



**The Abdus Salam  
International Centre for Theoretical Physics**



**2038-13**

**Conference: From DNA-Inspired Physics to Physics-Inspired Biology**

*1 - 5 June 2009*

**Physics Point of View. Topology and Dimensionality Effects**

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# DNA from the Polymer Physics Point of View: Topology and Dimensionality Effects

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[lpmv.epfl.ch](http://lpmv.epfl.ch)

[ipsb.epfl.ch](http://ipsb.epfl.ch)

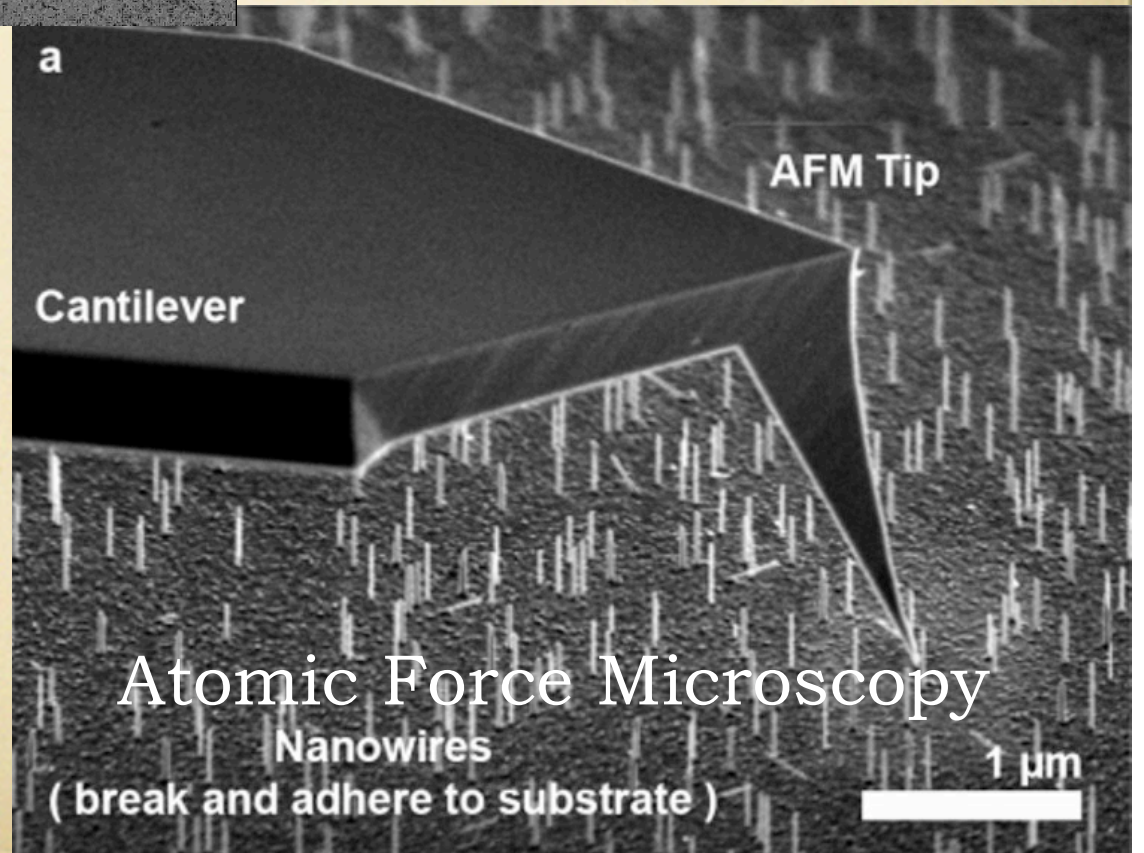
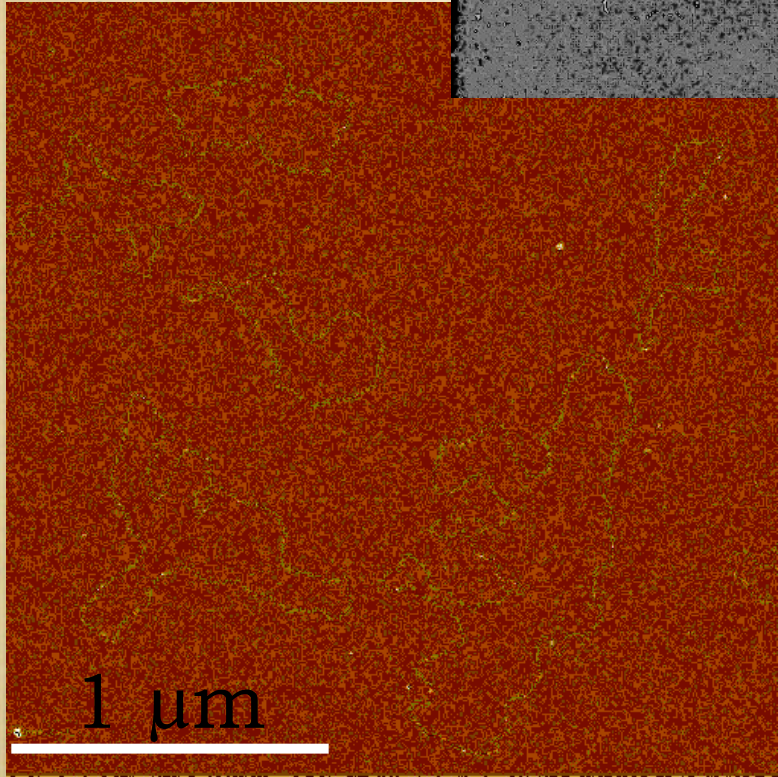
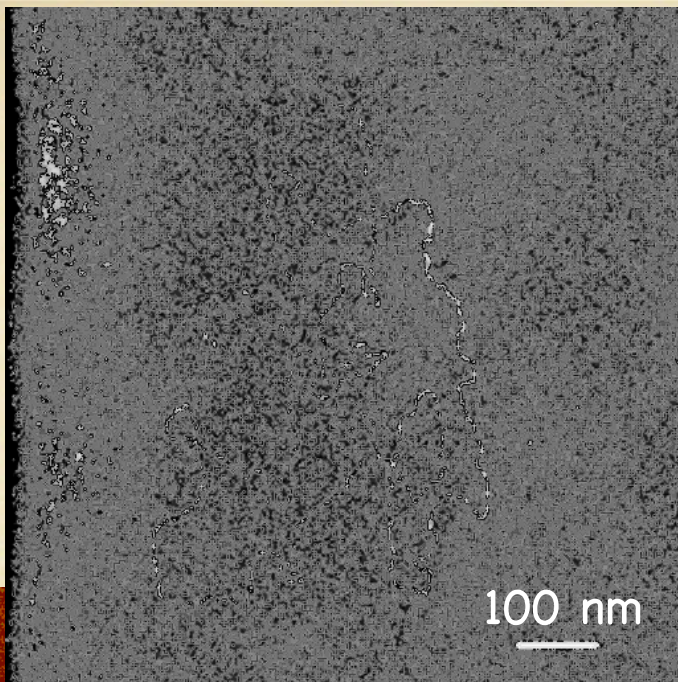
# Summary of the following publications

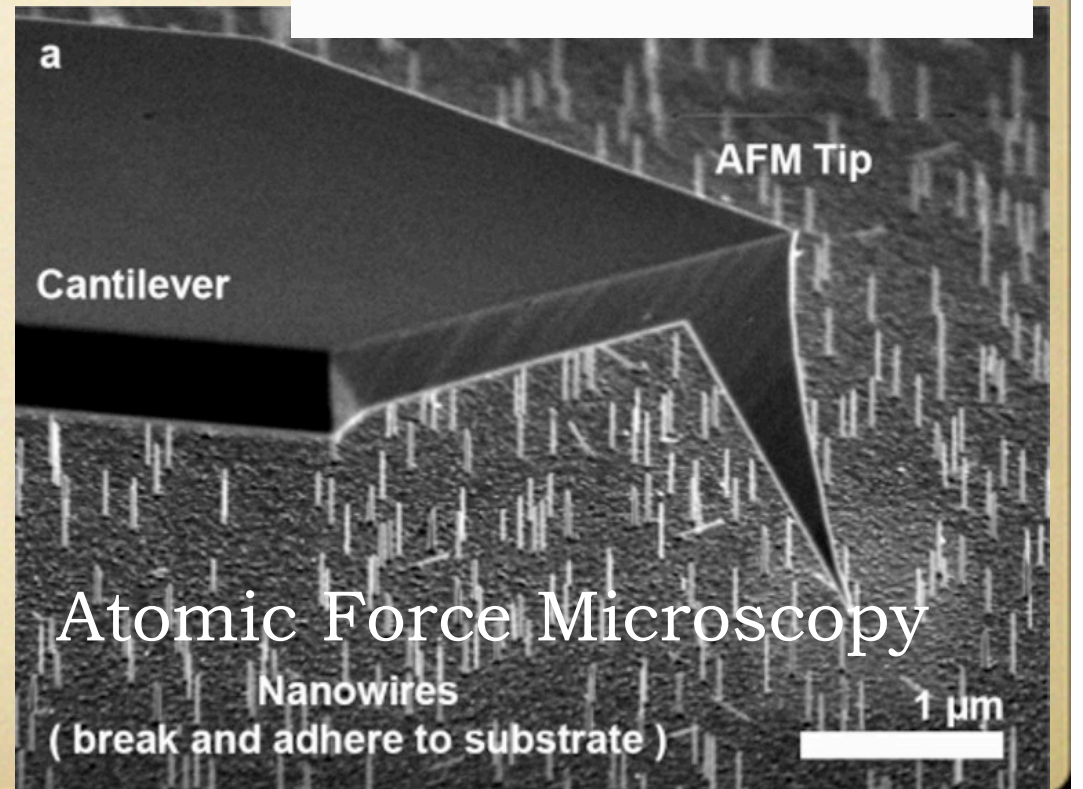
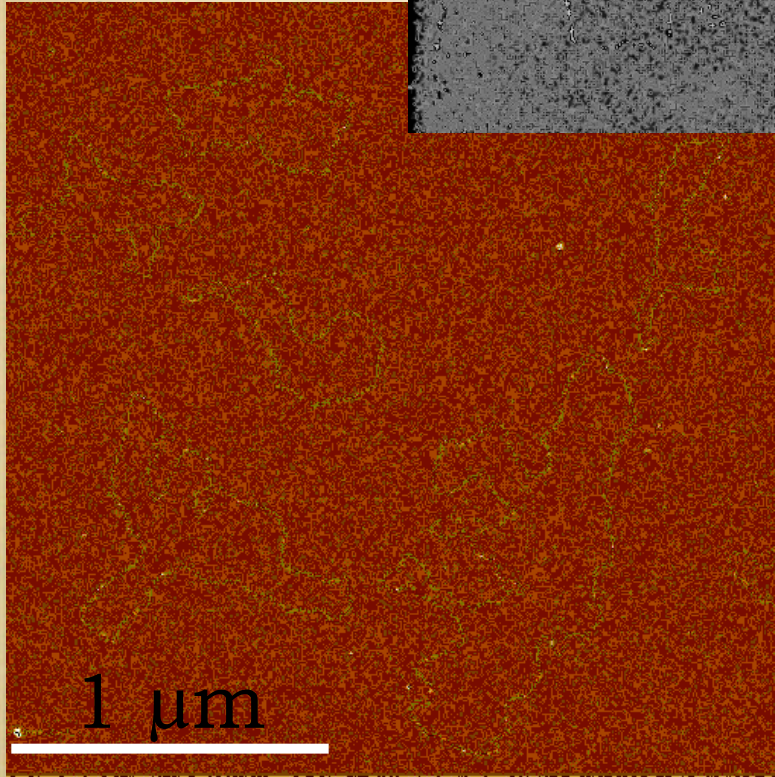
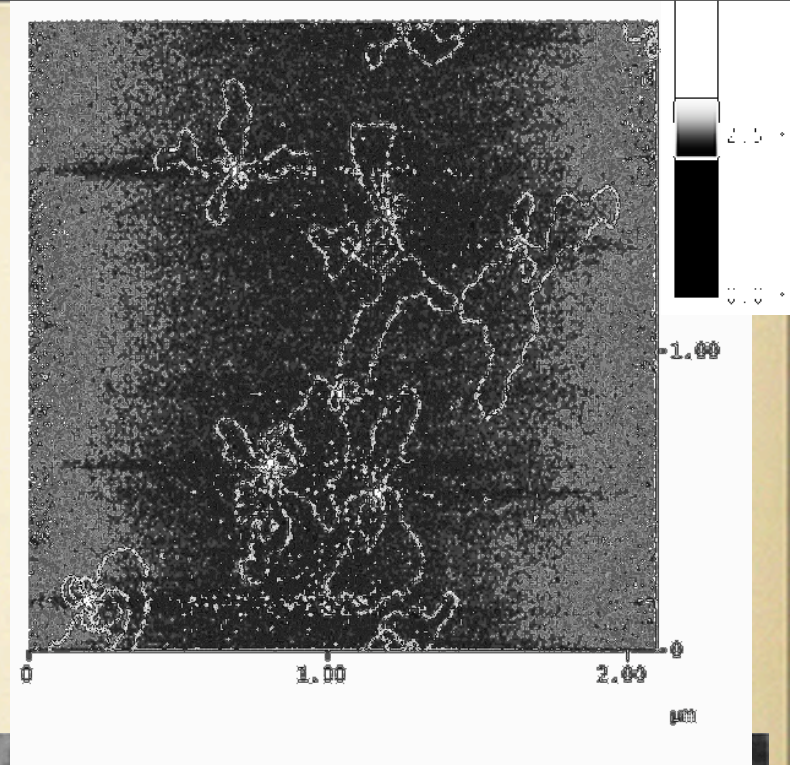
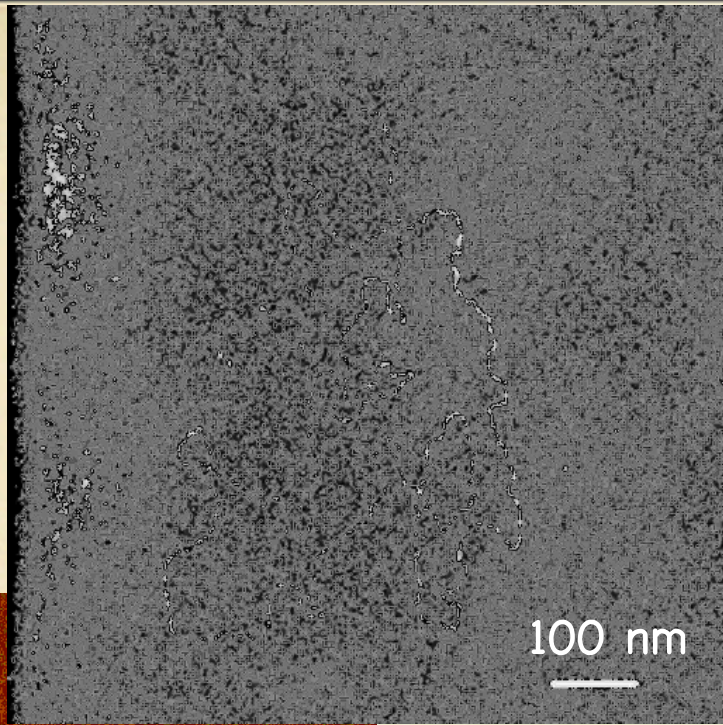
- Valle, Favre, De Los Rios, Rosa, Dietler, PRL, **88** (2005) 158105.
- Adamcik, Klinov, Witz, Sekatskii, Dietler, FEBS Lett., **580** (2006) 5671.
- Ercolini, Valle, Adamcik, Witz, Metzler, De Los Rios, Roca, Dietler, PRL, **98** (2007) 058102.
- Witz, Rechendorff, Adamcik, Dietler, PRL, **101** (2008) 148103.
- Witz, Rechendorff, Adamcik, Dietler, J. Chem. Phys., submitted

- Statistical Properties of 2D **double**-stranded circular DNA
- Statistical Properties of 2D **single**-stranded circular DNA

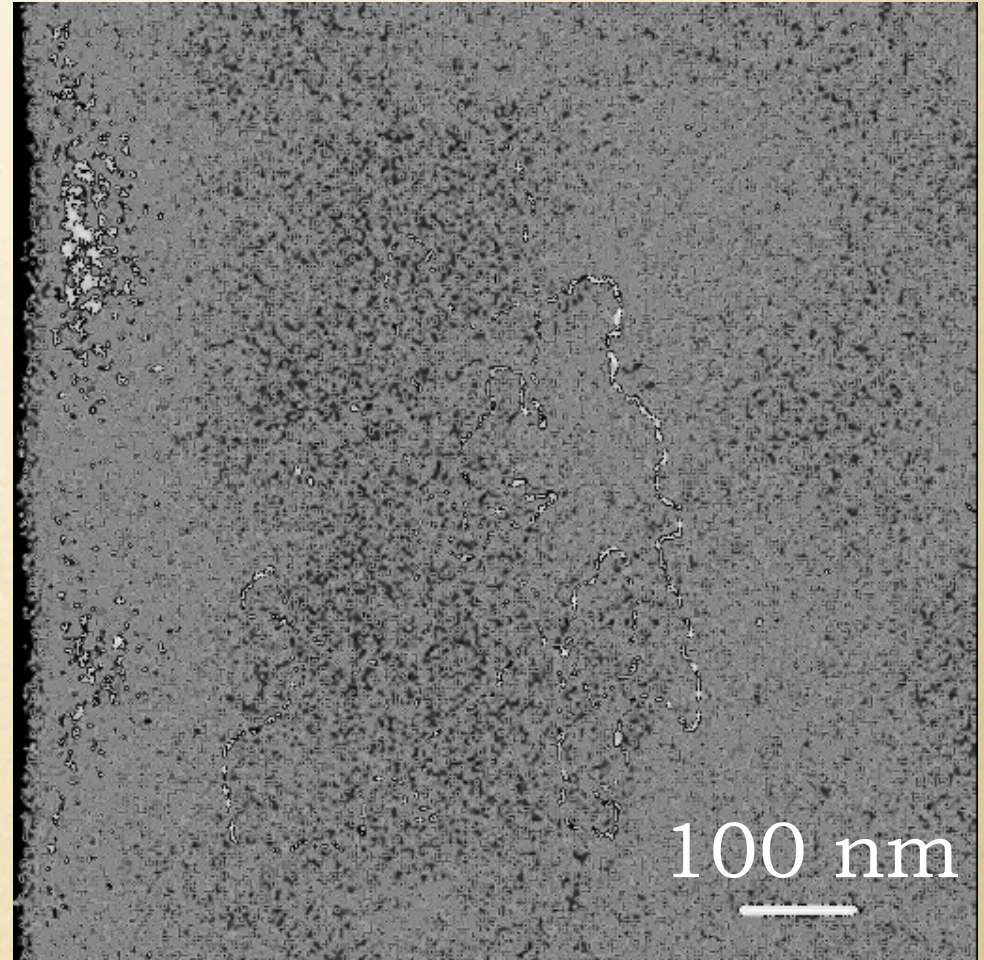
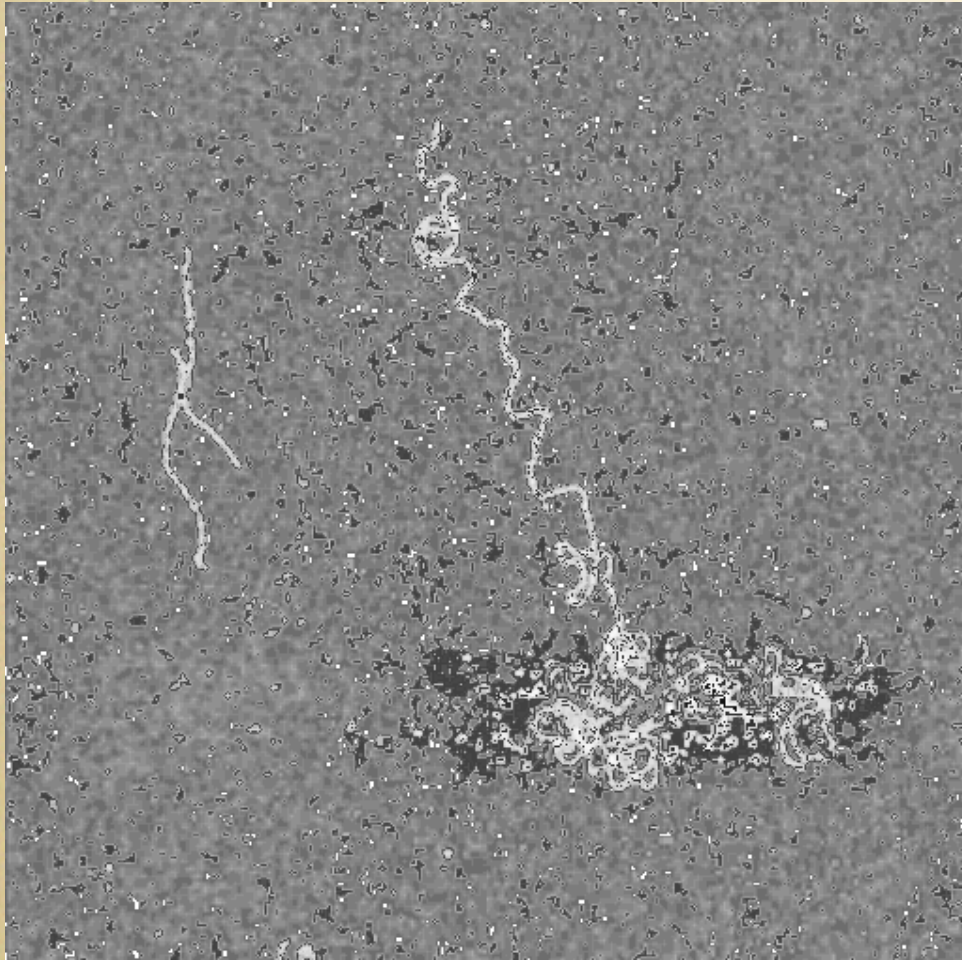
# Methods :

- ☑ Imaging of DNA by Atomic Force Microscopy
- ☑ Tracing the DNA molecules
- ☑ Statistical Properties in 2 D:
  - ☑ End-to-End Distance
  - ☑ Correlation Function
  - ☑ Distributions





# Analyzing DNA

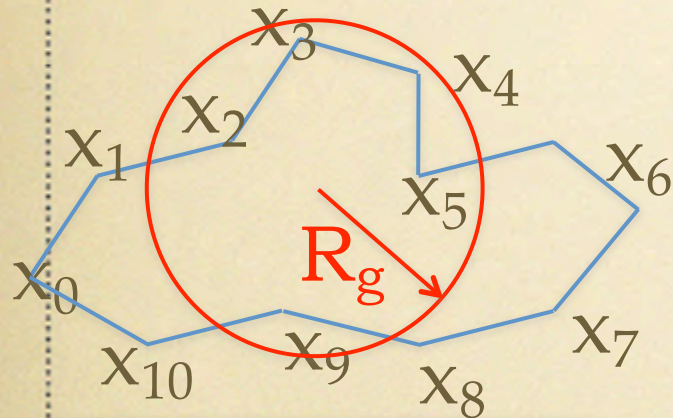




# Statistical properties of DNA

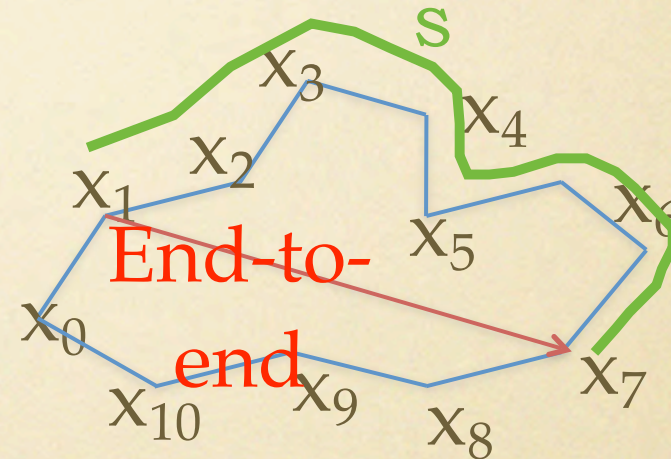
Scaling of the radius of

$$\langle R_G^2 \rangle = \frac{1}{N} \sum_{i=1}^N (r_i - r_{cm})^2 \sim L^{2\nu}$$



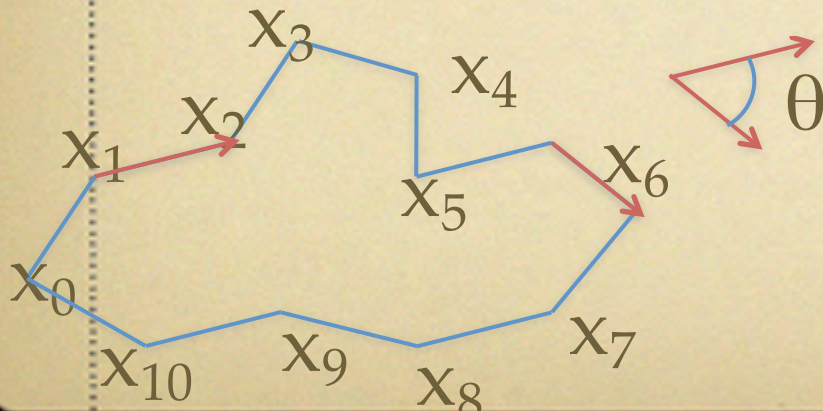
Scaling of the internal End-to-end distance

$$\langle \xi(s) \rangle = s^\nu$$

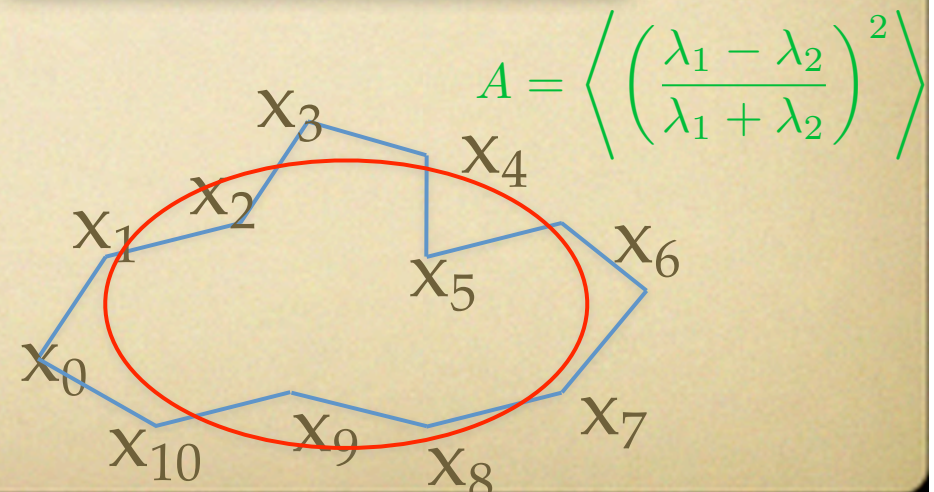


Directional correlation

$$\langle \cos \theta(s) \rangle = e^{(-s/2\ell_p)}$$



Shape properties: asphericity



# Statistical properties of DNA

## Scaling of the radius of

$$\langle R_G^2 \rangle = \frac{1}{N} \sum_{i=1}^N (r_i - r_{cm})^2 \sim L^{2\nu}$$

$$\langle \xi(s) \rangle = s^\nu$$

## Directional correlation

$$\langle \cos \theta(s) \rangle = e^{(-s/2\ell_p)}$$

$$\ell_p = 50 \text{ nm}$$

## Scaling of the internal End-to-end distance

SAW

$$\nu = 1 \quad \text{in 1D}$$

$$\nu = 0.75 \quad \text{in 2D}$$

$$\nu = 0.588 \quad \text{in 3D}$$

RW

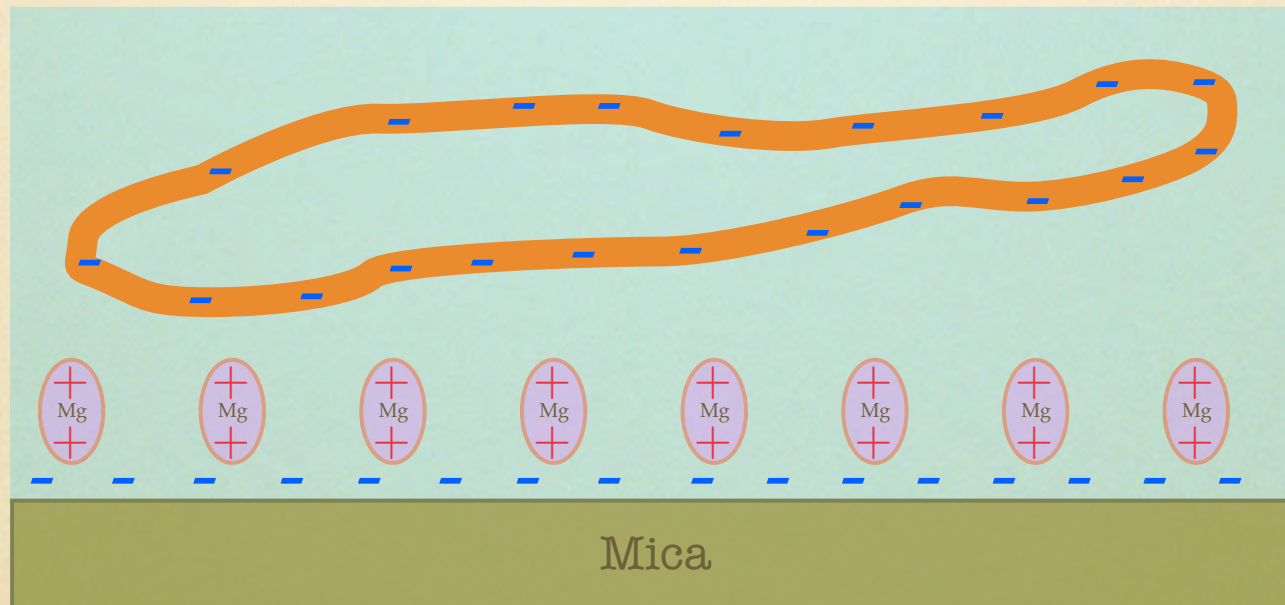
$$\nu = 0.5$$

## Shape properties: asphericity

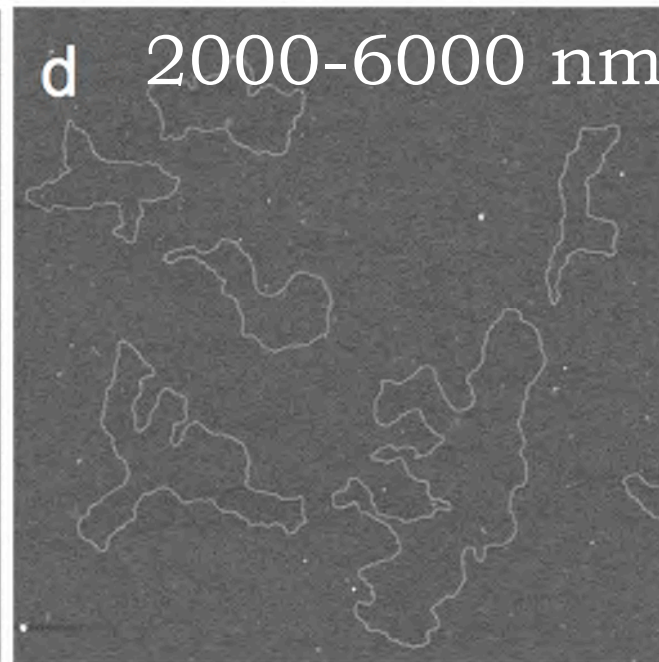
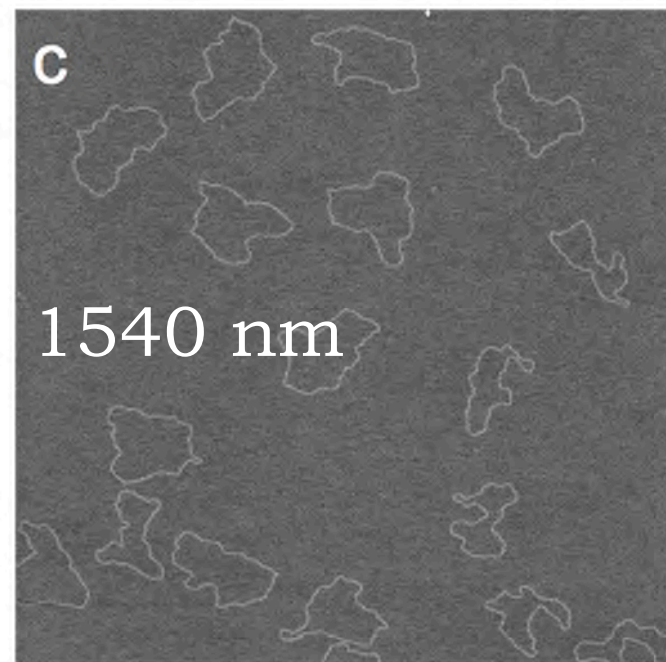
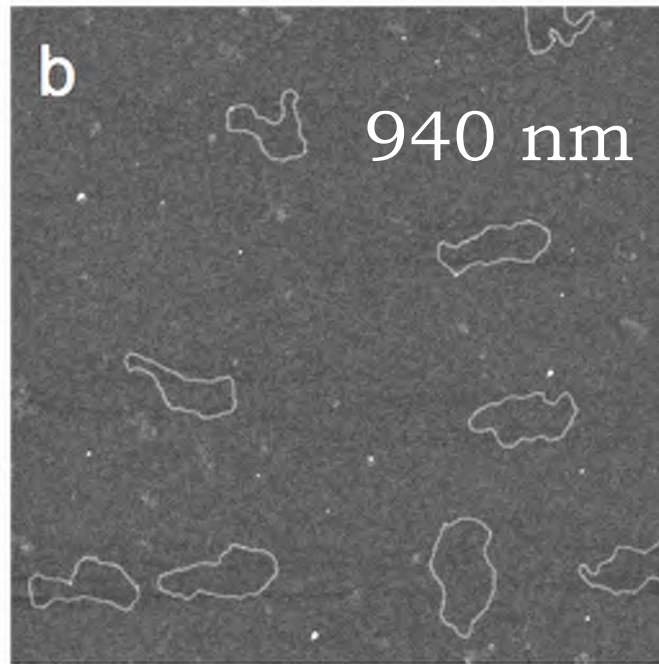
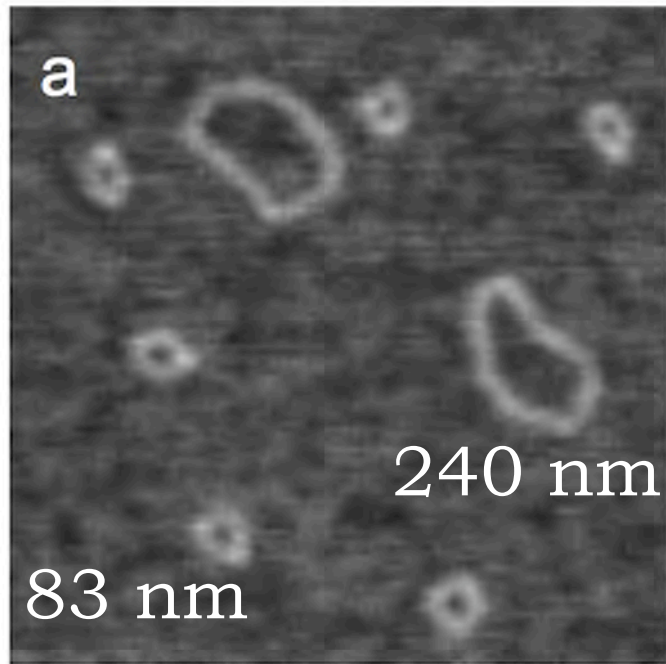
$$A = \left\langle \left( \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2 \right\rangle$$

Circle	A=0
Rod	A=1

# 2 D deposition of nicked dsDNA



# Circular dsDNA in 2D

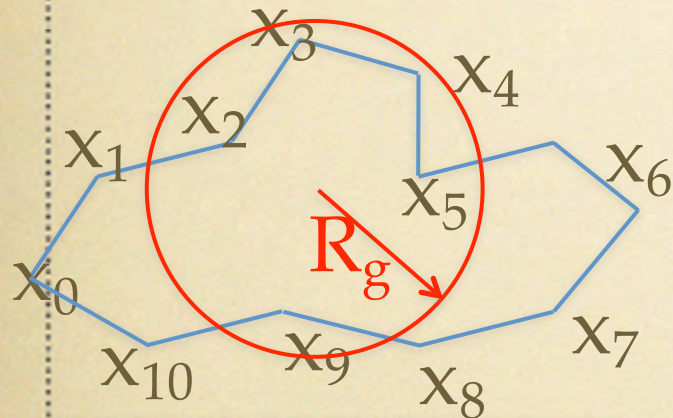


*Witz et al., PRL, 101,*  
*(2008) 148103.*

# Statistical properties of DNA

## Scaling of the radius of gyration

$$\langle R_G^2 \rangle = \frac{1}{N} \sum_{i=1}^N (r_i - r_{cm})^2 \sim L^{2\nu}$$



SAW

$$\nu = 1 \quad \text{in 1D}$$

$$\nu = 0.75 \quad \text{in 2D}$$

$$\nu = 0.588 \quad \text{in 3D}$$

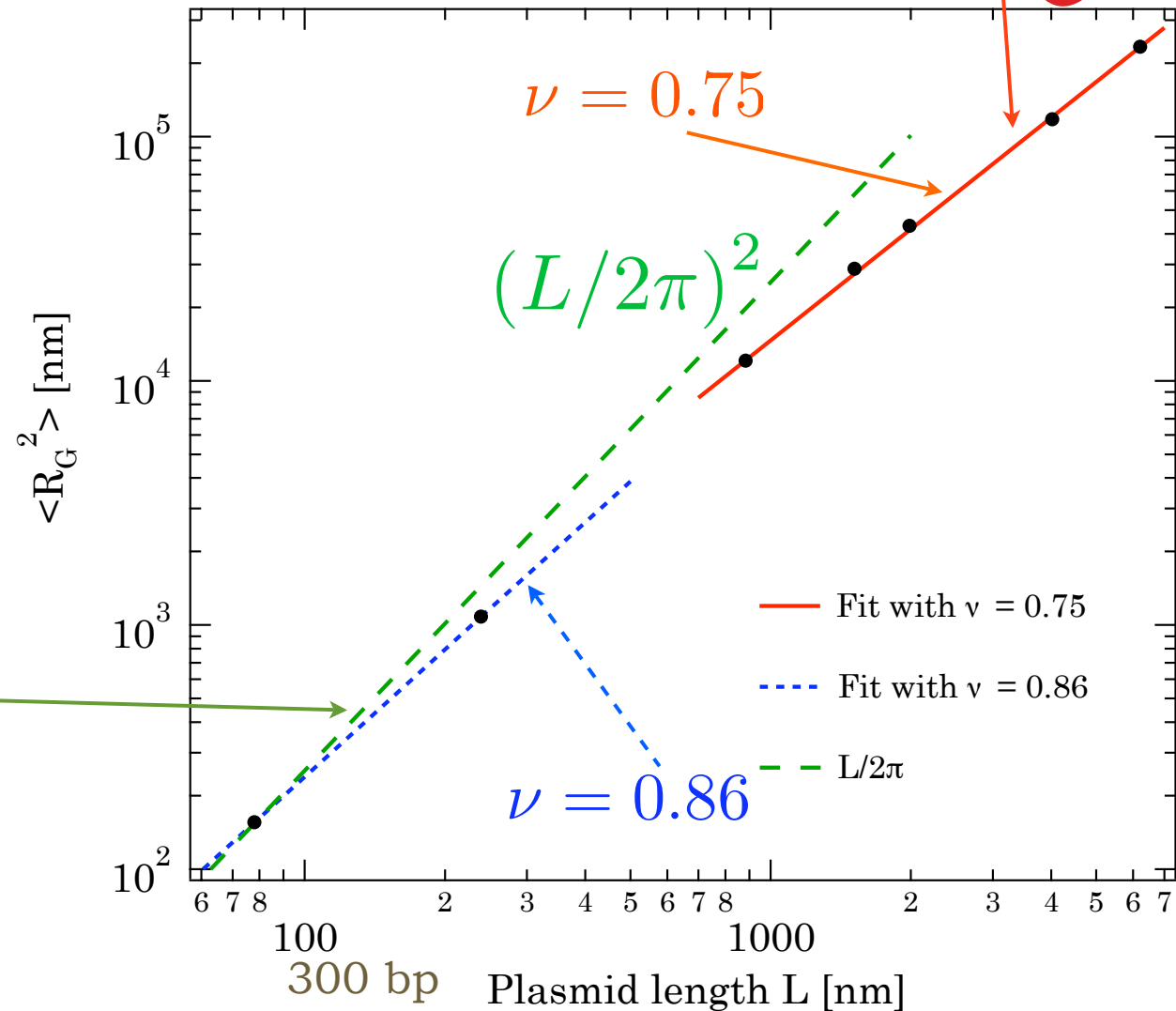
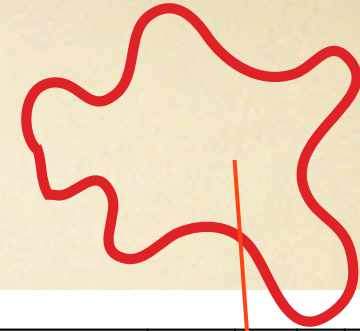
RW

$$\nu = 0.5$$

# Radius Of Gyration for Circular DNA in 2D

Prediction

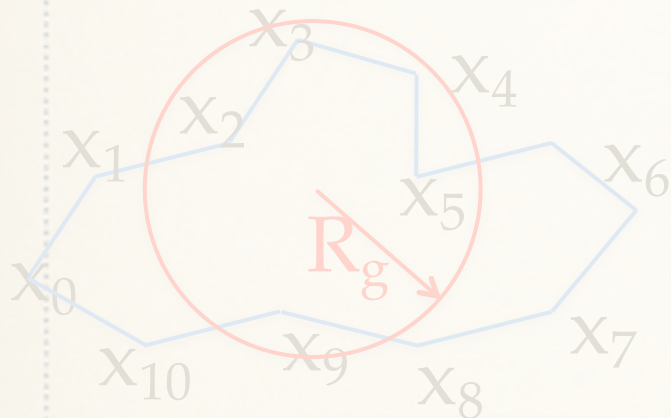
$$\langle R_g^2 \rangle \sim L^{2\nu}$$



# Statistical properties of DNA

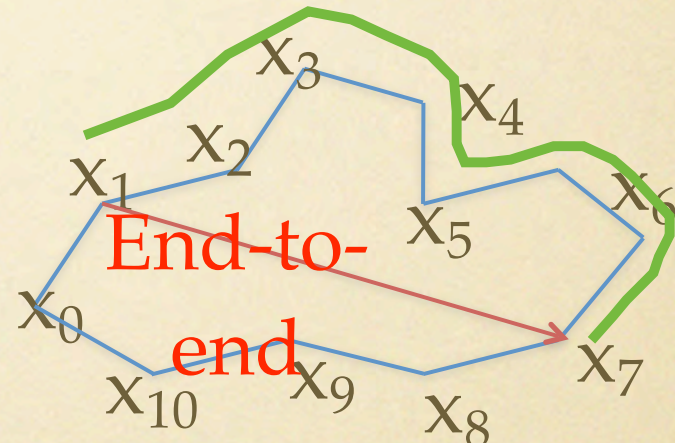
Scaling of the radius of gyration

$$\langle R_G^2 \rangle = \frac{1}{N} \sum_{i=1}^N (r_i - r_{cm})^2 \sim L^{2\nu}$$



Scaling of the internal End-to-end distance

$$\langle \xi(s) \rangle = s^\nu$$



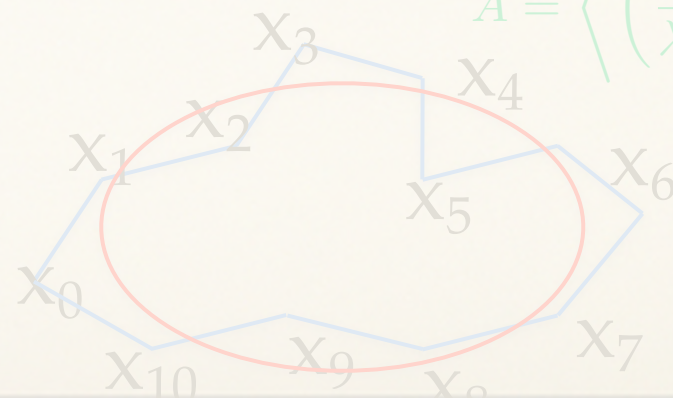
Directional correlation

$$\langle \cos \theta(s) \rangle = e^{(-s/2l_p)}$$



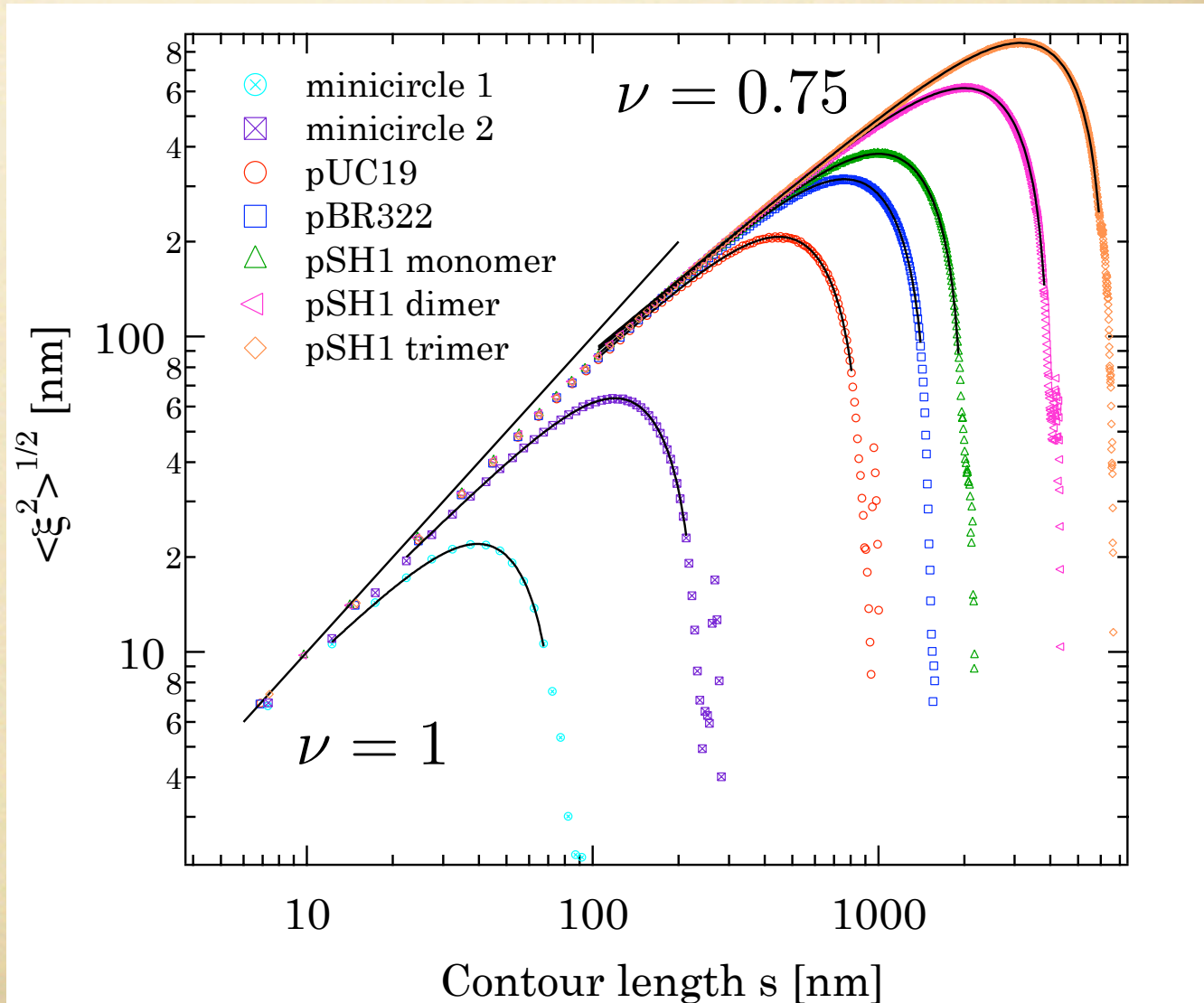
Shape properties: asphericity

$$A = \left\langle \left( \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2 \right\rangle$$



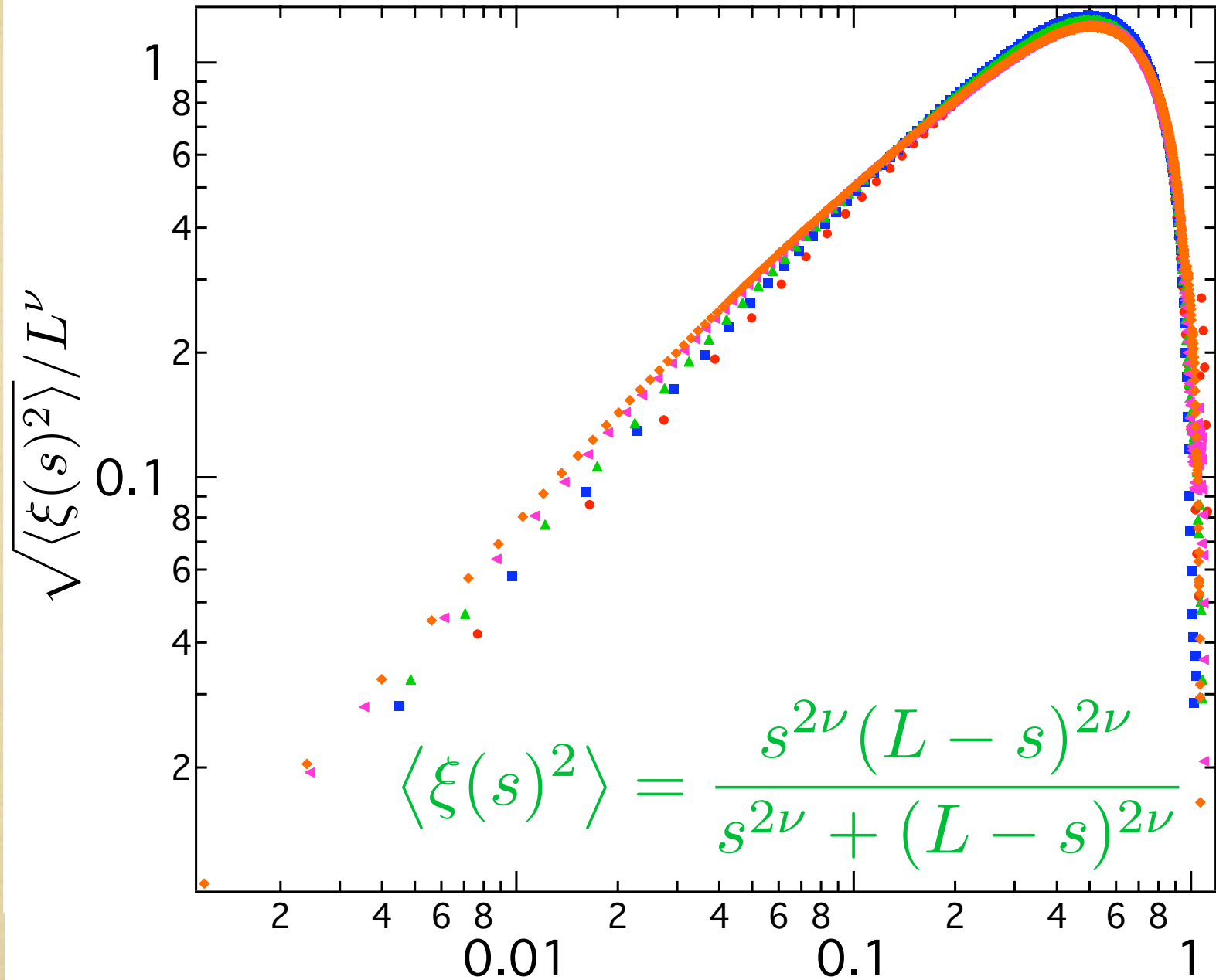
# Internal End-to-End Distance for Circular DNA

$$\langle r^2(s) \rangle \sim \frac{s^{2\nu} (L_o - s)^{2\nu}}{s^{2\nu} + (L_o - s)^{2\nu}}$$





# Rescaled Data

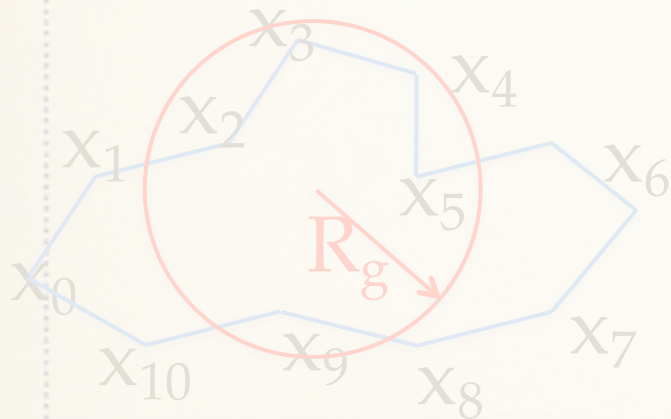


Plasmid rescaled internal contour length  $s/L$  [nm]

# Statistical properties of DNA

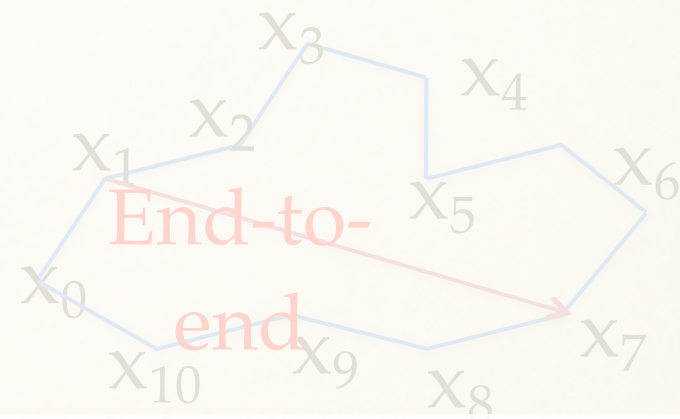
Scaling of the radius of gyration

$$\langle R_G^2 \rangle = \frac{1}{N} \sum_{i=1}^N (r_i - r_{cm})^2 \sim L^{2\nu}$$



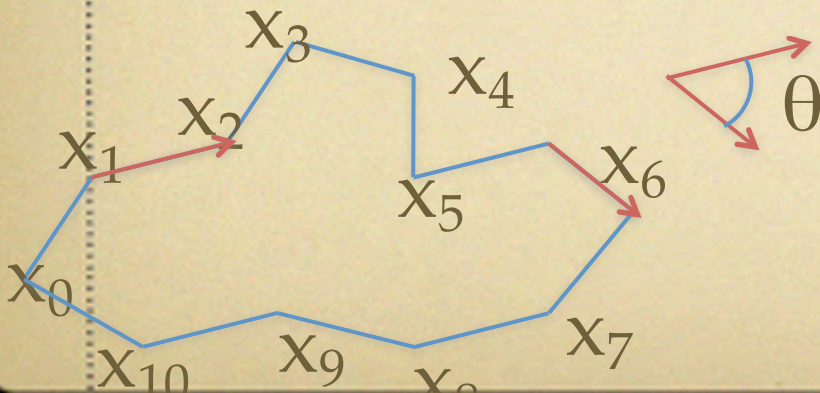
Scaling of the internal End-to-end distance

$$\langle \xi \rangle \sim L^\nu$$



Directional correlation

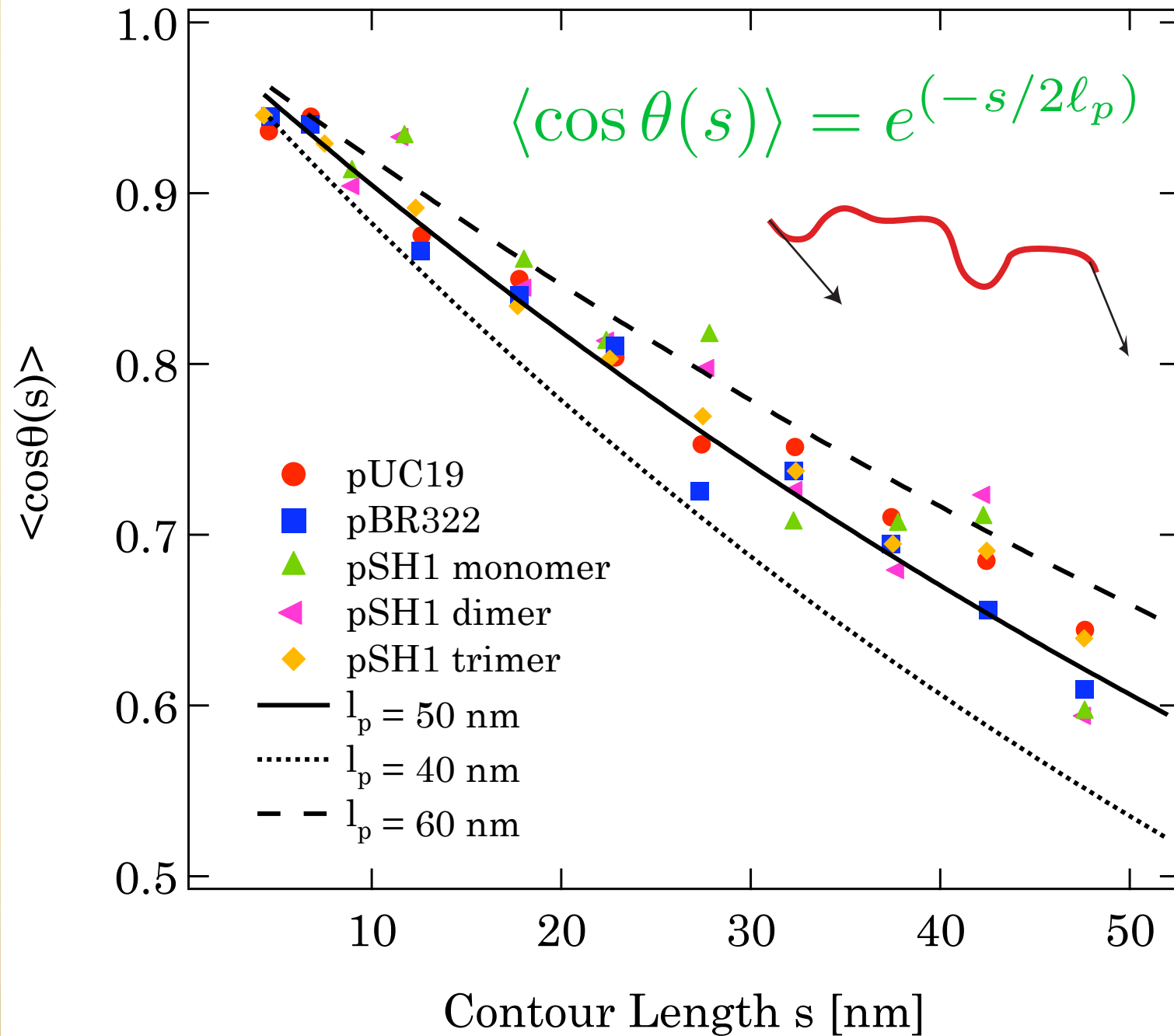
$$\langle \cos \theta(s) \rangle = e^{(-s/2\ell_p)}$$



Shape properties: asphericity

$$A = \left\langle \left( \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2 \right\rangle$$

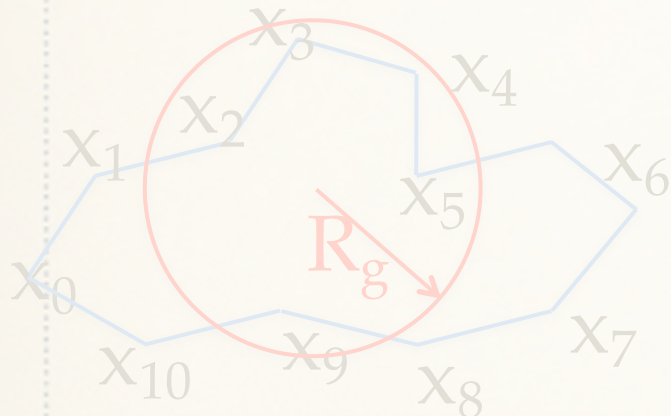
# Bond Correlation Function



# Statistical properties of DNA

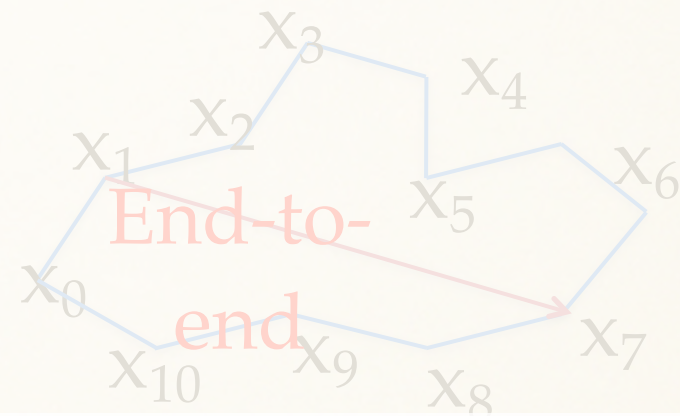
Scaling of the radius of gyration

$$\langle R_G^2 \rangle = \frac{1}{N} \sum_{i=1}^N (r_i - r_{cm})^2 \sim L^{2\nu}$$



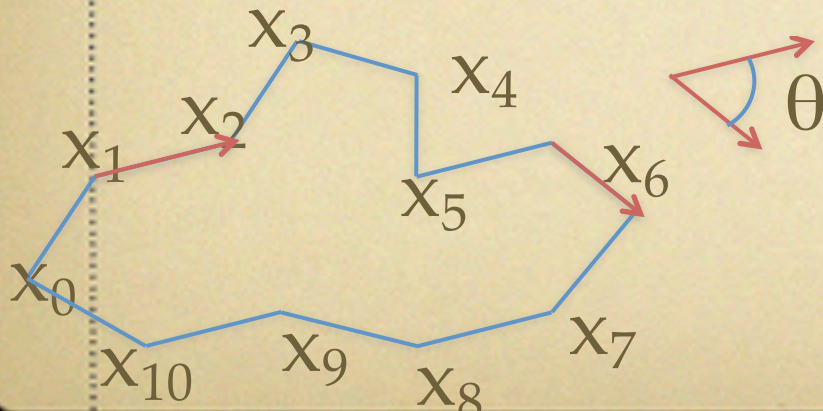
Scaling of the internal End-to-end distance

$$\langle \xi \rangle \sim L^\nu$$



Directional correlation

$$\langle \cos \theta(s) \rangle = e^{(-s/2\ell_p)}$$

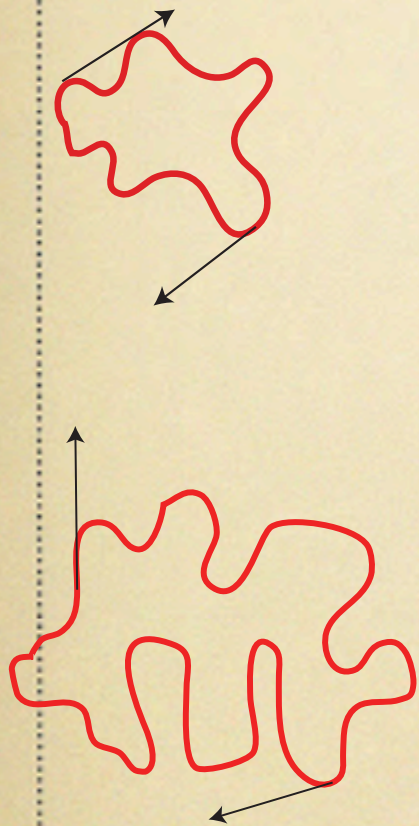


Shape properties: asphericity

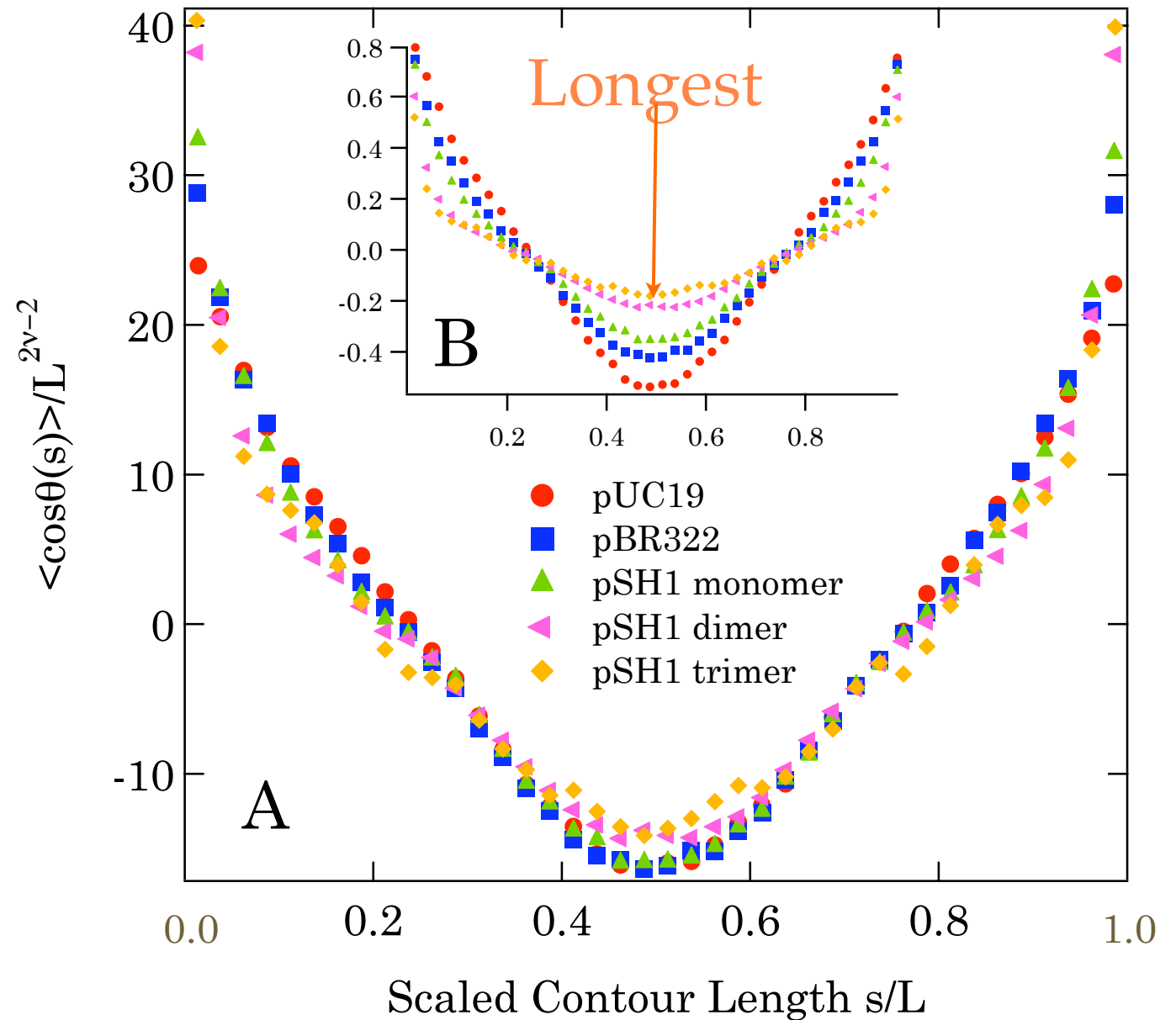
$$A = \left\langle \left( \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2 \right\rangle$$

# Bond Correlation Function of DNA

$$\langle \cos \theta(s) \rangle = \phi \left( \frac{s}{L_o}, \nu \right) L_o^{2\nu-2}$$

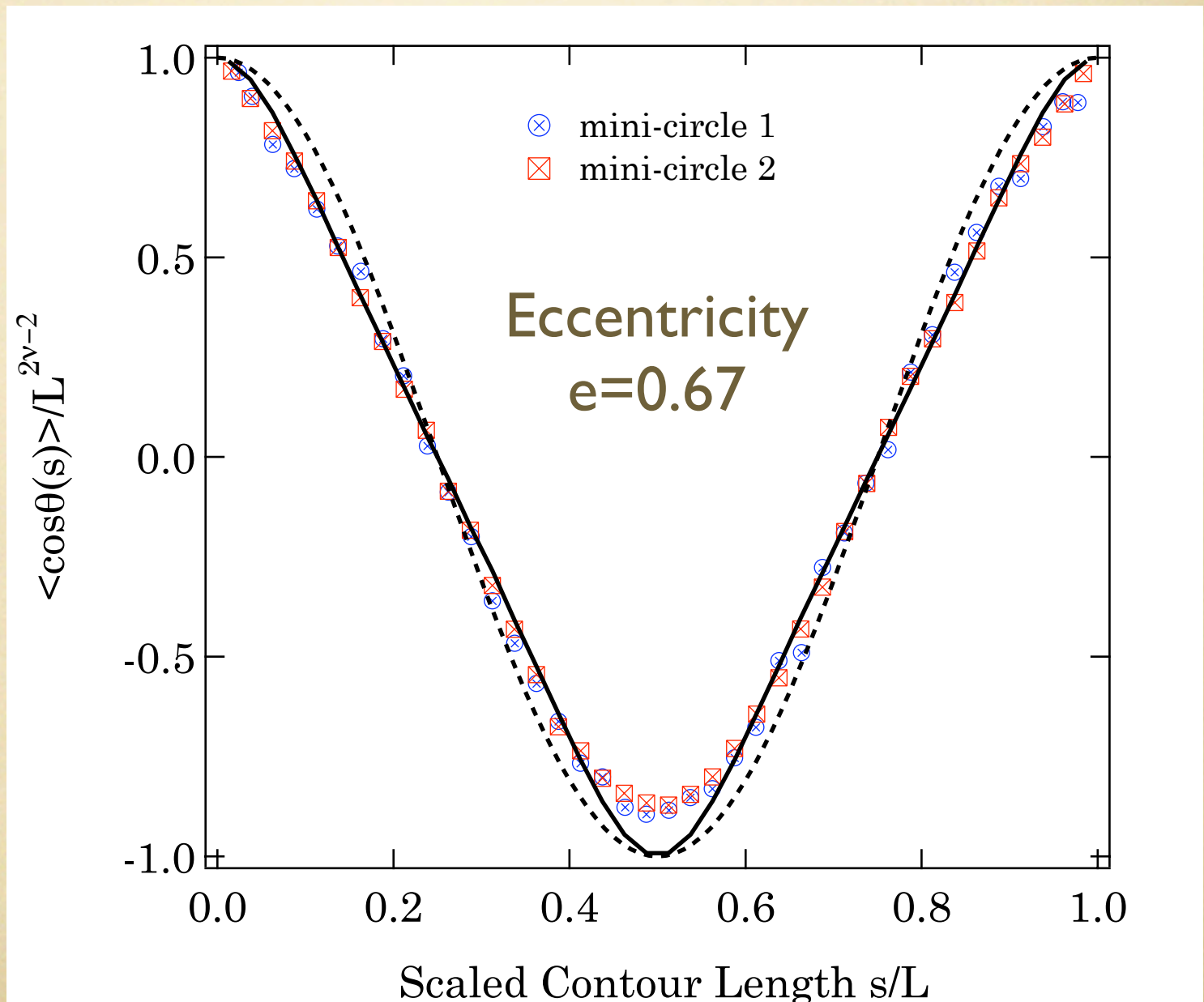
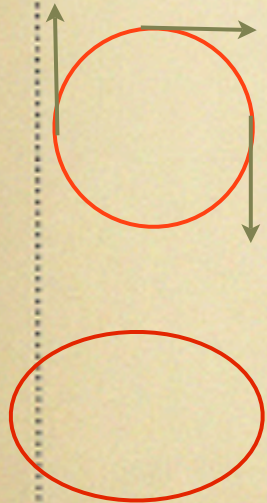


A. Baumgärtner,  
J. Chem. Phys., **76**,  
4275 (1982).

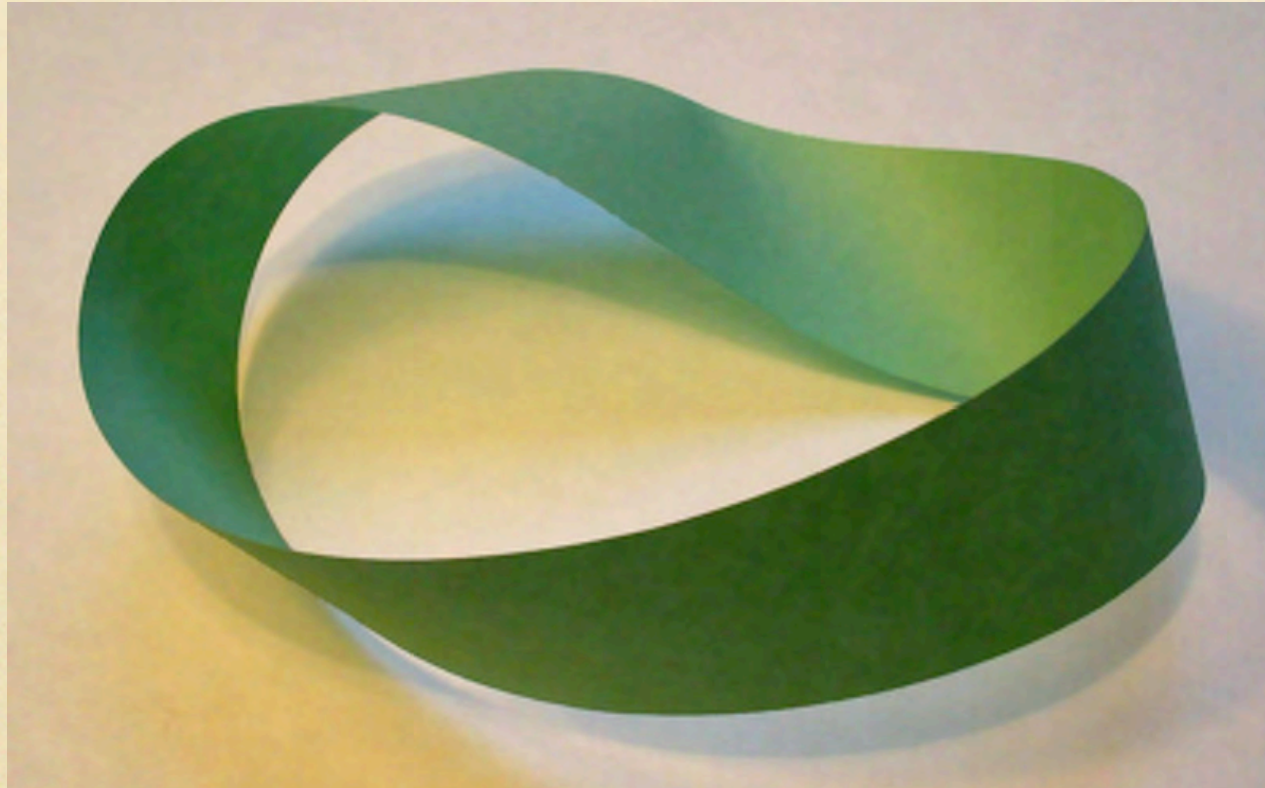


# Bond Correlation Function of DNA mini-circles

241 bp  
676 bp



# The double Helix Determines the Shape of Small DNA Circles



# Asphericity

## Gyration Tensor

$$T_{ij} = \frac{1}{N} \sum_{\ell=1}^N (x_{\ell i} - \langle x_i \rangle)(x_{\ell j} - \langle x_j \rangle)$$

$$R_G^2 = \text{Tr}(\mathbf{T}) = \sum_{i=0}^d \lambda_i$$

$$A_d = \frac{1}{d-1} \frac{\sum_{i>j}^d (\lambda_i - \lambda_j)^2}{(\sum_{i=1}^d \lambda_i)^2}$$

$$A_2 = \frac{(\lambda_2 - \lambda_1)^2}{(\lambda_1 + \lambda_2)^2}$$

$$A_2 = \left\langle \frac{(\lambda_2 - \lambda_1)^2}{(\lambda_1 + \lambda_2)^2} \right\rangle$$

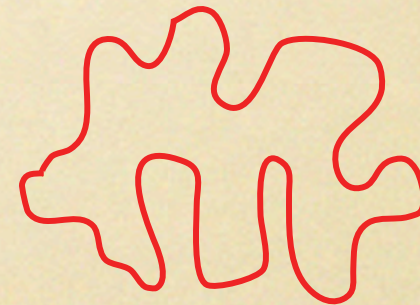


# Results for Circular DNA

Theoretical Values	Ring SAW 0.206	Ring RW 0.279	Bishop 1988
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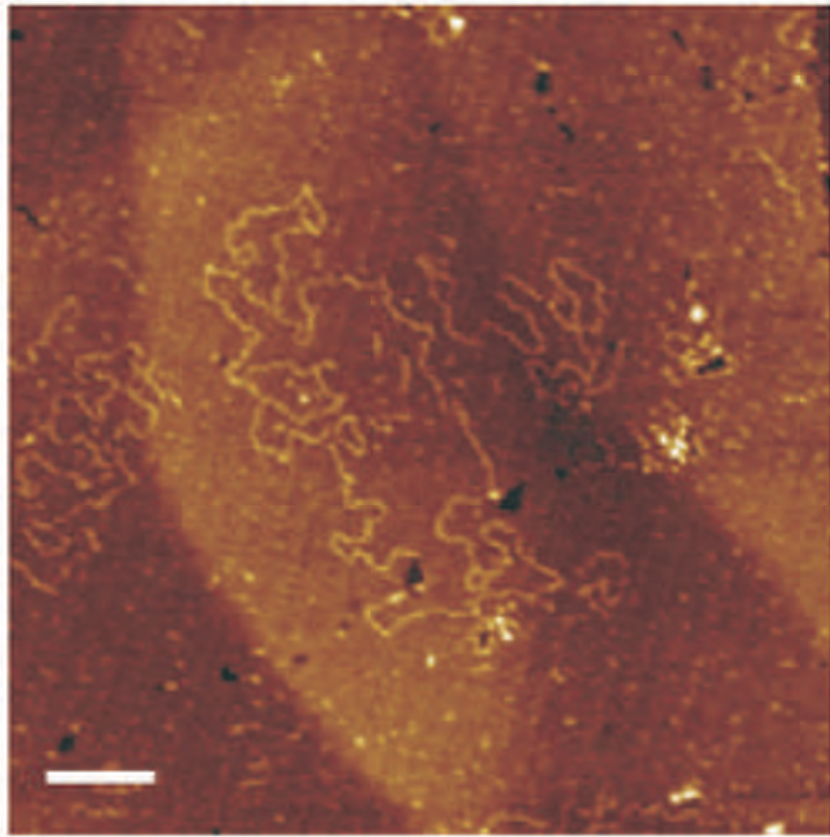
## Experimental Values

Plasmid	mini 1	mini 2	pUC19	pSH1	pBR322
A-value	0.083	0.13	0.28	0.299	0.265
Length (L/l <sub>p</sub> )	1.6	4.5	18	40-120	30



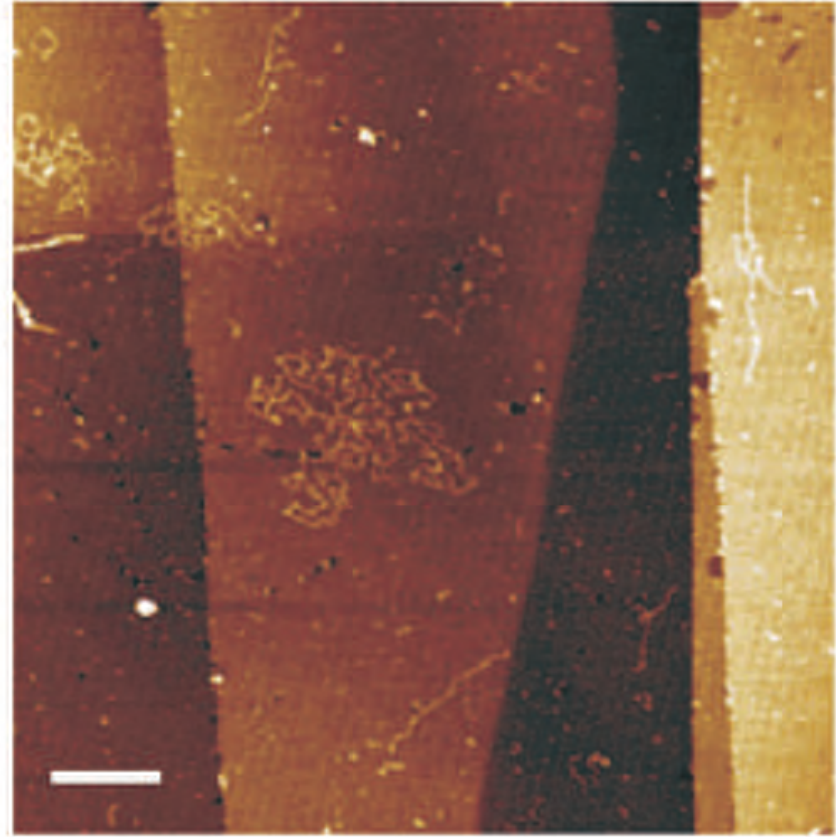
2 D Properties of  
circular ssDNA : one  
length (sorry !)

# AFM Images of ssDNA



100 nm

0 mM NaCl

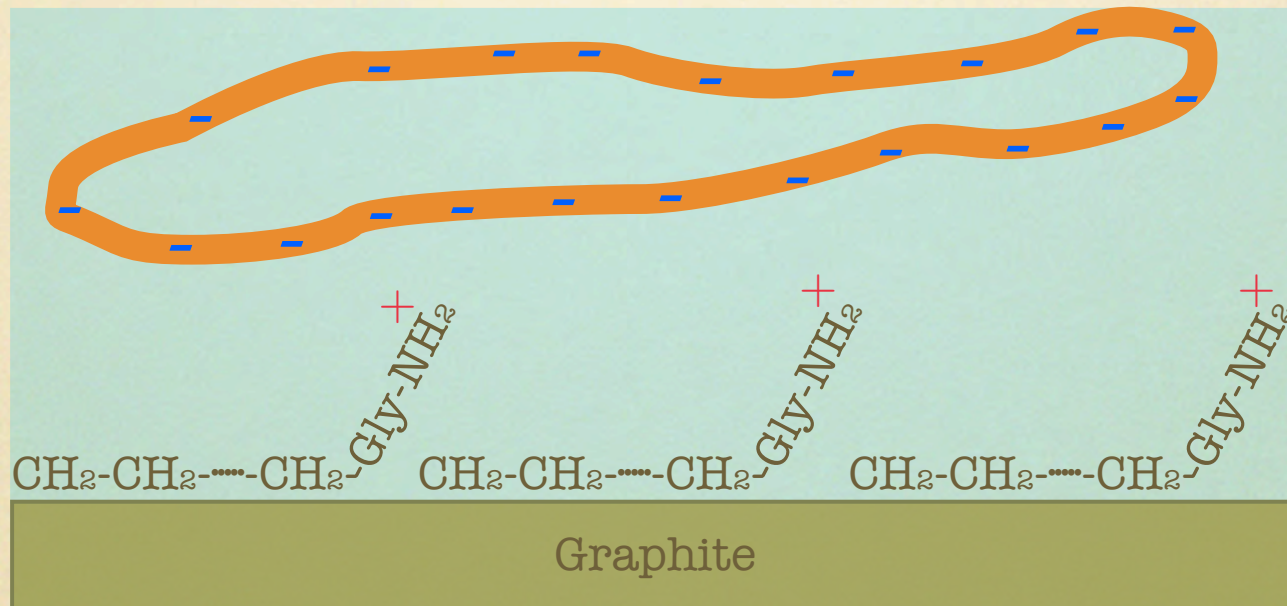


100 nm

10 mM NaCl

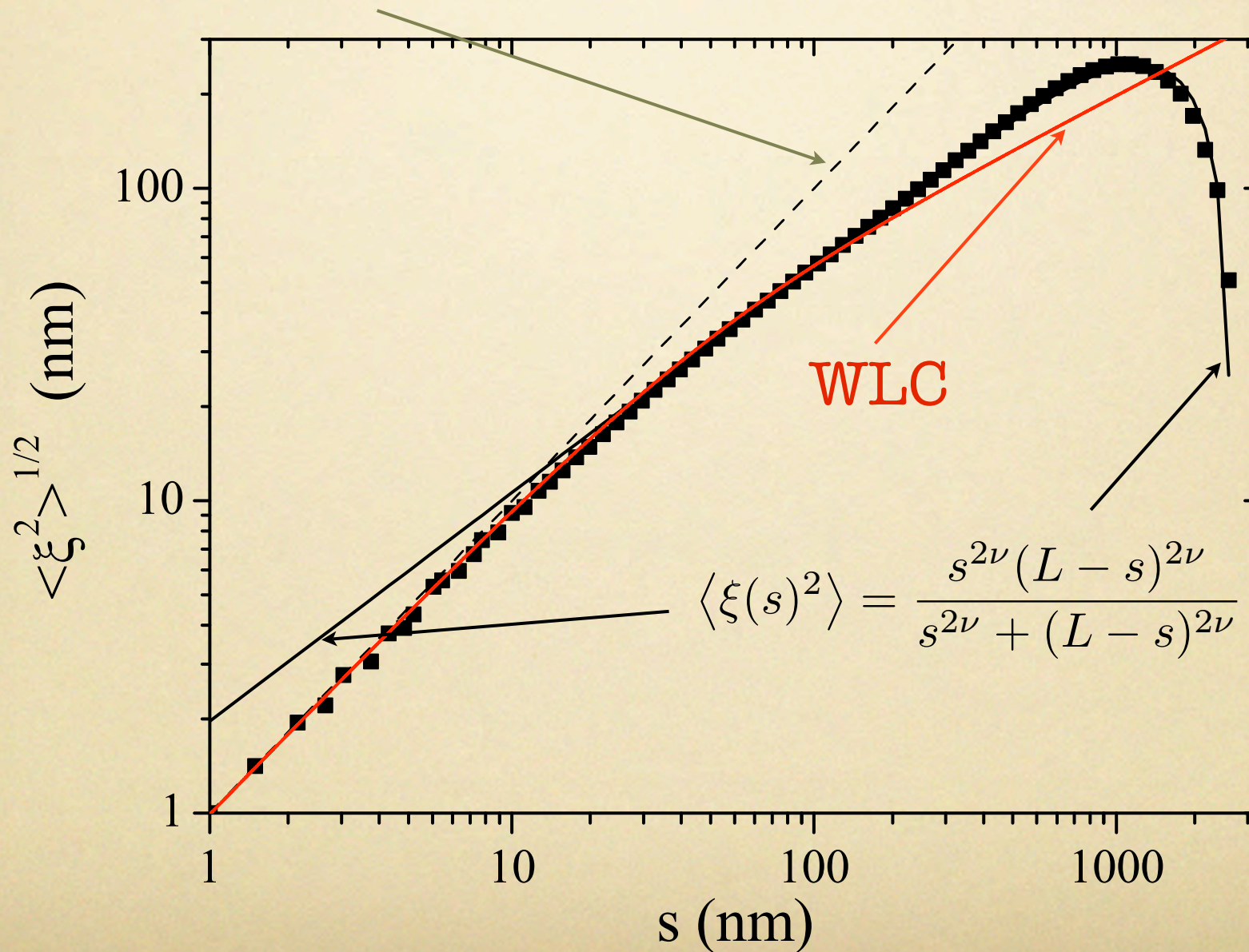
Virion  $\phi$  x174 : 5386 bp 10 mM Tris-HCl buffer, pH 7.8

# 2 D deposition of ssDNA



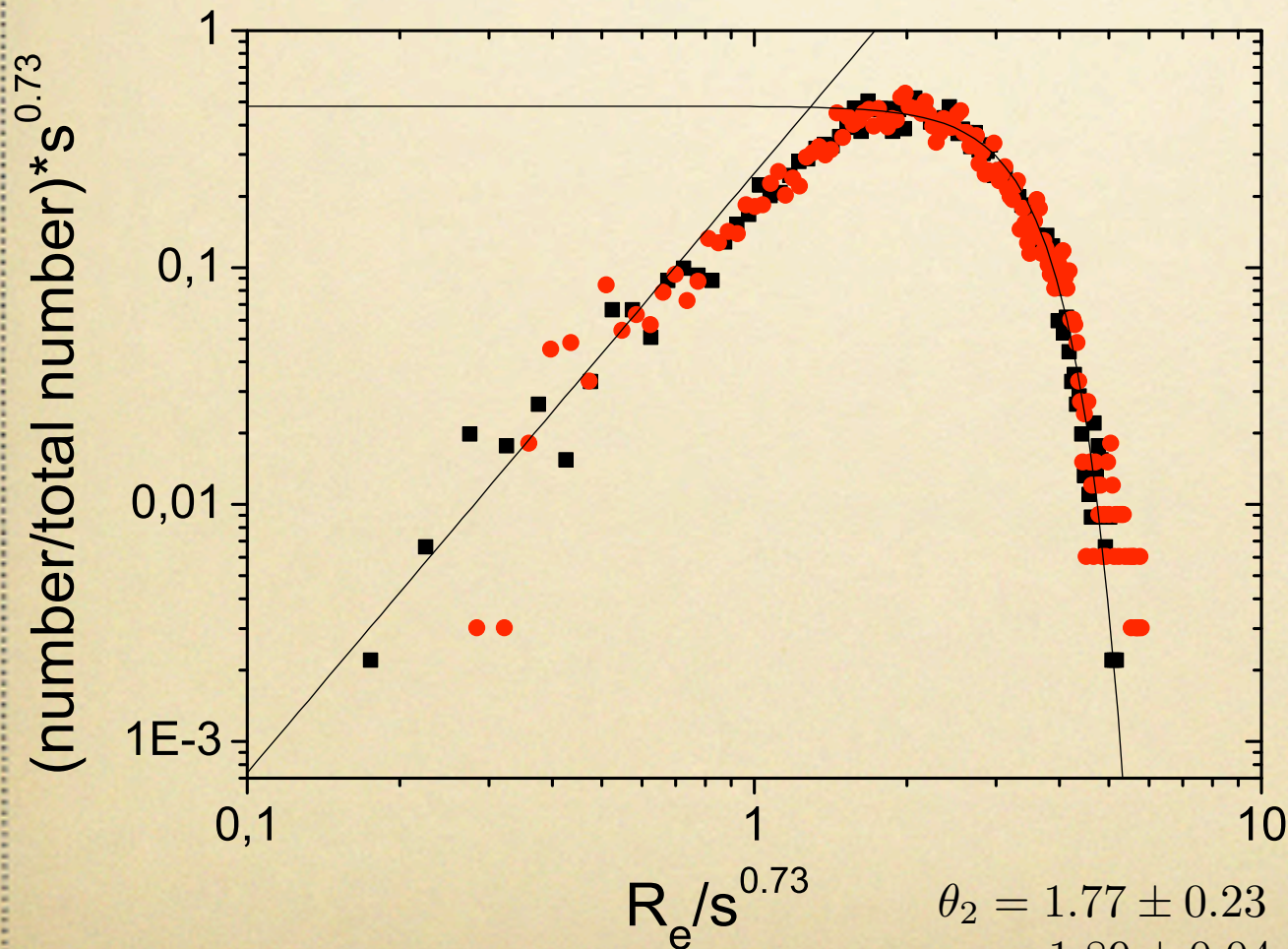
# Internal End-to-End Distance

Rigid Rod Regime (RRR)



# Distribution of the Internal End-to-End Distance for $s$ large $\gg l_p$

$$f_s(z) = \begin{cases} az^{d-1+\kappa} \exp(-bz^\delta) & \text{if } z \gg 1, \\ cz^{d-1+\theta_2} & \text{if } z \ll 1 \end{cases}$$



$$z = R_e / s^\nu$$

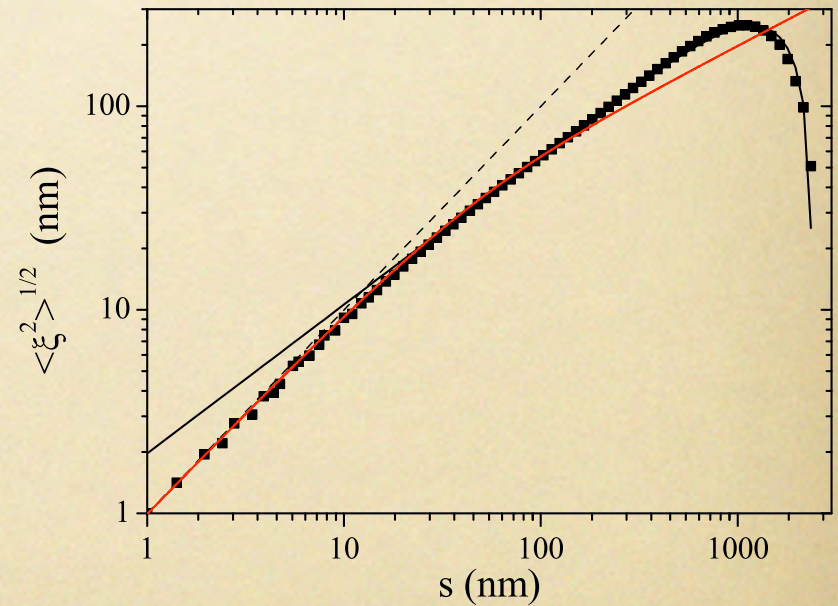
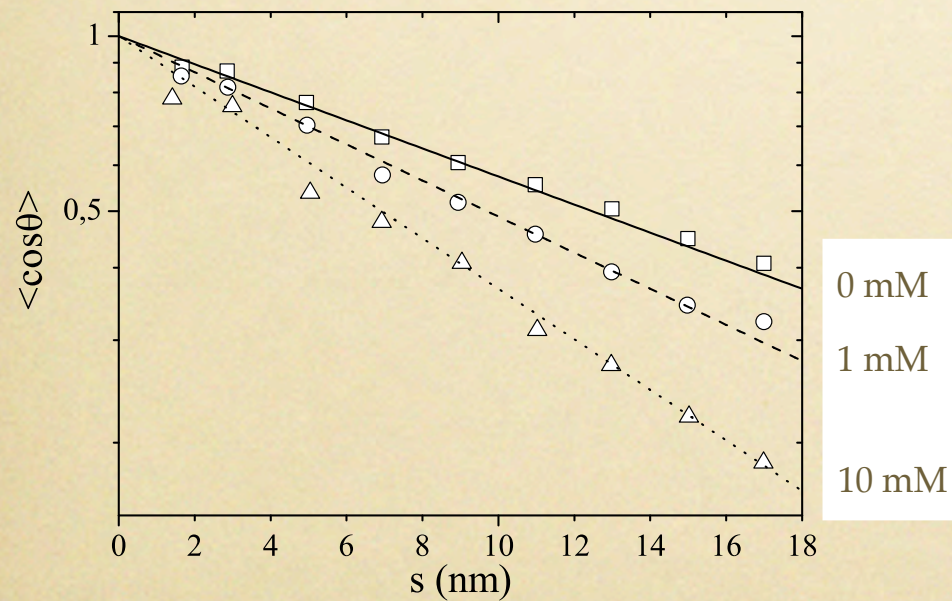
- $s_o = 210 \text{ nm} \approx 10 l_p$
- $s_o = 315 \text{ nm} \approx 15 l_p$

$$\theta_2 = 1.77 \pm 0.23 \quad (19/12 \simeq 1.583)$$

$$\kappa_2 = 1.89 \pm 0.04 \quad (1.93 \pm 0.27 \ \& \ 1.85 \pm 0.08)$$

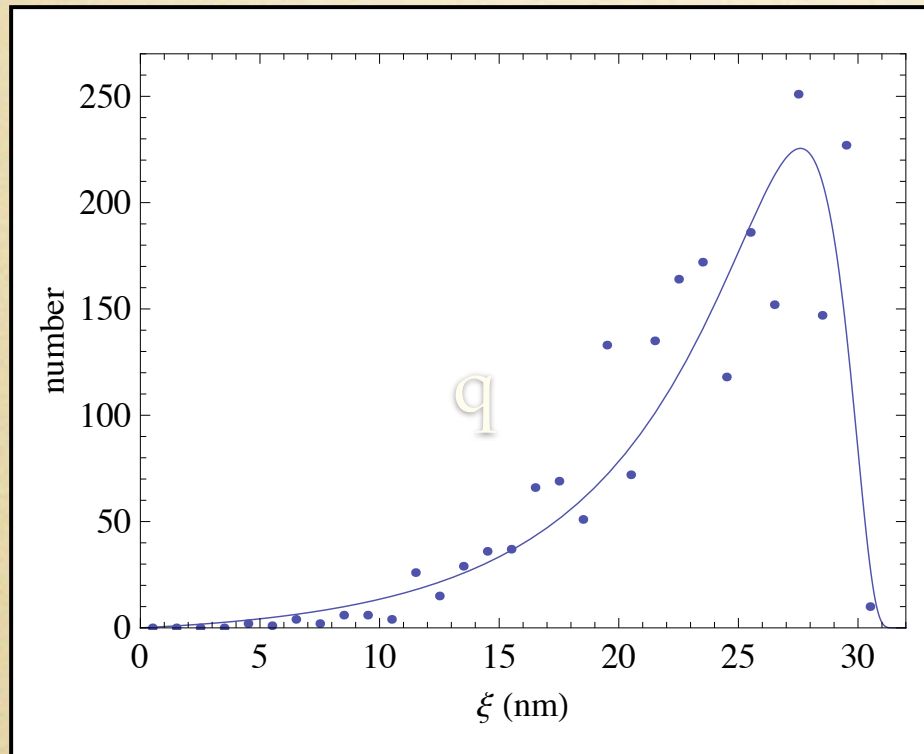
# Persistence Length from small $s$ (approx $lp$ )

- Correlation Function
- End-to-End Distance
- Ent-to-End Distance Distribution



# Persistence Length

- Correlation Function
- End-to-End Distance
- End-to-End Distance Distribution



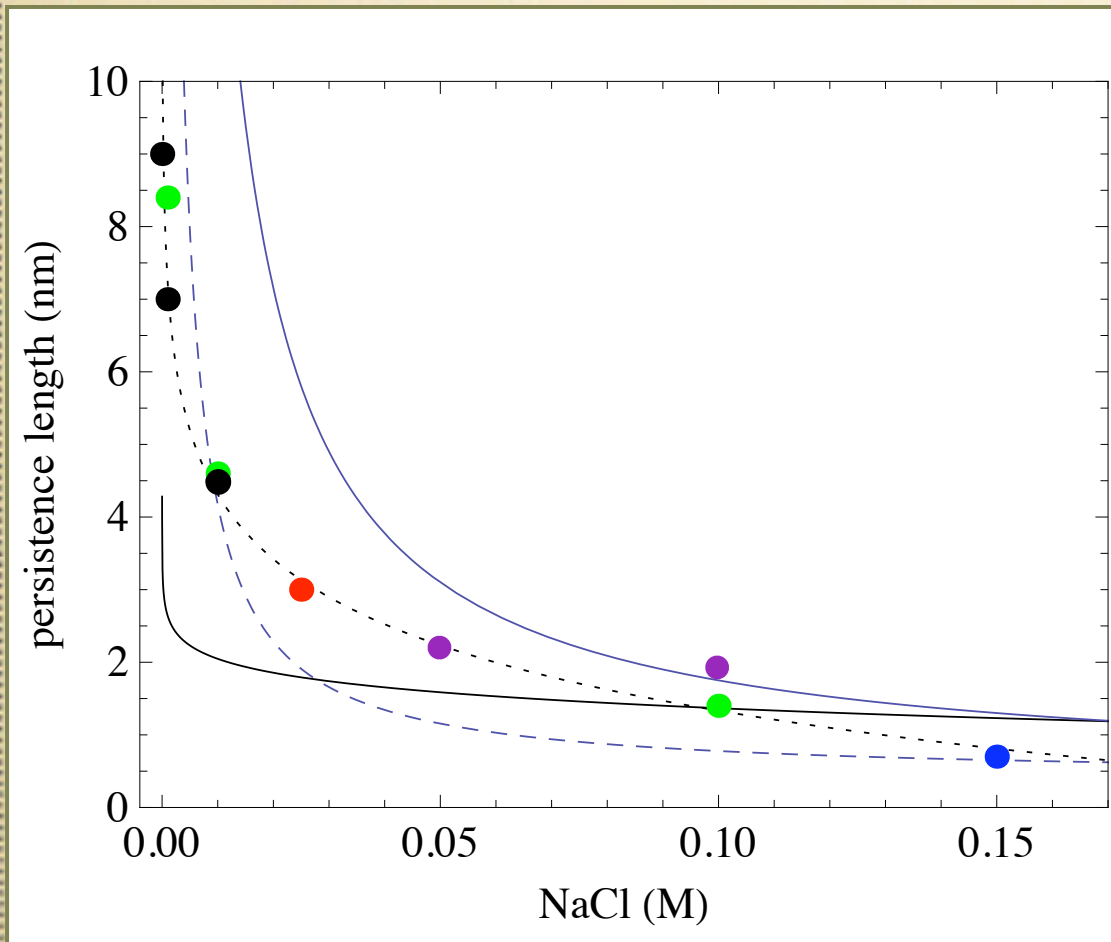
$$s = 32 \text{ nm} \sim 5 \text{ lp}$$

$$G(r) = \frac{1}{2\pi\mathcal{N}} \frac{2\kappa}{\sqrt{\pi}} \sum_{\ell=0}^{\infty} \frac{(2\ell-1)!!}{2^{\ell}\ell!} \frac{1}{[2\kappa(1-r)]^{5/4}} \\ \times \exp\left[-\frac{(\ell+1/4)^2}{2\kappa(1-r)}\right] D_{3/2}\left[2\frac{\ell+1/4}{\sqrt{2\kappa(1-r)}}\right]$$

$$\ell_p = \frac{2\kappa}{k_B T}$$



# Persistence Length vs. salt concentration



## Experiments

- Present Work
- Tinland, Pluen, Sturm, Weill
- Murphy, Rasnik, Cheng, Lohman, Ha
- Smith, Cui, Bustamante

## Theory

- ..... Manning, 1996
- - - Odijk, 1977
- Toan & Micheletti, 2006

# Collaborators

- Francesco Valle, ISMN CNR, Bologna
- Mélanie Favre, EPFL
- Jozef Adamcik, EPFL
- Erika Ercolini, EPFL
- Guillaume Witz, EPFL
- Kristian Rechendorff, EPFL
- Paolo De Los Rios, EPFL
- Ralf Metzler, TU Munich
- Joaquim Roca, Barcelona
- Andrzej Stasiak, Uni Lausanne
- Erwin Frey, LMU, Munich
- Bertrand Duplantier, Ecole Polytechnique, Paris
- Hirofumi Wada, Kyoto University,

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Novartis  
Foundation

