The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

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2036-7

International Workshop: Quantum Chromodynamics from Colliders to Super-High Energy Cosmic Rays

25 - 29 May 2009

Fully Unintegrated Parton Correlation Functions

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QCD at Cosmic Energies IV - May 26, 2009

Perturbative QCD

- Asymptotic freedom: Strong coupling becomes small over short time/distance scales!

$$\alpha_s(\underbrace{Q^2}_{\textbf{\textit{Hard scale}}}) << 1; \qquad Q >> \Lambda_{QCD}$$

- Use standard Feynman perturbation theory to make accurate first principles calculations.

pQCD and Factorization

- The Real World: Always involves both long and short time/distance scales.

- Factorization Theorem: Systematic separation of long and short distance scales in QFT.

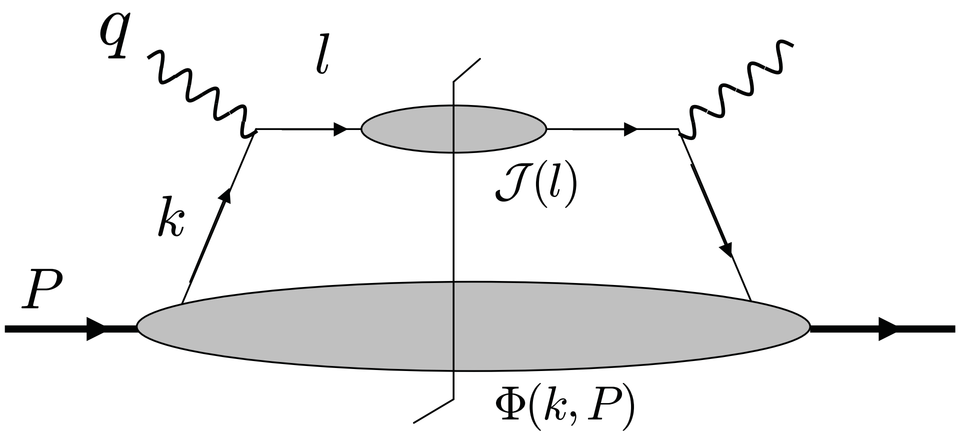
- Short distance part:
  - **Well-defined perturbation series in small coupling.**

- Long distance parts:
  - **Universal correlation functions.**

- Foundation of studies of nucleon structure in terms of QCD (quark/gluon) degrees of freedom.

DIS: Standard Approximations

- LO DIS: Before approximations:



$$P = \left(P^+, \frac{M_p^2}{2P^+}, \mathbf{0}_t\right)$$

$$q = \left(-xP^+, \frac{Q^2}{2xP^+}, \mathbf{0}_t\right)$$

- Parton model kinematics:

$$k^+ = xP^+ + \frac{M_J^2 + k_t^2}{2(q^- + k^-)}$$

$$k = (k^+, k^-, \mathbf{k}_\perp)$$

$$l = k + q = (k^+ - xP^+, q^- + k^-, \mathbf{k}_\perp)$$

$$l^2 \approx 0 \quad \Longrightarrow \quad k^+ \approx xP^+ \approx x_B P^+$$

Standard Approximations (cont.)

- Unapproximated structure tensor:

$$W^{\mu\nu}(q,P) = \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \,\mathrm{Tr}\left[\gamma^\mu \mathcal{J}(\underbrace{k+q}_{l})\gamma^\nu \Phi(k,P)\right]$$

- Approximate momentum inside subgraphs:

$$\underbrace{k \to (xP^+, k^-, \mathbf{k}_\perp);}_{\textit{Inside target subgraph}} \qquad \underbrace{k+q \to (l^+, q^-, \mathbf{0}_\perp)}_{\textit{Inside jet subgraph}}$$

- Use parton model approximation inside hard part.

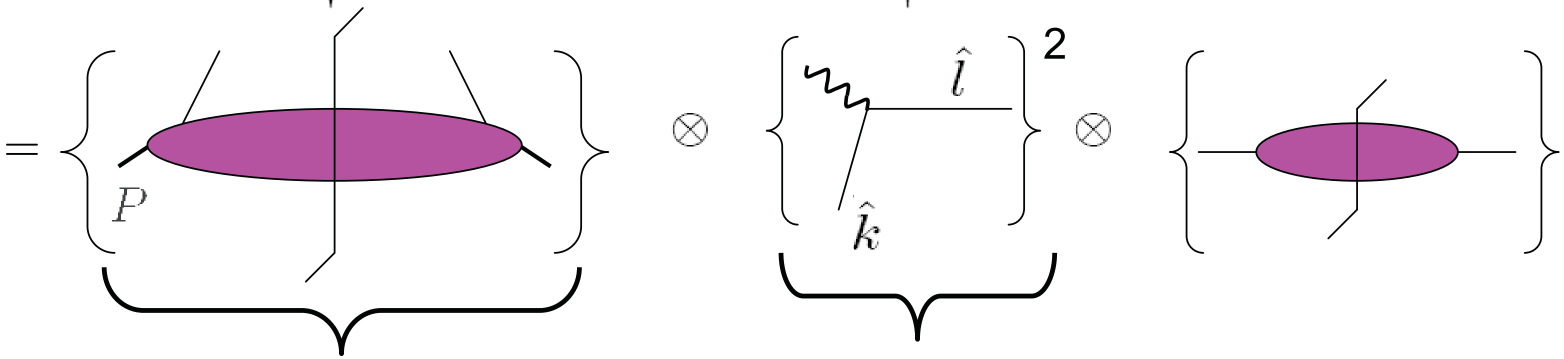
$$k \to \hat{k} = (xP^+, 0, \mathbf{0}_\perp); \qquad l \to \hat{l} = (0, q^-, \mathbf{0}_\perp)$$

- Project out largest Dirac components.

Factorized Graph

$$W^{\mu\nu}(q,P) \simeq$$

$$\frac{e_j^2}{4\pi}\underbrace{\left\{\int \frac{dk^- d^2\mathbf{k}_T}{(2\pi)^4}\Phi_j^+(x_{Bj}P^+, k^-, \mathbf{k}_T)\right\}}\,\underbrace{\mathrm{Tr}\left[\gamma^\mu\gamma^+\gamma^\nu\gamma^-\right]}\,\underbrace{\overbrace{\left\{\int dl^+ \mathcal{J}_j^-(l^+, q^-, \mathbf{0}_T)\right\}}^{\text{Set to one by unitarity:}}}$$



Parton Distribution???

LO partonic structure functions.

Note shift in kinematics !!

Varieties of parton correlation functions

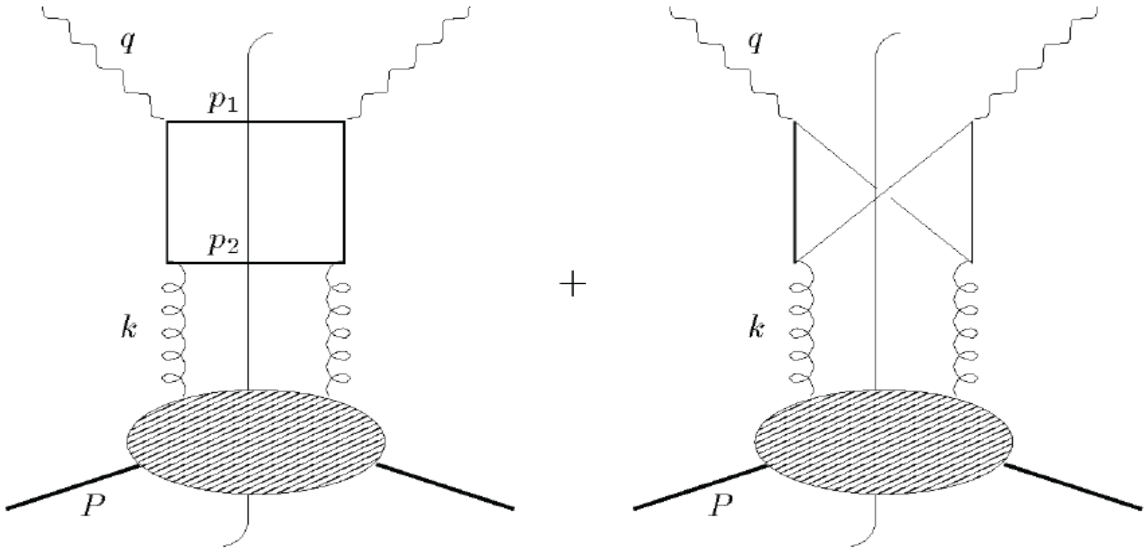
- Standard "Integrated" PCFs:
  - All small momentum components are integrated in definitions.
- Transverse momentum dependent (TMD) PCFs.
  - Only the minus component (smallest) is integrated definition.
  - Small-x
- Fully unintegrated PCFs.
  - Explicit dependence on exact parton momentum.
  - Needed for accurate treatment of final state kinematics.

Main Goal

- Rest of talk: *How to set up factorization that treats kinematics more accurately?*

PCFs and Hadron Kinematics

Explicit sample calculations: $c\bar{c}$ photoproduction



- Try:
  - Parton model kinematics.
  - Keeping k_T dependence, but approximating minus component. (TMD PDFs)
  - Exact kinematics. (Fully unintegrated PDFs)

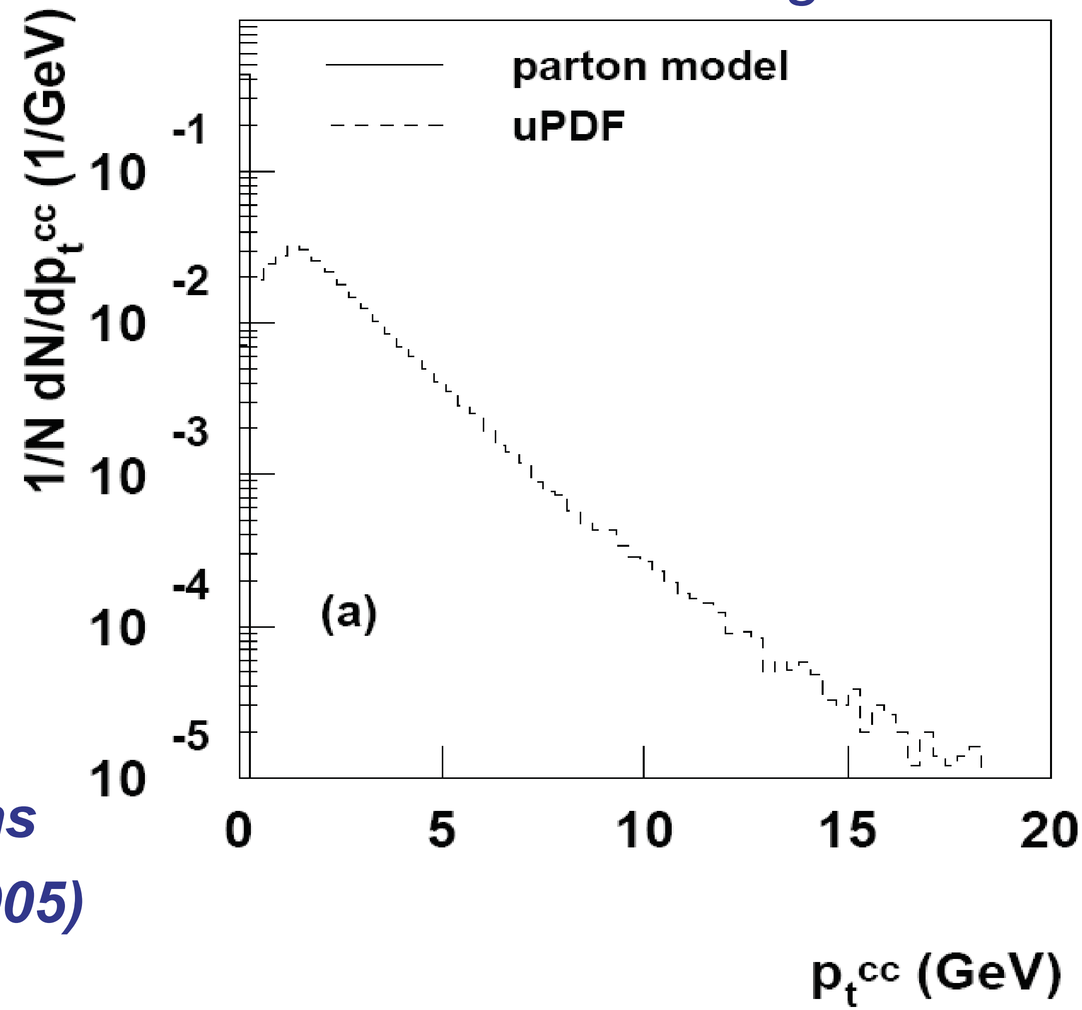
Errors in final state kinematics

Compare:

- Parton model kinematics
- TMD kinematics

(From Collins and Jung, 2005)

Cascade MC event generator



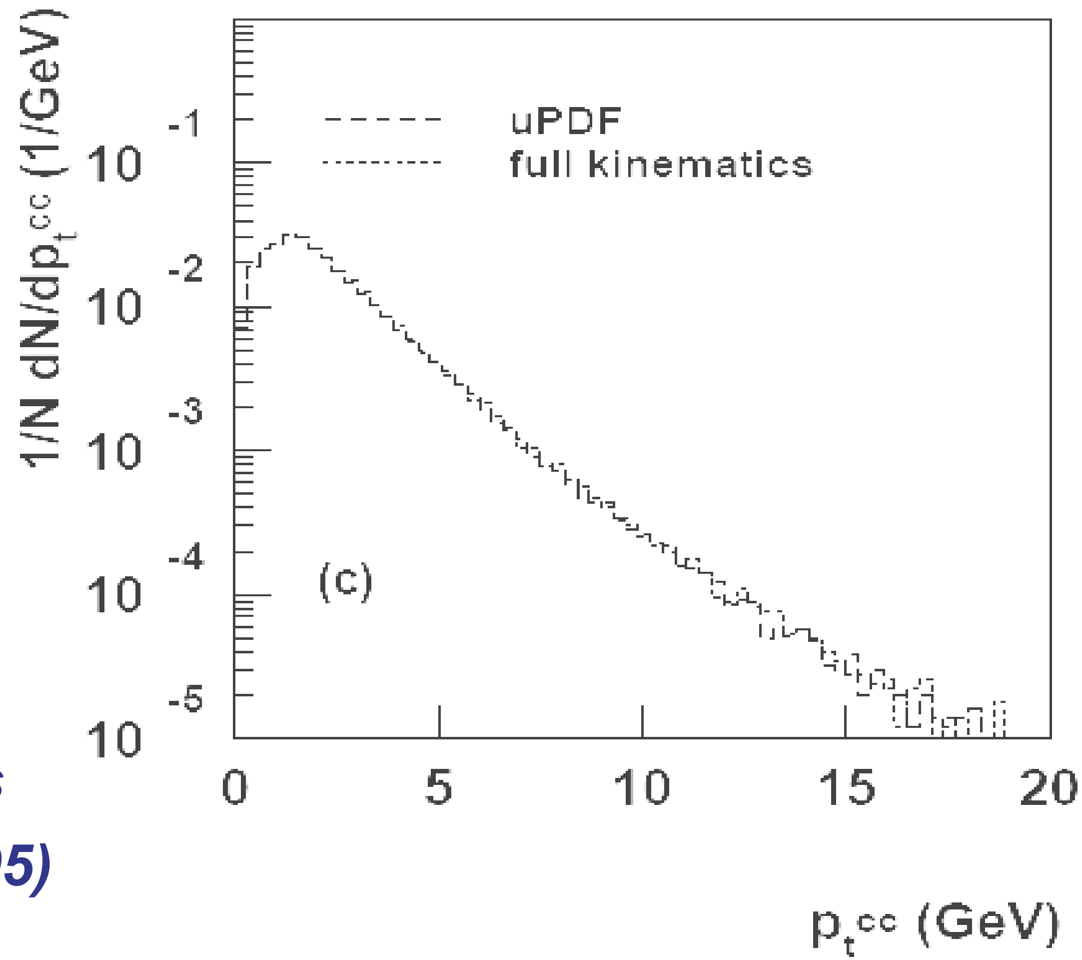
Errors in final state kinematics

Cascade MC event generator

Compare:

- ***TMD kinematics***
- ***Fully Unintegrated kinematics***

(From Collins and Jung, 2005)



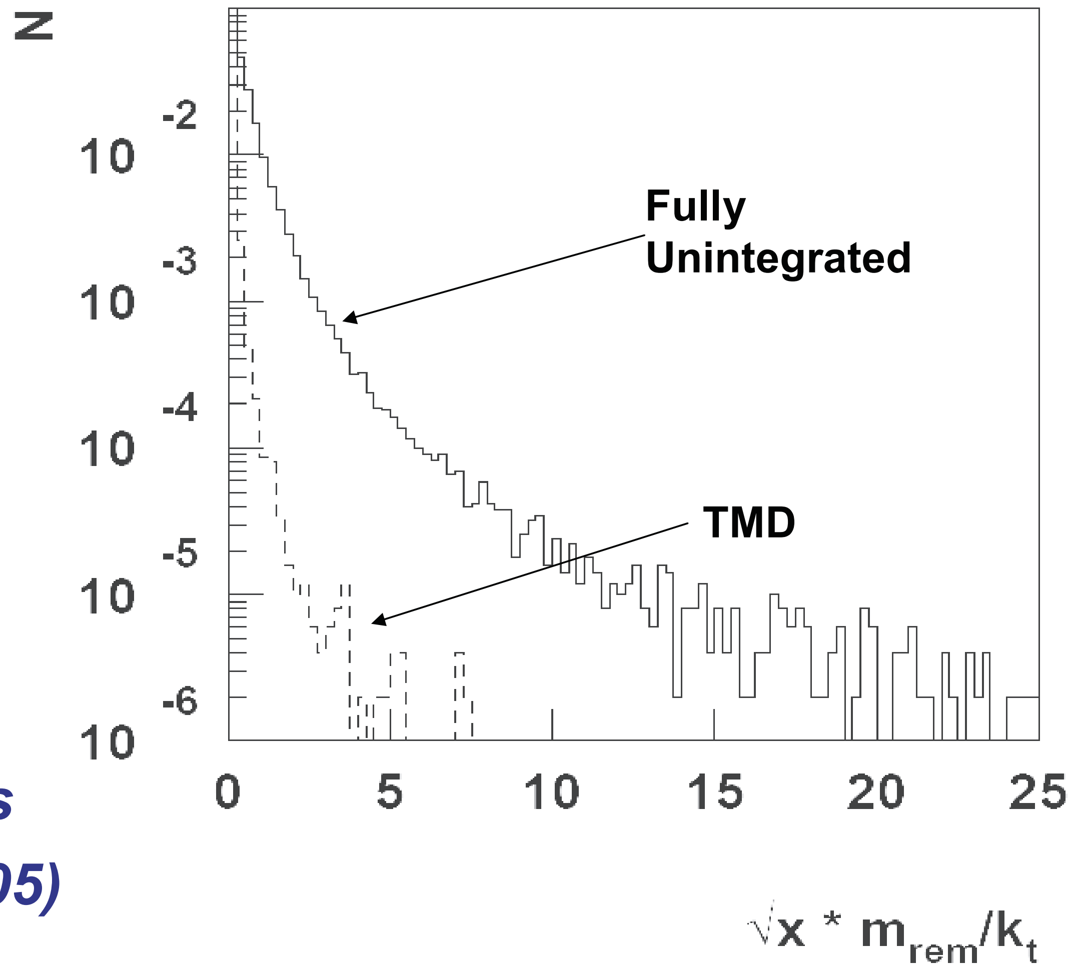
Errors in final state kinematics

Compare:

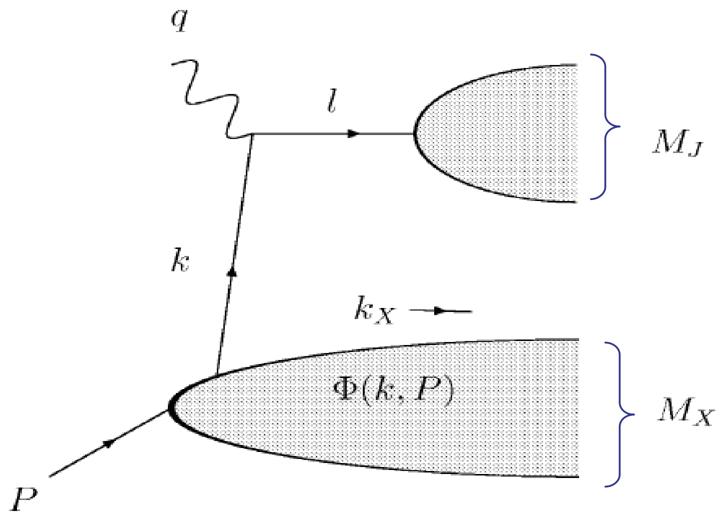
- ***TMD kinematics***
- ***Fully Unintegrated kinematics***

(From Collins and Jung, 2005)

Cascade MC event generator



Large-x DIS



$$s = (1-x)M_p^2 + \frac{Q^2}{x}(1-x)$$

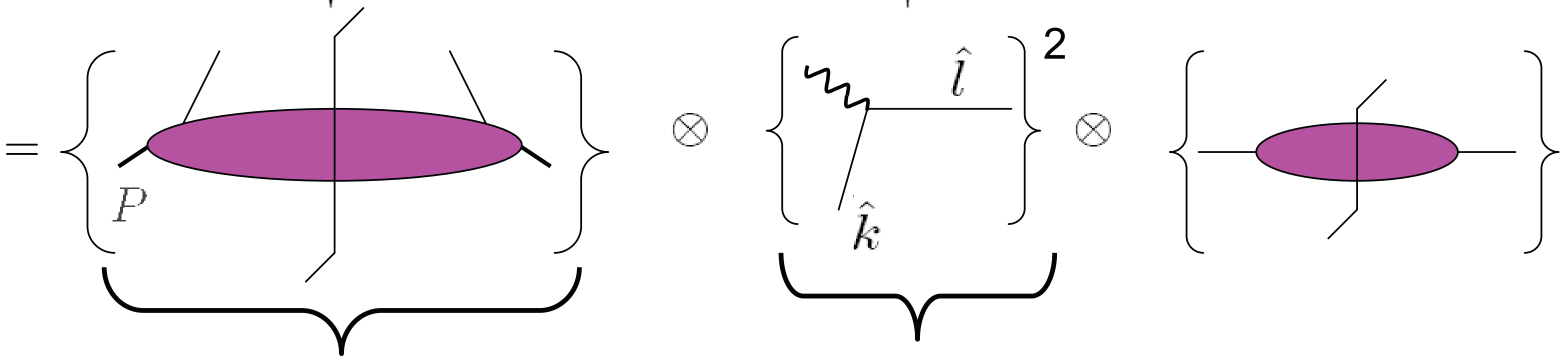
$$\Rightarrow \quad k_T^2 < \frac{(1-x)}{4}M_p^2 + \frac{Q^2}{4x}(1-x)$$

But k_T runs to order Q in the def. of the integrated PDF!

- **Definition of standard PDF becomes inconsistent**
- **Sensitive to remnant mass.**
- **Fully unintegrated PDFs needed.**

Standard Factorized Graph

$$W^{\mu\nu}(q,P) \simeq \frac{e_j^2}{4\pi}\left\{\int \frac{dk^- d^2\mathbf{k}_T}{(2\pi)^4}\Phi_j^+(x_{Bj}P^+,k^-,\mathbf{k}_T)\right\}\mathrm{Tr}\left[\gamma^\mu\gamma^+\gamma^\nu\gamma^-\right]\overbrace{\left\{\int dl^+ \mathcal{J}_j^-(l^+,q^-,\mathbf{0}_T)\right\}}^{\text{Set to one by unitarity:}}$$



Parton Distribution???

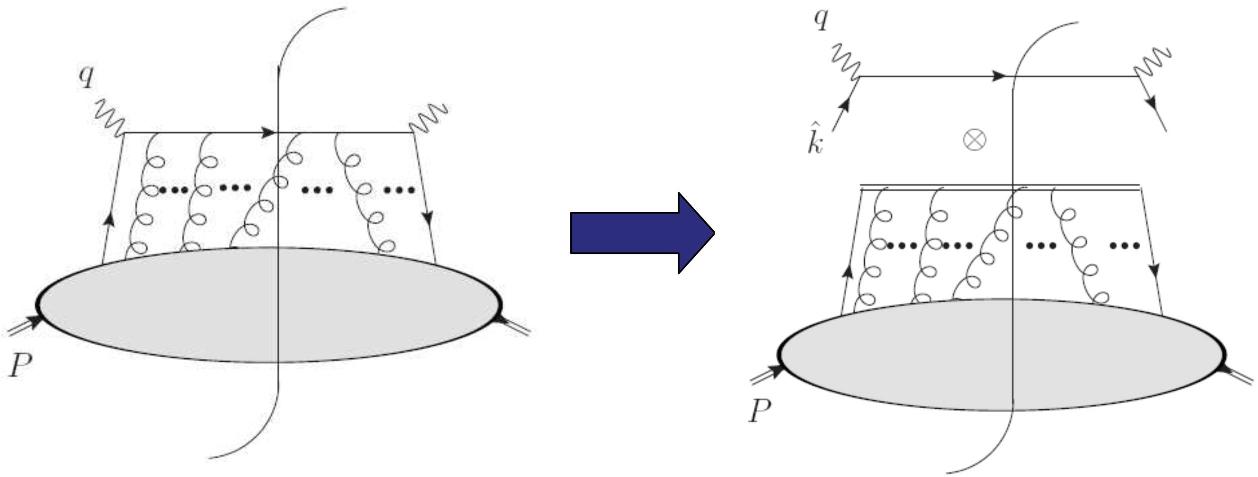
LO partonic structure functions.

- UV divergent – requires renormalization.

- Wilson lines needed for gauge invariance.

Wilson lines

- Ward identity: Target collinear gluons decouple from hard part.



- Ward identity → collinear lines extracted into overall factor.

- Wilson line becomes trivial in light-cone gauge.

Standard (Integrated) PDF

- Operator definition:

$$f(x,\mu) = \int \frac{dw^-}{4\pi}\, e^{-ixp^+w^-} \langle p|\, \bar{\psi}(0,w^-,\mathbf{0}_t) V_w^\dagger(u_\mathrm{J})\gamma^+ V_0(u_\mathrm{J})\psi(0)|p\rangle$$

$$u_\mathrm{J} = (0,1,\mathbf{0}_t)$$

- Light-like Wilson lines enforce gauge invariance.

$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

$$V_w^\dagger(u_\mathrm{J})V_0(u_\mathrm{J}) = P\exp\left(-ig\int_0^{w^-} d\lambda\, u_\mathrm{J}\cdot A(\lambda u_\mathrm{J})\right)$$

<u>*Summary So Far*</u>

- Details of kinematic approximations in standard factorization.

- Motivation to development more exact formalism.
  - Studies of underlying event?
  - Small-x?

- Emphasis on need for well-defined operator definition.

- <u>Basic Problem</u>: In standard treatment, factorization works after integrations, but final states changed.

Fully Unintegrated Factorization

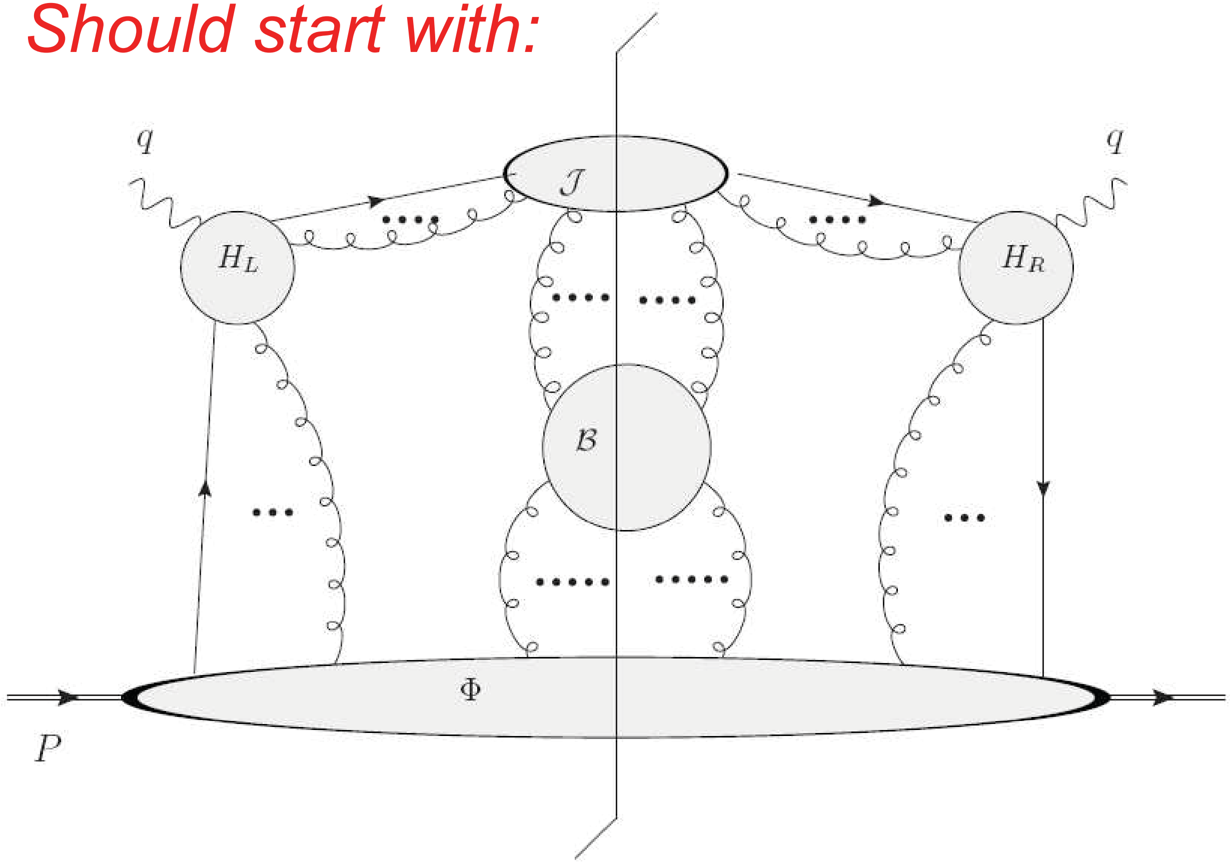
- Proposal: Set up a factorization formalism with *exact* kinematics for initial and final states.
  (Collins, TCR, Stasto, PRD77:085009,2008)
- Factorization should work *point-by-point in phase space*. *(correct of up to power suppressed terms.)*
- To obtain factorization, only apply approximations to the hard part.
- Hard scattering should involve ordinary functions.
- Need well-defined fully unintegrated parton correlation functions.

Factorization Strategy

- Consider general unapproximated Feynman diagrams. Classify leading regions.
- Apply approximations appropriate for each region starting with the smallest (but never changing final state momentum!).
- Obtain contribution from larger regions by subtracting smaller regions.
- Use Ward identities to disentangle soft and collinear gluons from hard part in sum over graphs.
- Identify well-defined operator definitions for the PCFs.

General Graphical Structure

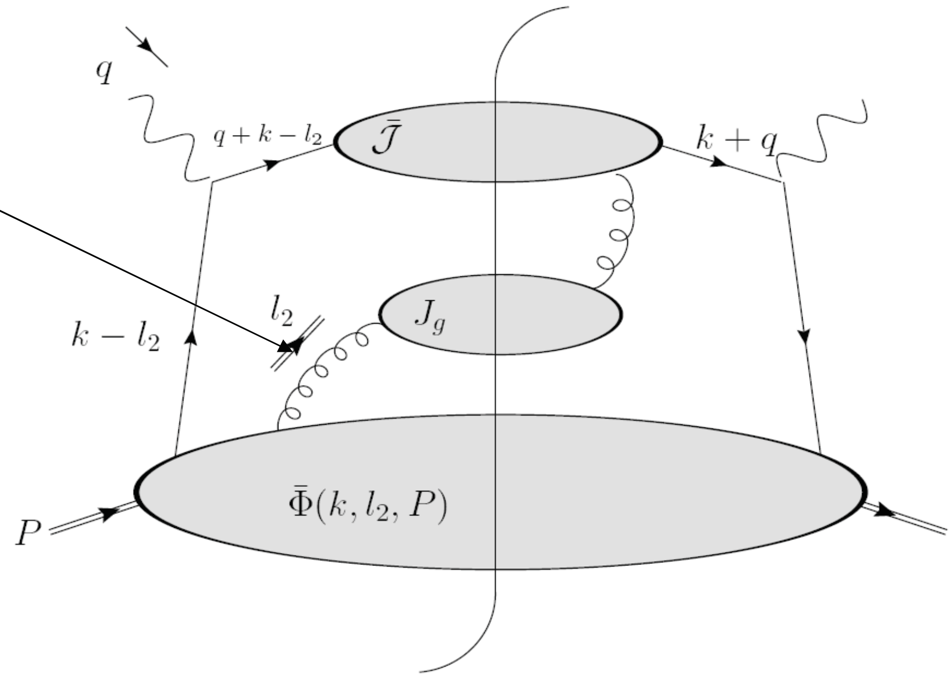
Should start with:

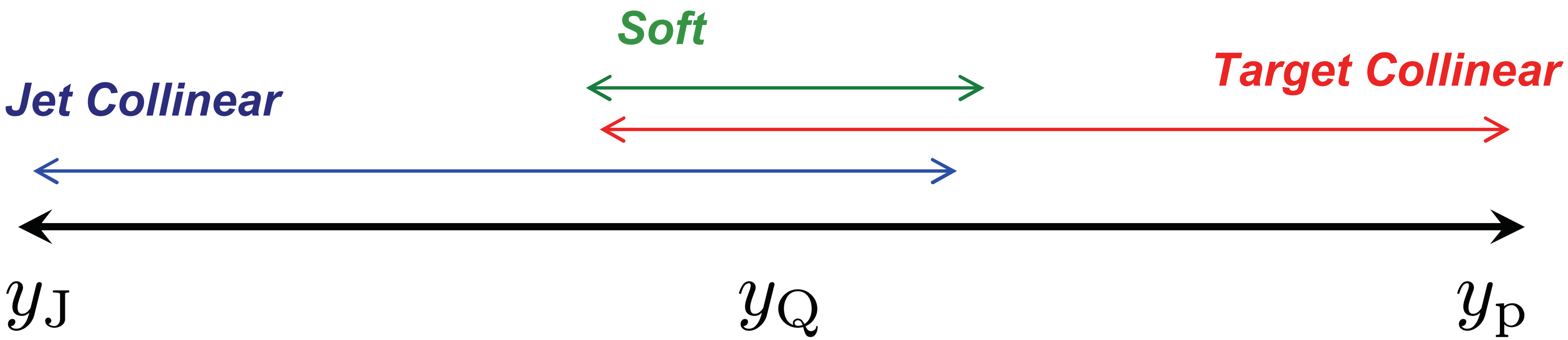


Must disentangle soft and collinear gluons to get factorization…

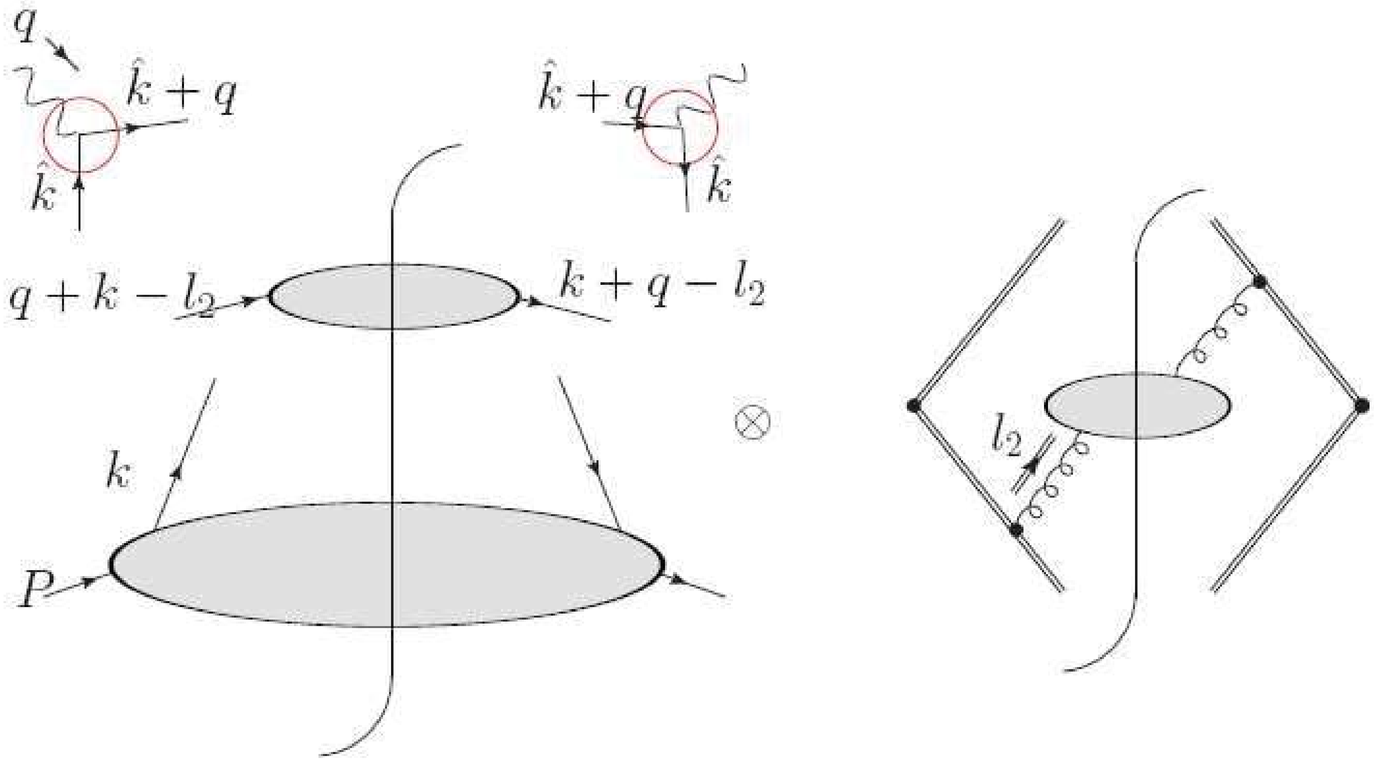
Graphical Example:

Single extra gluon: real emission





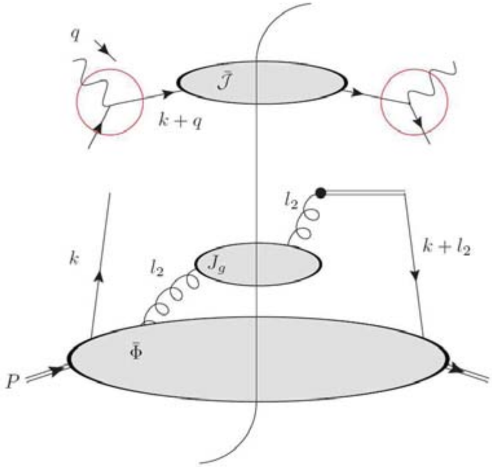
Soft Region: Factorized Structure



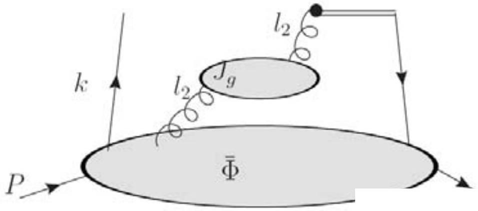
Graphical Example (Cont.): Subtractions

Consider target-collinear region

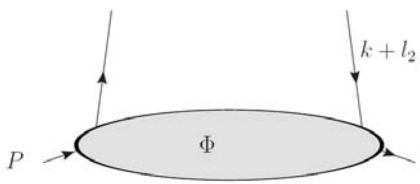
After applying Ward identities.



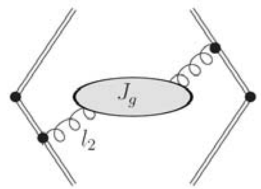
$F(k, P) =$



$-$



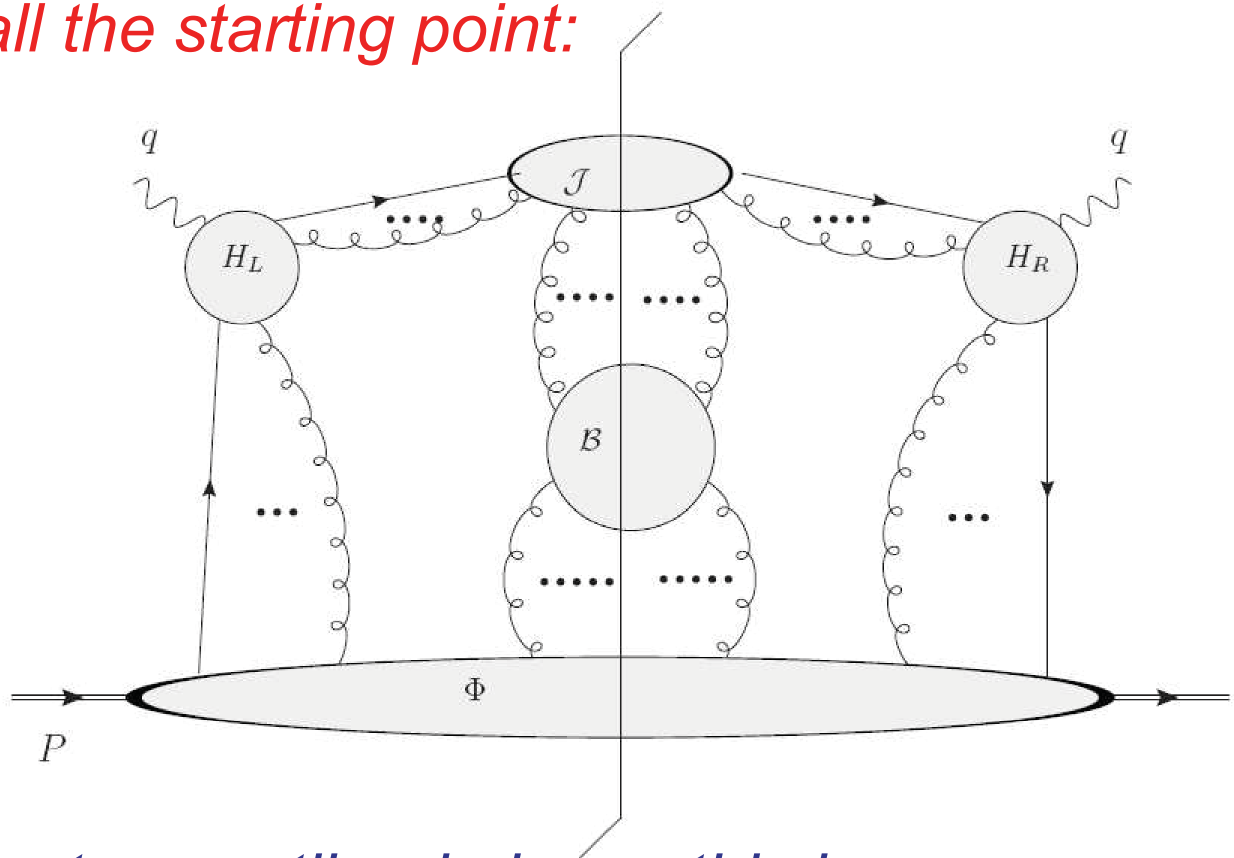
$\otimes$



The contribution to the PDF (target PCF) requires double counting subtractions.

Factorization

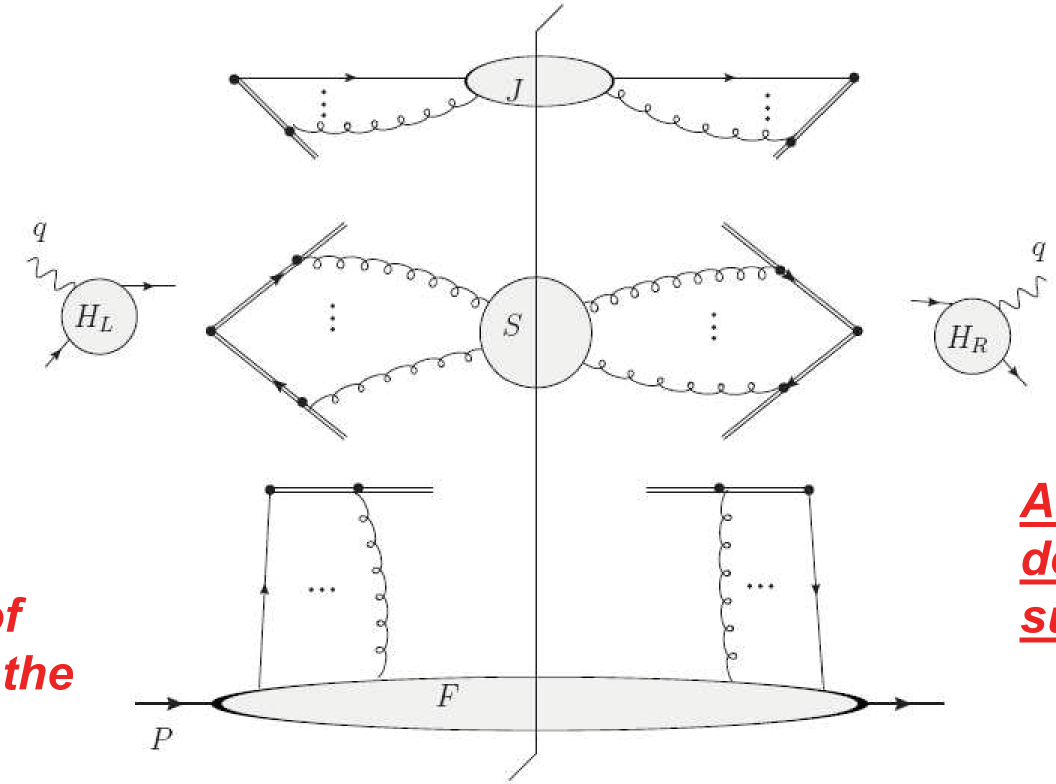
Recall the starting point:



After steps outlined above, this becomes…

(Ward identities + subtractions)

Topological Factorization:



Also need double counting subtractions.

A formula of this type is the goal.

$$\sigma = C \otimes F \otimes J \otimes S + \mathcal{O}((\Lambda/Q)^a |\sigma|), \qquad a > 0$$

Full Factorization

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_{\mathrm{T}}}{(2\pi)^4}\frac{d^4k_{\mathrm{J}}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\mathrm{T}}-k_{\mathrm{J}}-k_S)\times$$

$$\times\,|H(Q,\mu)|^2\;S(k_S,y_{\mathrm{T}},y_{\mathrm{J}},\mu)\;\underbrace{F_{\mathrm{mod}}(k_{\mathrm{T}},y_p,y_{\mathrm{T}},y_s,\mu)}\;\underbrace{J_{\mathrm{mod}}(k_{\mathrm{J}},y_{\mathrm{J}},y_s,\mu)}$$

Fourier Transform

$$\tilde{F}_{\mathrm{mod}}(w,y_p,y_{\mathrm{T}},y_s,\mu) \;=\; \frac{\langle p|\bar{\psi}(w)V_w^{\dagger}(n_s)I_{n_s;w,0}\dfrac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle_R}{\langle 0|I^{\dagger}_{n_{\mathrm{T}};w,0}V_w(n_{\mathrm{T}})V_w^{\dagger}(n_s)I_{n_s;w,0}V_0(n_s)V_0^{\dagger}(n_{\mathrm{T}})|0\rangle_R}$$

$$\tilde{J}_{\mathrm{mod}}(w,y_{\mathrm{J}},y_s,\mu) = \frac{\langle 0|\bar{\psi}(w)V_w^{\dagger}(-n_s)I_{-n_s;w,0}\gamma^- V_0(-n_s)\psi(0)|0\rangle_R}{\langle 0|I^{\dagger}_{-n_s;w,0}V_w(-n_s)V_w^{\dagger}(n_{\mathrm{J}})I_{n_{\mathrm{J}};w,0}V_0(n_{\mathrm{J}})V_0^{\dagger}(-n_s)|0\rangle_R}$$

Remarks on TMD PDFs:

- Generally accepted definition:

(Belitsky et. al Nucl. Phys.B 656, 165 2003)

Link at infinity

$$P(x, \mathbf{k}_t, \mu) =$$

$$\int \frac{dw^- \, d\mathbf{w}_t}{16\pi^3} \, e^{-ixp^+ w^- + i\mathbf{k}_t \cdot \mathbf{w}_t} \, \langle p | \, \bar{\psi}(0, w^-, \mathbf{w}_t) V_w^\dagger(n) I_{n;w,0} \gamma^+ V_0(n) \psi(0) \, | p \rangle$$

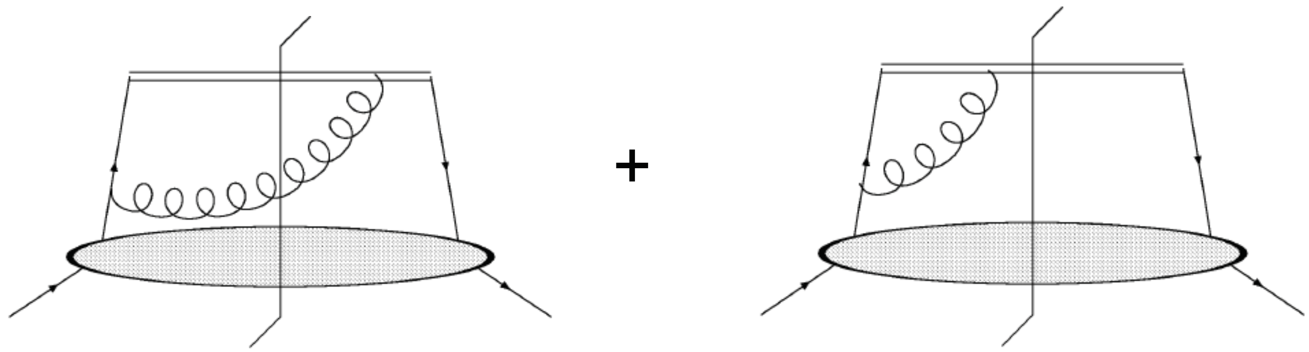
$$n = u_{\mathrm{J}} = (0, 1, \mathbf{0}_t)$$

- Fields are no longer evaluated along light-like separation.

- Connection at infinity needed for closed Wilson line / gauge invariance.

Complications with K_T - Factorization:

- Uncanceled light-cone divergences corresponding to gluons moving with infinite rapidity in minus direction.

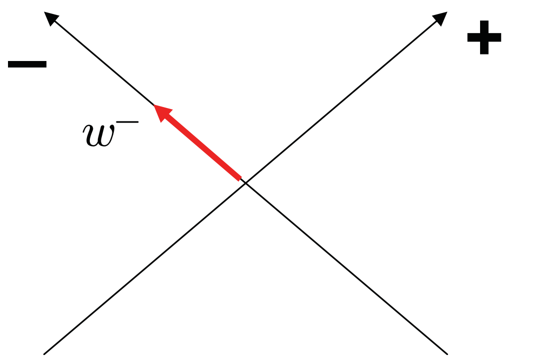


- Must use non-light-like Wilson lines!

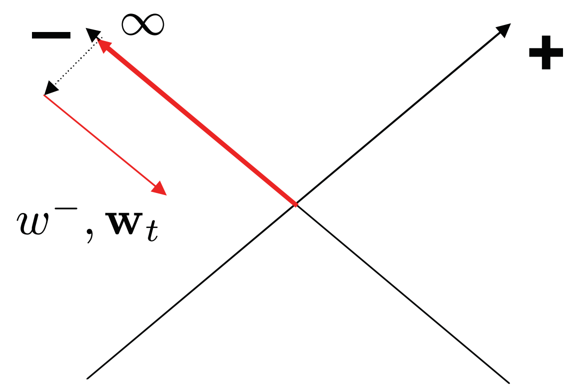
- Introduces new arbitrary parameter.
  - Predictability recovered with new evolution equations (e.g. CSS).

TMD PDFs: Wilson Lines

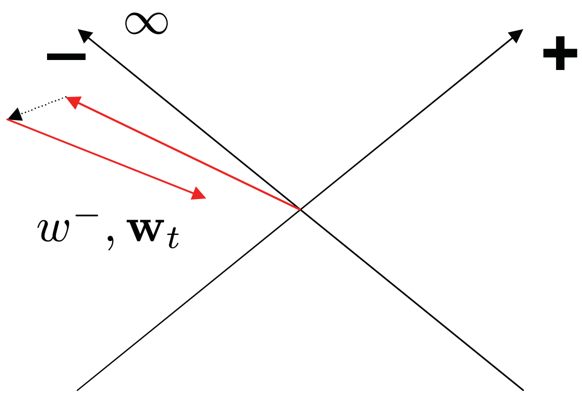
- Paths of Wilson lines:



Standard (Integrated)



Unintegrated First Try

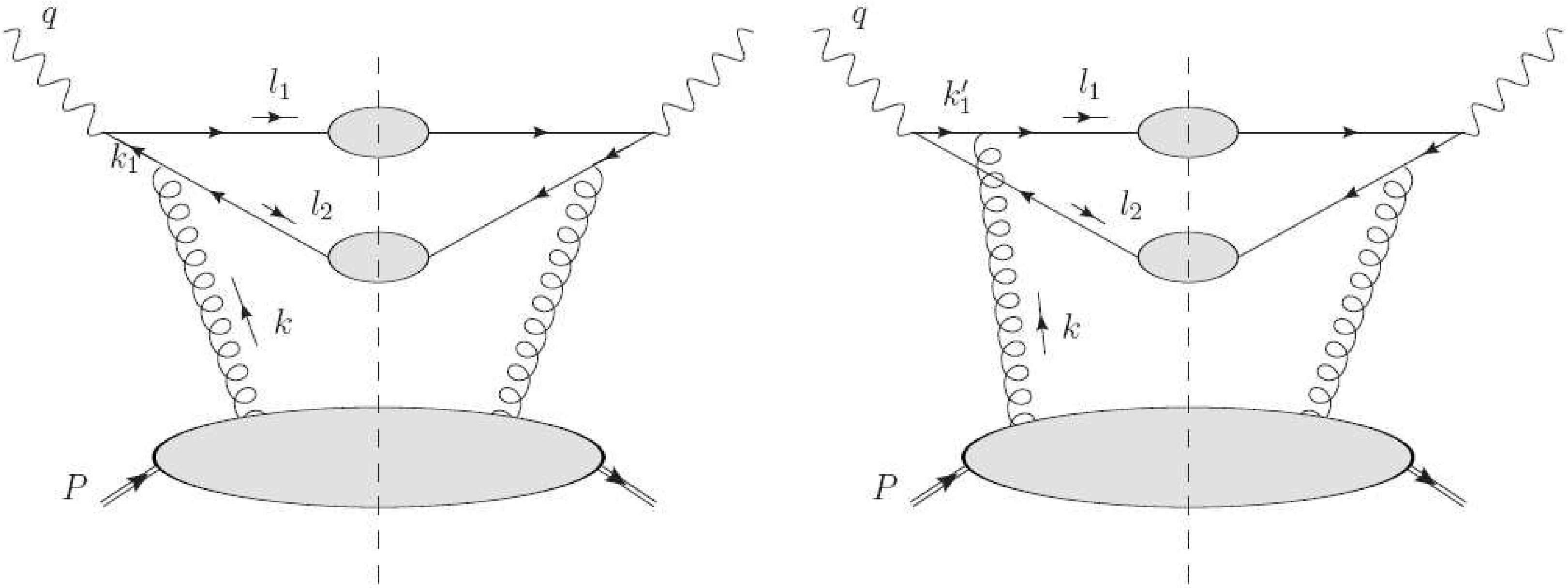


Unintegrated Correct

Same issues arise in treating fully unintegrated PCFs

Next-to-Leading Order

Wide angle jets:



Non-trivial hard
scattering matrix element.
Relies on details of LO treatment.

+ Cross terms

Next-to-Leading Order (Cont.)

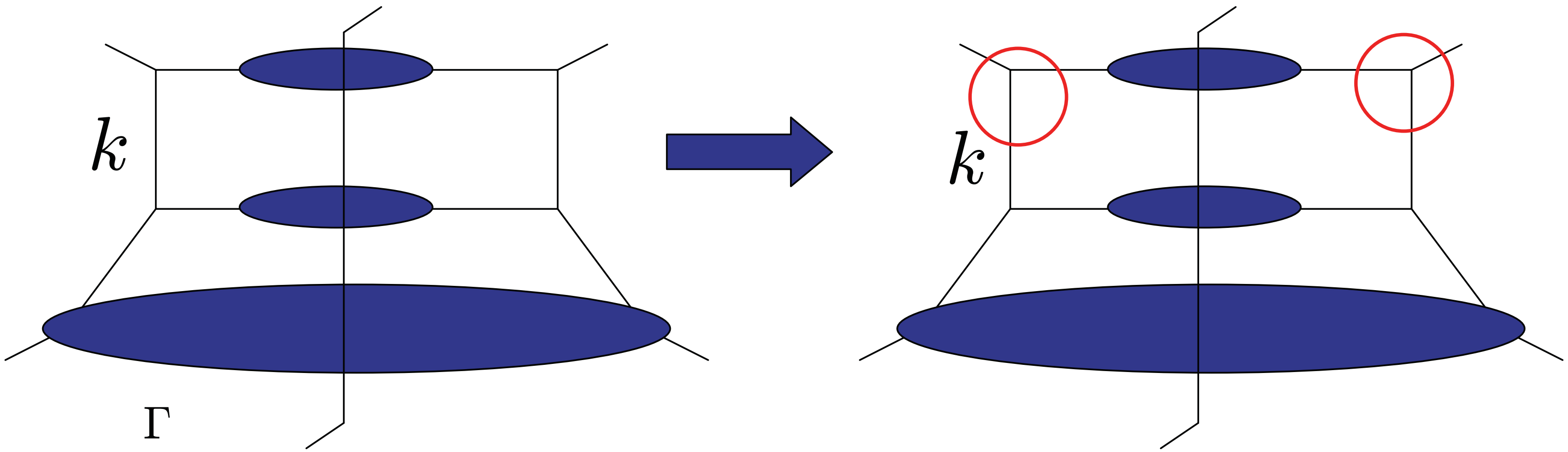
- NLO hard scattering overlaps with LO.
- Factorization requires double counting subtractions.
- Standard *integrated* factorization: Hard scattering involves generalized functions (distributions)!
- Exactly analogous subtractive formalism used in fully unintegrated case.
- Double counting subtractions give factorized expression with power suppressed errors point-by-point in phase.

Next-to-Leading Order (Cont.)

(Overview of subtraction approach)

- **Explicit implementation of subtractive formalism:**

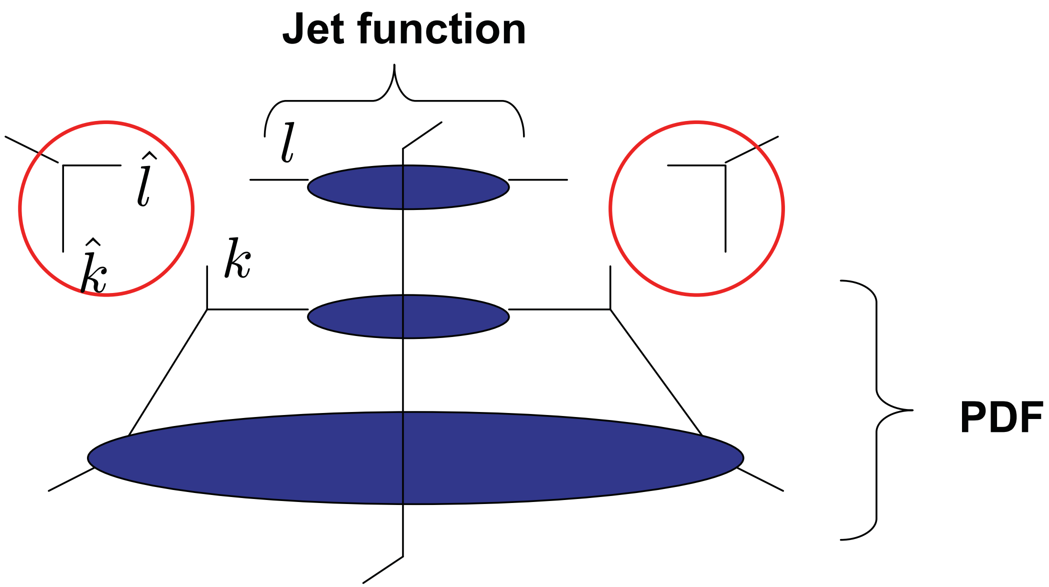
  $\phi^3_{(6)}$ - Scalar theory **(Collins, Zu, JHEP 03 (2005) 059)**



$$\Gamma = T_{LO}\Gamma + \mathcal{O}((\Lambda/Q)\Gamma)$$

Next-to-Leading Order (Cont.)

- Mapping of exact to approximate momentum variables:

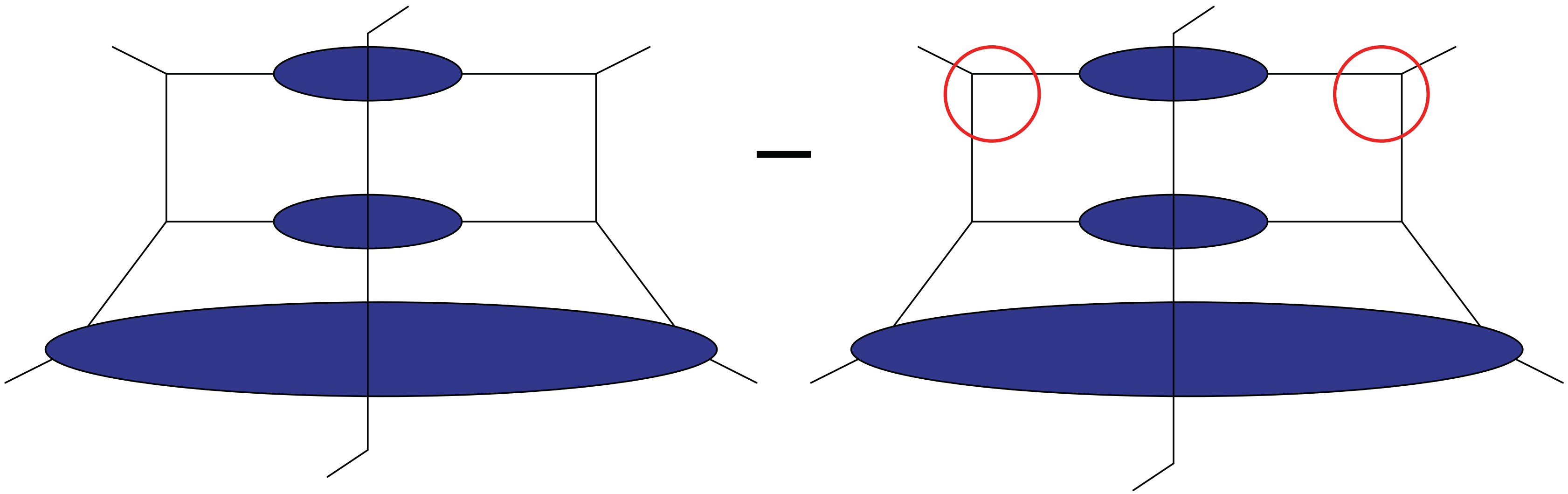


$$\hat{k} = (x_B P^+, 0, \mathbf{0}); \qquad \hat{l} = (0, q^-, \mathbf{0})$$

$$\sigma \sim C \otimes F(k, P) \otimes J_1(l)$$

Next-to-Leading Order (Cont.)

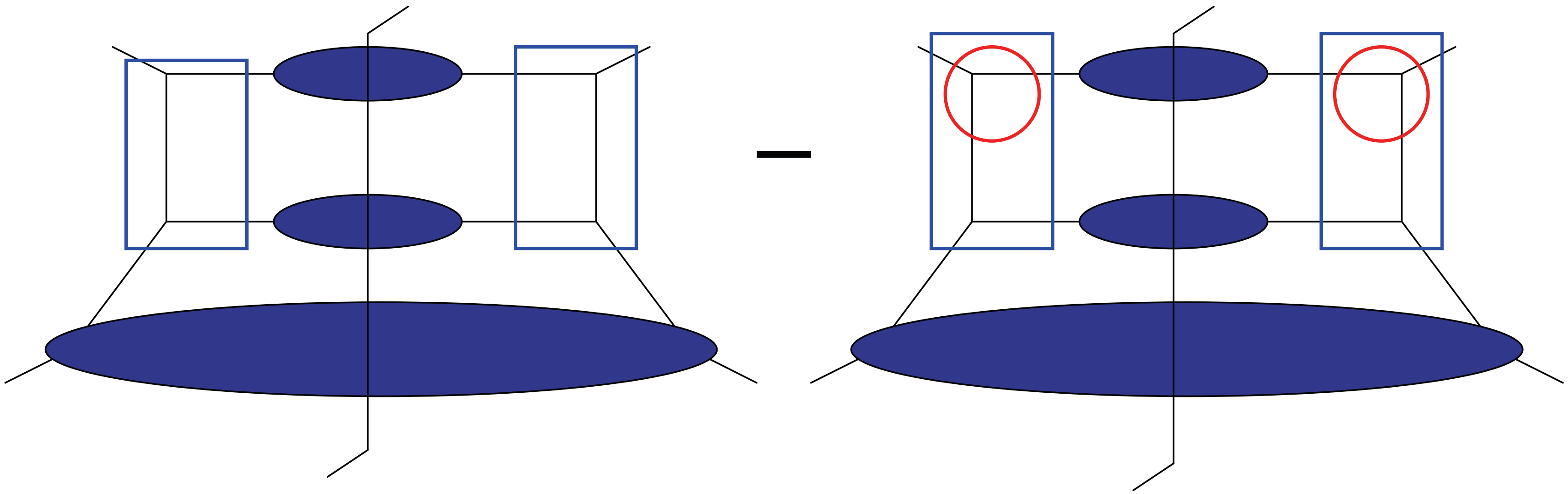
- Error in LO calculation:



$$\Gamma = T_{LO}\Gamma + \underbrace{(\Gamma - T_{LO}\Gamma)}_{\text{Non-vanishing for wide angles.}}$$

Next-to-Leading Order (Cont.)

- Apply approximation appropriate for wide-angle regime:



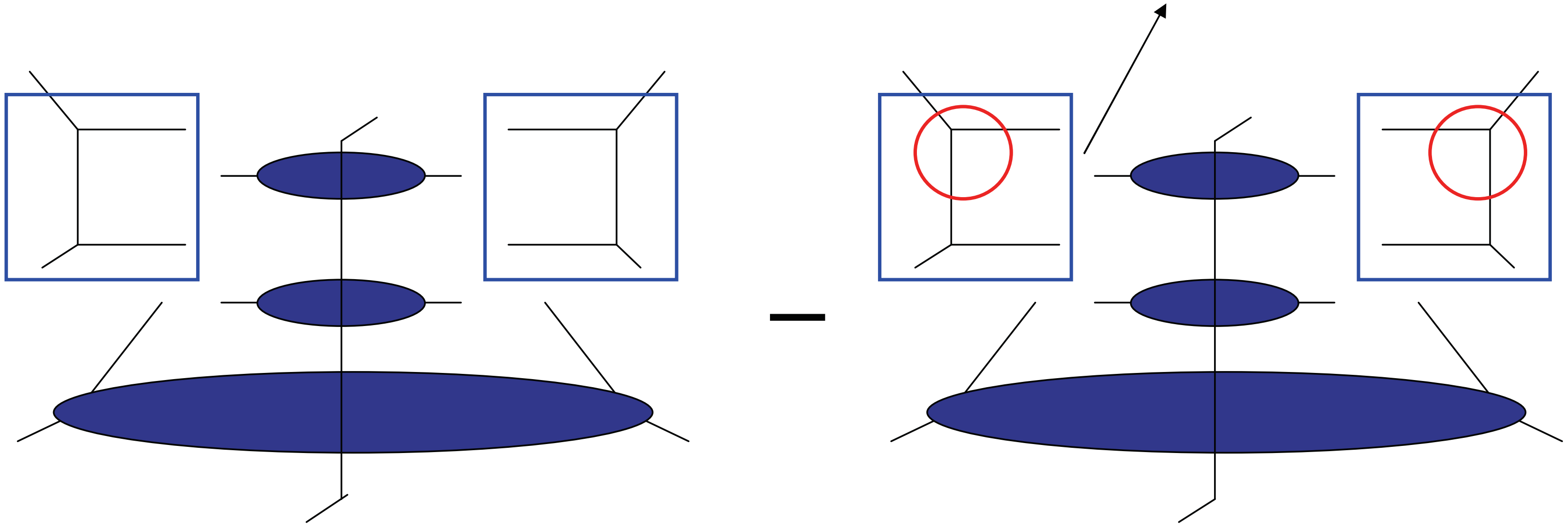
$$\Gamma = T_{LO}\Gamma + \underbrace{T_{NLO}(\Gamma - T_{LO}\Gamma)} + \mathcal{O}((\Lambda/Q)\Gamma)$$

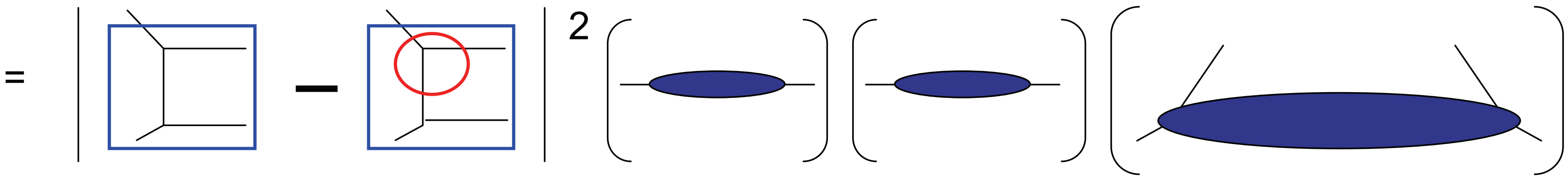
Calculate error term using approximation appropriate for wide angles.

Next-to-Leading Order (Cont.)

- Apply NLO approximations:

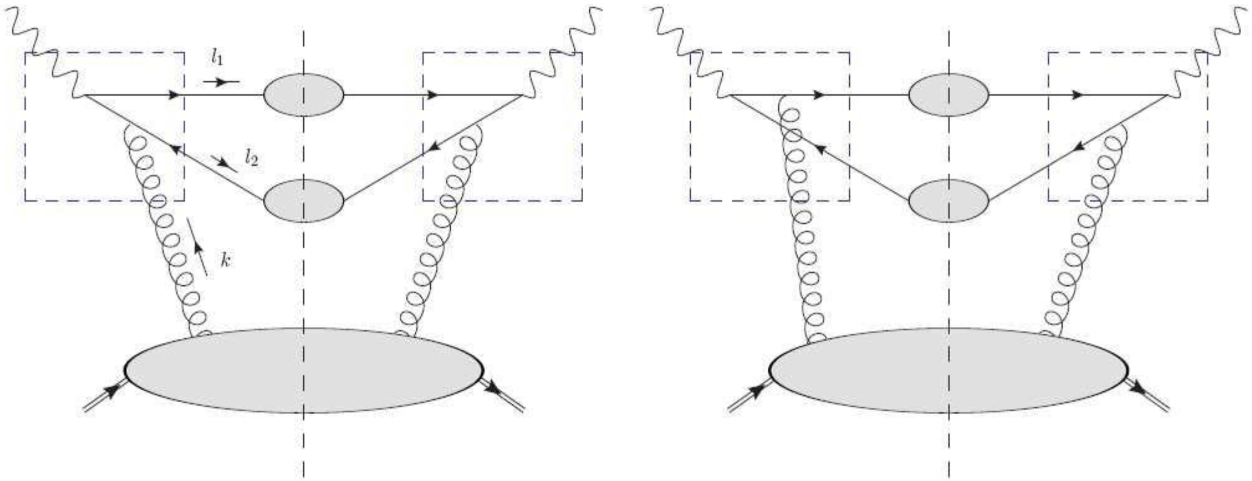
Layered approximations



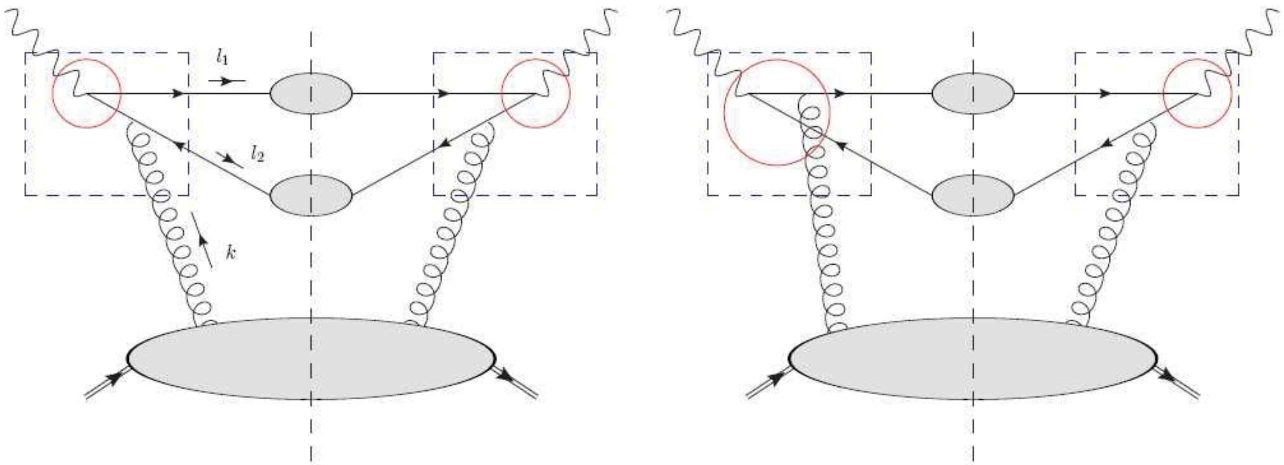


Next-to-Leading Order (Cont.)

(TCR, PRD78:074018,2008)

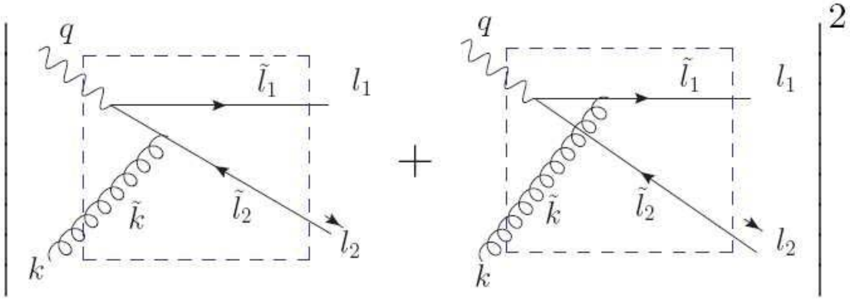


- Must subtract:

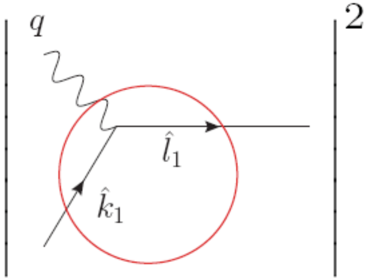


Fully Unintegrated Hard Scattering:

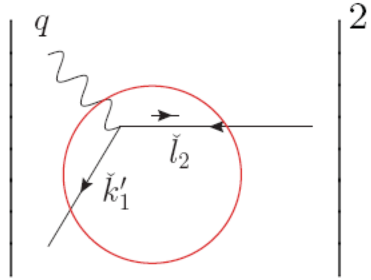
$$\tilde{W}^{\mu\nu}_{\gamma^* g\to q\bar{q}}(l_1, l_2, k) =$$



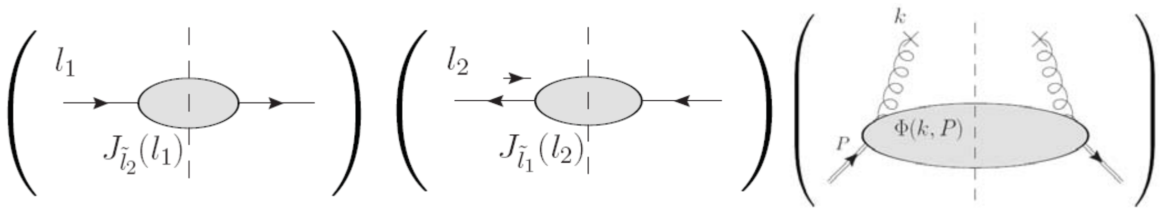
$$-\;\Xi_a(l_1, l_2, k)$$



$$-\;\bar{\Xi}_d(l_1, l_2, k)$$

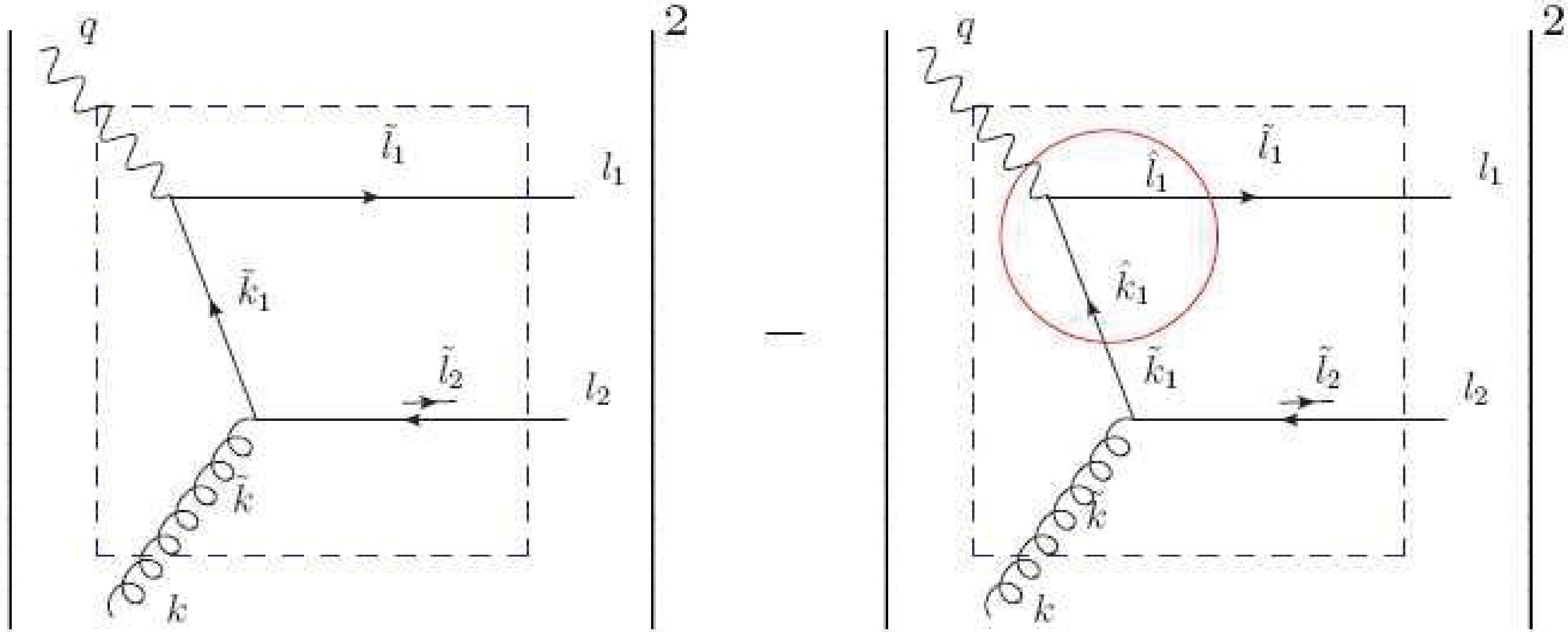


$$\Rightarrow\; \tilde{W}^{\mu\nu}_{\gamma^* g\to q\bar{q}}(l_1, l_2, k)$$



NLO Hard Scattering

- Structure of fully unintegrated hard scattering coefficient: layered approximations.



NLO Hard Scattering

- Explicit expressions have been obtained for including crossed diagrams and antiquark target.

- Mapping from exact to approximate variables in wide angle region:

$$\tilde{k} = \left(-q^+ + \frac{\tilde{l}_{1,t}^2}{2\tilde{l}_1^-} + \frac{\tilde{l}_{2,t}^2}{2\tilde{l}_2^-}, 0, \mathbf{0}_t \right)$$

$$\tilde{l}_1 = \left(\frac{\tilde{l}_{t,1}^2}{2\tilde{l}_1^-}, \tilde{l}_1^-, \tilde{\mathbf{l}}_{1,t} \right)$$

$$\tilde{l}_2 = \left(\frac{\tilde{l}_{t,2}^2}{2\tilde{l}_2^-}, \tilde{l}_2^-, \tilde{\mathbf{l}}_{2,t} \right)$$

$$\tilde{\mathbf{l}}_{1,t} = \mathbf{l}_{1,t} - \mathbf{k}_t/2$$

$$\tilde{l}_1^- = l_1^- - k^-/2$$

$$\tilde{\mathbf{l}}_{2,t} = \mathbf{l}_{2,t} - \mathbf{k}_t/2$$

$$\tilde{l}_2^- = l_2^- - k^-/2$$

What has been accomplished:

- Full factorization for scalar theory.

  Collins, Zu, JHEP 03 (2005) 059

- Abelian Gauge Theory: Detailed treatment of factorization for case of a single outgoing jet in DIS (SIDIS).

  (Collins, TCR, Stasto, PRD77:085009,2008)

  Suggestive of complete all-orders treatment in QCD

- Simplest NLO wide-angle hard scattering calculation in DIS. ***LO + NLO consistent with factorization point-by-point in phase space***

  (TCR, PRD78:074018,2008)

Outlook

- Practical Issues:
  - Need to work with experimentalists: Possibilities for exaction?
  - Need to finish calculation of NLO hard scattering.
  - Hadron-hadron collisions.
  - Simplifications?
  - Implementation in MCEGs.
- Open problems in formalism:
  - Evolution equations (CSS formalism??)
  - Relationship to other approaches?
  - Full extension to non-Abelian case.
  - General treatment of Ward identities.

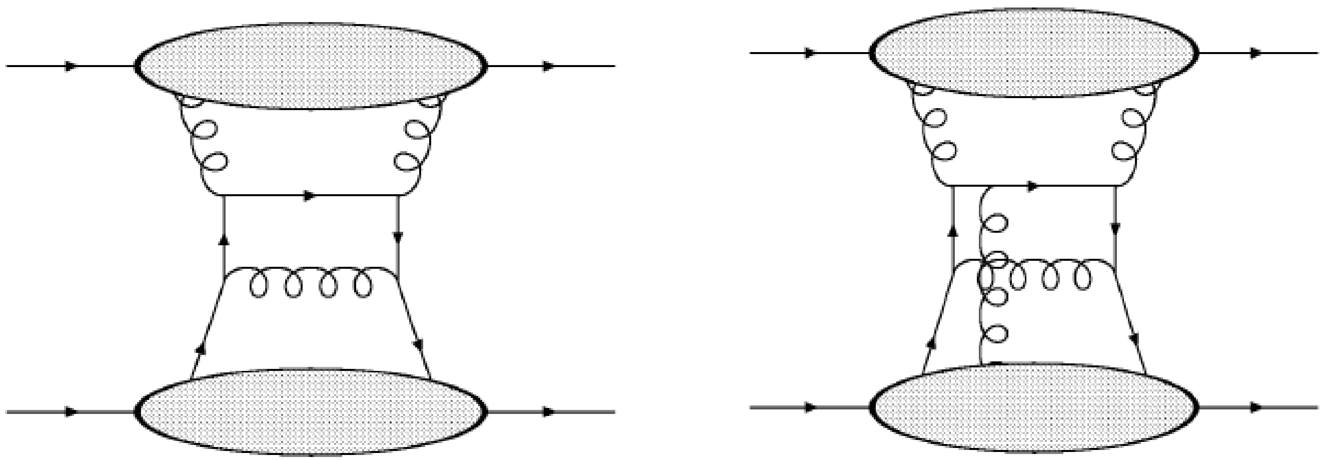
Backup Slides

K_T -Factorization in Hadron-Hadron Collisions

- Standard K_T-factorization generally fails in hadro-production of high-P_T particles.

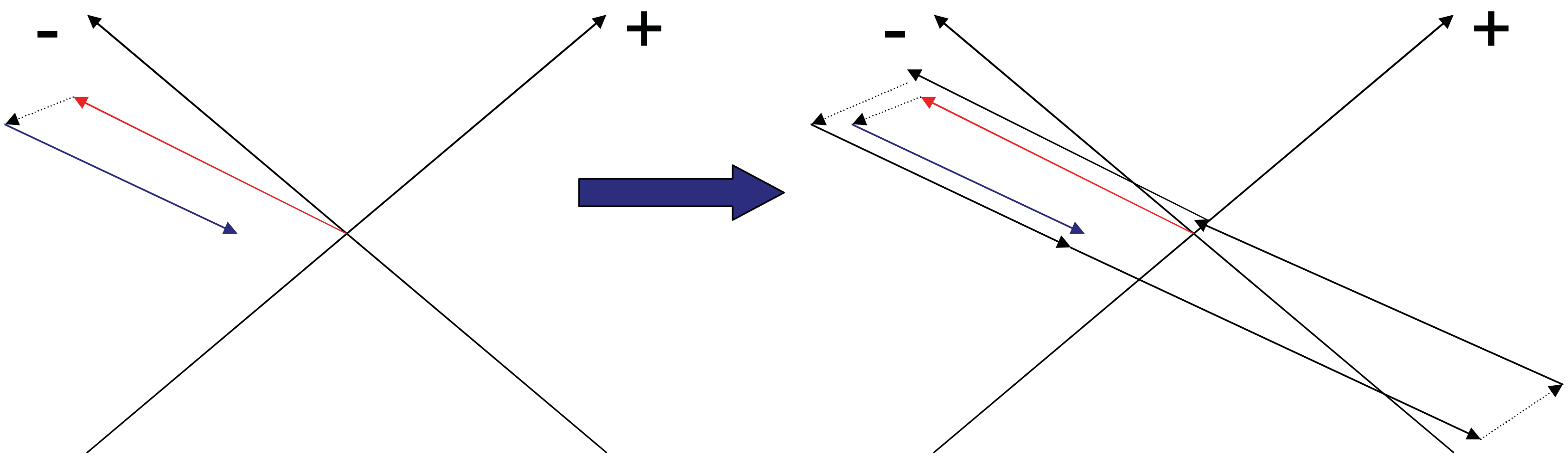
(Collins and Qiu, PRD75:114014,2007)

- Simple Wilson line structure in PDFs does not follow from Ward identity.



Hadron-Hadron Collisions (cont.)

- Can a generalization of K_T-factorization be formulated with well-defined but non-standard Wilson lines?



(Mulders et al, Eur.Phys.J.C47:147-162, 2006)