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Workshop: Eternal Inflation

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A new measure for eternal inflation

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A new measure for eternal inflation

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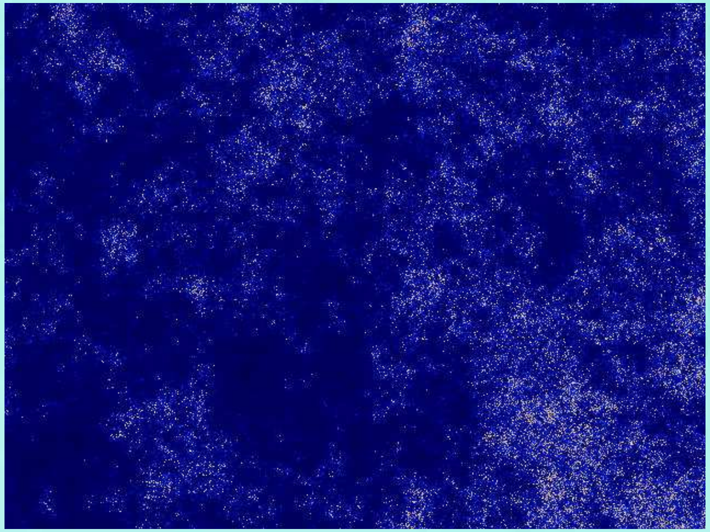
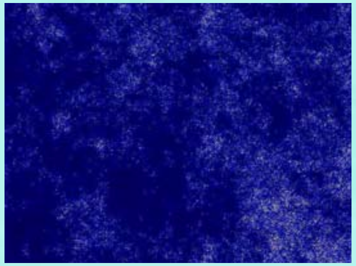


Trieste 08.06.2009

Probability measure in multiverse cosmology



- Motivation: “constants of nature”
- Inflation and “self-reproducing” space-time
  - Random-walk type (scalar field)
  - Tunneling type (landscape)
  - A toy model: de Sitter bubbles
- Predictions and the “measure problem”
  - Stochastic description of spacetime
  - Measure problem
  - Comparison of cutoff prescriptions
- New measure proposal: restrict in probability space to finite future
  - Proposal for scalar-field models
  - Proposal for landscape models
  - First results
- Summary

Are “constants of nature” environmental?

- Cosmology: “entire” history of the “entire” universe

- Explained today’s cosmological data! (abundances of light elements, density fluctuations, CMB, homogeneity, ...)
  ... assuming inflation + DM + DE

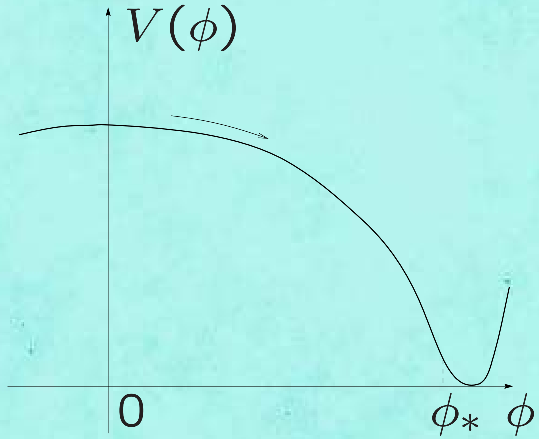
- Inflationary perturbations and/or landscape lead to eternal inflation

- Are *fine-tuned* “constants of nature” fundamental or environmental? (masses of elementary particles; cosmological constant $\Lambda \sim 10^{-120} M_{\text{Pl}}^4$; coupling constants for EM, weak, strong interactions)

Cosmological inflation

Models with one scalar field ϕ in Friedmann-Robertson-Walker (FRW) spacetime:

$$\mathcal{L} = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)$$
$$ds^2 = dt^2 - a^2(t)\,d\mathbf{x}^2$$



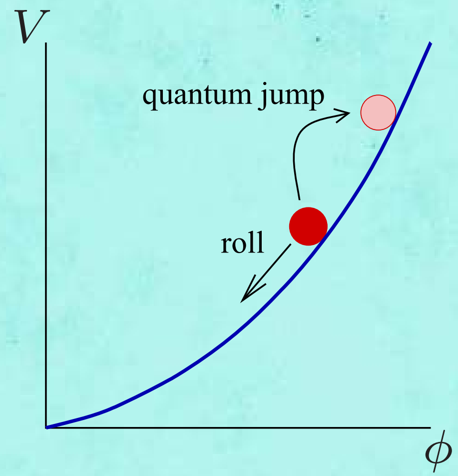
Evolution in the "slow roll" approximation:

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 \equiv H(\phi)^2 \approx \frac{8\pi}{3}V(\phi)$$
$$\frac{d\phi}{dt} \approx -\frac{V'(\phi)}{3H} = -\frac{H'(\phi)}{4\pi} \equiv v(\phi)$$

Exponential expansion: $H(t) \approx \text{const} \;\Rightarrow\; a(t) \sim \exp(Ht)$

Reheating near $\phi = \phi_*$ followed by "standard cosmology"

Inflation as a random walk



- Quantum fluctuations of ϕ generate "jumps" on top of the "slow roll"

[Linde 1983, Vilenkin 1983]

- Langevin equation for coarse-grained field ϕ:

[Starobinsky 1986]

$$\frac{d\phi}{dt} = -\frac{H'(\phi)}{4\pi} + \xi(\mathbf{x}, t); \qquad \left\langle \xi(\mathbf{x}, t)\, \xi\left(\mathbf{x}, t'\right) \right\rangle \approx \frac{1}{4\pi^2} H^3 \delta\left(t - t'\right)$$

Correlation function of "noise":

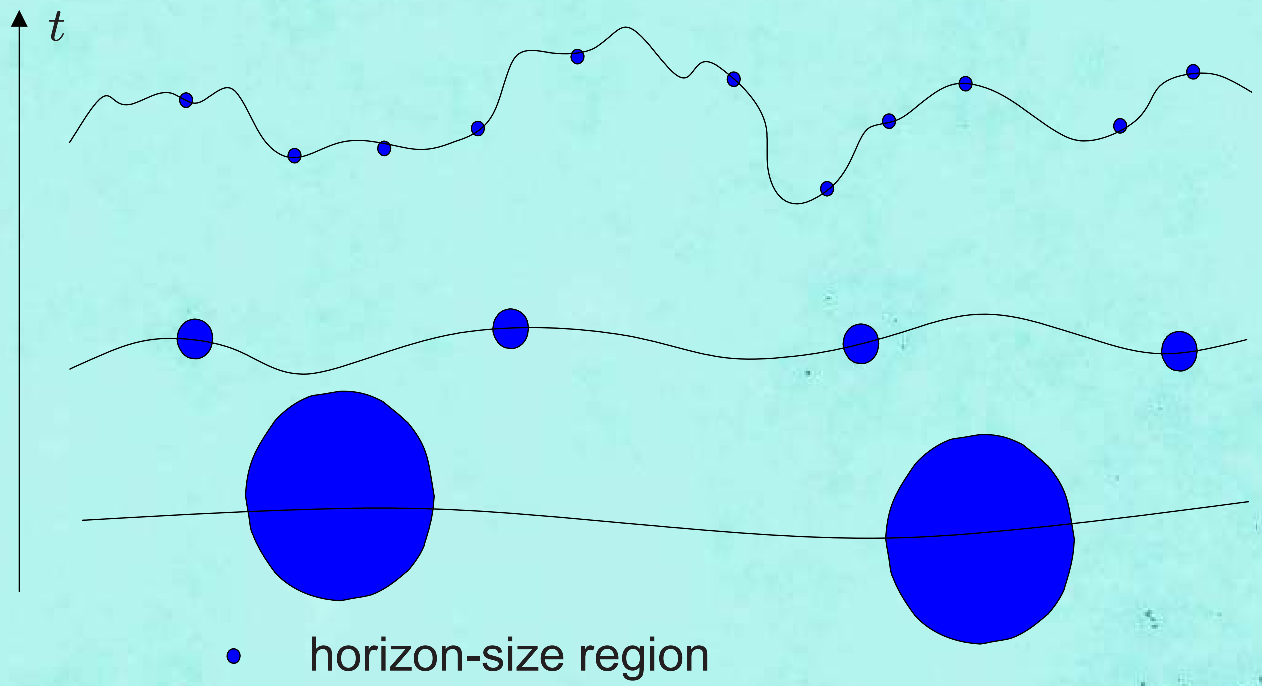
[Winitzki, Vilenkin 1999]

$$\langle \xi(\mathbf{x}, t)\, \xi(\mathbf{x}', t) \rangle \propto |\mathbf{x} - \mathbf{x}'|^{-4} \text{ for } H|\mathbf{x} - \mathbf{x}'| \gg 1$$

$$\langle \xi(\mathbf{x}, t)\, \xi(\mathbf{x}, t') \rangle \propto \exp(-2H|t - t'|) \text{ for } H|t - t'| \gg 1$$

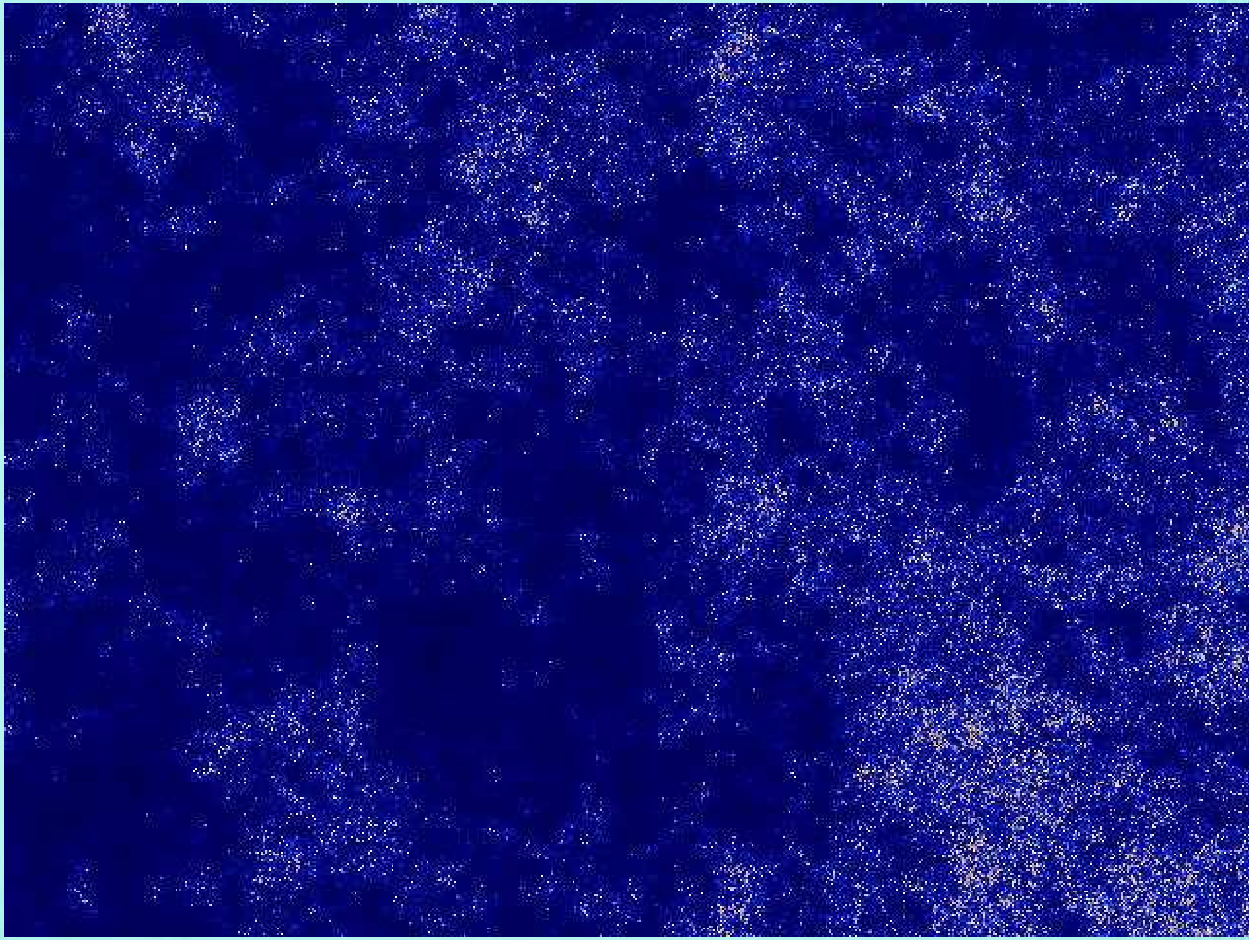
Evolution of field values

Surfaces $\phi =$ const:



Inhomogeneities develop on scales $\gtrsim H^{-1}$ in both space and time

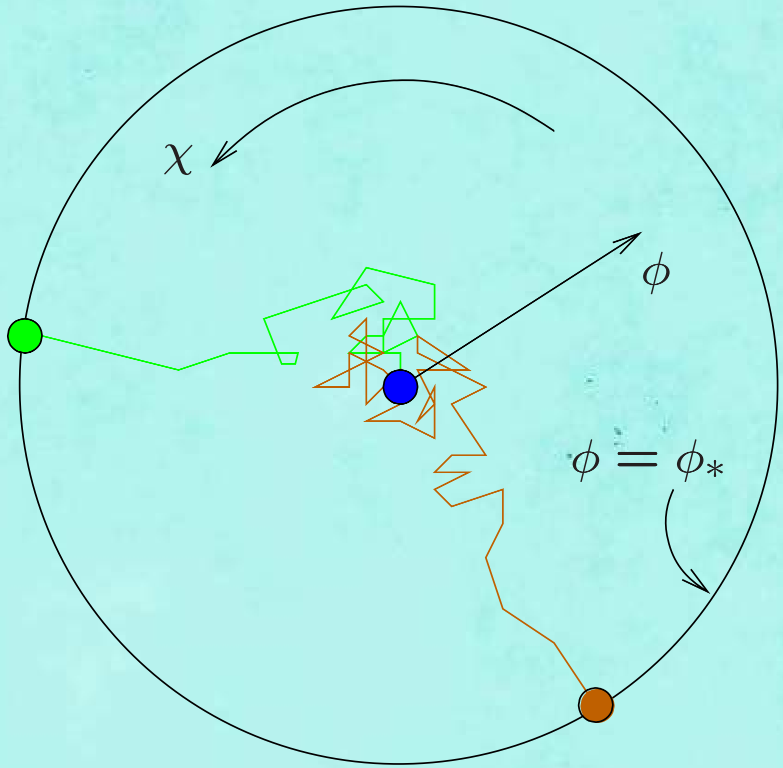
Fractal structure of the inflating domain



- Fractal dimension can be computed [Winitzki 2002]

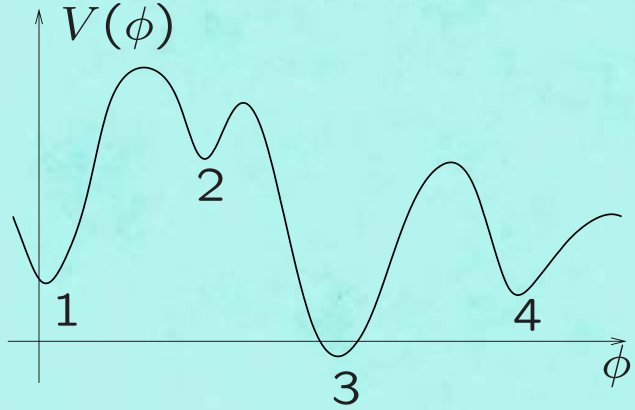
Distribution of observable parameters

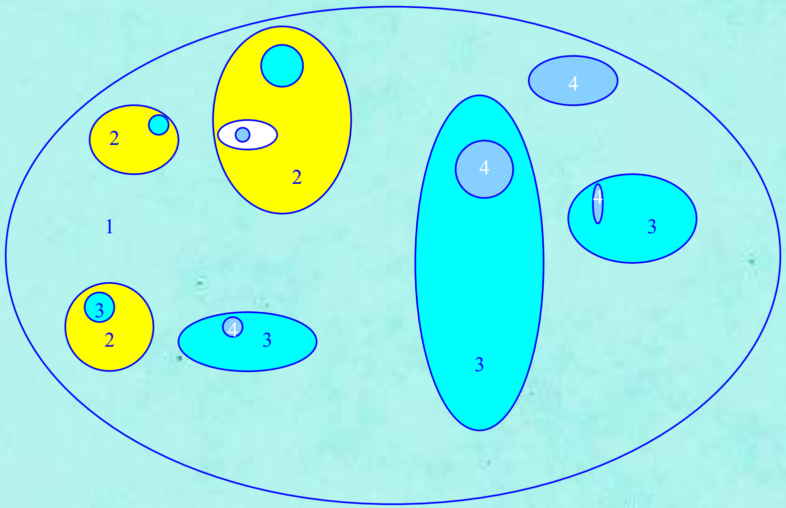
Models with several scalar fields (hybrid inflation, Brans-Dicke, etc.)



Have a *distribution* of χ along the reheating surface $\phi = \phi_*$
(parameter χ may be continuous or discrete)

Tunneling models: “recycling universe”

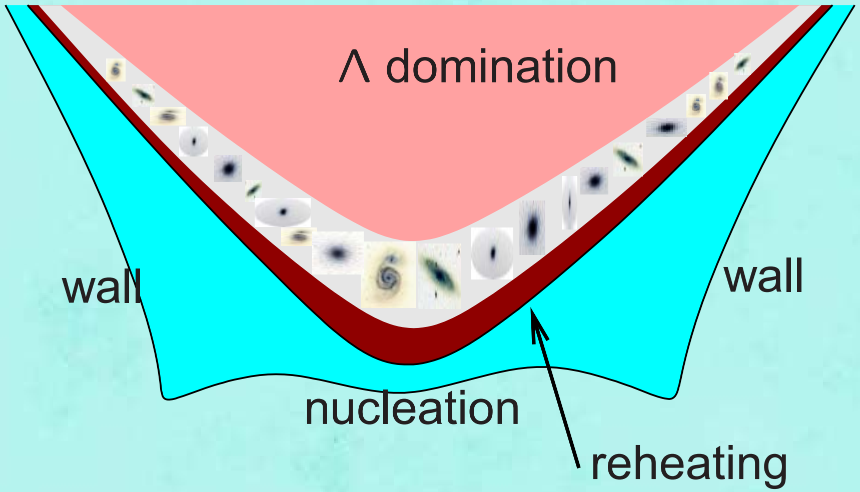


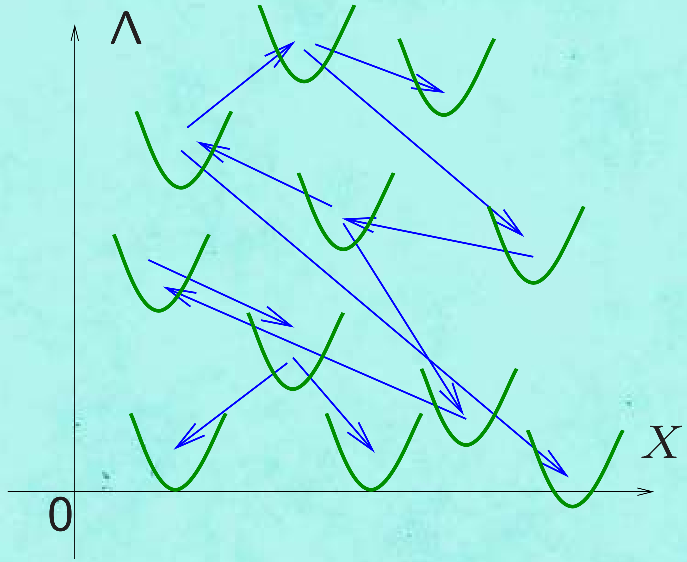


Each bubble contains an infinite number of other bubbles

(though “anti-de Sitter” states such as “3” collapse to singularity)

Tunneling models: landscape of string theory





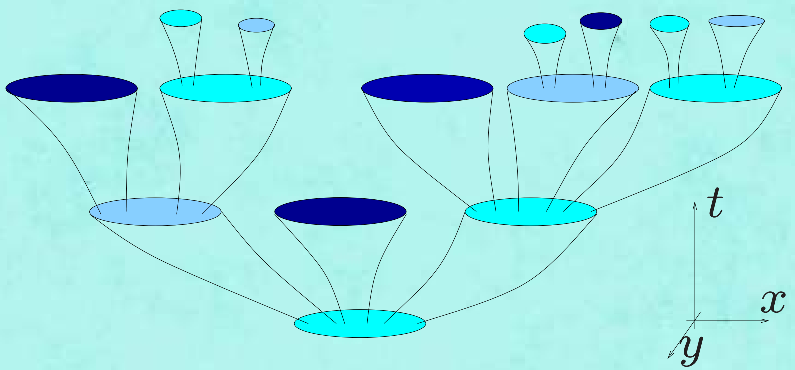
Each pocket universe is a “bubble” of FRW spacetime

Huge number of vacua (10^{500} or 10^{1000})

[Lerche, Lüst, Schellekens 1987]

Transition rates between vacua are known — *in principle*

Eternal inflation: qualitative features



Random walk inflation:

- independent domains of size $\gtrsim H^{-1}$ on timescales $\gtrsim H^{-1}$
- Reheating at $\phi = \phi_*$ followed by standard cosmology
- Inhomogeneous metric: $ds^2 = dt^2 - a^2(\mathbf{x}, t)\, d\mathbf{x}^2$

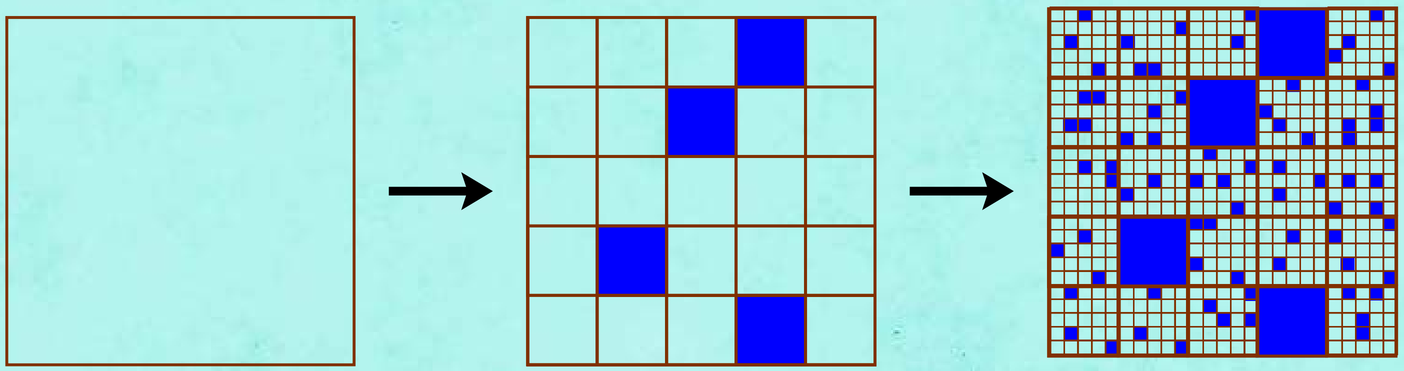
Tunneling-type inflation:

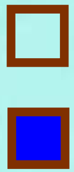
- independent domains of size $\gtrsim H_a^{-1}$ on timescales $\gtrsim \Gamma_{a\to b}^{-1}$
- Reheating within each bubble
- Evolution ends at “sinks”
- “Piecewise de Sitter” metric

- Inflation lasts arbitrarily long at some places – “eternal self-reproduction”

Eternal inflation in a box

Discrete spacetime simulation in 2+1 dimensions:



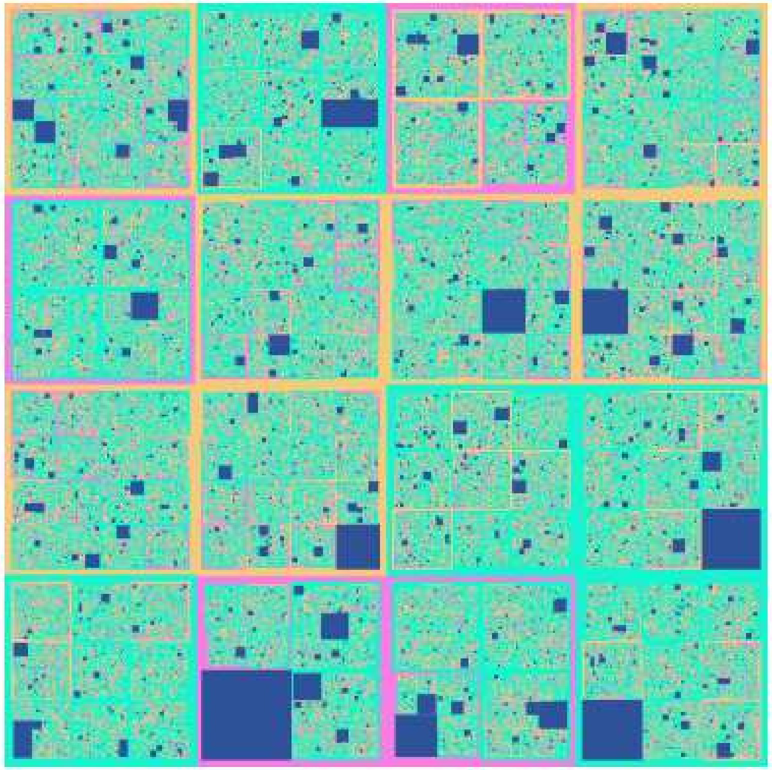


inflating H-regions

reheated H-regions

- Imitates bubble nucleation in de Sitter spacetime
- "Eternal inflation" = white squares multiply

Simulation with several bubble types



Predictions in eternal inflation

- Would like to obtain probability distribution for observables,
  — hoping to find that “constants of nature” are *not fine-tuned*

- Inflation generates an *infinite* 3-volume from a finite initial patch
  (where are we in the universe?)

- Cannot do statistics directly on an infinite set!
  (compare infinitely many apples to infinitely many oranges?)

The “measure problem”

- General approach:
  - Describe the evolution of the universe as a *stochastic process*
  - Introduce a *cutoff* to reduce an infinite 3-volume to finite
  - Compute the *limit distribution* as the cutoff is removed

- Results depend on the way the cutoff is introduced!

Stochastic description of random-walk inflation

Langevin equation:

$$\phi(t+\delta t) = \phi(t) + v(\phi)\delta t + \xi\sqrt{2D(\phi)\delta t}$$

Fokker-Planck equation for volume-weighted distribution $P_V(\phi, t)$:

$$\frac{\partial P_V}{\partial t} = \partial_\phi \left[\partial_\phi \left(D(\phi)P_V\right) - v(\phi)P_V\right] + 3HP_V$$

Boundary conditions:

$$P_V(\phi_{\text{Planck}}) = 0, \qquad \partial_\phi \left(DP_V\right)_{\phi=\phi_*} = 0$$

Late-time asymptotic distribution:

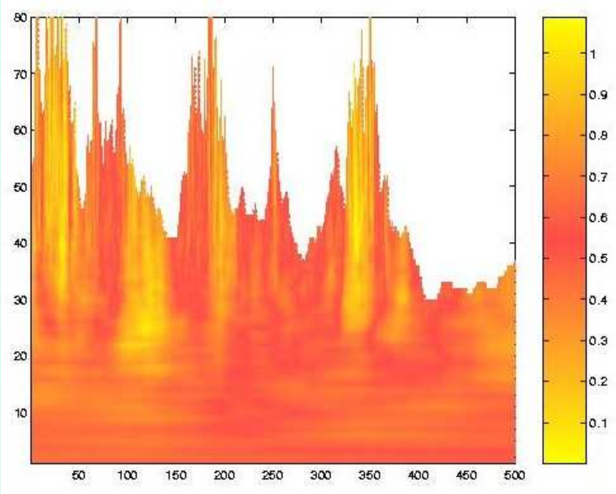
$$P_V(\phi, t) \approx f(\phi)e^{\gamma t}$$

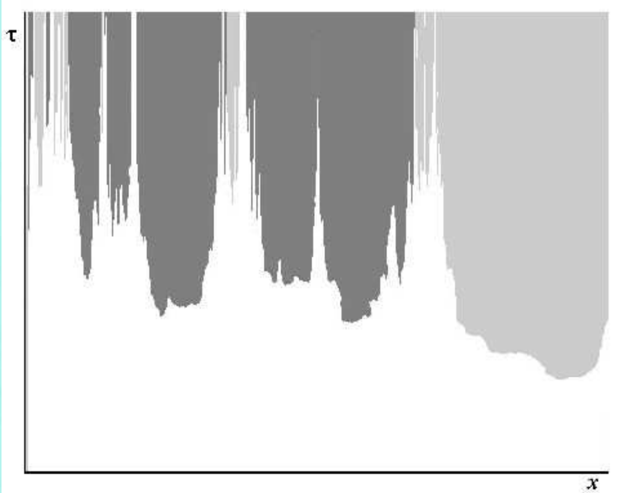
- Value of γ depends on choice of time coordinate t

- Eternal inflation is present if $\gamma > 0$, independent of the choice of t

[Winitzki 2002]

Simulation of random-walk inflation





- Model: two fields, χ fluctuates along the reheating surface $\phi = \phi_*$

[Vanchurin, Vilenkin, Winitzki 1998]

- Infinite portions of reheating surface generated near "spikes"

- Thermalized domains may be topologically disconnected [Winitzki 2004]

Stochastic description of tunneling-type inflation

Tunneling rate:

$$\Gamma_{a\to b} = O(1)\, H_a^{-4} \exp\left[-S_{I(a\to b)} - \frac{\pi}{H_a^2}\right]$$

Volume distribution $V_a(t)$:

$$\frac{\partial V_a}{\partial t} = \sum_b (-\Gamma_{a\to b} V_a + \Gamma_{b\to a} V_b) + 3 H_a V_a$$

Late-time asymptotic:

$$V_a(t) \approx f_a e^{\gamma t}$$

- Values of γ and f_a depend on choice of time coordinate t

Stochastic description + measure proposal = predictions

Gravitational constant in Brans-Dicke scenarios [Garcia-Bellido, Linde 1994]

Cosmological constant [Garriga, Vilenkin, *et al.* 1998-2008; Tegmark *et al.* 2003]

Amplitude of density fluctuations

[Garriga *et al.* 2005; Feldstein, Hall, Watari 2005]

Particle masses [Tegmark *et al.* 2003, 2005; Hall, Watari, Yanagida 2006]

Landscape probability distribution

[Vilenkin 2005-2008, Linde 2006-2008, Scherrer *et al.* 2007]

Measure proposals

Volume-based: Probability is proportional to 3-volume of reheating surface

[Linde 1994; Vilenkin 1995, 1998]

- Independent of initial conditions
- Need to specify a volume cutoff!
- Results depend sensitively on cutoff!

Worldline-based: Probability distribution along a single worldline

[Bousso et al. 2006-2008]

- Does not consider infinite reheated volume - no cutoff needed!
- Depends on initial conditions!

Volume cutoff proposals

- Equal-time cutoff: compute the volume thermalized before $t = t_{\text{max}}$, then set $t_{\text{max}} \to \infty$ [Linde et al. 1993]

  – Results depend sensitively on time slicing! ("gauge-dependent")

  – Youngness paradox, except when using scale factor $t = \ln a$

- "Spherical cutoff": take a sphere of radius R within the reheating surface, then set $R \to \infty$ [Vilenkin 1998; Vanchurin, Vilenkin, Winitzki 1999]

  – Gauge-independent, but difficult to implement calculations (need simulations of inflating spacetime through many e-folds!)

- "Stationary measure": Cutoff at time t_ε when volume distribution becomes stationary [Linde 2006]

  – Possible small dependence on time slicing; cannot use e-folding time!

  – Youngness paradox is avoided

[Bousso et al. 2008; Linde, Vanchurin, Winitzki 2008]

Problems with equal-time cutoff

Equal-time cutoff with proper time has “youngness paradox”

[Linde 1995, Tegmark 2004]

A delay in reheating is exponentially rewarded! (But the CMB temperature is not 100°C!)

There is no “correct” time slicing [Winitzki 2005]

Time $t = \ln a$ has advantages [Linde, Vilenkin, et al. 2008]

A new volume-based measure proposal

Need to cut the infinite volume of the reheating surface R, but without introducing any geometric bias

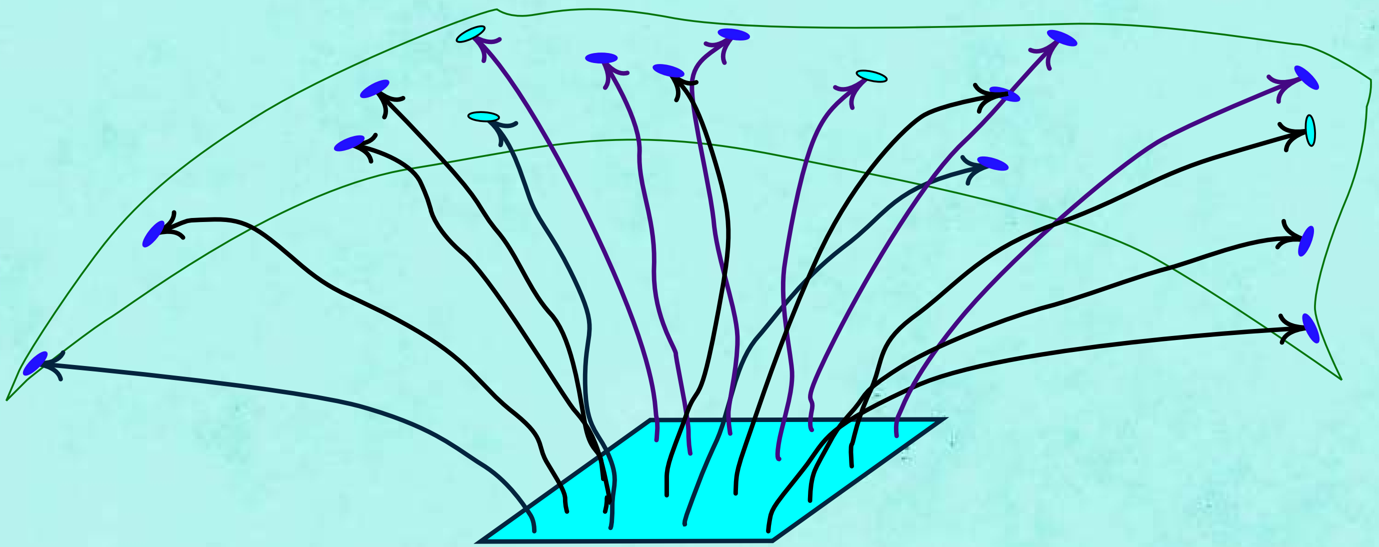
There is a small probability that R has a *finite* volume V

Proposal: *Reheating Volume* (RV) *cutoff* [Winitzki 2008]

- Consider the ensemble *conditioned in probability* on finite V
- Compute the volume-weighted distribution of cosmological parameters Q throughout V, e.g. $\langle Q \rangle = \frac{\int_R Q dV}{V}$ or more generally $p(Q|V)$
- RV cutoff defines $p(Q) = \lim_{V \to \infty} p(Q|V)$ if the limit exists

RV measure is applicable to any scenario where inflation ends globally with *nonzero* probability — need specific implementation in calculations

RV measure for random-walk inflation



Consider only events with *finite* reheating surface with final 3-volume V

- Distribution of observables is computed within finite volume V
- As $V \to \infty$, the limit distribution is independent of the initial state

RV measure for random-walk inflation: computations

Generating function for finite reheating volume (ϕ now denotes all fields):

$$g(z;\phi) \equiv \left\langle e^{-zV} \right\rangle_{V<\infty} \equiv \int_0^\infty e^{-zV}\,\text{Prob}(V;\phi)dV$$

Can be found by solving the nonlinear FP equation, [Winitzki 2008]

$$Dg_{,\phi\phi} + vg_{,\phi} + 3Hg\ln g = 0, \quad g(z;\phi_*) = e^{-zH^{-3}(\phi_*)}$$

The distribution $\text{Prob}(V;\phi)$ is found as inverse Laplace transform of g,

$$\text{Prob}(V;\phi) = \frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} e^{zV} g(z;\phi)dz$$

$\Rightarrow$ Can compute distributions of observables at finite V, then set $V \to \infty$:

$$\frac{\text{Prob}(\chi=\chi_1)}{\text{Prob}(\chi=\chi_2)} \equiv \lim_{V\to\infty} \frac{\text{Prob}(V;\phi=\phi_*,\chi=\chi_1)}{\text{Prob}(V;\phi=\phi_*,\chi=\chi_2)}$$

Result: obtain RV-regulated distribution of χ at reheating

RV measure for landscape

Vacua of types $j = 1, 2, ..., N$ and transition probabilities $\Gamma_{i \to j}$
Condition on finite *total number* of bubbles ($n_{\text{tot}} \equiv n_1 + n_2 + ...$)
Generating function for n_i, starting with bubble k:

$$g(z, q_1, q_2, ...; k) \equiv \sum_{n_1, n_2, ... < \infty} P(n_1, n_2, ...; k) z^{n_{\text{tot}}} q_1^{n_1} q_2^{n_2} ...$$

Function $g(z, q_1, q_2, ...; k)$ satisfies the equation

$$g^{1/\nu}(...; k) = \sum_j \Gamma_{k \to j} z q_j g(...; j) + \Gamma_{k \to k} g(...; k)$$

Note: $g(...; k) = 1$ for terminal bubbles k

Implement RV cutoff: Compute $\frac{\langle n_1 \rangle_{n_{\text{tot}}=n}}{\langle n_2 \rangle_{n_{\text{tot}}=n}}$ conditioned on fixed $n_{\text{tot}} < \infty$,

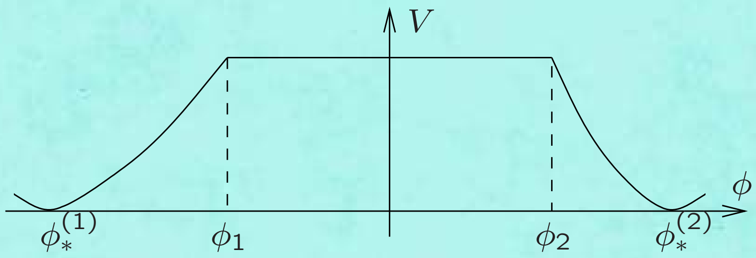
$$\frac{\langle n_1 \rangle_{n_{\text{tot}}=n}}{\langle n_2 \rangle_{n_{\text{tot}}=n}} = \left. \frac{\partial_z^n \partial_{q_1} g(z, q_i; k)}{\partial_z^n \partial_{q_2} g(z, q_i; k)} \right|_{z=0, q_i=1}$$

Then take the limit of the above as $n \to \infty$.

- The limit exists, is independent of the initial bubble k [Winitzki 2008]

RV measure: examples

- Toy model of random-walk inflation: [Winitzki 2008]



Probability ratio for exit at $\phi_*^{(1)}$ vs. $\phi_*^{(2)}$ (slow-roll expansion a_1, a_2):

$$\frac{P(2)}{P(1)} \approx O(1) \frac{H^{-3}(\phi_*^{(2)})}{H^{-3}(\phi_*^{(1)})} \frac{a_2^3}{a_1^3} \exp\left[3N_{12}\right], \text{ where } N_{12} \equiv \frac{\pi^2 \left(\phi_2 - \phi_1\right)^2}{\sqrt{2} H_0^2}.$$

- Toy model of landscape: one “top” vacuum ($j = 1$), many ($j = 2, ..., N_r$) “low” vacua, known transition rates $\kappa_{i \to j}$, transitions to “terminal” vacua $\kappa_{i \to T}$. Assuming $\kappa_{1 \to i} \gg \kappa_{j \to k}$.

$$\frac{p(j)}{p(k)} \approx \frac{\kappa_{1 \to j}}{\kappa_{1 \to k}} \left(\frac{\kappa_{j \to T}}{\kappa_{k \to T}} \right)^{\nu}, \quad j, k = 2, ..., N_r,$$

Features of the RV measure

Does not suffer from problems found with previous measures:

- No dependence on time slicing (use only intrinsic 3-volume of reheating 3-surface)
- No dependence on initial conditions (ensemble is dominated by long evolution in high-H regime)
- No youngness paradox (delay in reheating is suppressed in probability)
- No “Boltzmann brains” (explicit calculations made) [Winitzki 2008]

Calculations can be implemented through PDEs (slow-roll inflation) or algebraic equations (landscape)

Summary

Eternal inflation is generic in most inflationary scenarios (but not in braneworld)

A complicated structure of spacetime on very large scales

Physical considerations needed to choose between volume-based and worldline-based prescriptions

New volume-based measure proposal, broadly applicable, with good properties

Specific calculations can be implemented

First results encoraging; need further work to apply to various models and compare predictions