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Workshop: Eternal Inflation

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Global/Local Duality in the Measure Problem

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Global/Local Duality in the Measure Problem

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Outline

1. **The Measure Problem**
   - Global Measures
   - Local Measures
2. **The Duality**
   - Scale factor and fat geodesics
   - Lightcone time cut-off and the causal patch measure
   - It does not always work.
3. **Discussion**
   - Holography
   - The most stable vacuum

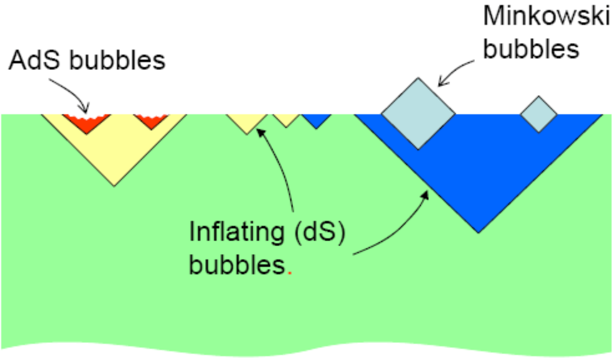
Infinite Occurence

Result of an observation : 1 or 2.

$$\frac{P_1}{P_2} = \frac{\langle N_1 \rangle}{\langle N_2 \rangle} = \frac{\infty}{\infty} \quad ???$$

How about an open FRW universe with standard big bang theory ?

No large scale homogeneity



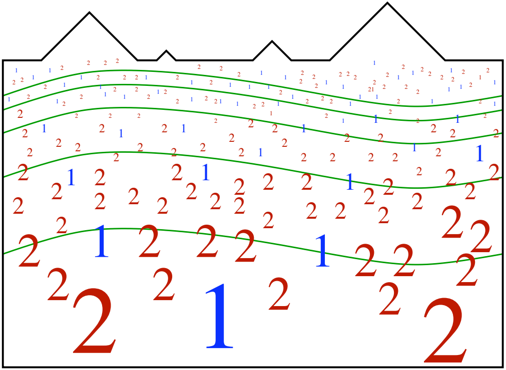
The Measure Problem

- Infinite Occurence
- No large scale homogeneity

Geometric cut-off.

- Global measure
  Specify a homogeneous state.
- Local measure
  Restrict to a finite region.

Geometric Cut-off



$$\frac{P_1}{P_2} = \lim_{t\to\infty} \frac{\langle N_1 \rangle_{\Sigma_t}}{\langle N_2 \rangle_{\Sigma_t}}$$

Proper time measure (Linde, 1986)

$$\begin{array}{ccc} \text{expansion} & \text{out-flow} & \text{in-flow} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$\frac{dV_i}{dt} = (3H_i - \sum_j \tilde{\Gamma}_{ji})V_i + \sum_j \tilde{\Gamma}_{ij} V_j$$

- Dominated by the fastest expanding vacuum.

$$H_* = Max\{H_i\} \ , \quad \frac{P_1}{P_2} = \frac{e^{-3H_* t_1}}{e^{-3H_* t_2}} \frac{\Gamma_{1*}}{\Gamma_{2*}} \ .$$

- Youngness Paradox.
  (Linde 1996, Guth 2004, Tegmark 2004, BFY 2007)

Alternatives ?

expansion out-flow in-flow

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{dV_i}{dt} = (3H_i - \sum_j \tilde{\Gamma}_{ji})V_i + \sum_j \tilde{\Gamma}_{ij} V_j$$

$$\frac{dV_i}{d\eta} = (3 - \sum_j \Gamma_{ji})V_i + \sum_j \Gamma_{ij} V_j$$

$$\eta = \frac{1}{3}\log(\frac{V}{V_{\mathrm{init}}})$$

Scale factor time (Garriga, Schwartz-Perlov, Vilenkin, Winitzki, 2005)

$$\frac{dV_i}{dt} = (3 - \sum_j \Gamma_{ji})V_i + \sum_j \Gamma_{ij} V_j$$

- Dominated by the most stable vacuum.

$$\sum_j \Gamma_{j*} = Min\{\sum_j \Gamma_{ji}\} , \quad \frac{P_1}{P_2} = \frac{\Gamma_{1*}}{\Gamma_{2*}} .$$

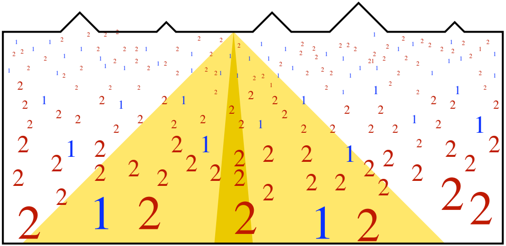
- Subtleties for non-expanding regions.
- No obvious contradictions.
- Motivated by holography.
  (J. Garriga and A. Vilenkin, 2008-2009)

One Geodesic

Which one ?

- A geodesic ends up in a special place.
  In a $\Lambda = 0$ region, the census taker. (L. Susskind, 2007)
  In an eternally inflating region.
  (V. Vanchurin and V. Vilenkin, 2006)
- A geodesic starts with specific initial conditions.
  (R. Bousso, 2006)
  Already finite.
  An Ensemble of histories.
  Predictions depend on initial conditions.

Counting Observers

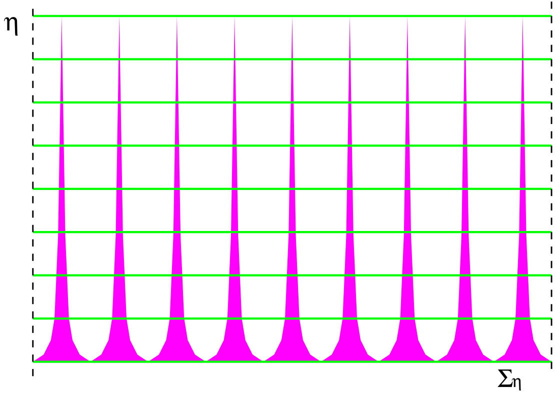


- Causal patch measure. (R. Bousso, 2006)
- Fat geodesics measure. (BFY, 2008)

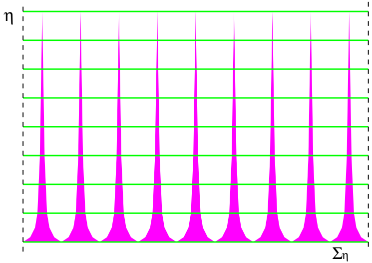
Scale factor and fat geodesics

Global/Local Duality

Fat geodesics measure + the most stable vacuum
= Scale factor measure. (BFY 2008)



Scale factor and fat geodesics (cont)



$$|dS_i(\eta)| = \frac{dN_i}{d\eta} = e^{3\eta}$$

$$\frac{|dS_i(\eta) \cap \tilde{S}_i|}{|dS_i(\eta)|} = e^{-3\eta}$$

Local: $\tilde{N}_i = |\tilde{S}_i|$

Global: $N_i(\eta) = |S_i(\eta)|$

$$\frac{\tilde{N}_i}{\tilde{N}_j} = \lim_{\eta \to \infty} \frac{N_i(\eta)}{N_j(\eta)}$$

Comparison

Global/Local Duality

Fat geodesics measure + the most stable vacuum
= Scale factor measure. (BFY 2008)

Fat geodesics measure

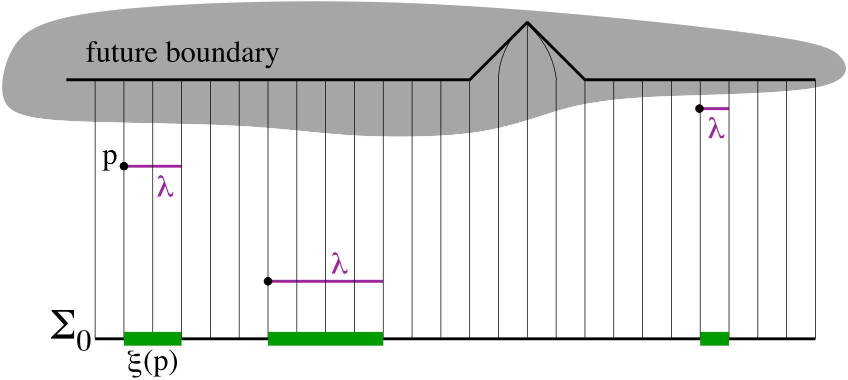
- Local
- Everywhere well-defined
- A redundant scale

Scale factor time measure

- Global
- Expanding regions
- Motivated by holography

Holography Motivated Scale Factor Cut-off

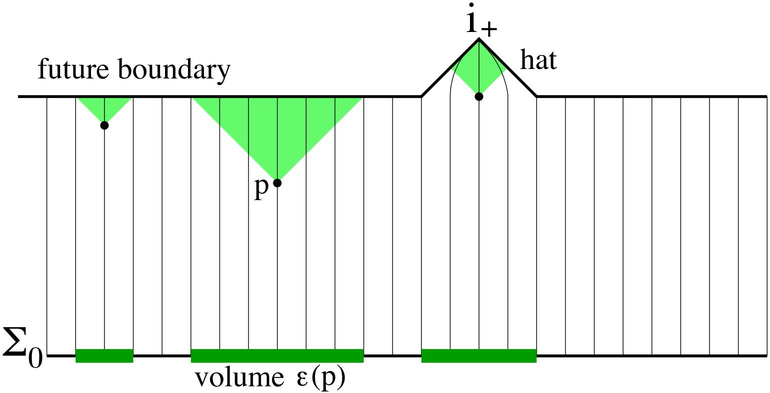
$$\eta = \frac{1}{3}\log\left(\frac{V}{V_{\text{init}}}\right)$$



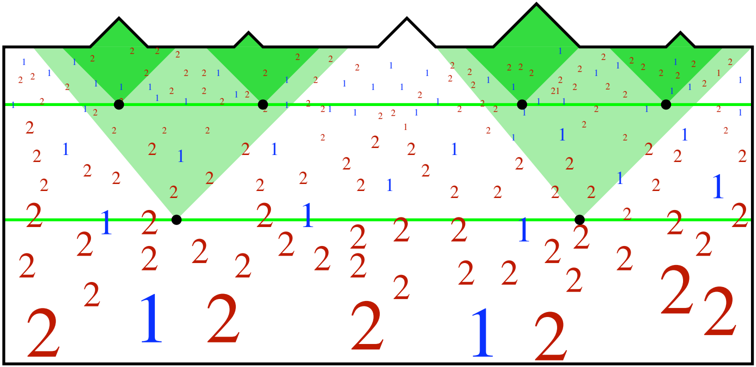
$$\eta = -\log(\text{number of geodesics in } \lambda)$$

Can we eliminate the redundant scale ?

$$\tau = -\log(\text{number of geodesics in the future lightcone})$$



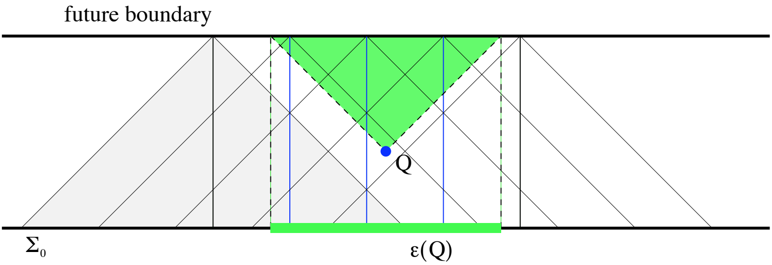
Lightcone time



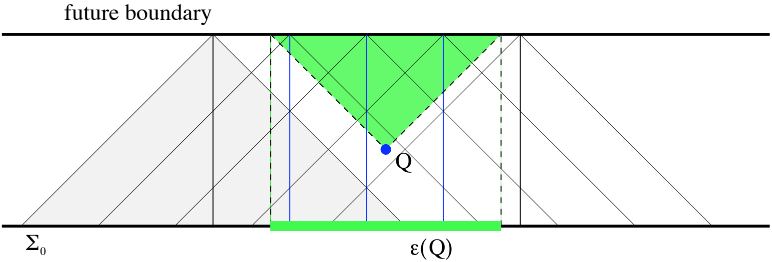
Lightcone time and the causal patch measure

Global/Local Duality

Causal patch measure + the most stable vacuum
= Lightcone time measure. (BY 2009)



Lightcone time and the causal patch measure



(Number of lightcones that include event Q)$=$ Exp$(-3\tau_Q)$.

$$\tilde{N}_i = \int \frac{dN_i(\tau)}{d\tau} e^{-3\tau} d\tau \propto N_i(\tau)$$

Lightcone time and the causal patch measure (cont)

Global/Local Duality

Causal patch measure + the most stable vacuum
= Lightcone time measure.

- Exactly the same prediction.
- Divergent in $\Lambda = 0$ vacuum.
- Prefer $\Lambda < 0$. (M. Salem 2009)
- Motivated by holography. (BFY 2006, Bousso 2009)

Does it always work ?

Duality.

Fat geodesics measure + the most stable vacuum
= Scale factor time measure.

Duality again !

Causal patch measure + the most stable vacuum
= Lightcone time measure.

Does this work ?

Proper time measure =
the fastest expanding vacuum + some local measure ?

No.

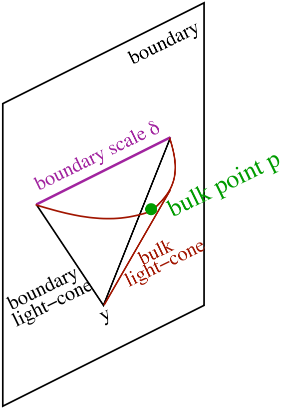
No.

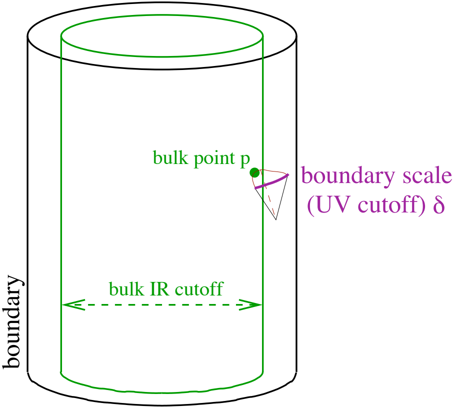
Proper time measure $\neq$
the fastest expanding vacuum $+$ some local measure.

The later gives no problematic predictions. (BY 2007, BFY 2008)

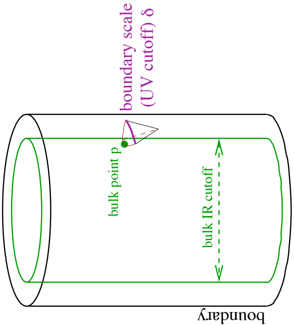
- Good predictions.
- The most stable vacuum.
- Holography.

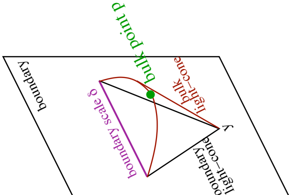
AdS/CFT

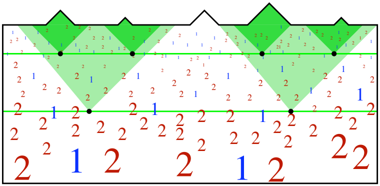

boundary
boundary scale δ
bulk point p
bulk
light–cone
boundary
light–cone
y


bulk point p
boundary scale
(UV cutoff) δ
bulk IR cutoff
boundary

CFT on the future boundary ?







- Bousso 2009
- Garriga and Vilenkin 2008-2009
- Freivogel and Kleban 2009

Future directions

- How to find it ? (Denef and Douglas 2006)
- Geometry of upward tunnelings.
- Other effects on recurrence time scale.

Thank you.

The graphs came from the following 3 papers.

- 0809.4257, J. Garriga and A. Vilenkin.
- 0901.4806, R. Bousso.
- 0904.2386, R. Bousso and I. Yang.