Introduction to spectral methods in Matlab I

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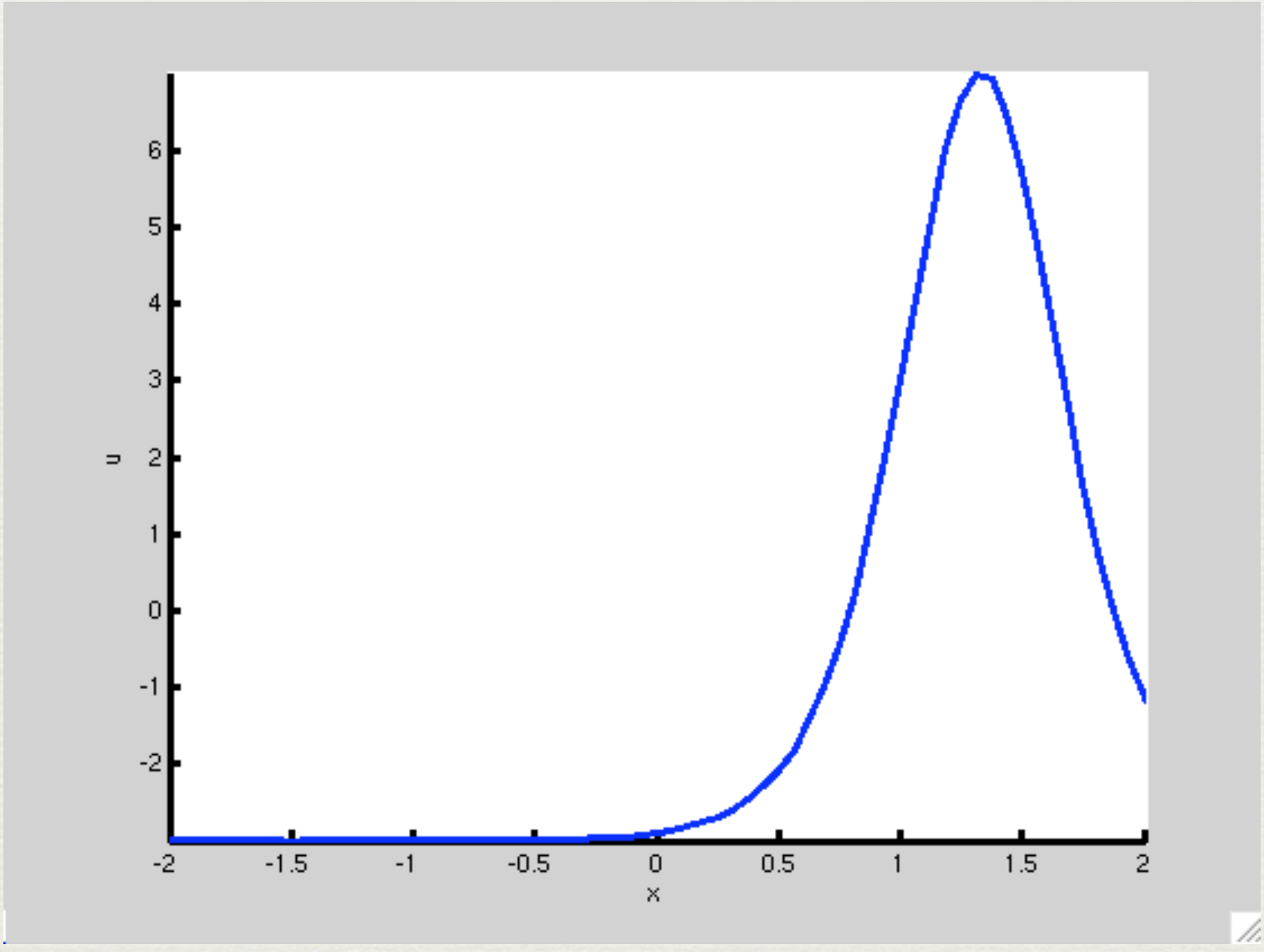
Introduction

- Goal: numerical solution of partial differential equations
- Example: Korteweg de Vries equation (1d waves in shallow water), solutions for given initial data (for instance solitons)

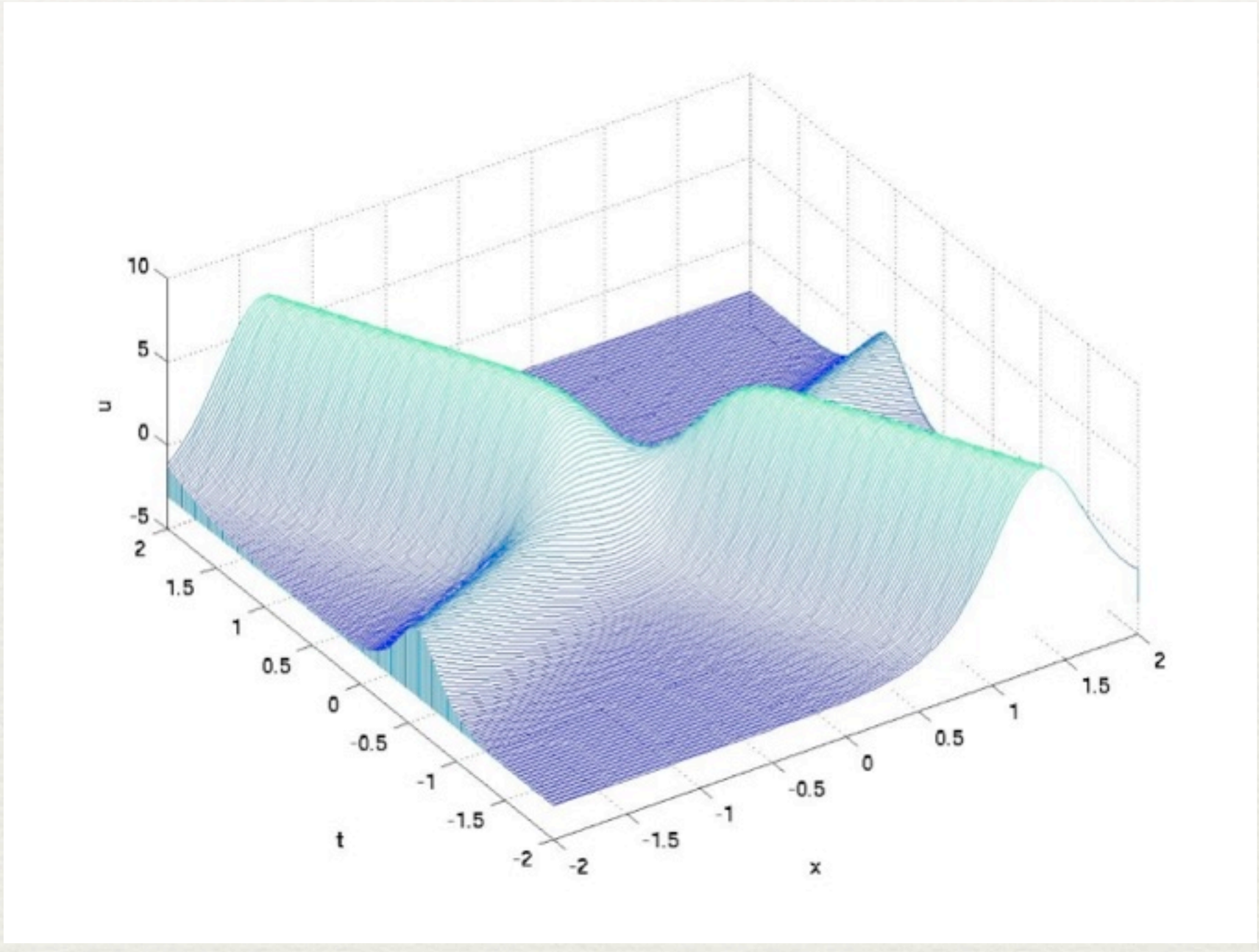
$$u_t + 6uu_x + \epsilon^2 u_{xxx} = 0$$

KdV 2-soliton



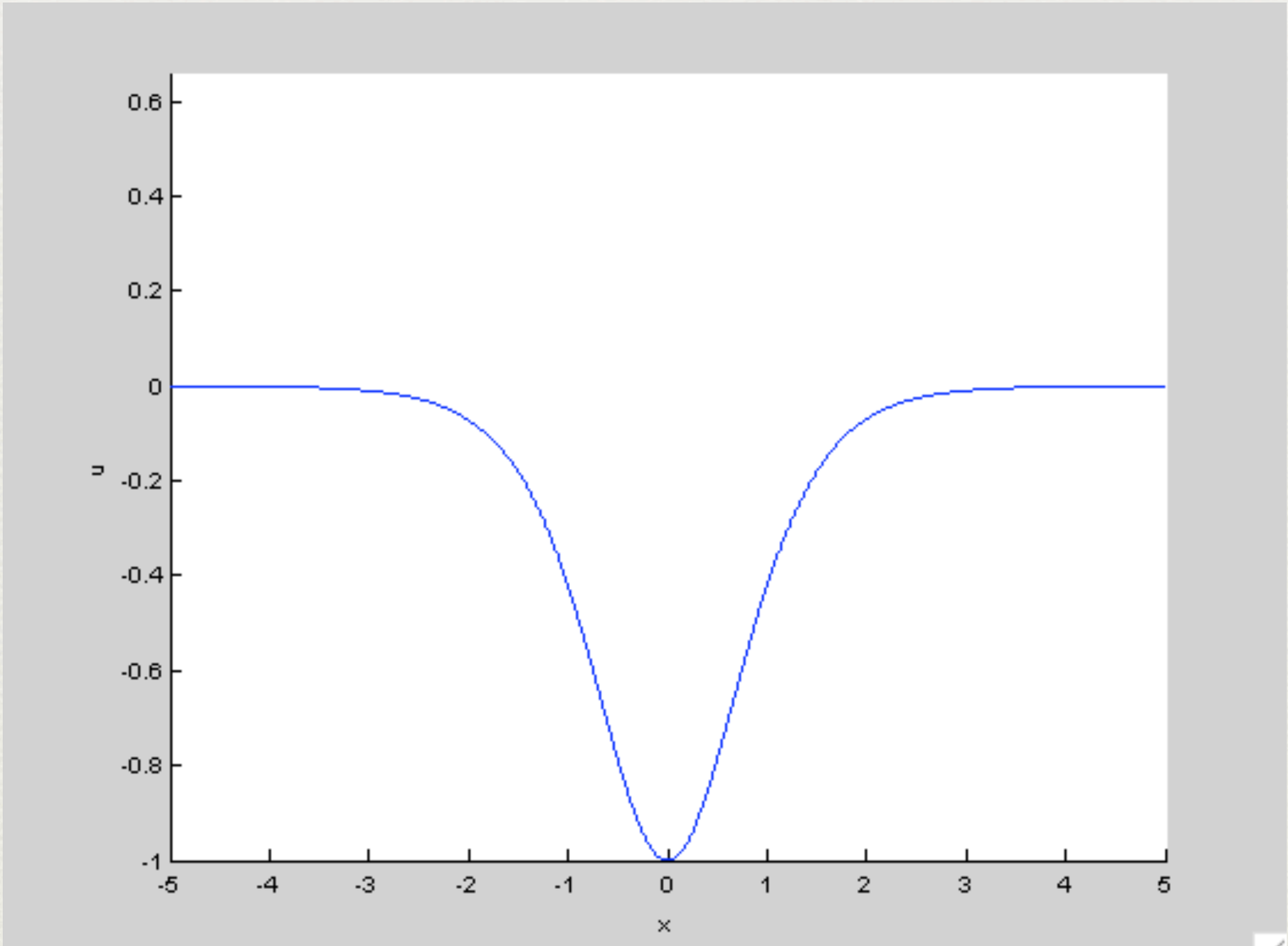
$\epsilon = 1$

KdV 2-soliton



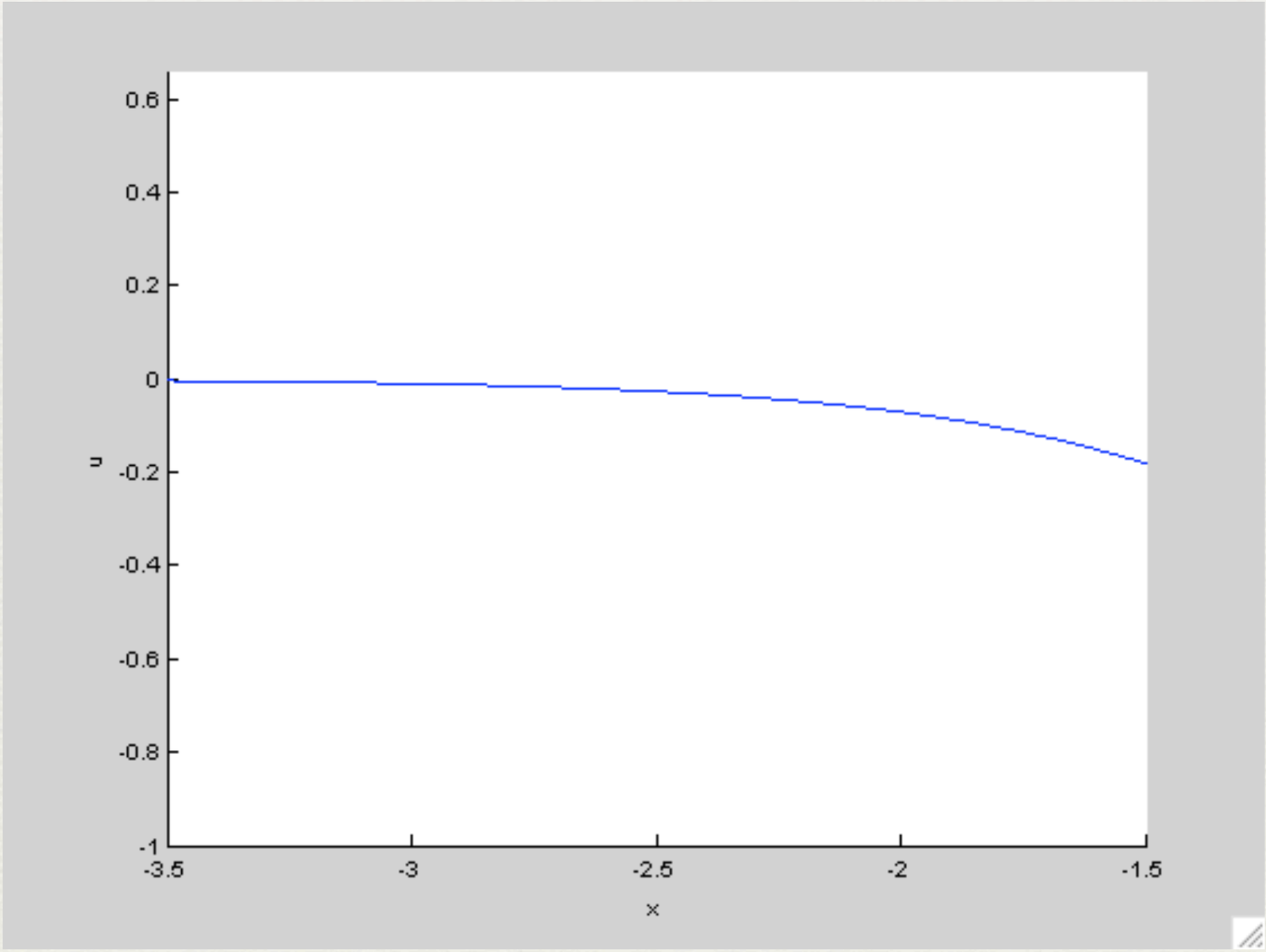
KdV, small dispersion

$$u(x,0) = -\mathrm{sech}^2 x$$

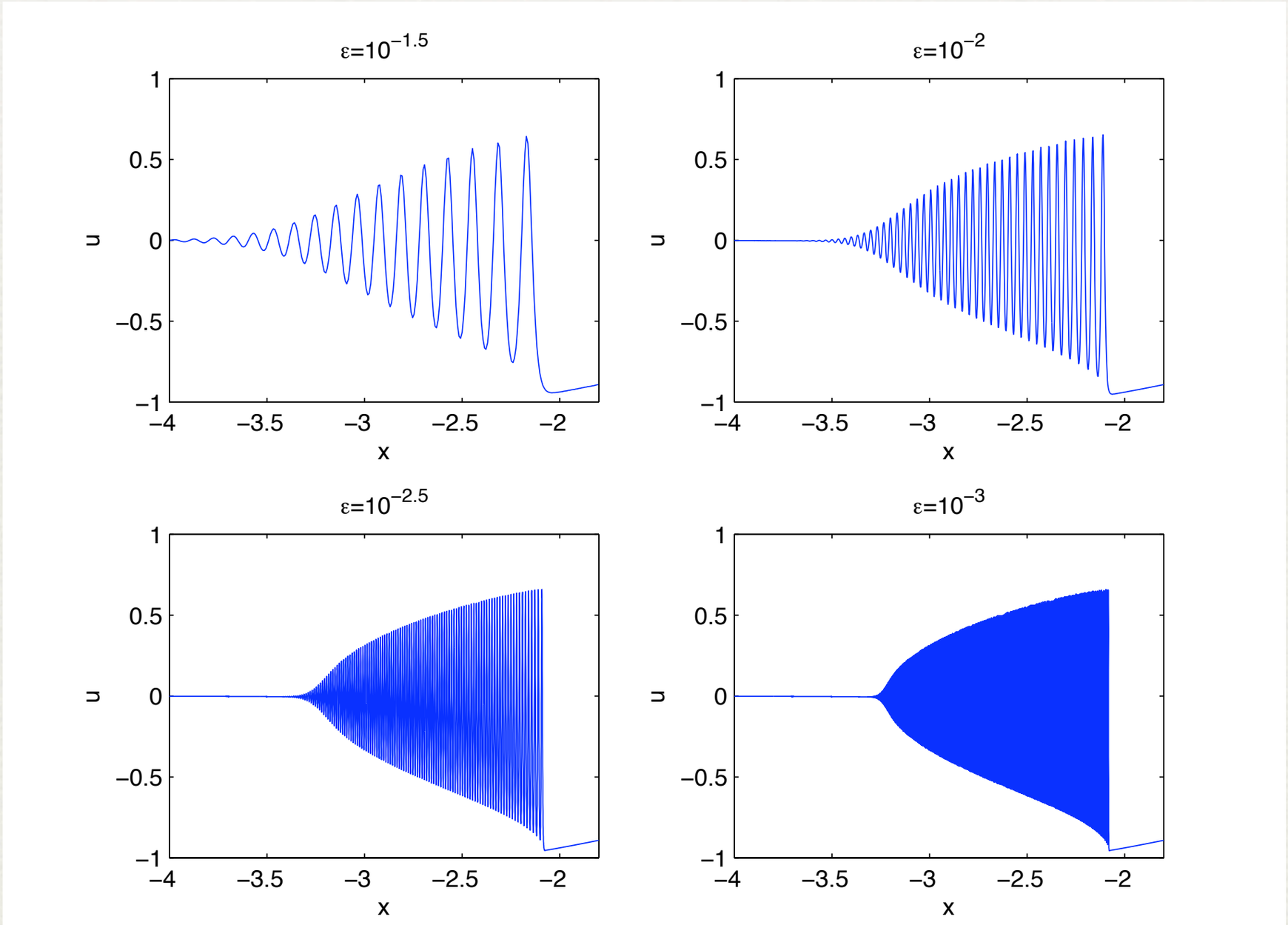


$$\epsilon \ll 1$$

Oscillatory zone



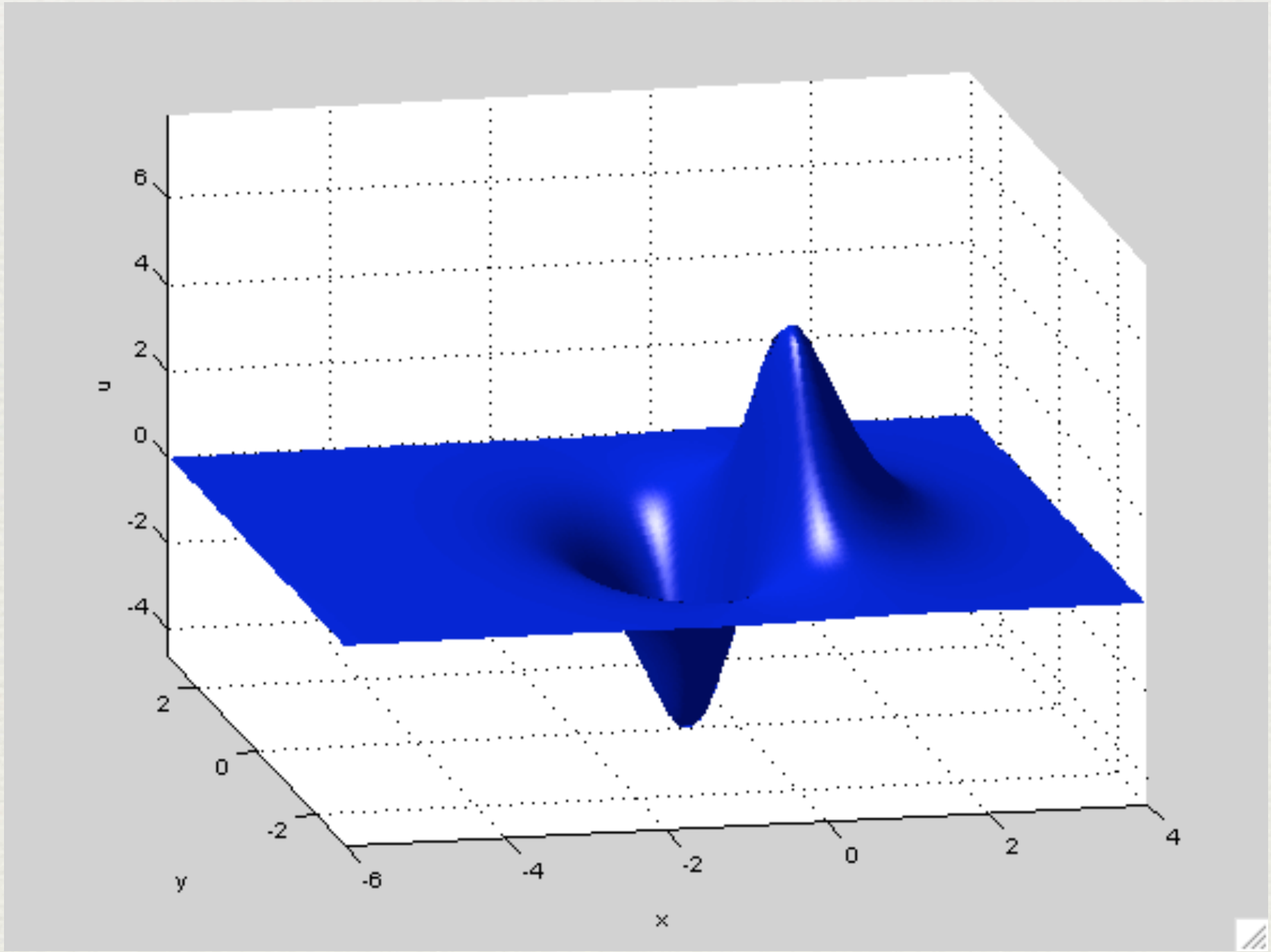
Different values of ϵ



Low dispersion KP I, ϵ=0.1

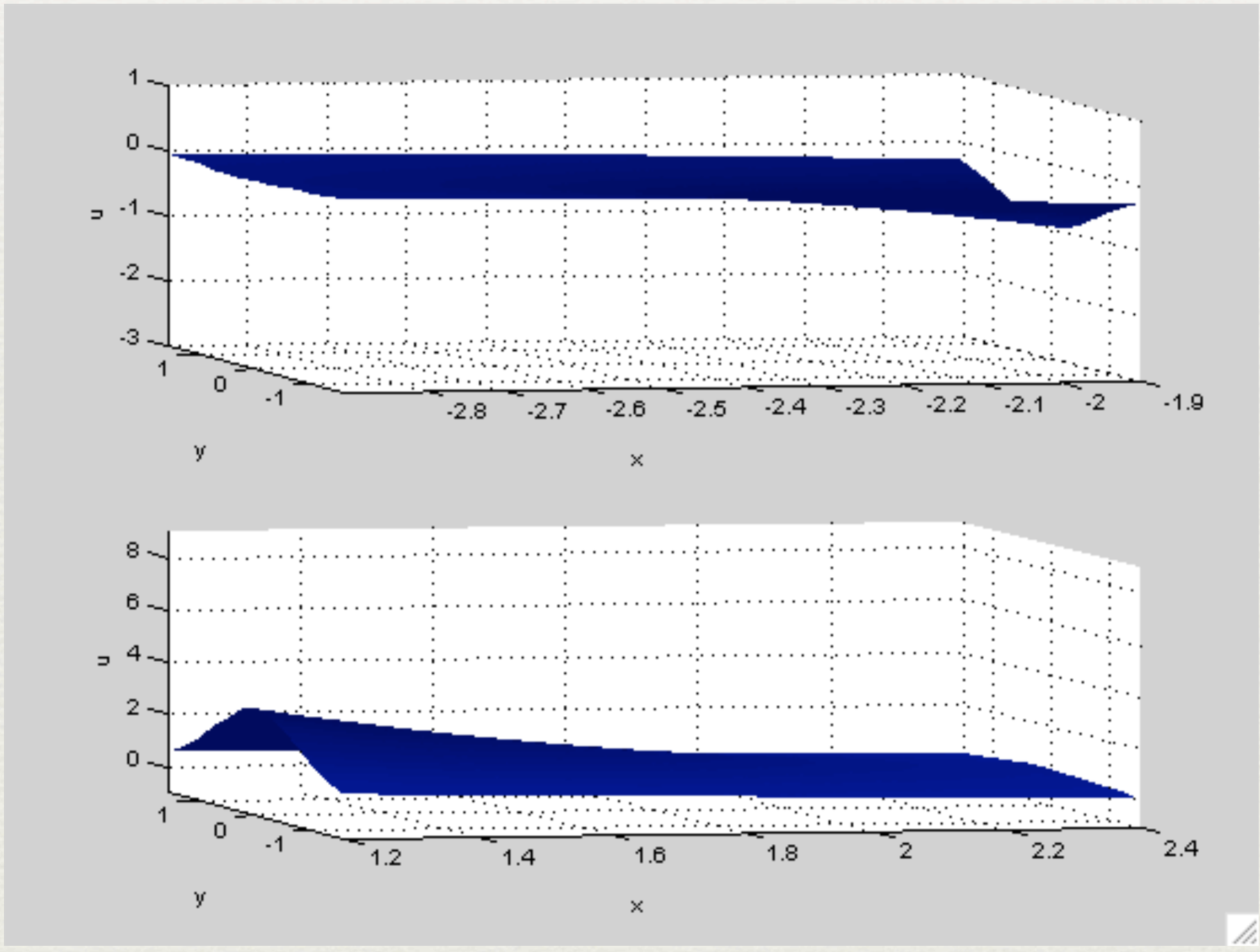
Kadomtsev-Petviashvili equation

$$(u_t + 6uu_x + \epsilon^2 u_{xxx})_x + \lambda u_{yy} = 0$$



$$u_0 = -6\partial_x \mathrm{sech}^2(x)$$

KP I solution, ϵ=0.01



Outline of the numerical part of the school I

$$u_t + 6uu_x + \epsilon^2 u_{xxx} = 0$$

- spatial derivatives:
  - C. K.: Introduction to spectral methods (Fourier (periodic settings), Chebychev (finite intervals))
  - F. Bornemann: advanced Chebychev methods

Outline of the numerical part of the school II

- time integation:
  - P. Matthews: stiff equations, stability and exponential integrators
  - A. Ostermann: exponential integrators and splitting
  - J. Frauendiener: structure preserving and geometric integrators

Introduction to spectral methods in Matlab

1. Introduction to Matlab (programming language)
2. Fourier methods for differential equations (periodic settings, rapidly decreasing functions)
3. Chebychev polynomials (finite intervals)

Literature (Matlab, Octave)

- Getting started with Matlab (Mathworks).
- D. Higham, N. Higham, *Matlab Guide,* SIAM 2005.
- C. Moler, *Numerical Computing with Matlab,* SIAM 2004.
- http://www.mathworks.com
- http://www.gnu.org/software/octave/

Literature (Spectral Methods)

- L. N. Trefethen, *Spectral Methods in Matlab*, SIAM 2000 (Codes at http://www.comlab.ox.ac.uk/nick.trefethen/spectral.html, free text at .../pdetext.html
- B. Fornberg: *A practical guide to pseudospectral methods,* CUP 1996.
- C. Canuto, M. Y. Hussaini, A. Quarteroni and T. A. Zang, *Spectral Methods in Fluid Dynamics*, Springer-Verlag 1988.

Matlab (*matrix laboratory)*

- Language written by Cleve Moler in the seventies to provide easy acces to linear algeba packages Linpack and Eispack
- today: Programming language based on matrix language with functionality of C, Fortran in very compact form (Lapack, Blas)
- distributed by Mathworks
- open source project *octave*

Matrices

- Creating a matrix: enter a list, colon operator, use function (zeros, ones, eye,...), special functions (pi, eps, i, Inf, NaN,...)
- addressing matrix elements, concatenation, deleting parts of a matrix
- manipulate matrices: sum, transpose, diag, inv, eig, entry-wise and matrix manipulation, logical subscripting, find
- control output: format, ;

Graphics

- Plots: real and complex vectors, matrices, hold, style, axis
- surface plots: mesh, surf, waterfall, contour,..., lighting
- videos

Programming

- Scripts and functions
- Flow control: if, while, for
- vectorizing, preallocation
- debugging
- demos for more information