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**From the GUT to the meV scale in the Standard Model of the Universe: Inflation,
Dark Matter and Dark Energy**

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The History of the Universe

It is a history of **EXPANSION** and cooling down.

EXPANSION: the space **itself** expands with the time.

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2, \quad a(t) = \text{scale factor.}$$

FRW: Homogeneous, isotropic and spatially **flat** geometry.

Cooling: temperature decreases as $1/a(t)$: $T(t) \sim 1/a(t)$.

The Universe underwent a succession of phase transitions towards the less symmetric phases.

Wavelengths **redshift** as $a(t)$: $\lambda(t) = a(t) \frac{\lambda(t_0)}{a(t_0)}$

Redshift z : $z + 1 = \frac{a(\text{today})}{a(t)}$, $a(\text{today}) \equiv 1$

The deeper you go in the past, the larger is the redshift and the smaller is $a(t)$.

Standard Cosmological Model: Λ CDM

Λ CDM = Cold Dark Matter + Cosmological Constant
begins by the Inflationary Era. **Explains** the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion:
Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
-

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t) d\vec{x}^2$: spatially **flat** geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2 a}{dt^2} > 0$.

During inflation the universe expands by at least sixty efolds: $e^{62} \simeq 10^{27}$. Inflation **lasts** $\simeq 10^{-36}$ sec and ends by $z \sim 10^{29}$ followed by a **radiation** dominated era.

Energy scale when inflation starts $\sim 10^{16}$ GeV (\Leftarrow CMB anisotropies) which **coincides** with the GUT scale.

Matter can be effectively described during inflation by a Scalar Field $\phi(t, x)$: the **Inflaton**.

Lagrangian: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2a^2(t)} - V(\phi) \right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3 M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Physics during Inflation

- **Out of equilibrium** evolution in a fastly expanding geometry. Vacuum energy **DOMINATES**. $a(t) \simeq e^{Ht}$.
- Extremely high energy density at the scale of $\lesssim 10^{16}$ GeV.
- **Explosive** particle production due to spinodal or parametric **instabilities**.
- Quantum non-linear phenomena eventually **shut-off** the instabilities and **stop** inflation. Radiation dominated era follows: $a(t) = \sqrt{t}$.
- Huge redshift classicalizes the dynamics: an **assembly** of (superhorizon) quantum modes behave as a classical and homogeneous inflaton field. Inflaton slow-roll.

D. Boyanovsky, H. J. de Vega, in *Astrofundamental Physics*, NATO ASI series vol. 562, 2000, Lectures at the Chalonge School, astro-ph/0006446.

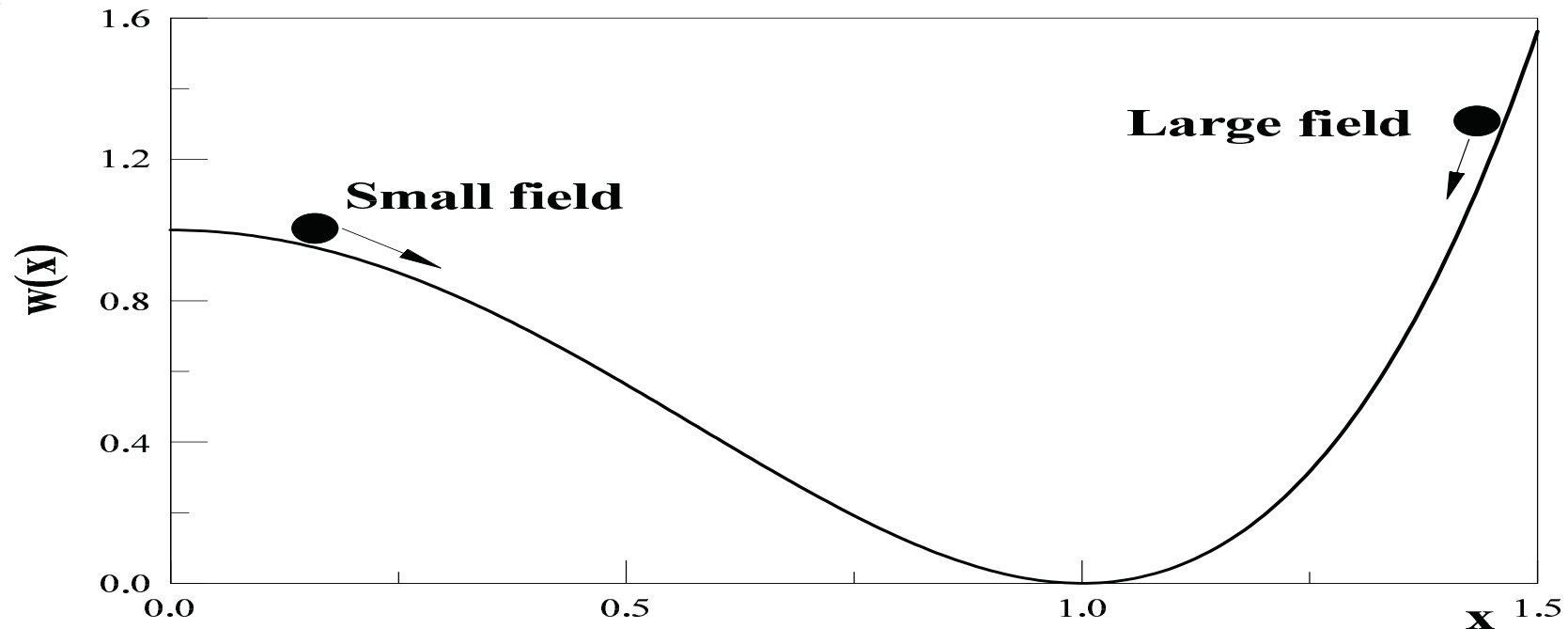
The Theory of Inflation

The inflaton is an **effective** field in the Ginsburg-Landau sense.

Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q$, $\sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)
- Fermi Theory of Weak Interactions (current-current).

Slow Roll Inflaton Models



$V(\text{Min}) = V'(\text{Min}) = 0$: inflation **ends** after a finite number of efolds. **Universal** form of the slow-roll inflaton potential:

$$V(\phi) = N M^4 w \left(\frac{\phi}{\sqrt{N} M_{Pl}} \right)$$

$N \sim 60$ number of efolds since horizon exit till end of inflation. M = energy scale of inflation.

Slow-roll is needed to produce enough efolds of inflation.

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N} M_{Pl}} \quad , \quad \tau = \frac{m t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m \sqrt{N}} \quad , \quad \left(m \equiv \frac{M^2}{M_{Pl}} \right)$$

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order **one**.

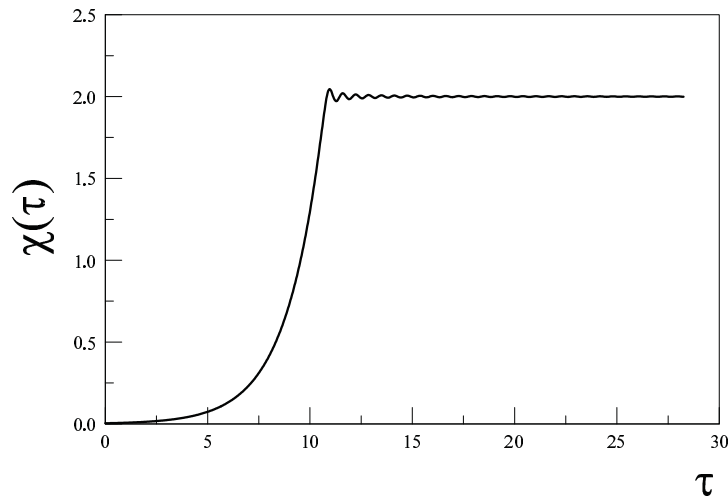
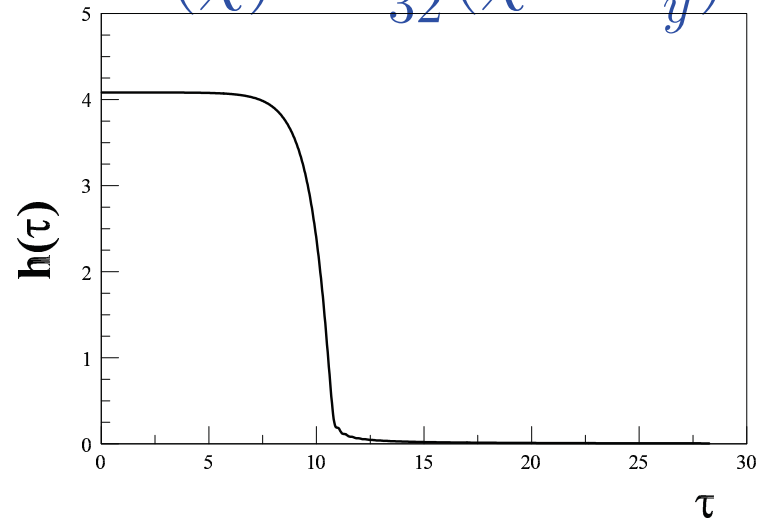
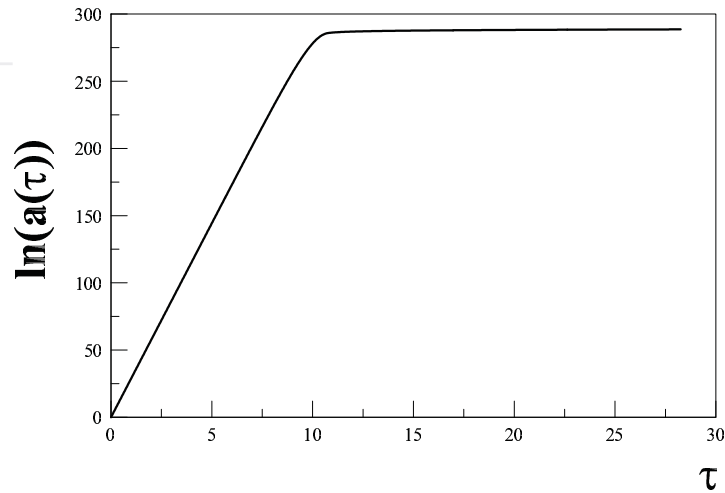
Evolution Equations:

$$\mathcal{H}^2(\tau) = \frac{1}{3} \left[\frac{1}{2 N} \left(\frac{d\chi}{d\tau} \right)^2 + w(\chi) \right] \quad ,$$
$$\frac{1}{N} \frac{d^2\chi}{d\tau^2} + 3 \mathcal{H} \frac{d\chi}{d\tau} + w'(\chi) = 0 \quad . \quad (1)$$

$1/N$ terms: **corrections** to slow-roll

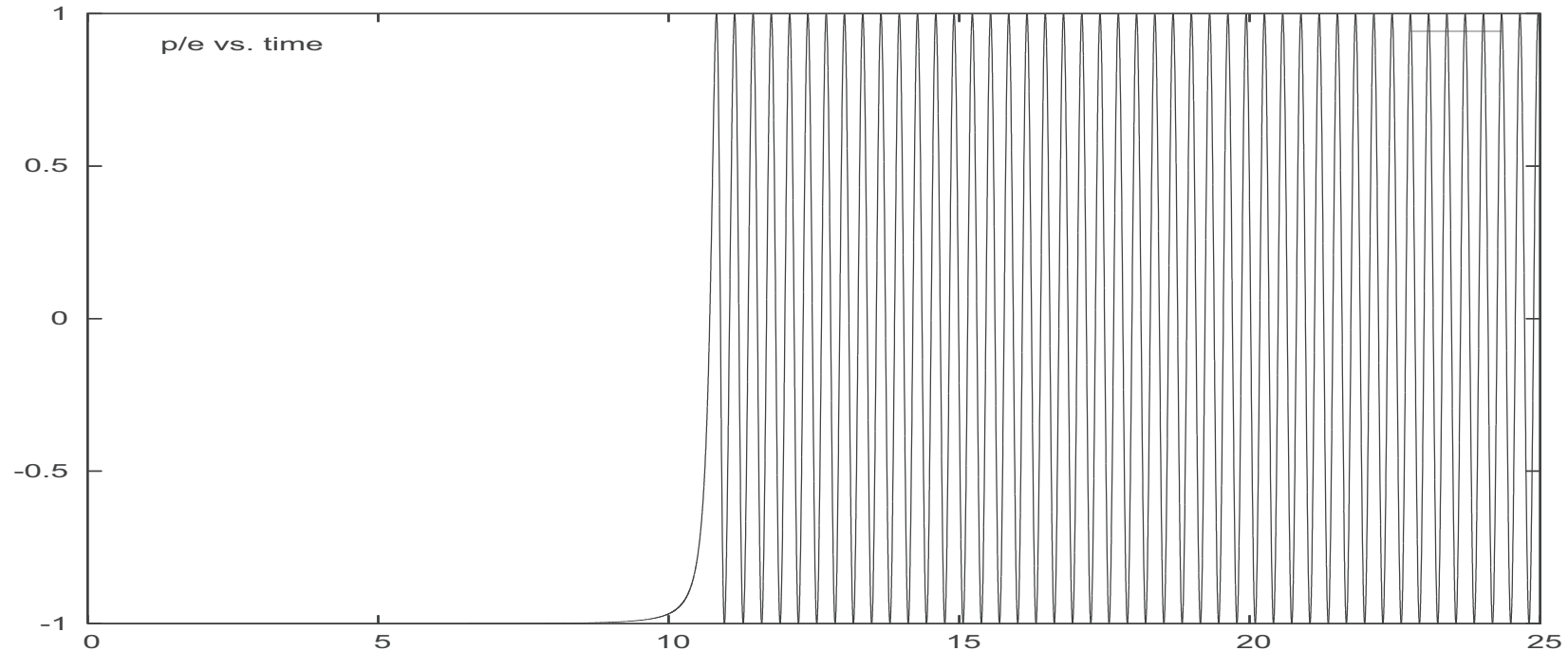
Higher orders in slow-roll are obtained **systematically** by expanding the solutions in $1/N$.

Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$



The **vacuum energy transforms into particles** and inflation is followed in this simplified approach by a matter dominated stage.

Equation of State: pressure/energy density



The equation of state is $p/e = -1$ during inflation.

p/e strongly oscillates between $+1$ and -1 during the matter dominated stage. We have in average $\langle p/e \rangle = 0$.

We have here neglected spatial gradient terms

$$\frac{(\nabla\phi)^2}{2a^2(t)}$$

since $a(t)$ grows exponentially during inflation.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2 k^{n_s-1}$.

To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\pi^2} \left(\frac{M}{M_{Pl}} \right)^4 \frac{w^3(\chi)}{w'^2(\chi)} .$$

Hence, for **all** slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\pi\sqrt{3}} \left(\frac{M}{M_{Pl}} \right)^2$$

The WMAP5 result: $|\Delta_{k\ ad}^{(S)}| = (0.494 \pm 0.1) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}} \right)^2 = 0.85 \times 10^{-5} \longrightarrow M = 0.70 \times 10^{16} \text{ GeV}$$

The inflation energy scale **turns to be** the grand unification energy scale !! We find the scale of inflation **without** knowing the tensor/scalar ratio r !!

The scale M is independent of the shape of $w(\chi)$.

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial **gravitons**.

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)}, \quad r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2$$

$$\frac{dn_s}{d \ln k} = -\frac{2}{N^2} \frac{w'(\chi) w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2} \frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2} \frac{[w'(\chi)]^2 w''(\chi)}{w^3(\chi)},$$

χ is the inflaton field at horizon exit.

$n_s - 1$ and r are **always** of order $1/N \sim 0.02$ (model indep.)

Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

We choose a polynomial for $w(\chi)$. A quartic $w(\chi)$ is renormalizable. Higher order polynomials are acceptable since inflation it is an effective theory.

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3 \chi^3 + G_4 \chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N M^4 w\left(\frac{\phi}{\sqrt{N} M_{Pl}}\right) = V_o \pm \frac{m^2}{2} \phi^2 + g \phi^3 + \lambda \phi^4 .$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}} \left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N} \left(\frac{M}{M_{Pl}}\right)^4$$

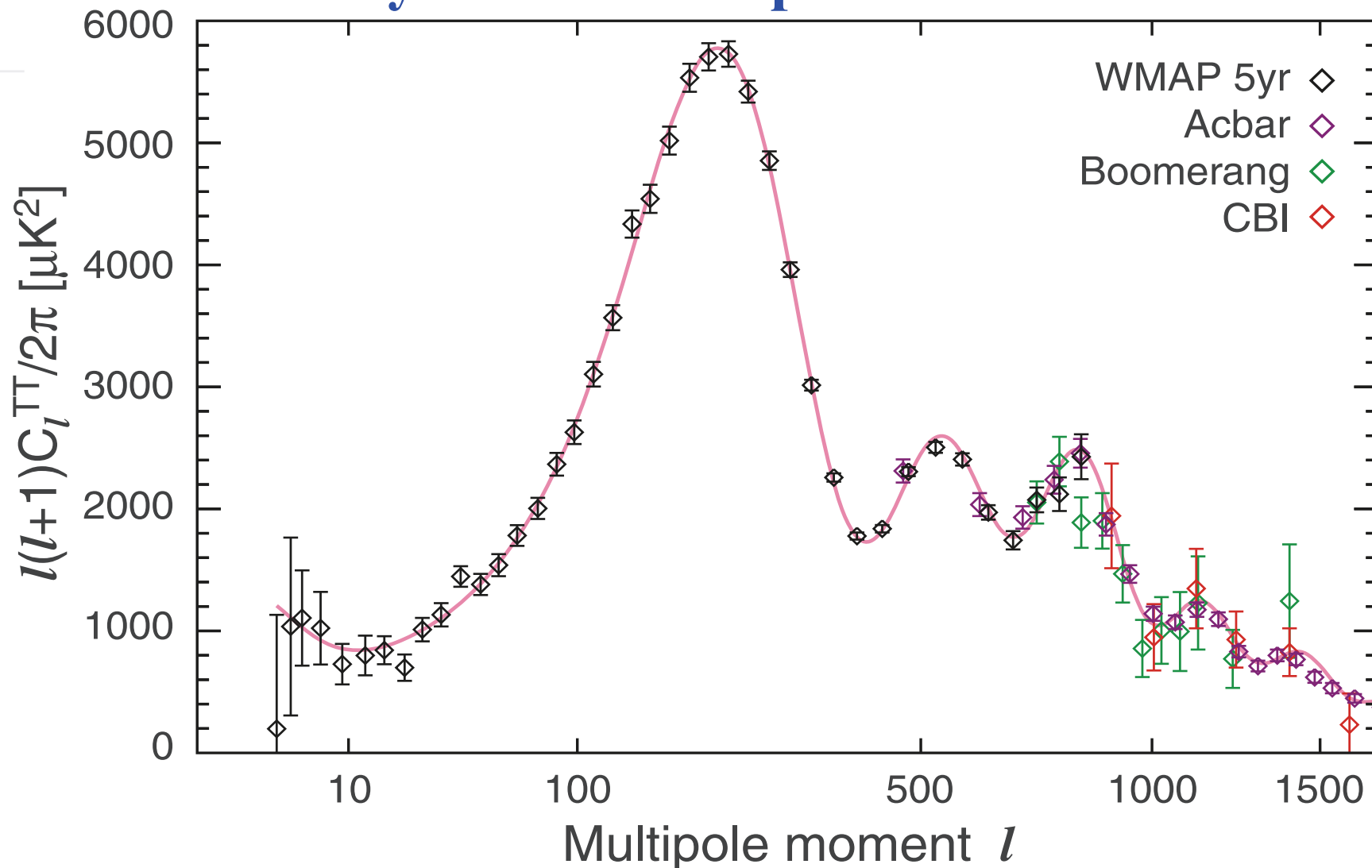
Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60 .$$

- Small couplings arise **naturally** as ratio of two energy scales: inflation and Planck.
- The inflaton is a **light** particle:

$$m = \frac{M^2}{M_{Pl}} \simeq 0.003 M \quad , \quad m = 2.5 \times 10^{13} \text{GeV}$$

WMAP 5 years data set plus other CMB data



Theory and observations **nicely agree** except for the lowest multipoles: **the quadrupole suppression**.

Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.

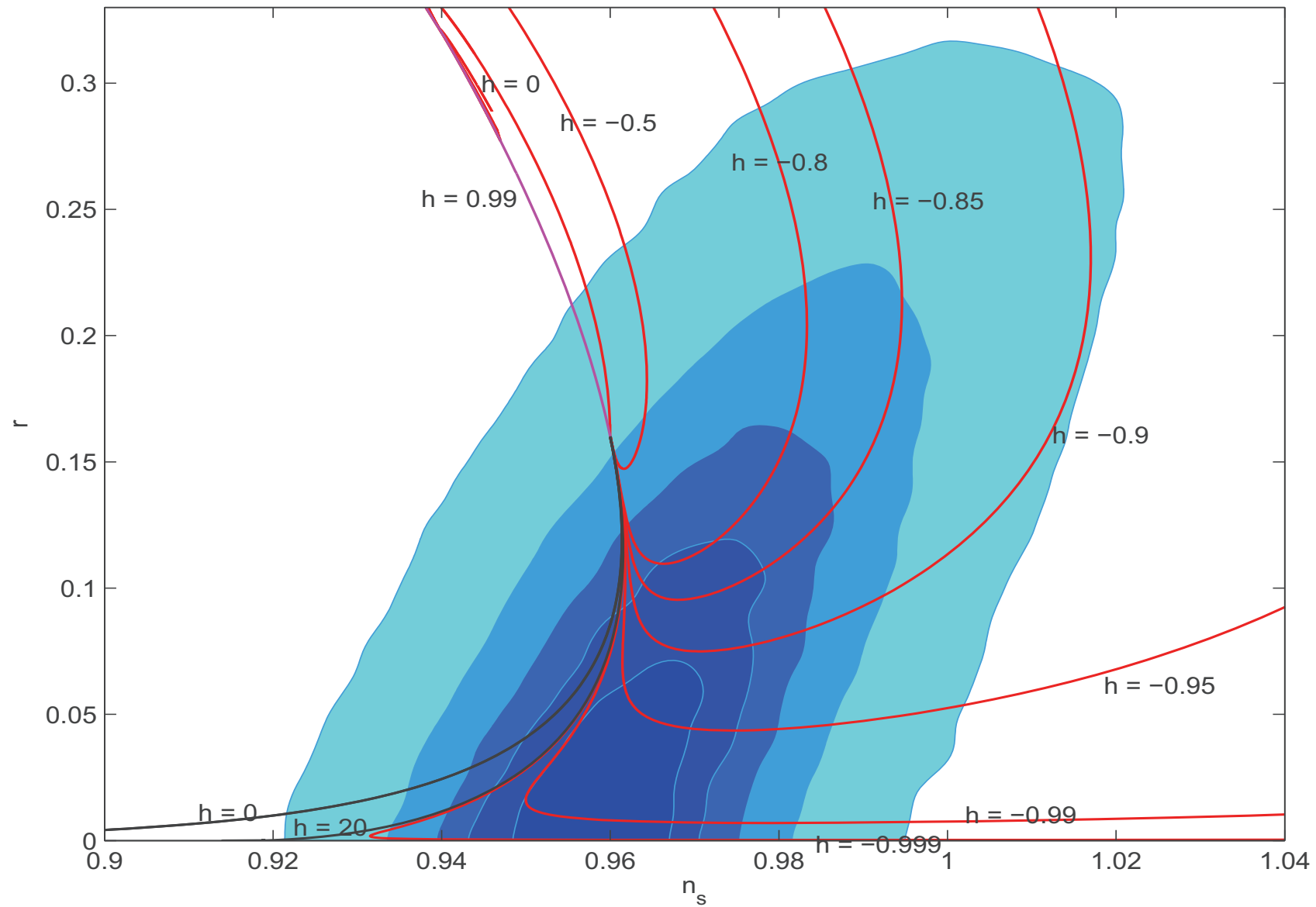
NEW: We imposed as a **hard constraint** that r and n_s are given by the inflaton potential.

Our analysis differs in **this crucial aspect** from previous MCMC studies of the WMAP data.

The color-filled areas correspond to 12%, 27%, 45%, 68% and 95% confidence levels according to the WMAP3 and Sloan data.

The color of the areas goes from the darker to the lighter for increasing CL.

MCMC Results for the double-well inflaton potential



MCMC Results for double–well inflaton potential

Bounds: $r > 0.023$ (95% CL) , $r > 0.046$ (68% CL)

Most probable values: $n_s \simeq 0.964$, $r \simeq 0.051 \Leftarrow$ measurable!!

The most probable double–well inflaton potential has a moderate nonlinearity with the quartic coupling $y \simeq 1.26 \dots$

The $\chi \rightarrow -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$.

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$$

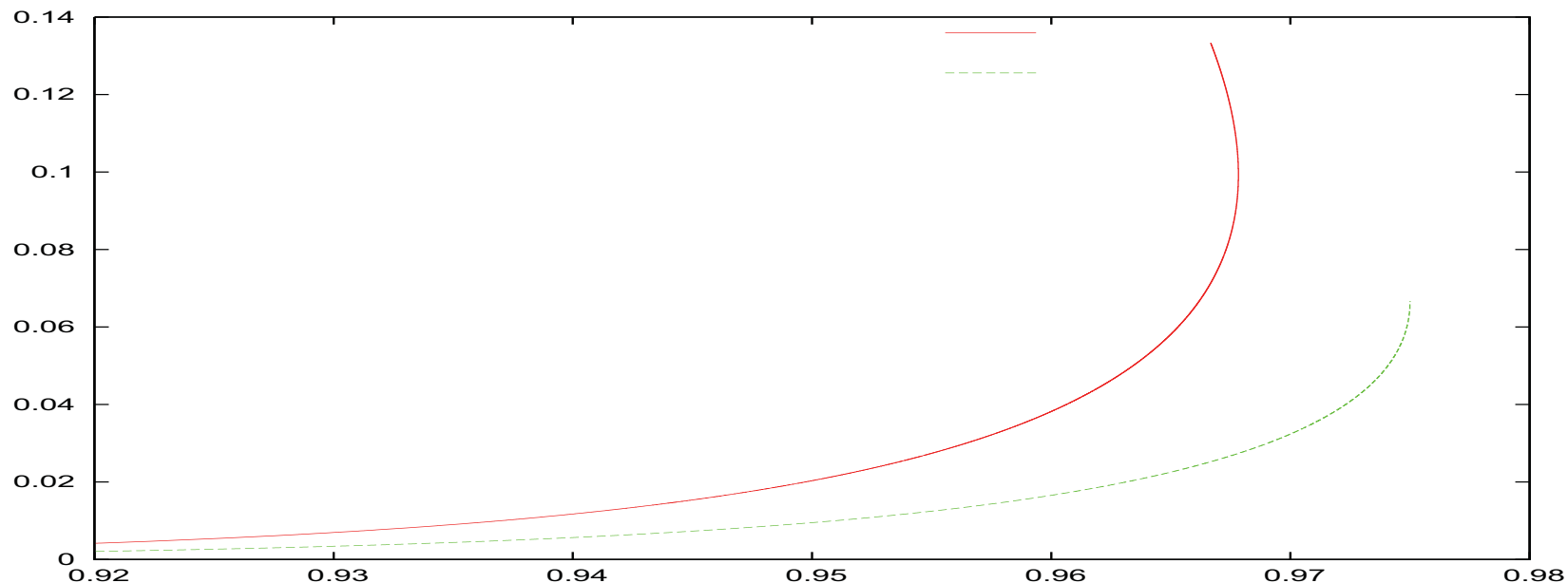
C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.

Similar results from WMAP5 data. Acbar08 data slightly increases $n_s < 1$ and r .

Higher Order Inflaton Potentials

Can higher order terms modify the physical results and the observable predictions?

We find that all $r = r(n_s)$ curves for double-well inflaton potentials in the Ginsburg-Landau spirit fall **inside** the **universal** banana region \mathcal{B} .



This gives a **lower bound** for r for **all** potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

The Energy Scale of Inflation

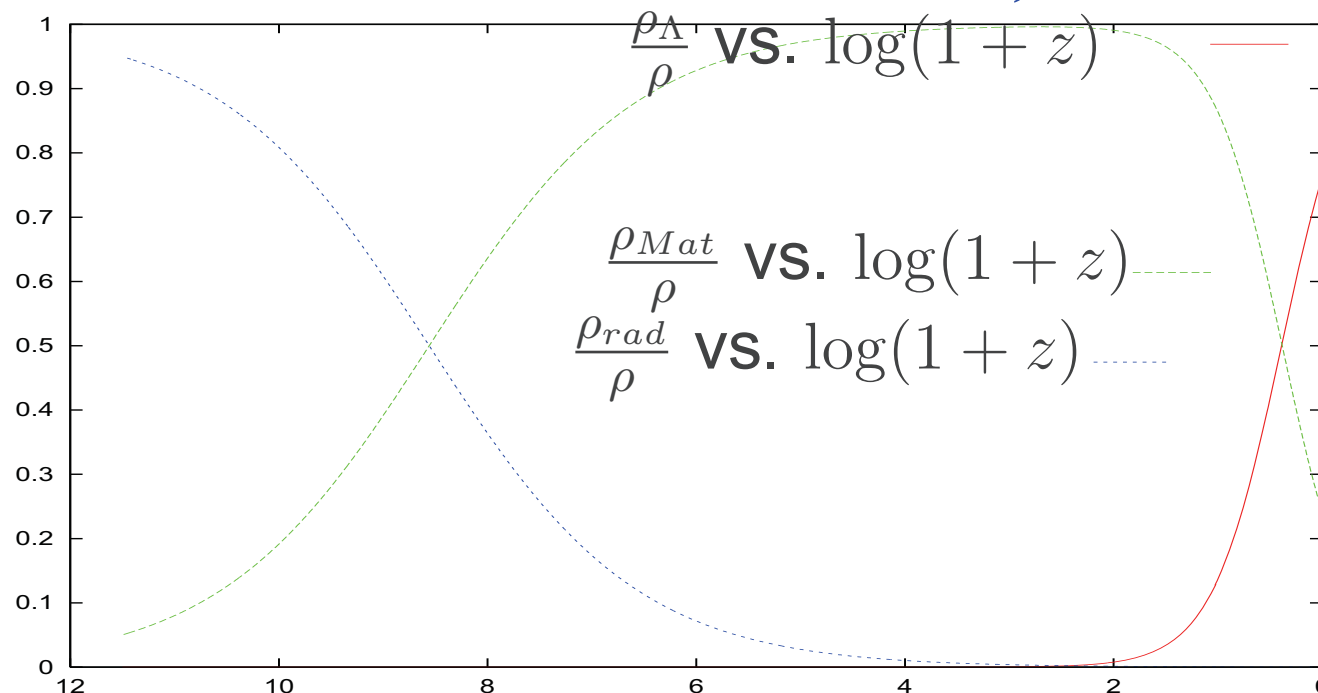
Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they **all meet** at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the **see-saw** mechanism: $m_\nu \sim \frac{M_{\text{Fermi}}^2}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least **three** independent ways.

Moreover, moduli potentials: $V_{moduli} = M_{\text{SUSY}}^4 v \left(\frac{\phi}{M_{Pl}} \right)$
resemble inflation potentials provided $M_{\text{SUSY}} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

The Universe is made of radiation, matter and dark energy



End of inflation: $z \sim 10^{29}$, $T_{reh} \lesssim 10^{16}$ GeV, $t \sim 10^{-36}$ sec.

E-W phase transition: $z \sim 10^{15}$, $T_{EW} \sim 100$ GeV, $t \sim 10^{-11}$ s.

QCD conf. transition: $z \sim 10^{12}$, $T_{QCD} \sim 170$ MeV, $t \sim 10^{-5}$ s.

BBN: $z \sim 10^9$, $T \simeq 0.1$ MeV, $t \sim 20$ sec.

Rad-Mat equality: $z \simeq 3050$, $T \simeq 0.7$ eV, $t \sim 57000$ yr.

CMB last scattering: $z \simeq 1100$, $T \simeq 0.25$ eV, $t \sim 370000$ yr.

Mat-DE equality: $z \simeq 0.47$, $T \simeq 0.345$ meV, $t \sim 8.9$ Gyr.

Today: $z = 0$, $T = 2.725$ K = 0.2348 meV, $t = 13.72$ Gyr.

Dark Matter

DM must be **non-relativistic** by structure formation ($z < 30$) in order to reproduce the observed small structures at $\sim 2 - 3$ kpc. DM particles can decouple being **ultrarelativistic** (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$. Consider particles that decouple **at or out** of LTE (LTE = local thermal equilibrium).

Distribution function:

$f_d[a(t) P_f(t)] = f_d[p_c]$ **freezes out** at decoupling.

$P_f(t) = p_c/a(t)$ = Physical momentum.

p_c = comoving momentum.

Velocity fluctuations:

$$y = P_f(t)/T_d(t) = p_c/T_d$$

$$\langle \vec{V}^2(t) \rangle = \left\langle \frac{\vec{P}_f^2(t)}{m^2} \right\rangle = \frac{\int \frac{d^3 P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2} f_d[a(t) P_f]}{\int \frac{d^3 P_f}{(2\pi)^3} f_d[a(t) P_f]} = \left[\frac{T_d}{m a(t)} \right]^2 \frac{\int_0^\infty y^4 f_d(y) dy}{\int_0^\infty y^2 f_d(y) dy} .$$

The formula for the Mass of the Dark Matter particles

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} f_d[a(t) P_f]$

g : # of internal degrees of freedom of the DM particle,
 $1 \leq g \leq 4$. For $z \lesssim 30 \Rightarrow$ DM particles are non-relativistic:

$$\rho_{DM}(t) = m g \frac{T_d^3}{a^3(t)} \int_0^\infty y^2 f_d(y) \frac{dy}{2\pi^2} .$$

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_\gamma (1 + z_d),$

g_d = effective # of UR degrees of freedom at decoupling,

$$T_\gamma = 0.2348 \text{ meV} , \quad 1 \text{ meV} = 10^{-3} \text{ eV}.$$

Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and we obtain for the **mass** of the DM particle:

$$m = 6.986 \text{ eV} \frac{g_d}{g \int_0^\infty y^2 f_d(y) dy} . \text{ Goal: determine } m \text{ and } g_d$$

Phase-space density invariant under universe expansion

Using again entropy conservation to replace T_d yields for the one-dimensional velocity dispersion,

$$\begin{aligned}\sigma_{DM}(z) &= \sqrt{\frac{1}{3} \langle \vec{V}^2 \rangle(z)} = \frac{2^{\frac{1}{3}}}{\sqrt{3}} \frac{1+z}{g_d^{\frac{1}{3}}} \frac{T_\gamma}{m} \sqrt{\frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy}} = \\ &= 0.05124 \frac{1+z}{g_d^{\frac{1}{3}}} \frac{\text{keV}}{m} \left[\frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy} \right]^{\frac{1}{2}} \frac{\text{km}}{\text{s}}.\end{aligned}$$

Phase-space density: $\mathcal{D} \equiv \frac{n(t)}{\langle \vec{P}_{phys}^2(t) \rangle^{\frac{3}{2}}} \stackrel{\text{non-rel}}{=} \frac{\rho_{DM}}{3 \sqrt{3} m^4 \sigma_{DM}^3}$

\mathcal{D} is computed **theoretically** from freezed-out distributions:

$$\mathcal{D} = \frac{g}{2 \pi^2} \frac{\left[\int_0^\infty y^2 F_d(y) dy \right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 F_d(y) dy \right]^{\frac{3}{2}}}$$

Theorem: The phase-space density \mathcal{D} can only **decrease** under self-gravity interactions (gravitational clustering) [Lynden-Bell, Tremaine, Henon, 1986].

Phase-space density invariant \mathcal{D}

Observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs) yields

$$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \frac{\text{keV}/\text{cm}^3}{(\text{km/s})^3} = (0.18 \text{ keV})^4 \quad \text{Gilmore et al. 07 and 08.}$$

During structure formation $z \lesssim 30$, \mathcal{D} **reduces** by a factor that we call Z . Since $\mathcal{D} = \rho_{DM}/[3 \sqrt{3} m^4 \sigma_{DM}^3]$,

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \frac{\rho_{DM}}{\sigma_{DM}^3} \quad N\text{-body simulations results: } 1000 > Z > 1.$$

$$\rho_{DM} = 1.107 \times \text{keV}/\text{cm}^3 = \text{average value today.}$$

We thus obtain **general formulas** for m and g_d :

$$m = (27)^{-\frac{1}{8}} \rho_{DM}^{\frac{1}{4}} \sigma_{DM}^{-\frac{3}{4}} = 0.2504 \text{ keV} \left(\frac{Z}{g} \right)^{\frac{1}{4}} \frac{\left[\int_0^\infty y^4 F_d(y) dy \right]^{\frac{3}{8}}}{\left[\int_0^\infty y^2 F_d(y) dy \right]^{\frac{3}{8}}}$$

$$g_d = 35.96 Z^{\frac{1}{4}} g^{\frac{3}{4}} \left[\int_0^\infty y^4 F_d(y) dy \int_0^\infty y^2 F_d(y) dy \right]^{\frac{3}{8}}$$

Mass Estimates of DM particles

Our previous formulas yield for relics decoupling **UR at LTE**:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}} \text{ keV} \begin{cases} 0.568 \\ 0.484 \end{cases}, \quad g_d = g^{\frac{3}{4}} Z^{\frac{1}{4}} \begin{cases} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{cases}.$$

Since $g = 1 - 4$, we see that $g_d > 100 \Rightarrow T_d > 100 \text{ GeV}$.

$1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$.

Example: for DM Majorana fermions ($g = 2$) $m \simeq 0.85 \text{ keV}$.

Sterile neutrinos ν as DM decoupling **out of LTE and UR**.

ν is a singlet Majorana fermion with a Majorana mass m_ν coupled with a Yukawa-type coupling $Y \sim 10^{-8}$ to a real scalar field χ . χ is more strongly coupled to the particles in the Standard Model. [Chikashige, Mohapatra, Peccei (1981), Gelmini, Roncadelli (1981), Schechter, Valle (1982), Shaposhnikov, Tkachev (2006), Boyanovsky (2008)]

DM particles decoupling out of LTE and UR

Distribution function: $F_d^\nu(y) = \tau \frac{g_{\frac{5}{2}}(y)}{\sqrt{y}}$, $g_{\frac{5}{2}}(y) \equiv \sum_{n=1}^{\infty} \frac{e^{-n y}}{n^{\frac{5}{2}}}$

$F_d^\nu(y)$ is enhanced for small y and suppressed for large y compared with Fermi-Dirac. We find for Sterile neutrinos

DM: $m_\nu \sim \left(\frac{Z}{\tau}\right)^{\frac{1}{4}} 0.434 \text{ keV}$, $g_d \sim \tau^{\frac{3}{4}} Z^{\frac{1}{4}} 185$.

Typical coupling range: $0.035 \lesssim \tau \lesssim 0.35$.

Generally speaking, thermalization is reached by the mixing of the particle modes and scattering between particles: the larger momentum modes are populated by a **cascade** towards the ultraviolet akin to a cascade in turbulence.

In case the DM particles decouple not yet being at LTE, their momentum distribution peaks at smaller momenta than at LTE since the UV cascade is not yet completed.

As a final result m can be **reduced** by a factor about ~ 2 and g_d even more. D. Boyanovsky, C. Destri, H. J. de Vega, PRD69,045003(2004), CD, HJdeV, PRD73,025014(2006)

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac{5}{2}} \pi^{\frac{7}{2}}}{45} g_d Y_\infty \left(\frac{T_d}{m}\right)^{\frac{3}{2}} e^{-\frac{p_c^2}{2m T_d}} = \frac{2^{\frac{5}{2}} \pi^{\frac{7}{2}}}{45} \frac{g_d Y_\infty}{x^{\frac{3}{2}}} e^{-\frac{y^2}{2x}}$$

$Y(t) = n(t)/s(t)$, $n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d$, $T_d < m$.

$$Y_\infty = \frac{1}{\pi} \sqrt{\frac{45}{8}} \frac{1}{\sqrt{g_d} T_d \sigma_0 M_{Pl}} \text{ late time limit of Boltzmann.}$$

σ_0 : thermally averaged total annihilation cross-section times the velocity.

From our general equations for m and g_d :

$$m = \frac{45}{4 \pi^2} \frac{\Omega_{DM} \rho_c}{g T_\gamma^3 Y_\infty} = \frac{0.748}{g Y_\infty} \text{ eV} \quad \text{and} \quad m^{\frac{5}{2}} T_d^{\frac{3}{2}} = \frac{45}{2 \pi^2} \frac{1}{g g_d Y_\infty} Z \frac{\rho_s}{\sigma_s^3}$$

Finally:

$$\sqrt{m T_d} = 1.47 \left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \text{ keV}. \quad m = 3.67 \text{ keV} Z^{\frac{1}{3}} \frac{g_d^{\frac{1}{12}}}{\sqrt{g}} \sqrt{\frac{\sigma_0}{\text{pb}}}$$

We used ρ_{DM} today **and** the decrease of the phase space density by a factor Z . $1 \text{ pb} = 10^{-36} \text{ cm}^2 = 0.257 / (10^5 \text{ GeV}^2)$.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d .

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV} \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3$ for $1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV}, \quad T_d < 10.2 \text{ keV}.$$

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.
H. J. de Vega, N. G. Sanchez, arXiv:0901.0922.

Only using ρ_{DM} today (**ignoring** the phase space density information) gives:

$$\sigma_0 = 0.16 \text{ pbarn} \frac{g}{\sqrt{g_d}} \frac{m}{T_d} \quad \text{http://pdg.lbl.gov}$$

Dark Energy

$76 \pm 5\%$ of the **present** energy of the Universe is Dark !

Current observed value:

$$\rho_{\Lambda} = \Omega_{\Lambda} \rho_c = (2.39 \text{ meV})^4, \quad 1 \text{ meV} = 10^{-3} \text{ eV}.$$

Equation of state $p_{\Lambda} = -\rho_{\Lambda}$ within observational errors.

Quantum zero point energy. Renormalized value is finite.

Bosons (fermions) give positive (negative) contributions.

Mass of the lightest particles $\sim 1 \text{ meV}$ is in the right scale.

Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions, majorons...

Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.

Dark energy **can be** a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).

We need to learn the **physics of light particles** ($< 1 \text{ MeV}$), also to understand dark matter !!

Summary and Conclusions

- We formulate inflation as an **effective** field theory in the Ginsburg-Landau spirit with energy scale

$M \sim M_{GUT} \sim 10^{16} \text{ GeV} \ll M_{Pl}$. Inflaton mass **small**:

$m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$. Infrared regime !!

- For all slow-roll models $n_s - 1$ and r are $1/N$, $N \sim 60$.

- MCMC analysis of WMAP+LSS data **plus** this theory input indicates a spontaneously broken inflaton

potential: $w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$, $y \simeq 1.26$.

- Lower Bounds: $r > 0.023$ (95% CL), $r > 0.046$ (68% CL). The most probable values are $r \simeq 0.051$ (\Leftarrow measurable !!) $n_s \simeq 0.964$.

- Model independent analysis of dark matter points to a particle mass at the **keV** scale. T_d may be $> 100 \text{ GeV}$. DM is cold.

Summary and Conclusions 2

- CMB quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The **Golden Age** of Cosmology and Astrophysics continues.

A wealth of data from WMAP (7 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy and Star formation. DM properties from astronomical observations. Better bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

The **Dark** Ages...Reionisation...the 21cm line...

Nature of **Dark** Energy? 76% of the energy of the universe.

Nature of **Dark** Matter? 83% of the matter in the universe.

Light DM particles are **strongly** favoured $m_{DM} \sim \text{keV}$.

Sterile neutrinos? Some **unknown light** particle ??

Need to learn about the **physics of light particles** ($< 1 \text{ MeV}$).

COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION



Planck time: $t \sim 10^{-44}$ sec

$t \sim 10^{-39}$ sec



inflation

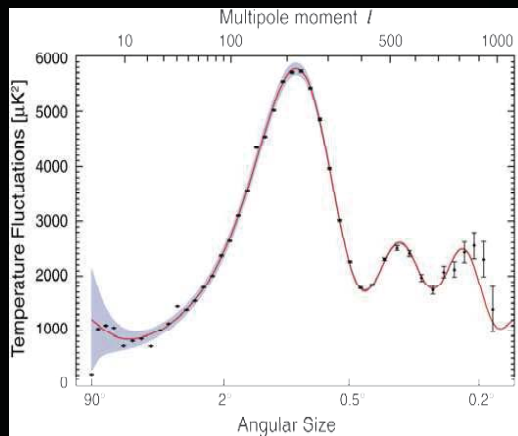
Fast roll inflation produces
the CMB quadrupole
suppression

Fast roll inflation

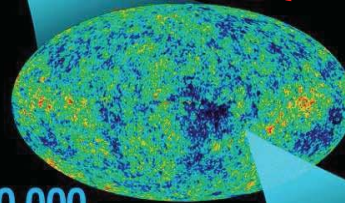
10^{-39} sec $\lesssim t \lesssim 10^{-38}$ sec

Slow roll inflation

10^{-38} sec $\lesssim t \lesssim 10^{-36}$ sec



380,000
years



13.7
billion
years



THANK YOU VERY MUCH
FOR YOUR ATTENTION!!

Little Bang vs. Big Bang

Similarities: baryon free, entropy is dominated by radiation, longitudinal expansion in RHIC and LHC similar to the Hubble expansion.

Differences: **Cosmology:**

Local Thermal Equilibrium: $1/H \sim 10^{-5} \text{ s} \gg t_{QCD} \sim 10^{-23} \text{ s}$,

Starting energy density \gg QCD phase transition energy density $\sim 1 \text{ GeV/fm}^3 \implies$

weakly interacting QGP was initially present due to asymptotic freedom.

URHIC: expansion time scale $\sim 10^{-22} \text{ sec} \sim 10 t_{QCD} \implies$ non-equilibrium effects **can be relevant**.

Strongly interacting *liquid* initially present ('color glass condensate').

Out of equilibrium Decoupling

Thermalization mechanism: k -modes **cascade** towards the UV till the thermal distribution is attained.

D. Boyanovsky, C. Destri, H. J. de Vega, PRD69, 045003 (2004), C. Destri, H. J. de Vega, PRD73, 025014 (2006)

Hence, **before** LTE is reached: **lower** momenta are **more** populated than at LTE.

An approximate description:

$$f_d(y) = f_{equil}(y/\xi) \theta(y_0 - y), \quad \xi < 1 \text{ out of equilibrium}$$

Modes with $p_c > y_0 T_d$ are empty. [$y = p_c/T_d$].

For fermions: $m = 6.99 \text{ eV } (g_d/g) F(\infty)/[\xi^3 F(y_0/\xi)]$

$$F(s) \equiv \int_0^s f_{equil}(w) w^2 dw \quad , \quad F(\infty)/[\xi^3 F(y_0/\xi)] > 1.$$

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} d\chi. \text{ We choose then } N = N[\chi].$$

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$$

produces inflation with $0 < \sqrt{y} \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

This is **small field** inflation.

From the above integral: $y = z - 1 - \log z$

where $z \equiv y \chi^2 / 8$ and we have $0 < y < \infty$ for $1 > z > 0$.

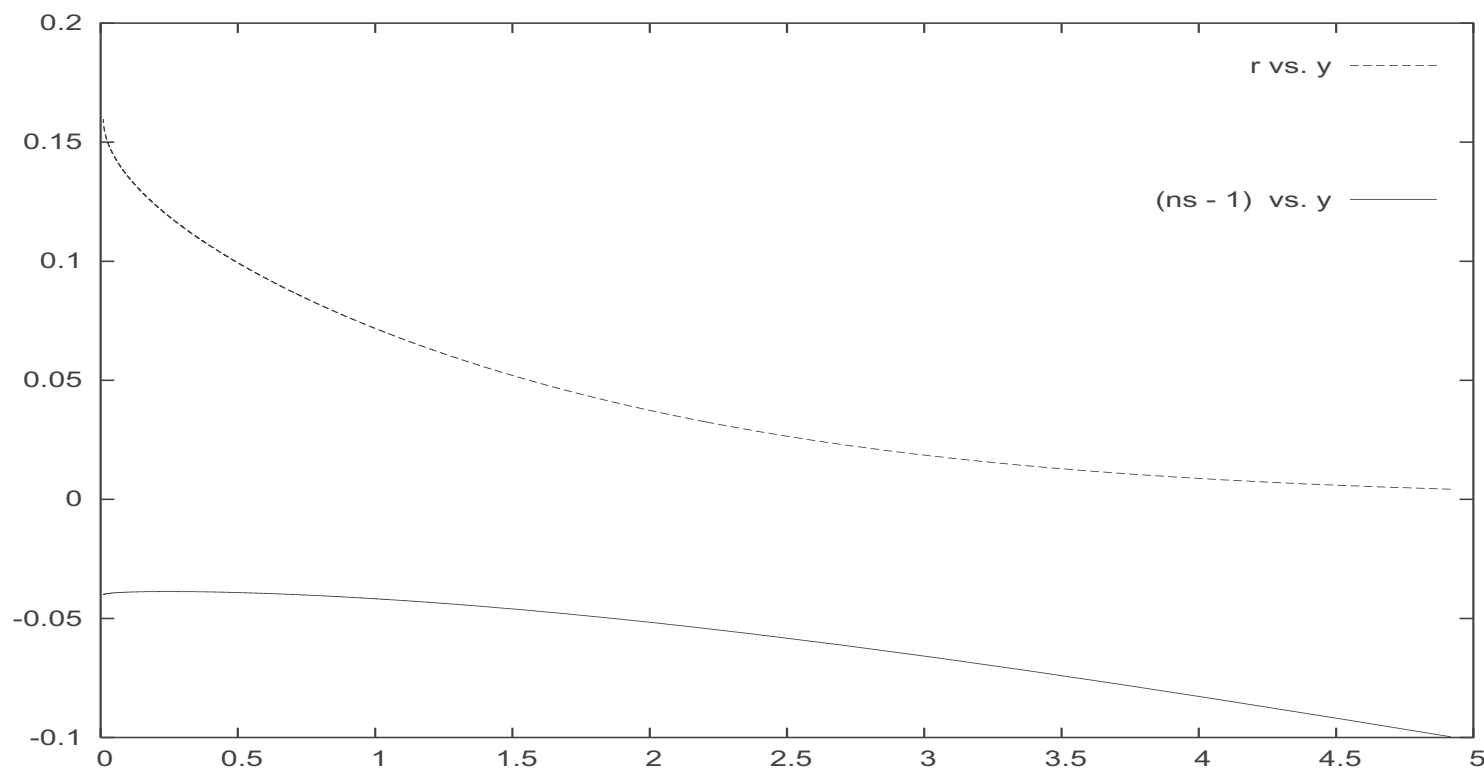
Spectral index n_s and the ratio r as functions of y :

$$n_s = 1 - \frac{y}{N} \frac{3z+1}{(z-1)^2}, \quad r = \frac{16y}{N} \frac{z}{(z-1)^2}$$

Binomial New Inflation: ($y = \text{coupling}$).

r decreases monotonically with y :

(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).

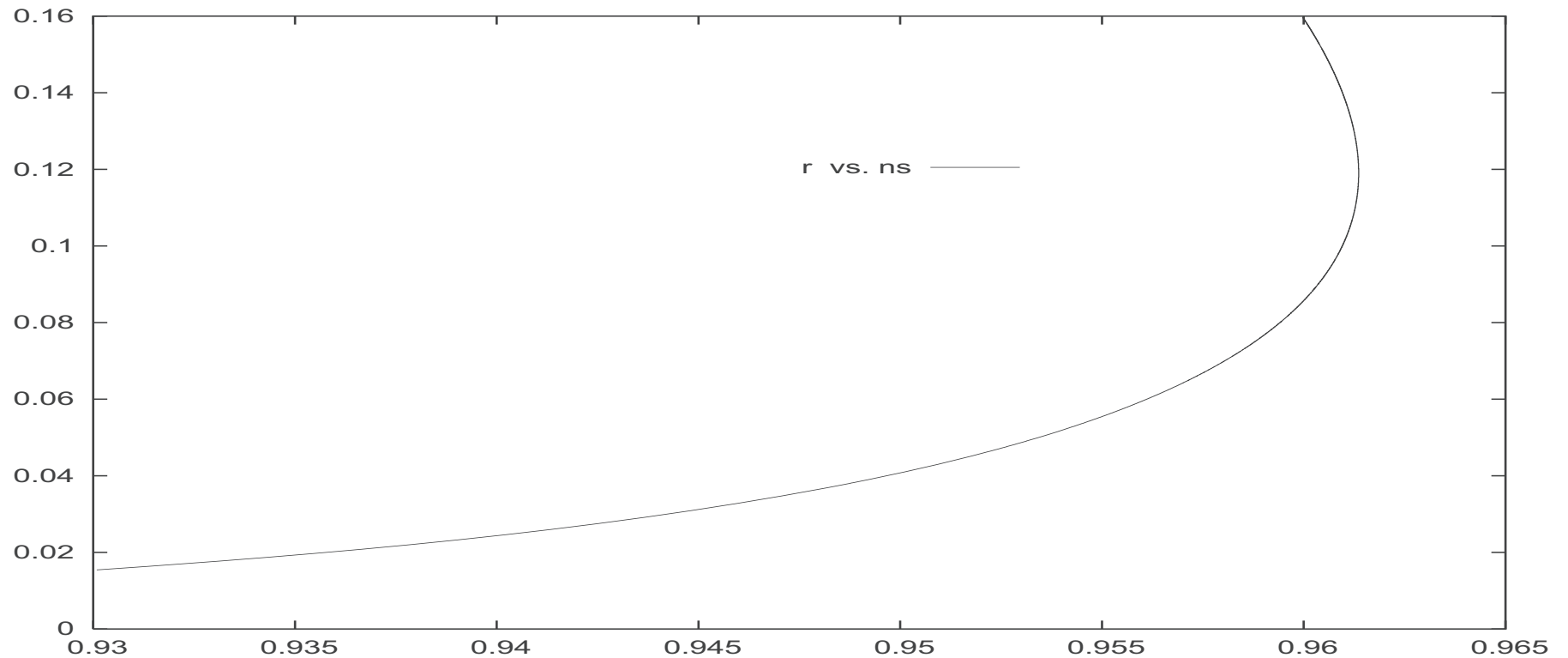


n_s first grows with y , reaches a **maximum value**

$n_{s, \text{maximum}} = 0.96139 \dots$ at $y = 0.2387 \dots$ and then n_s

decreases monotonically with y .

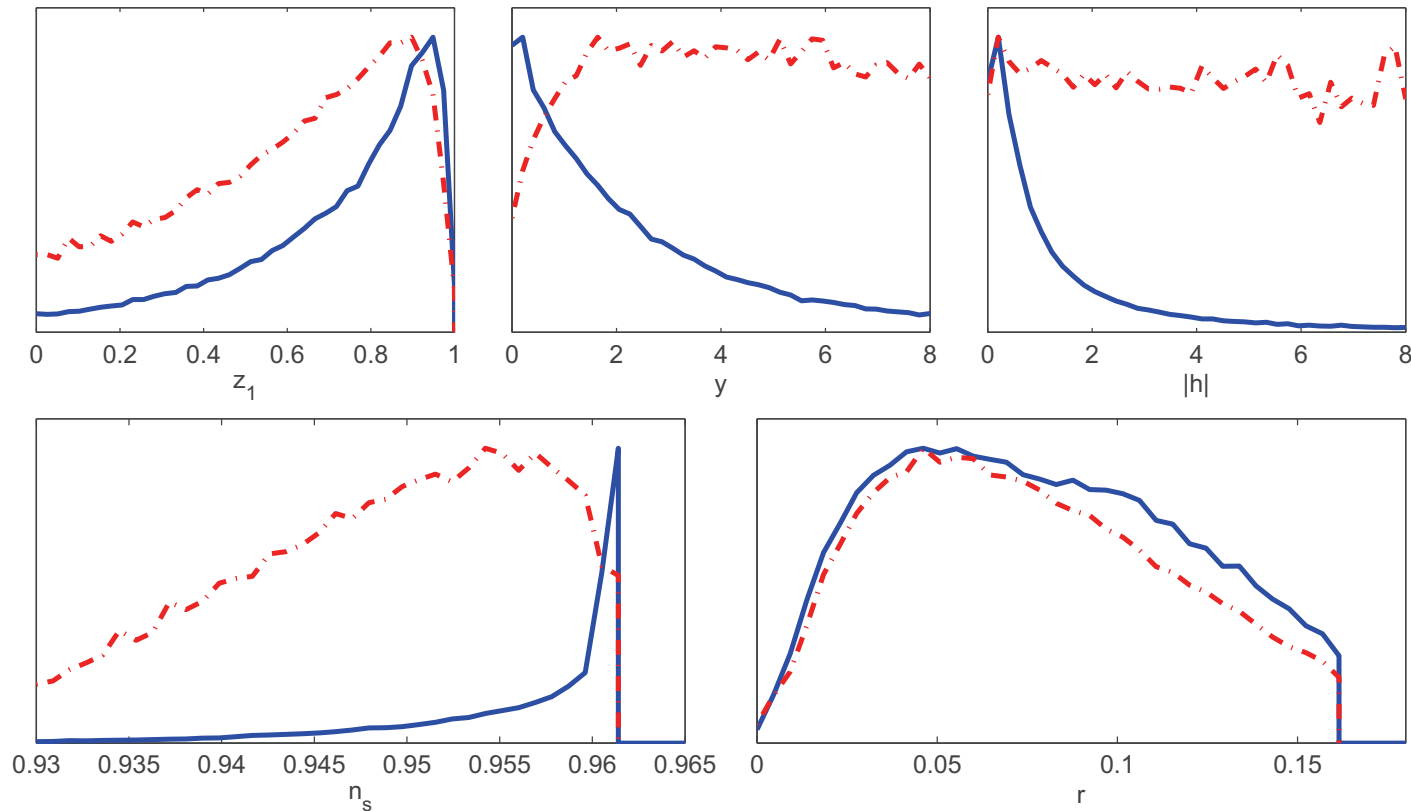
Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a **double valued** function of n_s .

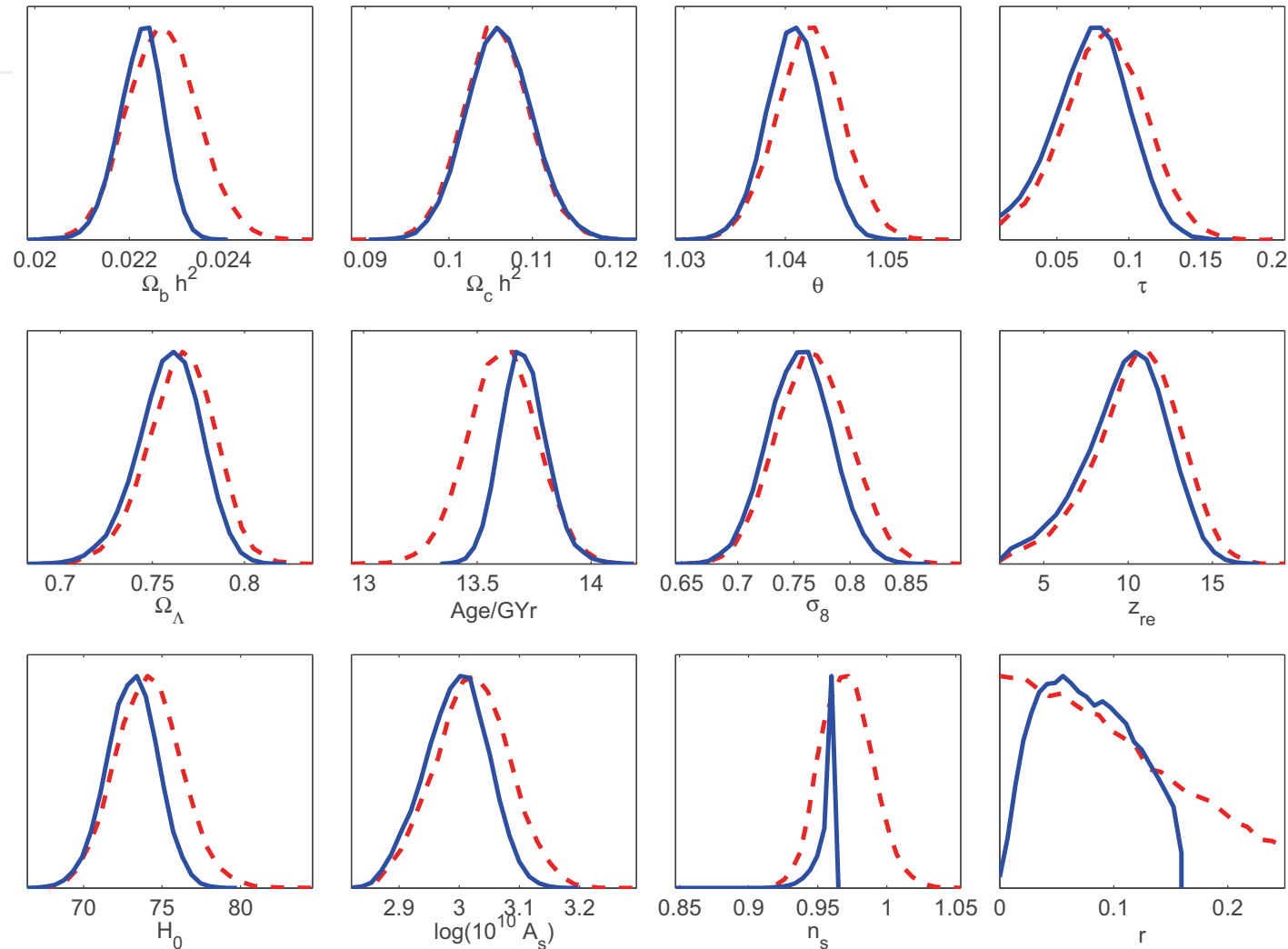
Probability Distributions. Trinomial New Inflation.



Probability distributions: solid blue curves
Mean likelihoods: dot-dashed red curves.

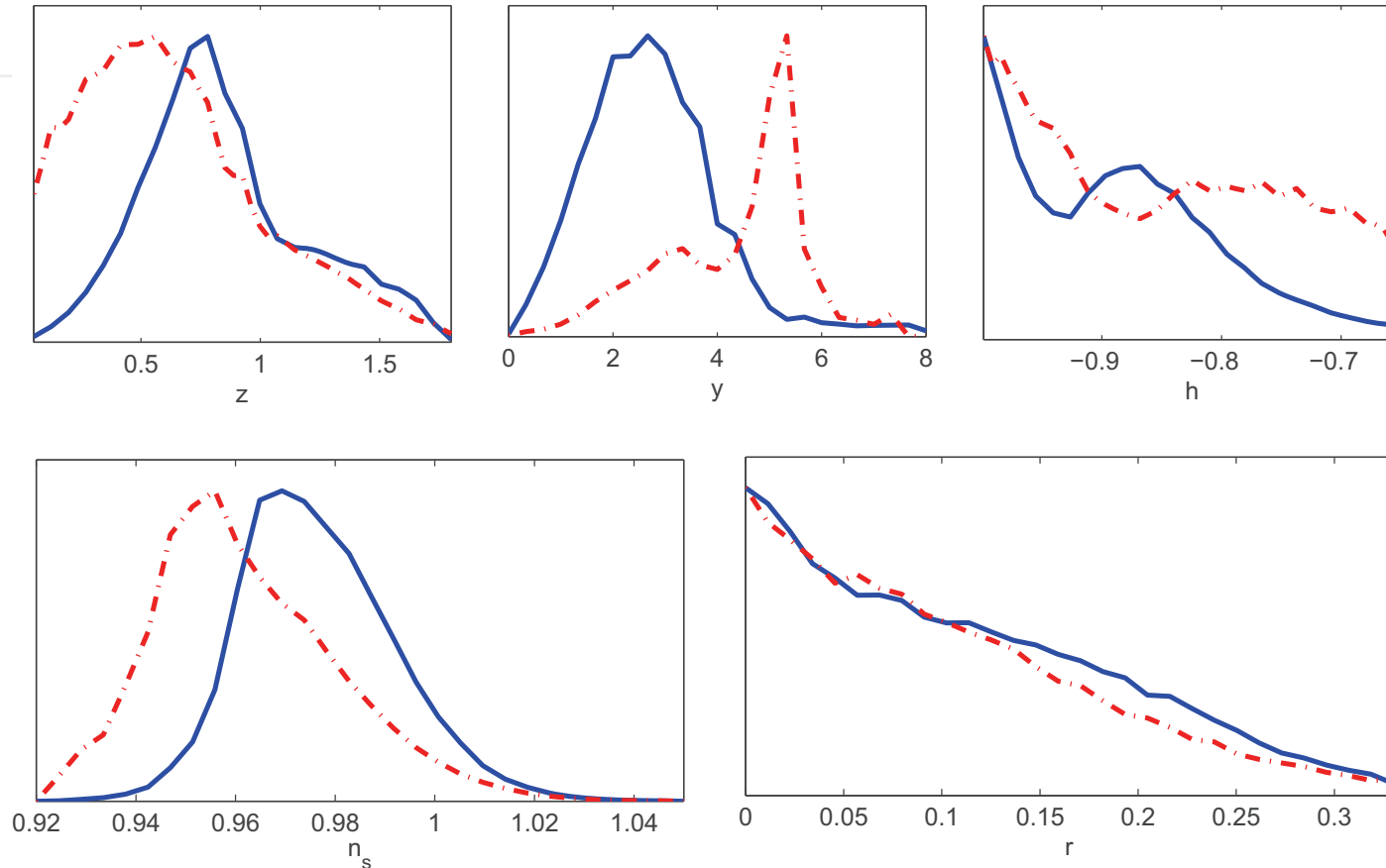
$$z_1 = 1 - \frac{y}{8 (|h| + \sqrt{h^2 + 1})^2} \chi^2 .$$

Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the Λ CDM+ r model (dashed red curves).
(curves normalized to have the maxima equal to one).

Probability Distributions. Trinomial Chaotic Inflation.



Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).

The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \rightarrow -\chi$ symmetry.

The QCD Phase Transition

Hubble scale then: $1/H \sim 10^{-5}$ sec

$\gg t_{QCD} \sim 1/[\alpha_s^2 T_{QCD}] \sim 10^{-23}$ sec \Rightarrow **Very fast!**.

Hence, the transition happens in **thermal equilibrium**.

Hubble radius then $c/H \sim 10$ km. \Rightarrow 1 pc today.

Probably a first order transition: Bubbles of the confined phase appear in the quark-gluon plasma \Rightarrow hadronization.

Supercooling very short $\sim 10^{-3} 1/H$.

Bubble separation $1 \text{ cm} \sim 10^{-6} c/H$.

Bubbles grow slowly and eventually fill the whole space.

Latent heat of the transition reheats the universe.

BBN happens at 200 secs $\gg 10^{-5}$ sec: signatures **ERASED**.

D. Boyanovsky, H. J. de Vega, D. J. Schwarz, Ann. Rev. Nucl. Part. Sci. 56, 441(2006), hep-ph/0602002.

Primordial Magnetic Fields

Astrophysical observations show the presence of large scale magnetic fields $\sim \mu G$ correlated on scales up to ~ 1 Mpc (cluster of galaxies).

Origin?: Dynamo mechanisms amplify seed magnetic fields. Typical growth rates $\Gamma \sim \text{Gyr}^{-1}$ over time scales $\sim 10 - 12$ Gyr

Origin of Seeds?: Inflation and/or phase transitions. If the electroweak and/or the chiral phase transitions occurred **out of equilibrium** they can be a significant source of primordial magnetic fields

Cosmic magnetic fields may be one of the **few observational relics** of primordial phase transitions.

D. Boyanovsky, H. J. de Vega, M. Simionato 'Large scale magnetogenesis from a non-equilibrium phase transition in the radiation dominated era', Phys.Rev.D67,123505(2003).

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x}, t) = \Phi_0(t) + \varphi(\vec{x}, t), \quad \Phi_0(t) \equiv \langle \phi(\vec{x}, t) \rangle, \quad \langle \varphi(\vec{x}, t) \rangle = 0$$

$$\varphi(\vec{x}, t) = \frac{1}{a(\eta)} \int \frac{d^3 k}{(2\pi)^3} \left[a_{\vec{k}} \chi_k(\eta) e^{i\vec{k} \cdot \vec{x}} + \text{h.c.} \right],$$

$a_{\vec{k}}^\dagger, a_{\vec{k}}$ are creation/annihilation operators,

$\chi_k(\eta)$ are mode functions. η = conformal time.

To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3H \dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0) \langle [\varphi(\mathbf{x}, t)]^2 \rangle = 0$$

where $g(\Phi_0) = \frac{1}{2} V'''(\Phi_0)$.

The mode functions obey:

$$\chi_k''(\eta) + \left[k^2 + M^2(\Phi_0) a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3 H_0^2 \eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi_k''(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} \right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0 \eta (1 - \epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$$\chi_k(\eta) = \frac{1}{2} \sqrt{-\pi\eta} i^{\nu+\frac{1}{2}} H_\nu^{(1)}(-k\eta), \quad H_\nu^{(1)}(z) = \text{Hankel function}$$

Quantum fluctuations: $\langle [\varphi(\mathbf{x}, t)]^2 \rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3} |\chi_k(\eta)|^2$

$$\frac{1}{2} \langle [\varphi(\mathbf{x}, t)]^2 \rangle = \left(\frac{H_0}{4\pi} \right)^2 \left[\Lambda_p^2 + \ln \Lambda_p^2 + \frac{1}{\Delta} + 2\gamma - 4 + \mathcal{O}(\Delta) \right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H$, $\frac{1}{\Delta} = \text{infrared pole}$.

$\langle \dot{\varphi}^2 \rangle$, $\langle (\nabla \varphi)^2 \rangle$ are **infrared finite**

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3 M_{Pl}^2} \left[\frac{1}{2} \dot{\Phi}_0^2 + V_R(\Phi_0) + \left(\frac{H_0}{4\pi} \right)^2 \frac{V_R''(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right) \right]$$

Quantum corrections are **proportional** to $\left(\frac{H}{M_{Pl}} \right)^2 \sim 10^{-9} !!$

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\pi} \right)^2 \frac{V_R''(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0) \left[1 + \left(\frac{H_0}{4\pi M_{Pl}} \right)^2 \frac{\eta_V}{\eta_V - \epsilon_V} \right]$$

in terms of slow-roll parameters

Very **DIFFERENT** from the one-loop effective potential in **Minkowski** space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R''(\Phi_0)]^2}{64\pi^2} \ln \frac{V_R''(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations **contribute** to the inflaton potential **and** to the primordial power spectra.

In de Sitter space-time: $\langle T_{\mu\nu} \rangle = \frac{1}{4} g_{\mu\nu} \langle T^\alpha_\alpha \rangle$

Hence, $V_{eff} = V_R + \langle T^0_0 \rangle = V_R + \frac{1}{4} \langle T^\alpha_\alpha \rangle$

Sub-horizon (Ultraviolet) contributions appear through the **trace anomaly** and only depend on the spin of the particle.

Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the **slow-roll parameters**.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3(4\pi)^2 M_{Pl}^2} \left(\frac{\eta_v - 4\epsilon_v}{\eta_v - 3\epsilon_v} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \quad \mathcal{T}_t = -\frac{717}{5}, \quad \mathcal{T}_F = \frac{11}{60}$$

→ the **graviton** (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta_{k,eff}^{(S)}|^2 = |\Delta_k^{(S)}|^2 \left\{ 1 + \frac{2}{3} \left(\frac{H_0}{4 \pi M_{Pl}} \right)^2 \left[1 + \frac{\frac{3}{8} r (n_s - 1) + 2 \frac{dn_s}{d \ln k}}{(n_s - 1)^2} + \frac{2903}{40} \right] \right\}$$
$$|\Delta_{k,eff}^{(T)}|^2 = |\Delta_k^{(T)}|^2 \left\{ 1 - \frac{1}{3} \left(\frac{H_0}{4 \pi M_{Pl}} \right)^2 \left[-1 + \frac{1}{8} \frac{r}{n_s - 1} + \frac{2903}{20} \right] \right\}.$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ **DOMINATES** as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta_k^{(S)}|^2$ are **ENHANCED** and the tensor fluctuations $|\Delta_k^{(T)}|^2$ **REDUCED**.

However, $\left(\frac{H}{M_{Pl}} \right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.