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Collider Phenomenology I

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Collider Phenomenology

— From basic knowledge to new physics searches

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ICTP Summer School

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Outline:

Lecture I: (a). Colliders and Detectors

(b). Basics Techniques and Tools

Lecture II: (a). An e^+e^- Linear Collider

(b). Hadron Colliders Physics

Lecture III: From Kinematics to Dynamics

Lecture IV: Search for New Physics at Hadron Colliders

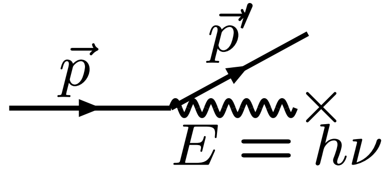
Main reference: TASI 04 Lecture notes
hep-ph/0508097,
plus the other related lectures in this school.

I. Colliders and Detectors

(A). High-energy Colliders:

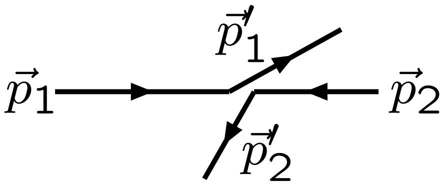
To study the deepest layers of matter,

we need the probes with highest energies.



Two parameters of importance:

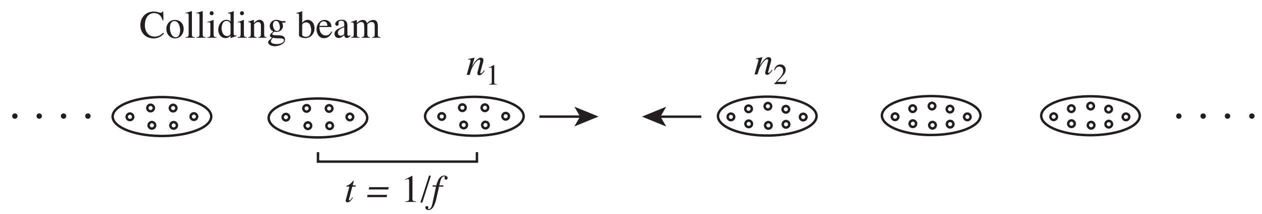
1. The energy:



$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{\mathrm{p}}_2 = 0. \end{cases}$$

2. The luminosity:



$$\mathcal{L} \propto f n_1 n_2 / a,$$

(a some beam transverse profile) in units of #particles/cm^2/s

$\Rightarrow$ 10^{33} cm^{-2}s $^{-1}$ = 1 nb^{-1} s^{-1} $\approx$ 10 fb^{-1}/year.

Current and future high-energy colliders:

Hadron Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	#/bunch (10^{10})	L (km)
Tevatron	1.96	2.1×10^{32}	9×10^{-5}	2.5	p: 27, $\bar{p}$: 7.5	6.28
LHC	14	10^{34}	0.01%	40	10.5	26.66

e^+e^- Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	polar.	L (km)
ILC	0.5–1	2.5×10^{34}	0.1%	3	80, 60%	14 – 33
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	80, 60%	33 – 53

(B). An e^+e^- Linear Collider

The collisions between e^- and e^+ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc.,

$\Longrightarrow$ it is suitable to create new particles after e^+e^- annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame,

$\Longrightarrow$ the total c.m. energy is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties,

$\Longrightarrow$ the scattering kinematics is well-constrained.

- Backgrounds low and well-undercontrol.
- It is possible to achieve high degrees of beam polarizations,

$\Longrightarrow$ chiral couplings and other asymmetries can be effectively explored.

Disadvantages

- Large synchrotron radiation due to acceleration,

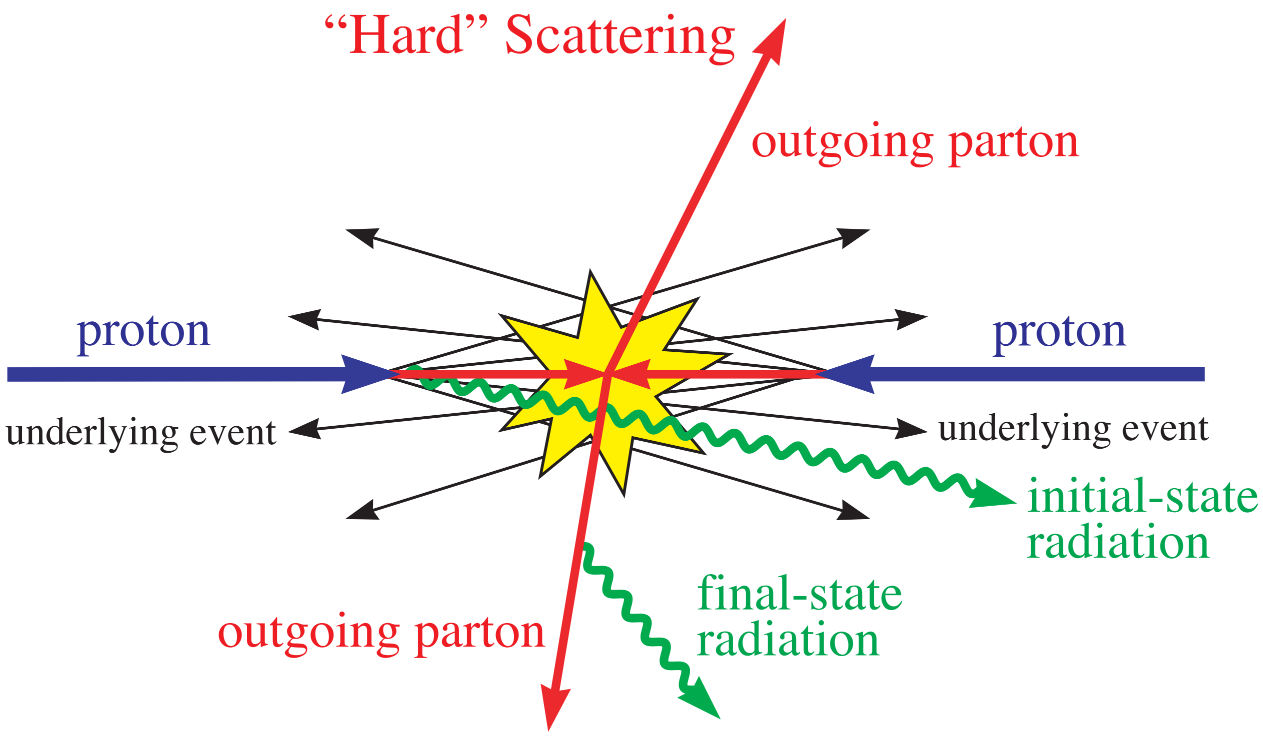
$$\Delta E \sim \frac{1}{R} \left(\frac{E}{m_e}\right)^4 .$$

  Thus, a multi-hundred GeV e^+e^- collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized; beamsstrahlung severe.

(C). Hadron Colliders

LHC: the next high-energy frontier



Advantages

- Higher c.m. energy, thus higher energy threshold:

$\sqrt{S} = 14$ TeV: $\quad M^2_{new} \sim s = x_1 x_2 S \quad \Rightarrow \quad M_{new} \sim 0.2\sqrt{S} \sim 3$ TeV.

- Higher luminosity: 10^{34}/cm^2/s $\Rightarrow$ 100 fb^{-1}/yr.

  Annual yield: 1B $W^{\pm}$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...
- Multiple (strong, electroweak) channels:

  $q\bar{q}'$, gg, qg, $b\bar{b} \to$ colored; $Q = 0, \pm 1$; $J = 0, 1, 2$ states;

  WW, WZ, ZZ, $\gamma\gamma \to I_W = 0, 1, 2$; $Q = 0, \pm 1, \pm 2$; $J = 0, 1, 2$ states.

<u>Disadvantages</u>

- Initial state unknown:

  colliding partons unknown on event-by-event basis;

  parton c.m. energy unknown: $E_{cm}^2 \equiv s = x_1 x_2 S$;

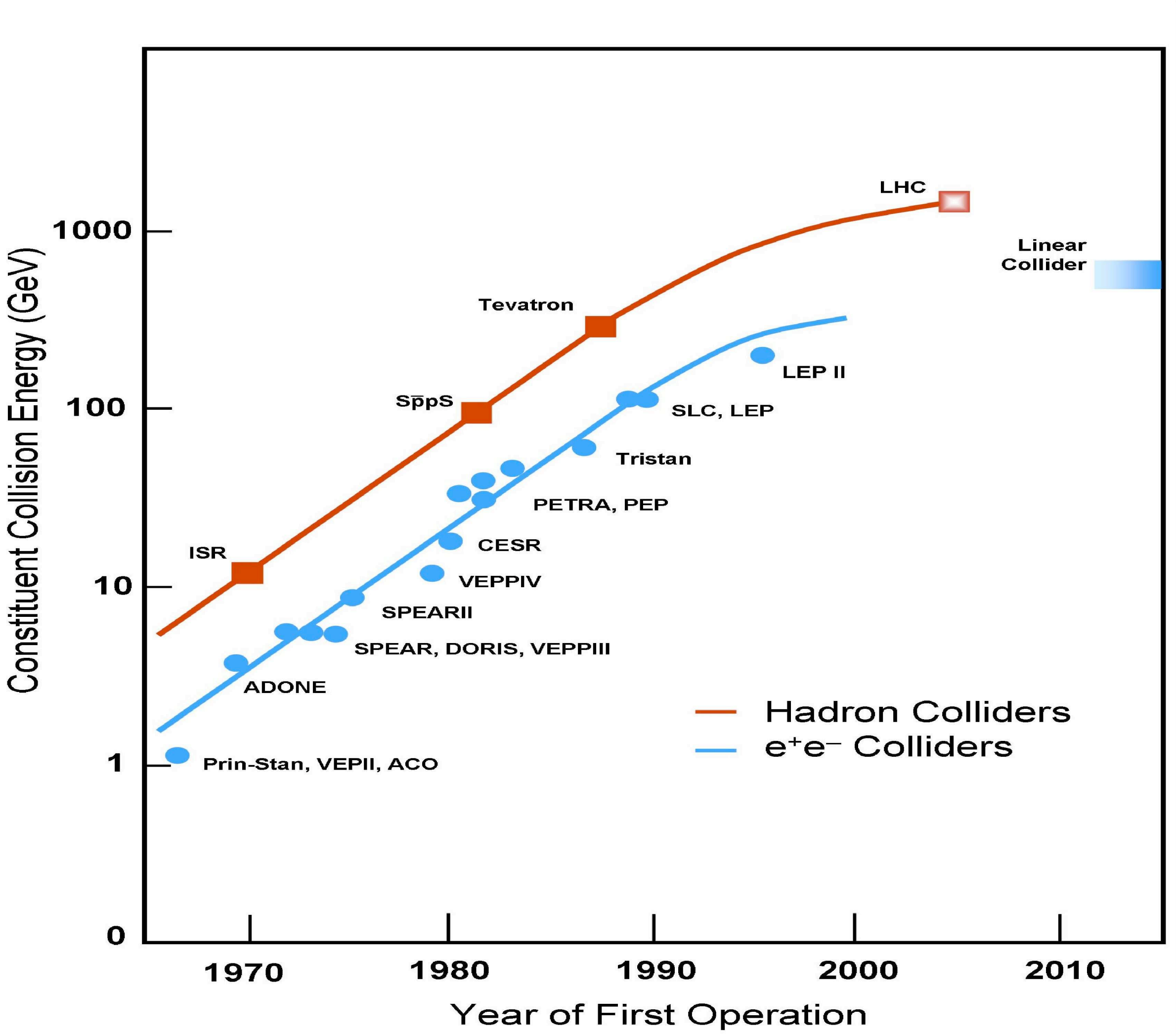
  parton c.m. frame unknown.

  $\Rightarrow$ largely rely on final state reconstruction.
- The large rate turns to a hostile environment:

  $\Rightarrow$ Severe backgrounds!

Our primary job !

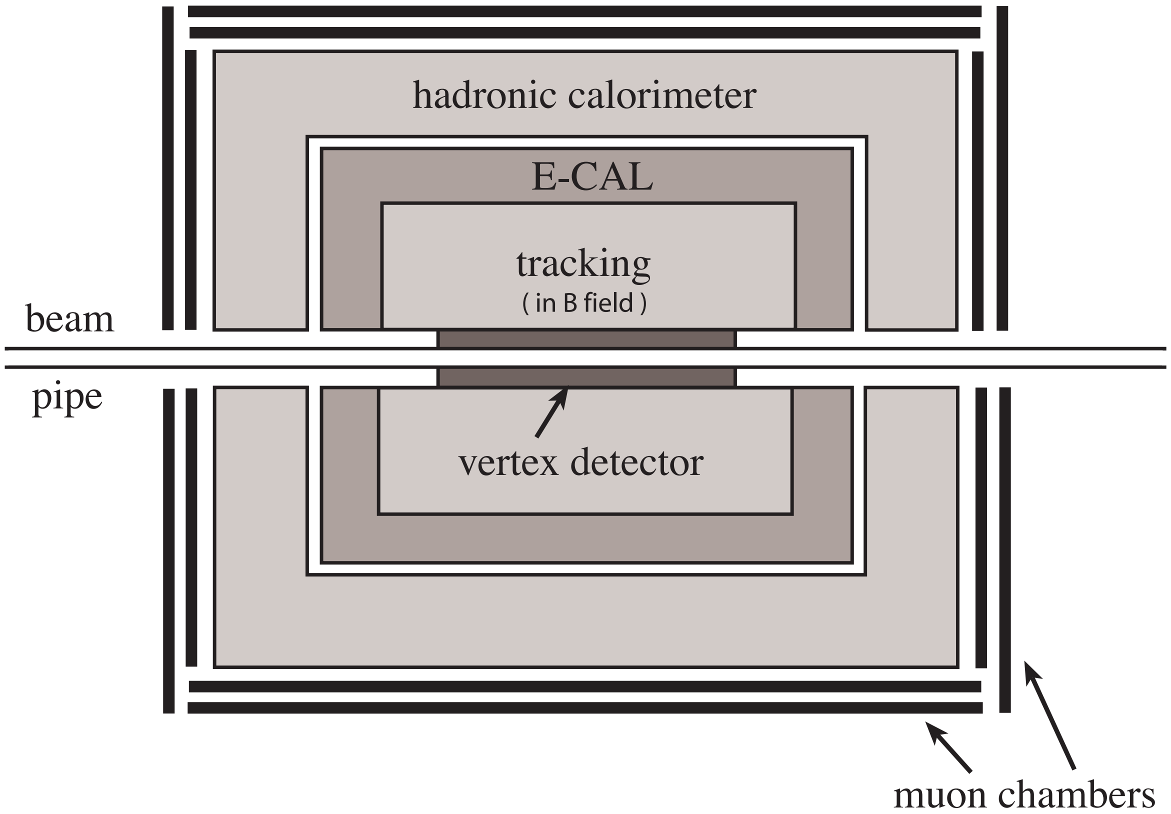
- *Path of the high-energy colliders:*



The CERN LHC will open a new eta of HEP.

(D). Particle Detection:

The detector complex:
Utilize the strong and electromagnetic interactions between detector materials and produced particles.



What we "see" as particles in the detector: (a few meters)

For a relativistic particle, the travel distance:

$$d = (\beta c\tau\ )\gamma \approx (300\ \mu m)(\frac{\tau}{10^{-12}\ s})\ \gamma$$

- stable particles directly "seen":

$$p,\ \bar{p},\ e^{\pm},\ \gamma$$

- quasi-stable particles of a life-time $\tau \geq 10^{-10}$ s also directly "seen":

$$n, \Lambda, K_L^0, ...,\ \mu^{\pm},\ \pi^{\pm}, K^{\pm}...$$

- a life-time $\tau \sim 10^{-12}$ s may display a secondary decay vertex, "vertex-tagged particles":

$$B^{0,\pm},\ D^{0,\pm},\ \tau^{\pm}...$$

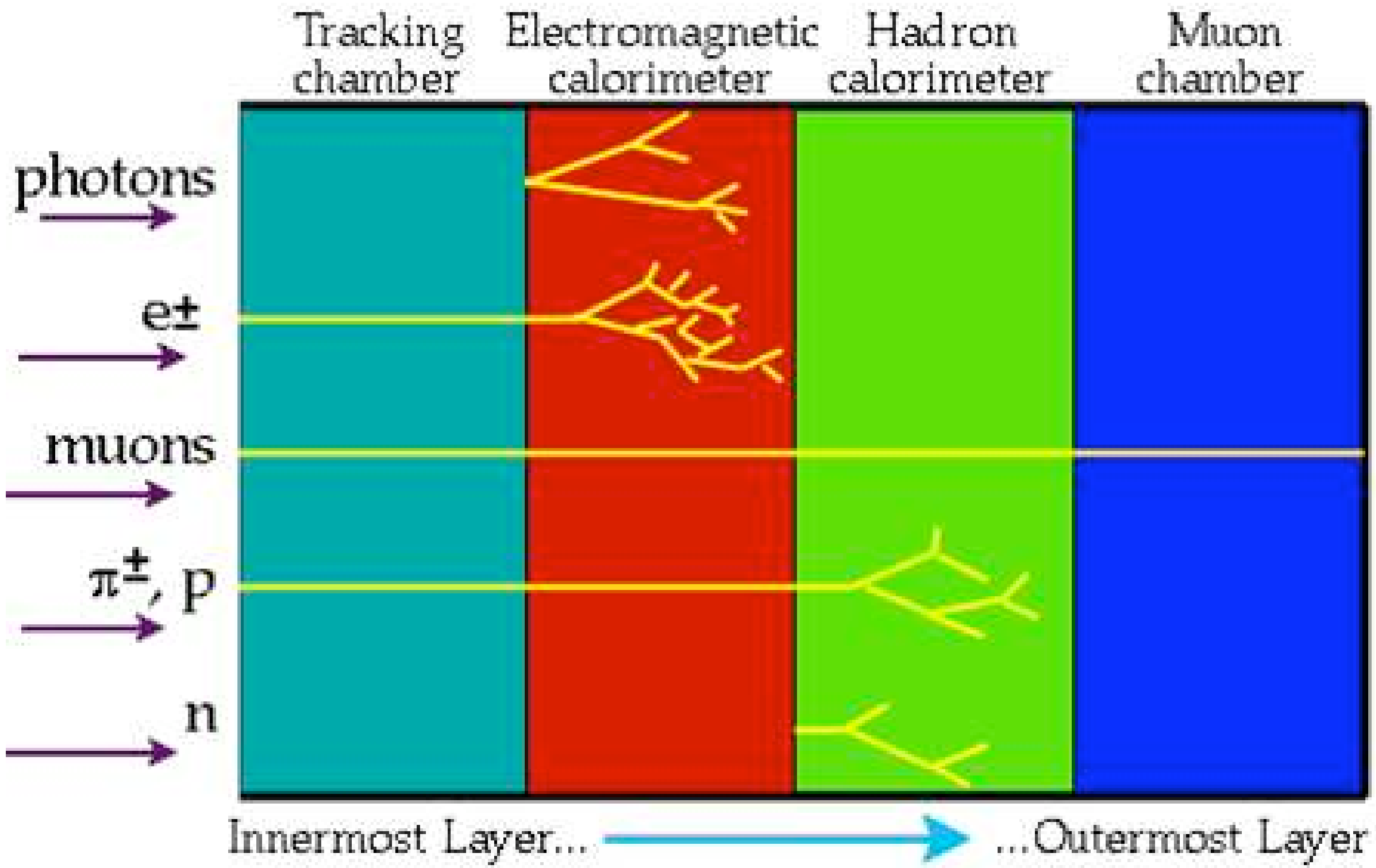
- short-lived not "directly seen", but "reconstructable":

$$\pi^0,\ \rho^{0,\pm}...\ ,\quad Z, W^{\pm}, t, H...$$

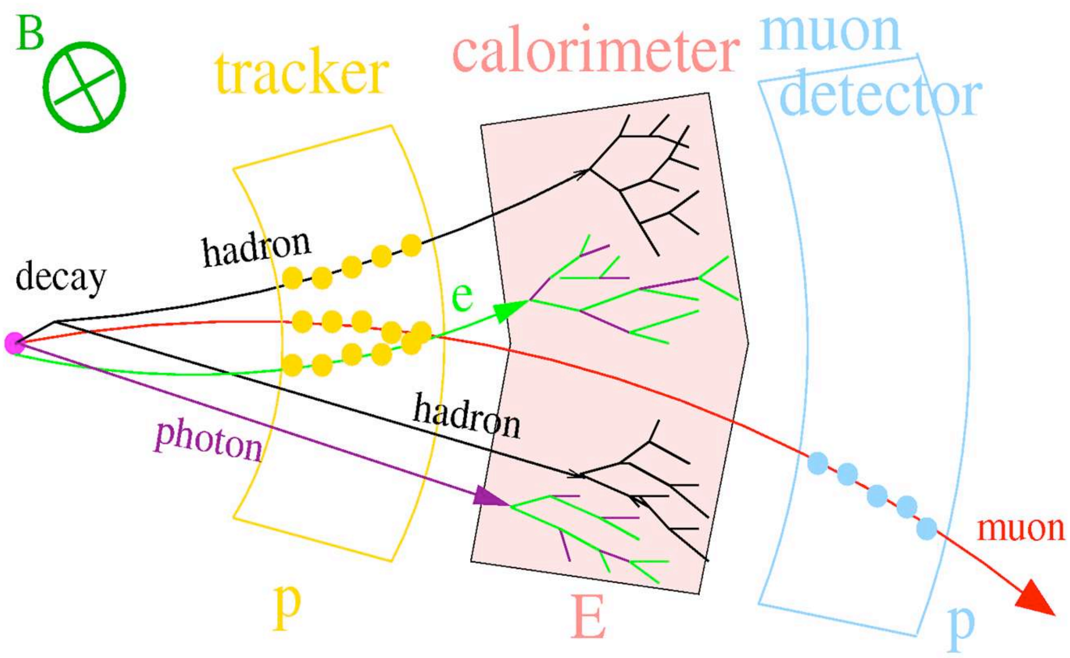
- missing particles are weakly-interacting and neutral:

$$\nu,\ \tilde{\chi}^0, G_{KK}...$$

† For stable and quasi-stable particles of a life-time $\tau \geq 10^{-10} - 10^{-12}$ s, they show up as



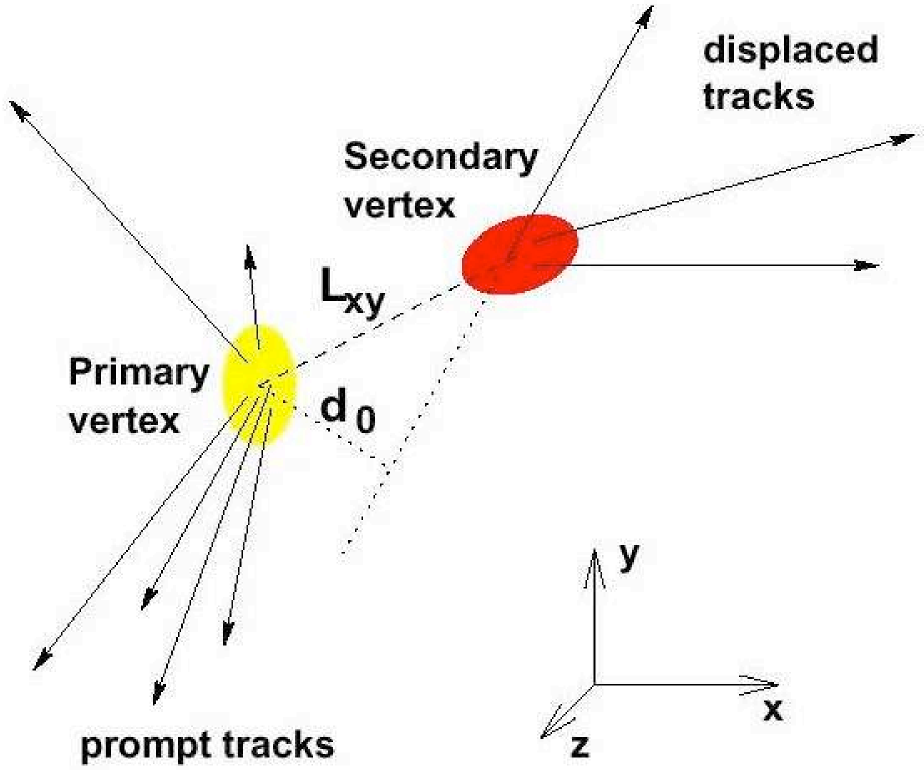
A closer look:



Theorists should know:

$$\text{For charged tracks}: \Delta \quad p/p \propto p,$$
$$\text{typical resolution}: \quad \sim p/(10^4\ GeV).$$
$$\text{For calorimetry}: \Delta \quad E/E \propto \frac{1}{\sqrt{E}},$$
$$\text{typical resolution}: \quad \sim (5-80\%)/\sqrt{E}.$$

† For vertex-tagged particles $\tau \approx 10^{-12}$ s,
heavy flavor tagging: the secondary vertex:



Typical resolution: $d_0 \sim 30 - 50\ \mu$m or so

$\Rightarrow$ Better have two (non-collinear) charged tracks for a secondary vertex;

Or use the "impact parameter" w.r.t. the primary vertex.

For theorists: just multiply a "tagging efficiency" $\epsilon_b \sim 40 - 60\%$ or so.

† For short-lived particles: $\tau < 10^{-12}$ s or so,
make use of final state kinematics to reconstruct the resonance.

† For missing particles:
make use of energy-momentum conservation to deduce their existence.

$$p_1^i + p_2^i = \sum_f^{obs.} p_f + p_{miss}.$$

But in hadron collisions, the longitudinal momenta unkown,
thus transverse direction only:

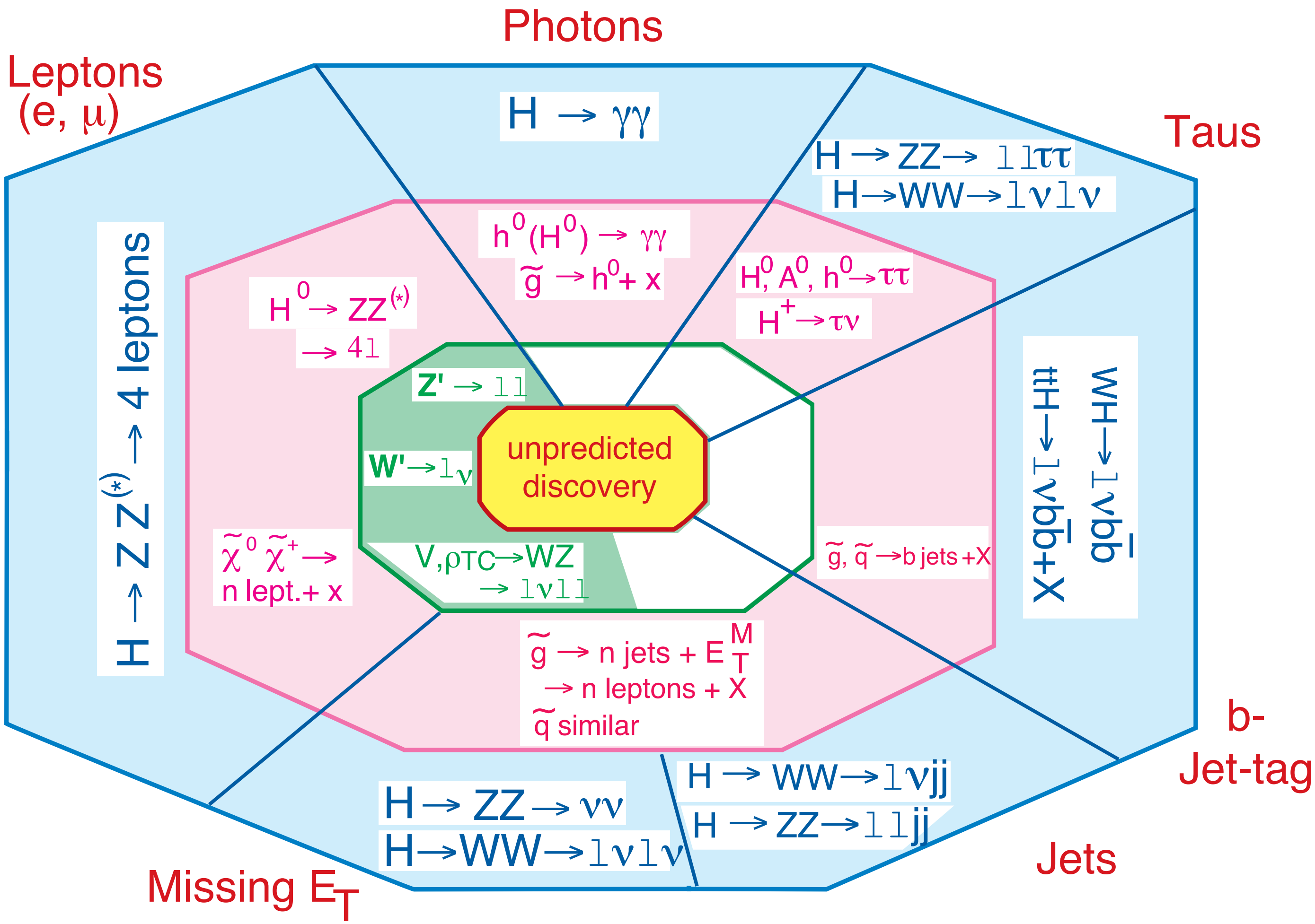
$$0 = \sum_f^{obs.} \vec{p}_{f\ T} + \vec{p}_{miss\ T}.$$

often called "missing p_T" ($\not{p}_T$) or "missing E_T" ($\not{E}_T$).

What we "see" for the SM particles (no universality – sorry!)

Leptons	Vetexing	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\surd$	$\surd$	$\vec{p}$
$\tau^\pm$	$\surd\times$	$\surd$	$e^\pm$	$h^\pm$; $3h^\pm$	$\mu^\pm$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$c \to D$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$b \to B$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$t \to bW^\pm$	b	$\surd$	$e^\pm$	$b +$ 2 jets	$\mu^\pm$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$W^\pm \to \ell^\pm \nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}'$	$\times$	$\surd$	$\surd$	2 jets	$\times$
$Z^0 \to \ell^+\ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\surd$	2 jets	$\times$

How to search for new particles?



y98014_416dPauss rd

Homework:

Exercise 1.1: For a π^0, μ^-, or a τ^- respectively, calculate its decay length for $E = 10$ GeV.

Exercise 1.2: An event was identified to have a $\mu^+\mu^-$ pair, along with some missing energy. What can you say about the kinematics of the system of the missing particles? Consider both an e^+e^- and a hadron collider.

Exercise 1.3: A 120 GeV Higgs boson will have a production cross section of 20 pb at the LHC. How many events per year do you expect to produce for the Higgs boson with an instantaneous luminosity 10^{33}/cm^2/s?
Do you expect it to be easy to observe and why?

II. Basic Techniques and Tools for Collider Physics

(A). Scattering cross section

For a $2 \to n$ scattering process:

$$\sigma(ab \to 1 + 2 + ...n) = \frac{1}{2s} \overline{\sum} |\mathcal{M}|^2 \; dPS_n,$$

$$dPS_n \equiv (2\pi)^4 \; \delta^4 \left(P - \sum_{i=1}^{n} p_i \right) \Pi_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left(\sum_{i=1}^{n} p_i \right)^2,$$

where $\overline{\sum} |\mathcal{M}|^2$: dynamics (dimension $4 - 2n$);

dPS_n: kinematics (Lorentz invariant, dimension $2n - 4$.)

For a $1 \to n$ decay process, the partial width in the rest frame:

$$\Gamma(a \to 1 + 2 + ...n) = \frac{1}{2M_a} \overline{\sum} |\mathcal{M}|^2 \; dPS_n.$$

$$\tau = \Gamma_{tot}^{-1} = (\sum_f \Gamma_f)^{-1}.$$

(B). Phase space and kinematics *

One-particle Final State $a + b \to 1$:

$$
\begin{aligned}
dPS_1 &\equiv (2\pi)\, \frac{d^3\vec{p}_1}{2E_1} \delta^4(P - p_1) \\
&\doteq \pi |\vec{p}_1| d\Omega_1 \delta^3(\vec{P} - \vec{p}_1) \\
&\doteq 2\pi\ \delta(s - m_1^2).
\end{aligned}
$$

where the first and second equal signs made use of the identities:

$$
|\vec{p}|d|\vec{p}| = EdE, \quad \frac{d^3\vec{p}}{2E} = \int d^4p\ \delta(p^2 - m^2).
$$

Kinematical relations:

$$
\begin{aligned}
\vec{P} &\equiv \vec{p}_a + \vec{p}_b = \vec{p}_1, \quad E_1^{cm} = \sqrt{s} \text{ in the c.m. frame,} \\
s &= (p_a + p_b)^2 = m_1^2.
\end{aligned}
$$

The "dimensinless phase-space volume" is $s(dPS_1) = 2\pi$.

*E.Byckling, K. Kajantie: Particle Kinemaitcs (1973).

Two-particle Final State $a + b \to 1 + 2$:

$$
\begin{aligned}
dPS_2 &\equiv \frac{1}{(2\pi)^2}\,\delta^4\left(P - p_1 - p_2\right)\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2} \\
&\doteq \frac{1}{(4\pi)^2}\,\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\,d\Omega_1 = \frac{1}{(4\pi)^2}\,\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\,d\cos\theta_1 d\phi_1 \\
&= \frac{1}{4\pi}\frac{1}{2}\,\lambda^{1/2}\left(1, \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) dx_1 dx_2.
\end{aligned}
$$

The magnitudes of the energy-momentum of the two particles are fully determined by the four-momentum conservation:

$$
|\vec{p}_1^{\,cm}| = |\vec{p}_2^{\,cm}| = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}},\;\; E_1^{cm} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}},\;\; E_2^{cm} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}},
$$

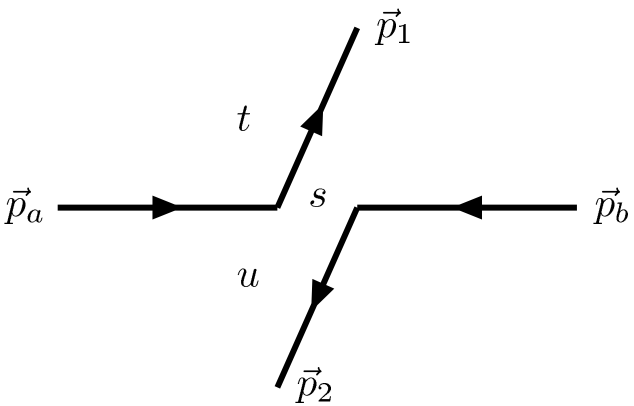
$$
\lambda(x, y, z) = (x - y - z)^2 - 4yz = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
$$

The phase-space volume of the two-body is scaled down with respect to that of the one-particle by a factor

$$
\frac{dPS_2}{s\ dPS_1} \approx \frac{1}{(4\pi)^2}.
$$

just like a “loop factor”.

Consider a $2 \to 2$ scattering process $p_a + p_b \to p_1 + p_2$,



the (Lorentz invariant) Mandelstam variables are defined as

$$
\begin{aligned}
s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\
t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos\theta_{a1}), \\
u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos\theta_{a2}), \\
& s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2.
\end{aligned}
$$

The two-body phase space can be thus written as

$$
dPS_2 = \frac{1}{(4\pi)^2} \frac{dt\ d\phi_1}{s\ \lambda^{1/2}\left(1, m_a^2/s, m_b^2/s\right)}.
$$

Exercise 2.1: Assume that $m_a = m_1$ and $m_b = m_2$. Show that

$$
\begin{aligned}
t &= -2p_{cm}^2(1 - \cos\theta_{a1}^*), \\
u &= -2p_{cm}^2(1 + \cos\theta_{a1}^*) + \frac{(m_1^2 - m_2^2)^2}{s},
\end{aligned}
$$

$p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$ is the momentum magnitude in the c.m. frame.
Note: t is negative-definite; $t \to 0$ in the collinear limit.

Exercise 2.2: A particle of mass M decays to two particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle is moving with a speed β_z? Compare the result with your expectation for the shape change for a basket ball.

Three-particle Final State $a + b \to 1 + 2 + 3$:

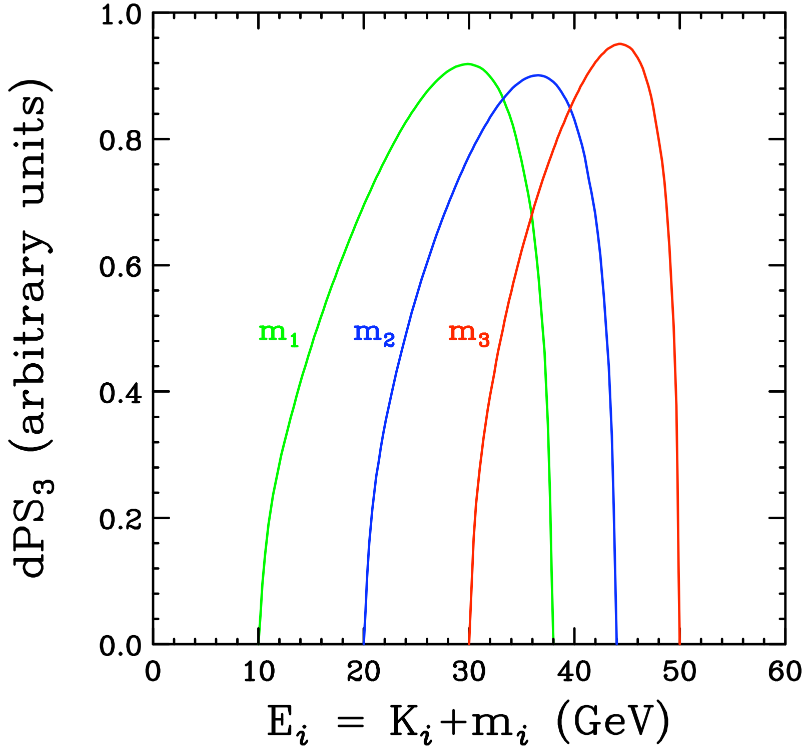
$$
\begin{aligned}
dPS_3 &\equiv \frac{1}{(2\pi)^5}\,\delta^4\left(P - p_1 - p_2 - p_3\right)\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}\frac{d^3\vec{p}_3}{2E_3} \\
&\doteq \frac{|\vec{p}_1|^2\ d|\vec{p}_1|\ d\Omega_1}{(2\pi)^3\ 2E_1}\ \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_2^{(23)}|}{m_{23}}\ d\Omega_2 \\
&= \frac{1}{(4\pi)^3}\ \lambda^{1/2}\left(1, \frac{m_2^2}{m_{23}^2}, \frac{m_3^2}{m_{23}^2}\right)\ 2|\vec{p}_1|\ dE_1\ dx_2 dx_3 dx_4 dx_5.
\end{aligned}
$$

$$
\begin{aligned}
d\cos\theta_{1,2} &= 2dx_{2,4}, \quad d\phi_{1,2} = 2\pi dx_{3,5}, \quad 0 \le x_{2,3,4,5} \le 1, \\
& |\vec{p}_1^{cm}|^2 = |\vec{p}_2^{cm} + \vec{p}_3^{cm}|^2 = (E_1^{cm})^2 - m_1^2, \\
& m_{23}^2 = s - 2\sqrt{s}E_1^{cm} + m_1^2, \quad |\vec{p}_2^{23}| = |\vec{p}_3^{23}| = \frac{\lambda^{1/2}(m_{23}^2, m_2^2, m_3^2)}{2m_{23}},
\end{aligned}
$$

The particle energy spectrum is not monochromatic.
The maximum value (the end-point) for particle 1 in c.m. frame is

$$
\begin{aligned}
E_1^{max} &= \frac{s + m_1^2 - (m_2 + m_3)^2}{2\sqrt{s}}, \quad m_1 \le E_1 \le E_1^{max}, \\
|\vec{p}_1^{max}| &= \frac{\lambda^{1/2}(s, m_1^2, (m_2 + m_3)^2)}{2\sqrt{s}}, \quad 0 \le p_1 \le p_1^{max}.
\end{aligned}
$$

With $m_i = 10,\ 20,\ 30,\ \sqrt{s} = 100$ GeV.



More intuitive to work out the end-point for the kinetic energy,
– recall the direct neutrino mass bound in β-decay:

$$K_1^{max} = E_1^{max} - m_1 = \frac{(\sqrt{s} - m_1 - m_2 - m_3)(\sqrt{s} - m_1 + m_2 + m_3)}{2\sqrt{s}}.$$

In general, the 3-body phase space boundaries are non-trivial.
That leads to the "Dalitz Plots".

One practically useful formula is:

`Exercise 2.3: A particle of mass` M `decays to 3 particles` $M \to abc$.
`Show that the phase space element can be expressed as`

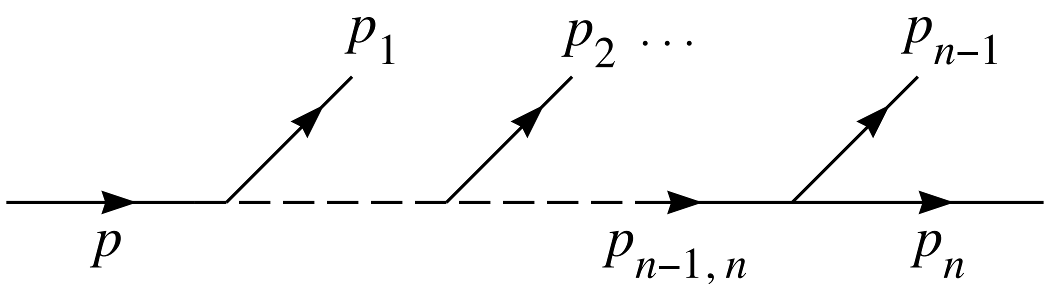
$$dPS_3 = \frac{1}{2^7 \pi^3}\; M^2 dx_a dx_b.$$

$$x_i = \frac{2E_i}{M},\ \ (i = a, b, c,\ \ \sum_i x_i = 2).$$

`where the integration limits for` $m_a = m_b = m_c = 0$ `are`

$$0 \le x_a \le 1, \quad 1 - x_a \le x_b \le 1.$$

Recursion relation $P \to 1 + 2 + 3 ... + n$:



$$dPS_n(P; p_1, ..., p_n) = dPS_{n-1}(P; p_1, ..., p_{n-1,n}) \, dPS_2(p_{n-1,n}; p_{n-1}, p_n) \frac{dm^2_{n-1,n}}{2\pi}.$$

For instance,

$$dPS_3 = dPS_2(i) \; \frac{dm^2_{prop}}{2\pi} \; dPS_2(f).$$

This is generically true, but particularly useful when the diagram has an s-channel particle propagation.

Breit-Wigner Resonance, the Narrow Width Approximation

An unstable particle of mass M and total widthΓ_V, the propagator is

$$R(s) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}.$$

Consider an intermediate state V^*

$$a \longrightarrow bV^* \longrightarrow b\ p_1 p_2.$$

By the reduction formula, the resonant integral reads

$$\int_{(m_*^{min})^2=(m_1+m_2)^2}^{(m_*^{max})^2=(m_a-m_b)^2} dm_*^2.$$

Variable change

$$\tan\theta = \frac{m_*^2 - M_V^2}{\Gamma_V M_V},$$

resulting in a flat integrand over θ

$$\int_{(m_*^{min})^2}^{(m_*^{max})^2} \frac{dm_*^2}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} = \int_{\theta^{min}}^{\theta^{max}} \frac{d\theta}{\Gamma_V M_V}.$$

In the limit

$$
\begin{aligned}
(m_1 + m_2) + \Gamma_V \ll M_V \ll m_a - \Gamma_V, \\
\theta^{min} = \tan^{-1}\frac{(m_1+m_2)^2 - M_V^2}{\Gamma_V M_V} \to -\pi, \\
\theta^{max} = \tan^{-1}\frac{(m_a-m_b)^2 - M_V^2}{\Gamma_V M_V} \to 0,
\end{aligned}
$$

then the Narrow Width Approximation

$$
\frac{1}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V}\ \delta(m_*^2 - M_V^2).
$$

Exercise 2.4: Consider a three-body decay of a top quark, $t \to bW^* \to b\ e\nu$. Making use of the phase space recursion relation and the narrow width approximation for the intermediate W boson, show that the partial decay width of the top quark can be expressed as

$$
\Gamma(t \to bW^* \to b\ e\nu) \approx \Gamma(t \to bW) \cdot BR(W \to e\nu).
$$

(C). Matrix element: The dynamics
Properties of scattering amplitudes

• Analyticity: A scattering amplitude is analytical except:
simple poles (corresponding to single particle states, bound states etc.);
branch cuts (corresponding to thresholds).

• Crossing symmetry: A scattering amplitude for a $2 \to 2$ process is symmetric among the s-, t-, u-channels.

• Unitarity:
S-matrix unitarity leads to :

$$-i(T - T^{\dagger}) = TT^{\dagger}$$

Partial wave expansion for $a + b \to 1 + 2$:

$$\begin{aligned}
\mathcal{M}(s,t) &= 16\pi \sum_{J=M}^{\infty} (2J+1) a_J(s) d^J_{\mu\mu'}(\cos\theta) \\
a_J(s) &= \frac{1}{32\pi} \int_{-1}^{1} \mathcal{M}(s,t)\ d^J_{\mu\mu'}(\cos\theta) d\cos\theta .
\end{aligned}$$

where $\mu = s_a - s_b,\ \ \mu' = s_1 - s_2,\ \ J = max(|\mu|, |\mu'|)$.

The partial wave amplitude have the properties:

(a). partial wave unitarity: $\mathrm{Im}(a_J) \geq |a_J|^2$, or $|Re(a_J)| \leq 1/2$,

(b). kinematical thresholds: $a_J(s) \propto \beta_i^{l_i}\ \beta_f^{l_f}\ \ (J = L + S)$.

$\Rightarrow$ well-known behavior: $\sigma \propto \beta_f^{2l_f+1}$.

Exercise 2.6: Appreciate the properties (a) and (b) by explicitly calculating the helicity amplitudes for

$$e^-_L e^+_R \to \gamma^* \to H^- H^+, \quad e^-_L e^+_{L,R} \to \gamma^* \to \mu^-_L \mu^+_R, \quad H^- H^+ \to G^* \to H^- H^+.$$

(D). Calculational Tools

Traditional "Trace" Techniques:

* You should be good at this — QFT course!

With algebraic symbolic manipulations:

* REDUCE
* FORM
* MATHEMATICA, MAPLE ...

Helicity Techniques:

More suitable for direct numerical evaluations.

* Hagiwara-Zeppenfeld: best for massless particles... (NPB)
* CalCul Method (by T.T. Wu et al., Parke-Mangano: Phys. Report);
* New techniques in loop calculations

(by Z.Bern, L.Dixon, W. Giele, N. Glover, K.Melnikov, F. Petriello ...)

`Exercise 2.5: Calculate the squared matrix element for` $\overline{\sum}|\mathcal{M}(f\bar{f}\to ZZ)|^2$, `in terms of` s, t, u, `in whatever technique you like.`

Calculational packages:

- Monte Carlo packages for phase space integration:

(1) VEGAS by P. LePage: adaptive important-sampling MC
 http://en.wikipedia.org/wiki/Monte-Carlo_integration

(2) SAMPLE, RAINBOW, MISER ...

- Automated software for matrix elements:

(1) REDUCE — an interactive program designed for general algebraic computations, including to evaluate Dirac algebra, an old-time program,
http://www.uni-koeln.de/REDUCE;
http://reduce-algebra.com.

(2) FORM by Jos Vermaseren: A program for large scale symbolic manipulation, evaluate fermion traces automatically,
and perform loop calculations,s commercially available at
http://www.nikhef.nl/ form

(3) FeynCalc and FeynArts: Mathematica packages for algebraic calculations in elementary particle physics.
http://www.feyncalc.org;
http://www.feynarts.de

(4) MadGraph: Helicity amplitude method for tree-level matrix elements available upon request or
http://madgraph.hep.uiuc.edu

Example:
Standard Model particles include:
Quarks: d u s c b t d u s c b t
Leptons: e- mu- ta- e+ mu+ ta+ ve vm vt ve vm vt
Bosons: g a z w+ w- h

Enter process you would like calculated in the form e+ e- $\rightarrow$ a.
(return to exit MadGraph.)

a a $\rightarrow$ w+ w-

Generating diagrams for 4 external legs
There are 3 graphs.
Writing Feynman graphs in file aa_wpwm.ps
Writing function AA_WPWM in file aa_wpwm.f.

- Automated evaluation of cross sections:

(1)MadGraph/MadEvent and MadSUSY:
Generate Fortran codes on-line!
http://madgraph.hep.uiuc.edu

(2) CompHEP: computer program for calculation of elementary particle processes in Standard Model and beyond. CompHEP has a built-in numeric interpreter. So this version permits to make numeric calculation without additional Fortran/C compiler. It is convenient for more or less simple calculations.

— It allows your own construction of a Lagrangian model!

http://theory.npi.msu.su/k̃ryukov

(3) GRACE and GRACE SUSY:
http://minami-home.kek.jp

(4) Pandora by M. Peskin:
C++ based package for e^+e^-, including beam effects.
http://www-sldnt.slac.stanford.edu/nld/new/Docs/Generators/PANDORA.htm
The program pandora is a general-purpose parton-level event generator which includes beamstrahlung, initial state radiation, and full treatment of polarization effects. (An interface to PYTHIA that produces fully hadronized events is possible.)

This version includes the SM physics processes:

$$\begin{aligned} e^+e^- &\rightarrow \ell^+\ell^-,\ q\bar{q},\ \gamma\gamma,\ t\bar{t},\ Z\gamma,\ ZZ,\ \ W^+W^- \\ &\rightarrow Zh, \nu\ \bar{\nu}h,\ e^+e^-h, \nu\ \bar{\nu}\gamma \\ \gamma\gamma &\rightarrow \ell^+\ell^-,\ q\bar{q},\ t\bar{t},\ e^+e^-,\ W^+W^-,\ h \\ e\gamma &\rightarrow e\gamma,\ eZ,\ \nu W \\ e^-e^- &\rightarrow e^-e^-. \end{aligned}$$

and some illustrative Beyond the SM processes:

$$\begin{aligned} e^+e^- &\rightarrow Z' \rightarrow \ell^+\ell^-,\ q\bar{q} \\ &\rightarrow KK - gravitons \rightarrow \ell^+\ell^-,\ q\bar{q},\ \gamma\gamma,\ ZZ,\ W^+W^- \\ &\rightarrow \gamma\ graviton \\ {}^{M} &\rightarrow \rho_{TC}W^+W^-. \end{aligned}$$

- Numerical simulation packages:

(1) PYTHIA:

PYTHIA is a Monte Carlo program for the generation of high-energy physics events, i.e. for the description of collisions at high energies between e^+, e^-, p and $\bar{p}$ in various combinations.
They contain theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial and final state parton showers, multiple interactions, fragmentation and decay.
http://www.thep.lu.se/ torbjorn/Pythia.html

(2) ISAJET

ISAJET is a Monte Carlo program which simulates $pp, \bar{p}p$, and ee interactions at high energies. It is based on perturbative QCD plus phenomenological models for parton and beam jet fragmentation.
http://www.phy.bnl.gov/ isajet

(3) HERWIG

HERWIG is a Monte Carlo program which simulates $pp, p\bar{p}$ interactions at high energies. It has the most sophisticated perturbative treatments, and possible NLO QCD matrix elements in parton showing.
http://hepwww.rl.ac.uk/theory/seymour/herwig/