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The cosmological constant as a lens on the measure problem

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The cosmological constant as a lens on the measure problem

(based in part on work with Andrea De Simone, Alan Guth, and Alexander Vilenkin)

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Alex Vilenkin

Presentation Organization

I. Outline

II. The “standard” anthropic landscape argument

III. The scale-factor cutoff measure

IV. The causal patch measure

V. The comoving probability measure

VI. Conclusions

The standard anthropic landscape argument

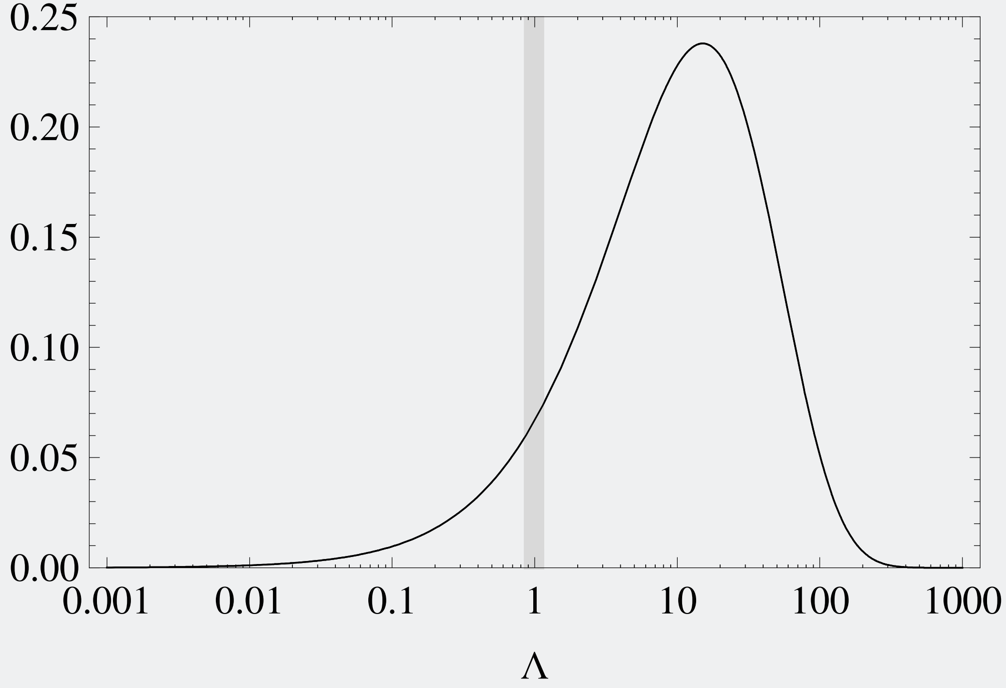
Standard anthropic landscape argument:

- An enormous landscape (read string landscape) contains a wide range of vacuum energies — including an essentially continuous distribution at $\mathcal{O}(10^{-123} M_P^4)$ with low-energy physics like ours.
- We measure such a small vacuum energy, despite its extremely unusual proximity to zero, because our existence relies on structure formation, which occurs only for such small values.

[Linde (1984); Banks (1985); Barrow and Tipler (1986); Weinberg (1987).]

The standard anthropic landscape argument

Weinberg *etal* made this argument precise, by calculating the collapse fraction into galaxies with $M \geq M_{\rm MW}$, making the proximity of our measurement to zero appear not too atypical:



[Martel, Shapiro, Weinberg (1998).]

The standard anthropic landscape argument

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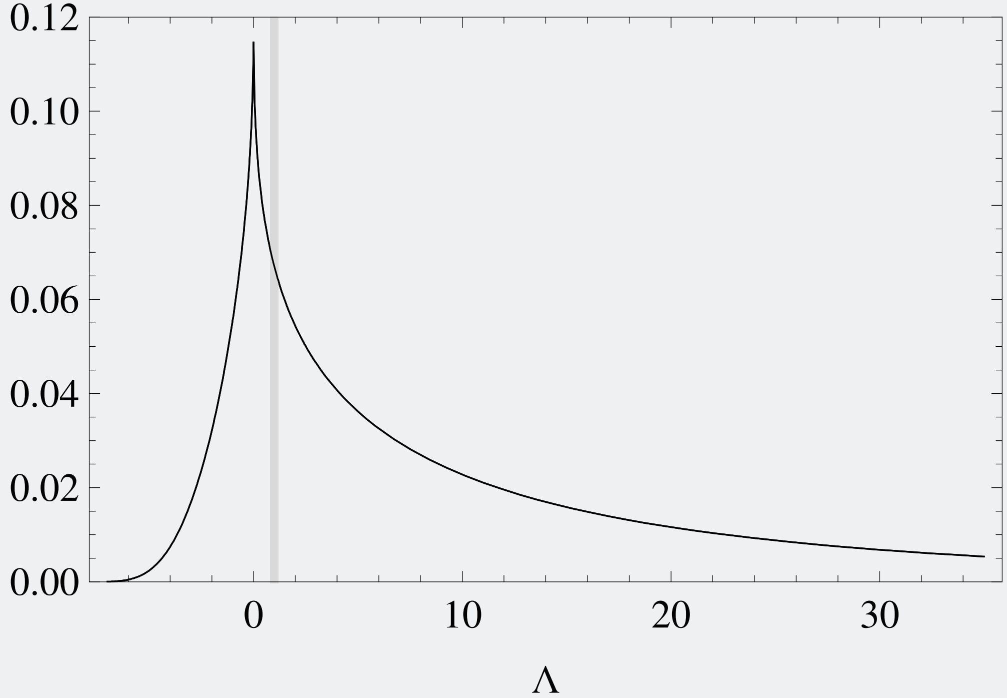


[Martel, Shapiro, Weinberg (1998).]

The standard anthropic landscape argument

But this isn't the end of the story...

Weinberg *etal*'s approach — to weight all comoving volumes along an FRW foliation equally — is known to suffer major pathologies:

- *Boltzmann brain domination:* observers arising due to fortuitous quantum fluctuations outnumber those arising from classical evolution of small inflationary perturbations.

  [Dyson, Kleban, Susskind (2002); Page (2006); Bousso and Freivogel (2006).]

- *Runaway inflation:* spacetime regions are weighted by a factor e^{3N}, where $N \gtrsim 60$ is the number of e-folds of inflation, giving exponential preference to observe extreme values of $\zeta(N)$ and $G(N)$.

  [Feldstein, Hall, Watari (2005); Garriga, Vilenkin (2006); Graesser, MPS (2007).]

Avoiding these pathologies necessarily changes the anthropic landscape prediction of the cosmological constant.

The standard anthropic landscape argument

To date, three measures are known (by me) to be able to circumvent these pathologies:

- *the "scale factor cutoff" measure*
- *the "causal patch" measure, and*
- *the "comoving probability" measure*.

Let us consider consider each of these from the perspective of its prediction for the cosmological constant.

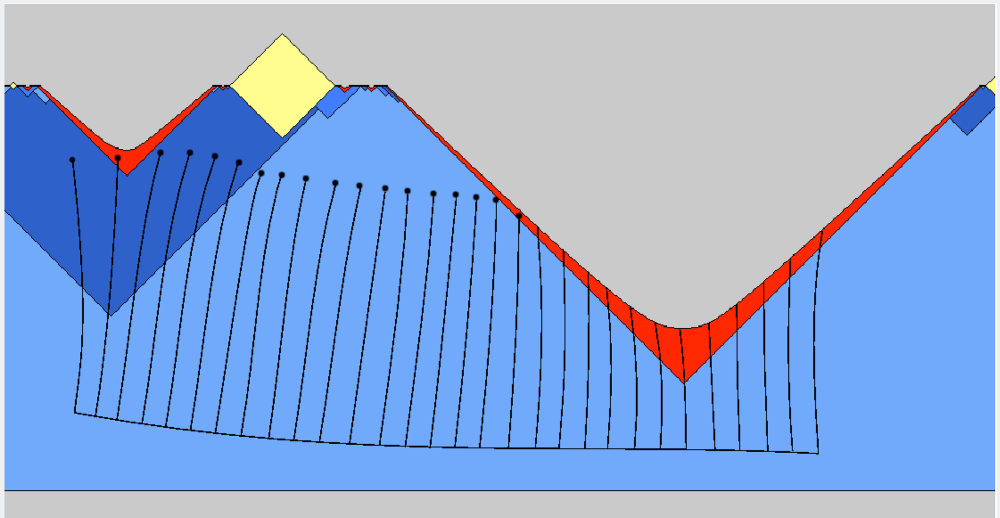
The scale-factor cutoff measure

The *scale-factor cutoff* measure is a form of global time cutoff, with "time" parametrized by

$$dt = H(\tau)\, d\tau\,,$$

where H is the Hubble rate and τ the proper time (i.e. $a(\tau) = e^t$).

[Garcia-Bellido, Linde, Linde (1994); Linde (2007); De Simone, Guth, MPS, Vilenkin (2008).]



The other obvious choice of time parametrization, $dt \propto d\tau$, suffers from the "youngness paradox" (and runaway inflation).

[Linde, Linde, Mezhlumian (1995); Guth (2000); Bousso, Freivogel, Yang (2007).]

The scale-factor cutoff measure

Since anthropic calculations focus on pocket universes where there is clear FRW symmetry, one can calculate an anthropic distribution of Λ using the FRW Hubble rate H to define scale-factor time — call this *"FRW" scale-factor time*.

For instance we can write

$$\frac{dP}{d\Lambda} \propto \lim_{t_c \to \infty} \int_{-\infty}^{t_c} e^{\gamma t_*} \, dt_* \int_{\tau_*}^{\tau_c(t_c, t_*)} \rho_{\rm obs}(\tau) \, a^3(\tau) \, d\tau \, ,$$

where $a(\tau_c)/a(\tau_*) \propto e^{t_c - t_*}$. The inner integral counts the number of observers in a comoving volume with reheating hypersurface at t_*, while the outer integral sums over such volumes. When in AdS vacua H becomes negative, we halt FRW scale-factor time.

[De Simone, Guth, MPS, Vilenkin (2008).]

The scale-factor cutoff measure

To generate predictions for Λ, here and below we use the following anthropic criteria:

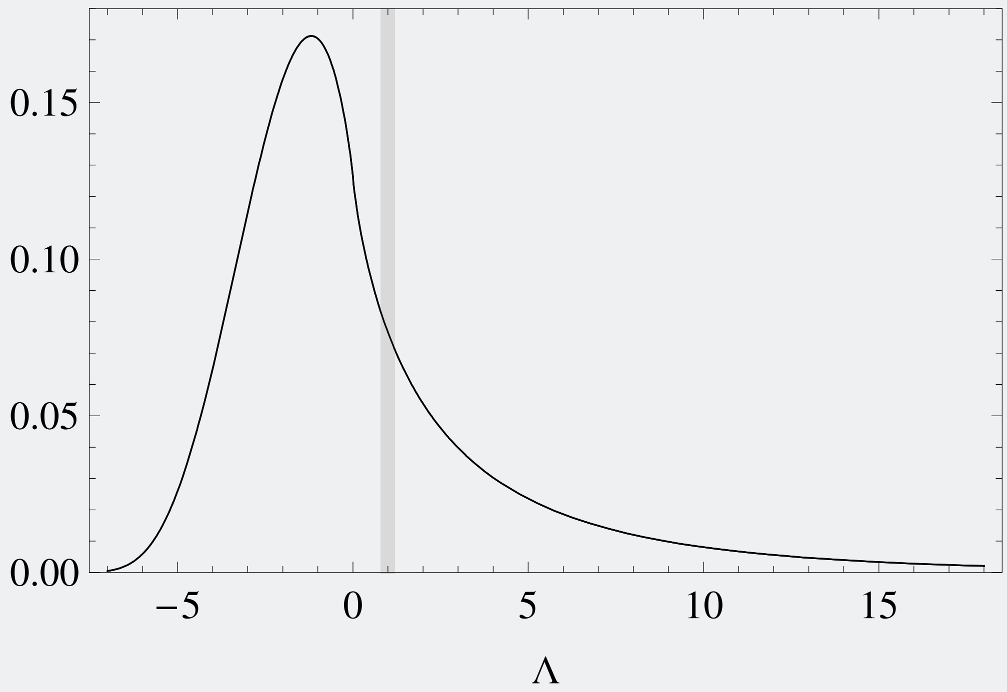
- observers arise 5 billion years after the formation of a Milky-Way mass galaxy (as determined by Press-Schechter analysis), i.e.

$$\rho_{\mathrm{obs}}(\Lambda,\tau) \propto \frac{1}{a^3}\,\frac{dF_{\mathrm{PS}}}{d\tau}\bigg|_{\tau-\Delta\tau},$$

  where F_{PS} is the Press-Schechter function evaluated at comoving scale enclosing $M = 10^{12} M_{\odot}$, and $\Delta\tau = 5\times 10^{9}$ yrs.

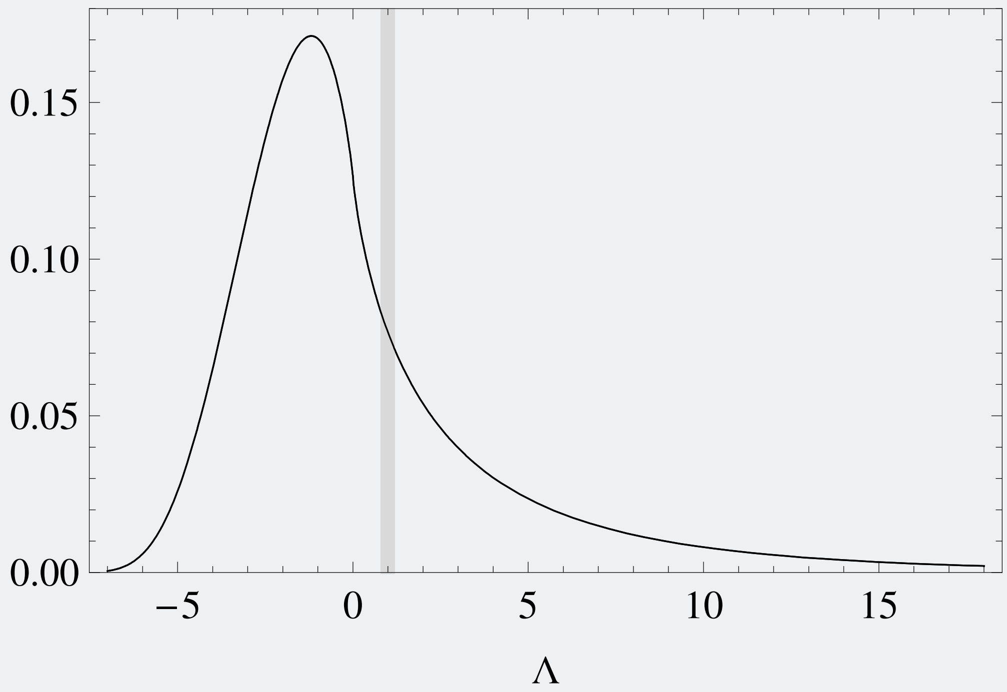
- when Λ is negative, observers do not arise after $\tau_{\mathrm{turn}} = \pi/3H_{\Lambda}$.

The result is in excellent agreement with observation:



- *25% of observers measure Λ closer to zero, 37% observe a less likely value, and 58% measure Λ to be further from the median.*
- The sharp suppression of large Λ stems from the need for observers to arise before exponential FRW expansion triggers the cutoff.

The result is in excellent agreement with observation:



Yet this prediction is made possible by our focusing on pockets with clear FRW symmetry, where H is unambiguously defined.

Generically, this will not be the case.

The scale-factor cutoff measure

One might generalize H according to

$$H = (1/3)\, u^{\mu}{}_{;\mu}\,,$$

where $u^{\mu}(x)$ is the four-velocity vector field along the congruence. At a point where geodesics intersect, the smallest scale-factor time is selected. Again, the scale-factor time is taken to halt if H becomes negative. We call this *"local" scale-factor time*.

This has the benefit of simplicity, but also raises some issues:

- in collapsing regions, the *congruence is extremely intricate*, giving scale-factor time very complicated local structure,
- it is hard to imagine how a theory of the multiverse on large scales will generate a measure concerned with such intricacy.

The scale-factor cutoff measure

To predict Λ, we average over this intricate structure by considering galaxies to start as spherical top-hat overdensities. Note that local scale-factor time halts when the overdensity *begins* to collapse.

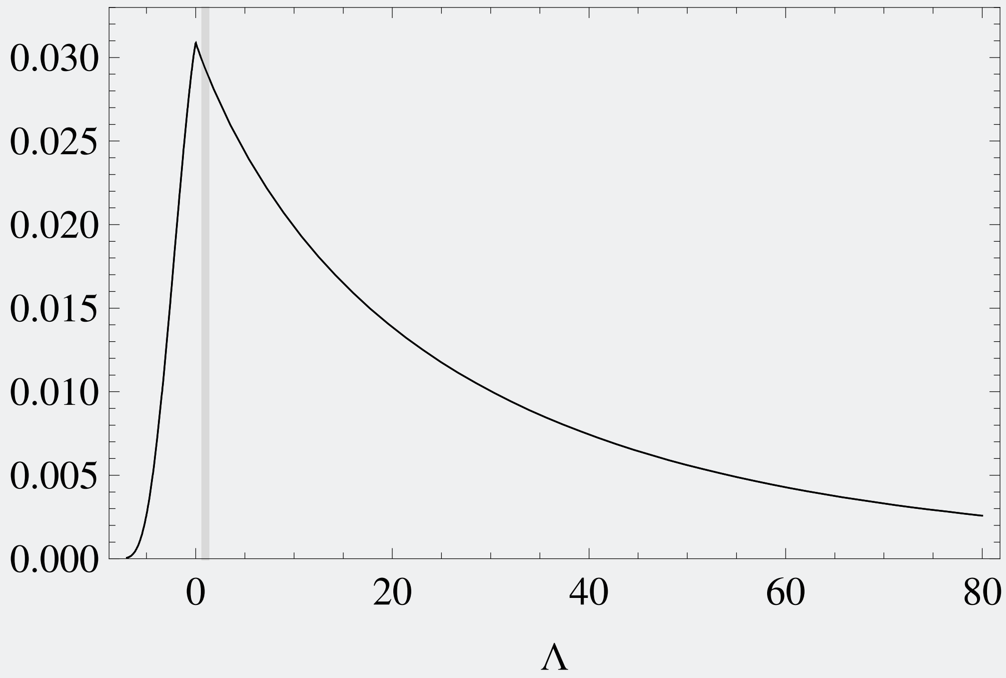
Footnote: Realistically, structure formation is hierarchical: small scales collapse before larger scales. When the region surrounding a given geodesic collapses, its scale-factor time halts. Thus, it would seem we cannot ignore structure formation on small scales. However, whether or not observers arise from a small collapsed structure depends on whether that structure combines with others to form a larger structure — ultimately a large galaxy. We model the requirement that small structures coalesce into larger ones as equivalent to requiring that structure formation occurs on the largest necessary scale, i.e. for a spherical top hat overdensity with $M = M_{\rm MW}$.

It can be shown that

$$\frac{dP}{d\Lambda} \propto \int_0^\infty \mathrm{erfc}\left[\frac{1 + (\rho_\Lambda/\rho_{\rm rec})\tilde{a}^3}{\sqrt{2}\,(5/3)\,\delta_{\rm rec}\,\tilde{a}}\right] \tilde{a}^{-\gamma-1} d\tilde{a}\,,$$

with some additional details to deal with negative Λ.

The local scale-factor time then predicts:

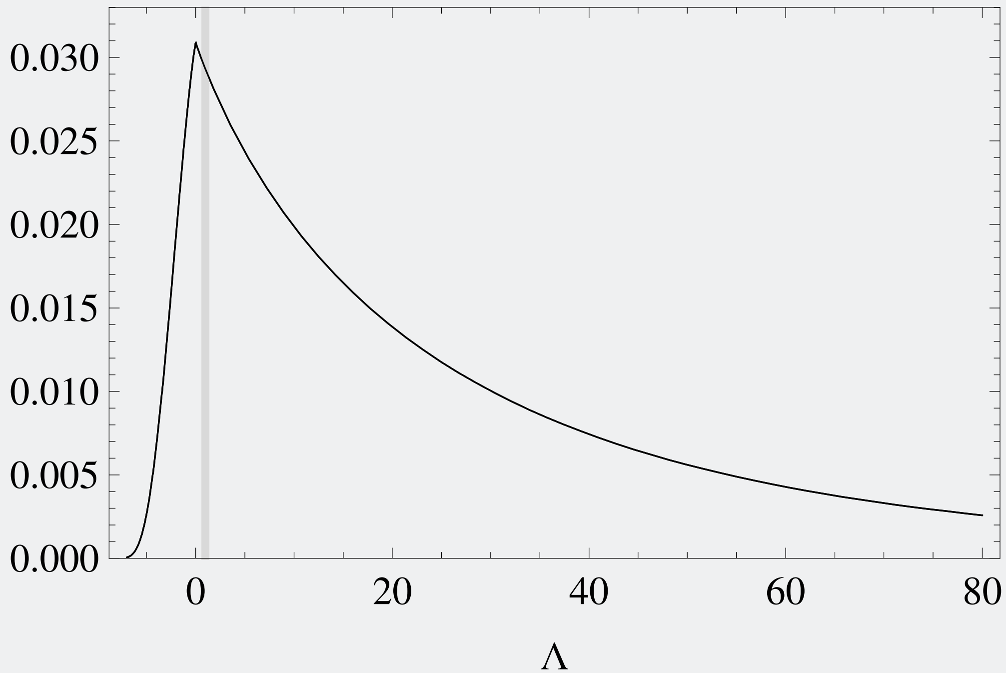


The proximity of our measurement of Λ to zero is not fully resolved;

- *only 6% of observers measure* Λ *to be closer to zero, and 96% of observers measure less likely values of* Λ*,*
- yet 23% of observers measure Λ to be “further from the median.”

The scale-factor cutoff measure

The local scale-factor time then predicts:



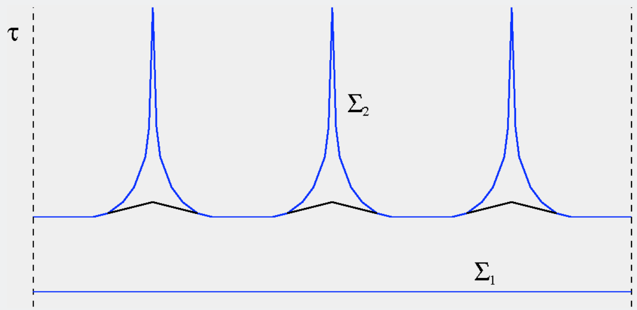
The result is noticeably poorer than Weinberg etal*:*

- local scale-factor time halts after the halo *begins* to collapse, requiring only that "turnaround" occurs before the cutoff,
- the measure is biased in favor of high cosmic matter densities.

The scale-factor cutoff measure

Are there other ways to unambiguously define the cutoff?

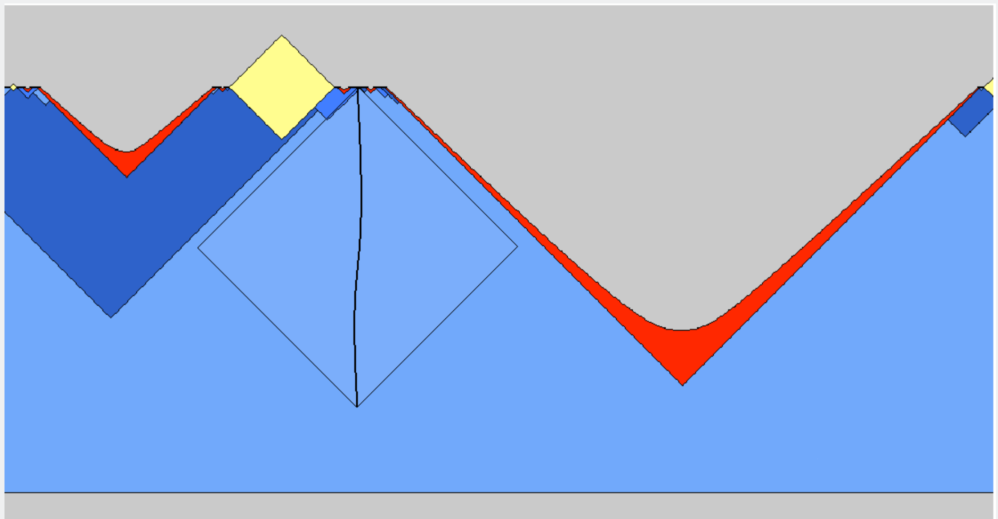
- spatial averaging — *tends to be complicated and ad hoc.*
- employ "local" scale-factor cutoff, but also excise the future lightcone of any point beyond the cutoff — *how to motivate?*

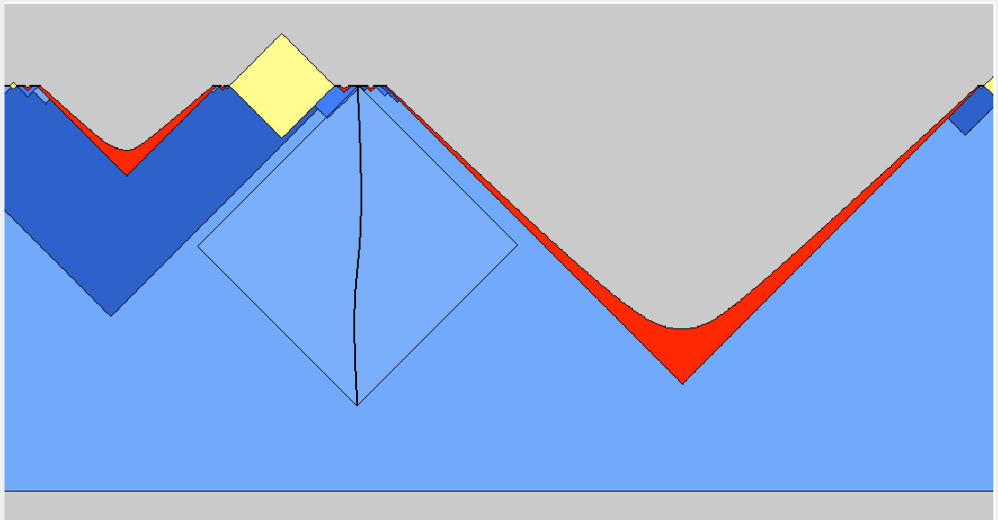


- employ "local" scale-factor cutoff, but when geodesics cross select the largest scale-factor time — *is the spacetime simply-connected?*
- replace the congruence with a "gas" of massless particles, and perform the cutoff when the density in the local static frame drops below a specified level — *does this work? Perturbations are undamped and net velocity flows persist.*

The *causal patch* measure is defined by

- selecting a worldline in a given de Sitter vacuum,
- considering all future histories of the worldline,
- creating an ensemble of causally-connected spacetimes according to those histories, and
- counting events according to their frequency in the ensemble.





Note that all the action occurs at the upper tip of the causal patch;

$$\frac{dP}{d\Lambda} \propto \int_0^\infty \rho_{\rm obs}(\tau)\, V_\diamond(\tau)\, d\tau\ ,$$

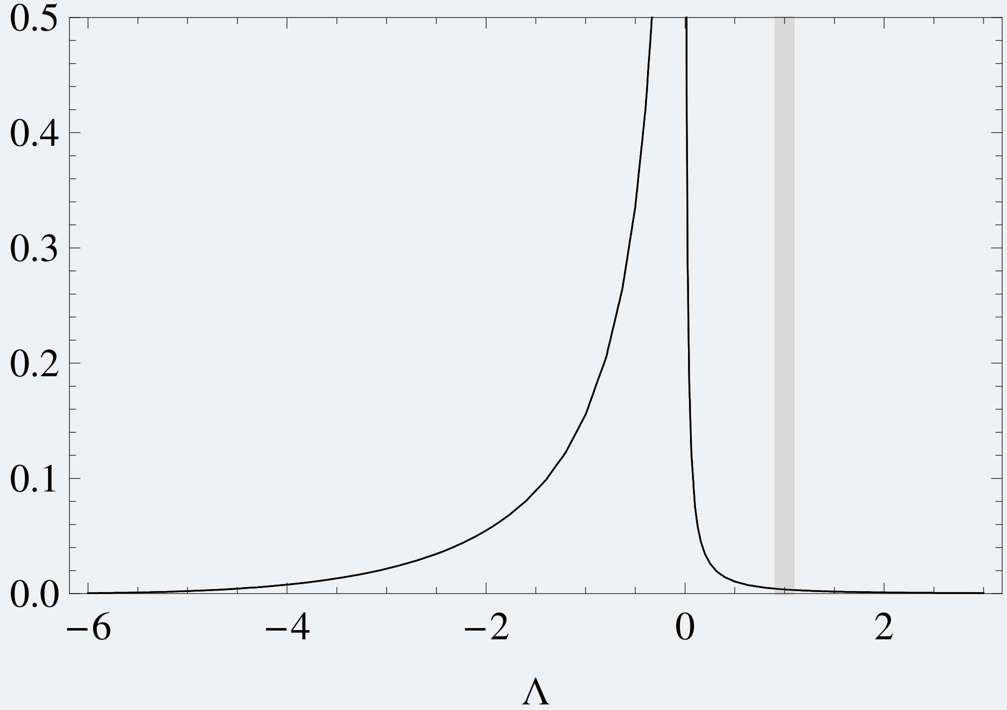
where $V_\diamond$ is the physical volume of the past lightcone,

$$V_\diamond(\tau) \propto -a^3(\tau)\, \eta^3(\tau)\ ,$$

for conformal time η (defined to approach zero as $\tau \to \infty$).

The causal patch measure

The causal patch measure benefits from some theoretical motivation, as well as simplicity. However it makes a very poor prediction for Λ:

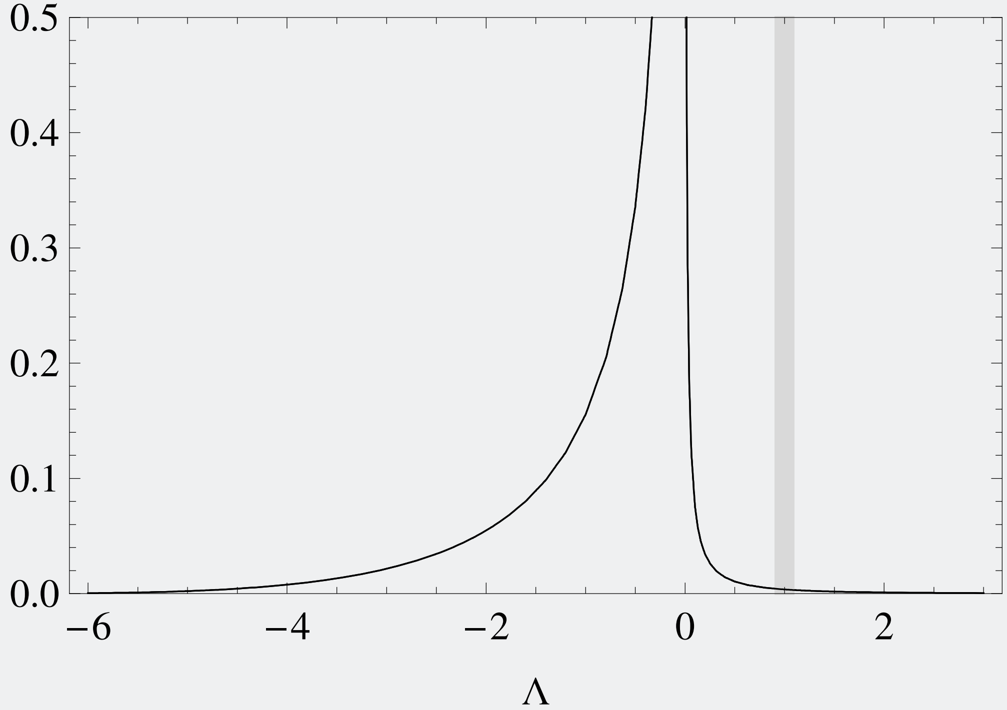


[MPS (2009).]

- We find 85% of observers measure Λ to be closer to zero (vast majority measuring negative Λ), *0.6% observe a less likely value of Λ, and 0.7% observe Λ to be further from the median*.

The causal patch measure

The causal patch measure benefits from some theoretical motivation, as well as simplicity. However it makes a very poor prediction for Λ:

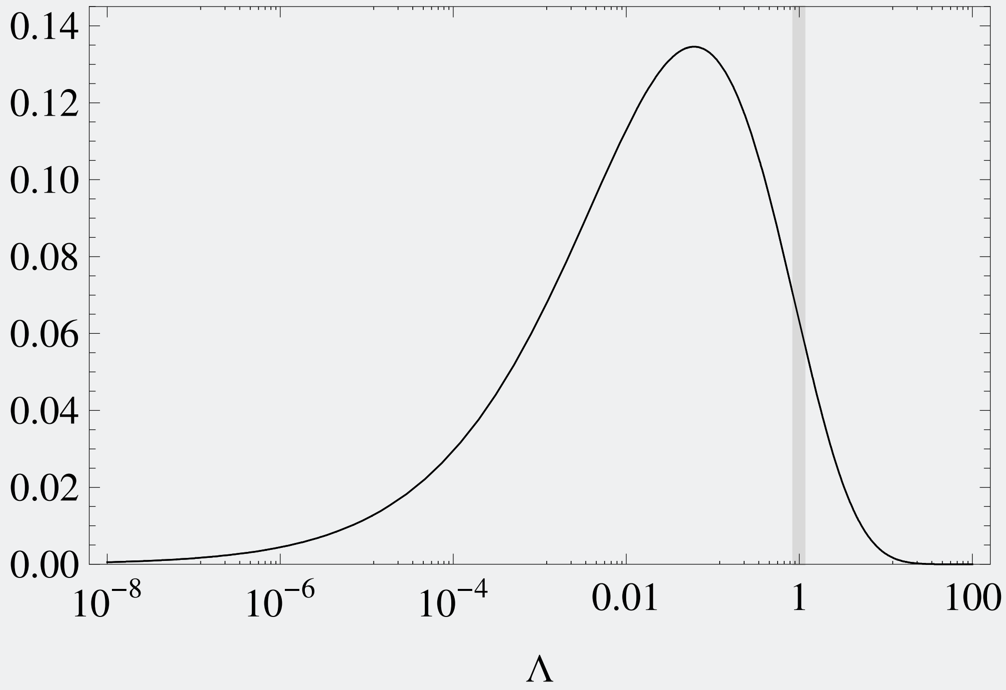


[MPS (2009).]

- relatively smaller values of Λ give relatively larger causal patches, but the effect is too strong,
- geometry of AdS generates larger causal patches than in dS.

What about the results of Bousso, Harnik, Kribs, and Perez (2007)?

They focus on positive Λ, where the problem is not nearly so bad. Furthermore, they estimate anthropic selection by studying entropy production, and compute within the “inner” causal diamond, each of which introduces a mild bias (both in the same direction).



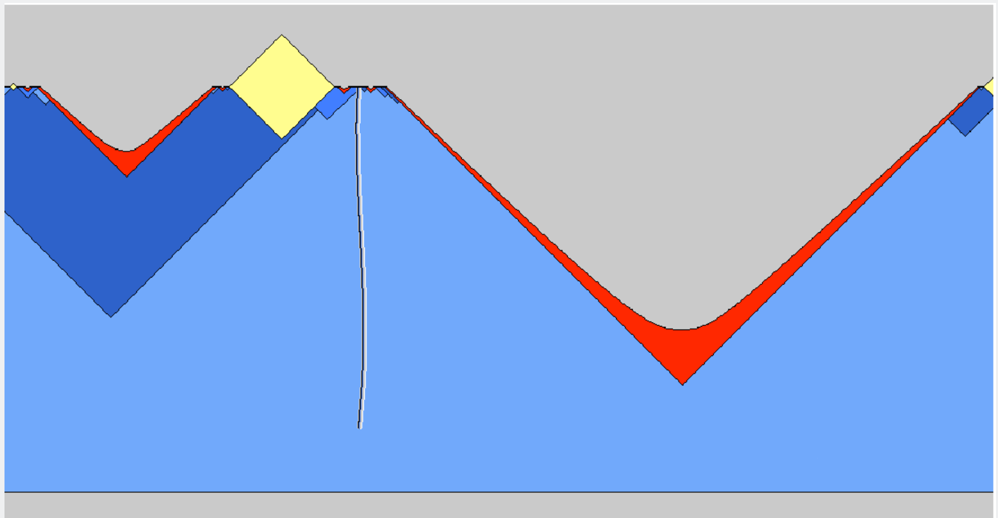
The comoving probability measure

The *comoving probability* measure is defined by

- selecting a worldline in a given de Sitter vacuum,
- considering all future histories of the worldline,
- assigning a small, fixed physical radius to events, and
- counting events according to the frequency at which they intersect the worldline.

Alternatively, one can assign a fixed physical radius to the worldline.

[Starobinsky (1986); Linde (2007); Bousso, Freivogel, Yang (2009).]



The comoving probability measure

When Λ is positive and the pocket universe has clear FRW symmetry, the comoving probability measure is equivalent (up to $\gamma - 3$ and initial conditions) to the FRW scale-factor cutoff:

$$\lim_{t_c \to \infty} \int_{-\infty}^{t_c} e^{\gamma t_*} \, dt_* \int_{\tau_*}^{\tau_c(t_c, t_*)} \rho_{\rm obs}(\tau) \, a^3(\tau) \, d\tau \Longrightarrow \int_{\tau_*}^{\infty} \rho_{\rm obs}(\tau) \, d\tau \, ,$$

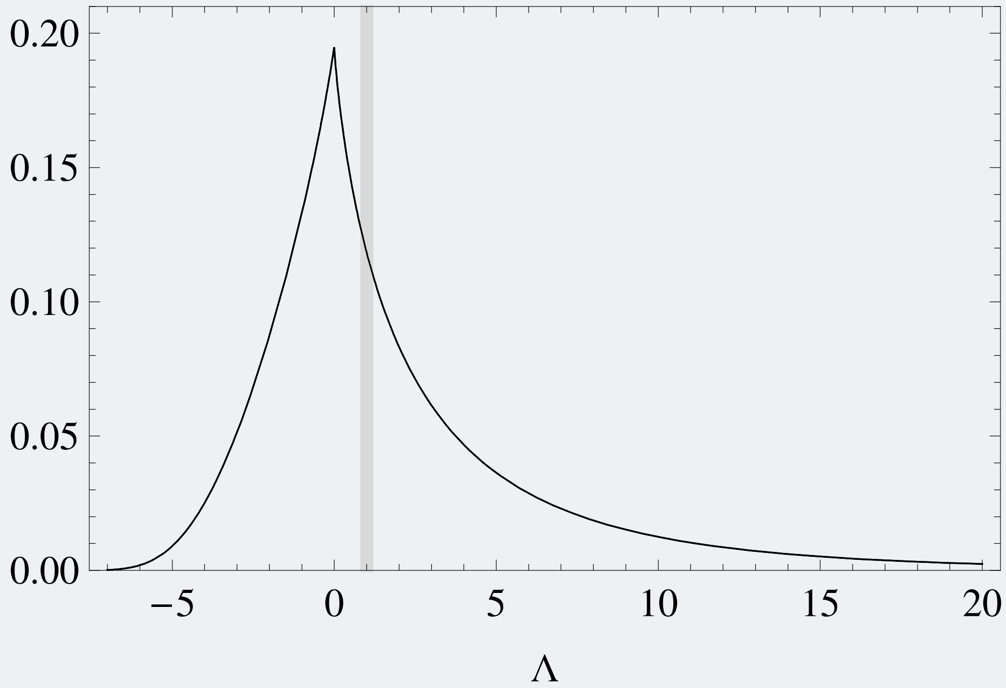
after integration by parts and substitution using $a(\tau_c)/a(\tau_*) \propto e^{t_c - t_*}$.

[Bousso, Freivogel, and Yang (2009).]

But the equivalence breaks down for negative Λ, where scale-factor time may halt and the simple relation $a(\tau_c)/a(\tau_*) \propto e^{t_c - t_*}$ cannot always be used.

The comoving probability measure

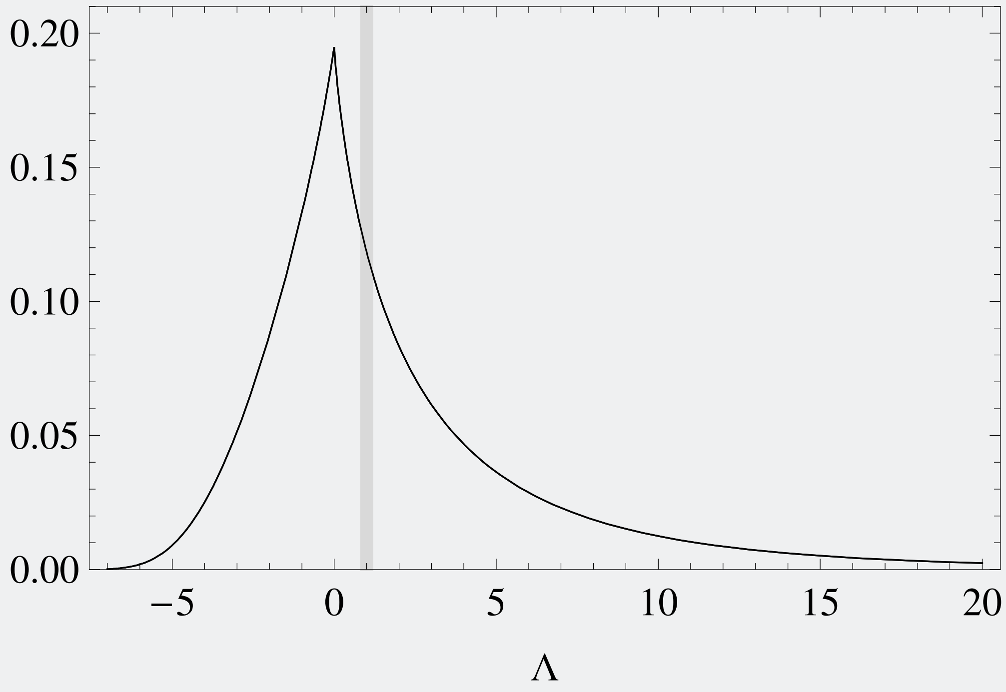
The comoving probability measure is simple, and gives an excellent prediction for Λ:



- We find that 31% of observers measure Λ to be closer to zero, 65% observe a less likely value of Λ, and 90% observe Λ to be further from the median.

The comoving probability measure

The comoving probability measure is simple, and gives an excellent prediction for Λ:



- Selection for higher density of observers apparently provides an ideal weight against increasing Λ.

Note added on comoving probability measure

Note added after talk:

The probability distribution displayed in the previous slides,

$$\frac{dP}{d\Lambda} \propto \int_0^\infty \rho_{\rm obs}(\tau)\, d\tau \,,$$

is not exactly what one would expect from the comoving probability measure (as defined on an earlier slide), because it does not account for the gravitational effect on the worldline due to structure formation (MPS thanks Raphael Bousso for pointing this out).

It would seem the correct probability that a worldline intersects a fixed-radius MW galaxy would be well-approximated by the collapse fraction (which represents the fraction of baryons that fall into such galaxies). Thus we expect this measure to reproduce Weinberg *etal*'s original results, displayed at the beginning of this talk.

Then we find that 16% of observers measure Λ to be closer to zero, 82% observe a less likely value of Λ, and 48% observe Λ to be further from the median.

There is still no Boltzmann brain problem, as the late-time de Sitter horizon protects the worldline from gravitational in-fall into a "Boltzmann galaxy."

Conclusions

- The standard anthropic landscape argument for the smallness of Λ turns a big problem into a little problem — but we cannot claim full understanding of Λ without a better understanding of the measure of the multiverse.
- Consideration of phenomenological pathologies — Boltzmann brain domination, runaway inflation, and youngness paradox — is a powerful tool for narrowing the set of measures.
- The value of Λ might also be used to give clues about the spacetime measure:
  - We find the standard causal patch measure to give an unsatisfactory prediction for Λ.
  - The FRW scale-factor cutoff gives an excellent prediction, but lacks in theoretical development.
  - The comoving probability measure is both simple and accurate, but does it point the way to a theory of the multiverse?

Here are all of the distributions together: