The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2040-6

Workshop: Eternal Inflation

8 - 12 June 2009

Dynamical compactification from higher dimensional de Sitter space

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Dynamical compactification from higher dimensional de Sitter space

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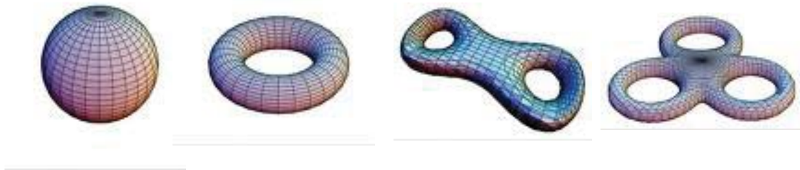
0904.3115

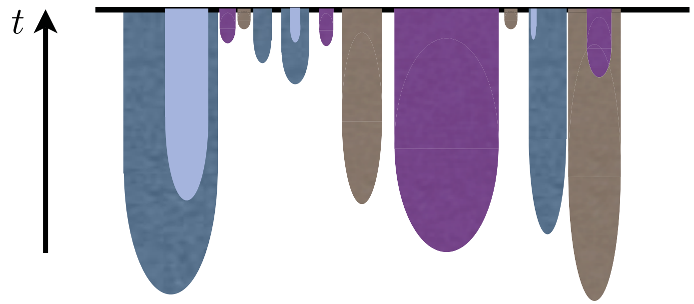
Landscapes and extra dimensions

- Extra dimensions = Landscapes of lower dimensional vacua.



? Why are some dimensions small and others large ?

- Eternal inflation - transitions within 4D EFT between vacua.



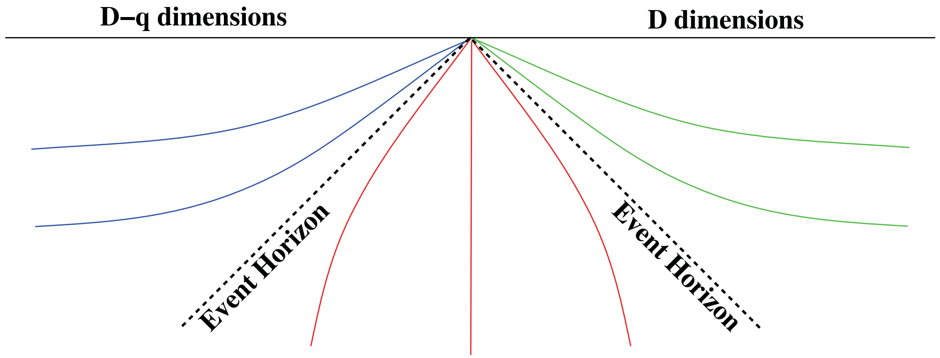
? What about the extra dimensions ?

- Do extra dimensions play a direct role in dynamics, or just provide the possibility of different 4D physics?

Dynamical Compactification

$$S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}^{(D)}} \left(\tilde{\mathcal{R}}^{(D)} - 2\Lambda - \frac{1}{2q!} \tilde{F}_q^2 \right)$$

- We will find non-singular black brane solutions that interpolate across event horizons between a D dimensional de Sitter space and a D-q dimensional open FRW universe with a stabilized q-sphere.



- These solutions can be nucleated out of D-dimensional dS space, explaining how extra dimensions became compact.
- Many types of lower-dimensional vacua exist and can be populated.

Previous work

S. B. Giddings and R. C. Myers, Phys. Rev. **D70**, 046005 (2004), hep-th/0404220.

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R. Bousso, Phys. Rev. **D60**, 063503 (1999), hep-th/9902183.

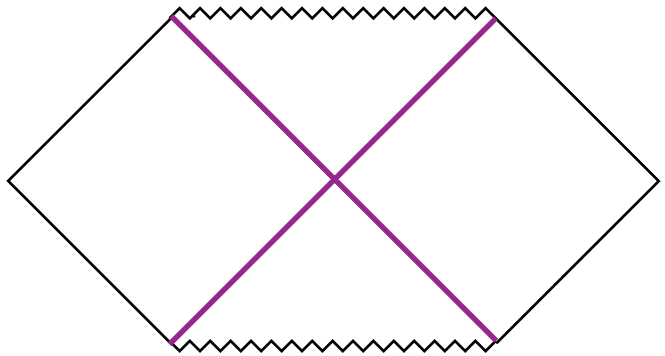
R. Bousso, O. DeWolfe, and R. C. Myers, Found. Phys. **33**, 297 (2003), hep-th/0205080.

G. W. Gibbons, G. T. Horowitz, and P. K. Townsend, Class. Quant. Grav. **12**, 297 (1995), hep-th/9410073.

G. W. Gibbons and D. L. Wiltshire, Nucl. Phys. **B287**, 717 (1987), hep-th/0109093.
H. Lu, S. Mukherji, and C. N. Pope, Int. J. Mod. Phys. **A14**, 4121 (1999), hep-th/9612224.
K. Behrndt and S. Forste, Nucl. Phys. **B430**, 441 (1994), hep-th/9403179.
E. A. Bergshoeff, A. Collinucci, D. Roest, J. G. Russo, and P. K. Townsend, Class. Quant. Grav. **22**, 4763 (2005), hep-th/0507143.

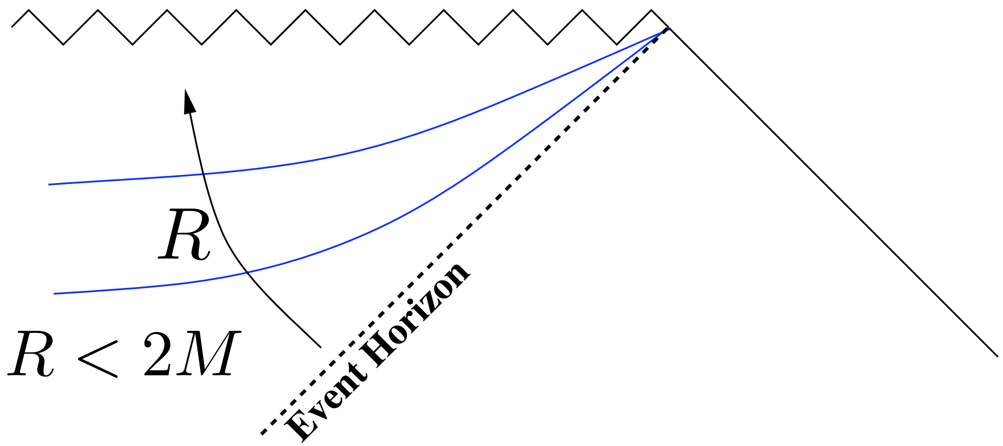
Cosmology inside a black hole

Each element of this picture can be understood from completely vanilla black holes in 4 dimensions.



Cosmology inside a black hole

$$ds^2 = -\frac{dR^2}{\left(\frac{2M}{R}-1\right)} + \left(\frac{2M}{R}-1\right)dt^2 + R^2 d\Omega_2^2$$



Near the horizon:

$$x = \frac{t}{4M}, \; \tau = \sqrt{16M^2 - 8MR}$$

$$ds^2 = \boxed{-d\tau^2 + \tau^2 dx^2} + \boxed{4M^2 d\Omega_2^2}$$

2D open FRW

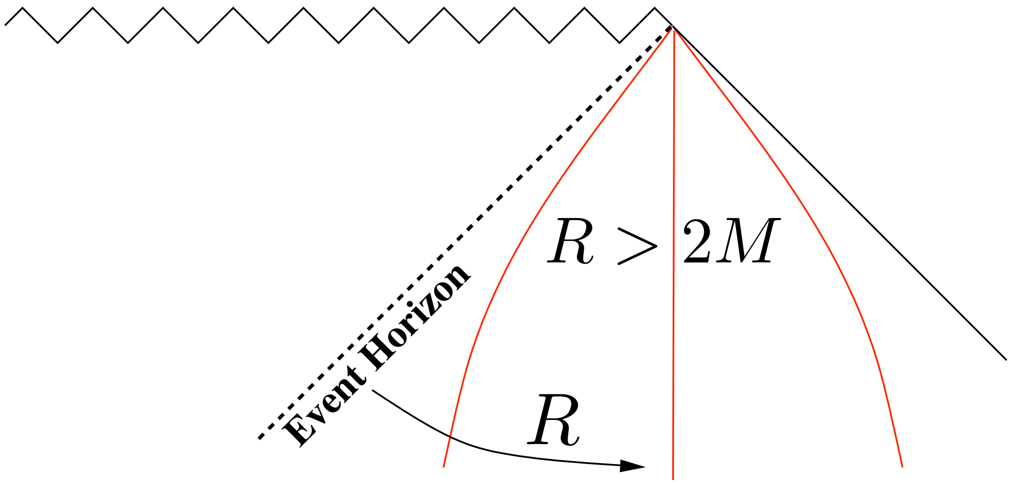
"compactified" 2-sphere

$\tau = 0$ 2D "Big-bang" is non-singular - just the event horizon.

Cosmology inside a black hole

Can continue across the horizon by taking $\tau \longrightarrow i\tau$, R is spacelike.

$$ds^2 = -\tau^2 dx^2 + d\tau^2 + 4M^2 d\Omega_2^2$$



- Event horizon separates 2D big-crunch cosmology from asymptotically flat 4D space.
- Can study in more detail.....

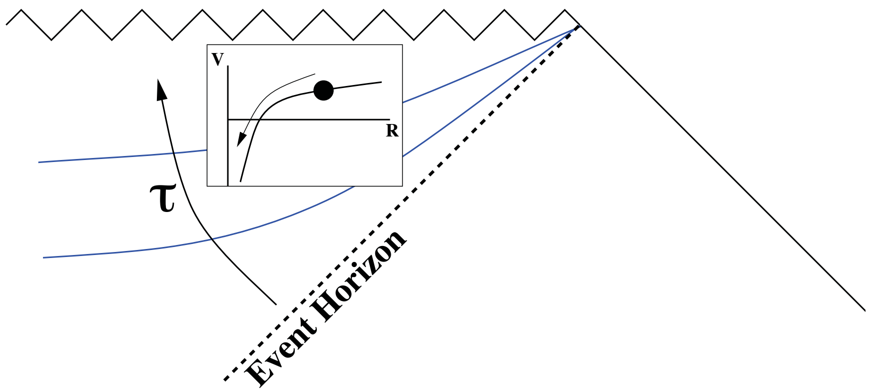
Dimensional reduction

$$ds^2 = -d\tau^2 + a^2(\tau)dx^2 + R^2(\tau)d\Omega_2^2$$

Einstein's equations

$$R'' + \frac{R'^2}{2R} = -\frac{1}{2R} \qquad a = R'$$

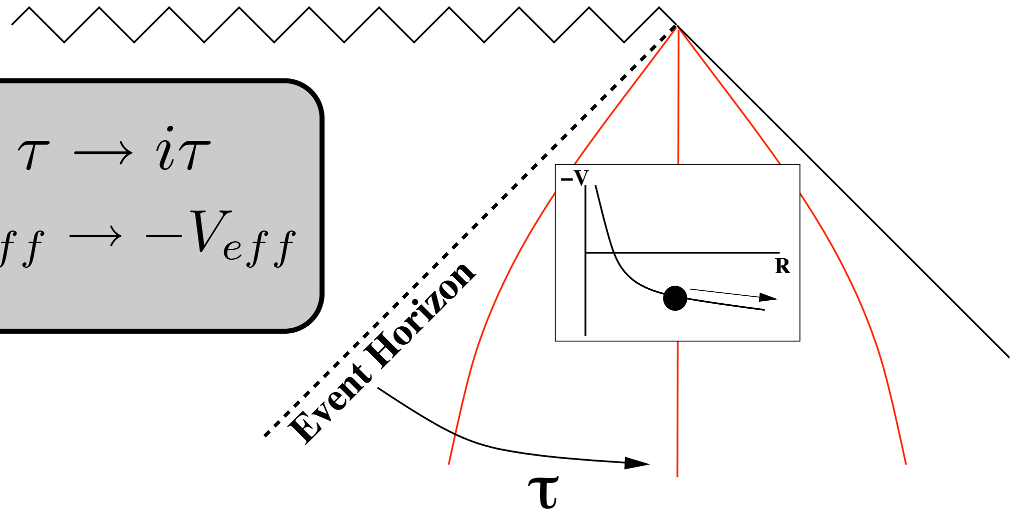
- R evolves in the potential $V_{eff} = \frac{1}{2}\log R$ ⇒ $R'' + \frac{R'^2}{2R} = -\frac{dV_{eff}}{dR}$
- Event horizon where $a = R' = 0$ ⇒ specify solution by R at the horizon



Going outside the horizon

- Continuing across the horizon:

$$\tau \longrightarrow i\tau$$
$$V_{eff} \longrightarrow -V_{eff}$$

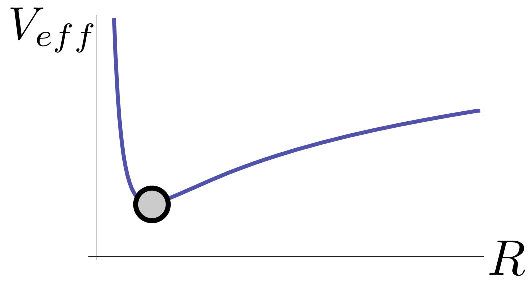


- This method of dimensionally reducing to a “radion” R living in lower dimensions (the open FRW) can be used to classify a wide variety of solutions.

Adding matter

- Add a 2-form: charge the black hole.

$$V_{eff} = \frac{1}{2}\log R + \frac{Q^2}{4R^2}$$



- Now, we can stabilize R: $AdS_2 \times S^2$ is a solution.

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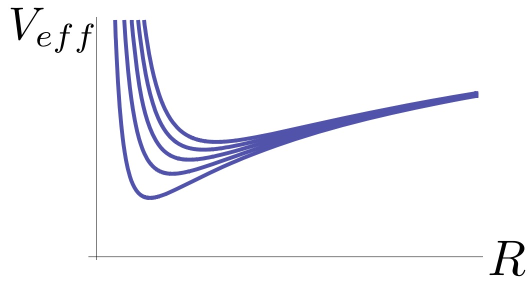

- Now, we can stabilize R: $AdS_2 \times S^2$ is a solution.
- There is a “landscape” of vacua, one for each Q.

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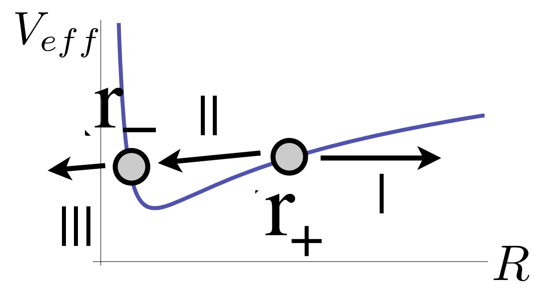
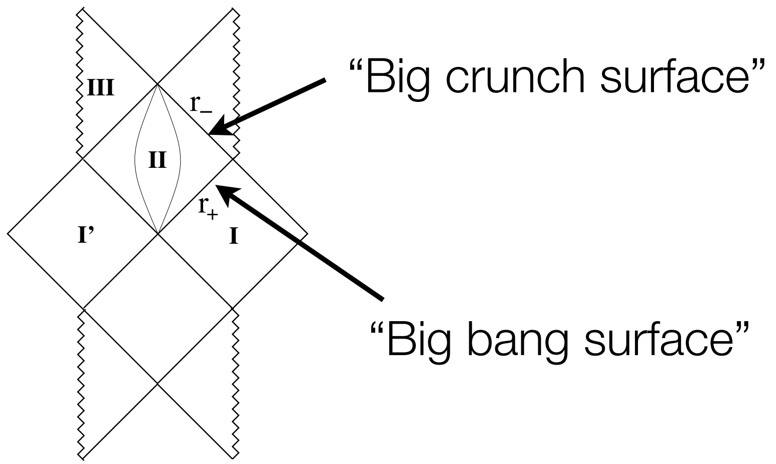


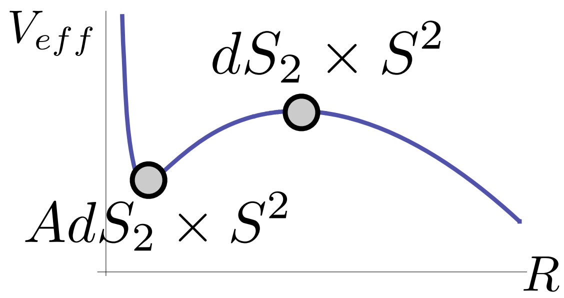
- Now, we can stabilize R: $AdS_2 \times S^2$ is a solution.
- There is a "landscape" of vacua, one for each Q.
- The black hole solutions can have multiple horizons.



Adding matter

- Add a cosmological constant

$$V_{eff} = \frac{1}{2}\log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4}R^2$$



- There are new “compactification” solutions.

Adding matter

- Add a cosmological constant

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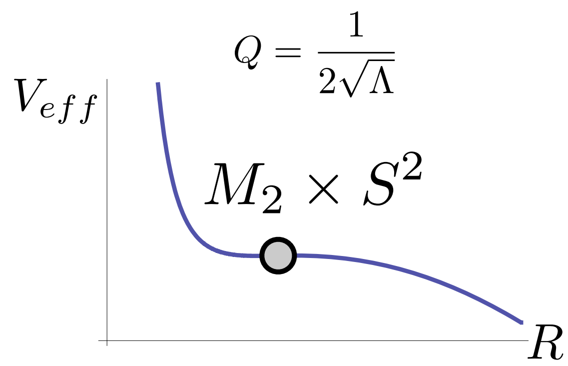

- Q is bounded.

Adding matter

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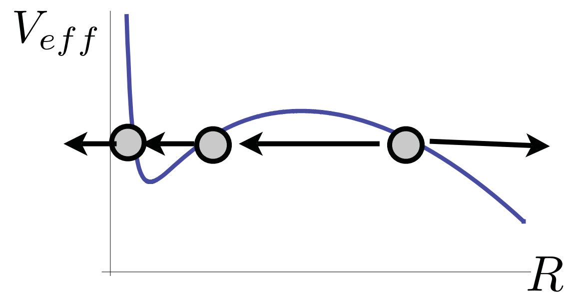
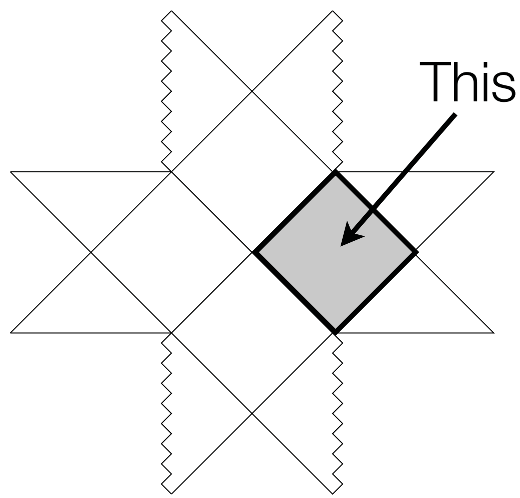


- Can have up to three horizons: 2 BH and 1 cosmological



This region interpolates between:

Adding matter

- Add a cosmological constant

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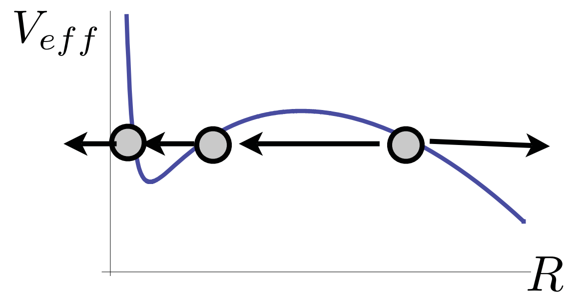

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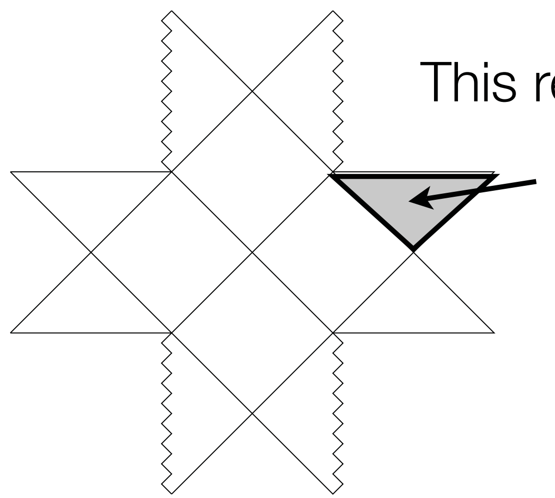


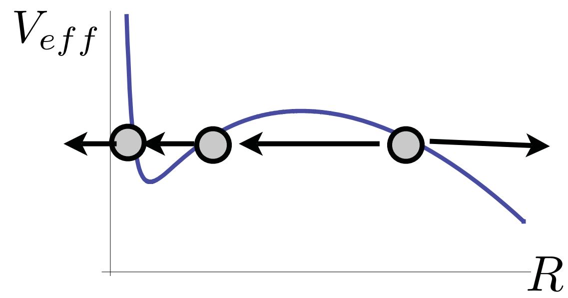
This region interpolates between:

4D dS and 2D FRW

Adding matter

- Add a cosmological constant

$$V_{eff} = \frac{1}{2}\log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4}R^2$$



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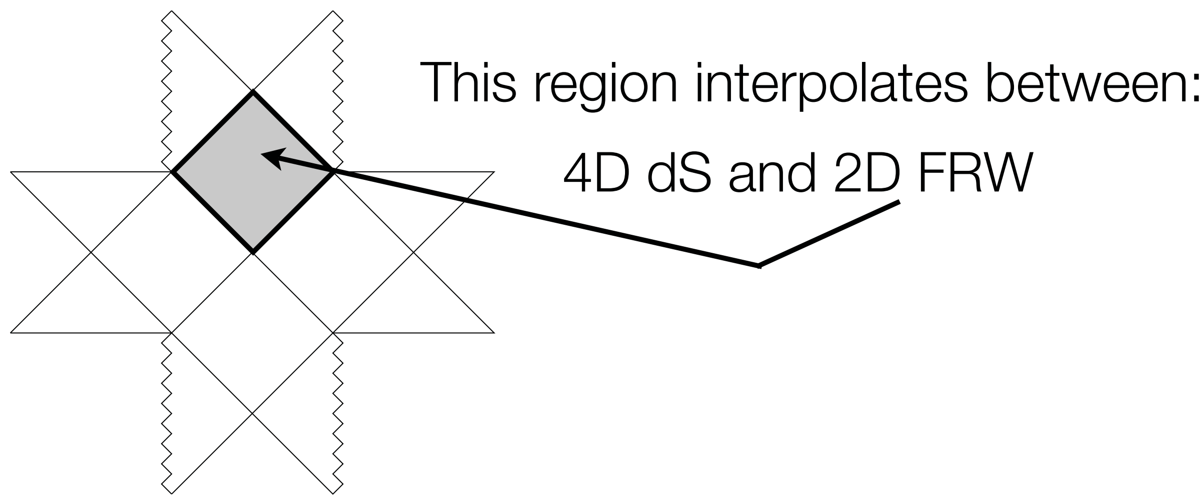

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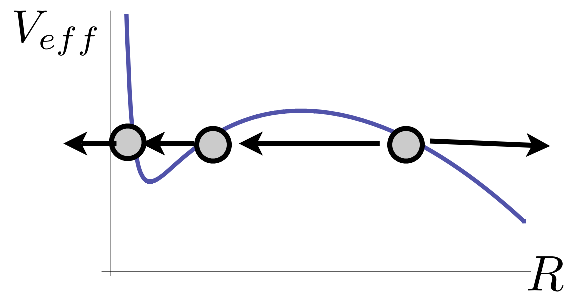


- Can have up to three horizons: 2 BH and 1 cosmological
- Charged black holes in de Sitter are “interpolating solutions.”
- The thermal properties of de Sitter space add interesting dynamics......

Black hole nucleation

- de Sitter space is semi-classically unstable to the nucleation of charged black holes.

$$\Gamma = A \exp\left[-(S_{inst} - S_{dS})\right]$$

- The 2D region inside of each black hole is spontaneously nucleated - An example of "Dynamical Compactification."
- Globally, an infinite number of black holes are nucleated, populating all possible 2D crunching universes.
- Future infinity of the dS space is split into many disconnected regions.

What if the lower dimensional FRW was 4D and didn't end in a crunch?

Now enters the magic of higher dimensional GR....

A very simple theory

$$S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}^{(D)}} \left(\tilde{\mathcal{R}}^{(D)} - 2\Lambda - \frac{1}{2q!} \tilde{F}_q^2 \right)$$

Dimensional reduction

- Assume q-dimensional spherical symmetry (D=q+p+2):

$$d\tilde{s}^2 = \tilde{g}^{p+2}_{\mu\nu}(\mathbf{x})dx^\mu dx^\nu + R^2(\mathbf{x})d\Omega_q^2$$

- For magnetic flux, Maxwell equations satisfied for:

$$F_q = Q\sin^{q-1}\theta_1 \ldots \sin\theta_{q-1} d\theta_1 \ldots \wedge d\theta_q$$

- Can integrate over the angular coordinates on the q-sphere and go to the Einstein frame of a p+2-dimensional theory:

$$S = \int d^{p+2}x\sqrt{-g}\left[\frac{M_{p+2}^p}{2}\mathcal{R} - \frac{M_{p+2}^{p-2}}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right]$$

$$M_{p+2} \equiv M_D\left(\mathrm{Vol}(S^q)\right)^{1/p} \qquad M_D R = \exp\left[\sqrt{\frac{p}{q(p+q)}}\frac{\phi}{M_{p+2}}\right]$$

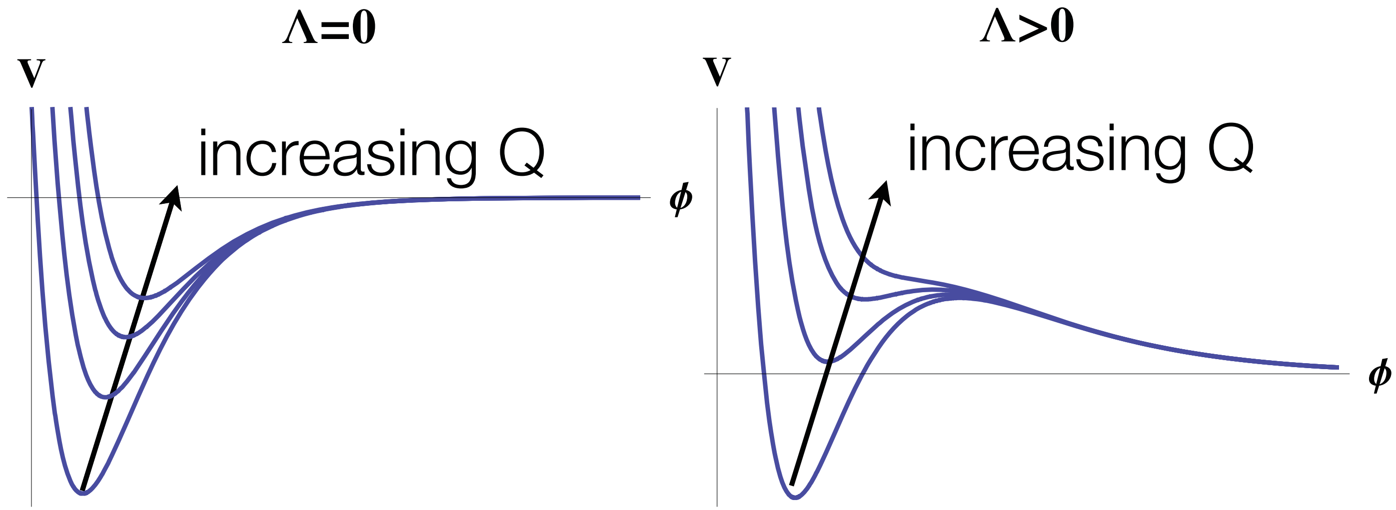
A landscape of lower-dimensional vacua

- The potential is given by:

curvature — cosmological constant

$$V(\phi) = \frac{M_{p+2}^{p} M_D^2}{2}\left[-q(q-1)\exp\left(-2\sqrt{\frac{p+q}{pq}}\frac{\phi}{M_{p+2}}\right) + \frac{2\Lambda}{M_D^2}\exp\left(-2\sqrt{\frac{q}{p(p+q)}}\frac{\phi}{M_{p+2}}\right)\right.$$
$$\left. + \frac{Q^2}{2}\exp\left(-2(p+1)\sqrt{\frac{q}{p(p+q)}}\frac{\phi}{M_{p+2}}\right)\right].$$

flux



A landscape of lower-dimensional vacua

- Can have lower dimensional vacua with positive, negative, or zero vacuum energy - our landscape.
- Possible to have 4D vacua (if q = D-4) with a small vacuum energy.
- The radius of the stabilized sphere is always less than $R \sim \Lambda^{-1/2}$
- The sphere can be small, so this is a true compactification.
- If there are multiple q-forms, there can be vacua with various numbers of compact and non-compact dimensions.

$$\frac{F_q^2}{2q!} \rightarrow \sum_{i=2}^{D-2} \frac{F_{q_i}^2}{2q_i!}$$

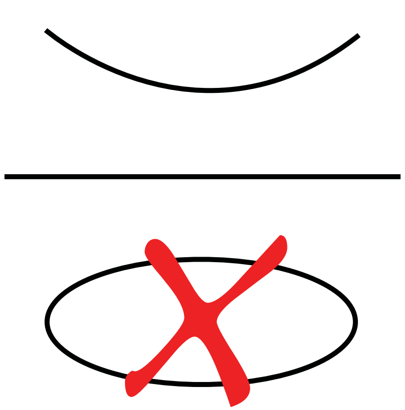
Solutions with a dynamical radion.

$$S = \int d^{p+2}x\sqrt{-g}\left[\frac{M_{p+2}^{p}}{2}\mathcal{R} - \frac{M_{p+2}^{p-2}}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right]$$

- We need to begin with an ansatz for the p+2 dimensional metric:

- Homogenous + isotropic = p+2 dimensional FRW with scalar field.

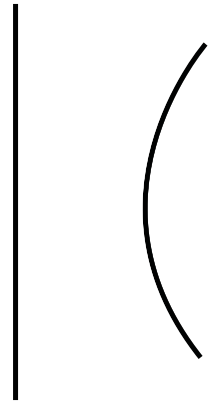
- spacelike — analytic continuation → - timelike



negative curvature (open)

no curvature (flat)

positive curvature (closed)

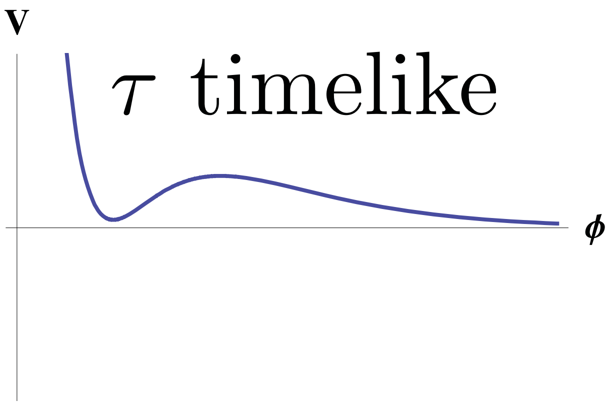


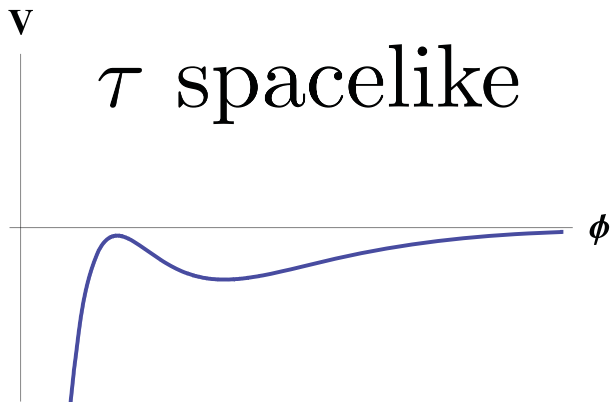
Solutions with a dynamical radion.

$$ds^2 = -d\tau^2 + a(\tau)^2 \left[d\chi^2 + S_k^2(\chi) d\Omega_p^2\right] \qquad S_k^2 = \{\chi, \sinh\chi\}$$

- Field and Friedmann equations:

$$\ddot{\phi} + (p+1)\frac{\dot{a}}{a}\dot{\phi} = \mp M_{p+2}^{2-p} V' \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{2}{M_{p+2}^2 p(p+1)}\left(\frac{\dot{\phi}^2}{2} \pm M_{p+2}^{2-p} V(\phi)\right) - \frac{k}{a^2}$$





Non-singular big-bang and big-crunch

- What about big-bang and big-crunch singularities (where a=0)?

$$\mathcal{R} = -\frac{\dot{\phi}^2}{M_{p+2}^2} + \frac{2(p+2)}{p}\frac{V(\phi)}{M_{p+2}^p}$$

- a=0 is a coordinate singularity if the field energy is finite. This requires

$$\dot{\phi} \to 0 \text{ as } a \to 0 \qquad \text{from} \qquad \ddot{\phi} + (p+1)\frac{\dot{a}}{a}\dot{\phi} = \mp M_{p+2}^{2-p} V'$$

- Possible for the open and flat cases. Scale factor has universal behavior:

open

$$a = \tau \text{ as } \tau \to 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 \to \frac{1}{a^2}$$

flat

$$a \propto e^{H\tau} \text{ as } \tau \to -\infty$$

$$\left(\frac{\dot{a}}{a}\right)^2 \to \pm \frac{2}{M_{p+2}^p p(p+1)} V(\phi)$$

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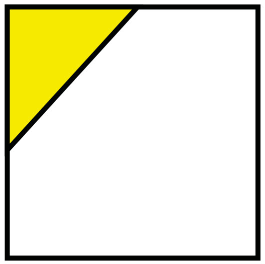
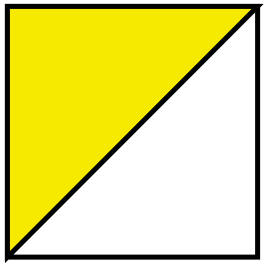
de Sitter:





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- Possible for the open and flat cases. Scale factor has universal behavior:

open

$$a = \tau \text{ as } \tau \to 0$$

flat

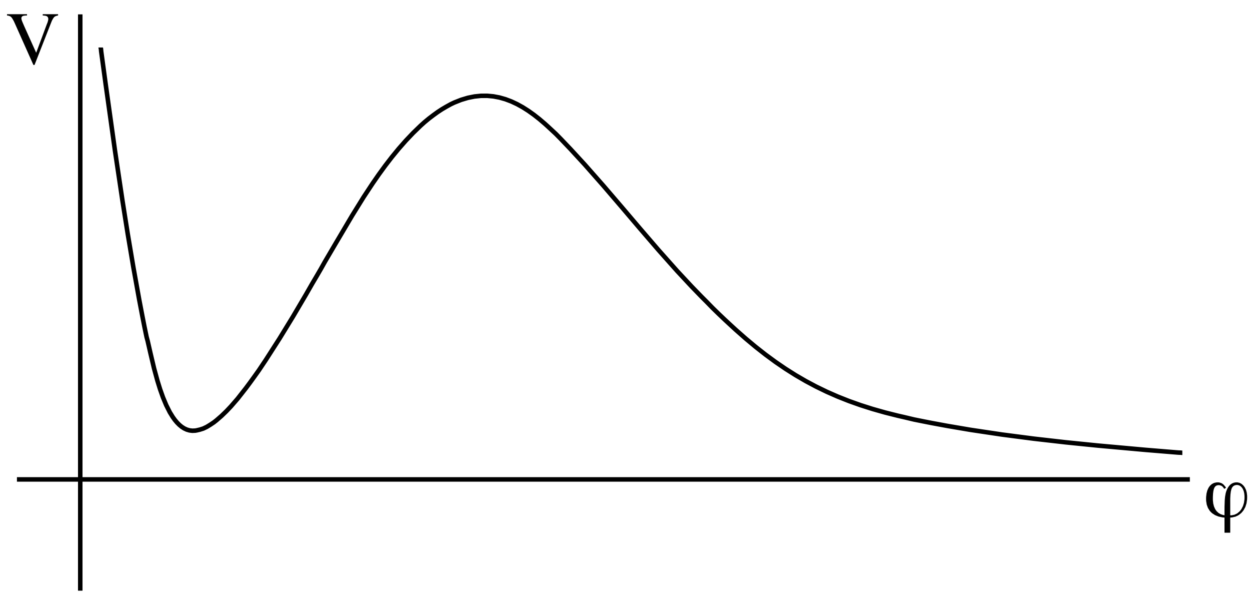
$$a \propto e^{H\tau} \text{ as } \tau \to -\infty$$

- Surface is always null, can be identified with an event horizon in D-dimensional geometry.

Classifying solutions

- Construct solutions by first specifying the radion potential (fix Λ and Q)
- Choose an open or flat metric ansatz.

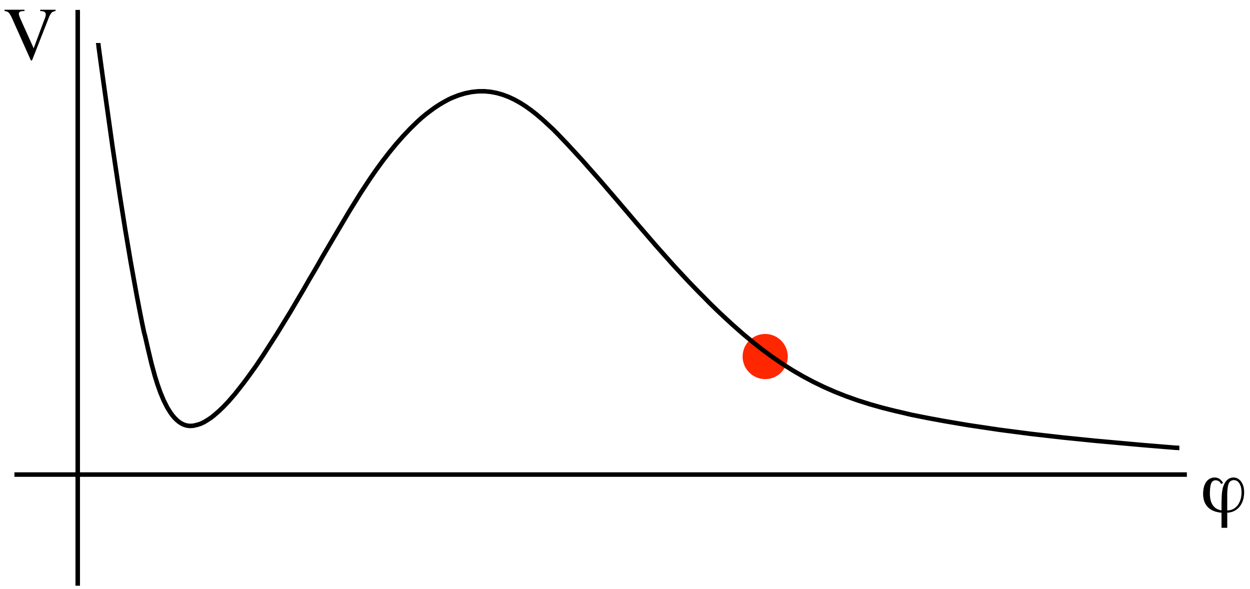
open FRW:



Classifying solutions

- Construct solutions by first specifying the radion potential (fix Λ and Q)
- Choose an open or flat metric ansatz.
- Match segments of timelike and spacelike τ across non-singular a=0 surfaces.

open FRW:



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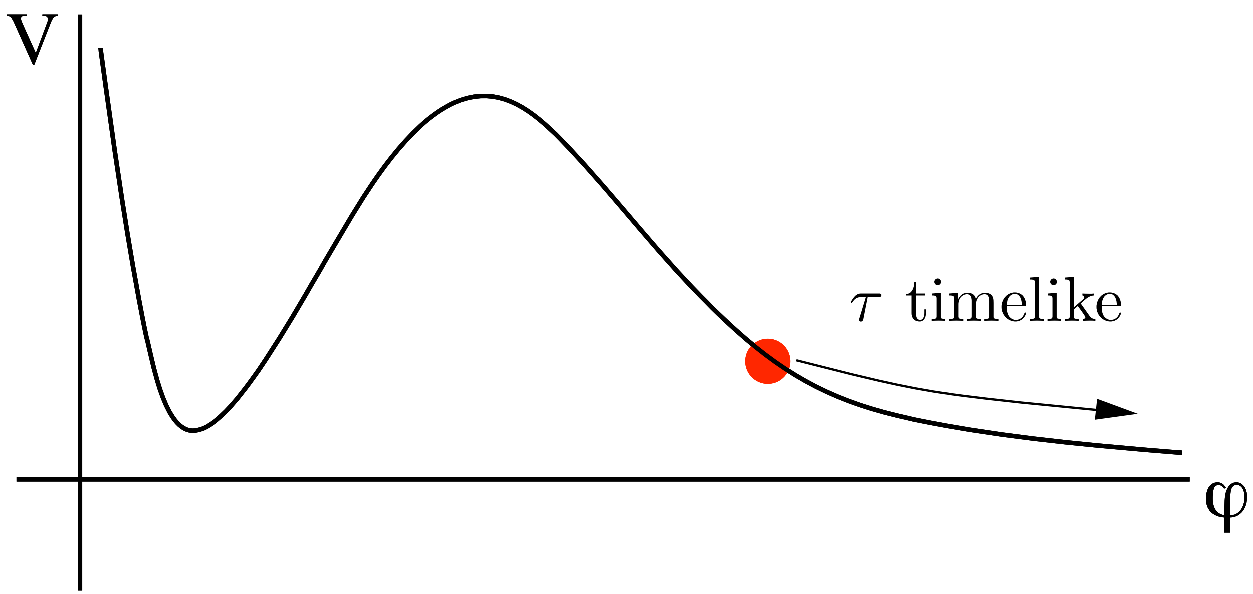

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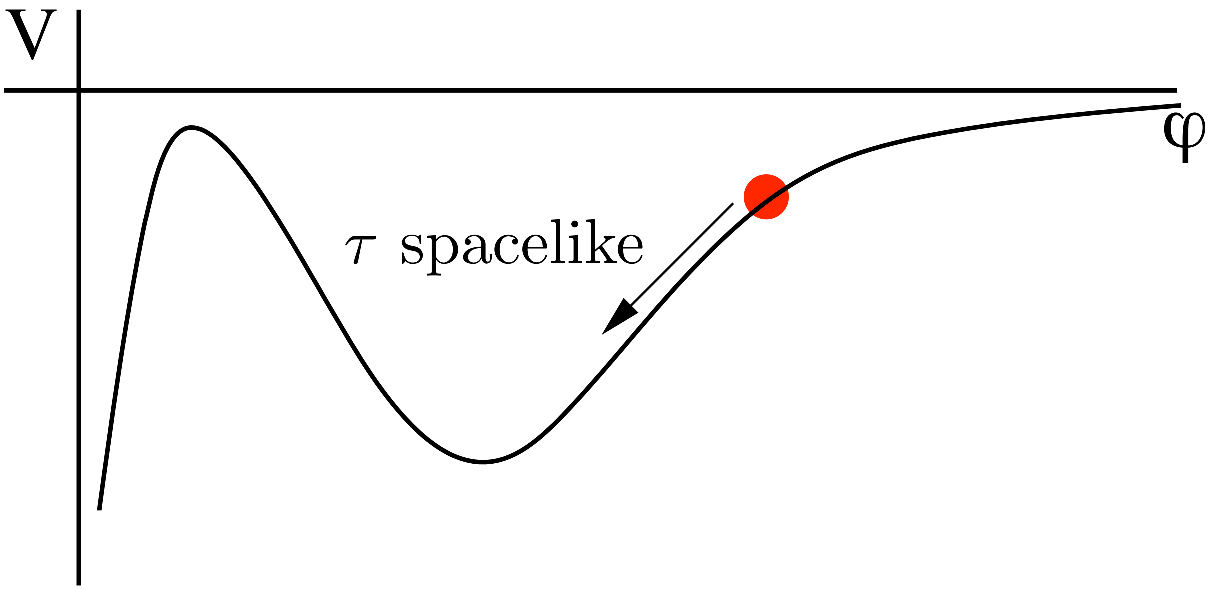
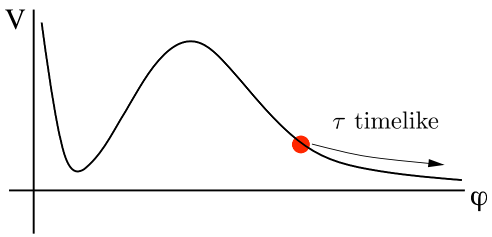


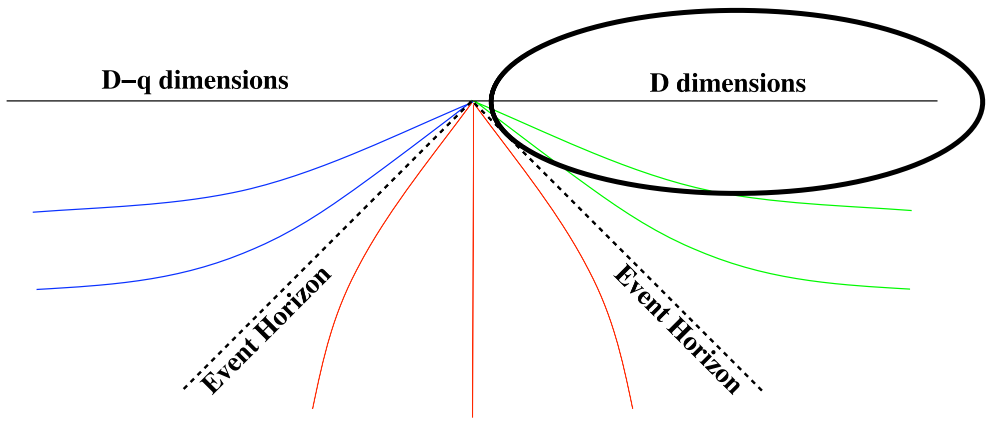
Timelike τ ①



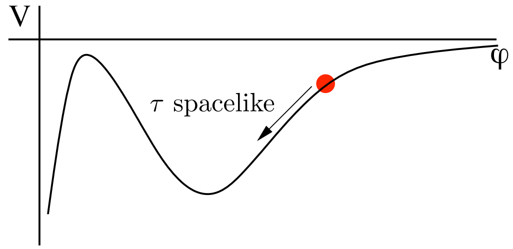
- At large ϕ the dominant term in the potential is

$$V \simeq M_{p+2}^{p} \Lambda \exp\left(-2\sqrt{\frac{q}{p(p+q)}}\frac{\phi}{M_{p+2}}\right)$$

- Exponential potentials admit attractor solutions.
- The metric describes the approach to D-dimensional de Sitter space as the radius of the q-sphere goes to infinity.



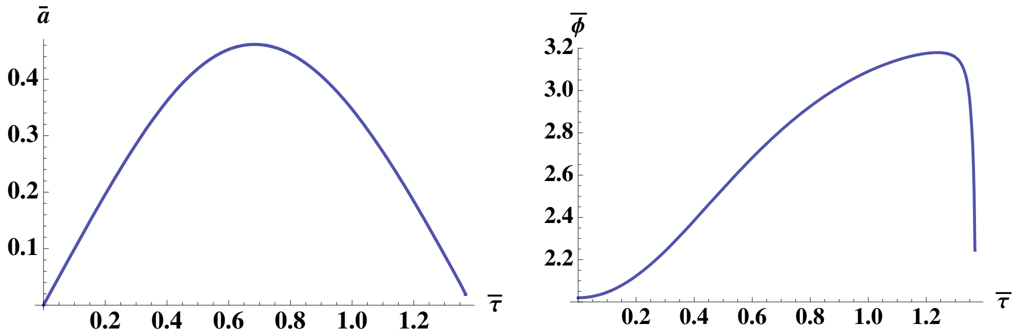
Spacelike τ ②



$$\frac{\ddot{a}}{a} = -\frac{1}{M_{p+2}^2 p(p+1)}\left(p\dot{\phi}^2 + 2M_{p+2}^{2-p}V\right)$$

- Scale factor is bounded. Generic choices of initial conditions lead to a singularity:

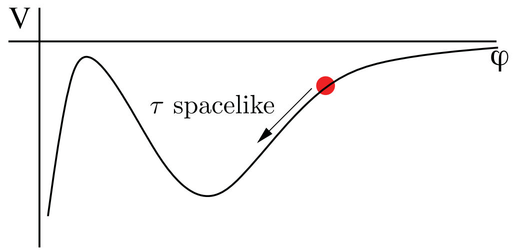
$$\ddot{\phi} + (p+1)\frac{\dot{a}}{a}\dot{\phi} = \mp M_{p+2}^{2-p}V'$$



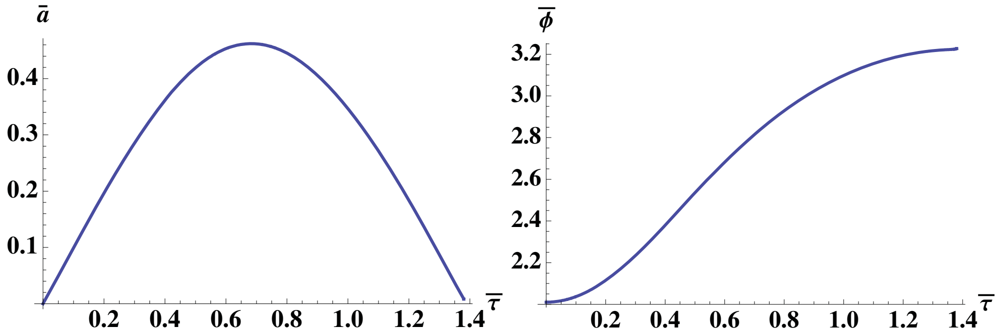
- Need to match the period of the scale factor to the barrier crossing time:

$$\Delta\tau_\phi \sim \frac{M_{p+2}^{p/2-1}}{\sqrt{|V''(\phi_{max})|}} \qquad \Delta\tau_a \sim \frac{M_{p+2}^{p/2}}{\sqrt{|V(\phi_{max})|}}$$

Spacelike τ ②

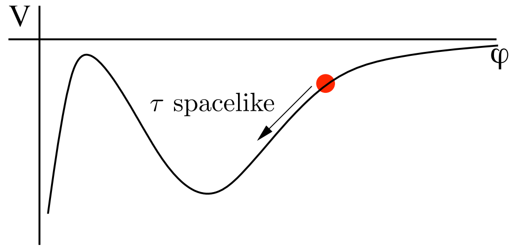


- For small enough Q, the periods can be adjusted by moving the endpoints. For each potential there can exist one set of non-singular endpoints:

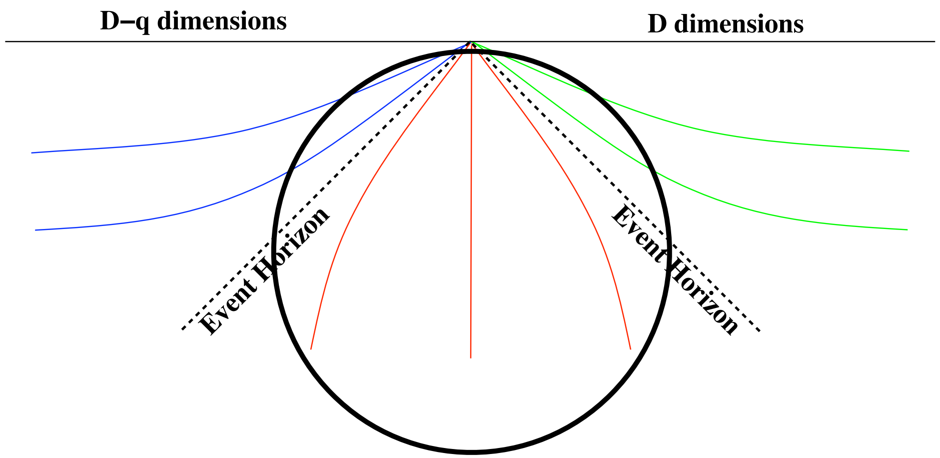


- There are two non-singular a=0 endpoints, and so two event horizons.

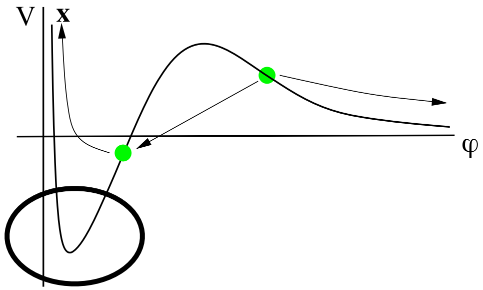
Spacelike τ ②



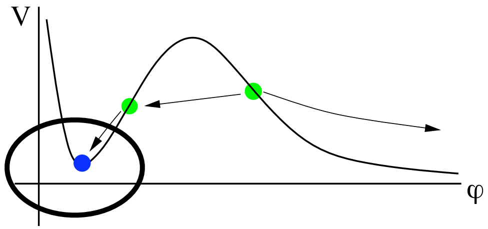
- The metric interpolates between the two event horizons.



Timelike $\mathcal{T}$ ③

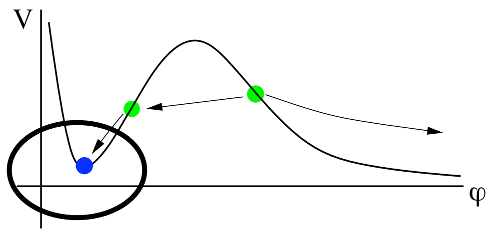


- For a negative minimum, there is always a spacelike singularity as perturbations are re-focused.

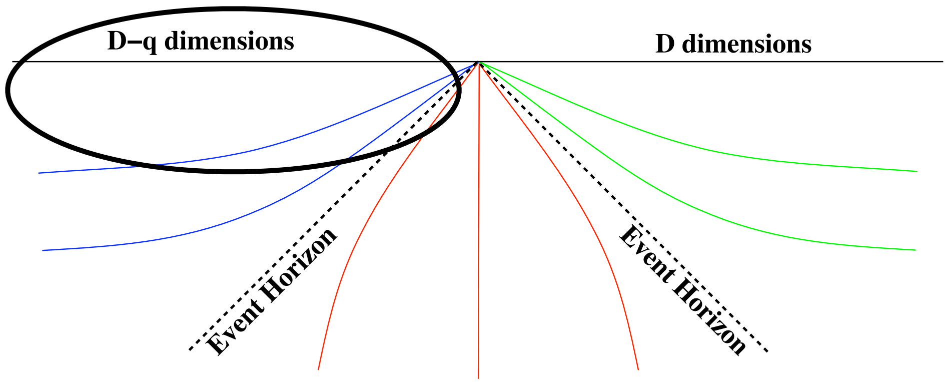


- For a zero or positive minimum,the field settles into the vacuum. There is no singularity.

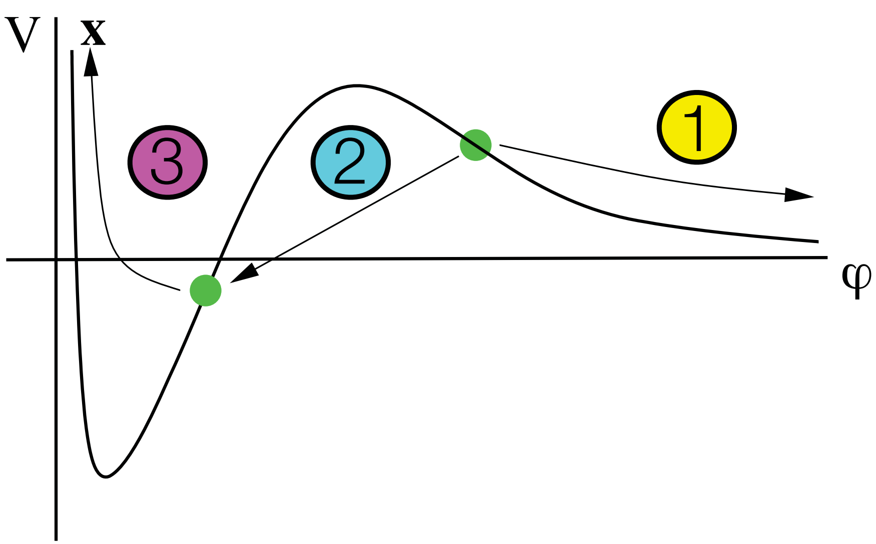
Timelike τ ③

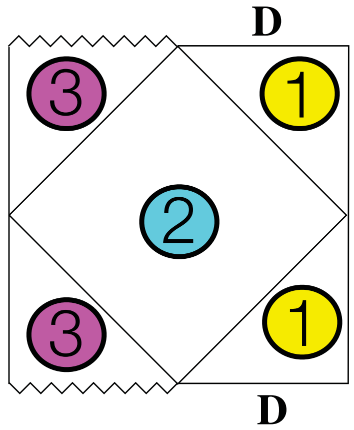


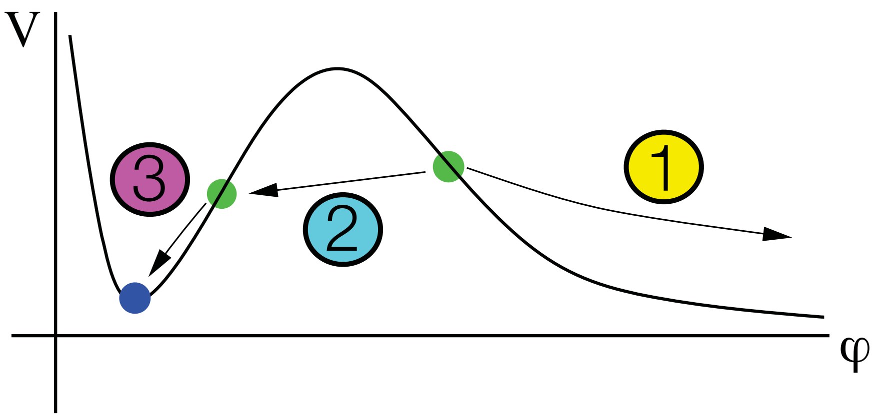
- In this region there is a D-q dimensional open FRW universe that evolves at late times to de Sitter: This could be how our universe began!

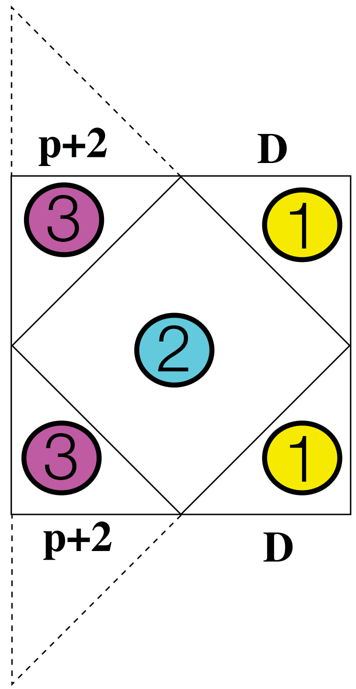


Interpolating solutions: open FRW ansatz









Classifying solutions

Many other solutions can be generated from other choices of the metric ansatz.

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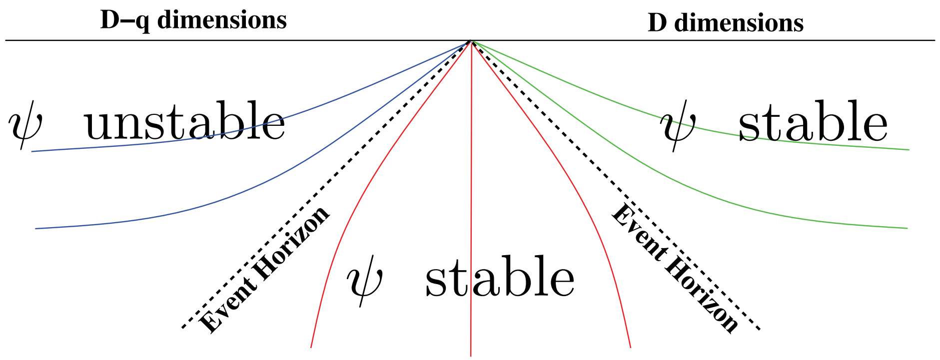
An aside: embedding Inflation

- Add a scalar:

$$S = \frac{M_D^{q+2}}{2} \int d^{q+4}x \sqrt{-\tilde{g}^{(q+4)}} \left(f(\psi)\tilde{\mathcal{R}}^{(q+4)} - 2\Lambda - \frac{h(\psi)}{2q!}\tilde{F}_q^2 \right) + \int d^{q+4}x \sqrt{-\tilde{g}^{(q+4)}} \left(-M_\psi^q k(\psi)\tilde{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V(\psi) \right)$$

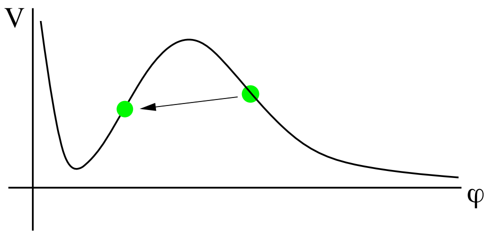
- The coupling to curvature and flux induces a negative mass squared for the scalar inside an event horizon:



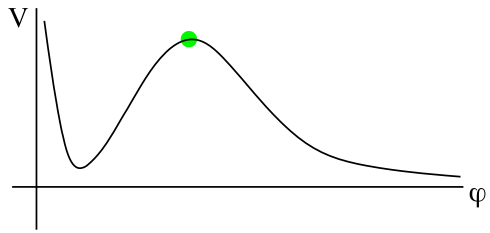
- This can drive an epoch of inflation.

Dynamical Compactification

- Two solutions that contain a non-singular p+2 dimensional region:



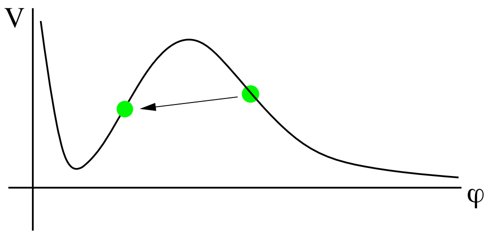
Interpolating



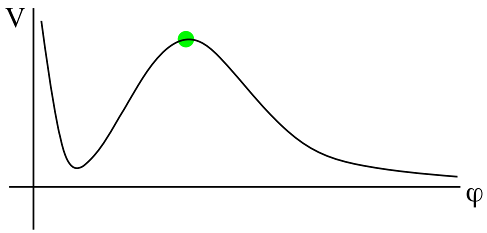
Compactification

Dynamical Compactification

- Two solutions that contain a non-singular p+2 dimensional region:



Coleman de Luccia



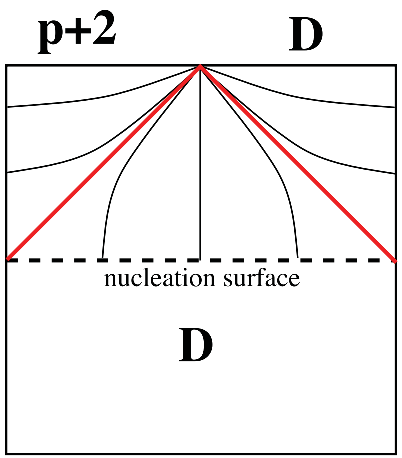
Hawking Moss

- These solutions are analogous to the charged dS black hole and compactification solution discussed earlier.
- Empty de Sitter space is unstable to the nucleation of these objects.
- We have answered our original question:

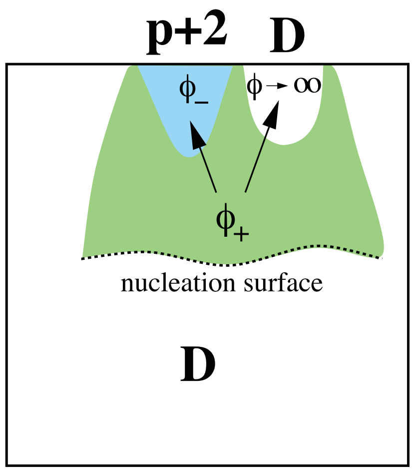
What if the lower dimensional FRW was 4D and didn't end in a crunch?

Dynamical compactification

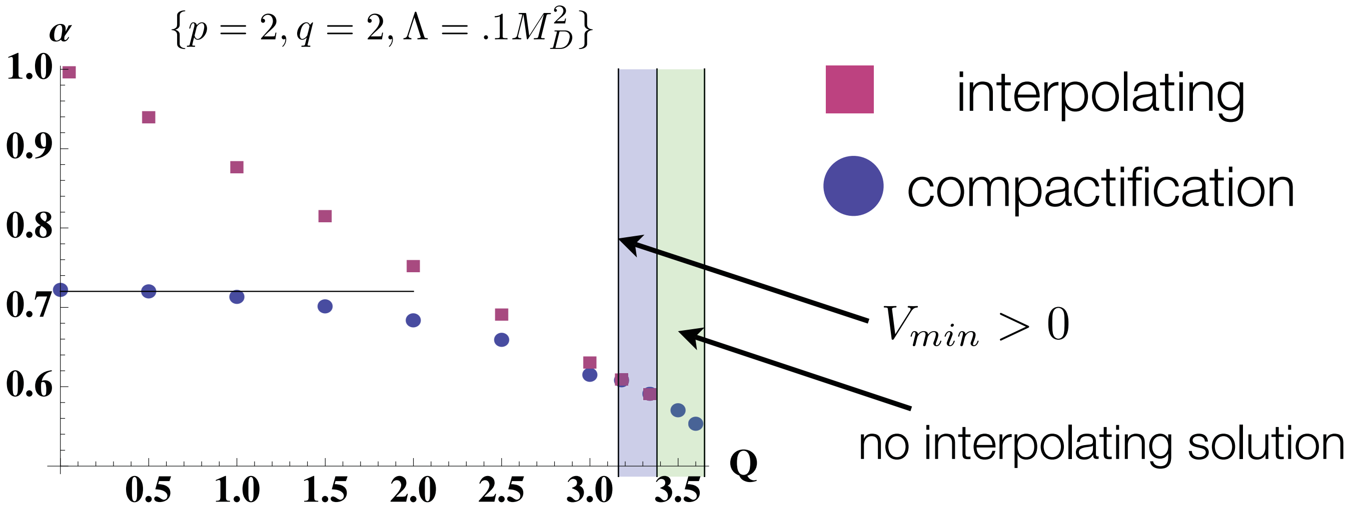
- Interpolating solution:



- Compactification solution:



Dynamical compactification: rates



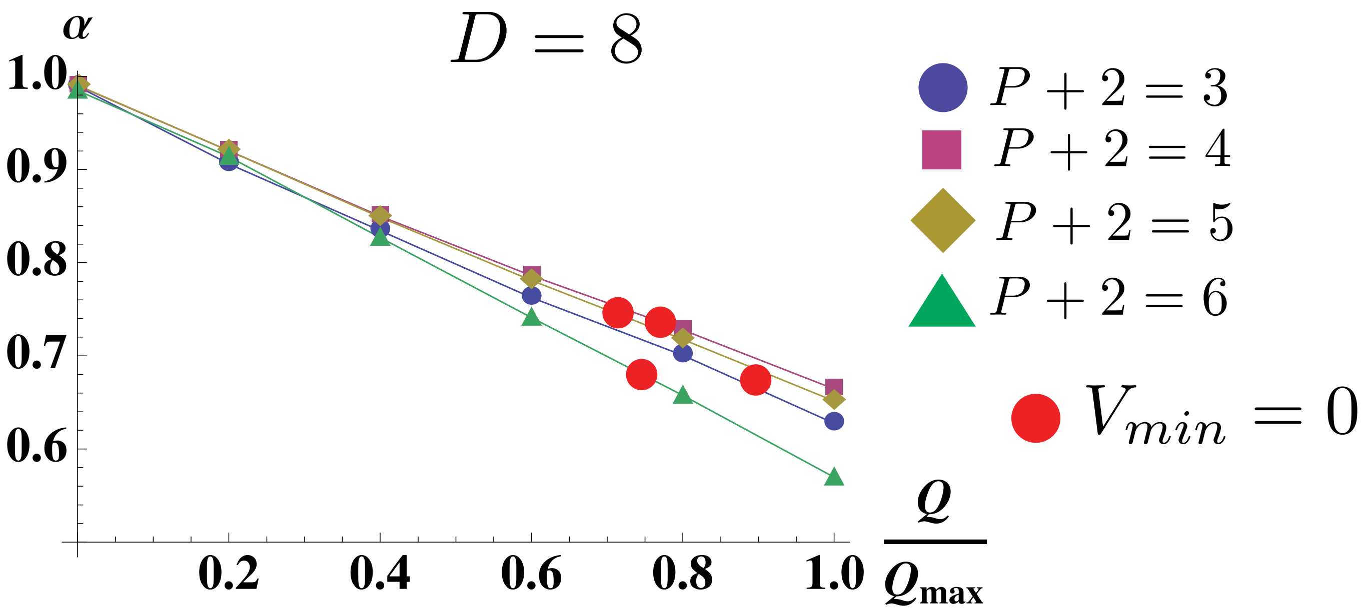
$$\Gamma = A \exp\left[S_{ds}^{(D)} (1 - \alpha) \right]$$

- Rates are suppressed by the de Sitter action.
- The rate for the interpolating solutions is higher when it exists.
- The rate is highest for small Q = lowest vacuum energy.

Dynamical compactification: rates

$$\frac{F_q^2}{2q!} \rightarrow \sum_{i=2}^{D-2} \frac{F_{q_i}^2}{2q_i!}$$

We can compare rates to vacua with different dimensionality:

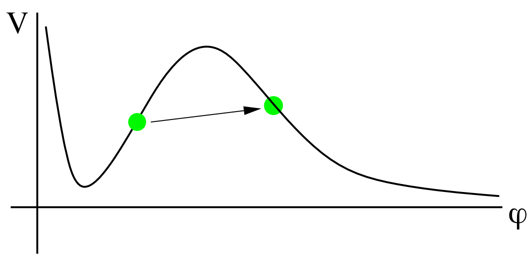


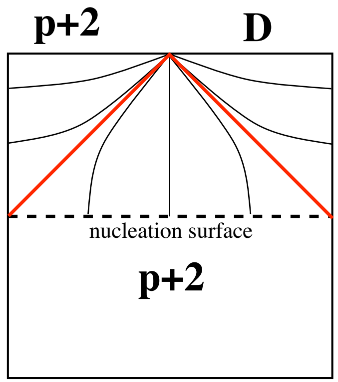
- No large disparity between different numbers of compactified dimensions.
- Unclear what to compare.....

Decompactification transitions (Giddings, Giddings+Myers)

- The p+2 dimensional de Sitter vacua decay back to D dimensional de Sitter space by the same instanton:

$$\Gamma = A \exp\left[-(S_{inst} - S_{dS}^{(p+2)})\right]$$



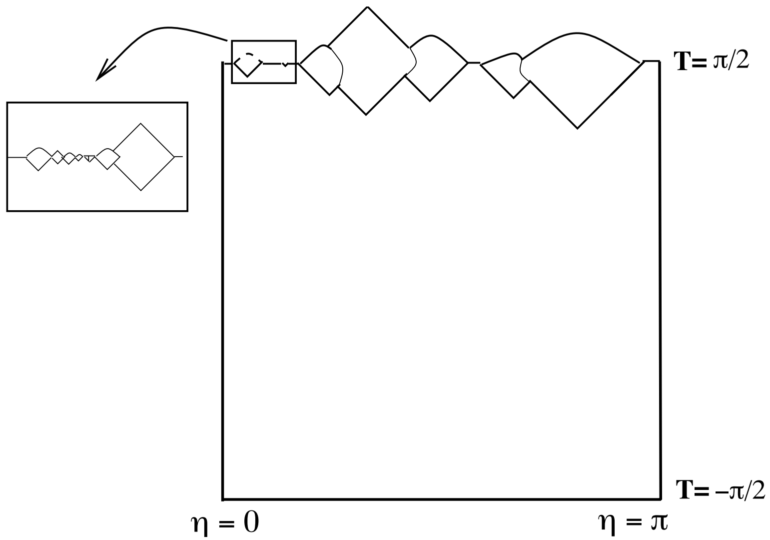


- The rate into a vacuum is always larger than the rate out

$$\frac{\Gamma_{in}}{\Gamma_{out}} = \exp\left[|S_{dS}^{(p+2)}| - |S_{dS}^{(D)}|\right] \qquad |S_{dS}^{(p+2)}| > |S_{dS}^{(D)}|$$

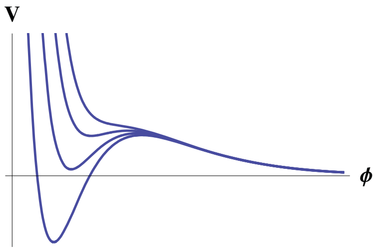
- Minkowski vacua are completely stable.

Global structure of the multiverse

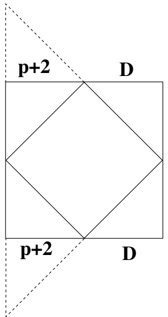


- Future infinity is fractally distributed among vacua with different vacuum energy and numbers of non-compact dimensions.
- Transitions occur back and forth between p+2 and D dimensions.
- Higher-dimensional eternal inflation!
- Connecting to predictions is a very difficult measure issue.

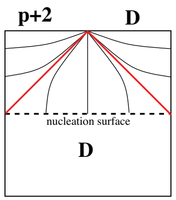
Summary



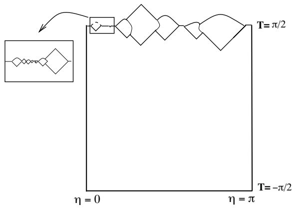
$$S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}^{(D)}} \left(\tilde{\mathcal{R}}^{(D)} - 2\Lambda - \frac{1}{2q!} \tilde{F}_q^2 \right) \longrightarrow$$ Landscape of vacua.



Solutions interpolate between D-dimensional dS and p+2 dimensional FRW across event horizons.



These solutions are nucleated from dS - Dynamical compactification.



Transitions back and forth populate the landscape of vacua.

Future directions

- Stability analysis.
  - For p+2 = 4, the compactification solutions have instabilities when q > 4 (Bousso, de Wolfe, Myers).
  - The endpoint of the instability may still be a compact manifold (warped sphere according to Kinoshita and Mukohyama).
  - What about the stability of the interpolating solutions?
  - What about thermodynamical stability? Can universes evaporate?

Future directions

- Stability analysis.
- Inhomogeneities.
  - Inevitably "collisions" between interpolating solutions will occur.
  - Field outside of brane will cause stimulated emission of small-charge branes (similar to Schwinger pair production).
  - These are multi-centered black brane solutions.
  - This changes the geometry - what happens to the homogeneity of the p+2 dimensional FRW inside the horizon?
  - Are there potentially observable effects?

Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
    - On the other side of the non-singular big-bang surface, extra dimensions become “large”.
    - Does this lead to any interesting effects?

Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
  - We have a catalog of nucleation rates. They have rather simple (and suggestive) properties.
  - Is it possible to go from this to statistical predictions for various fundamental parameters?
  - Requires an understanding of the measure - similar to eternal inflation.

Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
- What about standard 4D eternal inflation?
  - Membrane nucleation can occur inside of the locally 4D region, leading to the standard picture of 4D eternal inflation.
  - Subtleties due to interaction of flux d.o.f. with radion.

Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
- What about standard 4D eternal inflation?
- Other solutions?
  - Homogenous but anisotropic metric ansatz will generate different solutions.
  - A flat metric ansatz generates non-extremal black branes.
  - What about the other Bianchi types?
  - Bent branes?

The End.

Thanks!

Membrane nucleation inside of black branes.