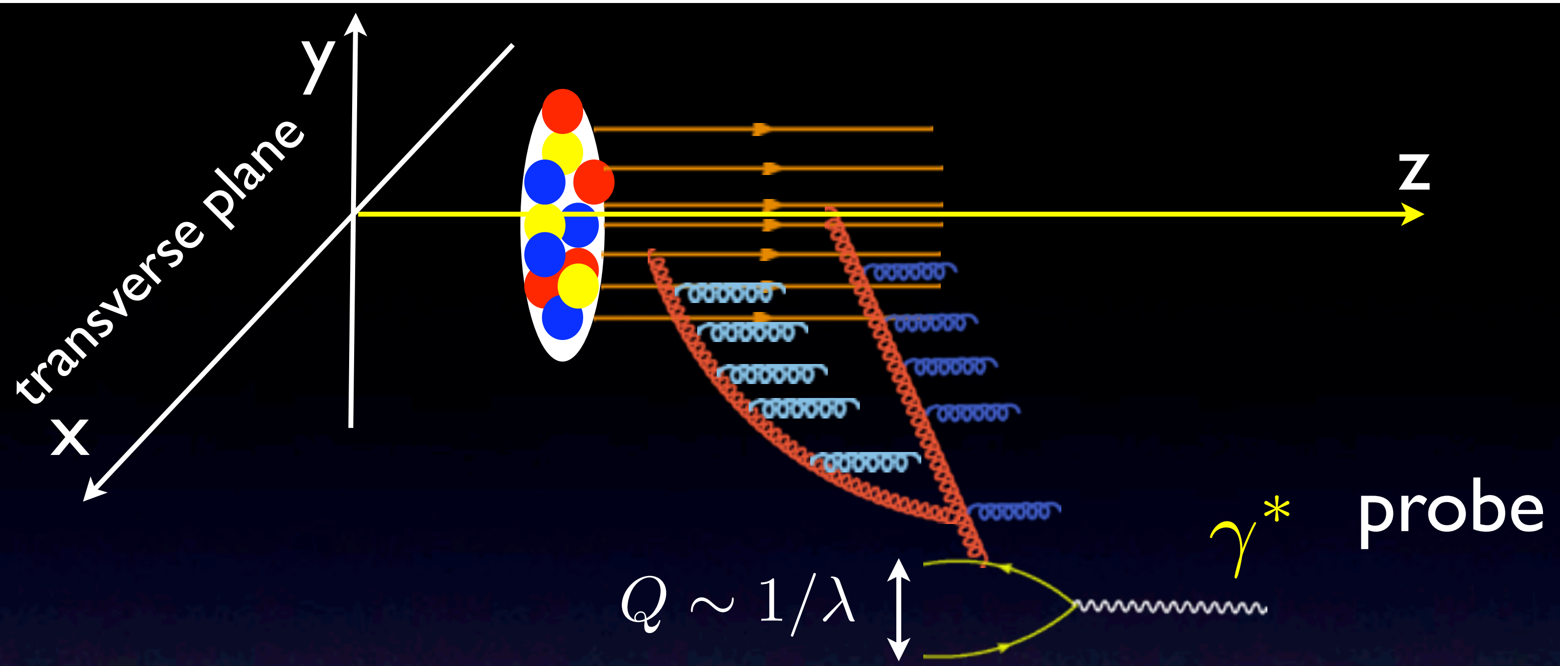


increase s
fixed Q

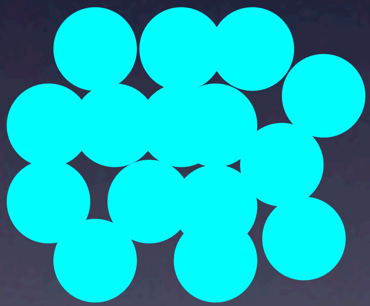


In the transverse plane

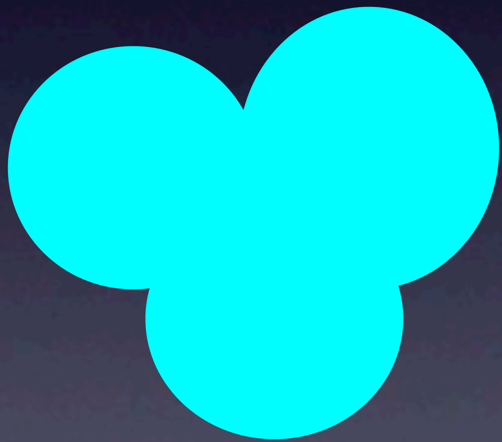
increase s
fixed Q



In the transverse plane



overlapping of gluons



increase s
fixed Q

recombination

decrease Q
fixed s

K.Golec-Biernat, L. Motyka, A.Stasto

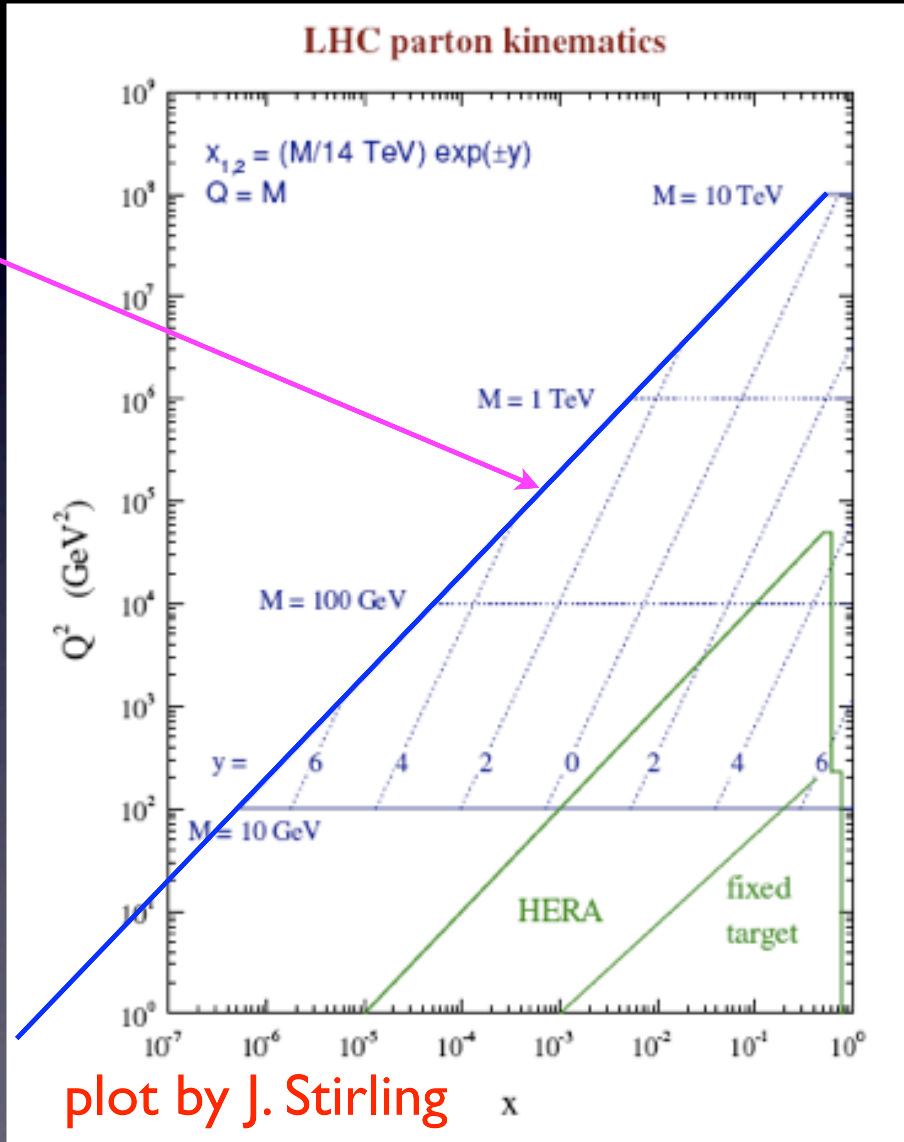
Parton saturation from nonlinear evolution equation:

- Geometrical scaling.
- Universal shape for the scattering amplitude for different initial conditions.
- Lack of infrared diffusion in the transverse momenta: saturation scale acts as a cutoff.
- Stability when the strong coupling runs: self-regulation of the equation.

Relation with statistical physics: geometrical scaling as travelling wave. The equation belongs to the same universality class as the Fisher-Kolmogorov-Petrovsky-Piskounov equation.
S. Munier, R. Peschanski

LHC kinematics

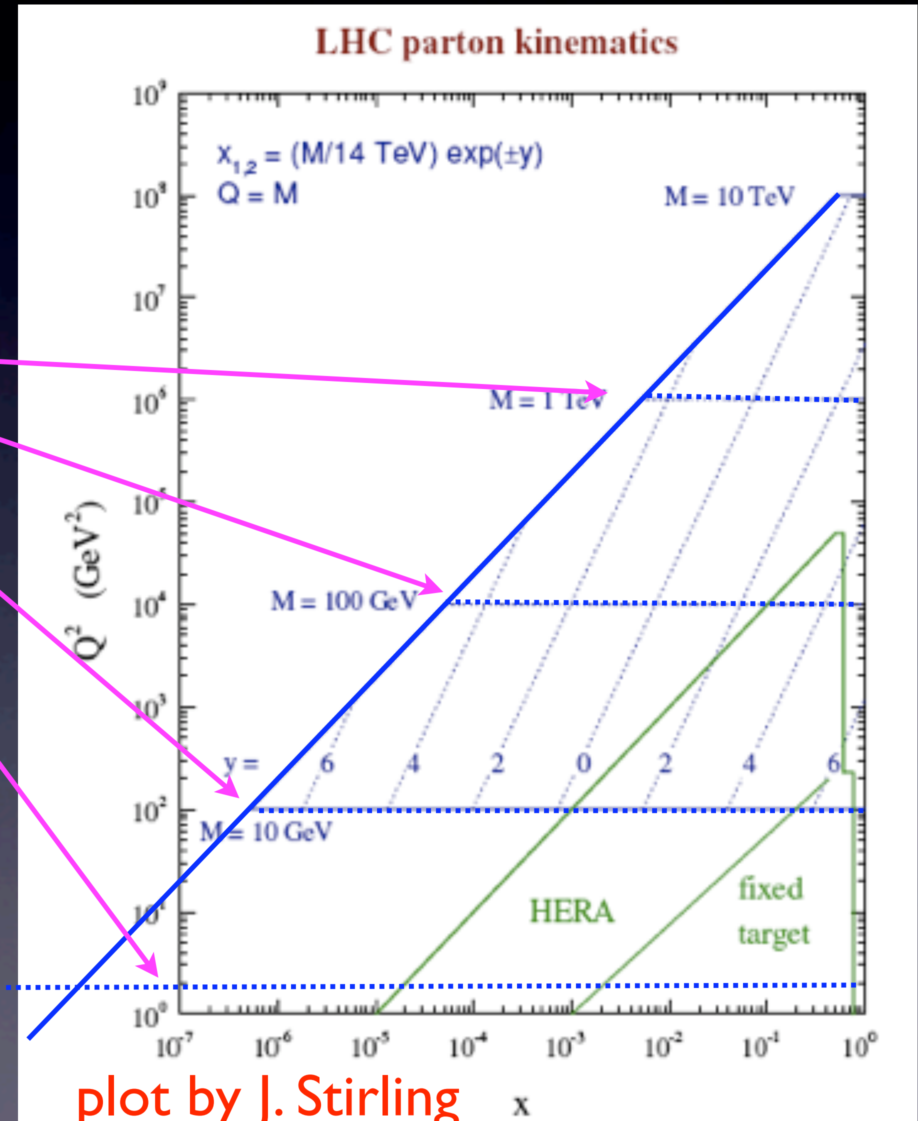
Kinematic boundary



LHC kinematics

Kinematic boundary

Different scales

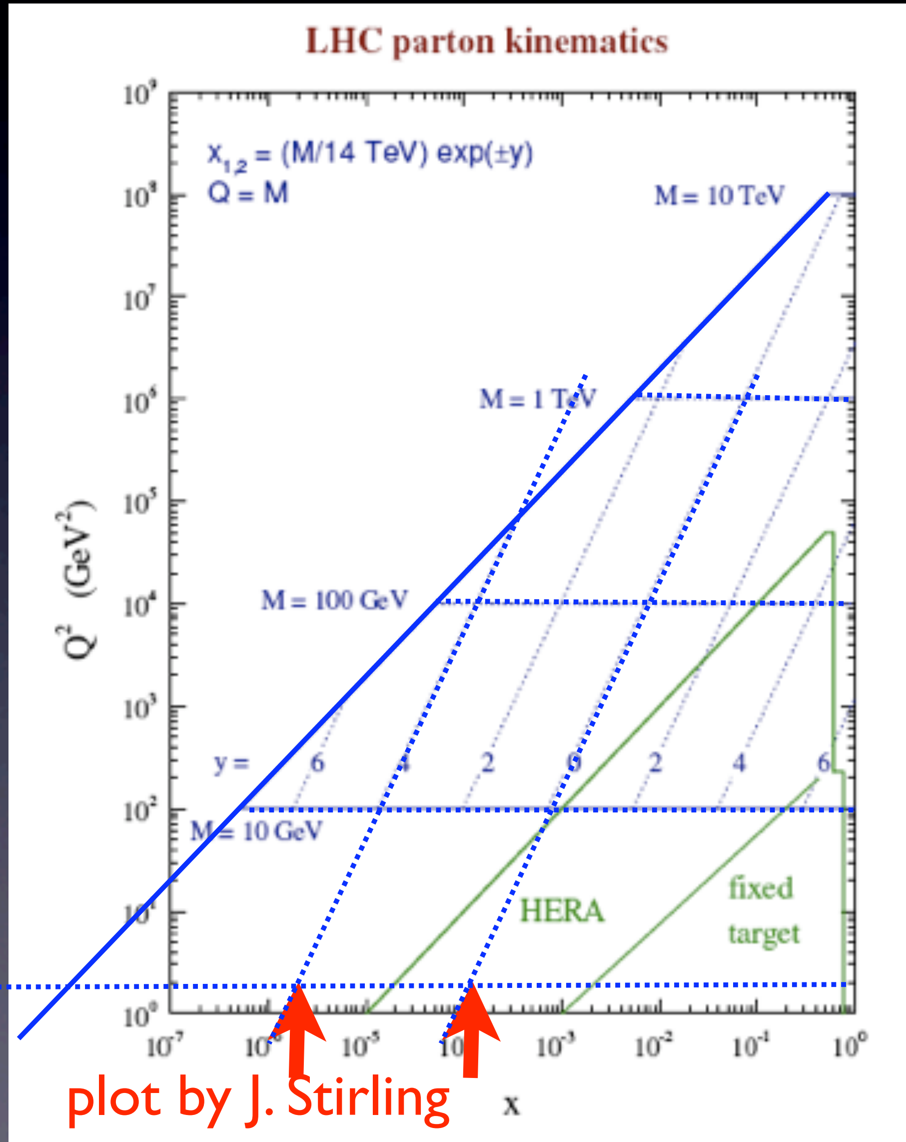


plot by J. Stirling

LHC kinematics

Kinematic boundary

Different scales
Different rapidities

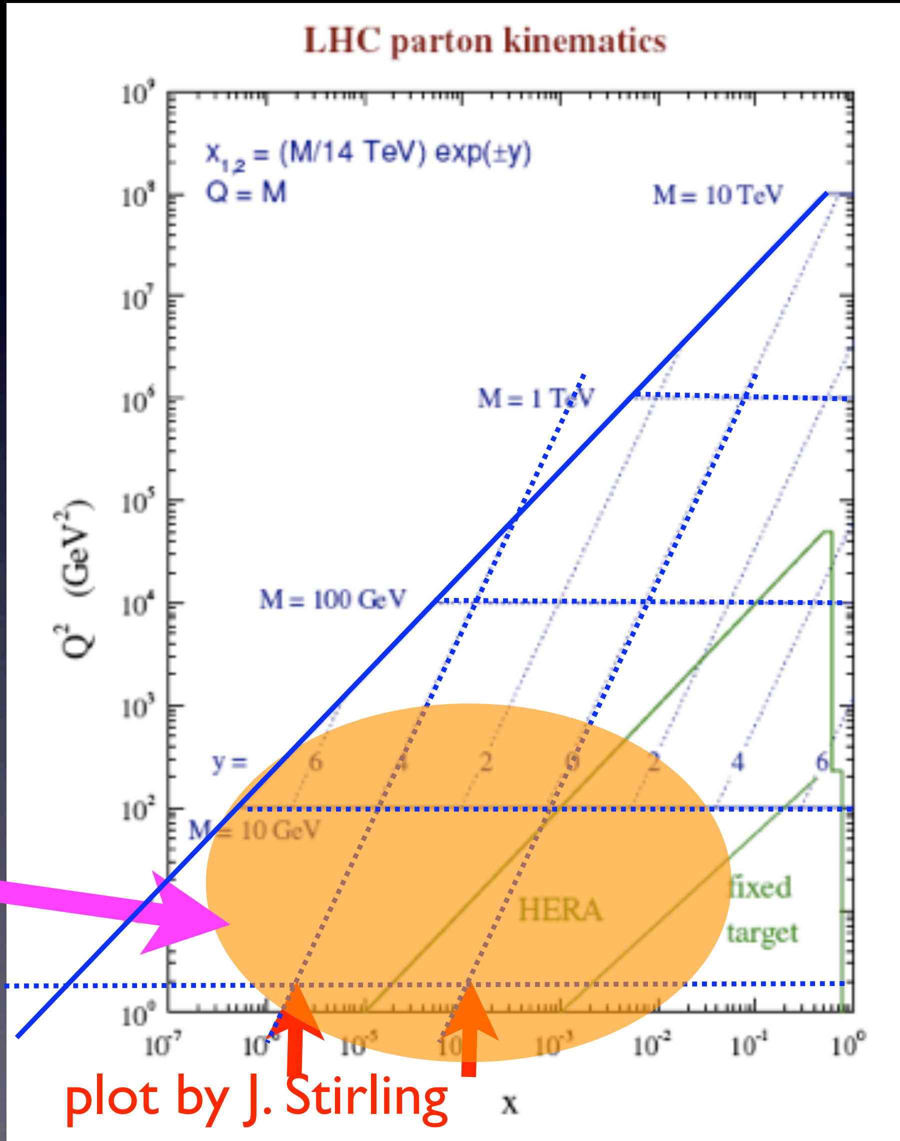


LHC kinematics

Kinematic boundary

Different scales
Different rapidities

Very large
region with
very small x!



plot by J. Stirling

Quantum Chromodynamics

QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_{\text{flavours}} \bar{q}_a (i D^\mu \gamma_\mu - m_f)_{ab} q_b$$

Field strength:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\mu A^a_\nu - g f^{abc} A^b_\mu A^c_\nu$$

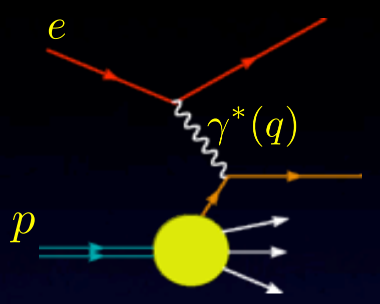
Color (a,b,c indices): charge in QCD

Degrees of freedom:

Gluon A^a_μ

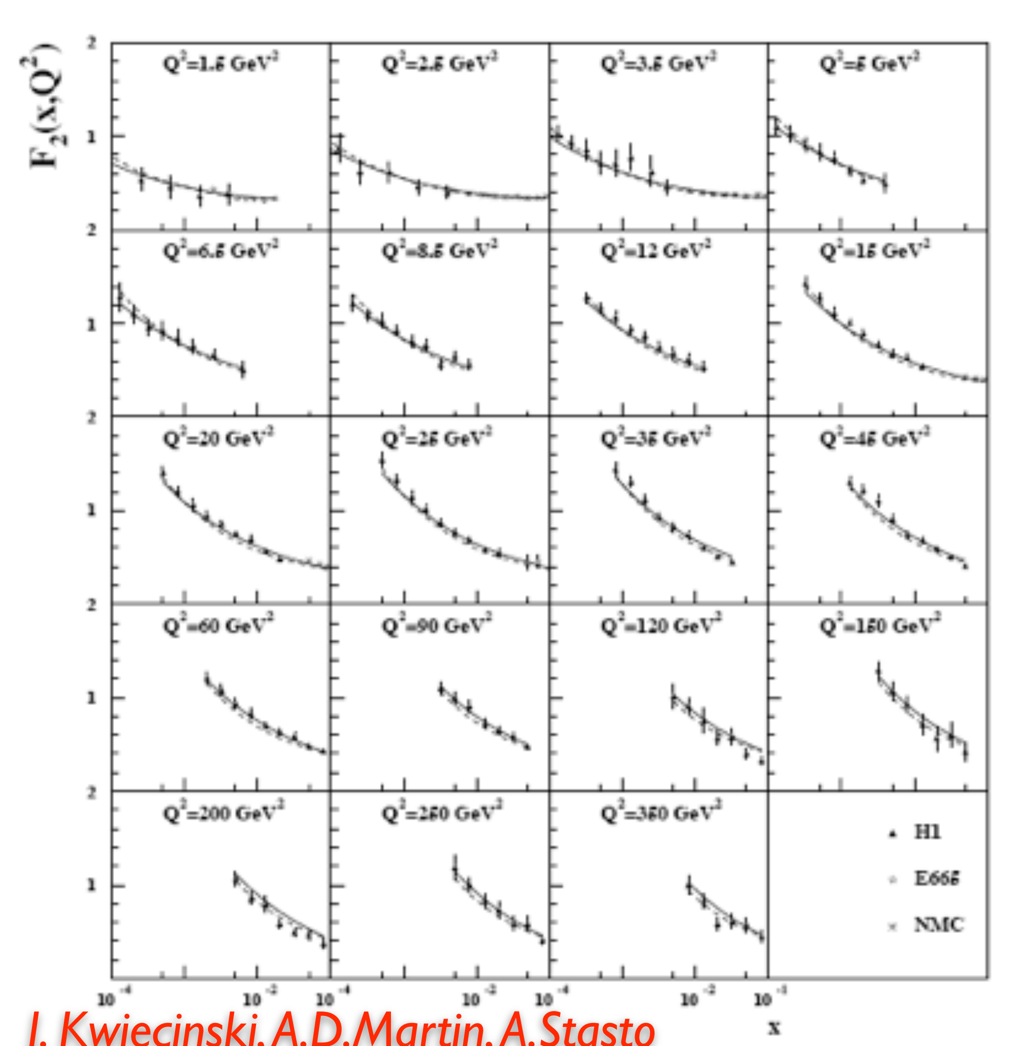
Quark q^a

Resummation: phenomenology



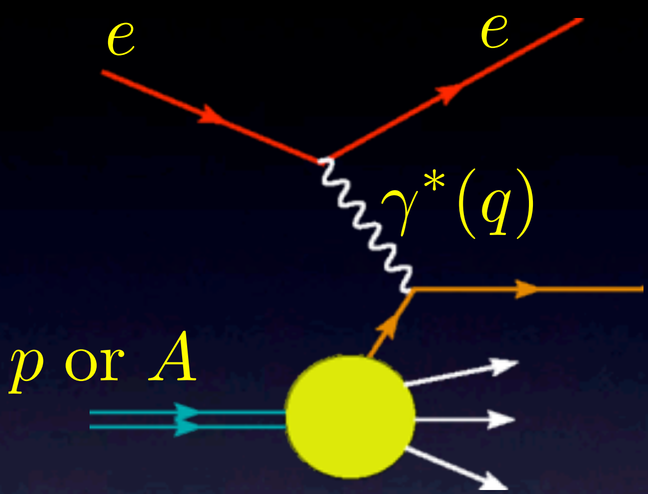
- Proton structure function as measured in ep scattering
- Obtain gluon density from resummed model.
- Calculate the cross section (two free parameters.)

HERA data on electron-proton DIS

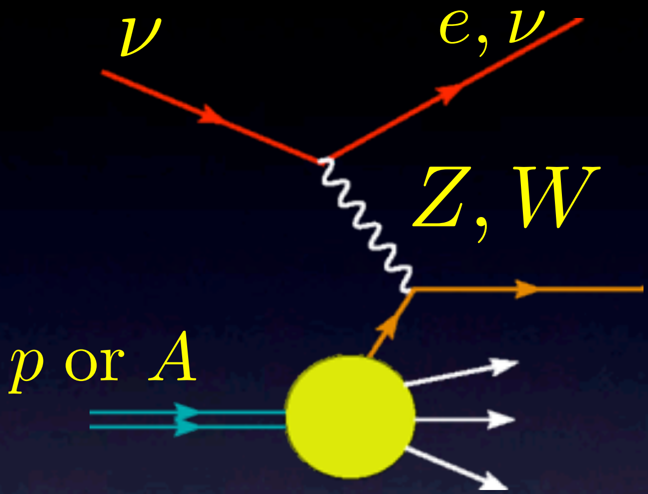


J. Kwiecinski, A.D. Martin, A. Stasto

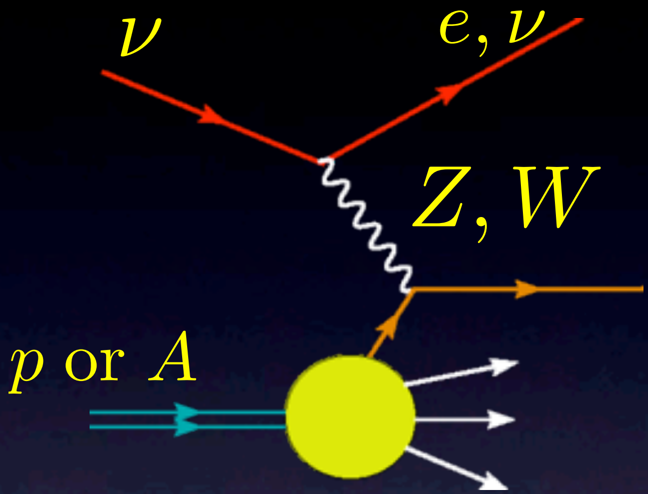
Applications: neutrino cross sections

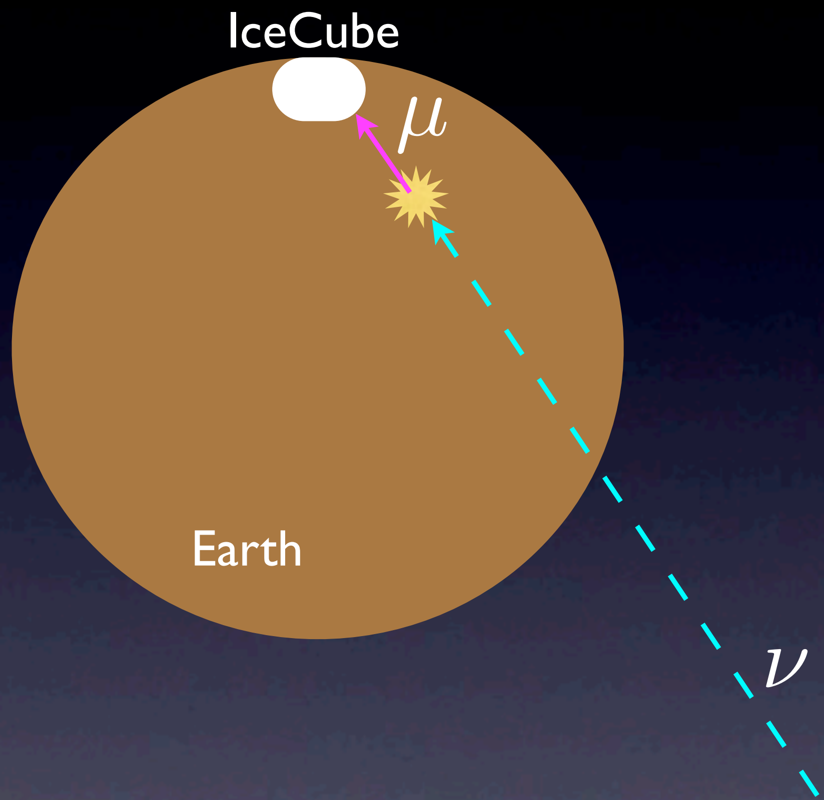


Applications: neutrino cross sections

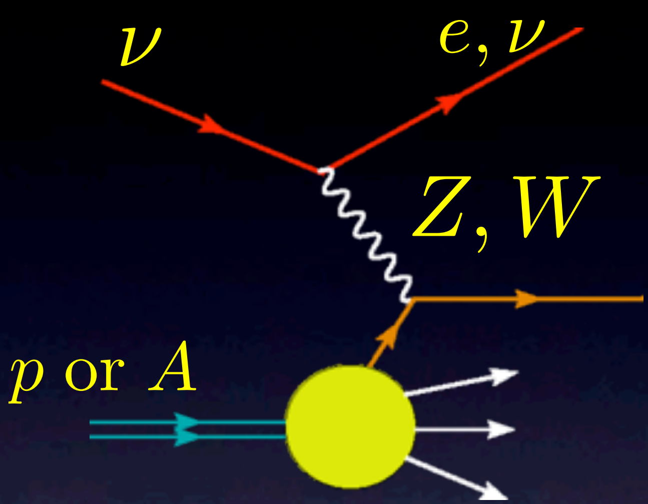


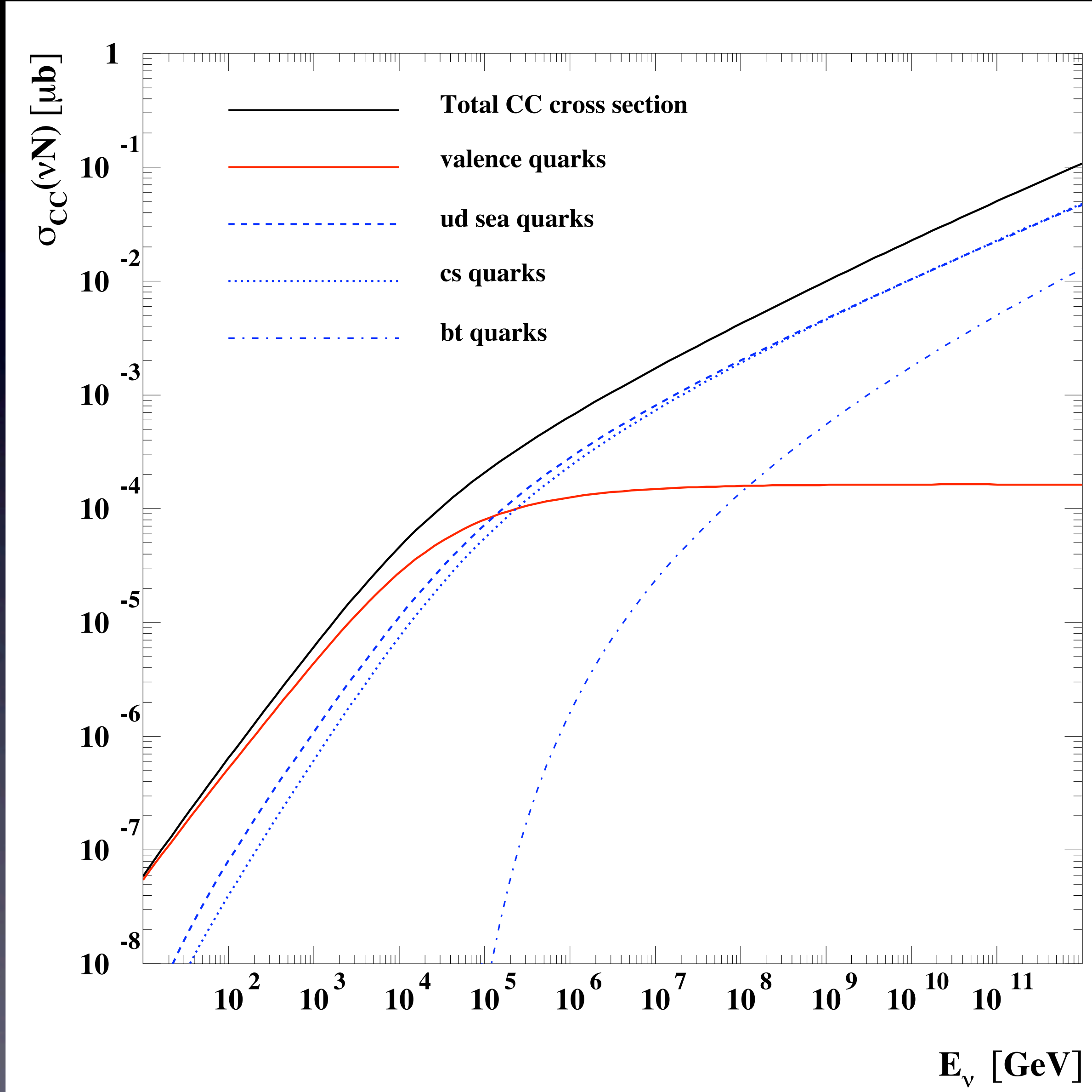
Applications: neutrino cross sections





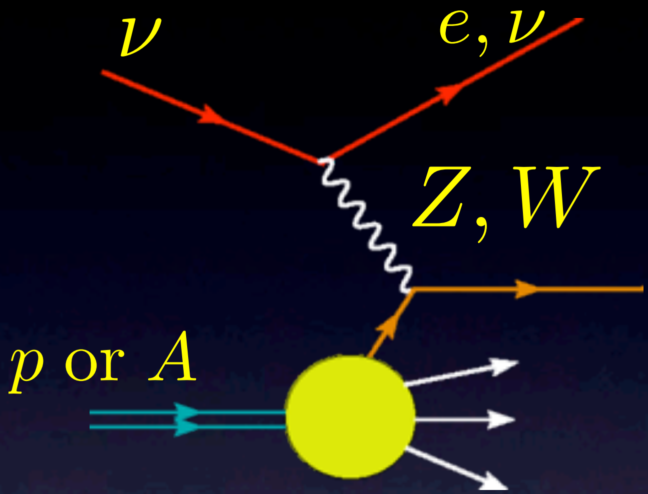
Applications: neutrino cross sections

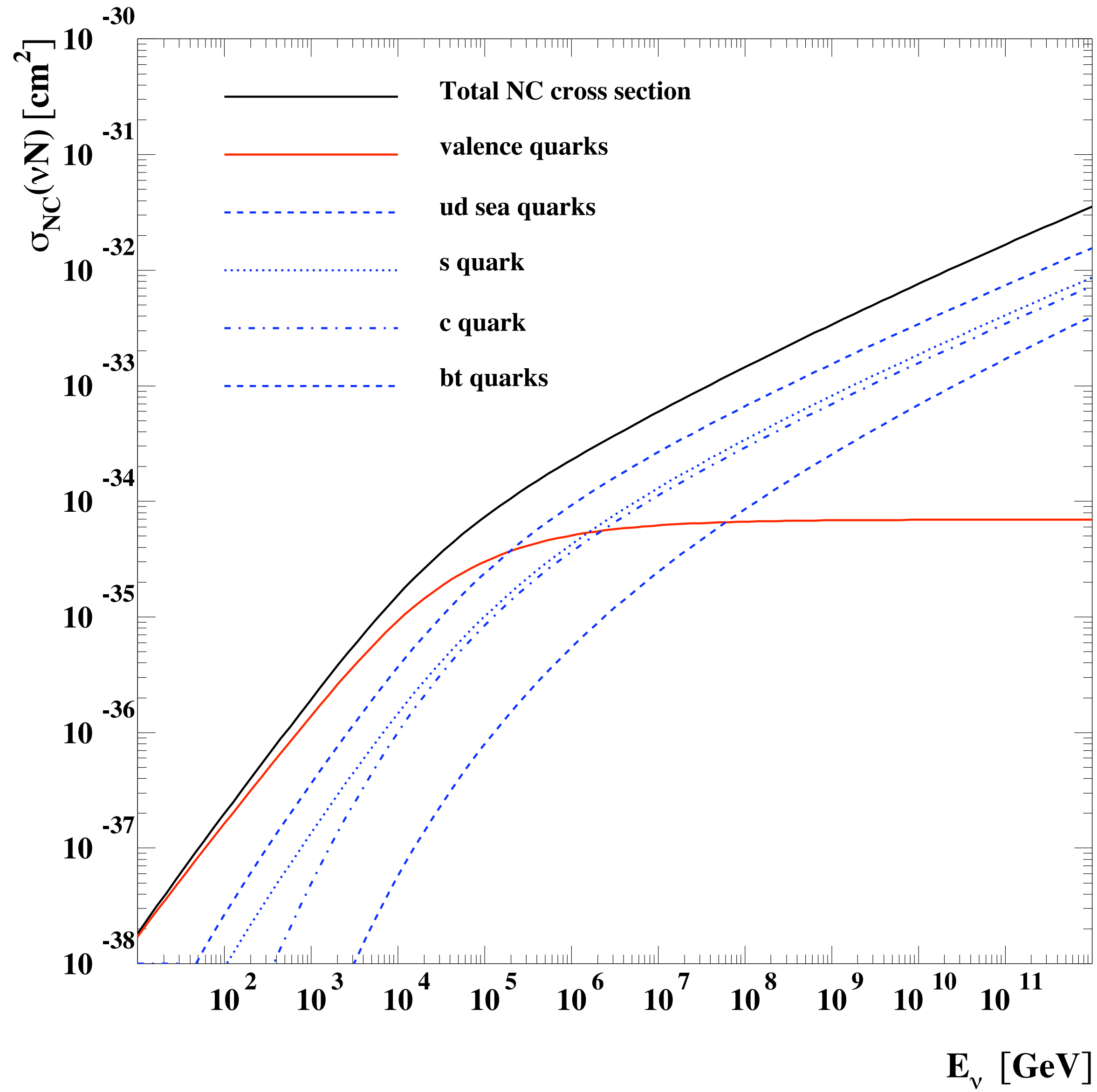




J. Kwiecinski, A.Martin, A.Stasto

Applications: neutrino cross sections

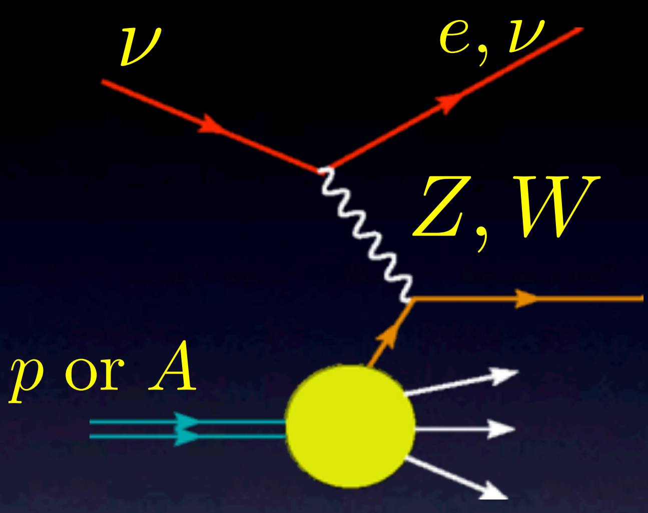




Behavior at high energies controlled dynamically by the resummed evolution equation, rather than the parametrized extrapolation.

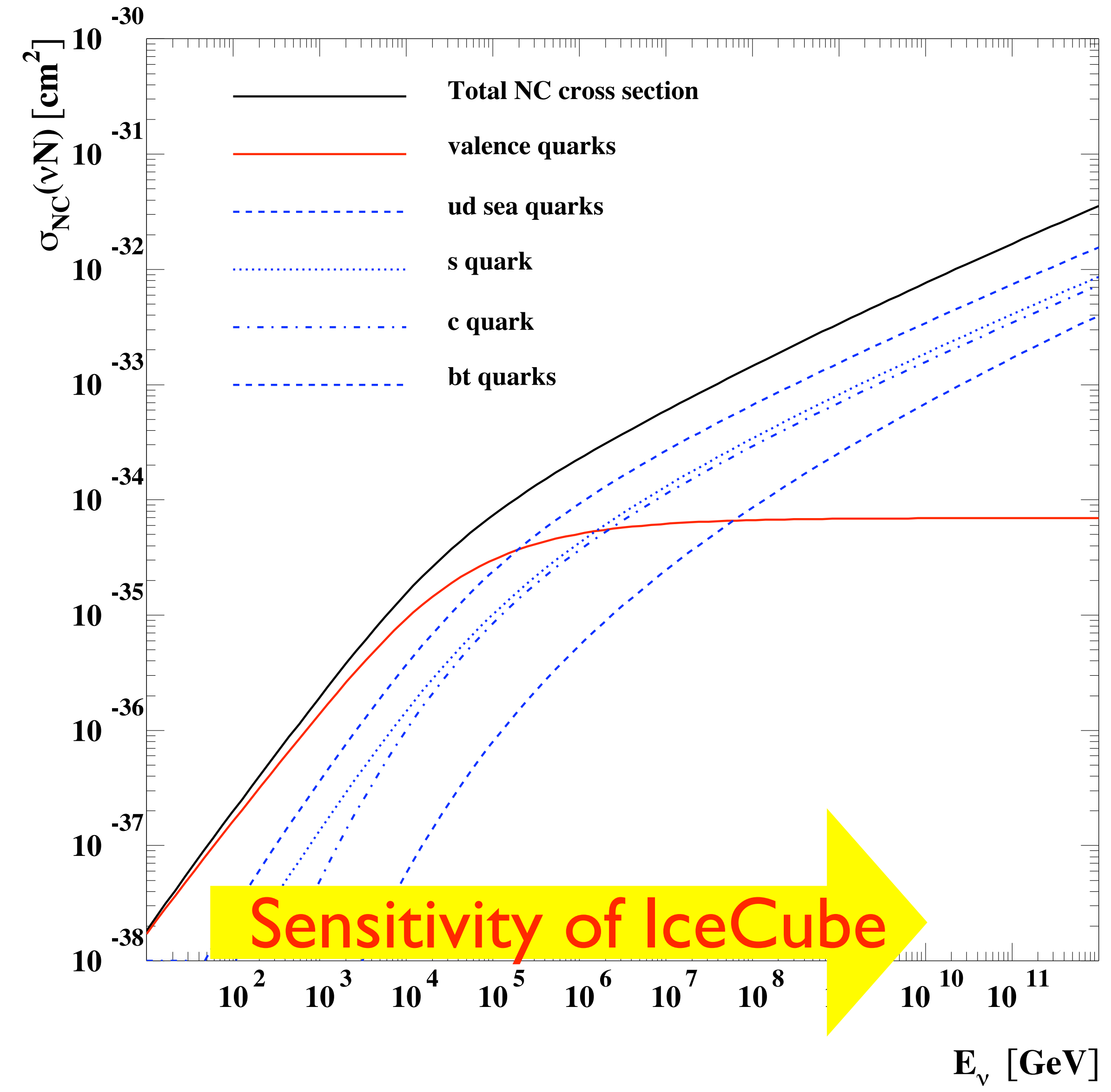
J. Kwiecinski, A.Martin, A.Stasto

Applications: neutrino cross sections

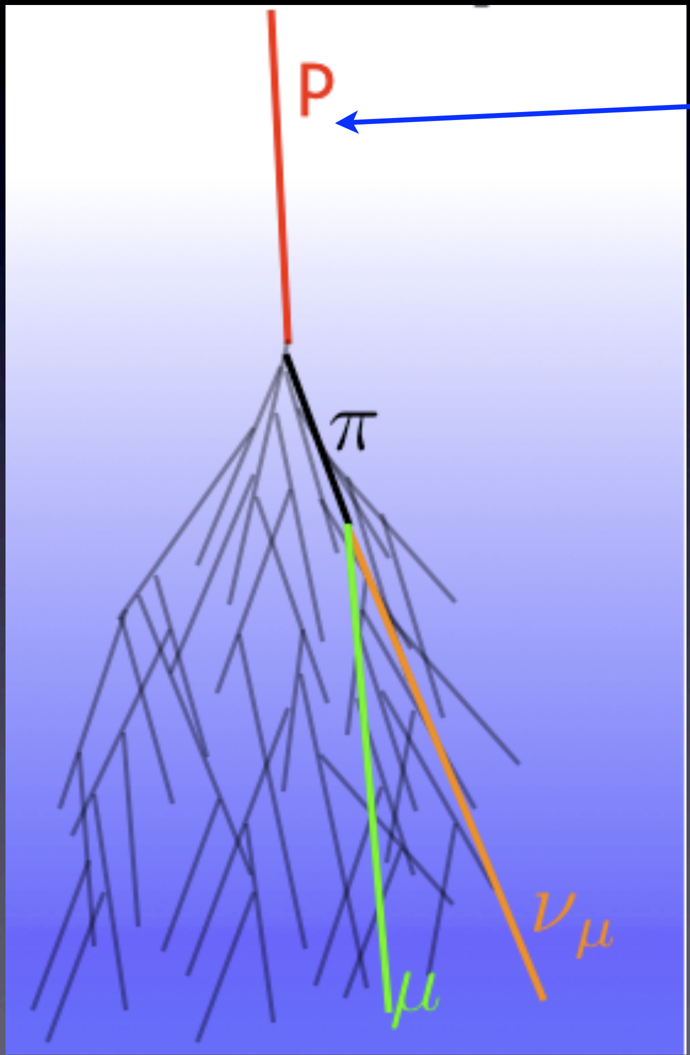


Behavior at high energies controlled dynamically by the resummed evolution equation, rather than the parametrized extrapolation.

J. Kwiecinski, A. Martin, A. Stasto



Production of atmospheric neutrinos



Very high energy cosmic ray proton

$p + \text{Air}$

π, K $\quad$ $D^\pm, D^0, D_s$

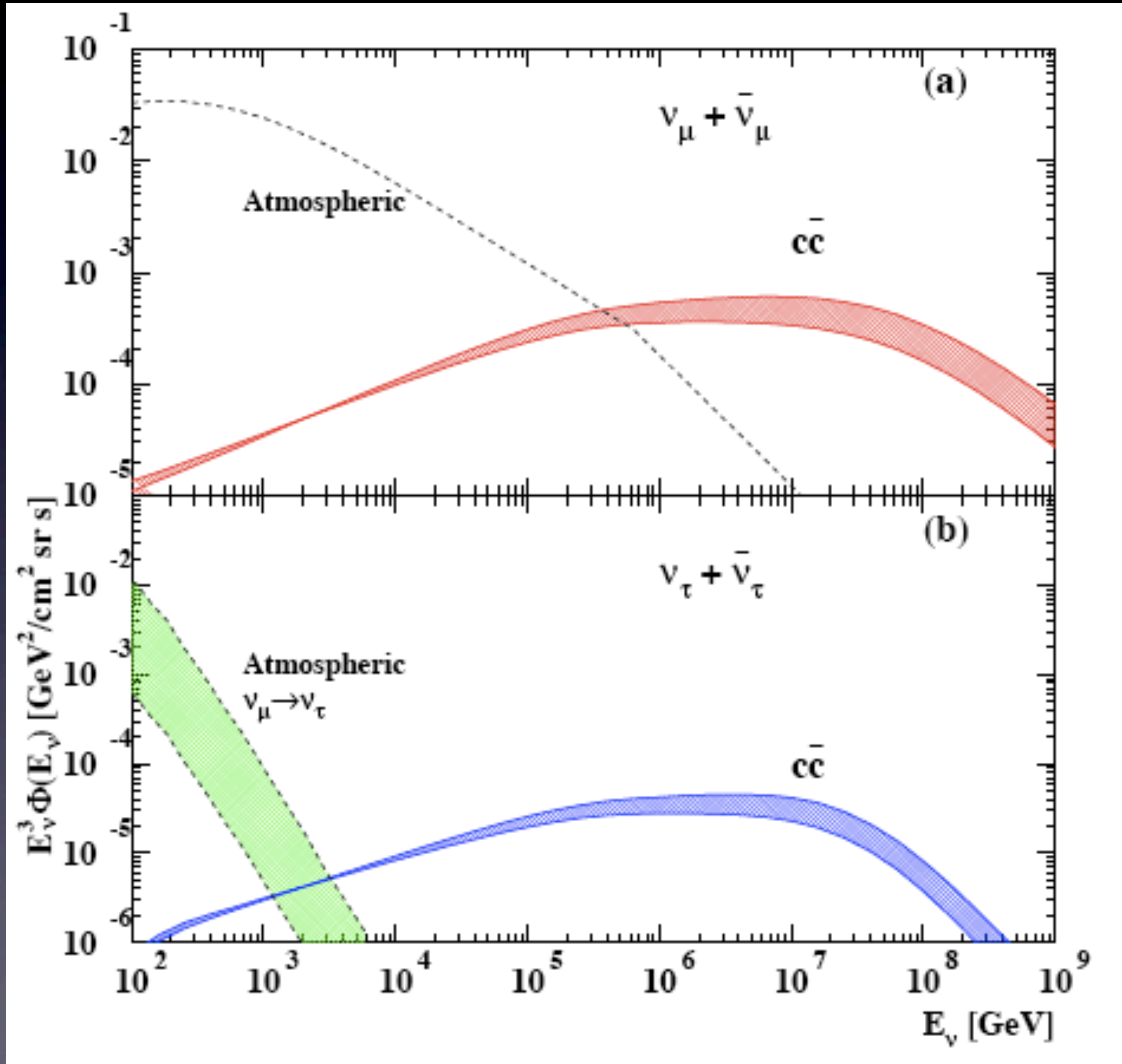
heavy mesons

ν $\quad$ ν high energy

prompt neutrino flux

Prompt neutrino flux

- Prompt neutrino flux vs conventional flux
- Prompt tau neutrino flux with the flux coming from oscillations
- Dominant contribution for E>500 TeV (for tau E>5 TeV)
- Sharp fall-off for E>1 EeV



A.Martin, M.Ryskin, A.Stasto

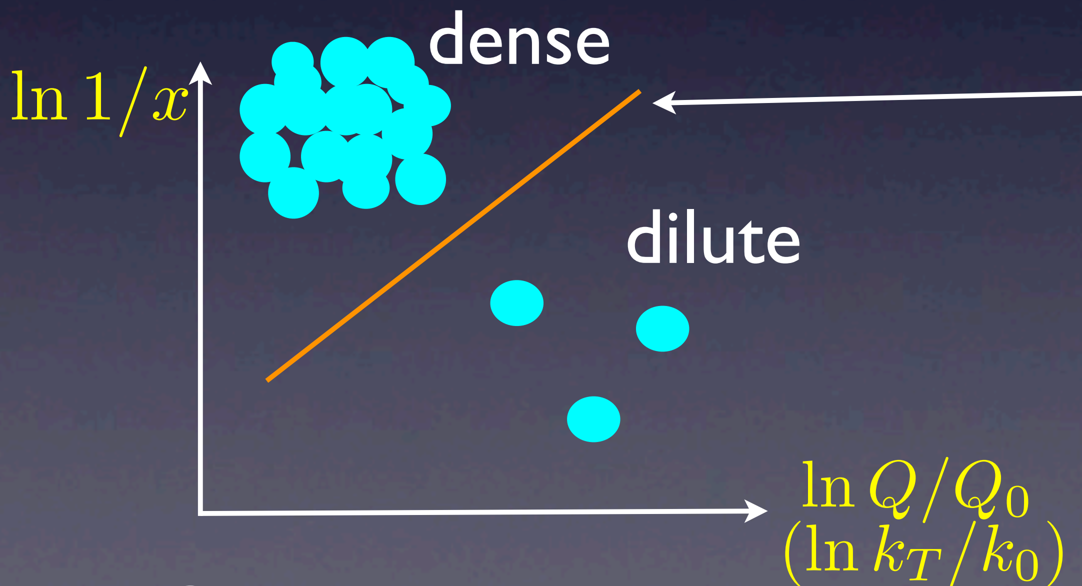
The evolution equation becomes nonlinear in density

$$\frac{df_g(x,k_T^2)}{d\ln 1/x} = \frac{\alpha_s N_c}{\pi}\int d^2k'_T \mathcal{K}(k_T,k'_T)\, f_g(x,k'_T) \quad \boxed{-\frac{\alpha_s N_c}{\pi}\left(f_g(x,k_T^2)\right)^2}$$

L.V.Gribov, E. Levin, M. Ryskin;
I.Balitsky,Y.Kovchegov; J.Jalilian-Marian,
E.Iancu,L.McLerran,H.Weigert, Leonidov

Note: Compare with Verhulst logistic equation for the population dynamics.

Color Glass Condensate *L.McLerran, R.Venugopalan*



saturation scale

$$Q_s(x) \sim x^{-\lambda}$$

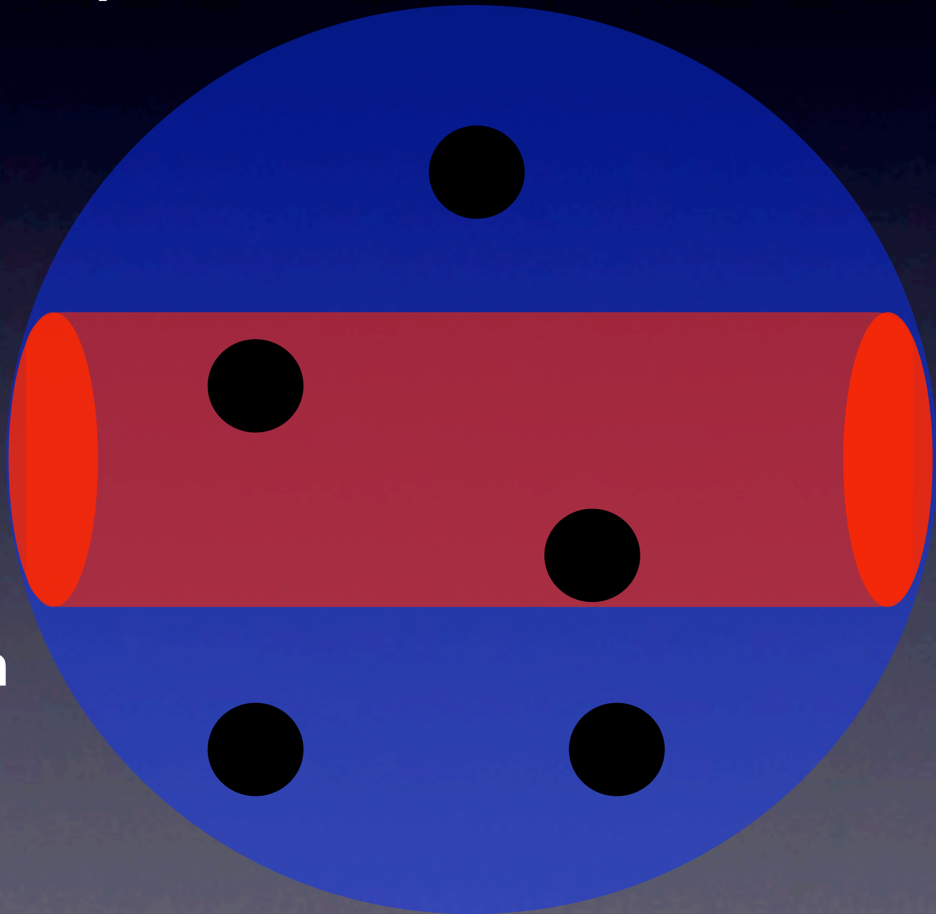
Dynamically generated

Gluon density saturates: ***parton saturation***

Gluon Saturation

At small x there are also other effects: the density of the gluons is so high that they start to recombine.

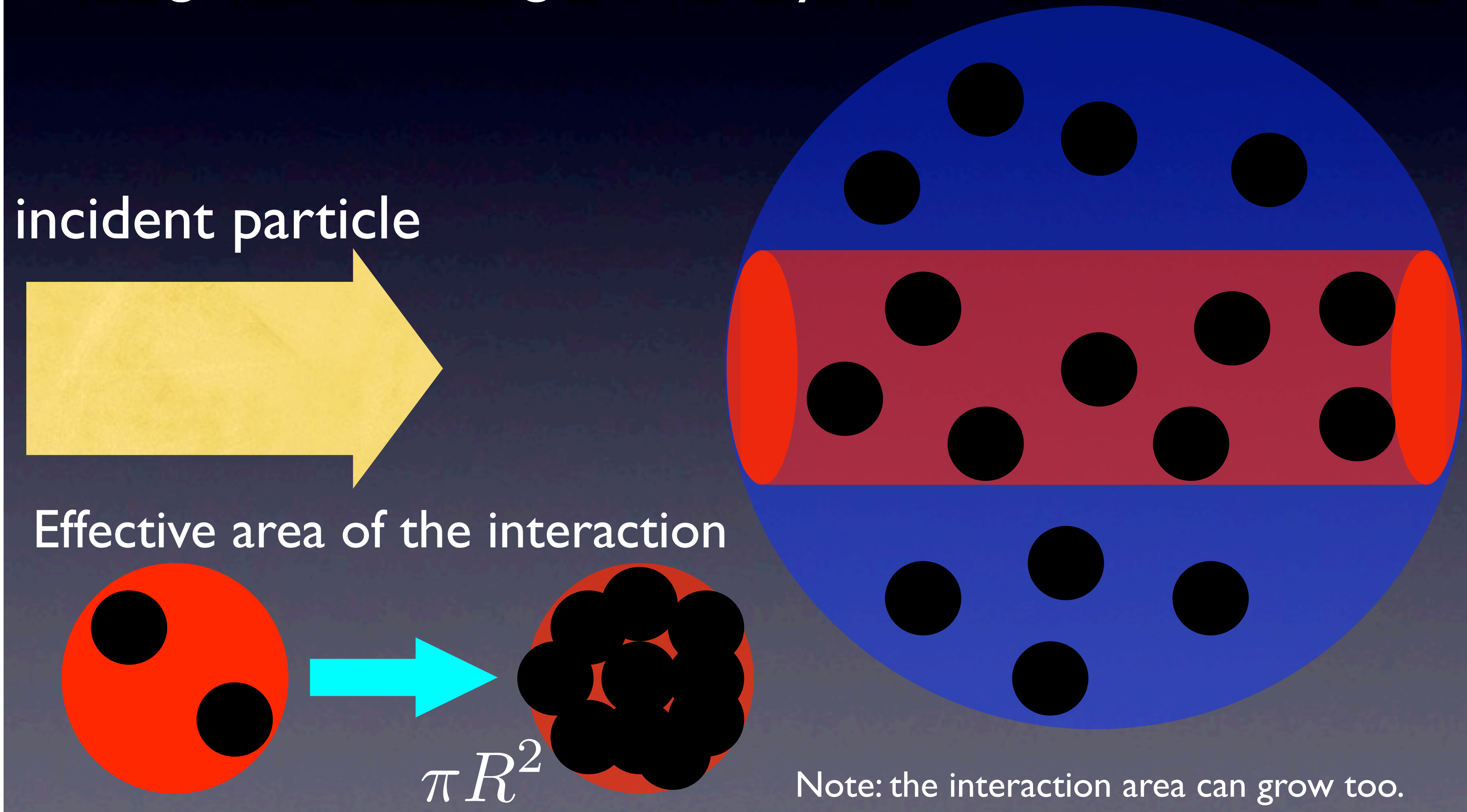
incident particle



Effective area of the interaction

Gluon Saturation

At small x there are also other effects: the density of the gluons is so high that they start to recombine.



Geometrical scaling vs data

K.Golec-Biernat, J.Kwiecinski, A.Stasto

K.Golec-Biernat, M.Wusthoff

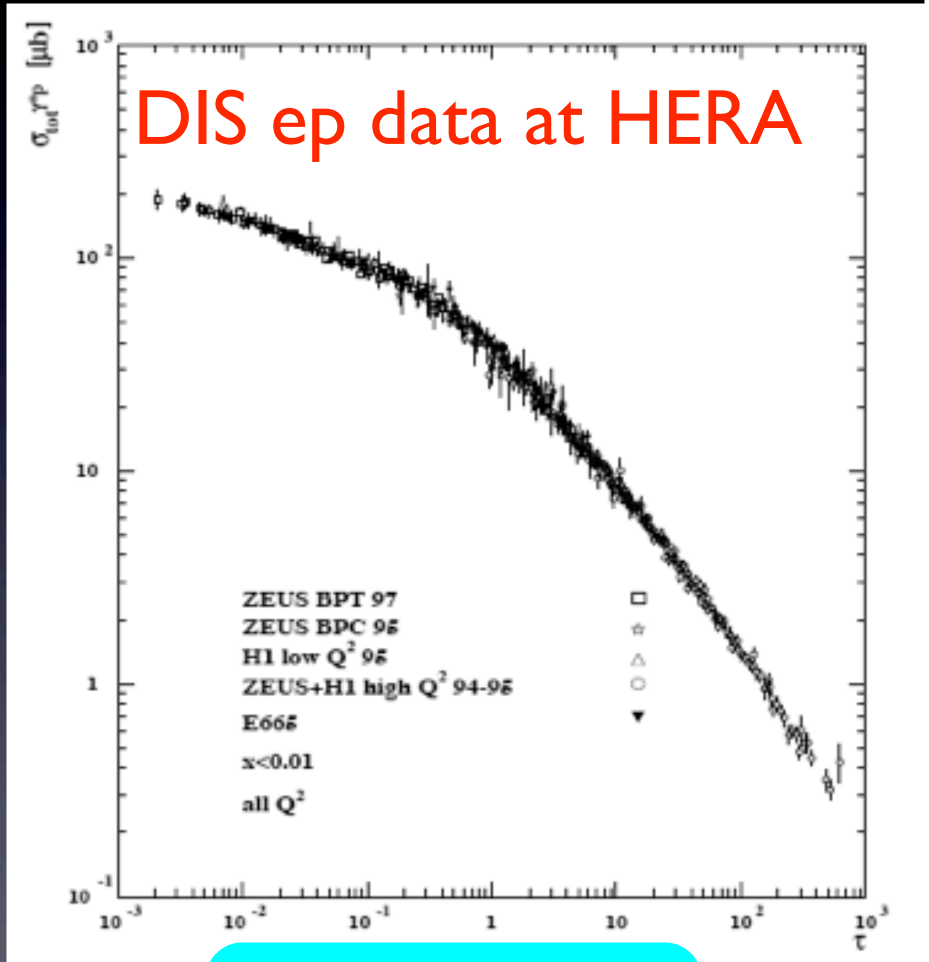
Saturation model: a few parameter model with a saturation scale and scaling property built in:

$$N(r,x) = N(r^2 Q_s(x)^2)$$

DIS ep

$$\sigma^{\gamma^* p}(Q^2/Q_s^2(x))$$

At small x.



$$\tau = Q^2/Q_s^2(x)$$

Geometrical scaling vs data

K.Golec-Biernat, J.Kwiecinski, A.Stasto

K.Golec-Biernat, M.Wusthoff

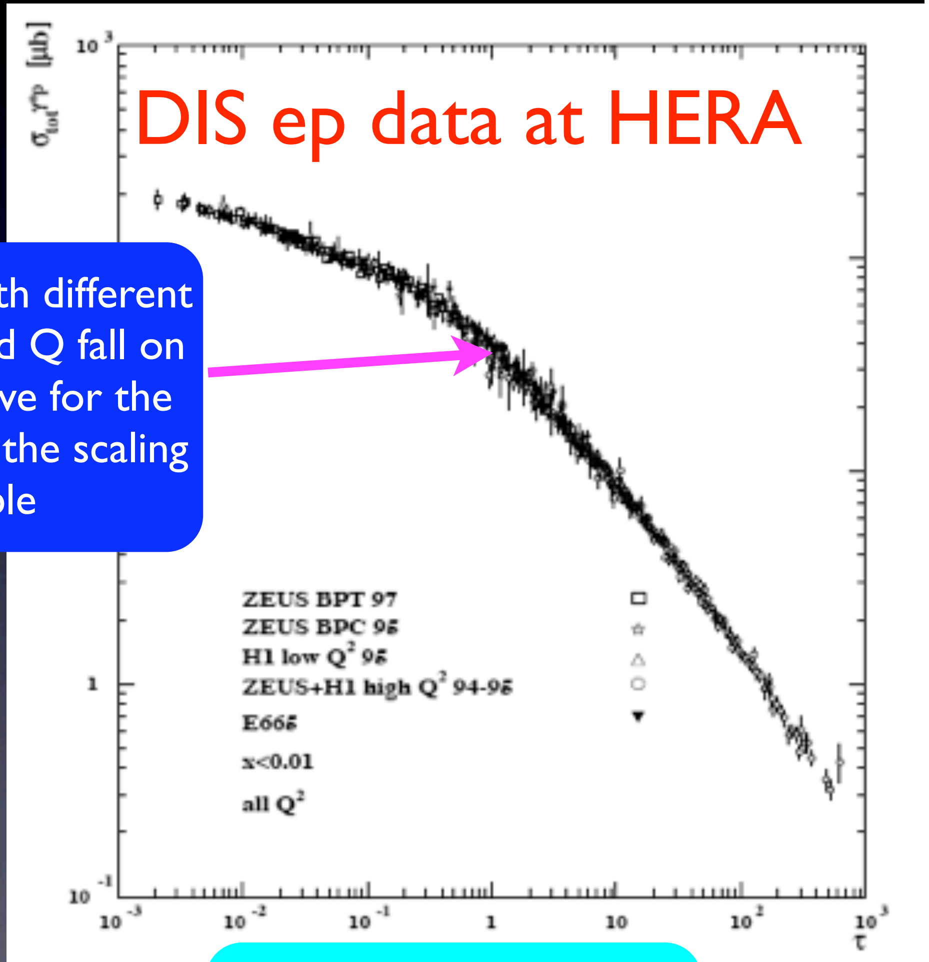
Saturation model: a few parameter model with a saturation scale and sc
property built in:

$$N(r,x) = N(r^2 Q_s(\ldots))$$

DIS ep

$$\sigma^{\gamma^* p}(Q^2/Q_s^2(x))$$

At small x.



Datapoints with different values of x and Q fall on the same curve for the same value of the scaling variable

$$\tau = Q^2/Q_s^2(x)$$

Geometrical scaling vs data

K.Golec-Biernat, J.Kwiecinski, A.Stasto

K.Golec-Biernat, M.Wusthoff

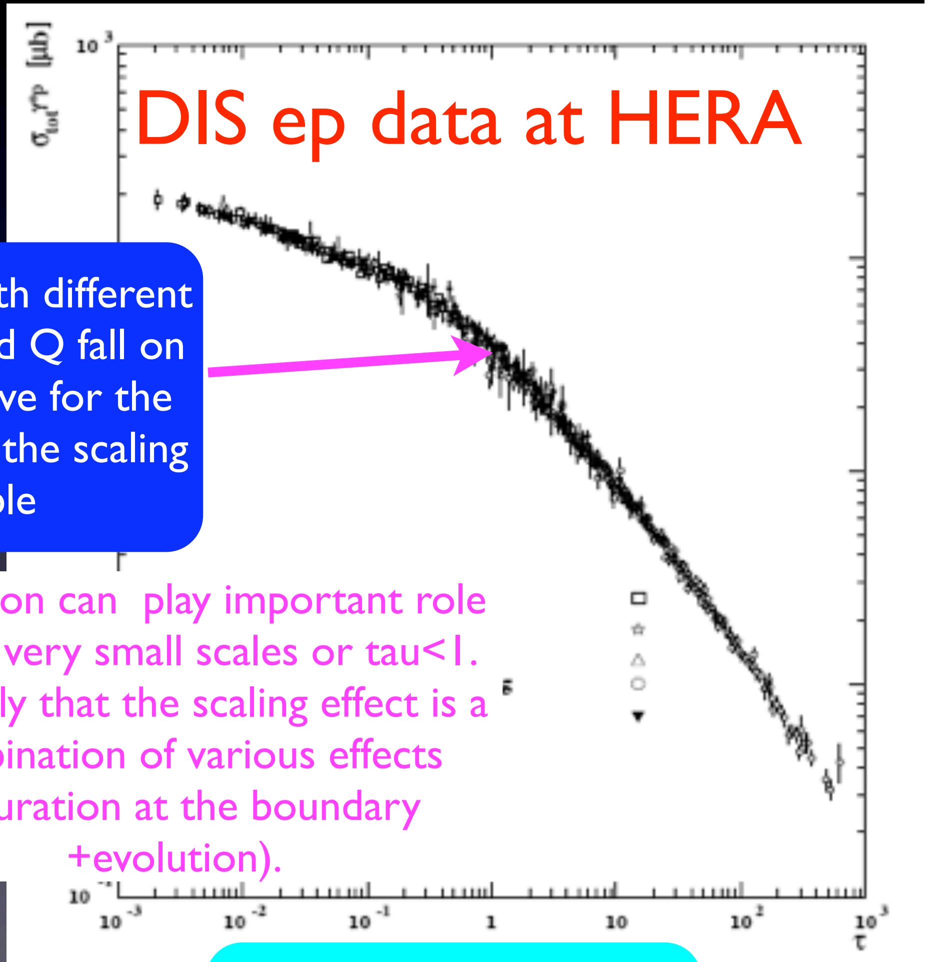
Saturation model: a few parameter model with a saturation scale and sc property built in:

$$N(r,x) = N(r^2 Q_s($$

DIS ep

$$\sigma^{\gamma^* p}(Q^2/Q_s^2(x))$$

At small x.



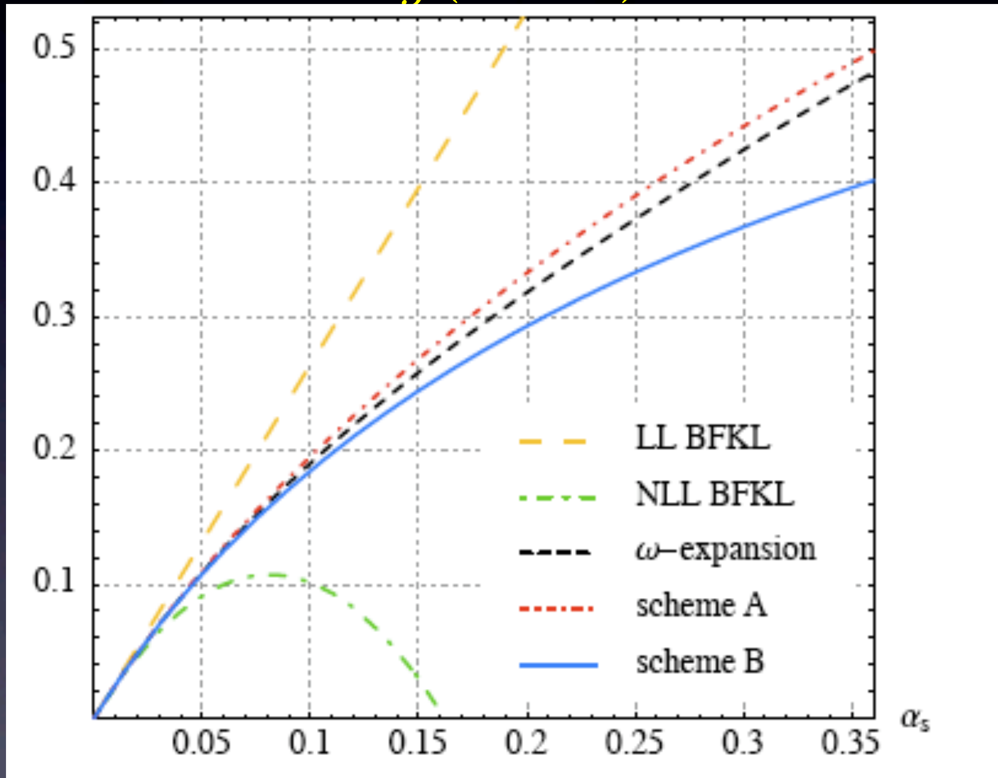
Datapoints with different values of x and Q fall on the same curve for the same value of the scaling variable

Saturation can play important role only at very small scales or tau<1. It is likely that the scaling effect is a combination of various effects (saturation at the boundary +evolution).

$$\tau = Q^2/Q_s^2(x)$$

Strong coupling limit

Recall the leading power $f_g(x, k_T) \sim x^{-\omega_P}$



What is the strong coupling limit?

Strong coupling limit

G.Veneziano, R.Janik, R.Peschanski, L.Lipatov, A.V.Kotikov, J.Polchinski, M.Strassler, R.Brower, C.Tan, L.Cornalba

Recall the leading power $f_g(x, k_T) \sim x^{-\omega_P}$

N=4 SYM

$$\omega_P = j - 1 \longrightarrow 1$$

The effective spin is j=2 in the strong coupling limit: the graviton.

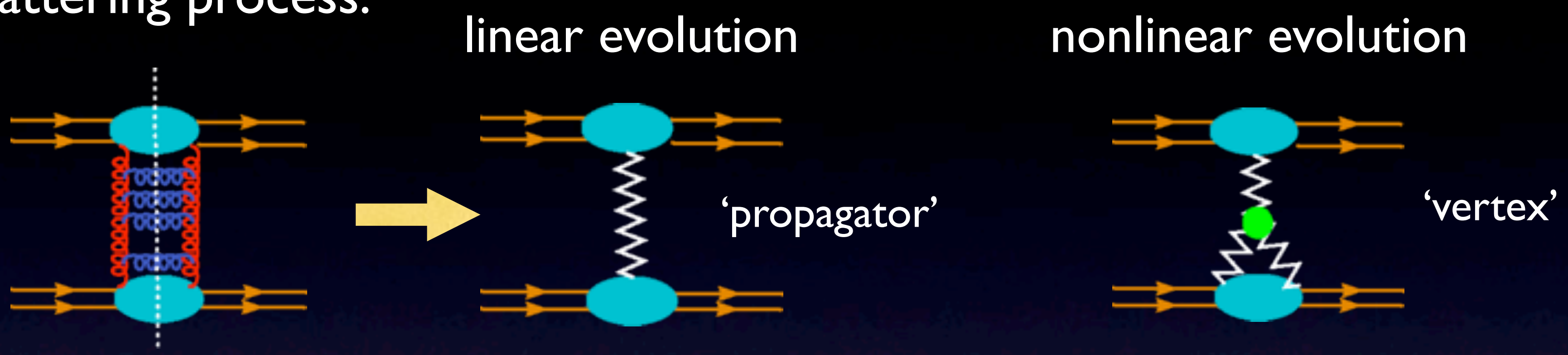
gauge theory $\longleftrightarrow$ string/gravity *J. Maldacena*

The resummed model provides with the interpolation between weak and strong coupling limits. *A.Stasto*

The role of the energy momentum constraint and the cancellations between real and virtual emissions of the soft gluons.

Effective field theory at high energies

Scattering process:



- At high energy effective degrees of freedom are compound states of gluons - quasiparticles.
- Effective field theory (Gribov's Reggeon field theory) with propagators and non-local vertices in the leading logarithmic approximation. *E. Levin, S. Bondarenko, A. Prygarin, M. Braun, I. Balitsky, L.Lipatov ,M. Lublinsky, A. Kovner, L.Motyka, A. Mueller, E.Iancu, Y.Hatta, L. McLerran, D.Triantafyllopoulos, A.Stasto...*
- Questions and problems:
    - Extension of this field theory to higher orders (resummation of the whole effective field theory) .
    - Phase space constraints in the case of the multichain interactions.