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Multi parton interactions in high energy hadronic collisions

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- Experimental evidence

- A functional approach. Cancellation of unitarity corrections

- Inclusive and "exclusive" cross sections

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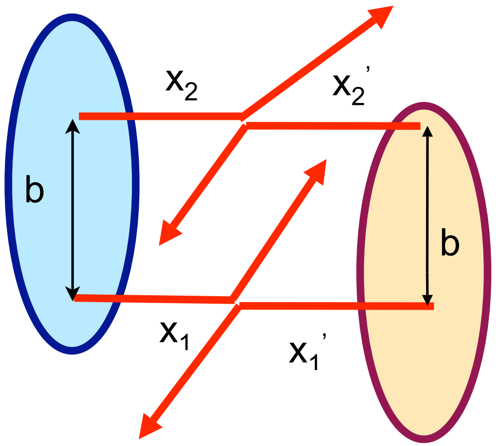
Experimental evidence

There have been only four dedicated measurements studying double parton scattering: the AFS experiment in proton-proton collisions at 63 GeV c.m. energy, UA2 in proton-antiproton collisions at 630 GeV c.m. energy and twice by CDF in proton-antiproton collisions at 1.8 TeV

The four jets final states have been used in the first three measurements and the photon + 3 jets in the last CDF one to extract the value of the effective cross section, which is defined by the integral



$$\frac{1}{\sigma_{eff}} = \int F(b)^2 d^2 b$$

where $F(b)$ is the density of parton pairs, normalized to one, with relative transverse distance b in the hadron

$$\sigma_D = \frac{\sigma_{jj}^2}{2\sigma_{eff}}, \quad 4 \text{ jets}$$

$$\sigma_D = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{eff}}, \quad \gamma + \text{ jets}$$

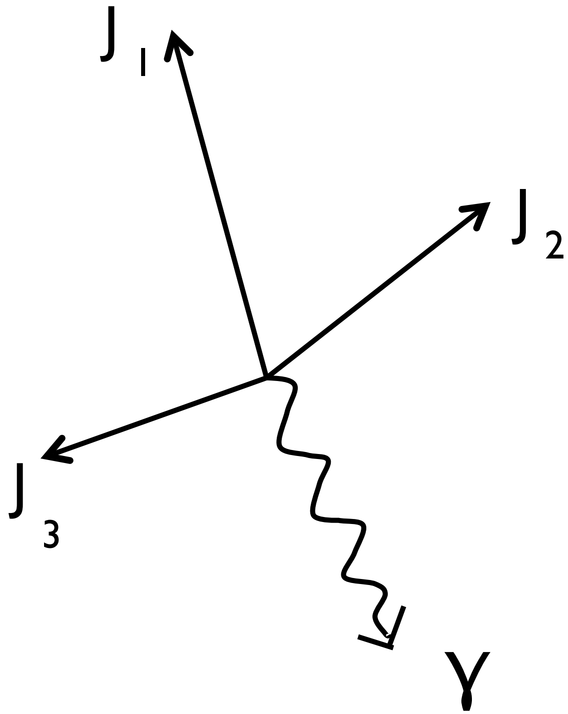
$$\sigma_{eff} \approx 5\text{mb} \quad (\text{AFS}) \quad 4 \text{ jets}$$

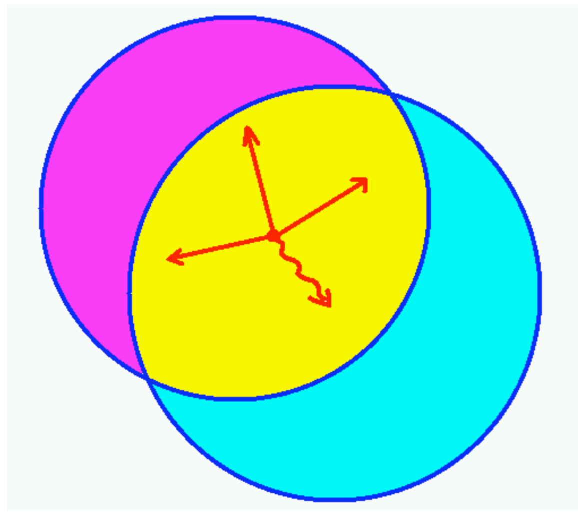
$$\sigma_{eff} > 8.3\text{mb} \quad (\text{UA2}) \quad 4 \text{ jets}$$

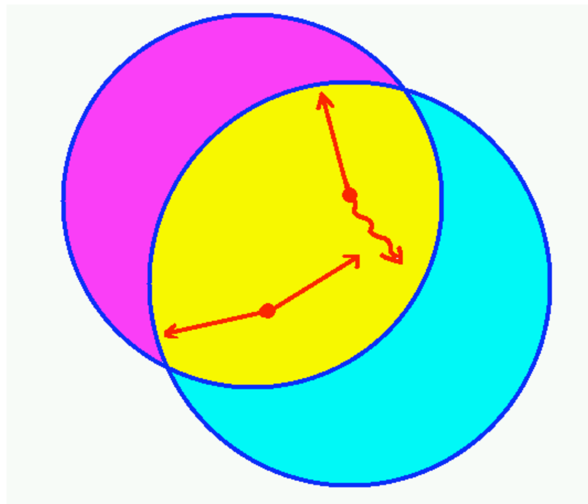
$$\sigma_{eff} = 12.1^{+10.7}_{-5.4}\text{mb} \quad (\text{CDF}) \quad 4 \text{ jets}$$

$$\sigma_{eff} = 14.1 \pm 1.7^{+1.7}_{-2.3}\text{mb} \quad (\text{CDF}) \quad \gamma + \text{jets}$$

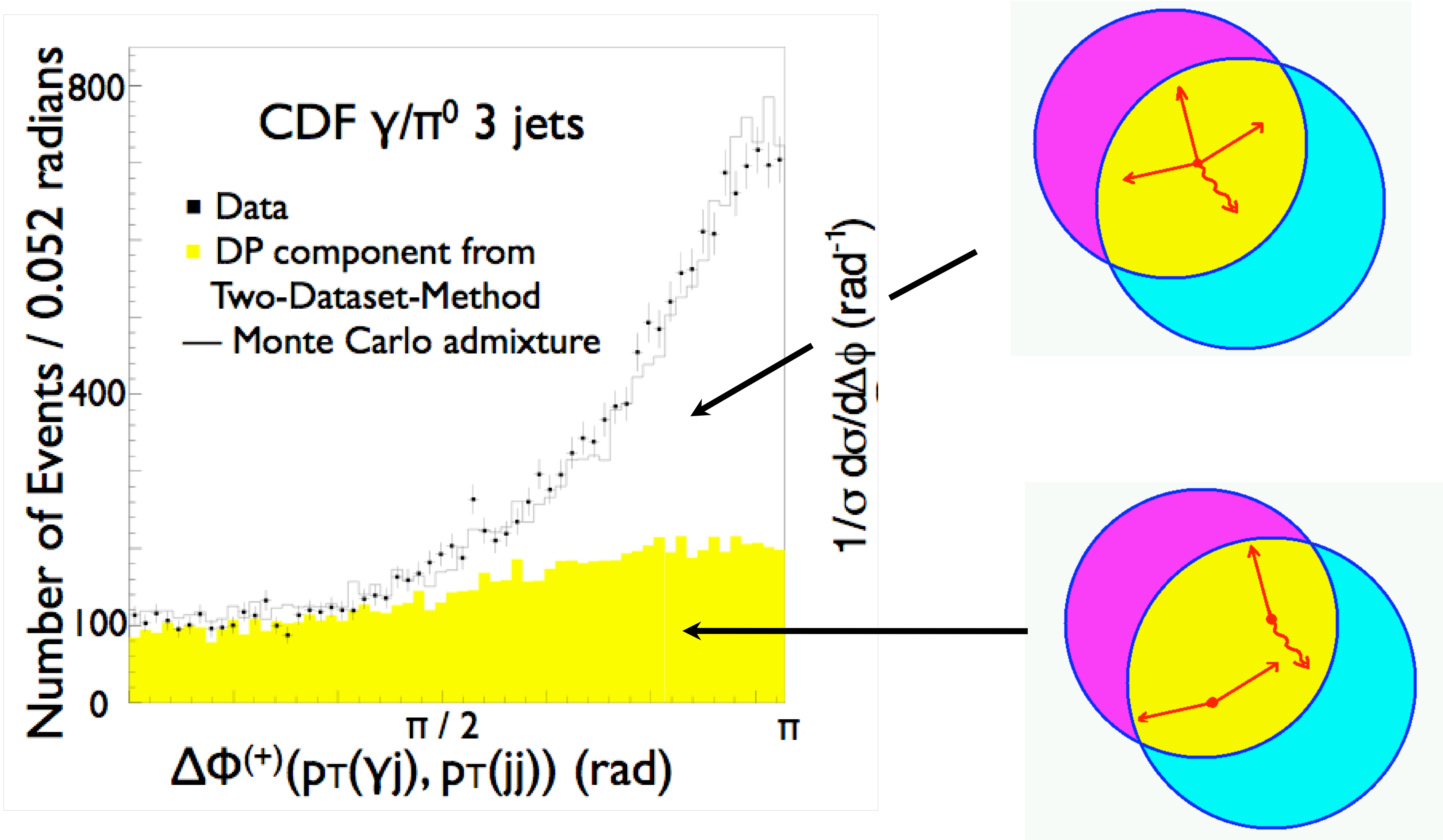
J_1 is the jet which carries most of the recoil of the photon transverse momentum. $J_2 J_3$ have decreasing transverse momenta



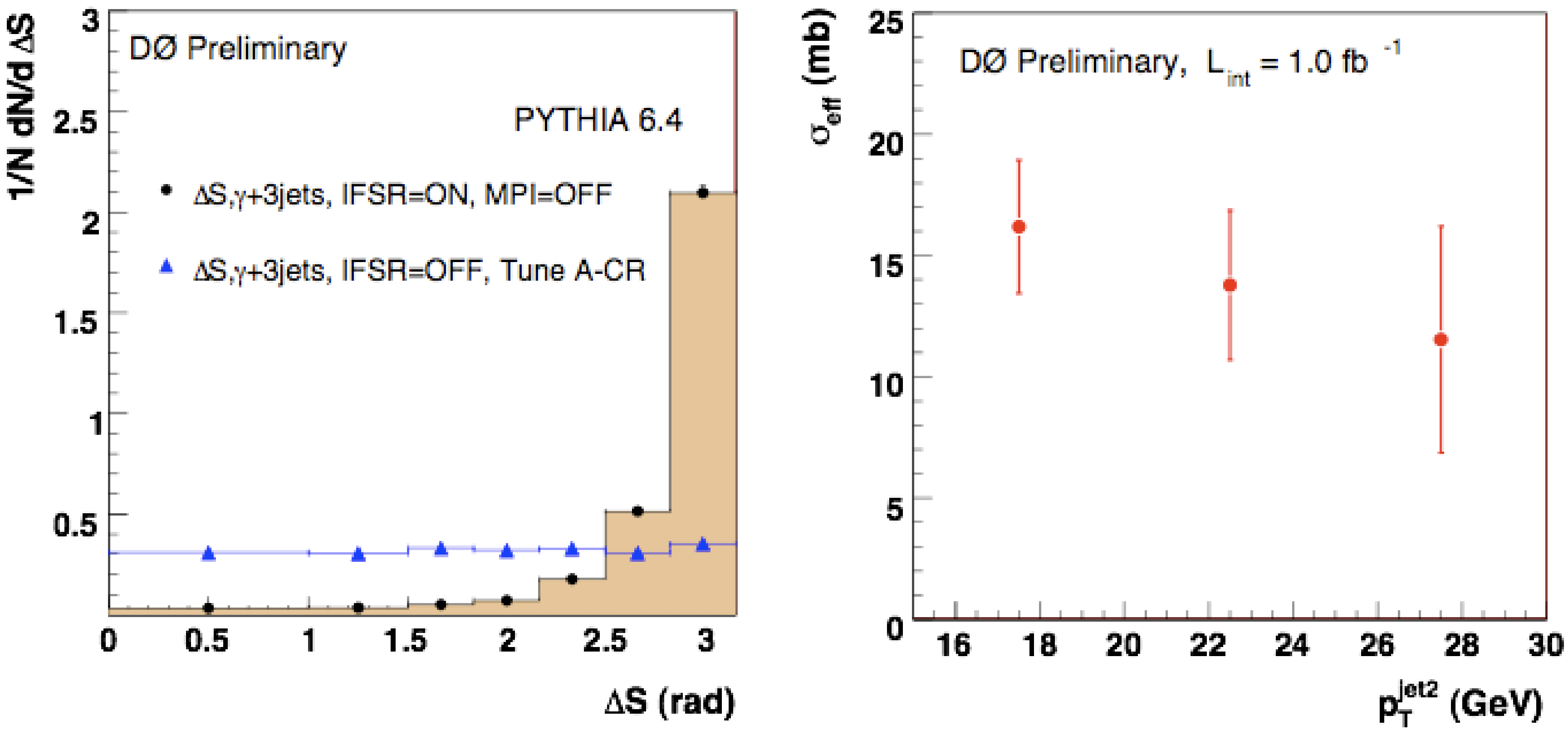




In the single scattering contribution the angle between γ and J_2 is mainly $>\pi/2$,
the double scattering contribution is on the contrary flat as a function of the angle between γ and J_2


CDF γ/π⁰ 3 jets
Data
DP component from Two-Dataset-Method
Monte Carlo admixture
Number of Events / 0.052 radians
800
400
100
0
π / 2
π
ΔΦ(+)(pT(γj), pT(jj)) (rad)
1/σ dσ/dΔφ (rad-1)

Very recently the CDF measurement has been repeated by D0 with a c.m. energy of 1.96 TeV



A functional approach to MPI

One may introduce the ***exclusive*** n-body parton ***distributions***

$$\boxed{W_n(u_1 \dots u_n)} \; , \quad \boxed{u_i \equiv (\mathbf{b}_i, x_i)}$$

which represents the probability to find the hadron in a configuration with *n* partons with coordinated $u_1 \dots u_n$, where $\mathbf{b}_i$ are the transverse parton coordinates and x_i the fractional momenta. The multi-parton generating functional is:

$$\boxed{\mathcal{Z}[J] = \sum_n \frac{1}{n!} \int J(u_1) \dots J(u_n) W_n(u_1 \dots u_n) du_1 \dots du_n,}$$

Probability conservation imposes the normalization condition $\boxed{\mathcal{Z}[1] = 1.}$

While the exclusive distributions are the coefficients of the expansion of $\mathcal{Z}$ around 0, the many-body densities, i.e. the ***inclusive distributions*** $\boxed{D_n(u_1 \dots u_n)}$ are the coefficients of the expansion of $\mathcal{Z}$ around 1:

$$\begin{aligned} D_1(u) &= \left.\frac{\partial \mathcal{Z}}{\partial J(u)}\right|_{J=1}, \\ D_2(u_1, u_2) &= \left.\frac{\partial^2 \mathcal{Z}}{\partial J(u_1) \partial J(u_2)}\right|_{J=1}, \\ &\dots \end{aligned}$$

One may introduce the logarithm of the generating functional $\boxed{\mathcal{F}[J] = \ln(\mathcal{Z}[J])}$

and, by expanding in the vicinity of J=1, one obtains the many-body ***parton correlations***

$$\mathcal{F}[J] = \int D(u)[J(u) - 1]du + \sum_{n=2}^{\infty} \frac{1}{n!} \int C_n(u_1 \ldots u_n) \big[J(u_1) - 1\big] \ldots \\ \ldots \big[J(u_n) - 1\big] du_1 \ldots du_n$$

A rather general expression for $\sigma_H = \int d^2\beta \sigma_H(\beta)$, where $\sigma_{inel} = \sigma_H + \sigma_{soft}$

is the following

Prob. to find the hadron *B* in a configuration with *m* partons

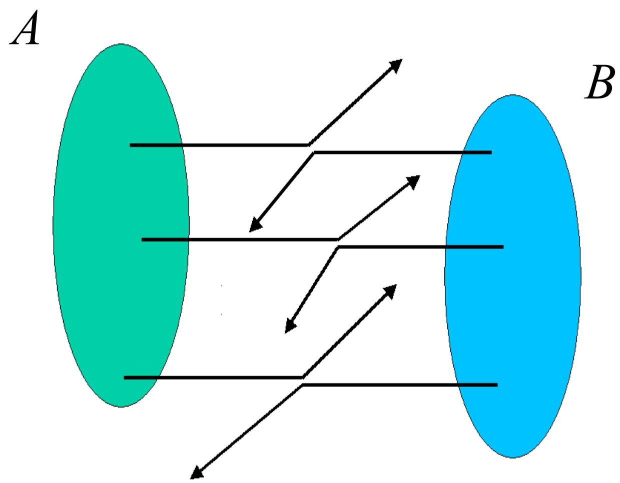
$$\sigma_H = \int d\beta \int \sum_n \frac{1}{n!} \frac{\partial}{\partial J(u_1)} \cdots \frac{\partial}{\partial J(u_n)} \mathcal{Z}_A[J] \quad \times \quad \sum_m \frac{1}{m!} \frac{\partial}{\partial J'(u'_1 - \beta)} \cdots \frac{\partial}{\partial J'(u'_m - \beta)} \mathcal{Z}_B[J'] \\ \times \quad \left\{1 - \prod_{i=1}^{n} \prod_{j=1}^{m} [1 - \hat{\sigma}_{i,j}(u, u')]\right\} \prod du du' \Big|_{J=J'=0}$$

Prob. to find the hadron *A* in a configuration with *n* partons

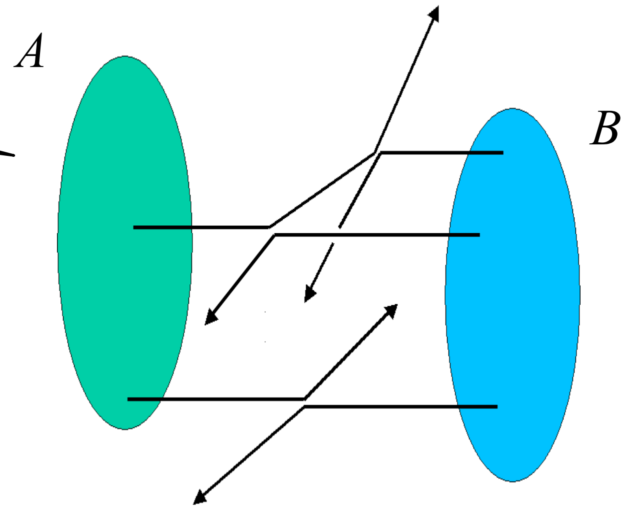
Prob. to have at least one interaction between the two configurations

This expression for σ_H includes all possible interaction with on-shell intermediate states, between any configuration with *n* partons of hadron *A* and any configuration with *m* partons of hadron *B*. ***The cross section complies with the AGK cancellation***, namely <N> $\sigma_H = \sigma_S$

Both disconnected collisions



and rescatterings



are included in σ_H

A simpler expression is obtained when keeping ***only disconneced collisions*** into account, which amounts removing all addenda with repeated indices in the interaction probability

$$\boxed{\left\{1-\prod_{i,j}^{n,m}[1-\hat{\sigma}_{ij}]\right\} \Rightarrow \sum_{ij}\hat{\sigma}_{ij}-\frac{1}{2!}\sum_{ij}\sum_{k\neq i,l\neq j}\hat{\sigma}_{ij}\hat{\sigma}_{kl}+\ldots}$$

Because of the symmetry of the derivative operators, the expression may be replaced by

$$\boxed{nm\hat{\sigma}_{11}-\frac{1}{2!}n(n-1)m(m-1)\hat{\sigma}_{11}\hat{\sigma}_{22}+\ldots}$$

The sums can be performed and the cross section is expressed in a ***closed analytic form***

$$\boxed{\sigma_H(\beta)=\Big[1-\exp\big(-\partial\cdot\hat{\sigma}\cdot\partial'\big)\Big]\mathcal{Z}_A[J]\mathcal{Z}_B[J']\Big|_{J=J'=1}}$$

where ***all disconnected collisions are included, while multi-parton correlations are kept into account at all orders***.

one may express the hard cross section as a sum of MPI

$$
\begin{aligned}
\sigma_H(\beta) &= \Big[1 - \exp\big(-\partial \cdot \hat{\sigma} \cdot \partial'\big)\Big] \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \,\Big|_{J=J'=1} \\
&= \sum_{N=1}^{\infty} \frac{\big(\partial \cdot \hat{\sigma} \cdot \partial'\big)^N}{N!} \, e^{-\partial \cdot \hat{\sigma} \cdot \partial'} \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \,\Big|_{J=J'=1}
\end{aligned}
$$

The average number of collisions is hence given by the single scattering inclusive cross section

$$
\begin{aligned}
\langle N \rangle \sigma_H(\beta) &= \sum_{N=1}^{\infty} \frac{N\big(\partial \cdot \hat{\sigma} \cdot \partial'\big)^N}{N!} \, e^{-\partial \cdot \hat{\sigma} \cdot \partial'} \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \,\Big|_{J=J'=1} \\
&= \partial_{J_1} \cdot \hat{\sigma} \cdot \partial_{J'_1} \sum_{N=0}^{\infty} \frac{\big(\partial \cdot \hat{\sigma} \cdot \partial'\big)^N}{N!} \, e^{-\partial \cdot \hat{\sigma} \cdot \partial'} \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \,\Big|_{J=J'=1} \\
&= \big(\partial_{J_1} \cdot \hat{\sigma} \cdot \partial_{J'_1}\big) \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \,\Big|_{J=J'=1} \\
&= \int D_A(x_1; b_1)\hat{\sigma}(x_1 x'_1) D_B(x'_1; b_1 - \beta) dx_1 dx'_1 d^2 b_1 \equiv \sigma_S(\beta)
\end{aligned}
$$

which is the cancellation of AGK

In the case of disconnected collisions, one analogously shows the validity the relations:

$$\boxed{\langle N\rangle\sigma_H=\sigma_S \quad \text{and} \quad \frac{1}{2}\langle N(N-1)\rangle\sigma_H=\sigma_D}$$

and more in general

$$\boxed{\frac{1}{K!}\langle N(N-1)\dots(N-K+1)\rangle\sigma_H=\sigma_K}$$

Notice that the relations above hold also when taking into account the effect of multi-parton correlations

In the case of of disconnected collisions all ***unitarity corrections cancel in the inclusive cross section***, which is given by the single scattering term expression (AGK cancellation).

The ***inclusive cross section*** is in fact given by the ***average number*** of partonic collision. The ***K-partons inclusive cross section*** is analogously given by the ***Kth moment of the distribution*** in the number of partonic collisions.

When only disconnected collisions are included in the interaction, the inclusive cross sections give the moments of the distribution in the number of hard collisions and are not affected by unitarity corrections. The statement holds for any correlation in the hadron sturcture.

Correlations and σ_H

Keeping into account disconnected multi-parton scatterings only and ***limiting correlations to the two-body case***, one may write a gaussian functional integral for the cross section, which ***allows obtaining a closed expression for σ_H***

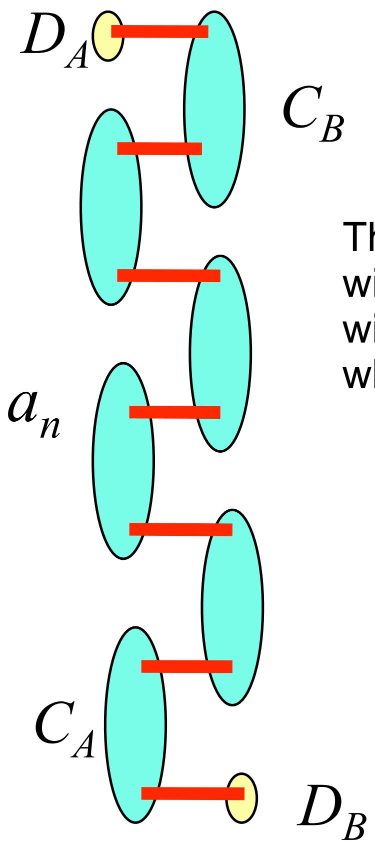
$$\mathcal{F}_{A,B}[J+1] = \int D_{A,B}(u)J(u)du + \frac{1}{2}\int C_{A,B}(u,v)J(u)J(v)dudv$$

D(u) is the average number of partons and *C(u,v)* the two-body correlation. The final expression obtained for σ_H is :

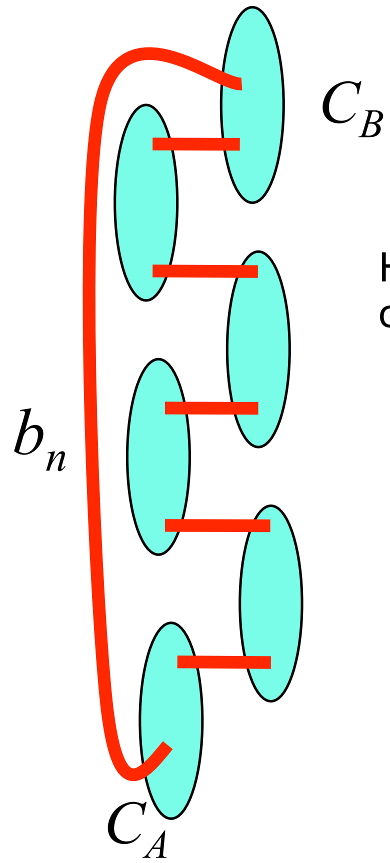
$$\boxed{\sigma_H(\beta) = 1 - \exp\left[-\tfrac{1}{2}\sum_n a_n - \tfrac{1}{2}\sum_n b_n/n\right]}$$

$$a_n = (-1)^{n+1}\int \quad D_A(u_1)\hat{\sigma}(u_1,u_1')C_B(u_1',u_2')\hat{\sigma}(u_2',u_2)C_A(u_2,u_3)\ldots$$
$$\cdots\hat{\sigma}(u_n,u_n')D_B(u_n')\prod_{i=1}^{n} du_i du_i'$$
$$b_n = (-1)^{n+1}\int \quad C_A(u_n,u_1)\hat{\sigma}(u_1,u_1')C_B(u_1',u_2')\ldots$$
$$\cdots C_B(u_{n-1}',u_n')\hat{\sigma}(u_n',u_n)\prod_{i=1}^{n} du_i du_i'$$

The structures a_n and b_n allow a graphic representation



The chain which starts with A may end either with A or with B, depending whether n is even or odd



Here only even values of n are possible

Inclusive and "exclusive" cross sections

In proton-proton collisions, the ***inclusive cross sections*** are basically the ***moments of the distribution of the number of MPI***.

The most basic information on the distribution in the number of collisions, the average number, is hence given by the single scattering inclusive cross section of the QCD parton model. Analogously the K-parton scattering inclusive cross section gives the Kth moment of the distribution in the number of collisions and is related directly to the K-body parton distribution of the hadron structure.

A way alternative to the set of moments, to provide the whole information of the distribution, is represented by the set of the different terms of the probability distribution of multiple collisions. Correspondingly, in addition to the set of the inclusive cross sections, one may consider the set of the ***"exclusive" cross sections***, where one selects the ***events where only a given number of collisions are present***. The cross sections called here "exclusive" are in fact partially inclusive cross sections, since one sums over all partons outside the given phase space interval and on the soft fragments.

Interestingly, in its study of MPI, ***the CDF experiment did not measure the inclusive cross section of double parton scattering***. The events selected where in fact only those which contained just double parton collisions, while all events with triple scatterings (about 17% of the sample of all events with double parton scatterings) where removed.

The resulting quantity measured by CDF is hence different with respect to the inclusive cross sections usually discussed in large momentum transfer physics. In fact ***CDF measured the "exclusive" double parton scattering cross section.***

While the inclusive cross sections are linked directly to the multi-parton structure of the hadron the link of the "exclusive“ cross sections with the hadron structure is more elaborate.

The requirement of having only events with a given number of hard collisions implies that the corresponding cross section (being proportional to the probability of not having any further hard interaction) ***depends on the whole series of multiple hard collisions.***

Notice that in the infrared limit the number of collisions becomes infinite and each exclusive cross section, corresponding to a finite number of partonic collisions, becomes as a consequence exceedingly small. ***All “exclusive” cross sections are hence well behaved in the infrared limit.***

The number of hard partonic collisions which can be observed directly is nevertheless limited, which allows to discuss the ***"exclusive" cross sections*** by ***expanding in the number of elementary interactions.***

One hence has:

$$\sigma_H \equiv \sum_{N=1}^{\infty} \tilde{\sigma}_N, \qquad \sigma_K \equiv \sum_{N=K}^{\infty} \frac{N(N-1)\ldots(N-K+1)}{K!}\tilde{\sigma}_N$$

σ_K K-parton scattering inclusive cross section

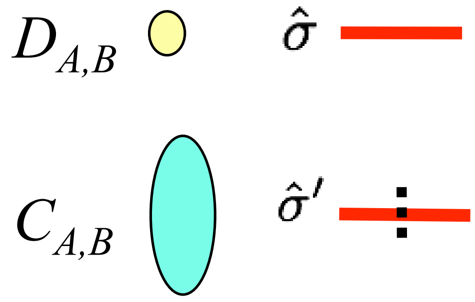
$\tilde{\sigma}_N$ N-parton scattering “exclusive” cross sections, where one selects the events where only N collisions are present

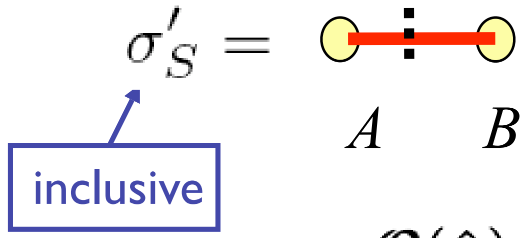
Notice that the relation above represents also a set of ***sum rules connecting the inclusive and the "exclusive" cross sections***

At order $\hat{\sigma}^3$ one obtains :

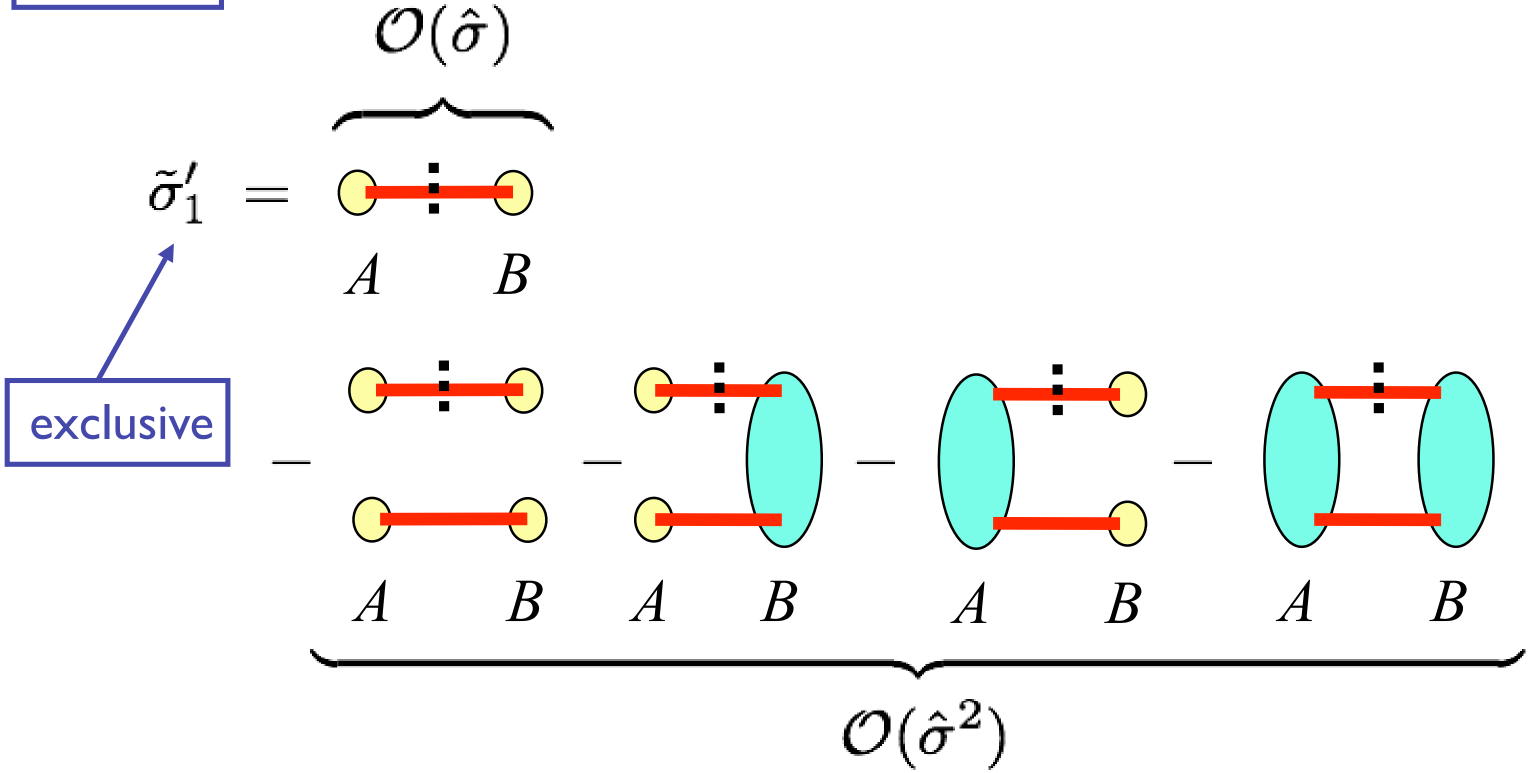
$$\begin{aligned} \tilde{\sigma}_1 &= \sigma_S - 2\sigma_D + 3\sigma_T \\ \tilde{\sigma}_2 &= \sigma_D - 3\sigma_T \\ \tilde{\sigma}_3 &= \sigma_T \end{aligned}$$

Single scattering differential "exclusive" cross sections
at order $\hat{\sigma}^2$ including two-body correlations:

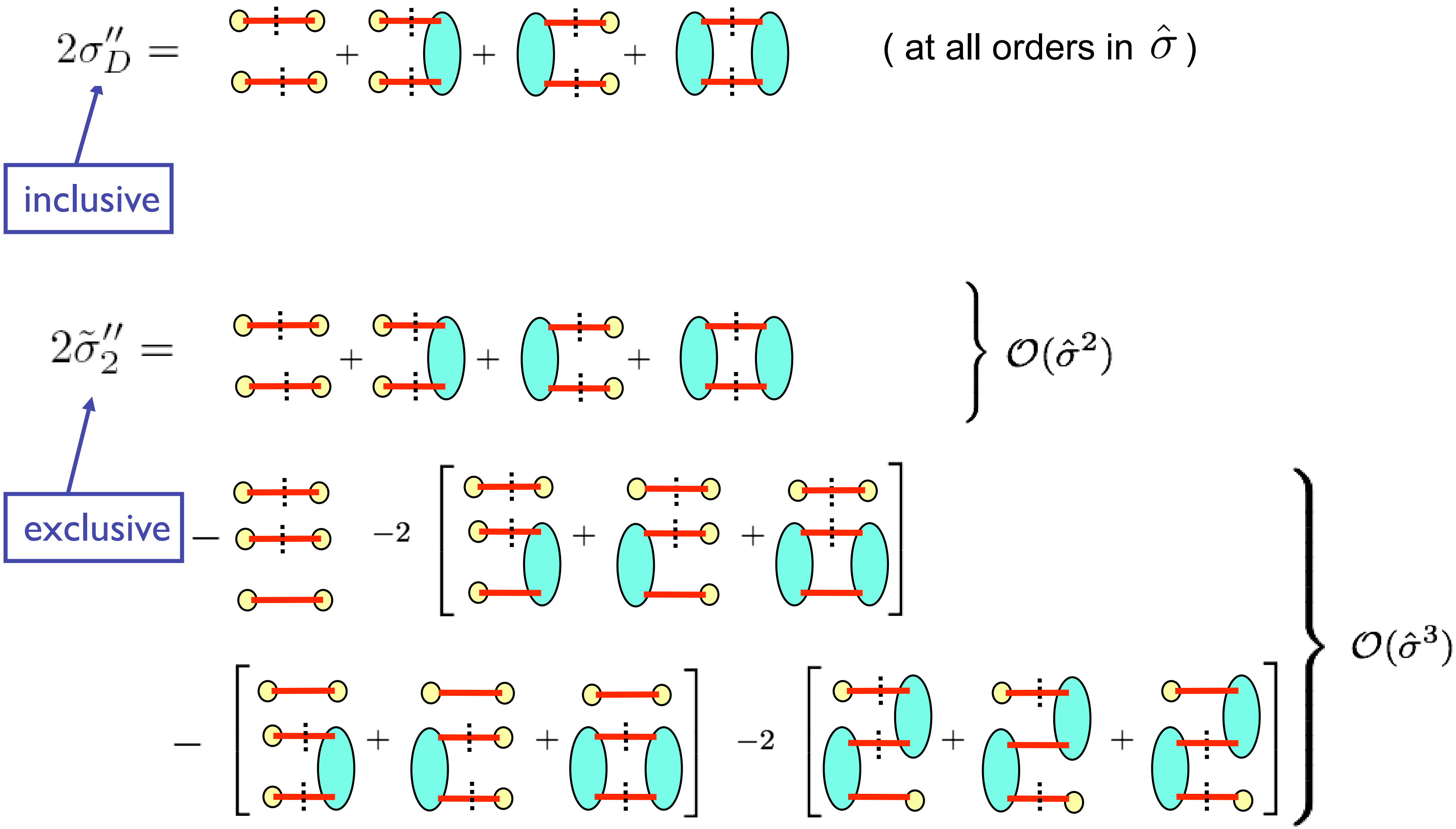




(at all orders in $\hat{\sigma}$)



Double parton scattering differential "exclusive" cross sections at order $\hat{\sigma}^3$
taking into account two-body parton correlations



MPI add incoherently in the final cross section, leading to a purely ***probabilistic picture*** of the process in each phase space interval, in such a way that one may associate a ***different probability distribution to each different phase space choice*** to observe the final state.

Any final state phase space window identifies an interval in momentum transfer and in fractional momenta, which identifies the domain of definition of the probability distribution of multiple collisions.

Notice that the same interval in momentum transfer and fractional momenta represents also the integration domain of the integrated terms, which appear in the "exclusive" differential cross sections. Integrated terms appear in the "exclusive" differential cross sections because of the normalization of the probability distribution. ***Normalization*** hence ***fixes unambiguously the integration limits of the virtual terms***, which must coincide with the kinematical limits imposed to the real terms by the choice adopted to select the final state.

One can show that ***the components of the hadron structure***, namely the terms D and C, ***are well defined quantities, as they do not mix when the kinematical limits adopted to selelect the final state are changed***. The effect, to modify the kinematical limits adopted to selelect the final state, is in fact only to change the integration domain of each term, namely to probe the multi-parton structure of the hadron in different domains in x and Q^2.

By restricting the phase space interval of the observed final state, $\tilde{\sigma}_1$ is well expressed by the term linear in $\hat{\sigma}$. In that limit the probability of interaction is well approximated by the average of the distribution, while $\tilde{\sigma}_2$ is negligibly small and ***the single parton scattering "exclusive" cross section is well represented by the single scattering expression of the simple QCD parton model***.

When the phase space volume is increased, the single parton scattering ***"exclusive" cross section becomes increasingly different*** from the prediction of the single scattering expression of the simple QCD parton model and the difference allows a direct measure of the importance of correlations.

By summing the expressions of $\tilde{\sigma}_1, \tilde{\sigma}_2$ etc. at a given order in $\hat{\sigma}$ with the proper multiplicity factors one obtains the inclusive single parton scattering cross section, which is correctly given by the QCD parton model expression. ***By comparing the measured inclusive cross section with a sum of the measured "exclusive" cross sections***, taken with the proper multiplicity factors, up to a given order in the number of collisions, ***one hence has a direct indication of the importance of higher order unitarity corrections*** in a given phase space interval.

Inclusive and "exclusive" cross sections result from independent measurements and, if in a given phase space interval only single and double collisions give sizable contributions, the following relation between the measured integrated cross sections holds:

$$\begin{aligned} \sigma_S &= \tilde{\sigma}_1 + 2\tilde{\sigma}_2 \\ \sigma_D &= \tilde{\sigma}_2 \end{aligned} \qquad \text{at } \mathcal{O}(\hat{\sigma}^2)$$

By increasing the phase space interval triple collisions may become important and, in such a case, the relation becomes

$$\begin{aligned} \sigma_S &= \tilde{\sigma}_1 + 2\tilde{\sigma}_2 + 3\tilde{\sigma}_3 \\ \sigma_D &= \tilde{\sigma}_2 + 3\tilde{\sigma}_3 \\ \sigma_T &= \tilde{\sigma}_3 \end{aligned} \qquad \text{at } \mathcal{O}(\hat{\sigma}^3)$$

By checking the validity of the sum rules above, one hence controls the effects of unitarity. In a phase space interval, where only single and double collisions give sizable contributions, one may hence obtain ***information on the multi-parton correlations*** by looking at the difference between the single scattering inclusive and "exclusive" differential cross sections

$$\frac{d\sigma_S}{dyd\mathbf{p}_t} - \frac{d\tilde{\sigma}_1}{dyd\mathbf{p}_t} = \frac{d\sigma_S}{dyd\mathbf{p}_t}\frac{\sigma_S}{\sigma_{eff}}$$

One may hence obtain the value of the effective cross section by measuring the single scattering inclusive and "exclusive" cross sections. By comparing the behavior, as a function of the fractional momenta, of the difference on the left hand side of the equation with the right hand side one checks the dependence of the effective cross section on *x*

In the case where ***partons*** are ***correlated only in the transverse coordinates*** one may express the effective cross section as a function of the correlation parameters

$$
\begin{aligned}
D(x,b) &= G(x)f(b) \\
f(b) &= g(b,R^2) \\
C(x_1,x_2;b_1,b_2) &= G(x_1)G(x_2)h(b_1,b_2) \\
h(b_1,b_2) &= c\cdot g(B,R^2/2)\bar{h}(b,\lambda^2) \\
\bar{h}(b,\lambda^2) &= \frac{d}{d\gamma}\bar{g}(b,\lambda^2/\gamma)\Big|_{\gamma=1}
\end{aligned}
$$

Where $G(x)$ represents the usual one-body parton distribution, $g(b,R^2)$ is a Gaussian, λ is the correlation length, c the correlation strength and $h(b,\lambda^2)$ integrates to zero.
Notice that in this way the multi-parton distribution, after integrating on the transverse parton coordinates is strictly Poissonian with average number $G(x)$.

Correlations are defined by the ***deviation of the multiparton distribution from the Poissonian***. The multiparton distribution density depends on the fractional momenta and on the transverse parton coordinates. The uncorrelated distribution is hence a Poissonian at fixed values of fractional momenta *x* and at fixed values of the transverse parton coordinates **b**. A distribution correlated only on **b** is hence different from a Poissonian only as a function of the transverse parton coordinates, while the distribution is still a Poissonian after integrating on the transverse parton coordinates, which is achieved by requiring that the integral of the correlation terms on the transverse coordinates give zero.

$$W(x_1, \mathrm{b}_1, \ldots x_n, \mathrm{b}_n) = \frac{1}{n!}\langle W(x_1, \mathrm{b}_1)\rangle \ldots \langle W(x_n, \mathrm{b}_n)\rangle\, e^{-\langle W\rangle}$$

distribution ***uncorrelated*** in *x* and **b**

$$W(x_1, \ldots x_n) = \int W(x_1, \mathrm{b}_1, \ldots x_n, \mathrm{b}_n) \prod_{i=1}^{n} d\mathrm{b}_i = \frac{1}{n!}\langle W(x_1)\rangle \ldots \langle W(x_n)\rangle\, e^{-\langle W\rangle}$$

distribution ***correlated*** in **b**

if partons are correlated only in **b**, the distribution is a Poissonian after integrating on the transverse parton coordanates

With a ***Gaussian parton density*** one obtains

$$\frac{1}{\sigma_{eff}} = \frac{3}{8\pi\bar{R}^2}\left\{1 + c\cdot e\frac{16\times 3\bar{s}^2}{(4+3\bar{s}^2)^2} + c^2\cdot e^2\frac{2}{3\bar{s}^2}\right\}$$

Where $\bar{R}$ is the root mean square hadron radius, measured in the generalized parton distributions, $\bar{s} \equiv \lambda/\bar{R}$ and *e* is the Euler number

Explicit expressions can be found also for the scale factor characterizing triple parton collisions and in the case of an ***exponential parton density***.

$$\frac{1}{\sigma_{eff}} = \frac{3}{7\pi\bar{R}^2}\left\{1 + c\cdot 14\eta F\left(\frac{1}{12}\frac{x_0^2}{\bar{s}^2}\right) + c^2\cdot\frac{14}{15}\eta^2\left(\frac{1}{12}\frac{x_0^2}{\bar{s}^2}\right)\right\}$$

where $x_0 \approx 2.386$, $\eta \approx 3.456$ and $F(a)$ is an explicit function of the variable a

Concluding summary

In the regime of MPI ***the picture of the interaction is probabilistic*** and different final state phase space intervals are characterized by different probability distributions of MPI

One may hence construct (and measure) ***two different sets of cross sections***, the ***inclusive*** cross sections, which are ***related to the moments*** of the probability distribution of MPI, and the ***"exclusive"*** cross sections, which are ***related to the different terms*** of the probability distribution.

All different terms of the distribution of MPI contribute (each with a different multiplicity factor) to the inclusive cross sections, which give directly a measurement of the multiparton distributions.

Each single term of the distribution of gives an "exclusive" cross section.

Notice that when the number of interactions become very large each single term of the probability distribution becomes very small. ***All "exclusive" cross sections are hence well behaved in the infrared region***.

Inclusive and "exclusive" cross section are measured independently.

The average number of MPI depends on the final state phase space interval and one may control the number of MPI by adjusting the final state phase space interval

Inclusive and "exclusive" cross sections are linked by sum rules, which are saturated by a different number of terms in each final state phase space interval. The number of terms needed to saturate the sum rules are a quantitative measure of the importance of the unitarity corrections.

For a given number of hard interactions the non perturbative input to the "exclusive" cross sections is expressed explicitly in terms of the non perturbative input to the inclusive cross sections.

Information on the ***correlations*** (defined by the difference with respect to the Poissonian distribution) ***may be obtained from the difference between inclusive and "exclusive cross sections***.

In the case of transverse correlations, ***explicit expressions***, including all effects of two-body parton correlations, ***have been worked out both for the inclusive and the exclusive cross sections***