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Workshop: Eternal Inflation

8 - 12 June 2009

Precision Simulations of Bubble Collisions in Eternal Inflation

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June 12, 2009

Abdus Salam International Center for Theoretical Physics

Trieste, Italy

Easther, Giblin, Hui, Lim: arxiv:09xx.xxxx

Outline

- Goals and Aspirations
- Computational Strategy
- Models:
  - Two-minima model
  - Multiple vacua
  - Vacua-in-a-row
- Extensions

Motivation

- Observational Signatures:
    - Signatures of Collisions
    - Gravitational Radiation
    - ??
- Numerical methods (3+1 lattice simulations, preheating, turbulence, etc) applicable to *any* scalar field problem?

Prior Art

- Simulating first order phase transitions:
  - Hawking, Moss, Stewart (1982)
  - Kosowksy, Turner, Watkins, Kamionkowski (1991, 1992, 1992, 1993)
- Observational Effects of Bubble Collisions:
  - Chang, Kleban, Levi (2007, 2008)
  - Aguirre, Johnson, Shomer (2007, 2007)
- Eternal Inflation Bubble Simulations:
  - Aguirre, Johnson, Tysanner (2008)

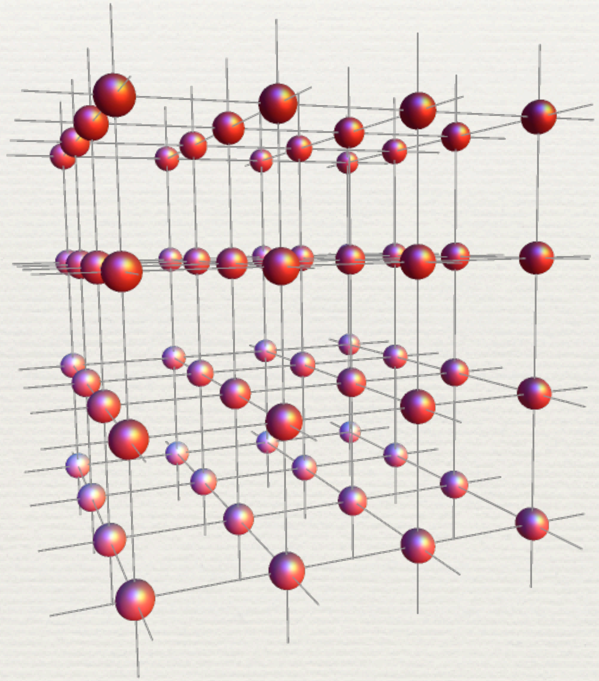
Computational Strategy

- We use a (slightly) modified version of LATTICEEASY
  - Modifications to allow for higher resolution
- Evolves scalar fields, ϕ_i, on a 3-dimensional lattice,

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{1}{a^2}\nabla^2\phi_i + \frac{\partial V(\phi)}{\partial \phi_i} = 0$$

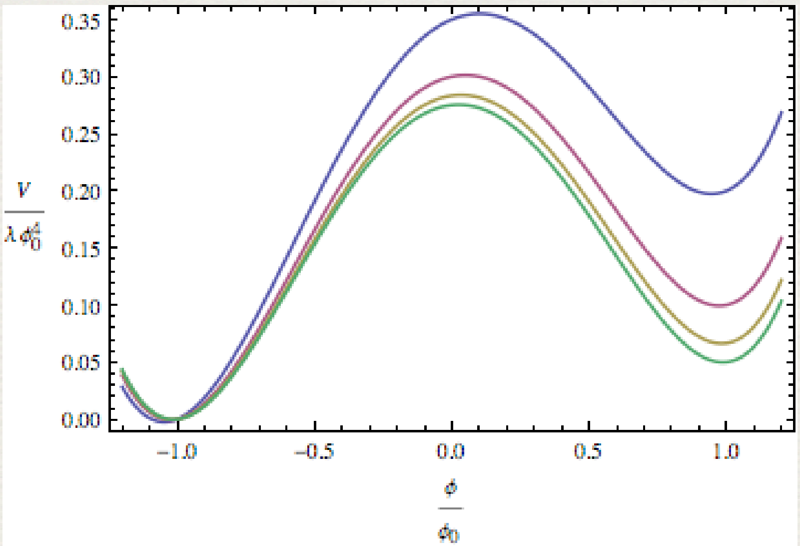
- Coupled to FRW gravity,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

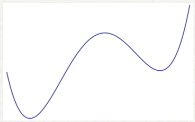


Two minima model

$$V(\phi) = \frac{\lambda}{8}\left(\phi^2 - \phi_0^2\right)^2 + \epsilon\lambda\phi_0^3(\phi + \phi_0) + \alpha\lambda\phi_0^4$$



$$\epsilon = \begin{cases} 1/10 & \text{blue} \\ 1/20 & \text{red} \checkmark \\ 1/30 & \text{yellow} \\ 1/40 & \text{green} \end{cases}$$



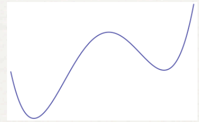
Coleman De Luccia

$$V(\phi) = \frac{\lambda}{8}\left(\phi^2 - \phi_0^2\right)^2 + \epsilon\lambda\phi_0^3(\phi + \phi_0) + \alpha\lambda\phi_0^4$$

- We have:
  - a potential with degenerate minima, U_0
  - a small symmetry breaking term
- So we can (can we?) use the CDL instanton, "thin wall":

$$\phi'' + \frac{3}{\rho}\phi' = \frac{dU}{d\phi} \rightarrow \phi'' = \frac{dU_0}{d\phi}$$

$$\phi = \phi_0 \tanh\left[\frac{\sqrt{\lambda}\phi}{2}(\rho - R_0)\right] \qquad R_0 = \frac{1}{\sqrt{\lambda}\phi_0\epsilon}$$



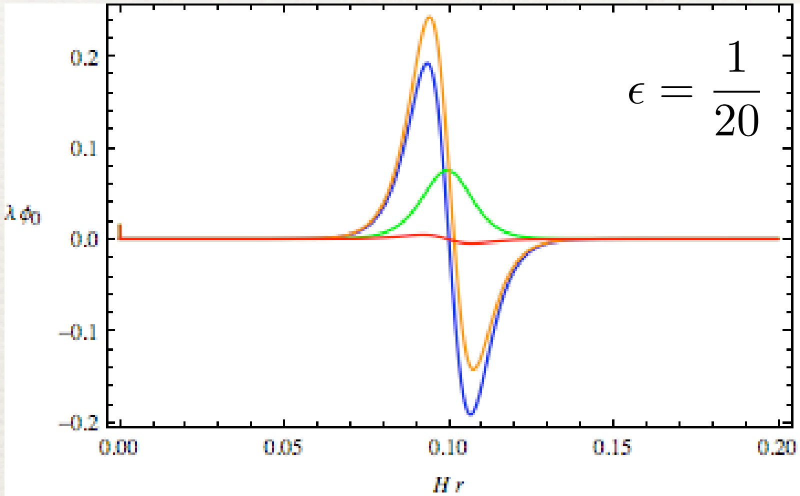
Constraints

- GUT-scale vacuum energy: $\alpha\lambda\phi_0^4 \approx 10^{-20} m_{pl}^4$
- Initial Bubble Radii, R_0, some fraction of the Hubble Length: $\sqrt{\frac{8\pi}{3}} \frac{10^{-10}}{m_{pl}} R_0 \approx 0.1$

Suggestions

- Almost-degenerate vacua: $\epsilon \ll 1$

Trust?

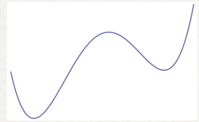


$$\phi'' \rightarrow \text{blue}$$
$$\frac{3}{\rho}\phi' \rightarrow \text{green}$$
$$\frac{dU}{d\phi} \rightarrow \text{orange}$$
$$\phi'' + \frac{3}{\rho}\phi' - \frac{dU}{d\phi} \rightarrow \text{red}$$

- Although we derived

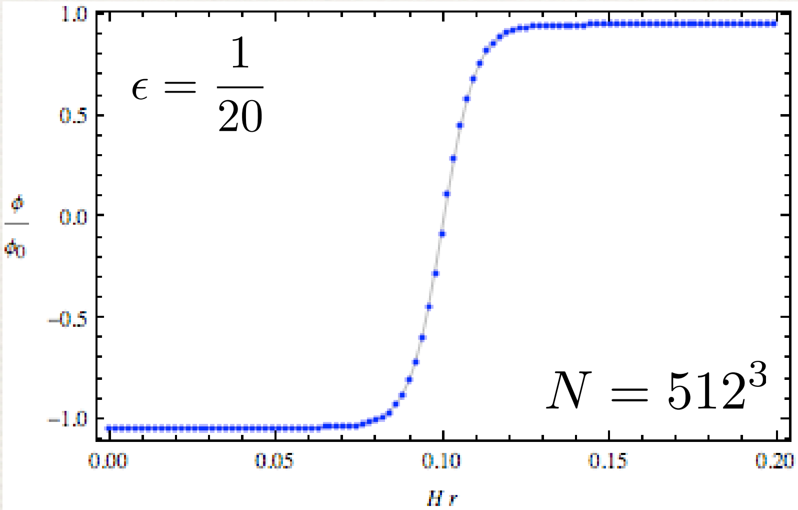
$$\phi = \phi_0 \tanh\left[\frac{\sqrt{\lambda}\phi_0^2}{2}(\rho - R_0)\right]$$

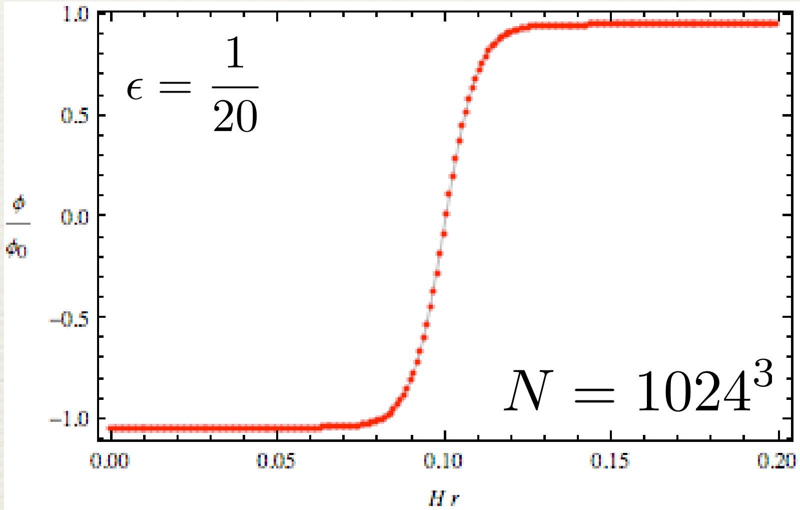
using CDL, the approximate solution is a good solution to the full equations of motion



Resolution?

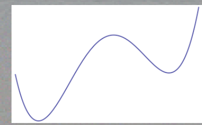
- We have to make sure that there are sufficient grid-points along each lattice side:





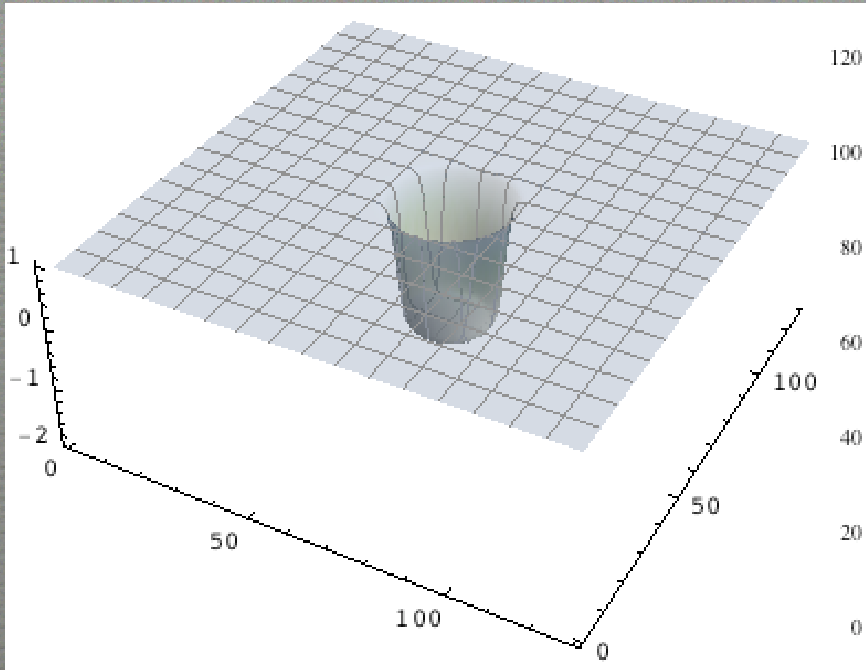
- We aim to use $N = 512^3$ and $N = 1024^3$ so that we have sufficient resolution along each edge

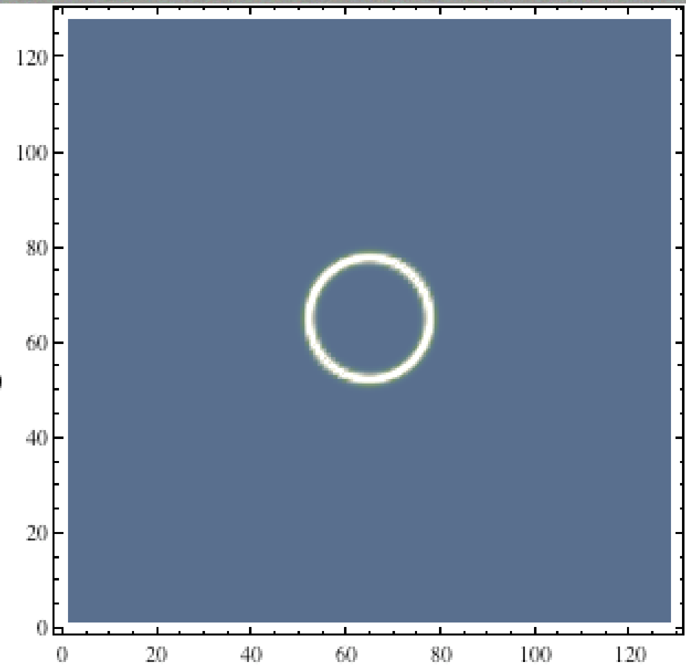
- Technical Aside: a 1024 lattices is 8Gb large!

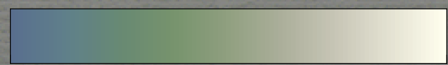


Field Profile (y-z plane)

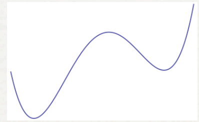
Energy Density



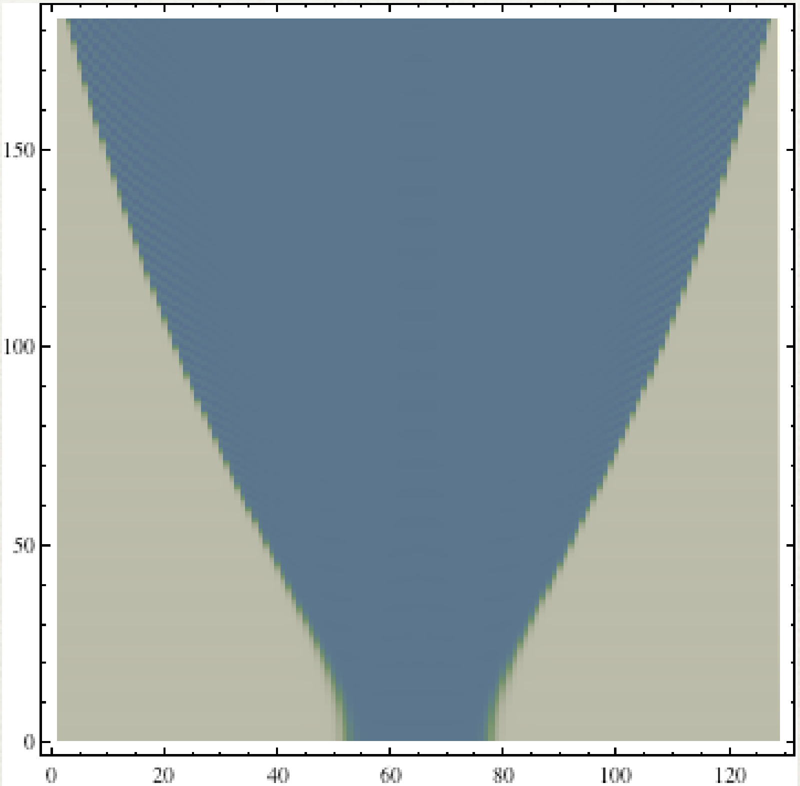


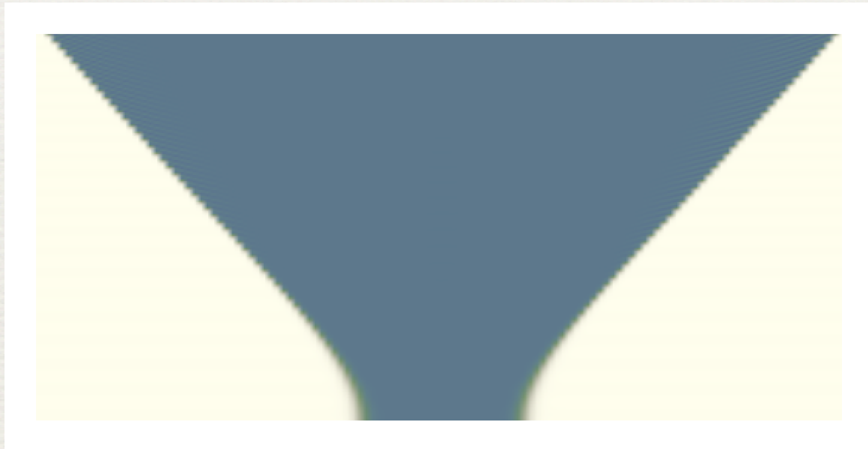


One Bubble

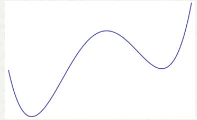


One Bubble





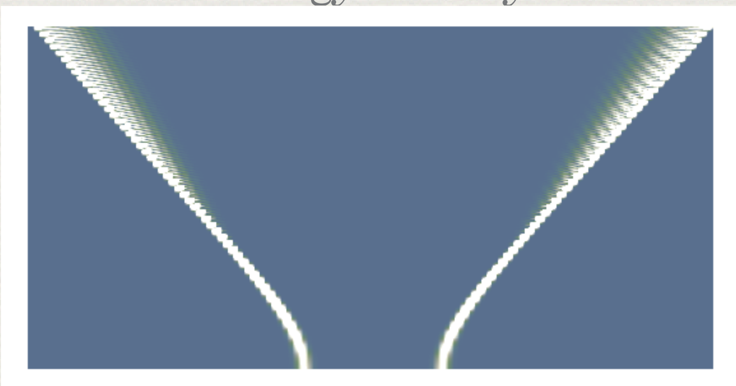
Comoving vs Conformal
Field Configuration



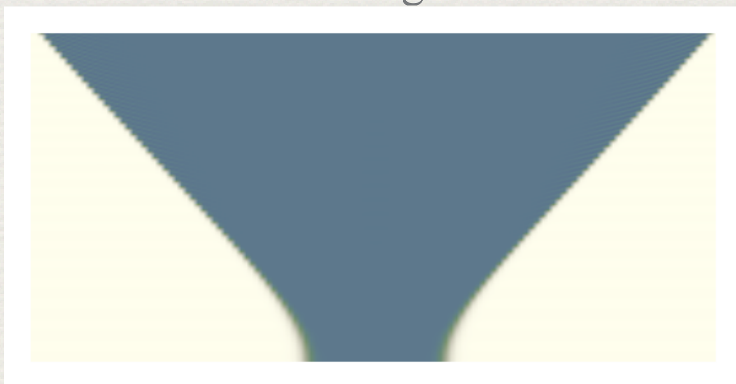
Conformal Diagrams

When you plot the energy density, you can see that most of the energy lies along the domain wall between the bubble and the bulk

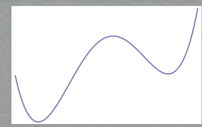
Conformal Diagram:
Energy Density



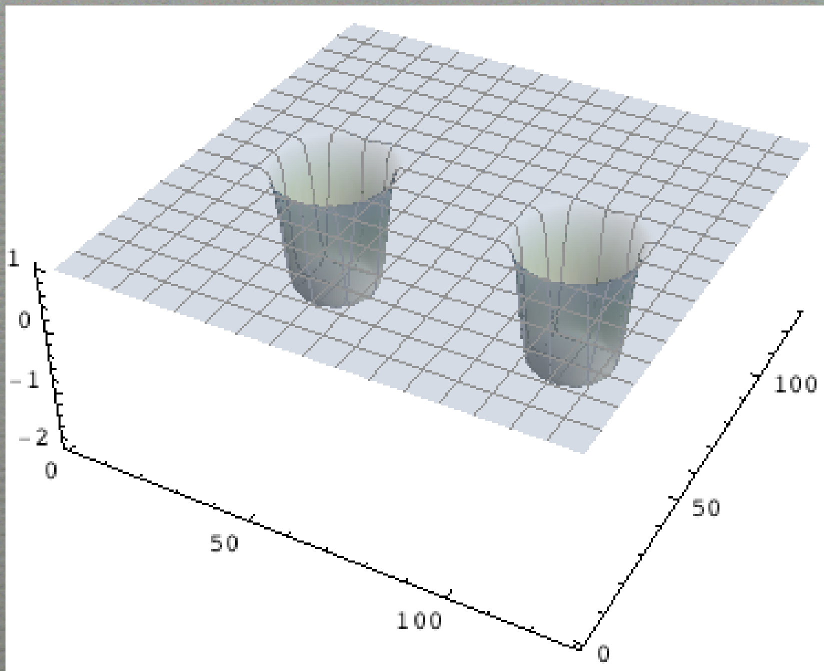
Conformal Diagram:
Field Configuration



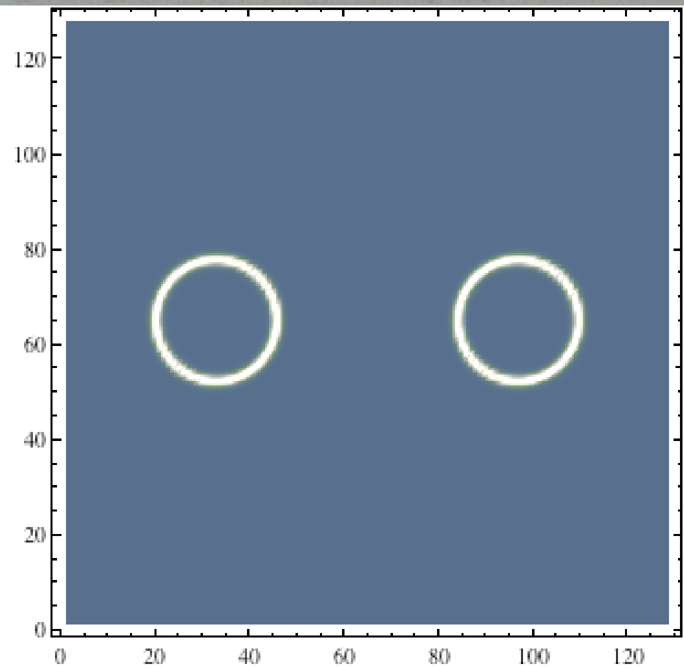
What about bubble collisions?



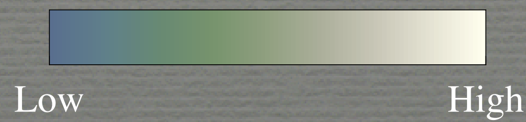
Field Profile (y-z plane)



Energy Density

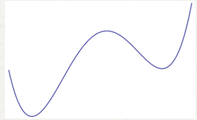


Two Bubbles



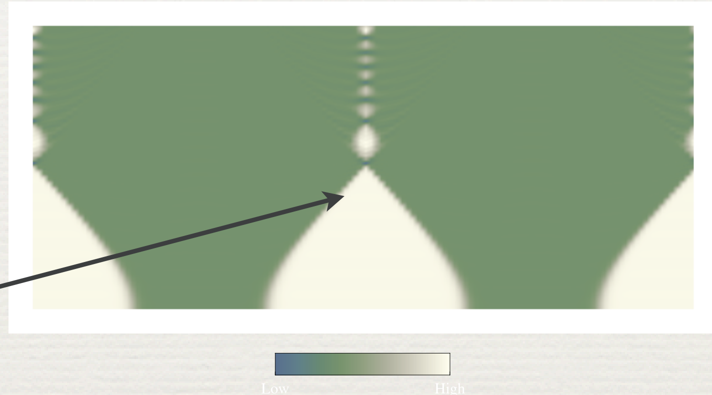
Field Configuration
z-τ plane

Energy Density
z-τ plane



Interaction Plane

Conformal Diagram:
Field Configuration



- Along the interaction plane, regions of spacetime are *classically* returned to the upper minima (pictured in beige here)

- This is reminiscent of...

  - Hawking, Moss, Stewart (1982)
  - Kosowsky, Turner (1993)

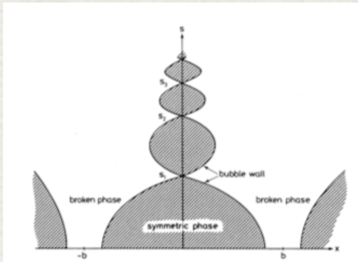


FIG. 1. The collision of two bubbles in the case $\alpha=0$. The dashed region denotes the old, symmetric phase outside the bubbles which nucleate at $x=b$ and $x=-b$ on $s=0$.

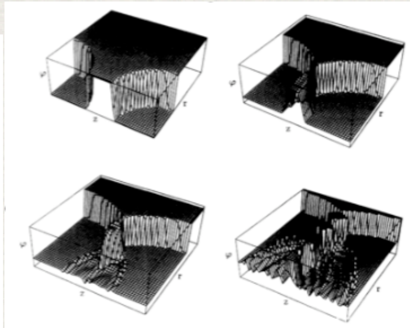
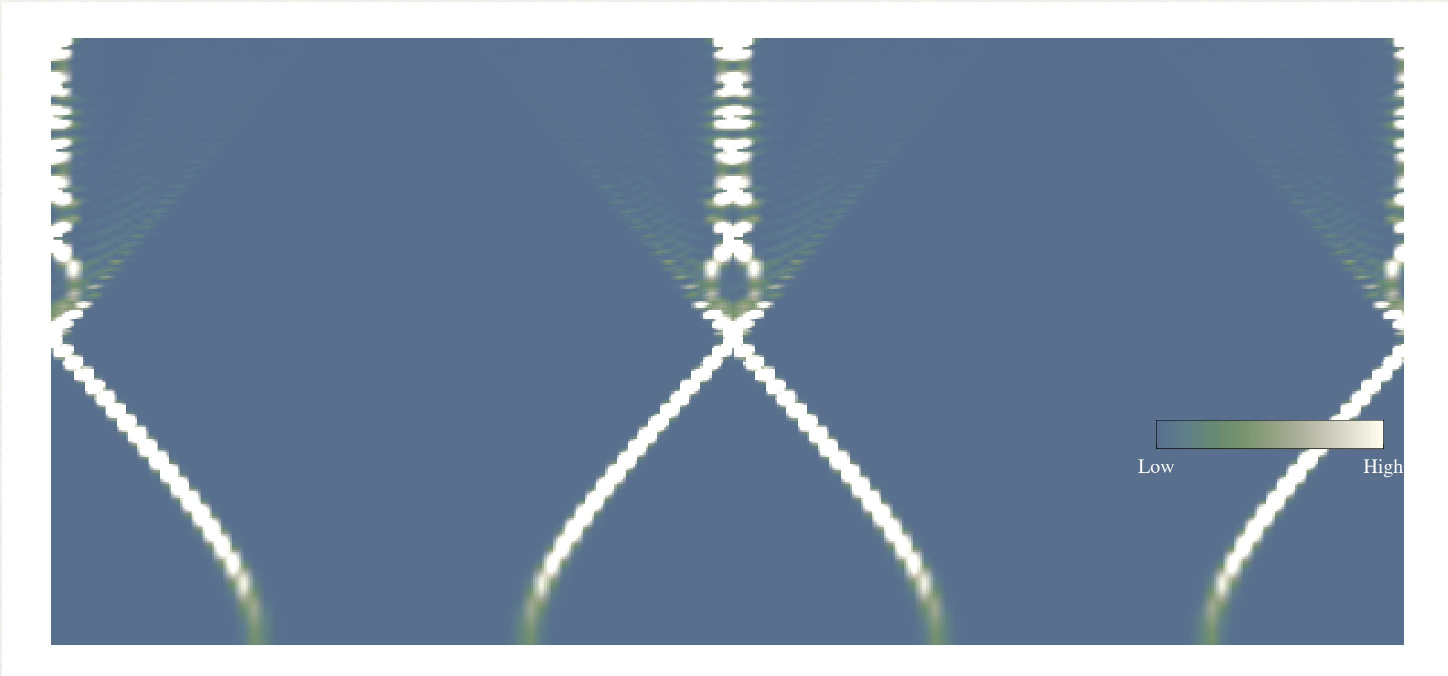


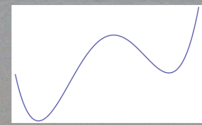
FIG. 2. Evolution of two identical vacuum bubbles. From left to right and top to bottom, $t=36$, 60, 72, and 96. The plots are 100 units in the r and z directions; each square is 2×2 dimensionless units.

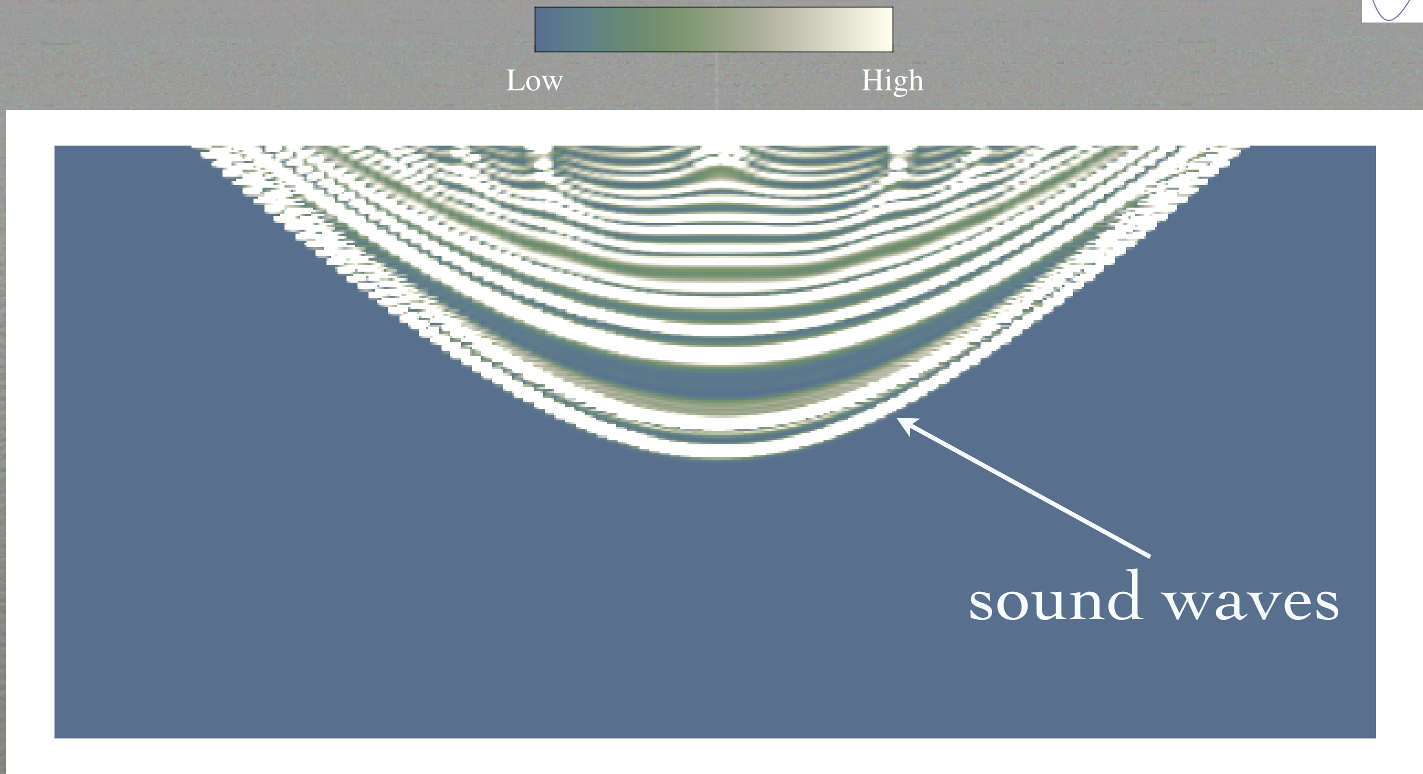
Interaction Plane

- We also see some (although very little) energy being "radiated" toward the center of each bubble.

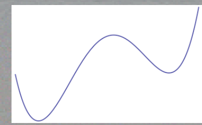


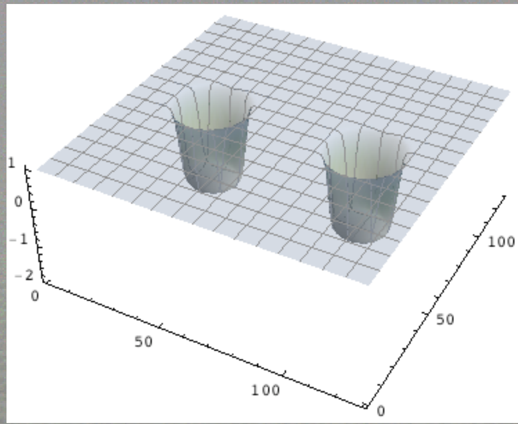
Conformal Diagram: Energy Density

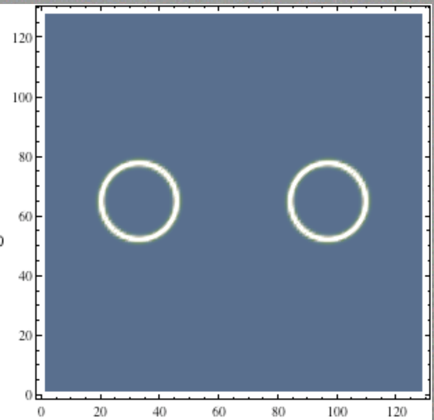


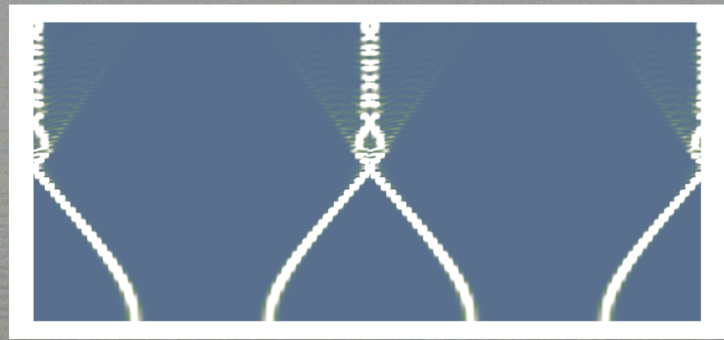


Conformal Diagram: Energy Density y-t plane

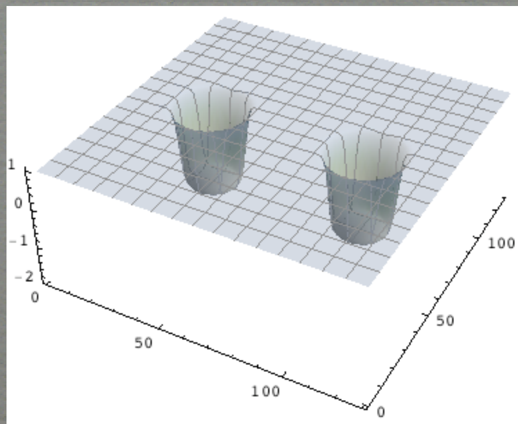


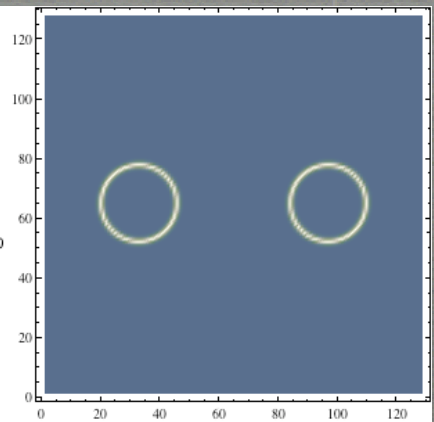


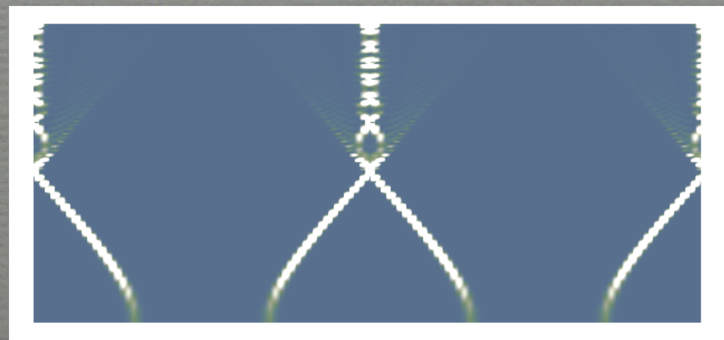


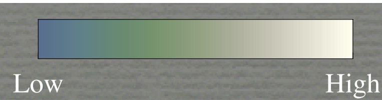


Field Profile (y-z plane) Energy Density Conformal Diagram: Energy Density

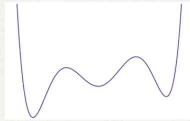






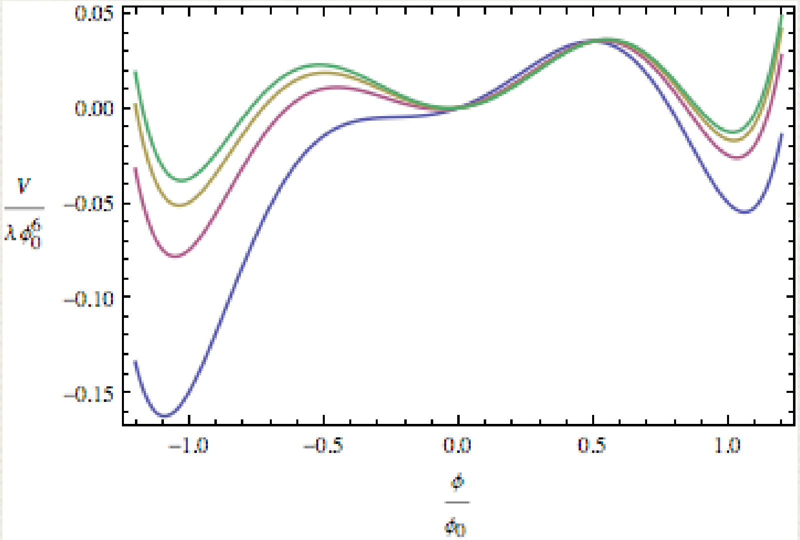


Resolution?

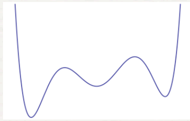


A Model with Three Minima

$$V(\phi) = \frac{\lambda}{4}\phi^2\left(\phi^2 - \phi_0^2\right)^2 - \epsilon\phi_0^4\phi\lambda\left(\phi - \frac{\phi_0}{2}\right) + \alpha\lambda\phi_0^6$$



$$\epsilon = \begin{cases} 1/10 & \text{blue} \\ 1/20 & \text{red} \\ 1/30 & \text{yellow} \\ 1/40 & \text{green} \checkmark \end{cases}$$



CDL

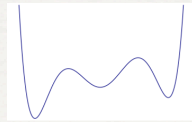
$$V(\phi) = \frac{\lambda}{4}\phi^2\left(\phi^2 - \phi_0^2\right)^2 - \lambda\phi_0^4\phi\left(\phi - \frac{\phi_0}{2}\right) + \alpha\lambda\phi_0^6$$

- We have:
  - a potential with degenerate minima,
  - a small symmetry breaking term

- There are two transitions, so we have two CDL solutions:

$$R_1 = \frac{3}{2\sqrt{2\lambda}\phi_0^2\epsilon} \qquad R_2 = \frac{1}{2\sqrt{2\lambda}\phi_0^2\epsilon}$$

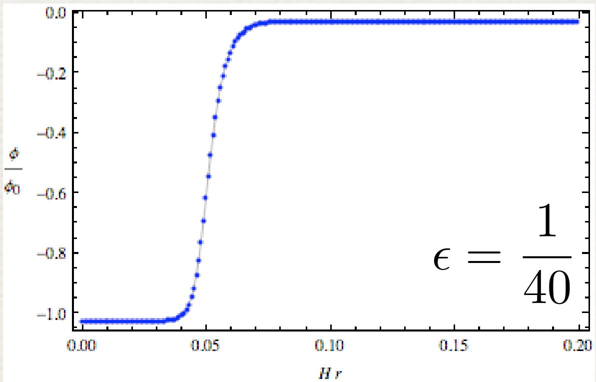
$$\phi = -\frac{\phi_0}{\sqrt{1 + 2e^{\sqrt{2\lambda}\phi_0^2(\rho - R_2)}}} \qquad \phi = \frac{\phi_0}{\sqrt{1 + 2e^{\sqrt{2\lambda}\phi_0^2(\rho - R_1)}}}$$

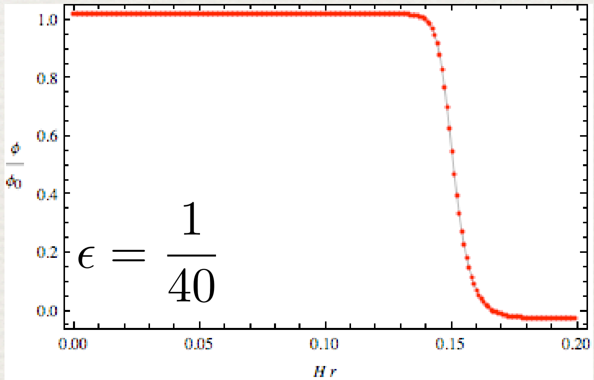


the field profile

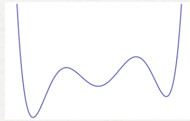
$$\phi = -\frac{\phi_0}{\sqrt{1 + 2e^{\sqrt{2\lambda}\phi_0^2(\rho - R_2)}}}$$

$$\phi = \frac{\phi_0}{\sqrt{1 + 2e^{\sqrt{2\lambda}\phi_0^2(\rho - R_1)}}}$$





- Using 1024^3, we can get reasonable resolution along the field profile



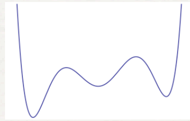
Same Constraints as before

- GUT-scale vacuum energy: $\alpha\lambda\phi_0^6 \approx 10^{-20} m_{pl}^4$

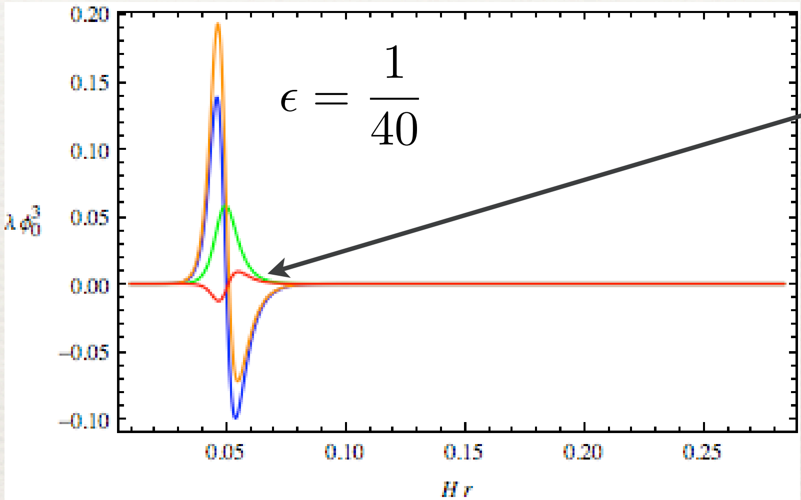
- Initial Bubble Radii, R_0, some fraction of the Hubble Length: $\sqrt{\frac{8\pi}{3}} \frac{10^{-10}}{m_{pl}} \frac{R_1 + R_2}{2} \approx 0.1$

with the same suggestion

- Almost-degenerate vacua: $\epsilon \ll 1$



Just so that you (might) trust us...



the exact solution is in good agreement

ϕ'' → blue

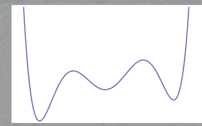
$\frac{3}{\rho}\phi'$ → green

$\frac{dU}{d\phi}$ → orange

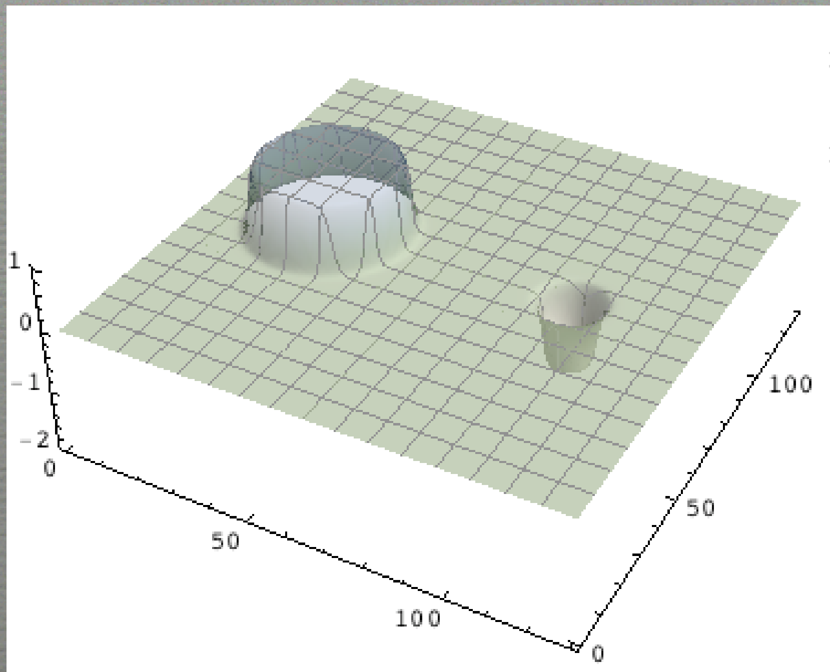
$\phi'' + \frac{3}{\rho}\phi' - \frac{dU}{d\phi}$ → red

<u>Moral</u>

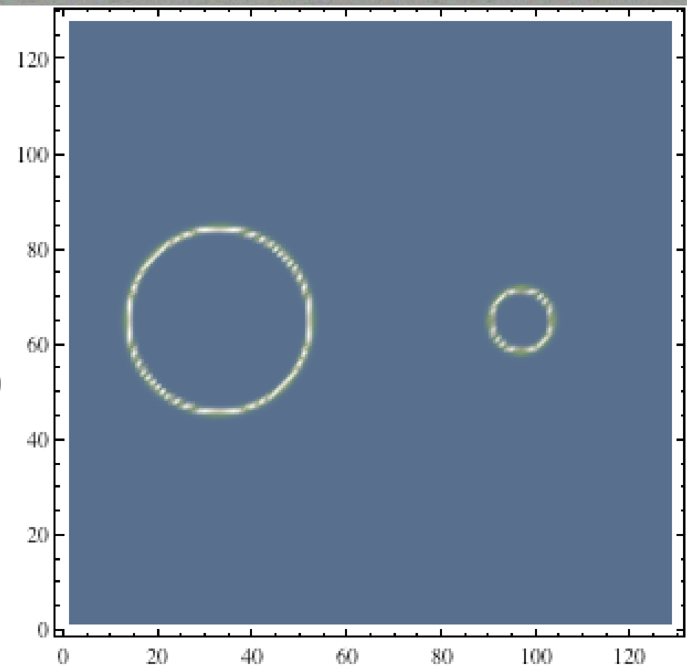
The the CDL solution is usually a good approximation, but it needs to be explicitly checked



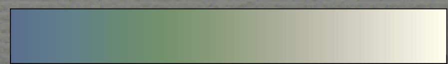
Field Profile (y-z plane)



Energy Density

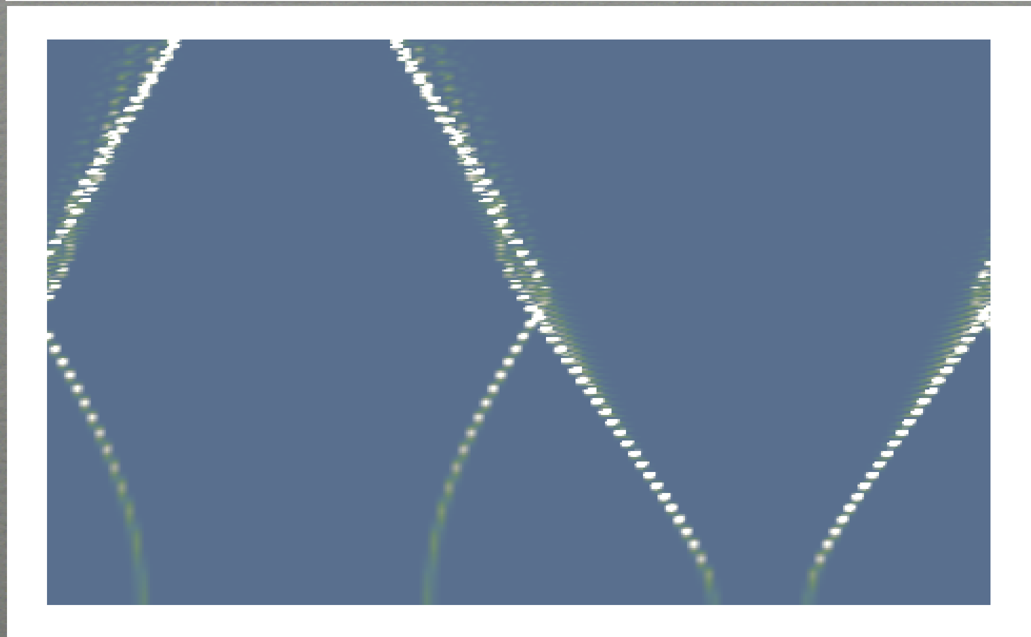


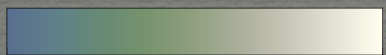
Two Bubbles



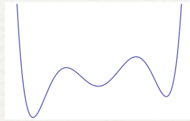
Field
Configuration
z-τ plane

Energy Density
z-τ plane

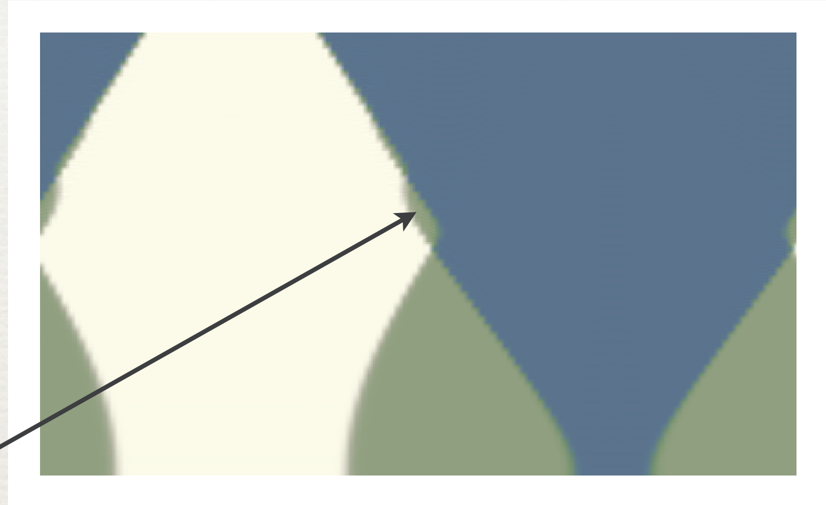


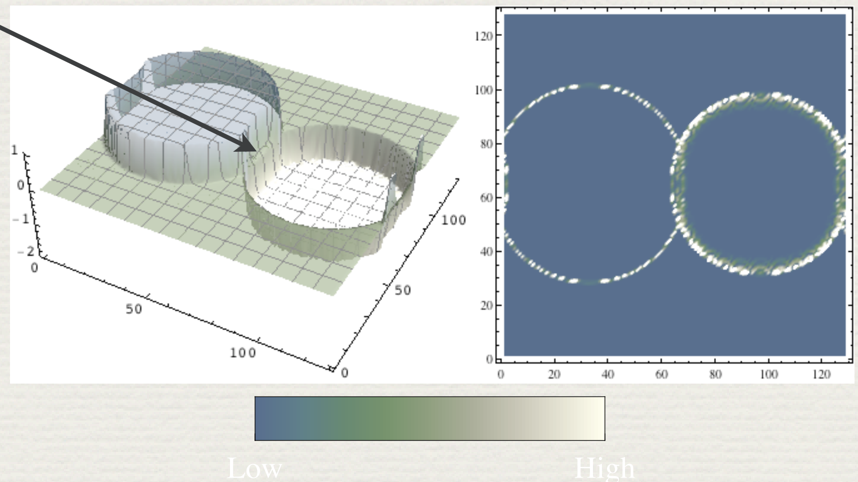


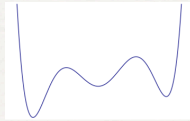
The highest minimum



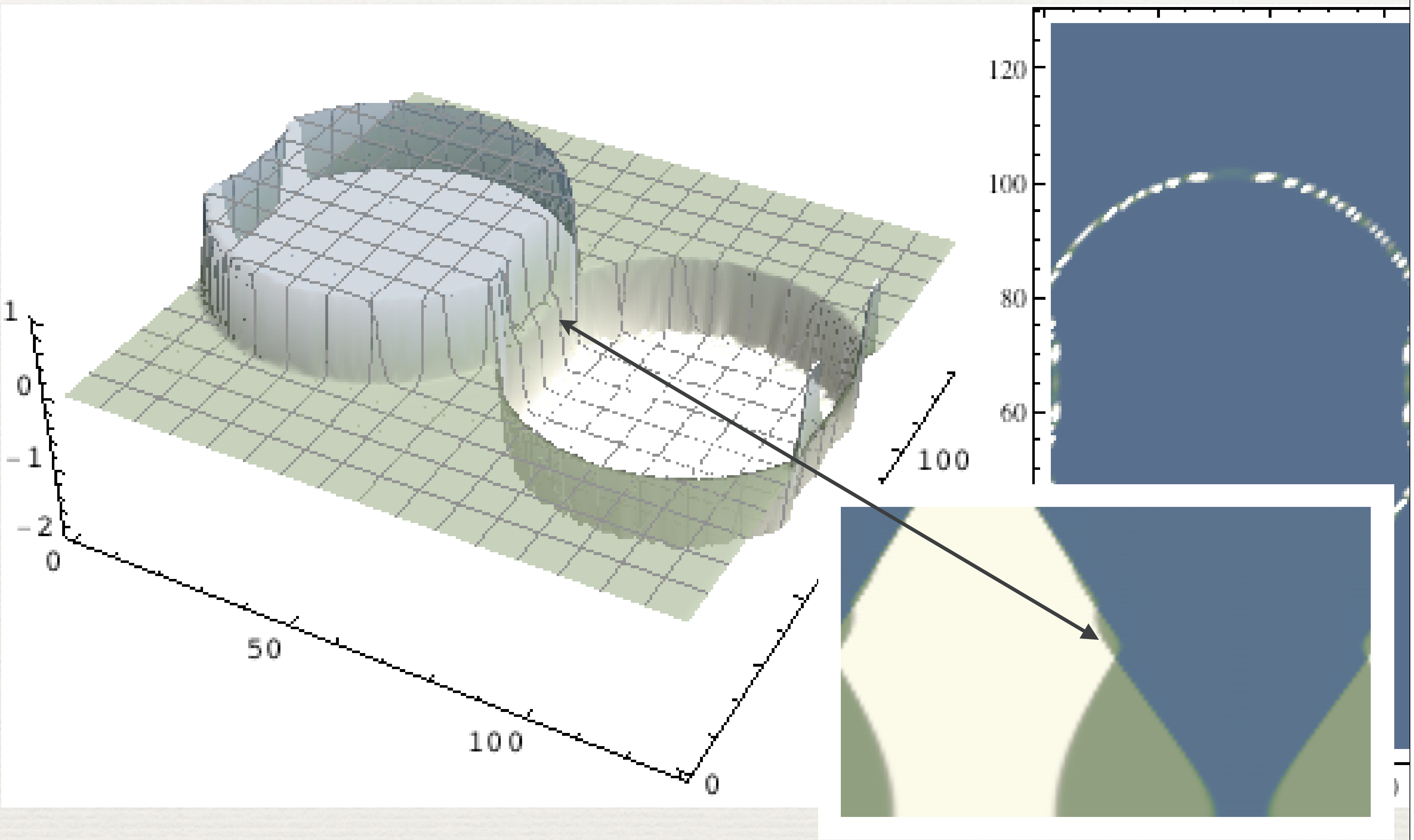
- We recover an (almost)expected picture
  - Kleban et al (2007)
- But we still see regions where the field is in the highest energy metastable local minima





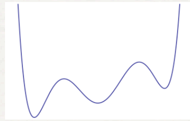


Hard to see?



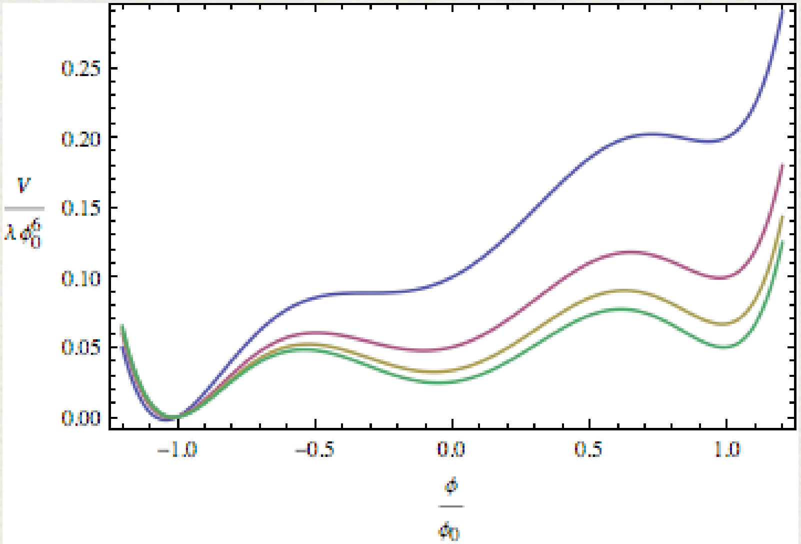
Generic quality?

- Even in the case of complicated potentials, the field can *classically* move into different metastable states
- In both cases so far, the field had *already* been in that vacuum (just before the collision).
- What if it's not?



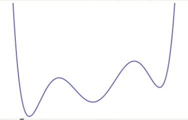
A(nother) Model with Three Minima

$$V(\phi) = \frac{\lambda}{4}\phi^2\left(\phi^2 - \phi_0^2\right)^2 + \epsilon\phi_0^5\lambda(\phi + \phi_0) + \alpha\lambda\phi_0^6$$

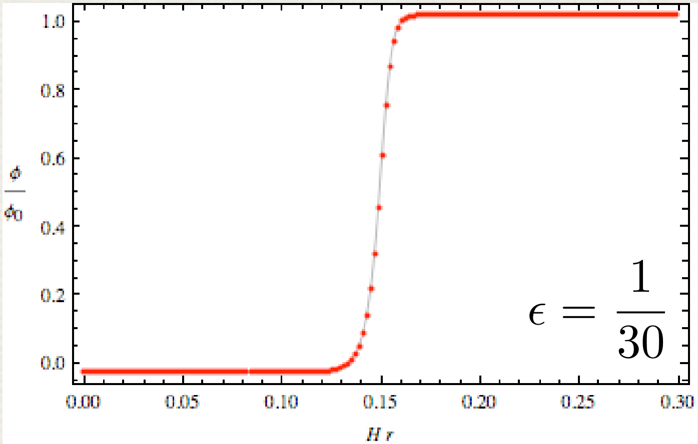


$$\epsilon = \begin{cases} 1/10 & \text{blue} \\ 1/20 & \text{red} \\ 1/30 & \text{yellow} ✓ \\ 1/40 & \text{green} \end{cases}$$

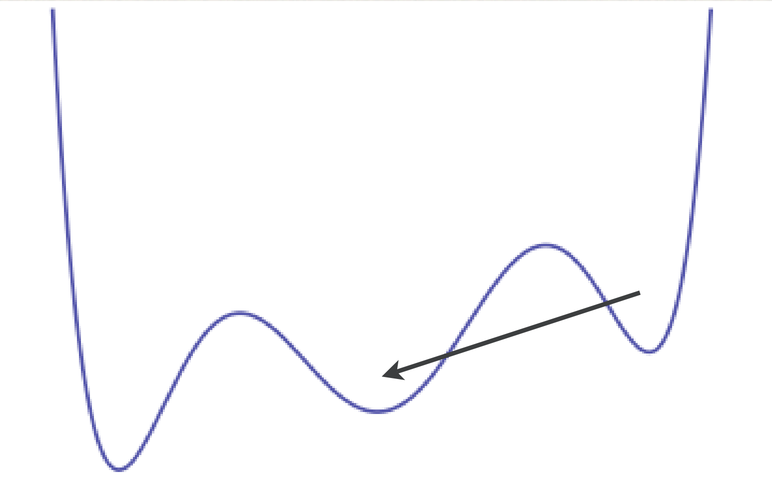
We (basically) know the CDL solution

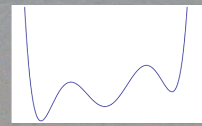


$$\phi = \frac{\phi_0}{\sqrt{1 + 2e^{-\sqrt{2\lambda}\phi_0^2(\rho - R_0)}}}$$

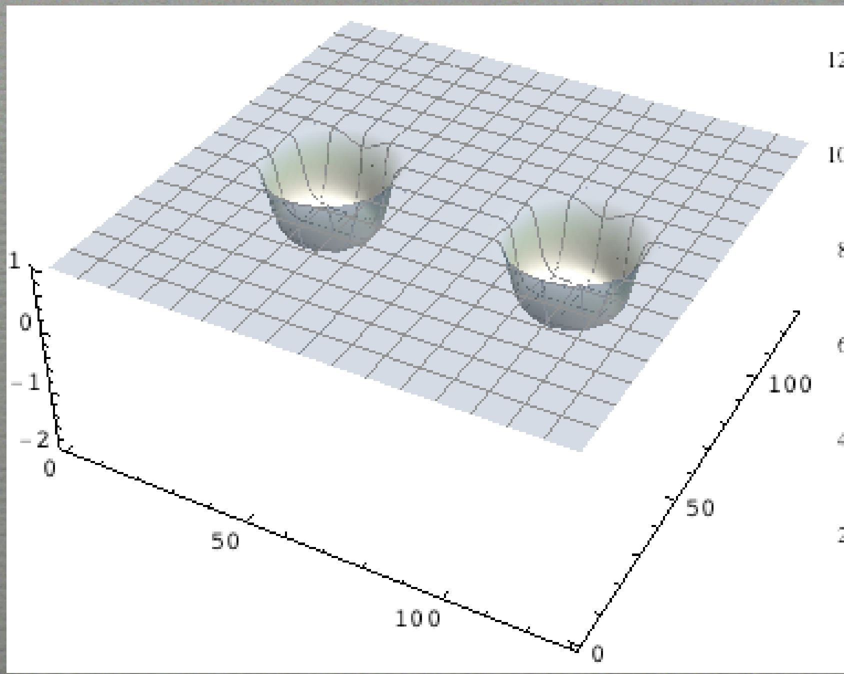


- We will start in the highest of the three potentials
- We will nucleate two bubbles in the middle minima

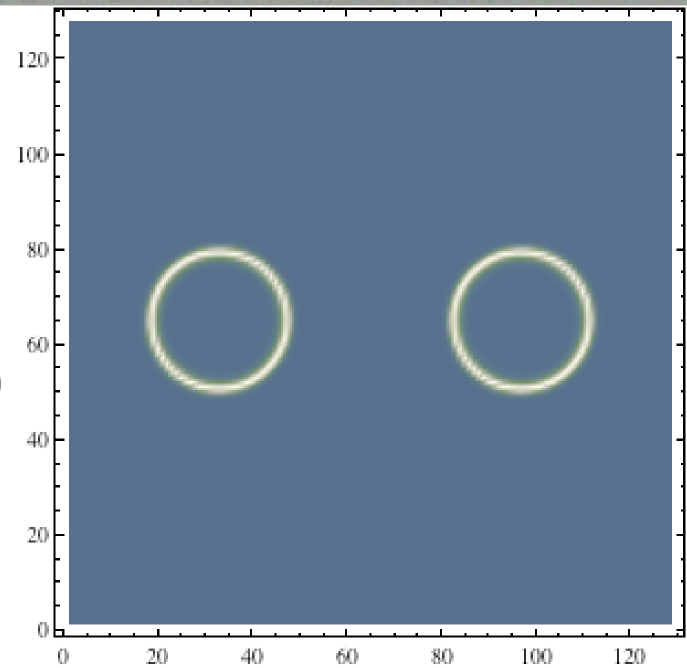




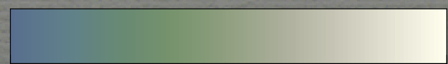
Field Profile (y-z plane)



Energy Density

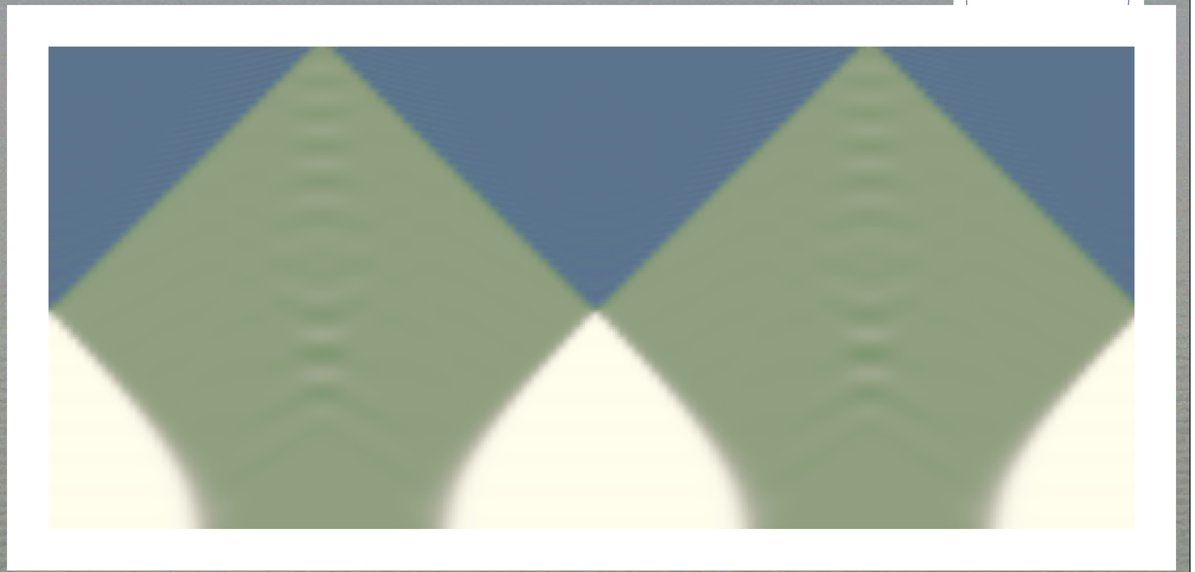


Two Bubbles

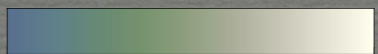


Low High

Field Configuration
z-τ plane



Energy Density
z-τ plane



A new way to form bubbles: Classical Transitions

- For (at least some) generic set of parameters, we seem to see regions of spacetime being moved via classical processes, not quantum
  - Due to proximity in field space?
  - Related to tunneling rate??
- How generic is this??

Future

- More bubbles (trivial extension)
  - arbitrary configuration possible!
- How much energy is transported into the bubble? (metric stitching)
- More toy models
- Use scalar fields as a source for gravitational radiation
  - a la preheating
- Couple these fields to massive/massless fields

Concluding Remarks

- We need to be careful when making dimensional reductions
  - energy flow orthogonal to line between colliding bubbles
- We need to understand where the field "can go" during a bubble collision
  - if there's a close lower energy minima in the vicinity...

Fin