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**Summer School on Particle Physics in the LHC Era**

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**Collider Phenomenology II**

Tao HAN

*Department of Physics University of Wisconsin-Madison  
1150 University Avenue Madison  
WI 5370-1390  
U.S.A.*

# Collider Phenomenology

— From basic knowledge  
to new physics searches

Tao Han

University of Wisconsin – Madison

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## Outline:

Lecture I: (a). Colliders and Detectors  
(b). Basics Techniques and Tools

Lecture II: (a). An  $e^+e^-$  Linear Collider  
(b). Hadron Colliders Physics

Lecture III: From Kinematics to Dynamics

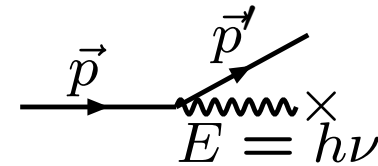
Lecture IV: Search for New Physics at Hadron Colliders

Main reference: TASI 04 Lecture notes  
hep-ph/0508097,  
plus the other related lectures in this school.

## I. Colliders and Detectors

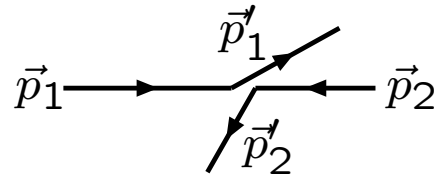
### (A). High-energy Colliders:

To study the deepest layers of matter,  
we need the probes with highest energies.



Two parameters of importance:

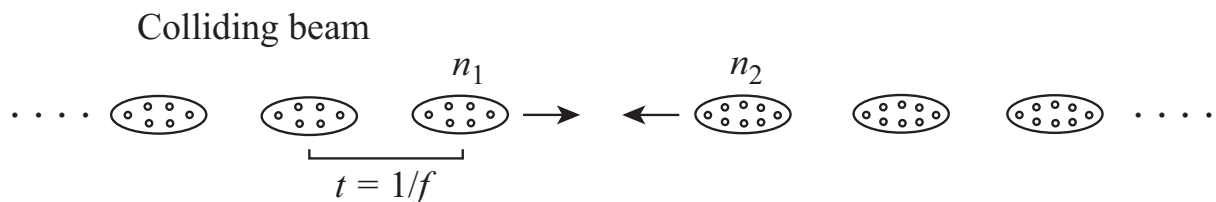
1. The energy:



$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). & \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{p}_2 = 0. \end{cases}$$

## 2. The luminosity:



$$\mathcal{L} \propto f n_1 n_2 / a,$$

( $a$  some beam transverse profile) in units of #particles/cm<sup>2</sup>/s  
 $\Rightarrow 10^{33} \text{ cm}^{-2} \text{ s}^{-1} = 1 \text{ nb}^{-1} \text{ s}^{-1} \approx 10 \text{ fb}^{-1} / \text{year}.$

Current and future high-energy colliders:

Hadron Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm <sup>-2</sup> s <sup>-1</sup> )	$\delta E/E$	$f$ (MHz)	#/bunch (10 <sup>10</sup> )	L (km)
Tevatron	1.96	$2.1 \times 10^{32}$	$9 \times 10^{-5}$	2.5	$p$ : 27, $\bar{p}$ : 7.5	6.28
LHC	14	$10^{34}$	0.01%	40	10.5	26.66

$e^+e^-$ Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm <sup>-2</sup> s <sup>-1</sup> )	$\delta E/E$	$f$ (MHz)	polar.	L (km)
ILC	0.5–1	$2.5 \times 10^{34}$	0.1%	3	80, 60%	14 – 33
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	80, 60%	33 – 53

## (B). An $e^+e^-$ Linear Collider

The collisions between  $e^-$  and  $e^+$  have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc.,  
 $\Rightarrow$  it is suitable to **create new particles** after  $e^+e^-$  annihilation.
- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame,  
 $\Rightarrow$  the **total c.m. energy** is fully exploited to reach the highest possible physics threshold.
- With well-understood beam properties,  
 $\Rightarrow$  the **scattering kinematics** is well-constrained.
- **Backgrounds low** and well-undercontrol.
- It is possible to achieve high degrees of **beam polarizations**,  
 $\Rightarrow$  chiral couplings and other asymmetries can be effectively explored.

## Disadvantages

- Large synchrotron radiation due to acceleration,

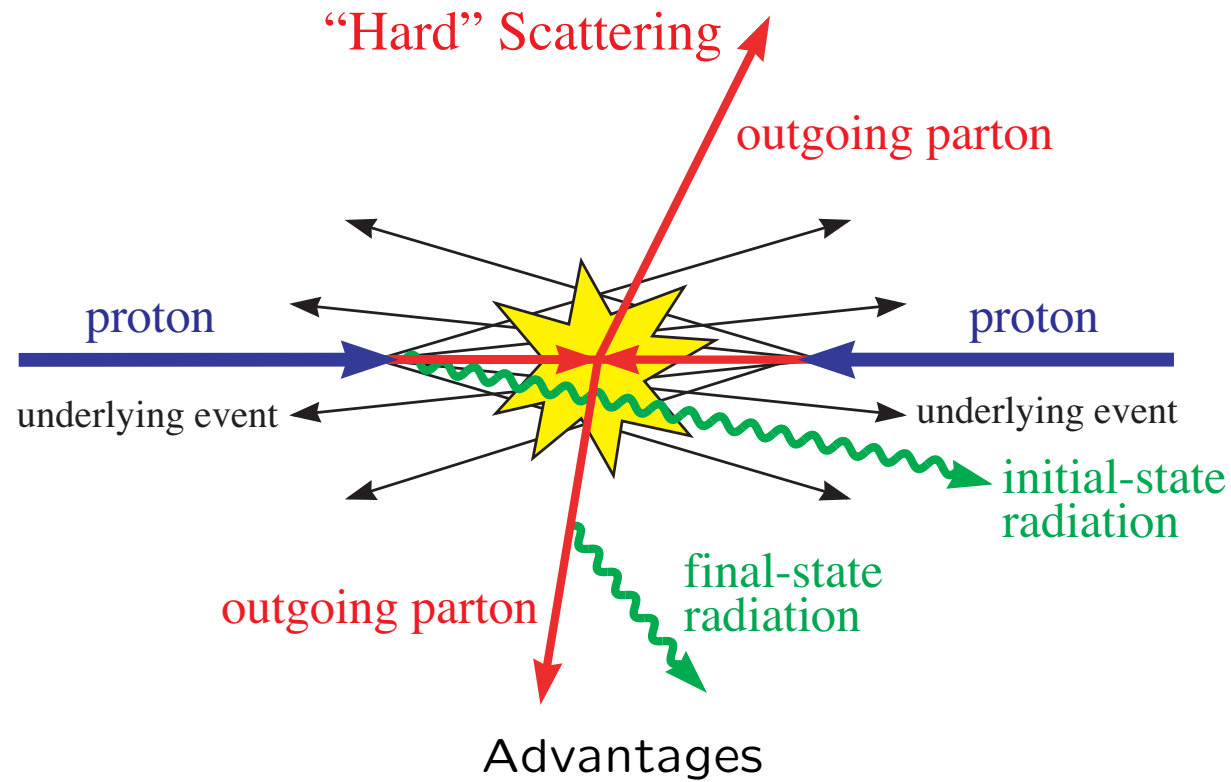
$$\Delta E \sim \frac{1}{R} \left( \frac{E}{m_e} \right)^4 .$$

Thus, a multi-hundred GeV  $e^+e^-$  collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized; beamsstrahlung severe.

## (C). Hadron Colliders

LHC: the next high-energy frontier



- Higher c.m. energy, thus higher energy threshold:

$$\sqrt{S} = 14 \text{ TeV}: \quad M_{new}^2 \sim s = x_1 x_2 S \quad \Rightarrow \quad M_{new} \sim 0.2\sqrt{S} \sim 3 \text{ TeV}.$$

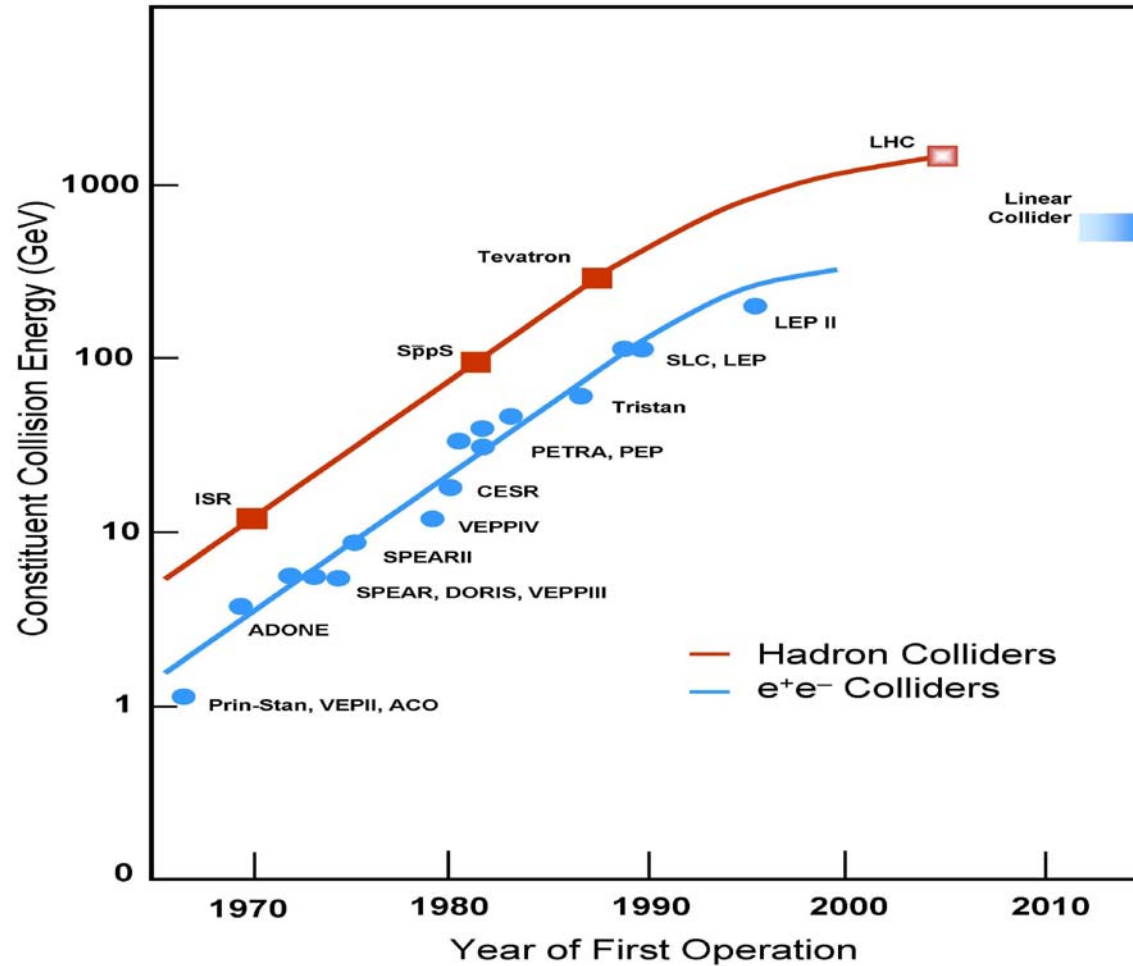
- Higher luminosity:  $10^{34}/\text{cm}^2/\text{s} \Rightarrow 100 \text{ fb}^{-1}/\text{yr}$ .  
Annual yield:  $1\text{B } W^\pm$ ;  $100\text{M } t\bar{t}$ ;  $10\text{M } W^+W^-$ ;  $1\text{M } H^0\ldots$
- Multiple (strong, electroweak) channels:  
 $q\bar{q}', gg, qg, b\bar{b} \rightarrow \text{colored}; Q = 0, \pm 1; J = 0, 1, 2 \text{ states};$   
 $WW, WZ, ZZ, \gamma\gamma \rightarrow I_W = 0, 1, 2; Q = 0, \pm 1, \pm 2; J = 0, 1, 2 \text{ states}.$

### Disadvantages

- Initial state unknown:  
colliding partons unknown on event-by-event basis;  
parton c.m. energy unknown:  $E_{cm}^2 \equiv s = x_1 x_2 S$ ;  
parton c.m. frame unknown.  
 $\Rightarrow$  largely rely on final state reconstruction.
- The large rate turns to a hostile environment:  
 $\Rightarrow$  Severe backgrounds!

Our primary job !

- *Path of the high-energy colliders:*

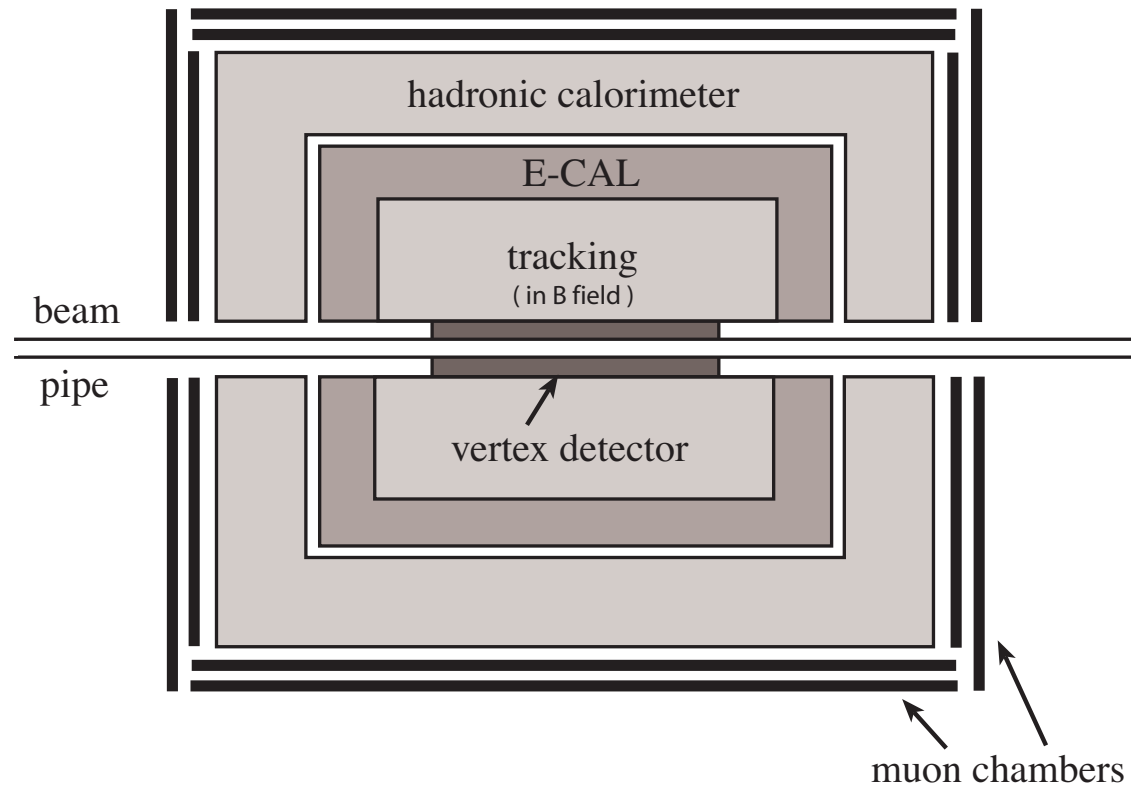


The CERN LHC will open a new era of HEP.

## (D). Particle Detection:

The detector complex:

Utilize the **strong and electromagnetic interactions** between detector materials and produced particles.



What we “see” as particles in the detector: (a few meters)

For a relativistic particle, the travel distance:

$$d = (\beta c \tau) \gamma \approx (300 \text{ } \mu\text{m}) \left( \frac{\tau}{10^{-12} \text{ s}} \right) \gamma$$

- stable particles directly “seen”:

$$p, \bar{p}, e^{\pm}, \gamma$$

- quasi-stable particles of a life-time  $\tau \geq 10^{-10} \text{ s}$  also directly “seen”:

$$n, \Lambda, K_L^0, \dots, \mu^{\pm}, \pi^{\pm}, K^{\pm} \dots$$

- a life-time  $\tau \sim 10^{-12} \text{ s}$  may display a secondary decay vertex, “vertex-tagged particles”:

$$B^{0,\pm}, D^{0,\pm}, \tau^{\pm} \dots$$

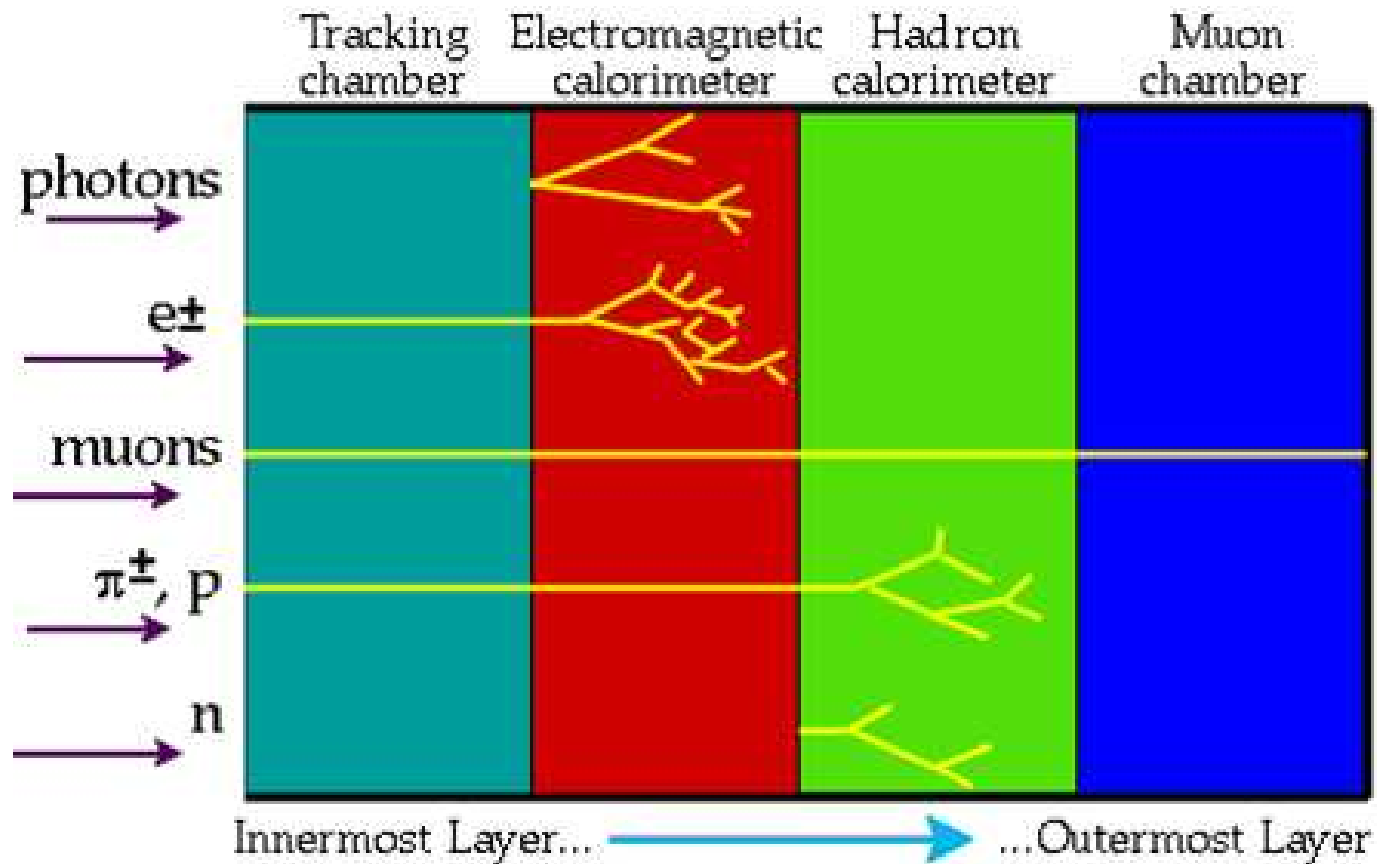
- short-lived not “directly seen”, but “reconstructable”:

$$\pi^0, \rho^{0,\pm} \dots, Z, W^{\pm}, t, H \dots$$

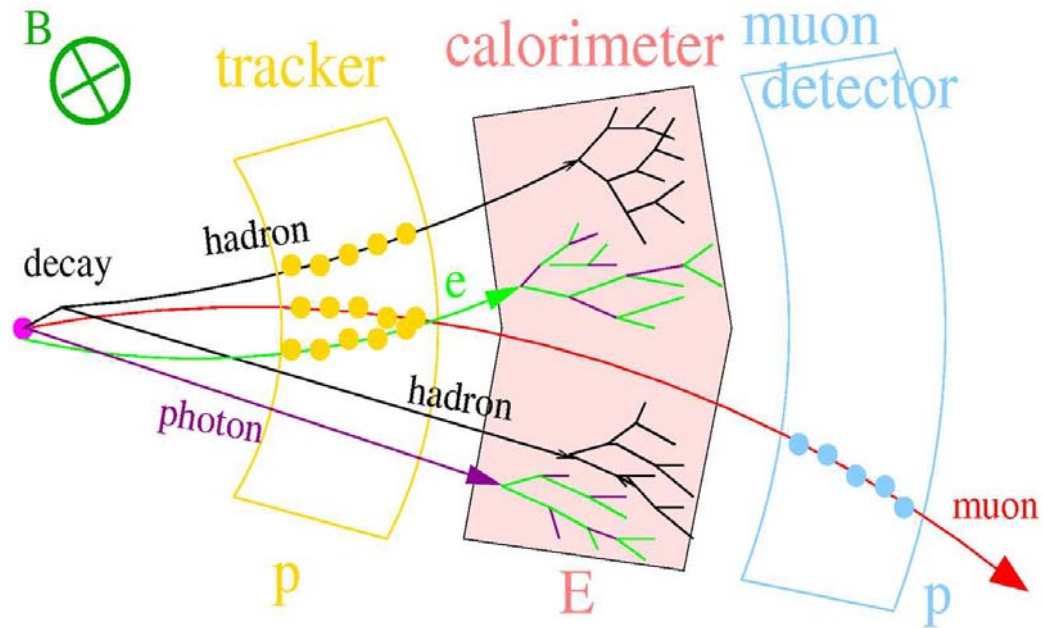
- missing particles are weakly-interacting and neutral:

$$\nu, \tilde{\chi}^0, G_{KK} \dots$$

† For **stable and quasi-stable particles** of a life-time  $\tau \geq 10^{-10} - 10^{-12}$  s, they show up as



A closer look:

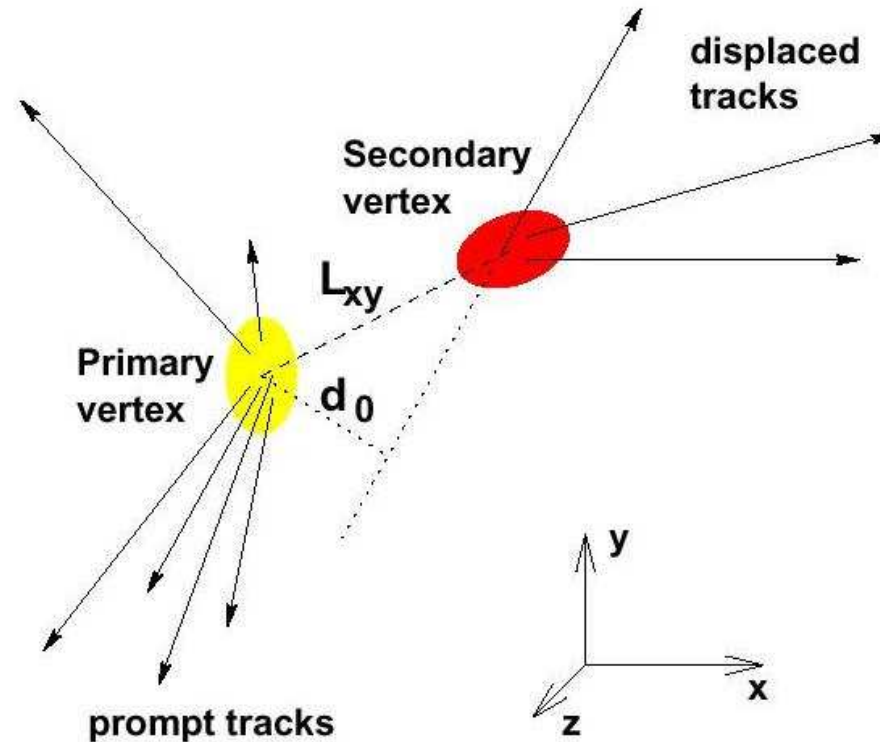


Theorists should know:

For charged tracks :  $\Delta p/p \propto p$ ,  
 typical resolution :  $\sim p/(10^4 \text{ GeV})$ .

For calorimetry :  $\Delta E/E \propto \frac{1}{\sqrt{E}}$ ,  
 typical resolution :  $\sim (5 - 80\%)/\sqrt{E}$ .

† For vertex-tagged particles  $\tau \approx 10^{-12}$  s,  
heavy flavor tagging: the secondary vertex:



Typical resolution:  $d_0 \sim 30 - 50 \mu\text{m}$  or so

⇒ Better have two (non-collinear) charged tracks for a secondary vertex;  
Or use the “impact parameter” w.r.t. the primary vertex.

For theorists: just multiply a “tagging efficiency”  $\epsilon_b \sim 40 - 60\%$  or so.

† For **short-lived particles**:  $\tau < 10^{-12}$  s or so,  
make use of final state kinematics to reconstruct the resonance.

† For **missing particles**:  
make use of energy-momentum conservation to deduce their existence.

$$p_1^i + p_2^i = \sum_f^{obs.} p_f + p_{miss}.$$

But in hadron collisions, the longitudinal momenta unknown,  
thus transverse direction only:

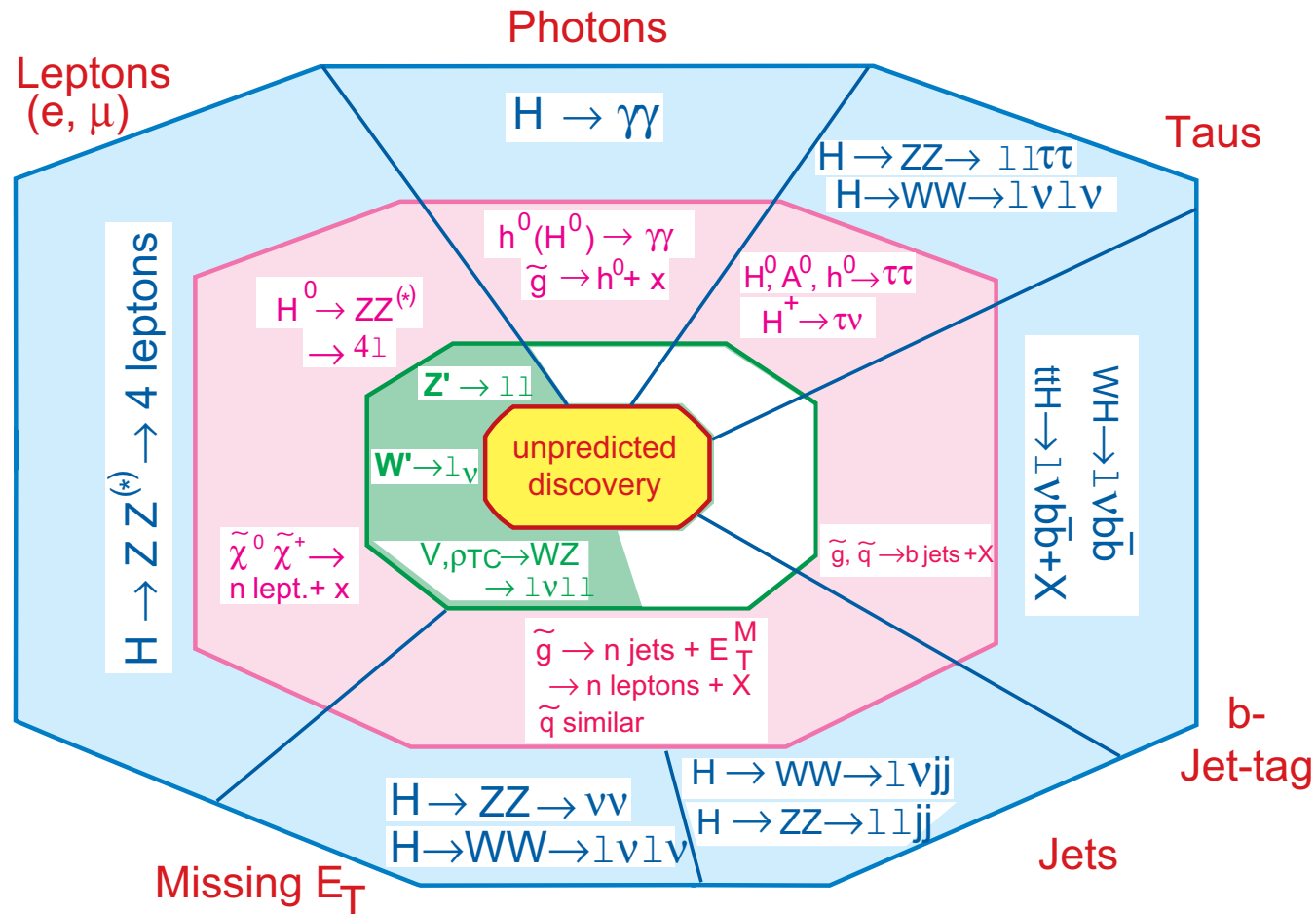
$$0 = \sum_f^{obs.} \vec{p}_{fT} + \vec{p}_{missT}.$$

often called “missing  $p_T$ ” ( $\cancel{p}_T$ ) or “missing  $E_T$ ” ( $\cancel{E}_T$ ).

What we “see” for the SM particles  
(no universality – sorry!)

Leptons	Vetexing	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	$E$	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\checkmark$	$\checkmark$	$\vec{p}$
$\tau^\pm$	$\checkmark \times$	$\checkmark$	$e^\pm$	$h^\pm; 3h^\pm$	$\mu^\pm$
$\nu_e, \nu_\mu, \nu_\tau$	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
$u, d, s$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$
$c \rightarrow D$	$\checkmark$	$\checkmark$	$e^\pm$	$h's$	$\mu^\pm$
$b \rightarrow B$	$\checkmark$	$\checkmark$	$e^\pm$	$h's$	$\mu^\pm$
$t \rightarrow bW^\pm$	$b$	$\checkmark$	$e^\pm$	$b + 2 \text{ jets}$	$\mu^\pm$
Gauge bosons					
$\gamma$	$\times$	$\times$	$E$	$\times$	$\times$
$g$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$
$W^\pm \rightarrow \ell^\pm \nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\rightarrow q\bar{q}'$	$\times$	$\checkmark$	$\checkmark$	2 jets	$\times$
$Z^0 \rightarrow \ell^+ \ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\rightarrow q\bar{q}$	$(b\bar{b})$	$\checkmark$	$\checkmark$	2 jets	$\times$

# How to search for new particles?



## Homework:

Exercise 1.1: For a  $\pi^0$ ,  $\mu^-$ , or a  $\tau^-$  respectively, calculate its decay length for  $E = 10 \text{ GeV}$ .

Exercise 1.2: An event was identified to have a  $\mu^+\mu^-$  pair, along with some missing energy. What can you say about the kinematics of the system of the missing particles? Consider both an  $e^+e^-$  and a hadron collider.

Exercise 1.3: A 120 GeV Higgs boson will have a production cross section of 20 pb at the LHC. How many events per year do you expect to produce for the Higgs boson with an instantaneous luminosity  $10^{33}/\text{cm}^2/\text{s}$ ?  
Do you expect it to be easy to observe and why?

## II. Basic Techniques and Tools for Collider Physics

### (A). Scattering cross section

For a  $2 \rightarrow n$  scattering process:

$$\sigma(ab \rightarrow 1 + 2 + \dots n) = \frac{1}{2s} \sum |\mathcal{M}|^2 dPS_n,$$

$$dPS_n \equiv (2\pi)^4 \delta^4 \left( P - \sum_{i=1}^n p_i \right) \prod_{i=1}^n \frac{1}{(2\pi)^3} \frac{d^3 \vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left( \sum_{i=1}^n p_i \right)^2,$$

where  $\sum |\mathcal{M}|^2$ : dynamics (dimension  $4 - 2n$ );

$dPS_n$ : kinematics (Lorentz invariant, dimension  $2n - 4$ .)

For a  $1 \rightarrow n$  decay process, the partial width in the rest frame:

$$\Gamma(a \rightarrow 1 + 2 + \dots n) = \frac{1}{2M_a} \sum |\mathcal{M}|^2 dPS_n.$$

$$\tau = \Gamma_{tot}^{-1} = \left( \sum_f \Gamma_f \right)^{-1}.$$

## (B). Phase space and kinematics \*

One-particle Final State  $a + b \rightarrow 1$ :

$$\begin{aligned} dPS_1 &\equiv (2\pi) \frac{d^3\vec{p}_1}{2E_1} \delta^4(P - p_1) \\ &\doteq \pi |\vec{p}_1| d\Omega_1 \delta^3(\vec{P} - \vec{p}_1) \\ &\doteq 2\pi \delta(s - m_1^2). \end{aligned}$$

where the first and second equal signs made use of the identities:

$$|\vec{p}| d|\vec{p}| = E dE, \quad \frac{d^3\vec{p}}{2E} = \int d^4p \delta(p^2 - m^2).$$

Kinematical relations:

$$\begin{aligned} \vec{P} &\equiv \vec{p}_a + \vec{p}_b = \vec{p}_1, \quad E_1^{cm} = \sqrt{s} \text{ in the c.m. frame,} \\ s &= (p_a + p_b)^2 = m_1^2. \end{aligned}$$

The “dimensionless phase-space volume” is  $s(dPS_1) = 2\pi$ .

\*E.Byckling, K. Kajantie: Particle Kinematics (1973).

Two-particle Final State  $a + b \rightarrow 1 + 2$ :

$$\begin{aligned}
 dPS_2 &\equiv \frac{1}{(2\pi)^2} \delta^4(P - p_1 - p_2) \frac{d^3\vec{p}_1}{2E_1} \frac{d^3\vec{p}_2}{2E_2} \\
 &\doteq \frac{1}{(4\pi)^2} \frac{|\vec{p}_1^{cm}|}{\sqrt{s}} d\Omega_1 = \frac{1}{(4\pi)^2} \frac{|\vec{p}_1^{cm}|}{\sqrt{s}} d\cos\theta_1 d\phi_1 \\
 &= \frac{1}{4\pi} \frac{1}{2} \lambda^{1/2} \left(1, \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) dx_1 dx_2, \\
 d\cos\theta_1 &= 2dx_1, \quad d\phi_1 = 2\pi dx_2, \quad 0 \leq x_{1,2} \leq 1,
 \end{aligned}$$

The magnitudes of the energy-momentum of the two particles are fully determined by the four-momentum conservation:

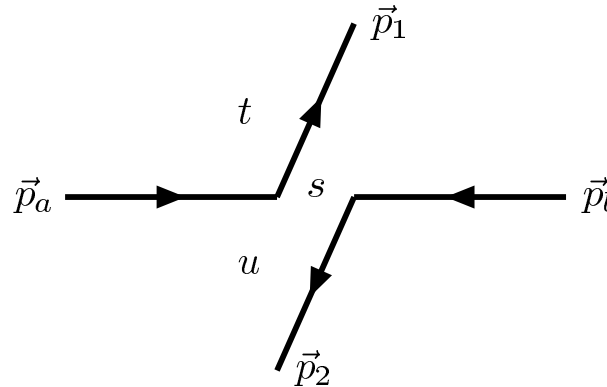
$$\begin{aligned}
 |\vec{p}_1^{cm}| = |\vec{p}_2^{cm}| &= \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}}, \quad E_1^{cm} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2^{cm} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}}, \\
 \lambda(x, y, z) &= (x - y - z)^2 - 4yz = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
 \end{aligned}$$

The phase-space volume of the two-body is scaled down with respect to that of the one-particle by a factor

$$\frac{dPS_2}{s \, dPS_1} \approx \frac{1}{(4\pi)^2}.$$

just like a “loop factor”.

Consider a  $2 \rightarrow 2$  scattering process  $p_a + p_b \rightarrow p_1 + p_2$ ,



the (Lorentz invariant) Mandelstam variables are defined as

$$\begin{aligned}
 s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\
 t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos \theta_{a1}), \\
 u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos \theta_{a2}), \\
 s + t + u &= m_a^2 + m_b^2 + m_1^2 + m_2^2.
 \end{aligned}$$

The two-body phase space can be thus written as

$$dPS_2 = \frac{1}{(4\pi)^2} \frac{dt \, d\phi_1}{s \, \lambda^{1/2} \left(1, m_a^2/s, m_b^2/s\right)}.$$

Exercise 2.1: Assume that  $m_a = m_1$  and  $m_b = m_2$ . Show that

$$\begin{aligned}t &= -2p_{cm}^2(1 - \cos \theta_{a1}^*), \\u &= -2p_{cm}^2(1 + \cos \theta_{a1}^*) + \frac{(m_1^2 - m_2^2)^2}{s},\end{aligned}$$

$p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$  is the momentum magnitude in the c.m. frame.  
Note:  $t$  is negative-definite;  $t \rightarrow 0$  in the collinear limit.

Exercise 2.2: A particle of mass  $M$  decays to two particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle is moving with a speed  $\beta_z$ ? Compare the result with your expectation for the shape change for a basket ball.

Three-particle Final State  $a + b \rightarrow 1 + 2 + 3$ :

$$\begin{aligned}
 dPS_3 &\equiv \frac{1}{(2\pi)^5} \delta^4(P - p_1 - p_2 - p_3) \frac{d^3\vec{p}_1}{2E_1} \frac{d^3\vec{p}_2}{2E_2} \frac{d^3\vec{p}_3}{2E_3} \\
 &\doteq \frac{|\vec{p}_1|^2 d|\vec{p}_1| d\Omega_1}{(2\pi)^3 2E_1} \frac{1}{(4\pi)^2} \frac{|\vec{p}_2^{(23)}|}{m_{23}} d\Omega_2 \\
 &= \frac{1}{(4\pi)^3} \lambda^{1/2} \left( 1, \frac{m_2^2}{m_{23}^2}, \frac{m_3^2}{m_{23}^2} \right) 2|\vec{p}_1| dE_1 dx_2 dx_3 dx_4 dx_5.
 \end{aligned}$$

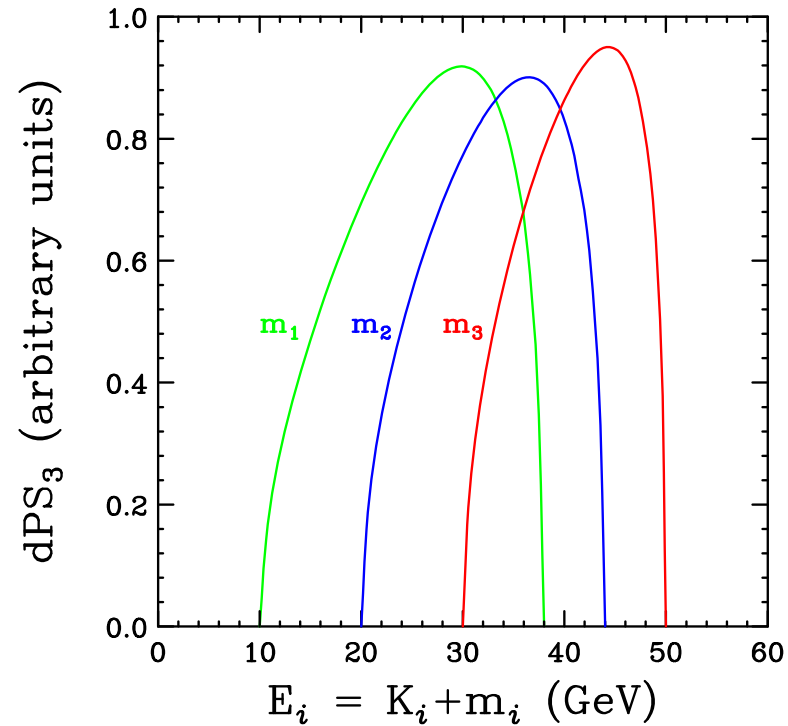
$$\begin{aligned}
 d\cos\theta_{1,2} &= 2dx_{2,4}, \quad d\phi_{1,2} = 2\pi dx_{3,5}, \quad 0 \leq x_{2,3,4,5} \leq 1, \\
 |\vec{p}_1^{cm}|^2 &= |\vec{p}_2^{cm} + \vec{p}_3^{cm}|^2 = (E_1^{cm})^2 - m_1^2, \\
 m_{23}^2 &= s - 2\sqrt{s}E_1^{cm} + m_1^2, \quad |\vec{p}_2^{23}| = |\vec{p}_3^{23}| = \frac{\lambda^{1/2}(m_{23}^2, m_2^2, m_3^2)}{2m_{23}},
 \end{aligned}$$

The particle energy spectrum is not monochromatic.

The maximum value (the end-point) for particle 1 in c.m. frame is

$$\begin{aligned}
 E_1^{max} &= \frac{s + m_1^2 - (m_2 + m_3)^2}{2\sqrt{s}}, \quad m_1 \leq E_1 \leq E_1^{max}, \\
 |\vec{p}_1^{max}| &= \frac{\lambda^{1/2}(s, m_1^2, (m_2 + m_3)^2)}{2\sqrt{s}}, \quad 0 \leq p_1 \leq p_1^{max}.
 \end{aligned}$$

With  $m_i = 10, 20, 30$ ,  $\sqrt{s} = 100$  GeV.



More intuitive to work out the end-point for the kinetic energy,  
 – recall the direct neutrino mass bound in  $\beta$ -decay:

$$K_1^{max} = E_1^{max} - m_1 = \frac{(\sqrt{s} - m_1 - m_2 - m_3)(\sqrt{s} - m_1 + m_2 + m_3)}{2\sqrt{s}}.$$

In general, the 3-body phase space boundaries are non-trivial.  
That leads to the “Dalitz Plots”.

One practically useful formula is:

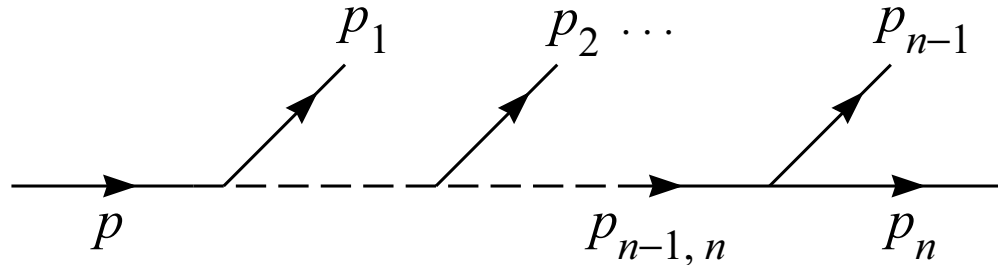
Exercise 2.3: A particle of mass  $M$  decays to 3 particles  $M \rightarrow abc$ .  
Show that the phase space element can be expressed as

$$dPS_3 = \frac{1}{2^7 \pi^3} M^2 dx_a dx_b.$$
$$x_i = \frac{2E_i}{M}, \quad (i = a, b, c, \quad \sum_i x_i = 2).$$

where the integration limits for  $m_a = m_b = m_c = 0$  are

$$0 \leq x_a \leq 1, \quad 1 - x_a \leq x_b \leq 1.$$

Recursion relation  $P \rightarrow 1 + 2 + 3 \dots + n$ :



$$dPS_n(P; p_1, \dots, p_n) = dPS_{n-1}(P; p_1, \dots, p_{n-1, n}) \\ dPS_2(p_{n-1, n}; p_{n-1}, p_n) \frac{dm_{n-1, n}^2}{2\pi}.$$

For instance,

$$dPS_3 = dPS_2(i) \frac{dm_{prop}^2}{2\pi} dPS_2(f).$$

This is generically true, but particularly useful when the diagram has an  $s$ -channel particle propagation.

## Breit-Wigner Resonance, the Narrow Width Approximation

An unstable particle of mass  $M$  and total width  $\Gamma_V$ , the propagator is

$$R(s) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}.$$

Consider an intermediate state  $V^*$

$$a \rightarrow b V^* \rightarrow b p_1 p_2.$$

By the reduction formula, the resonant integral reads

$$\int_{(m_*^{min})^2 = (m_1 + m_2)^2}^{(m_*^{max})^2 = (m_a - m_b)^2} dm_*^2.$$

Variable change

$$\tan \theta = \frac{m_*^2 - M_V^2}{\Gamma_V M_V},$$

resulting in a flat integrand over  $\theta$

$$\int_{(m_*^{min})^2}^{(m_*^{max})^2} \frac{dm_*^2}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} = \int_{\theta^{min}}^{\theta^{max}} \frac{d\theta}{\Gamma_V M_V}.$$

In the limit

$$\begin{aligned}(m_1 + m_2) + \Gamma_V &\ll M_V \ll m_a - \Gamma_V, \\ \theta^{min} &= \tan^{-1} \frac{(m_1 + m_2)^2 - M_V^2}{\Gamma_V M_V} \rightarrow -\pi, \\ \theta^{max} &= \tan^{-1} \frac{(m_a - m_b)^2 - M_V^2}{\Gamma_V M_V} \rightarrow 0,\end{aligned}$$

then the Narrow Width Approximation

$$\frac{1}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V} \delta(m_*^2 - M_V^2).$$

Exercise 2.4: Consider a three-body decay of a top quark,  $t \rightarrow bW^* \rightarrow b \ e \nu$ . Making use of the phase space recursion relation and the narrow width approximation for the intermediate  $W$  boson, show that the partial decay width of the top quark can be expressed as

$$\Gamma(t \rightarrow bW^* \rightarrow b \ e \nu) \approx \Gamma(t \rightarrow bW) \cdot BR(W \rightarrow e \nu).$$

## (C). Matrix element: The dynamics

### Properties of scattering amplitudes

- **Analyticity:** A scattering amplitude is analytical except: simple poles (corresponding to single particle states, bound states etc.); branch cuts (corresponding to thresholds).
- **Crossing symmetry:** A scattering amplitude for a  $2 \rightarrow 2$  process is symmetric among the  $s$ -,  $t$ -,  $u$ -channels.
- **Unitarity:**  
S-matrix unitarity leads to :

$$-i(T - T^\dagger) = TT^\dagger$$

Partial wave expansion for  $a + b \rightarrow 1 + 2$ :

$$\mathcal{M}(s, t) = 16\pi \sum_{J=M}^{\infty} (2J+1) a_J(s) d_{\mu\mu'}^J(\cos\theta)$$

$$a_J(s) = \frac{1}{32\pi} \int_{-1}^1 \mathcal{M}(s, t) d_{\mu\mu'}^J(\cos\theta) d\cos\theta.$$

where  $\mu = s_a - s_b$ ,  $\mu' = s_1 - s_2$ ,  $J = \max(|\mu|, |\mu'|)$ .

By Optical Theorem:  $\sigma = \frac{1}{s} \text{Im} \mathcal{M}(\theta = 0) = \frac{16\pi}{s} \sum_{J=M}^{\infty} (2J+1) |a_J(s)|^2$ .

The partial wave amplitude have the properties:

- (a). partial wave unitarity:  $\text{Im}(a_J) \geq |a_J|^2$ , or  $|\text{Re}(a_J)| \leq 1/2$ ,
- (b). kinematical thresholds:  $a_J(s) \propto \beta_i^{l_i} \beta_f^{l_f}$  ( $J = L + S$ ).

$\Rightarrow$  well-known behavior:  $\sigma \propto \beta_f^{2l_f+1}$ .

Exercise 2.6: Appreciate the properties (a) and (b) by explicitly calculating the helicity amplitudes for

$$e_L^- e_R^+ \rightarrow \gamma^* \rightarrow H^- H^+, \quad e_L^- e_{L,R}^+ \rightarrow \gamma^* \rightarrow \mu_L^- \mu_R^+, \quad H^- H^+ \rightarrow G^* \rightarrow H^- H^+.$$

## (D). Computational Tools

### Traditional “Trace” Techniques:

- \* You should be good at this — QFT course!

With algebraic symbolic manipulations:

- \* REDUCE
- \* FORM
- \* MATHEMATICA, MAPLE ...

### Helicity Techniques:

More suitable for direct numerical evaluations.

- \* Hagiwara-Zeppenfeld: best for massless particles... (NPB)
- \* CalCul Method (by T.T. Wu et al., Parke-Mangano: Phys. Report);
- \* New techniques in loop calculations  
(by Z.Bern, L.Dixon, W. Giele, N. Glover, K.Melnikov, F. Petriello ...)

Exercise 2.5: Calculate the squared matrix element for  $\overline{\Sigma} |\mathcal{M}(f\bar{f} \rightarrow ZZ)|^2$ , in terms of  $s, t, u$ , in whatever technique you like.

Calculational packages:  
check up at <http://www.ippp.dur.ac.uk/montecarlo/BSM>

- Monte Carlo packages for phase space integration:

(1) VEGAS by P. LePage: adaptive important-sampling MC

[http://en.wikipedia.org/wiki/Monte-Carlo\\_integration](http://en.wikipedia.org/wiki/Monte-Carlo_integration)

(2) SAMPLE, RAINBOW, MISER ...

- Automated software for matrix elements:

(1) REDUCE — an interactive program designed for general algebraic computations, including to evaluate Dirac algebra, an old-time program,

<http://www.uni-koeln.de/REDUCE>;

<http://reduce-algebra.com>.

(2) FORM by Jos Vermaseren: A program for large scale symbolic manipulation, evaluate fermion traces automatically, and perform loop calculations, commercially available at

<http://www.nikhef.nl/form>

(3) FeynCalc and FeynArts: Mathematica packages for algebraic calculations in elementary particle physics.

<http://www.feyncalc.org>;

<http://www.feynarts.de>

(4) MadGraph: Helicity amplitude method for tree-level matrix elements available upon request or

<http://madgraph.hep.uiuc.edu>

- Automated evaluation of cross sections:

(1) MadGraph/MadEvent and MadSUSY:

Generate Fortran codes on-line!

<http://madgraph.hep.uiuc.edu>

(2) CompHEP: computer program for calculation of elementary particle processes in Standard Model and beyond. CompHEP has a built-in numeric interpreter. So this version permits to make numeric calculation without additional Fortran/C compiler. It is convenient for more or less simple calculations.

— It allows your own construction of a Lagrangian model!

<http://theory.npi.msu.su/~kryukov>

(3) GRACE and GRACE SUSY:

<http://minami-home.kek.jp>

(4) AlpGen (M. Mangano et al.):

<http://mlm.home.cern.ch/mlm/alpgen/>

SM matrix elements

(5) SHERPA (F. Krauss et al.):

Generate Fortran codes on-line! Merging with MC generators (see next).

<http://www.sherpa-mc.de/>

(6) Pandora by M. Peskin:

C++ based package for  $e^+e^-$ , including beam effects.

<http://www-sldnt.slac.stanford.edu/nld/new/Docs/Generators/PANDORA.htm>

The program pandora is a general-purpose parton-level event generator which includes beamstrahlung, initial state radiation, and full treatment of polarization effects. (An interface to PYTHIA that produces fully hadronized events is possible.)

- Cross sections at NLO packages:

MC(at)NLO (B. Webber et al.):

<http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/>

- Numerical simulation packages:

(1) PYTHIA:

PYTHIA is a Monte Carlo program for the generation of high-energy physics events, i.e. for the description of collisions at high energies between  $e^+$ ,  $e^-$ ,  $p$  and  $\bar{p}$  in various combinations.

They contain theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial and final state parton showers, multiple interactions, fragmentation and decay.

<http://www.thep.lu.se/~torbjorn/Pythia.html>

(2) ISAJET

ISAJET is a Monte Carlo program which simulates  $pp$ ,  $\bar{p}p$ , and  $ee$  interactions at high energies. It is based on perturbative QCD plus phenomenological models for parton and beam jet fragmentation.

<http://www.phy.bnl.gov/~isajet>

(3) HERWIG

HERWIG is a Monte Carlo program which simulates  $pp$ ,  $p\bar{p}$  interactions at high energies. It has the most sophisticated perturbative treatments, and possible NLO QCD matrix elements in parton showering.

<http://hepwww.rl.ac.uk/theory/seymour/herwig/>

### III. An $e^+e^-$ Linear Collider (ILC)

#### (A.) Simple Formalism

Event rate of a reaction:

$$\begin{aligned} R(s) &= \sigma(s)\mathcal{L}, \quad \text{for constant } \mathcal{L} \\ &= \mathcal{L} \int d\tau \frac{dL}{d\tau} \sigma(\hat{s}), \quad \tau = \frac{\hat{s}}{s}. \end{aligned}$$

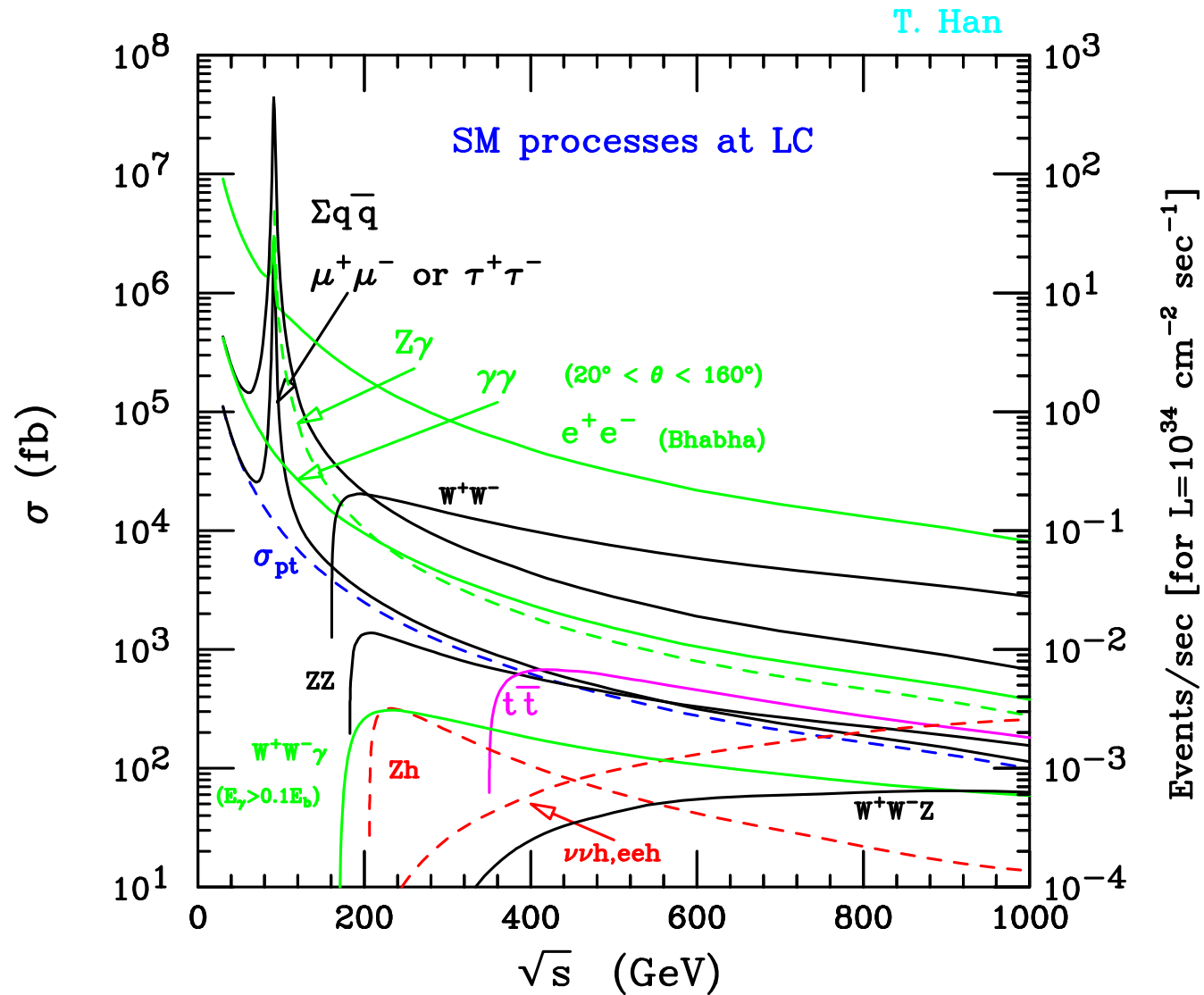
As for the differential production cross section of two-particle  $a, b$ ,

$$\frac{d\sigma(e^+e^- \rightarrow ab)}{d\cos\theta} = \frac{\beta}{32\pi s} \overline{\sum |\mathcal{M}|^2}$$

where

- $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$ , is the speed factor for the out-going particles in the c.m. frame, and  $p_{cm} = \beta\sqrt{s}/2$ ,
- $\overline{\sum |\mathcal{M}|^2}$  the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)
- unpolarized beams so that the azimuthal angle trivially integrated out,

Total cross sections and event rates for SM processes:



## (B). Resonant production: Breit-Wigner formula

$$\frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}$$

If the energy spread  $\delta\sqrt{s} \ll \Gamma_V$ , the line-shape mapped out:

$$\sigma(e^+e^- \rightarrow V \rightarrow X) = \frac{4\pi(2j+1)\Gamma(V \rightarrow e^+e^-)\Gamma(V \rightarrow X)}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2} \frac{s}{M_V^2},$$

If  $\delta\sqrt{s} \gg \Gamma_V$ , the narrow-width approximation:

$$\begin{aligned} \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2} &\rightarrow \frac{\pi}{M_V \Gamma_V} \delta(s - M_V^2), \\ \sigma(e^+e^- \rightarrow V \rightarrow X) &= \frac{4\pi^2(2j+1)\Gamma(V \rightarrow e^+e^-)BF(V \rightarrow X)}{M_V^3} \frac{dL(\hat{s} = M_V^2)}{d\tau} \end{aligned}$$

Exercise 3.1: sketch the derivation of these two formulas, assuming a Gaussian distribution for  $dL/d\tau$ .

### Away from resonance

For finite-angle scattering:

$$\sigma \sim \frac{1}{s} \quad \text{or} \quad \sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$

### (C). Fermion production:

Common processes:  $e^-e^+ \rightarrow f\bar{f}$ .

For most of the situations, the scattering matrix element can be casted into a  $V \pm A$  chiral structure of the form (sometimes with the help of Fierz transformations)

$$\mathcal{M} = \frac{e^2}{s} Q_{\alpha\beta} [\bar{v}_{e^+}(p_2) \gamma^\mu P_\alpha u_{e^-}(p_1)] [\bar{\psi}_f(q_1) \gamma_\mu P_\beta \psi'_{\bar{f}}(q_2)],$$

where  $P_\mp = (1 \mp \gamma_5)/2$  are the  $L, R$  chirality projection operators, and  $Q_{\alpha\beta}$  are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.

With this structure, the scattering matrix element squared:

$$\begin{aligned} \overline{\sum} |\mathcal{M}|^2 &= \frac{e^4}{s^2} [(|Q_{LL}|^2 + |Q_{RR}|^2) u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2) t_i t_j \\ &\quad + 2\text{Re}(Q_{LL}^* Q_{LR} + Q_{RR}^* Q_{RL}) m_f m_{\bar{f}} s], \end{aligned}$$

where  $t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2$  and  $u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2$ .

**Exercise 3.2:** Verify this formula.

## (D). Typical size of the cross sections:

- The simplest reaction

$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s}.$$

In fact,  $\sigma_{pt} \approx 100 \text{ fb}/(\sqrt{s}/\text{TeV})^2$  has become standard units to measure the size of cross sections.

- The  $Z$  resonance prominent (or other  $M_V$ ),
- At the ILC  $\sqrt{s} = 500 \text{ GeV}$ ,

$$\sigma(e^+e^- \rightarrow e^+e^-) \sim 100\sigma_{pt} \sim 40 \text{ pb}.$$

(angular cut dependent.)

$$\sigma_{pt} \sim \sigma(ZZ) \sim \sigma(t\bar{t}) \sim 400 \text{ fb};$$

$$\sigma(u, d, s) \sim 9\sigma_{pt} \sim 3.6 \text{ pb};$$

$$\sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb}.$$

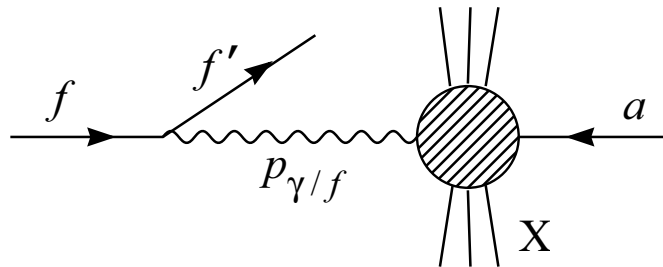
and

$$\sigma(ZH) \sim \sigma(WW \rightarrow H) \sim \sigma_{pt}/4 \sim 100 \text{ fb};$$

$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40 \text{ fb}.$$

## (E). Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \rightarrow e^+, \gamma^*e^- \rightarrow e^+e^-.$$

The dominant features are due to the result of a  $t$ -channel singularity, induced by the collinear photon splitting:

$$\sigma(e^-a \rightarrow e^-X) \approx \int dx P_{\gamma/e}(x) \sigma(\gamma a \rightarrow X).$$

The so called the effective photon approximation.

For an electron of energy  $E$ , the probability of finding a collinear photon of energy  $xE$  is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

**Exercise 3.3:** Try to derive this splitting function.

We see that:

- $m_e$  enters the log to regularize the collinear singularity;
- $1/x$  leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a “photon collider” ...

(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons,  $V = W^\pm, Z$ .

Consider a fermion  $f$  of energy  $E$ , the probability of finding a (nearly) collinear gauge boson  $V$  of energy  $xE$  and transverse momentum  $p_T$  (with respect to  $\vec{p}_f$ ) is approximated by

$$P_{V/f}^T(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$
$$P_{V/f}^L(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Although the collinear scattering would not be a good approximation until reaching very high energies  $\sqrt{s} \gg M_V$ , it is instructive to consider the qualitative features.

## (F). Beam polarization:

One of the merits for an  $e^+e^-$  linear collider is the possible high polarization for both beams.

Consider first the longitudinal polarization along the beam line direction. Denote the average  $e^\pm$  beam polarization by  $P_\pm^L$ , with  $P_\pm^L = -1$  purely left-handed and  $+1$  purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes  $\mathcal{M}_{\sigma_e-\sigma_{e^+}}$ :

$$\overline{\sum} |\mathcal{M}|^2 = \frac{1}{4} [(1 - P_-^L)(1 - P_+^L) |\mathcal{M}_{--}|^2 + (1 - P_-^L)(1 + P_+^L) |\mathcal{M}_{-+}|^2 \\ + (1 + P_-^L)(1 - P_+^L) |\mathcal{M}_{+-}|^2 + (1 + P_-^L)(1 + P_+^L) |\mathcal{M}_{++}|^2].$$

Since the electroweak interactions of the SM and beyond are chiral: Certain helicity amplitudes can be suppressed or enhanced by properly choosing the beam polarizations: e.g.,  $W^\pm$  exchange ...

Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator.

If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction,  $-1 < P_{\pm}^T < 1$ , then there will be one additional term in the limit  $m_e \rightarrow 0$ ,

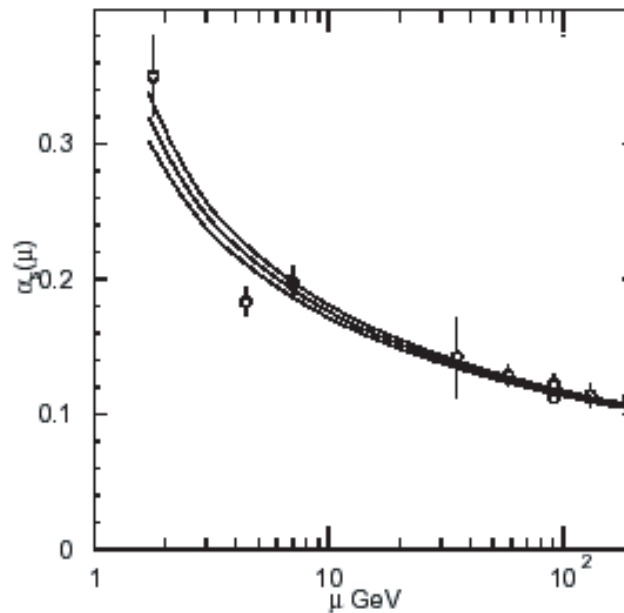
$$\frac{1}{4} 2 P_{-}^T P_{+}^T \operatorname{Re}(\mathcal{M}_{-+} \mathcal{M}_{+-}^*).$$

The transverse polarization is particularly important when the interactions produce an asymmetry in azimuthal angle, such as the effect of CP violation.

## II'. Perturbative QCD

(A). Running of the strong coupling:

$$\alpha_s(Q_R^2) = \frac{12\pi}{(33 - 2n_f) \ln \frac{Q_R^2}{\Lambda_{QCD}^2}}, \quad 11n_c - 2n_f > 0.$$



Significant implications (D. Gross, D. Politzer, F. Wilczek, Nobel Prize 2004):

- † Confinement at low energies (hadrons: the observable world);
- † Asymptotic freedom at high energies (quarks, gluons and perturbation techniques);
- † Possibility of Grand Unification; Description of the early universe.

## (B). Parton Distribution Functions (PDF)

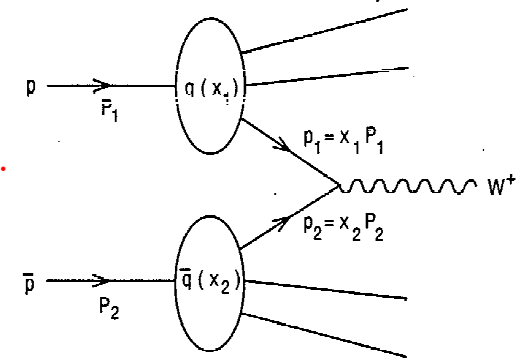
- Factorization theorem:

In high energy collisions involving a hadron, the total cross sections can be factorized into two factors:

- (1). hard subprocess of parton scattering with a large scale  $\mu^2 \gg \Lambda_{QCD}^2$ ;
- (2). “parton distribution functions” (hadronic structure with  $Q^2 < \mu^2$ .)

Observable cross sections at hadron level:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2) \hat{\sigma}_{parton}(s).$$



†  $\hat{\sigma}_{parton}(s)$  is theoretically calculated by perturbation theory (in the SM or models beyond the SM).

Ultra violet (UV) divergence (beyond leading order) is renormalized;  
 Infra-red (IR) divergence is cancelled by soft gluon emissions;  
 Co-linear divergence (massless) is factorized into PDF  
 – The essence of “factorization theorem”.

†  $P(x, Q^2)$  is the “Parton Distribution Functions” (PDF): The probability of finding a parton  $P$  with a momentum fraction  $x$  inside a proton.

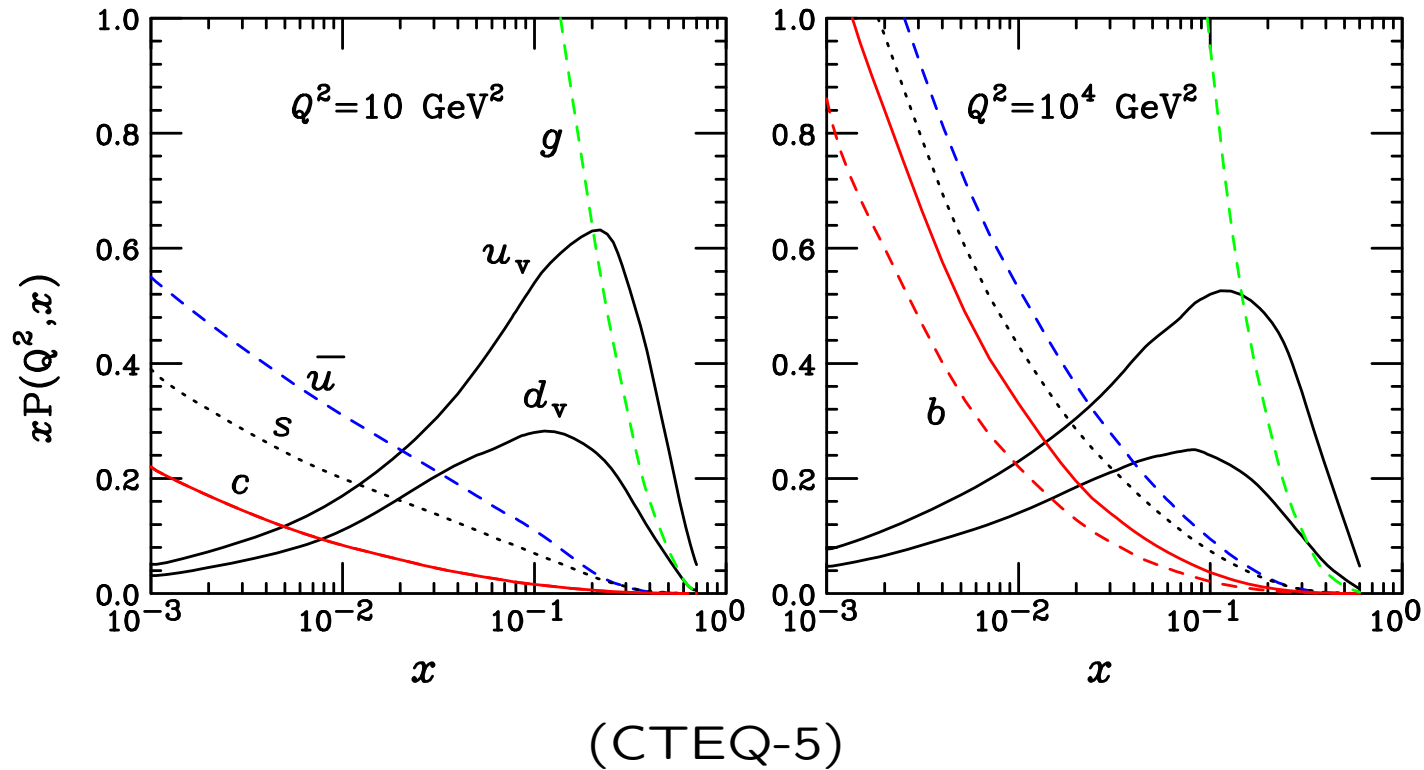
$P(x, Q^2)$  cannot be calculated from first principles, only extracted by fitting data, assuming a boundary condition at  $Q_0^2 \sim (2 \text{ GeV})^2$ .

The PDF's should match the parton-level cross section  $\hat{\sigma}_{parton}(s)$  at a given order in  $\alpha_s$ .

†  $Q^2$  is the “factorization scale”, below which it is collinear physics. It is NOT uniquely determined, leading to intrinsic uncertainty in QCD perturbation predictions. But its uncertainty is reduced with higher order calculations.

Several dedicated groups are developing PDF's:  
CTEQ (Michigan State U.); MRS (Durham U.) ... ..

Typical quark/gluon parton distribution functions:



Better understanding of the SM cross section, in particular in QCD are crucial for observing new physics as deviations from the SM.

## (C). Jets and fragmentation functions

Upon production of a colored parton (quark/gluon):

† At the scale  $\Lambda_{QCD} \sim 10^{-24}\text{s}$  or 1 fm, the parton “hadronizes (fragments)” into massive hadrons  $\pi, n, p, K \dots$

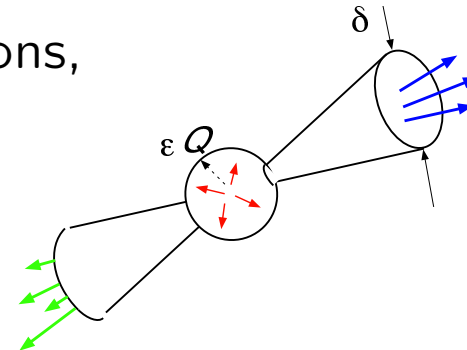
The “fragmentation function” is like the reverse of the PDF:

$$\frac{d\sigma(pp \rightarrow hX)}{dE_h} = \sum_q \int \frac{d\sigma(pp \rightarrow qX)}{dE_q} \frac{dE_q}{E_q} f_q^h(z, Q^2)$$

where  $z = E_h/E_q$ .

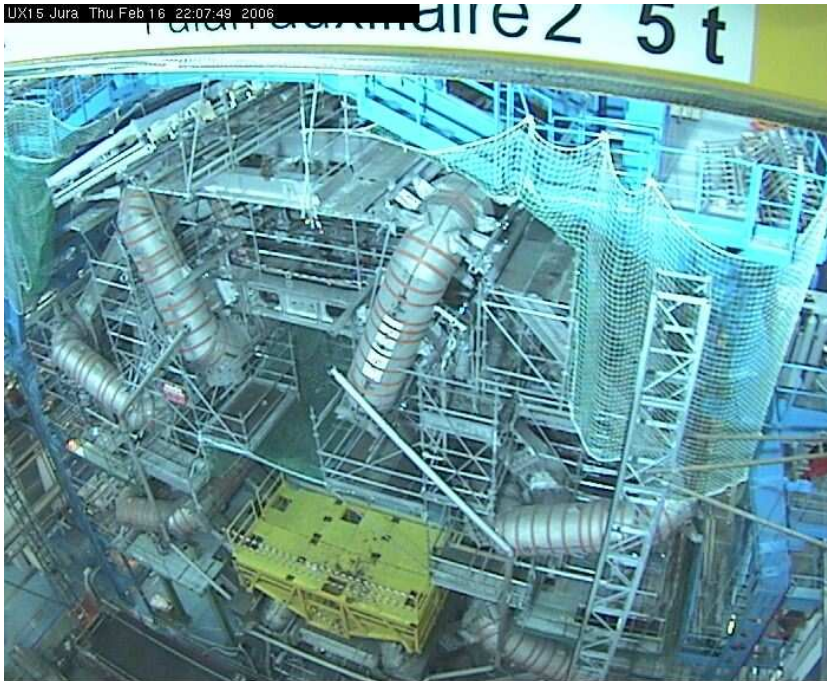
Non-perturbative and can't be calculated from first principles.

† For most of the purposes in high energy collisions, we do not need to keep track of the individual hadrons, and thus the collective and collimated hadrons form a “jet”.



### III. Hadron Collider Physics

(A). New HEP frontier: the LHC  
Major discoveries and excitement ahead ...



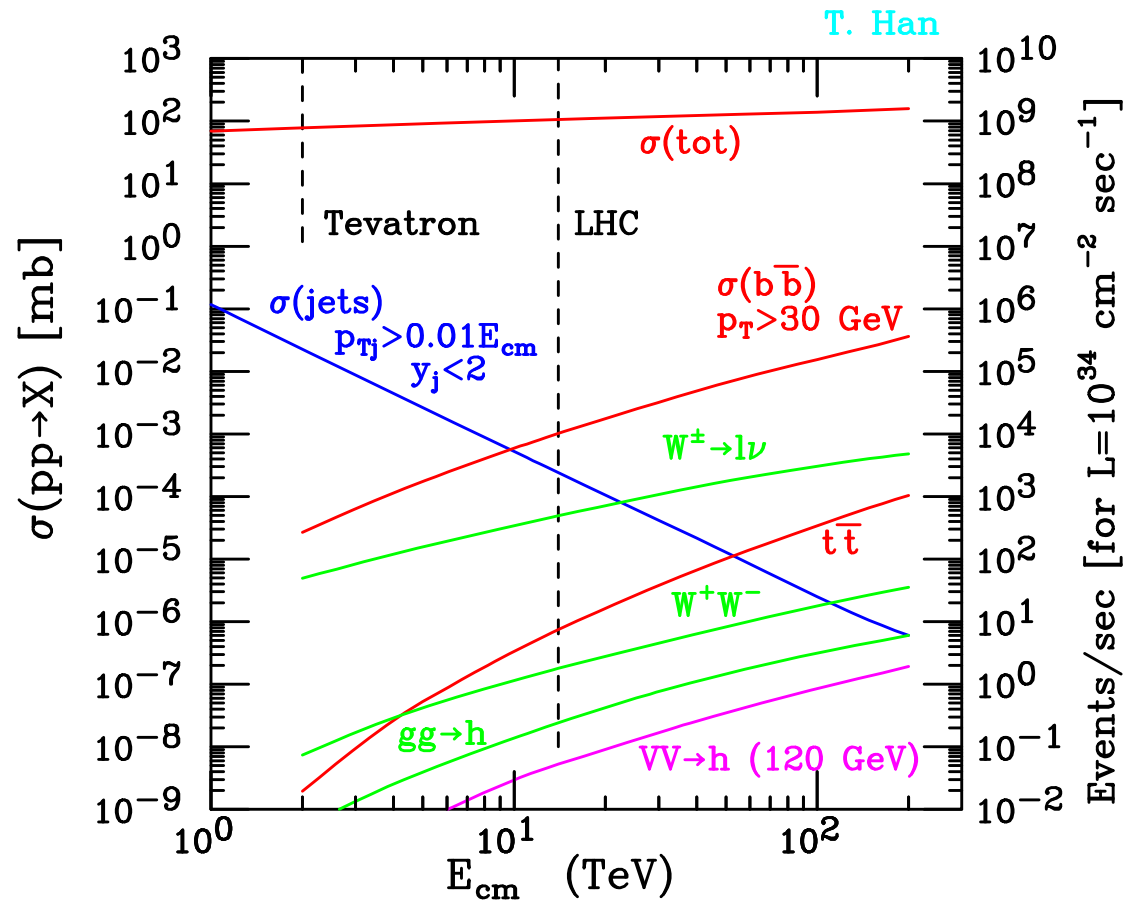
ATLAS (90m underground)



CMS

(start in the Fall of 2009.)

# LHC Event rates for various SM processes:



$$10^{34} / \text{cm}^2 / \text{s} \Rightarrow 100 \text{ fb}^{-1} / \text{yr}.$$

Annual yield # of events =  $\sigma \times L_{\text{int}}$ :

10B  $W^\pm$ ; 100M  $t\bar{t}$ ; 10M  $W^+W^-$ ; 1M  $H^0$ ...

Great potential to open a new chapter of HEP!

Theoretical challenges:

## Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.

The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi/m_\pi^2 \sim 120 \text{ mb.}$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7 \left(\frac{s}{\text{GeV}^2}\right)^{0.0808} & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2} \ln^2 \frac{s}{s_0} & \text{Froissart bound.} \end{cases}$$

(b) Perturbative hadronic cross section:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2) \hat{\sigma}_{parton}(s).$$

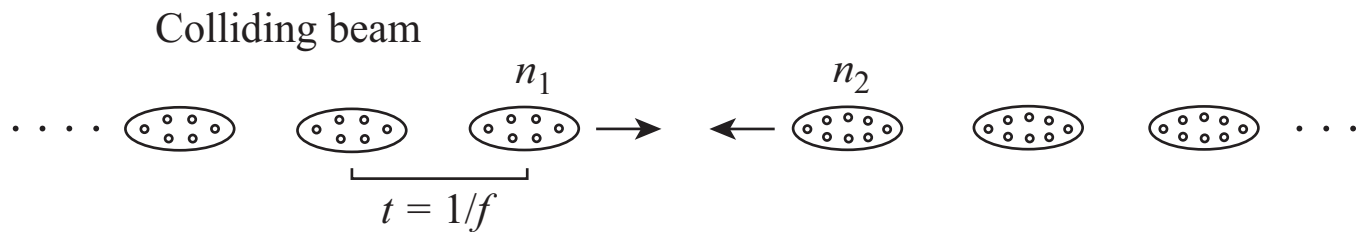
- Accurate (higher orders) partonic cross sections  $\hat{\sigma}_{parton}(s)$ .

- Parton distributions functions to the extreme (density):

$$Q^2 \sim (a \text{ few TeV})^2, \quad x \sim 10^{-3} - 10^{-6}.$$

## Experimental challenges:

- The large rate turns to a hostile environment:
  - $\approx 1$  billion event/sec: impossible read-off !
  - $\approx 1$  interesting event per 1,000,000: selection (triggering).
  - $\approx 25$  overlapping events/bunch crossing:



$\Rightarrow$  Severe backgrounds!

## Triggering thresholds:

	ATLAS	
Objects	$\eta$	$p_T$ (GeV)
$\mu$ inclusive	2.4	6 (20)
$e$ /photon inclusive	2.5	17 (26)
Two $e$ 's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
$\tau$ /hadrons	2.5	43 (65)
$\cancel{E}_T$	4.9	100
Jets+ $\cancel{E}_T$	3.2, 4.9	50,50 (100,100)

$$(\eta = 2.5 \Rightarrow 10^\circ; \quad \eta = 5 \Rightarrow 0.8^\circ.)$$

With optimal triggering and kinematical selections:

$$p_T \geq 30 - 100 \text{ GeV}, \quad |\eta| \leq 3 - 5; \quad \cancel{E}_T \geq 100 \text{ GeV}.$$

## (B). Special kinematics for hadron colliders

Hadron momenta:  $P_A = (E_A, 0, 0, p_A)$ ,  $P_B = (E_A, 0, 0, -p_A)$ ,

The parton momenta:  $p_1 = x_1 P_A$ ,  $p_2 = x_2 P_B$ .

Then the parton c.m. frame moves randomly, even by event:

$$\beta_{cm} = \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :}$$
$$y_{cm} = \frac{1}{2} \ln \frac{1 + \beta_{cm}}{1 - \beta_{cm}} = \frac{1}{2} \ln \frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty).$$

The four-momentum vector transforms as

$$\begin{pmatrix} E' \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma \beta_{cm} \\ -\gamma \beta_{cm} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}$$
$$= \begin{pmatrix} \cosh y_{cm} & -\sinh y_{cm} \\ -\sinh y_{cm} & \cosh y_{cm} \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}.$$

This is often called the “boost”.

One wishes to design final-state kinematics **invariant under the boost**:

For a four-momentum  $p \equiv p^\mu = (E, \vec{p})$ ,

$$\begin{aligned} E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \\ p^\mu &= (E_T \cosh y, \quad p_T \sin \phi, \quad p_T \cos \phi, \quad E_T \sinh y), \\ \frac{d^3 \vec{p}}{E} &= p_T dp_T d\phi \, dy = E_T dE_T d\phi \, dy. \end{aligned}$$

Due to random boost between Lab-frame/c.m. frame event-by-event,

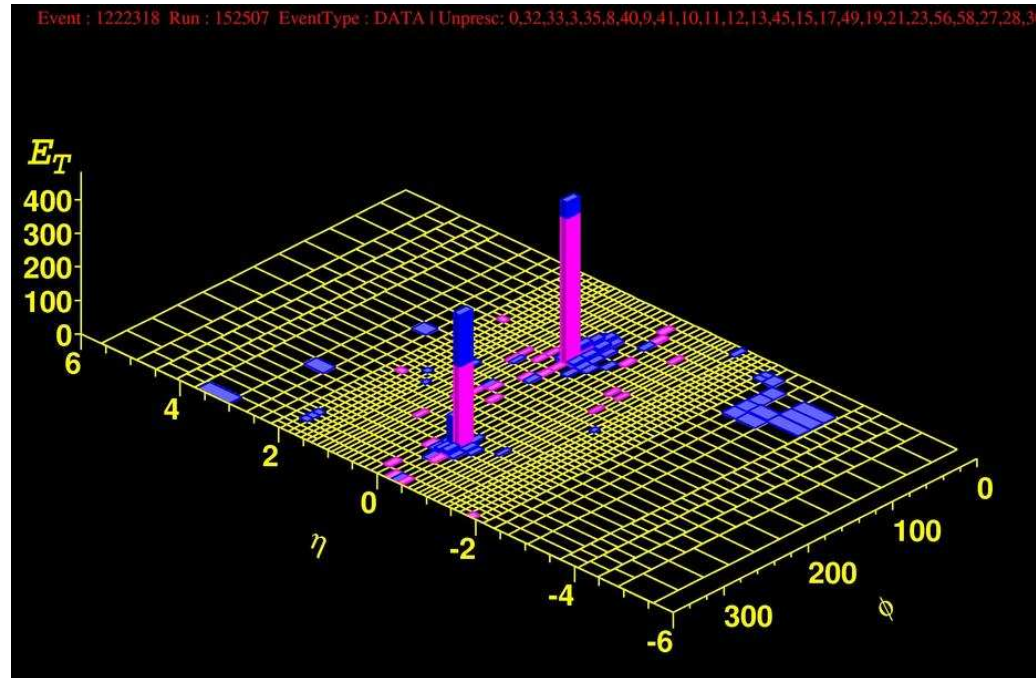
$$y' = \frac{1}{2} \ln \frac{E' + p'_z}{E' - p'_z} = \frac{1}{2} \ln \frac{(1 - \beta_{cm})(E + p_z)}{(1 + \beta_{cm})(E - p_z)} = y - y_{cm}.$$

In the massless limit, rapidity  $\rightarrow$  pseudo-rapidity:

$$y \rightarrow \eta = \frac{1}{2} \ln \frac{1 + \cos \theta}{1 - \cos \theta} = \ln \cot \frac{\theta}{2}.$$

**Exercise 4.1:** Verify all the above equations.

The “Lego” plot:



A CDF di-jet event on a lego plot in the  $\eta - \phi$  plane.

$\phi, \Delta y = y_2 - y_1$  is boost-invariant.

Thus the “separation” between two particles in an event

$\Delta R = \sqrt{\Delta\phi^2 + \Delta y^2}$  is boost-invariant,  
and lead to the “cone definition” of a jet.

(C). Before considering the LHC,

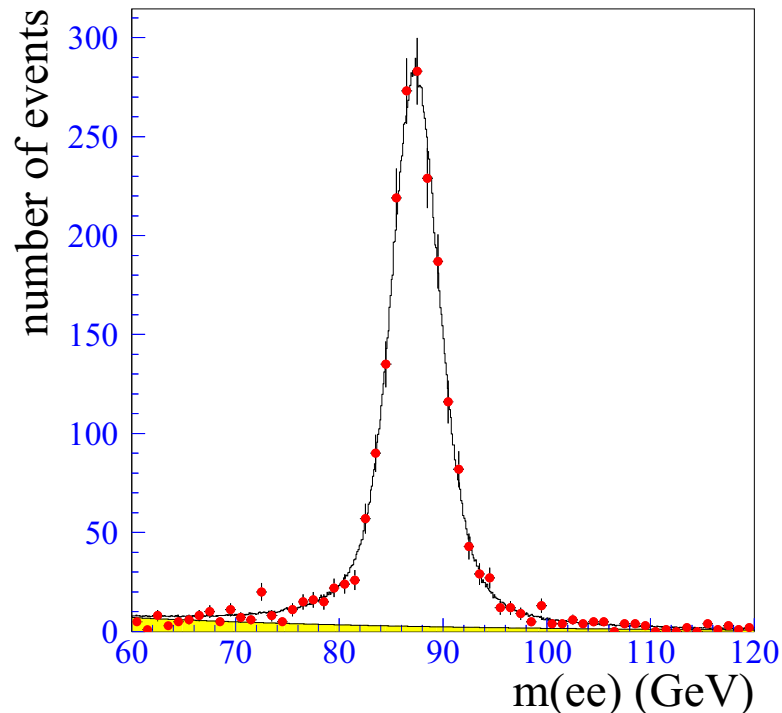
Appreciate the beautiful results from the Tevatron!

At the Tevatron Run II:

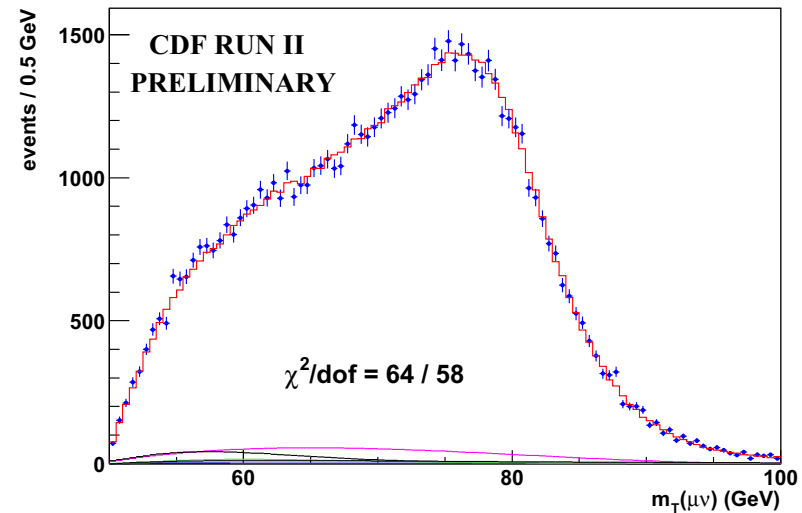
Peak luminosity record high  $\approx 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ ;

Integrated luminosity over  $1 \text{ fb}^{-1}$ , leading the HEP frontier.

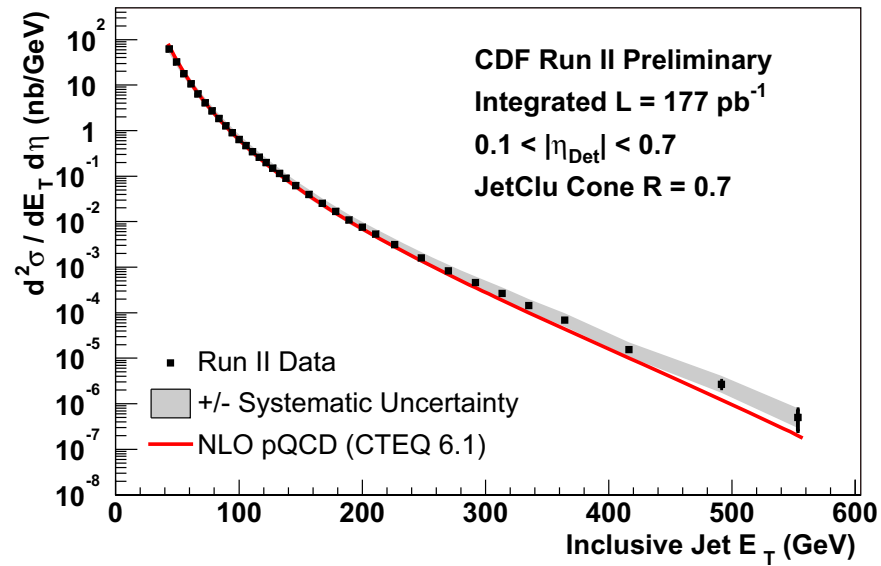
D0  $Z \rightarrow e^+e^-$



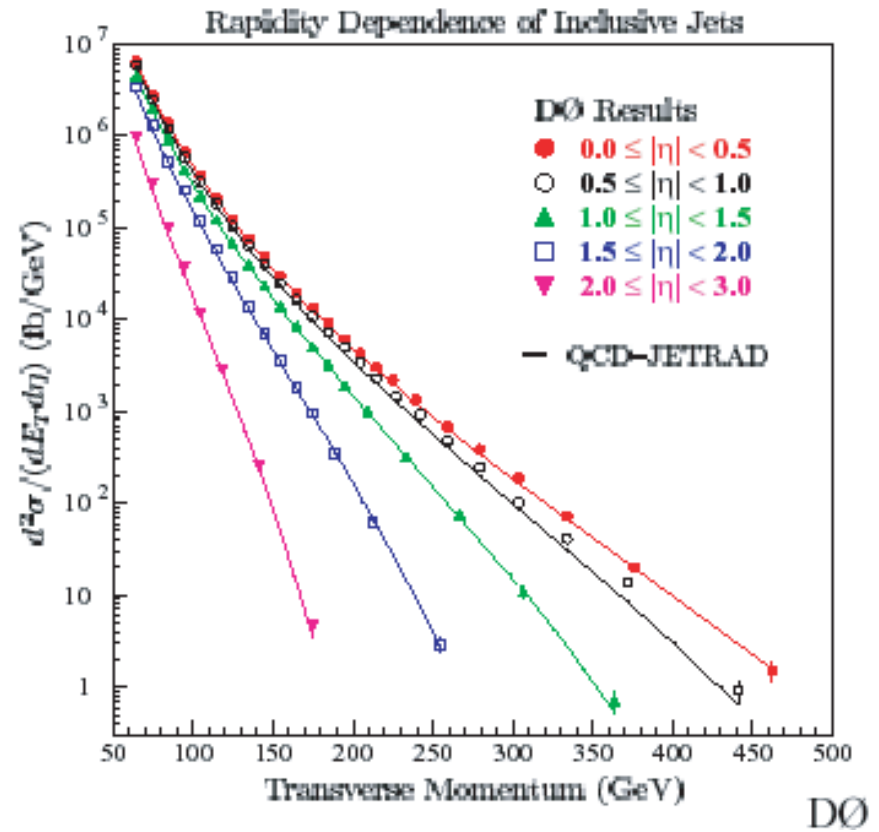
CDF  $W \rightarrow \mu\nu$



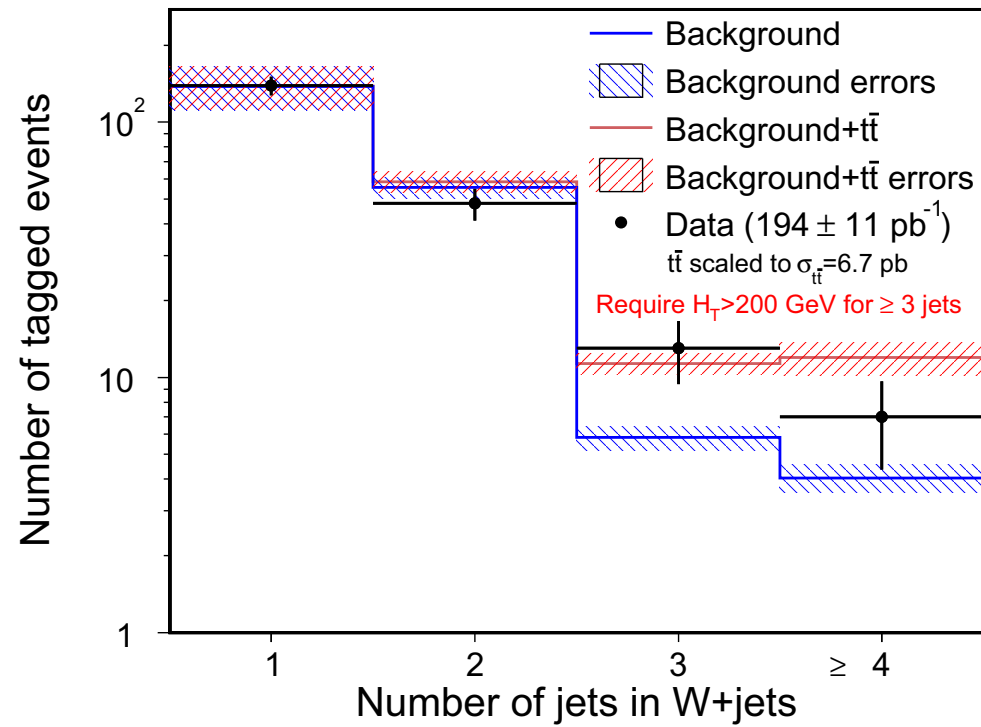
## CDF 1-jet inclusive



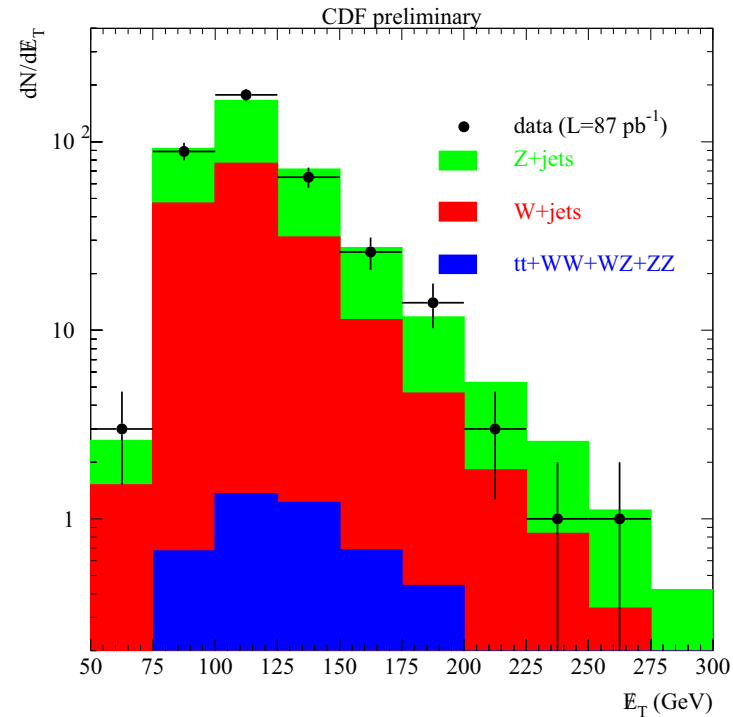
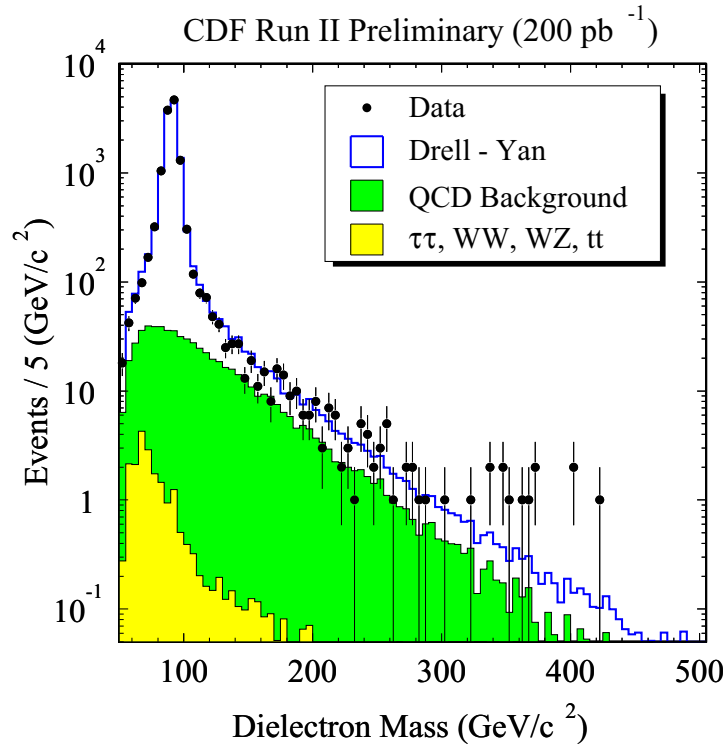
## D0 1-jet in rapidity ranges



## CDF $W$ +jets sample and top-quark events



CDF  $M_{ee}$  and  $E_T$



LHC – first (crucial) steps:  
Re-discover the Standard Model.

- $Z/\gamma^*$ ,  $W^\pm$  Drell-Yan rate and spectrum;
- jet inclusive to  $p_T^j \sim 300 - 500$  GeV;
- near thresholds of  $WW$ ,  $WZ$ ,  $ZZ$ ,  $W\gamma$ ,  $\gamma$ +jets.

## VI. From Kinematics to Dynamics

(A). Characteristic observables:  
Crucial for uncovering new dynamics.

Selective experimental events

$\Rightarrow$  Characteristic kinematical observables  
(spatial, time, momenta phase space)

$\Rightarrow$  Dynamical parameters  
(masses, couplings)

Energy momentum observables  $\Rightarrow$  mass parameters

Angular observables  $\Rightarrow$  nature of couplings;

Production rates, decay branchings/lifetimes  $\Rightarrow$  interaction strengths.

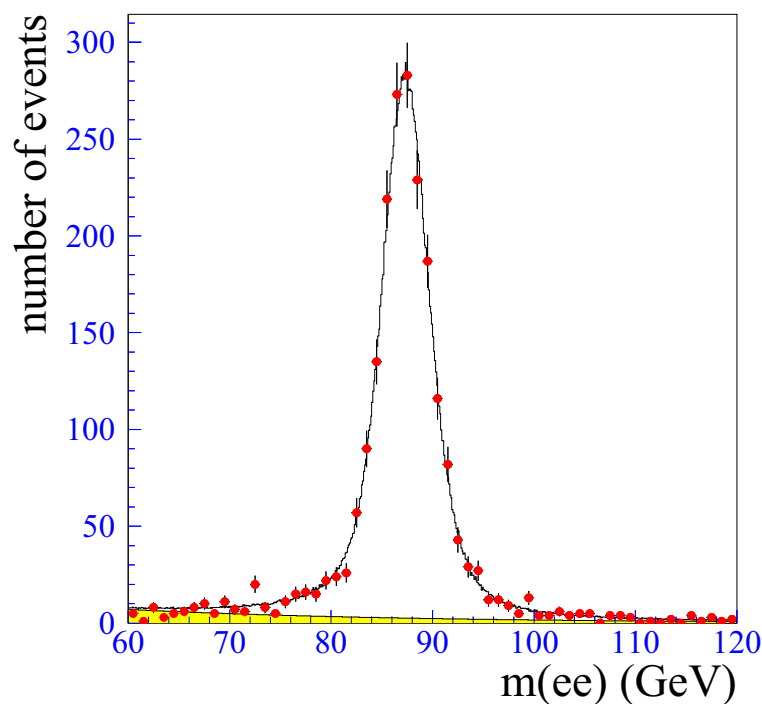
## (B). Kinematical features:

(a).  $s$ -channel singularity: bump search we do best.

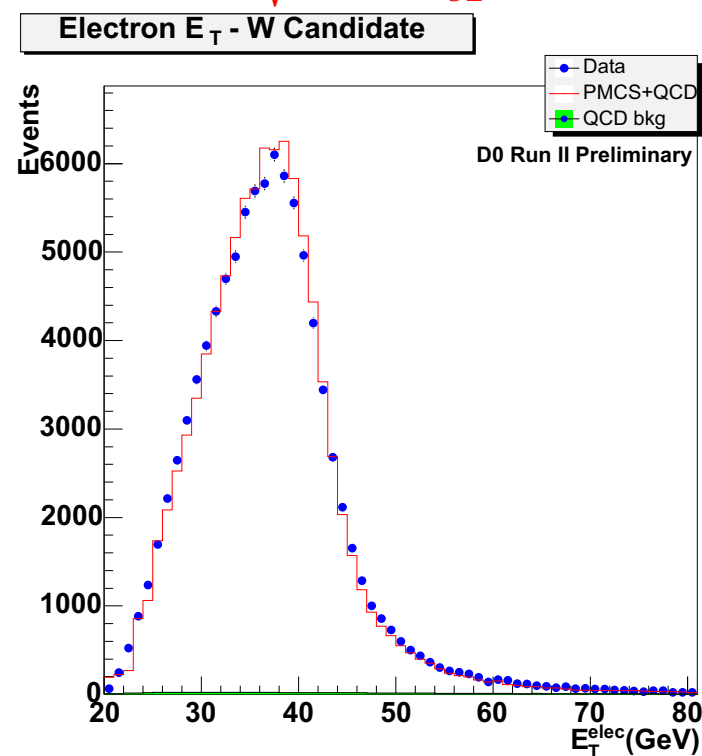
- invariant mass of two-body  $R \rightarrow ab$ :  $m_{ab}^2 = (p_a + p_b)^2 = M_R^2$ .

combined with the two-body Jacobian peak in transverse momentum:

$$\frac{d\hat{\sigma}}{dm_{ee}^2 dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{1}{m_{ee}^2 \sqrt{1 - 4p_{eT}^2/m_{ee}^2}}$$



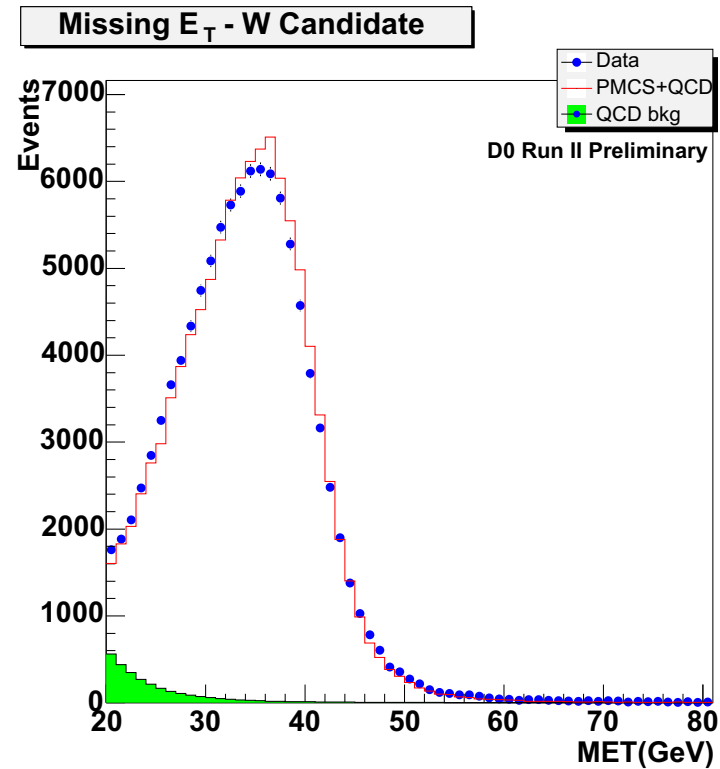
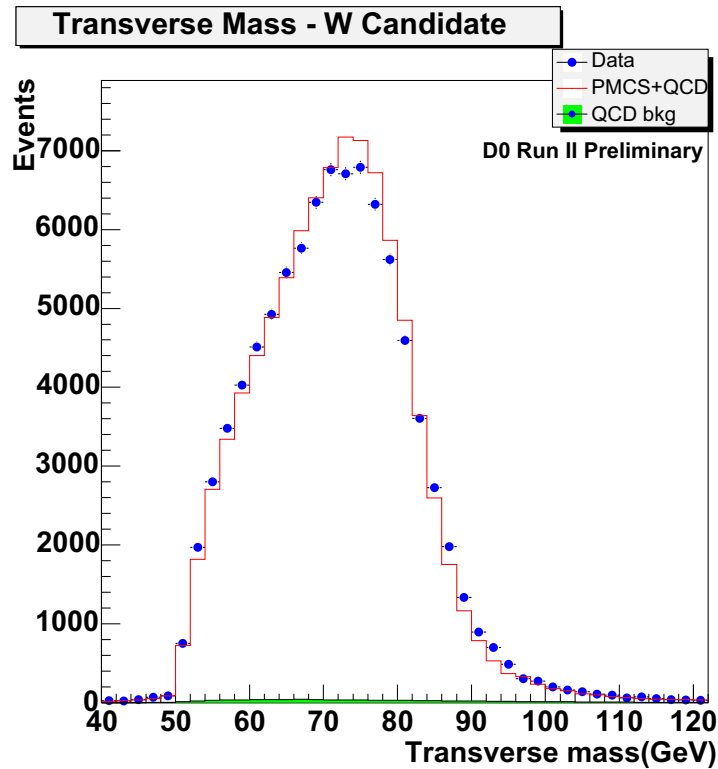
$Z \rightarrow e^+e^-$



$W \rightarrow e\nu$

- “transverse” mass of two-body  $W^- \rightarrow e^- \bar{\nu}_e$  :

$$\begin{aligned}
 m_{e\nu T}^2 &= (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \\
 &= 2E_{eT}E_T^{miss}(1 - \cos \phi) \leq M_W^2.
 \end{aligned}$$



If  $p_{WT} = 0$ , then  $m_{e\nu T} = 2E_{eT} = 2E_T^{miss}$ .

Exercise 5.1: For a two-body final state kinematics, show that

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1-4p_{eT}^2/s}} \frac{d\hat{\sigma}}{d\cos\theta^*}.$$

where  $p_{eT} = p_e \sin\theta^*$  is the transverse momentum and  $\theta^*$  is the polar angle in the c.m. frame. Comment on the apparent singularity at  $p_{eT}^2 = s/4$ .

Exercise 5.2: Show that for an on-shell decay  $W^- \rightarrow e^- \bar{\nu}_e$ :

$$m_{e\nu}^2 \equiv (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \leq M_W^2.$$

Exercise 5.3: Show that if  $W/Z$  has some transverse motion,  $\delta P_V$ , then:

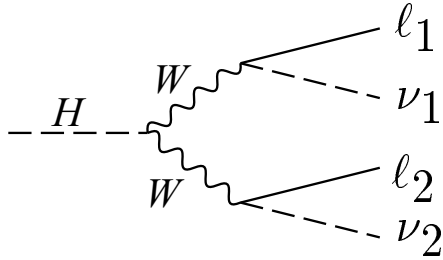
$$\begin{aligned} p'_{eT} &\sim p_{eT} [1 + \delta P_V/E_V], \\ m_{e\nu}^{\prime 2} &\sim m_{e\nu}^2 [1 + (\delta P_V/E_V)^2], \\ m_{ee}^2 &= m_{ee}^2. \end{aligned}$$

- $H^0 \rightarrow W^+W^- \rightarrow j_1j_2 e^- \bar{\nu}_e$  :

cluster transverse mass (I):

$$\begin{aligned}
 m_{WW\ T}^2 &= (E_{W_1T} + E_{W_2T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{miss})^2 \\
 &= (\sqrt{p_{jjT}^2 + M_W^2} + \sqrt{p_{e\nu T}^2 + M_W^2})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{miss})^2 \leq M_H^2.
 \end{aligned}$$

where  $\vec{p}_T^{miss} \equiv \vec{\not{p}}_T = -\sum_{obs} \vec{p}_T^{obs}$ .



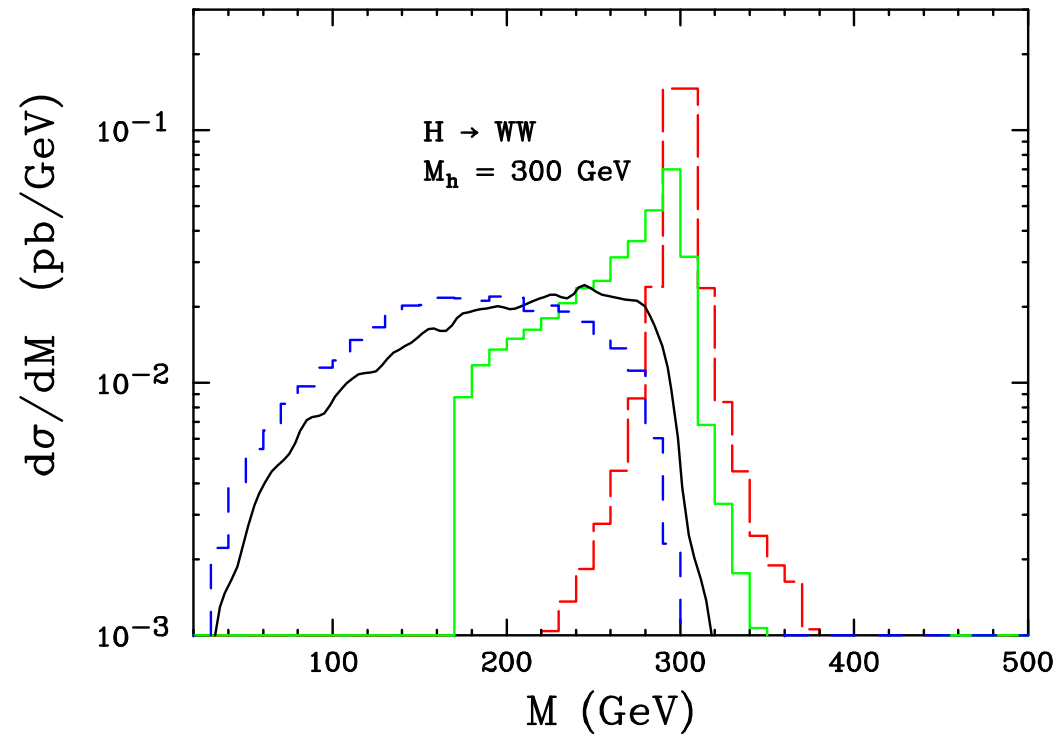
- $H^0 \rightarrow W^+W^- \rightarrow e^+ \nu_e e^- \bar{\nu}_e$  :

“effective” transverse mass:

$$\begin{aligned}
 m_{eff\ T}^2 &= (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{miss})^2 \\
 m_{eff\ T} &\approx E_{e1T} + E_{e2T} + E_T^{miss}
 \end{aligned}$$

cluster transverse mass (II):

$$\begin{aligned}
 m_{WW\ C}^2 &= \left( \sqrt{p_{T,\ell\ell}^2 + M_{\ell\ell}^2} + \not{p}_T \right)^2 - (\vec{p}_{T,\ell\ell} + \vec{\not{p}}_T)^2 \\
 m_{WW\ C} &\approx \sqrt{p_{T,\ell\ell}^2 + M_{\ell\ell}^2} + \not{p}_T
 \end{aligned}$$



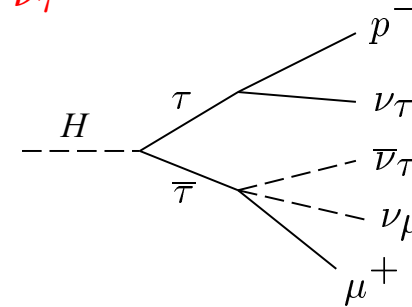
$M_{WW}$  invariant mass ( $WW$  fully reconstructable): - - - - -  
 $M_{WW, T}$  transverse mass (one missing particle  $\nu$ ): —————  
 $M_{eff, T}$  effective trans. mass (two missing particles): - - - - -  
 $M_{WW, C}$  cluster trans. mass (two missing particles): —————

YOU design an optimal variable/observable for the search.

- cluster transverse mass (III):

$$H^0 \rightarrow \tau^+ \tau^- \rightarrow \mu^+ \bar{\nu}_\tau \nu_\mu, \quad \rho^- \nu_\tau$$

A lot more complicated with (many) more  $\nu's$ ?



Not really!

$\tau^+ \tau^-$  ultra-relativistic, the final states from a  $\tau$  decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau / E_\tau = 2m_\tau / m_H \approx 1.5^\circ \quad (m_H = 120 \text{ GeV}).$$

We can thus take

$$\vec{p}_{\tau^+} = \vec{p}_{\mu^+} + \vec{p}_+^{\nu's}, \quad \vec{p}_+^{\nu's} \approx c_+ \vec{p}_{\mu^+}.$$

$$\vec{p}_{\tau^-} = \vec{p}_{\rho^-} + \vec{p}_-^{\nu's}, \quad \vec{p}_-^{\nu's} \approx c_- \vec{p}_{\rho^-}.$$

where  $c_\pm$  are proportionality constants, to be determined.

This is applicable to any decays of fast-moving particles, like

$$T \rightarrow Wb \rightarrow \ell\nu, \quad b.$$

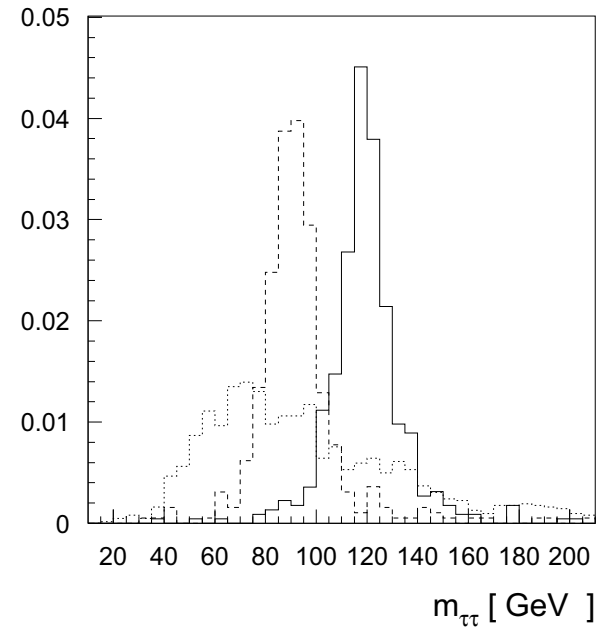
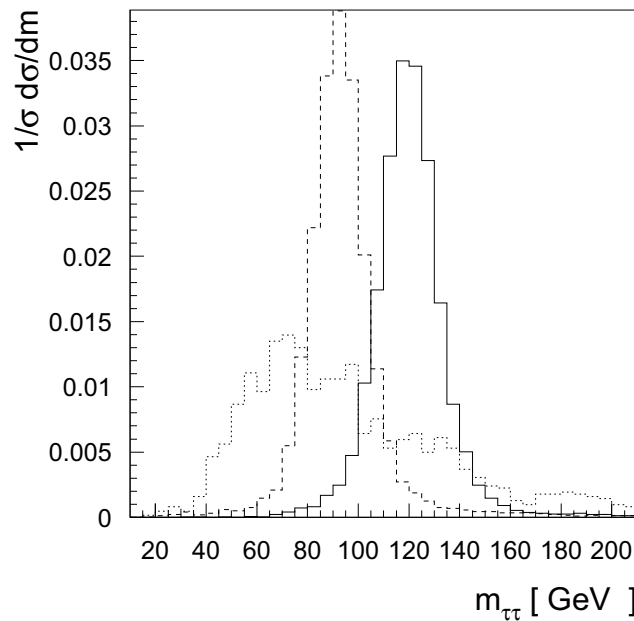
Experimental measurements:  $p_{\rho^-}$ ,  $p_{\mu^+}$ ,  $\not{p}_T$ :

$$\begin{aligned} c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x &= (\not{p}_T)_x, \\ c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y &= (\not{p}_T)_y. \end{aligned}$$

Unique solutions for  $c_{\pm}$  exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

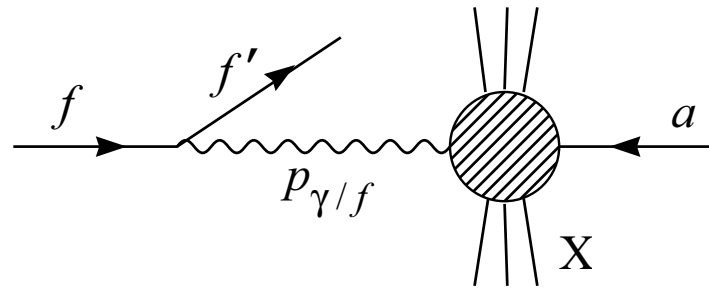
Physically, the  $\tau^+$  and  $\tau^-$  should form a finite angle,  
or the Higgs should have a non-zero transverse momentum.



(b).  $t$ -channel singularity: splitting.

- Gauge boson radiation off a fermion:

The familiar Weizsäcker-Williams approximation



$$\sigma(fa \rightarrow f'X) \approx \int dx \, dp_T^2 \, P_{\gamma/f}(x, p_T^2) \, \sigma(\gamma a \rightarrow X),$$

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{E^2}{m_e^2},$$

† The kernel is the same as  $q \rightarrow qg^*$   $\Rightarrow$  generic for parton splitting;

† The high energy enhancement  $\ln(E/m_e)$  reflects the collinear behavior.

- Generalize to massive gauge bosons:

$$P_{V/f}^T(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P_{V/f}^L(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Special kinematics for massive gauge boson fusion processes:

For the accompanying jets,

At low- $p_{jT}$ ,

$$\left. \begin{aligned} p_{jT}^2 &\approx (1-x)M_V^2 \\ E_j &\sim (1-x)E_q \end{aligned} \right\} \text{forward jet tagging}$$

At high- $p_{jT}$ ,

$$\left. \begin{aligned} \frac{d\sigma(V_T)}{dp_{jT}^2} &\propto 1/p_{jT}^2 \\ \frac{d\sigma(V_L)}{dp_{jT}^2} &\propto 1/p_{jT}^4 \end{aligned} \right\} \text{central jet vetoing}$$

has become important tools for Higgs searches, single-top signal etc.

### (C). Charge forward-backward asymmetry $A_{FB}$ :

The coupling vertex of a vector boson  $V_\mu$  to an arbitrary fermion pair  $f$

$$i \sum_{\tau}^{L,R} g_{\tau}^f \gamma^{\mu} P_{\tau} \quad \rightarrow \quad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

$$A_{FB}^{i,f} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4} \mathcal{A}_i \mathcal{A}_f,$$
$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}.$$

where  $N_F$  ( $N_B$ ) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion  $\vec{p}_i$ .

At hadronic level:

$$A_{FB}^{\text{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \left( P_q(x_1) P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1) P_q(x_2) \right) \text{sign}(x_1 - x_2)}{\int dx_1 \sum_q \left( P_q(x_1) P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1) P_q(x_2) \right)}.$$

Perfectly fine for  $Z/Z'$ -type:

In  $p\bar{p}$  collisions,  $\vec{p}_{proton}$  is the direction of  $\vec{p}_{quark}$ .

In  $pp$  collisions, however, what is the direction of  $\vec{p}_{quark}$ ?

It is the boost-direction of  $\ell^+ \ell^-$ .

How about  $W^\pm/W'^\pm$ -type?

In  $p\bar{p}$  collisions,  $\vec{p}_{proton}$  is the direction of  $\vec{p}_{quark}$ , AND:

$$u(\Leftarrow) \bar{d}(\Leftarrow) \rightarrow W^+ \rightarrow \ell^+(\Leftarrow) \nu(\Leftarrow).$$

So (known):  $\ell^+$  ( $\ell^-$ ) goes along the direction with  $\bar{q}$  ( $q$ )

$\Rightarrow$  OK at the Tevatron.

But don't have a good idea for  $pp$  collisions yet ...

## (D). CP asymmetries $A_{CP}$ :

To non-ambiguously identify  $CP$ -violation effects,  
one must rely on **CP-odd variables**.

Definition:  $A_{CP}$  vanishes if **CP-violation interactions** do not exist  
(for the relevant particles involved).

This is meant to be in contrast to an observable:  
that'd be modified by the presence of CP-violation,  
but is not zero when CP-violation is absent.

$$\text{e.g. } M_{(\chi^\pm \chi^0)}, \quad \sigma(H^0, A^0), \dots$$

Two ways:

a). Compare the rates between a process and its **CP-conjugate process**:

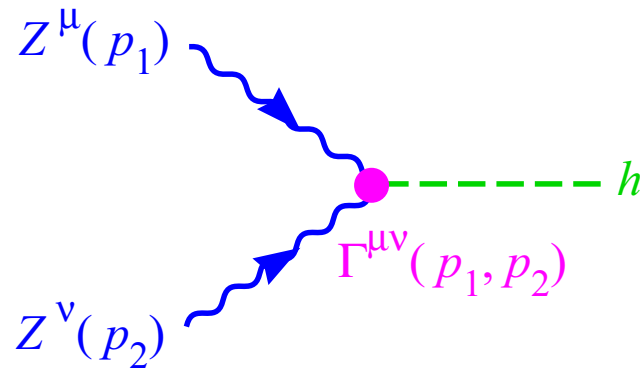
$$\frac{R(i \rightarrow f) - R(\bar{i} \rightarrow \bar{f})}{R(i \rightarrow f) + R(\bar{i} \rightarrow \bar{f})}, \quad \text{e.g.} \quad \frac{\Gamma(t \rightarrow W^+ q) - \Gamma(\bar{t} \rightarrow W^- \bar{q})}{\Gamma(t \rightarrow W^+ q) + \Gamma(\bar{t} \rightarrow W^- \bar{q})}.$$

b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M} \sim M_1 + M_2 \sin \theta,$$

$$A_{CP} = \sigma^F - \sigma^B = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

E.g. 1:  $H \rightarrow Z(p_1)Z^*(p_2) \rightarrow e^+(q_1)e^-(q_2), \mu^+\mu^-$



$$\Gamma^{\mu\nu}(p_1, p_2) = i\frac{2}{v} h [a M_Z^2 g^{\mu\nu} + b (p_1^\mu p_2^\nu - p_1 \cdot p_2 g^{\mu\nu}) + \tilde{b} \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}]$$

$a = 1, b = \tilde{b} = 0$  for SM.

In general,  $a, b, \tilde{b}$  complex form factors,  
describing new physics at a higher scale.

For  $H \rightarrow Z(p_1)Z^*(p_2) \rightarrow e^+(q_1)e^-(q_2)$ ,  $\mu^+\mu^-$ , define:

$$O_{CP} \sim (\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2),$$

$$\text{or } \cos \theta = \frac{(\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2)}{|\vec{p}_1 - \vec{p}_2| |\vec{q}_1 \times \vec{q}_2|}.$$

E.g. 2:  $H \rightarrow t(p_t)\bar{t}(p_{\bar{t}}) \rightarrow e^+(q_1)\nu_1 b_1, e^-(q_2)\nu_2 b_2$ .

$$-\frac{m_t}{v}\bar{t}(a + b\gamma^5)t \ H$$

$$O_{CP} \sim (\vec{p}_t - \vec{p}_{\bar{t}}) \cdot (\vec{p}_{e+} \times \vec{p}_{e-}).$$

thus define an asymmetry angle.

Still need optimal thinking about the asymmetry definition.

E.g. 3:  $\tilde{g}_1\tilde{g}_2 \rightarrow q\tilde{Q}_1, q\tilde{Q}_1 \rightarrow \tilde{\chi}^\pm\tilde{\chi}^\mp + jets \rightarrow e^\pm(q_1)e^\mp(q_2) + jets$ .

probing CP phases  $\theta_3, \theta_2, \theta_\mu$  etc., by

$$O_{CP} \sim \vec{p}_j \cdot (\vec{p}_{e+} \times \vec{p}_{e-}).$$

We must “purify” the sample for the initial state.

### III. The Search for New Physics at Hadron Colliders

We are entering a “data-rich” era:

Electroweak precision constraints;

muon  $g - 2$ ;  $\mu \rightarrow e\gamma\dots$ ; neutron/electron EDMs;

Neutrino masses and mixing;

$K/B$  rare decays and CP violation:  $B \rightarrow X_s\gamma$ ;  $J/\psi K_S$ ,  $\phi K_S$ ,  $\eta' K_S$ ;

Nucleon stability;

Direct/Indirect dark matter searches;

Cosmology constraints on  $m_\nu$ , dark matter and dark energy.

Yet more to come:

**Tevatron:** EW, top sector, Higgs (?), new particle searches...

**LHC:** Higgs studies, comprehensive new particle searches...

**LC:** more on top sector, precision Higgs and new light particles...

**High energy cosmic rays:** AUGER, ICECUBE ... ..

Tevatron has reached a record-high luminosity:

$$2 \times 10^{32} / \text{cm}^2 / \text{s} \Rightarrow 2 \text{ fb}^{-1} / \text{yr} / \text{detector}.$$

continue on till 2010–2011 (?)

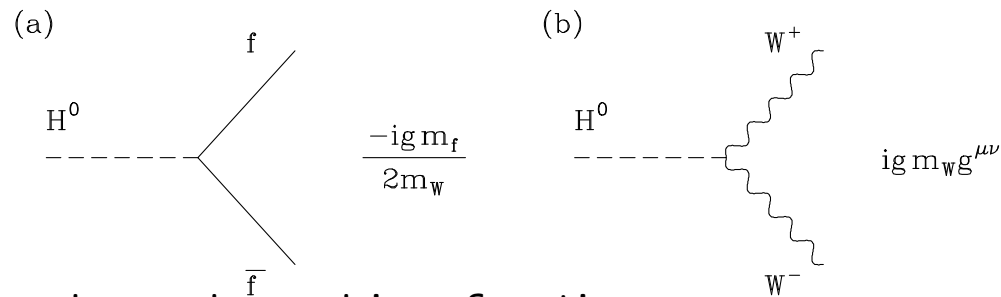
LHC will start in the Fall 2009, initially at  $5 \oplus 5 \text{ TeV}$ .

and  $\sim 200 \text{ pb}^{-1} / \text{detector}$ .

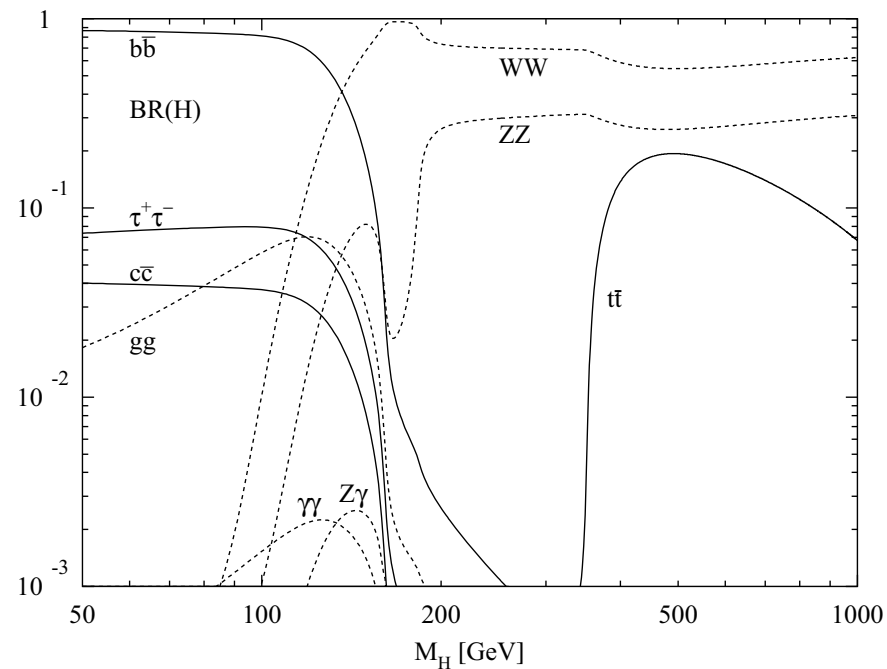
In (almost) ANY TeV scale new physics scenario,  
the LHC will significantly contribute!

## (A). Higgs Searches at the Tevatron and the LHC:

The crucial features: Couplings proportional to masses.

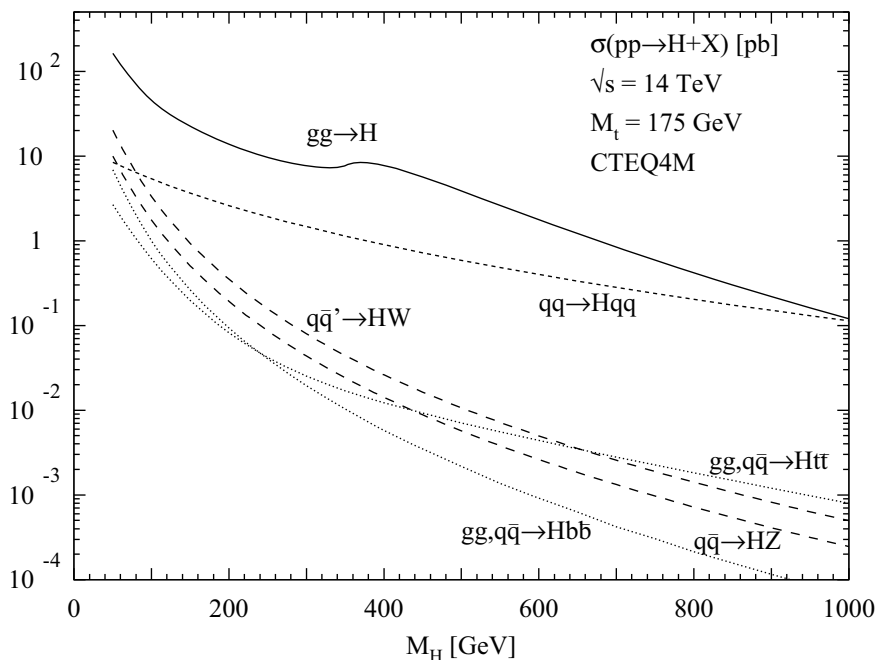
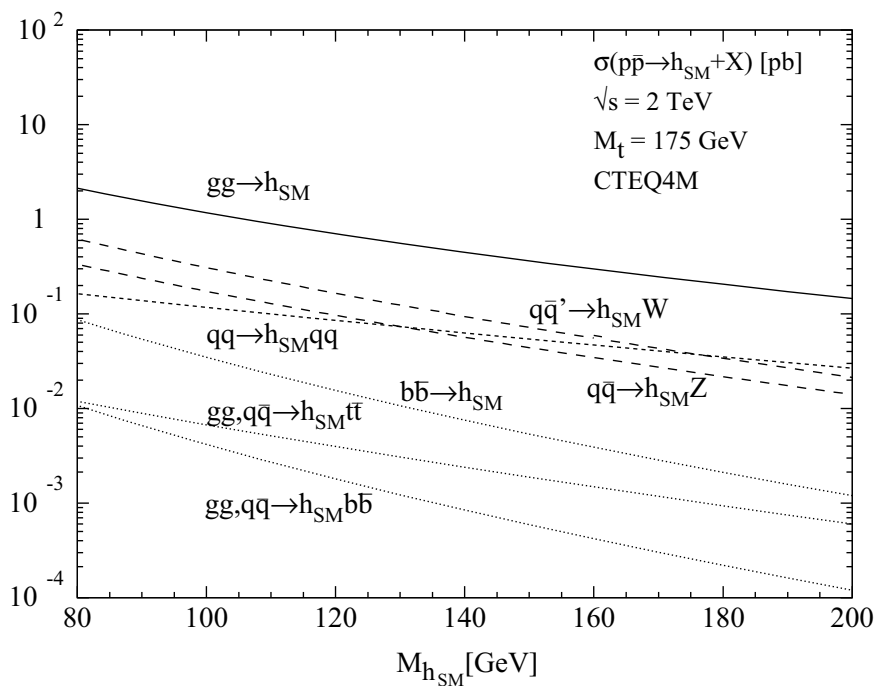


SM Higgs boson decay branching fractions:



preferably to heavier particles.

## SM Higgs boson production rates:



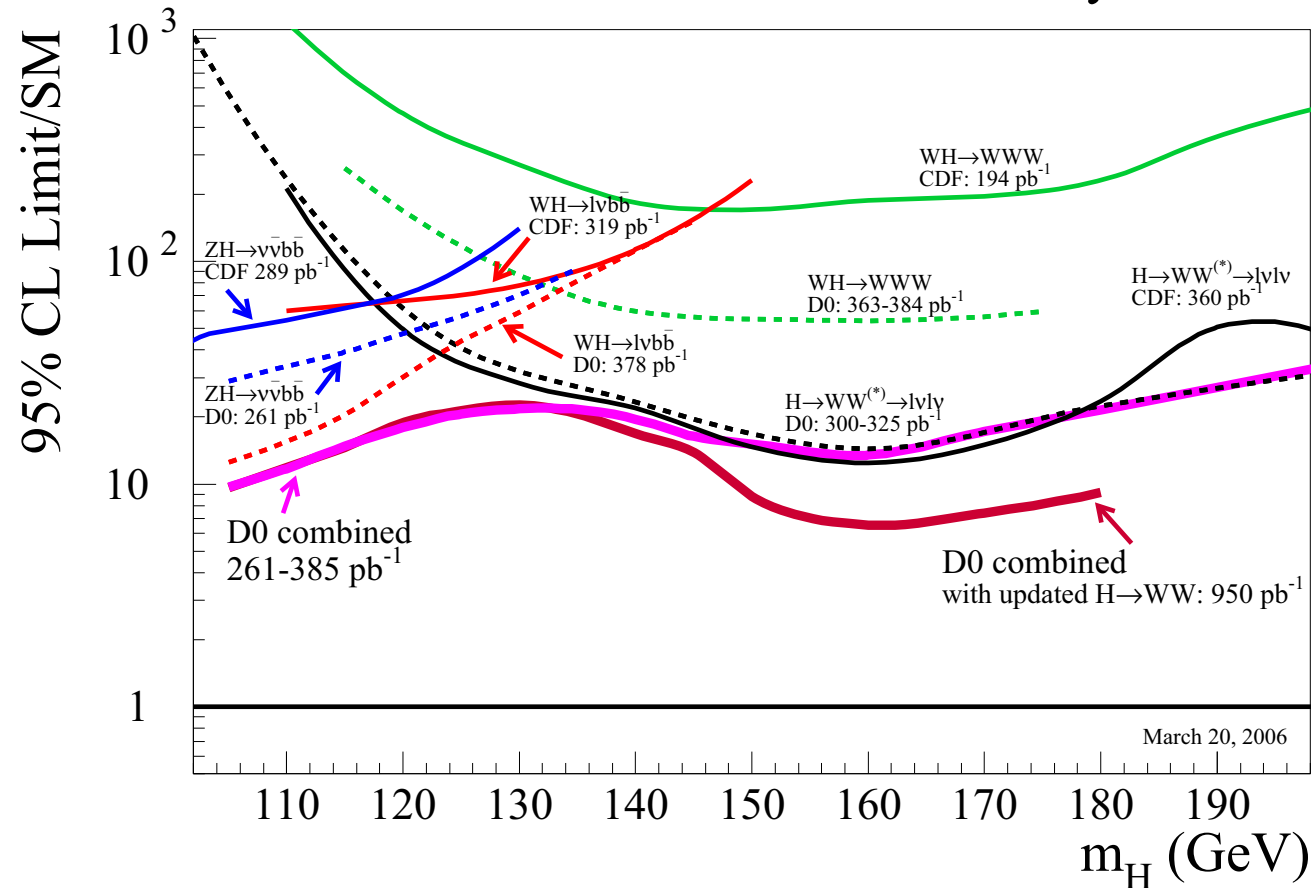
- At the Tevatron: hundreds of Higgs bosons may have been produced, for  $m_h \lesssim 200$  GeV with  $1 \text{ fb}^{-1}$ .
- At the LHC: hundreds of thousand may be produced, for  $m_h \lesssim 700$  GeV with  $100 \text{ fb}^{-1}$ .

- Higgs first shot at the Tevatron:

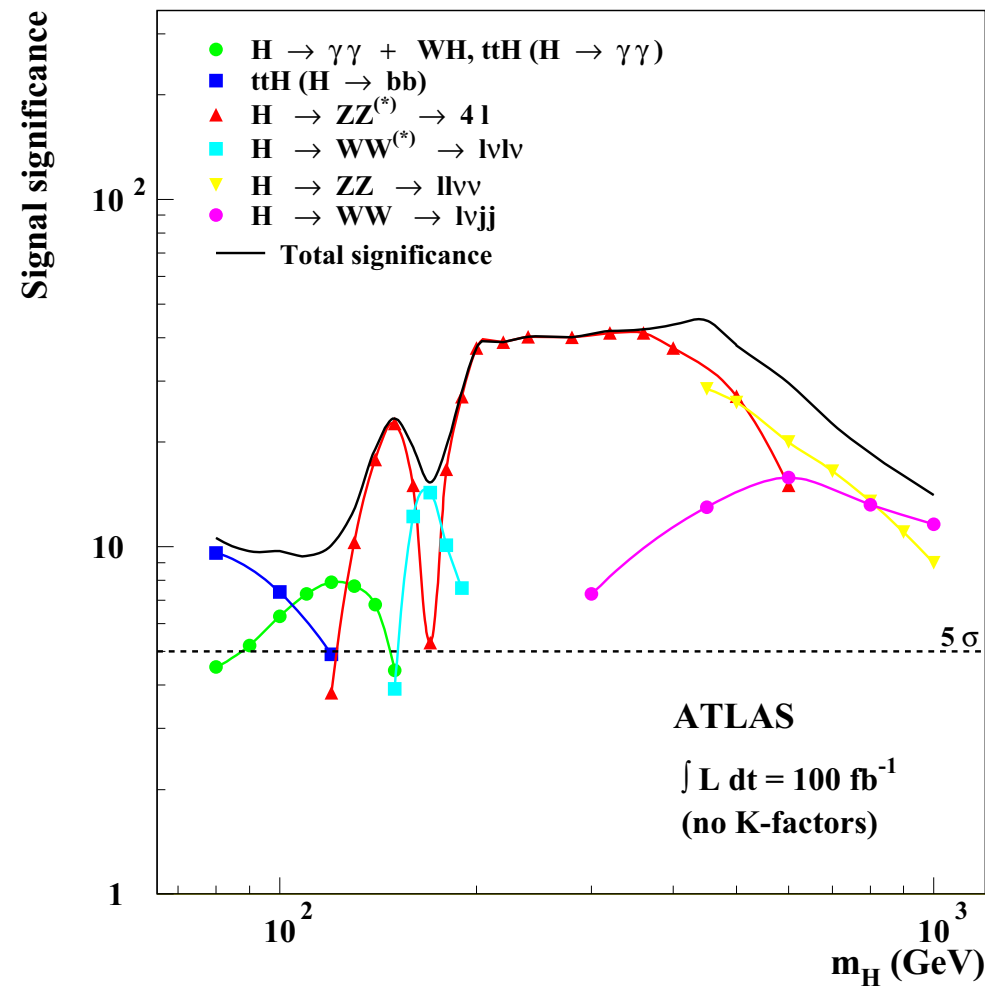
$$q\bar{q}' \rightarrow Wh, Zh, \quad h \rightarrow b\bar{b}$$

$$gg \rightarrow h, \quad h \rightarrow WW^*, ZZ^*, \tau^+\tau^-$$

### Tevatron Run II Preliminary



- SM Higgs fully covered at the LHC:

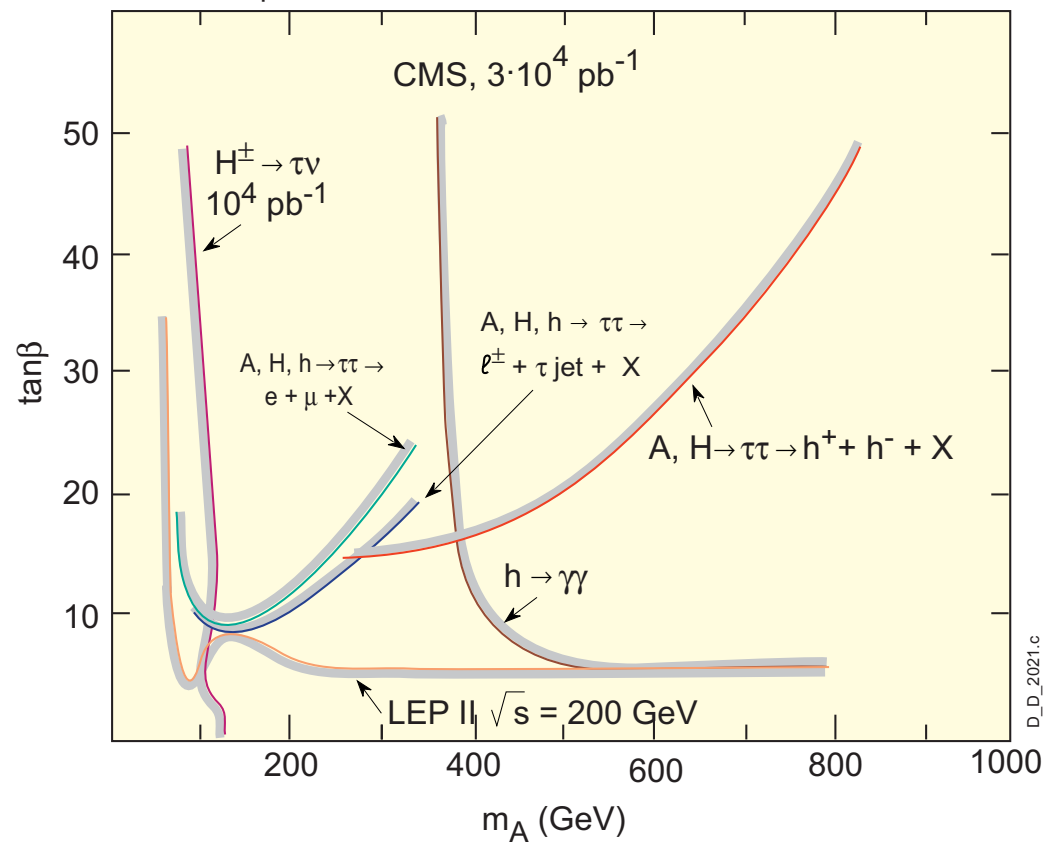


ATLAS report: combining multiple channels,  
 $10\sigma$  observation achievable.

## Significance contours for SUSY Higgses

Regions of the MSSM parameter space ( $m_A$ ,  $\tan\beta$ )  
explorable through various SUSY Higgs channels

- $5\sigma$  significance contours
- two-loop / RGE-improved radiative corrections
- $m_{\text{top}} = 175$  GeV,  $m_{\text{SUSY}} = 1$  TeV, no stop mixing ;

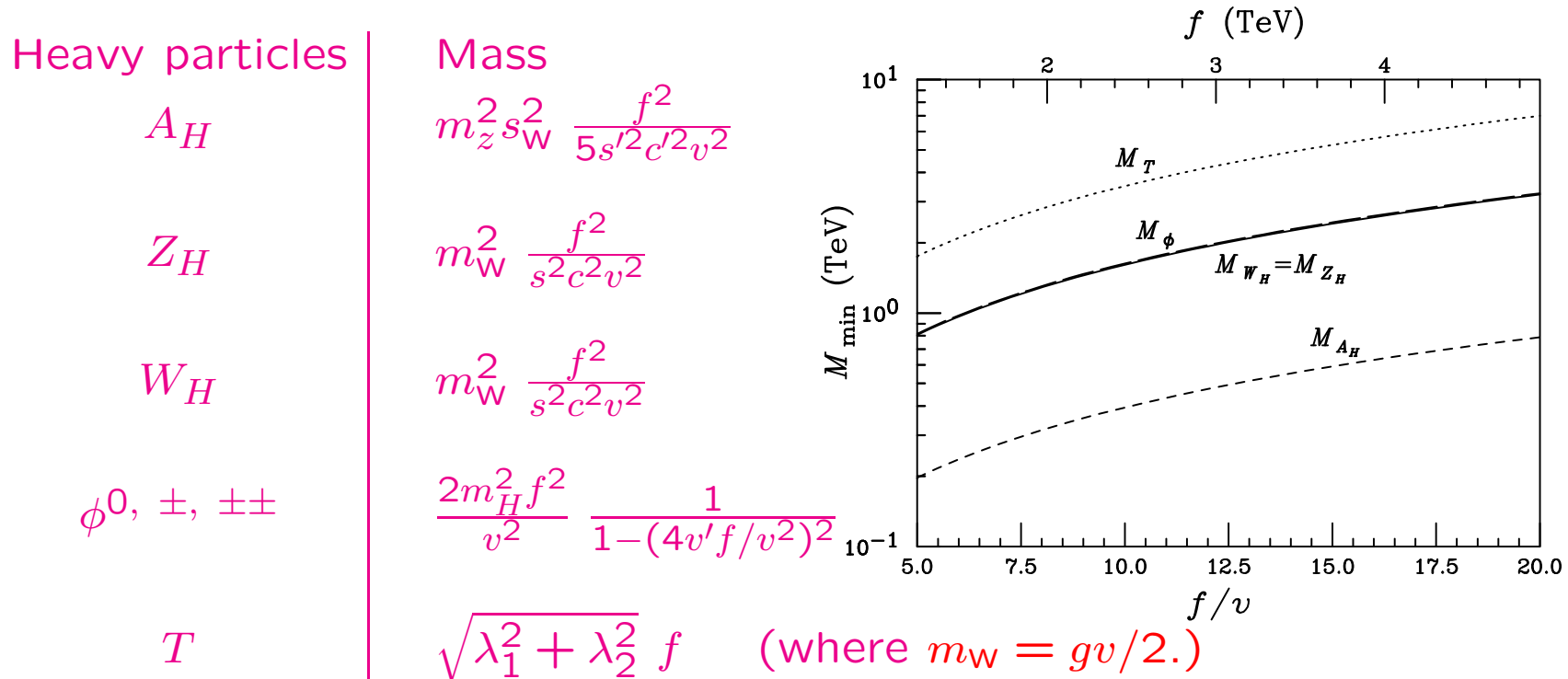


## (B). Weak Scale Supersymmetry

## (C). New gauge bosons and heavy fermions

Little Higgs models as an example

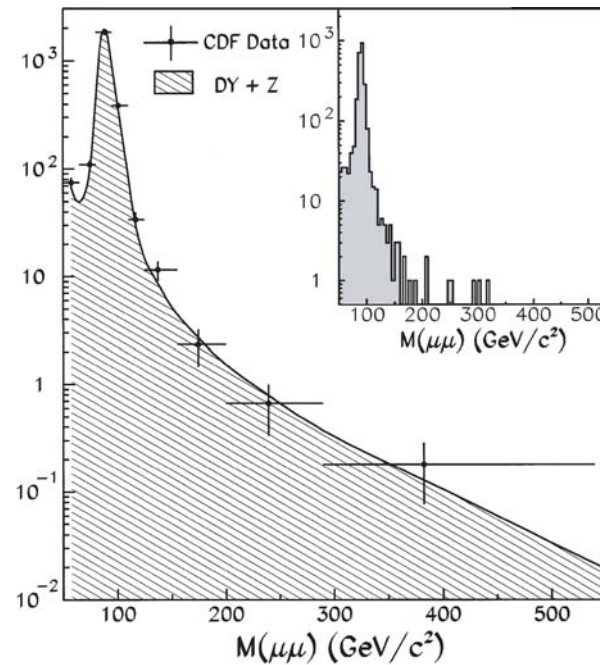
In the Littlest Higgs model:\*



\*Arkani-Hamed, Cohen, Katz, Nelson, hep-ph/0206021.

- New gauge bosons in DY process:

Recall CDF searches for a  $Z' \rightarrow \mu^+ \mu^-$ : [PRL 79, (1997)]

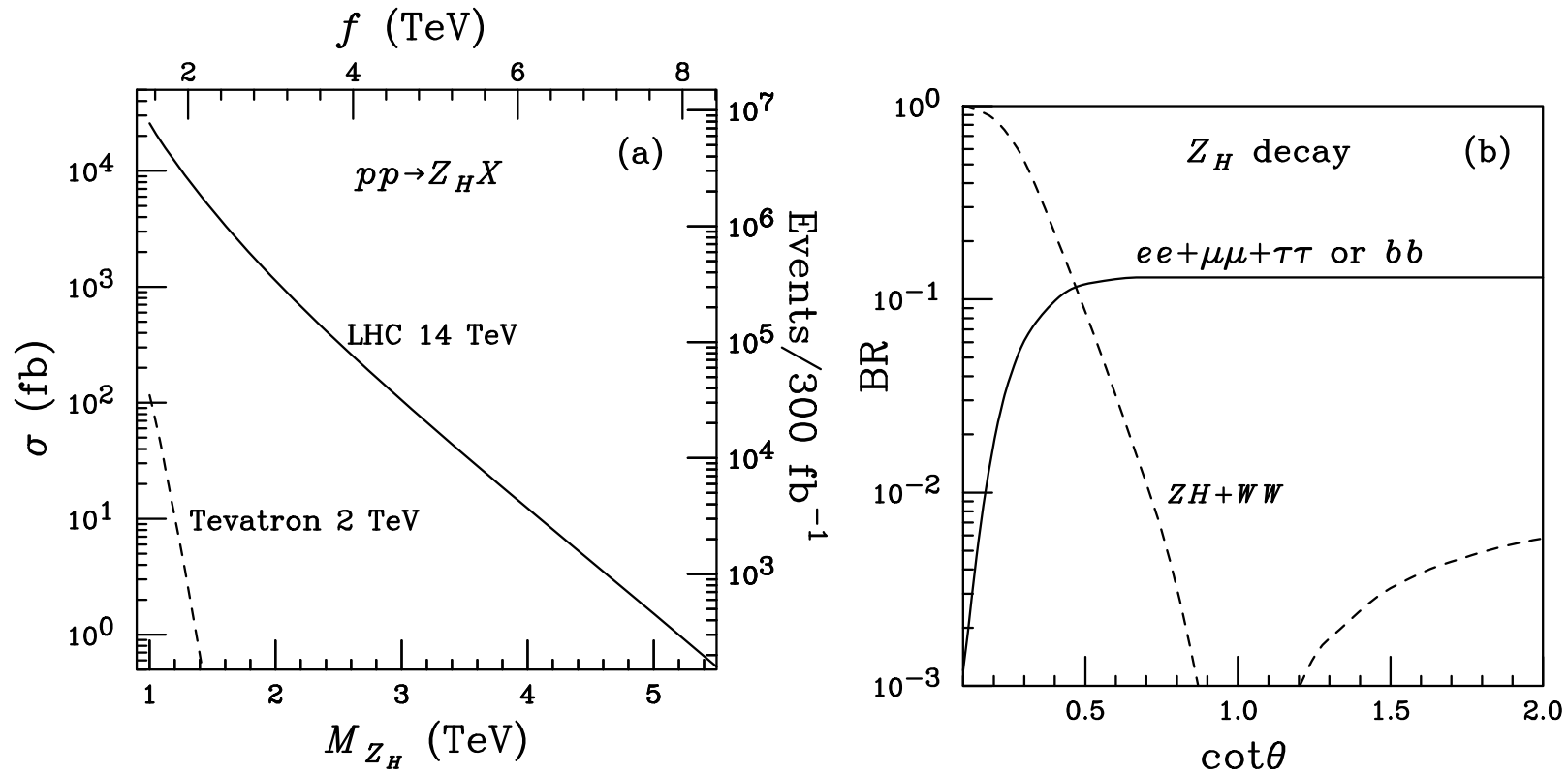


including:

$$\begin{aligned}
 p\bar{p} &\rightarrow Z, \gamma \rightarrow \mu^+ \mu^- X, \\
 p\bar{p} &\rightarrow W^+ W^- \rightarrow \mu^+ \nu_\mu \mu^- \bar{\nu}_\mu X, \\
 p\bar{p} &\rightarrow b\bar{b} \rightarrow \mu^+ \mu^- + \text{hadrons} + X, \\
 p\bar{p} &\rightarrow t\bar{t} \rightarrow W^+ b W^- \bar{b} \rightarrow \mu^+ \nu_\mu \mu^- \bar{\nu}_\mu b\bar{b} X.
 \end{aligned}$$

$$\sigma < 40 \text{ fb} \Rightarrow M_{Z'} > 600 \text{ GeV}.$$

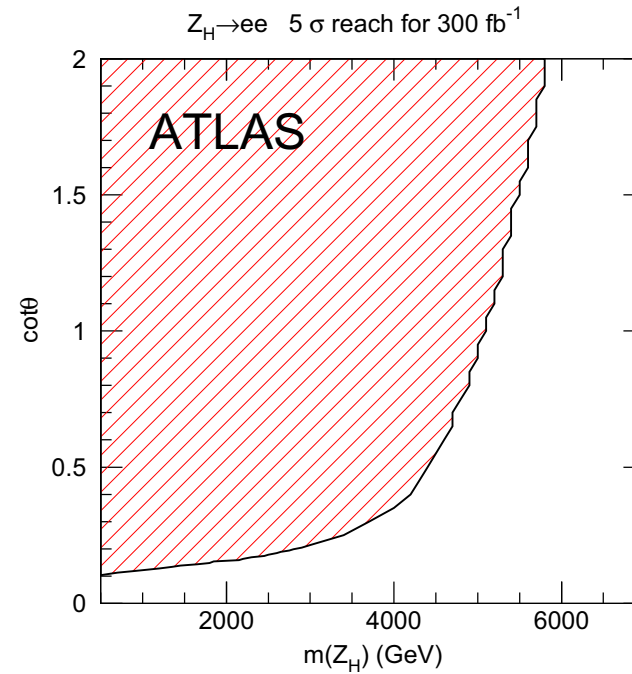
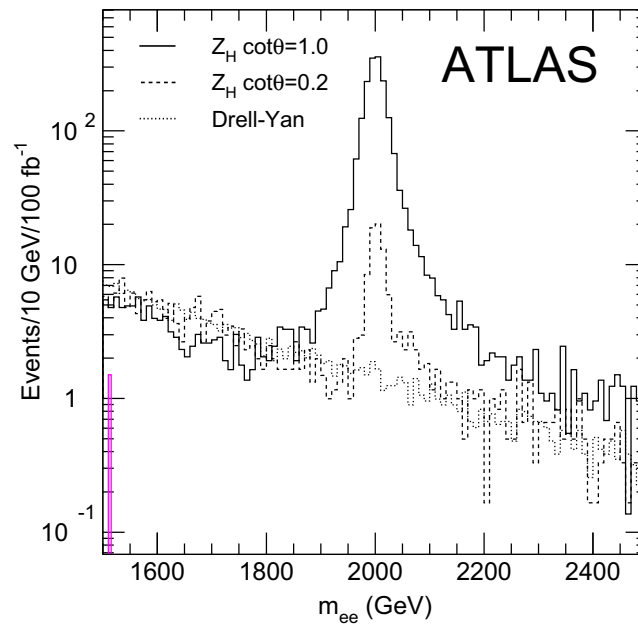
- $Z_H/W_H$  robust new state
- DY production rate large



Tevatron: not quite accessible (except for  $A_H$ );

LHC:  $M_{Z_H} \sim 5$  TeV or  $f \sim 8$  TeV.

ATLAS simulations for  $Z \rightarrow \ell^+ \ell^-$ :



Reach  $M_{Z_H} \sim$  several TeV for  $\cot\theta > 0.1$ :

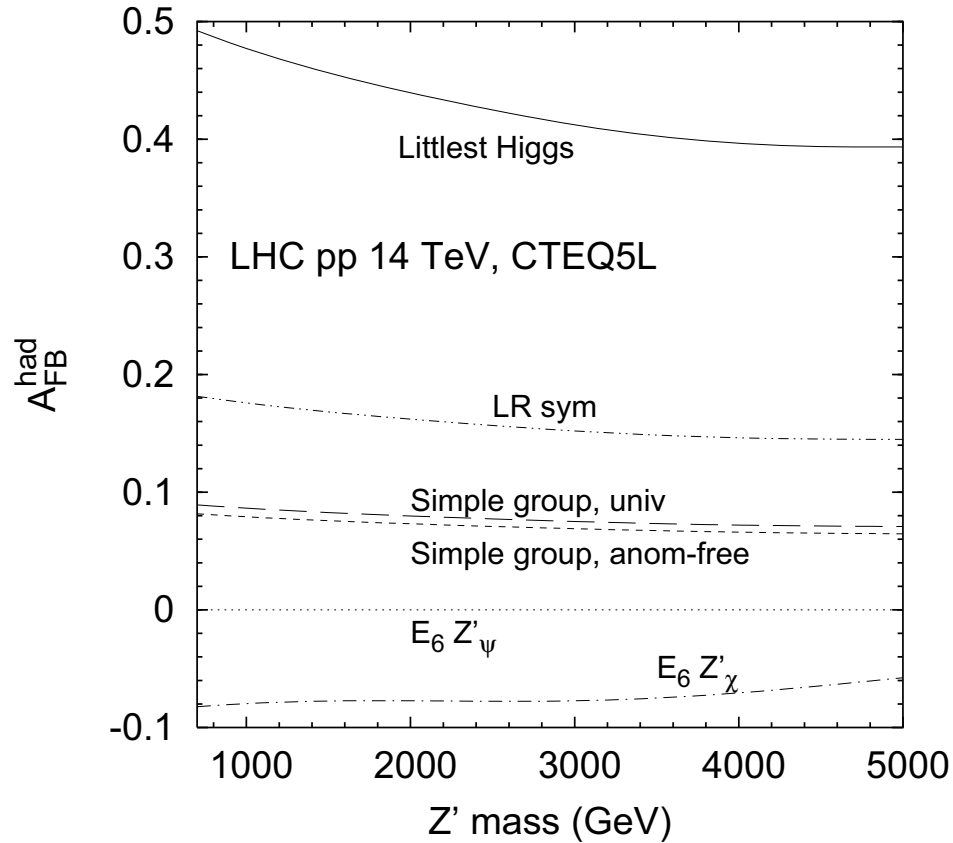
Cross-sections measure  $\cot\theta$ :  $N(\ell^+ \ell^-)$  versus  $N(Zh)$ .

Mass peak  $M_{Z_H}$  determines  $f$ .

Significant differences for FB asymmetry among  $Z'$ 's:

$$A_{FB}^{i,f} = \frac{3}{4} A_i A_f, \quad A_i = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}.$$

$$A_{FB}^{\text{had}} = \frac{\int dx_1 \sum_{q=u,d} A_{FB}^{qe} (F_q(x_1) F_{\bar{q}}(x_2) - F_{\bar{q}}(x_1) F_q(x_2)) \text{sign}(x_1 - x_2)}{\int dx_1 \sum_{q=u,d,s,c} (F_q(x_1) F_{\bar{q}}(x_2) + F_{\bar{q}}(x_1) F_q(x_2))},$$



- Heavy quark signals:

Recall the top-quark searches at hadron colliders

The leading production channels:

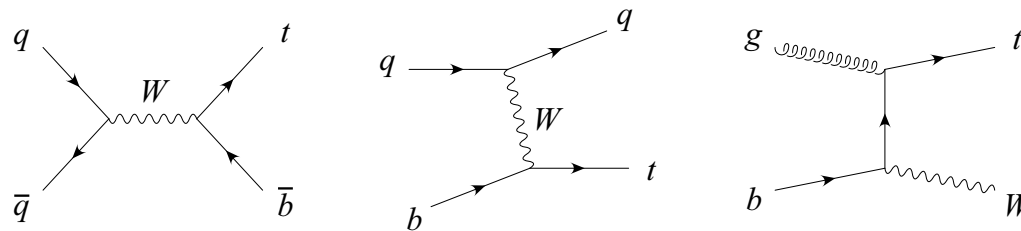
$$q\bar{q} \rightarrow t\bar{t}, \quad \text{Tevatron 90\%; LHC 10\%}$$

$$gg \rightarrow t\bar{t}, \quad \text{Tevatron 10\%; LHC 90\%}$$

$$\text{with } t\bar{t} \rightarrow W^+b \ W^- \bar{b} \rightarrow \dots$$

Top-quark discovered (1993):  $m_t \approx 178 \text{ GeV}$ .

Interesting sub-leading (electroweak) production channels: the single-top



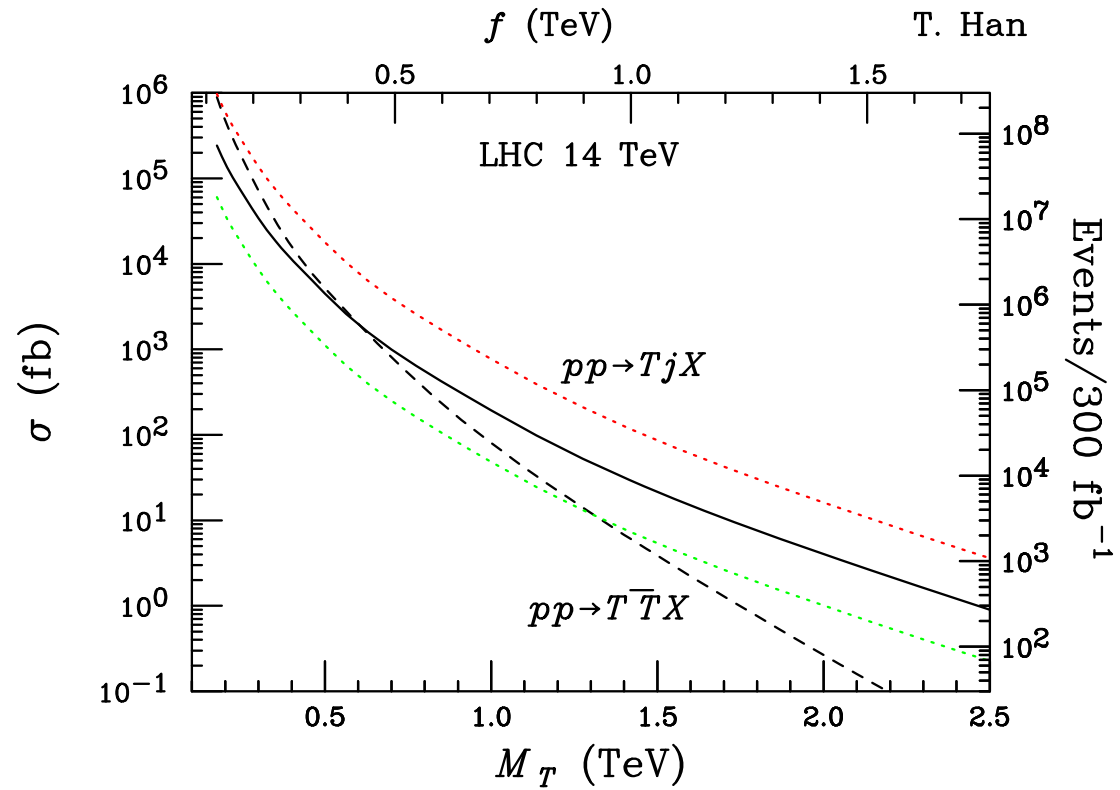
$$q\bar{q}' \rightarrow W^* \rightarrow t\bar{b}, \quad \text{a lot smaller}$$

$$gb \rightarrow tW, \quad \text{smaller too}$$

$$qb \rightarrow q'W^*b \rightarrow q' t \quad 1/3 \text{ of QCD.}$$

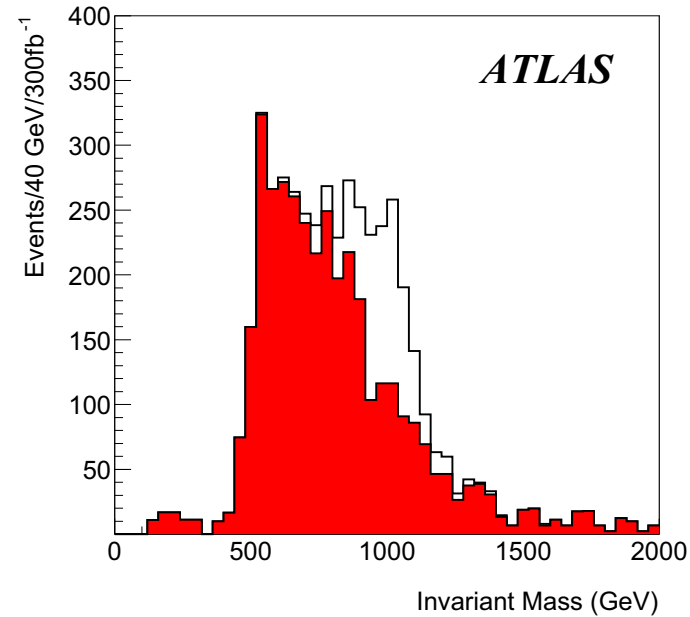
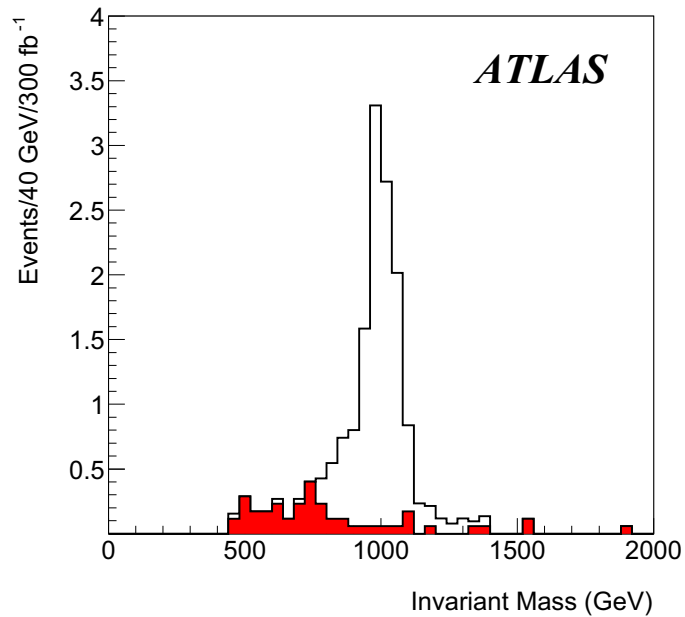
Recently observed at the Tevatron: measure  $V_{tb}$  and test  $tbW_L$  coupling.

## The heavy $T$ signal at the LHC



$gg \rightarrow T\bar{T}$  phase-space suppression;  
 $qb \rightarrow q'T$  via  $t$ -channel  $W_L b \rightarrow T$ .

ATLAS simulations for  $T \rightarrow tZ$ ,  $bW$ :



Reach  $M_T \sim 1$  (2) TeV for  $x_\lambda = 1$  (2).

Cross-sections measure coupling  $x_\lambda$ .

Mass peak  $M_T$  determines  $f$ :  $v/f = m_t/M_T(x_\lambda + x_\lambda^{-1})$   
 $\Rightarrow$  check consistency with  $f$  from  $M_{ZH}$ .

## (D). LHC–Dark Matter connection:

The most likely DM candidates seem to be of particle-physics origin, but beyond the SM. †

See Konstantin's lectures.

†For recent review, H.Baer and X.Tata (2008).

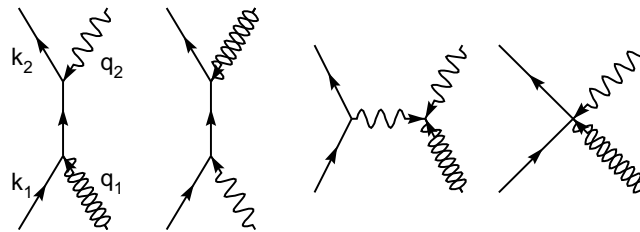
## (E). Deep into extra-dimensions at the LHC:

### ● Collider Searches for Extra Dimensions:

#### A. Collider Signals I (ADD)

Real KK Emission: Missing Energy Signature

a.  $e^+e^- \rightarrow \gamma + KK$  ( $\gamma$ +missing energy)



n – dim : at LEP2

$n = 4$   $M_S > 730$  (GeV)

$n = 6$   $M_S > 520$  (GeV)

b.  $p\bar{p} \rightarrow jet + KK$  (mono-jet+missing energy)

n – dim :	at Tevatron	at LHC
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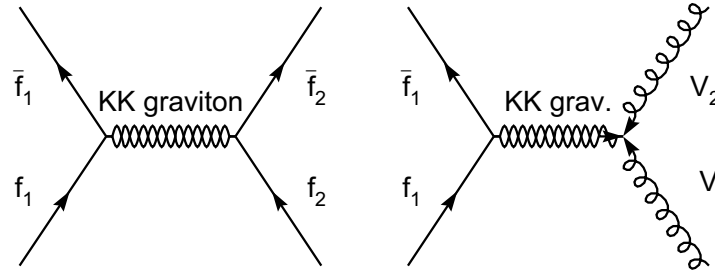
$n = 4$	$M_S > 900$ (GeV)	3400
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$n = 6$	$M_S > 810$ (GeV)	3300
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## B. Collider Signals II (ADD)

### Virtual KK Graviton Effects

On four-particle contact interactions:



Sum over virtual KK exchanges:

$$\begin{aligned}
 i\mathcal{M} &\sim \bar{f}\mathcal{O}_i f \bar{f}\mathcal{O}_j f \int_0^\infty \frac{dm_{\vec{n}}^2}{s - m_{\vec{n}}^2 + i\epsilon} \kappa^2 \rho(m_{\vec{n}}) \\
 &\sim \frac{s^2}{M_S^4} \bar{f}\mathcal{O}_i f \bar{f}\mathcal{O}_j f.
 \end{aligned}$$

Again, effective coupling  $\kappa^2 \sim \frac{1}{M_{pl}^2} \rightarrow \frac{1}{M_S^2}$  !

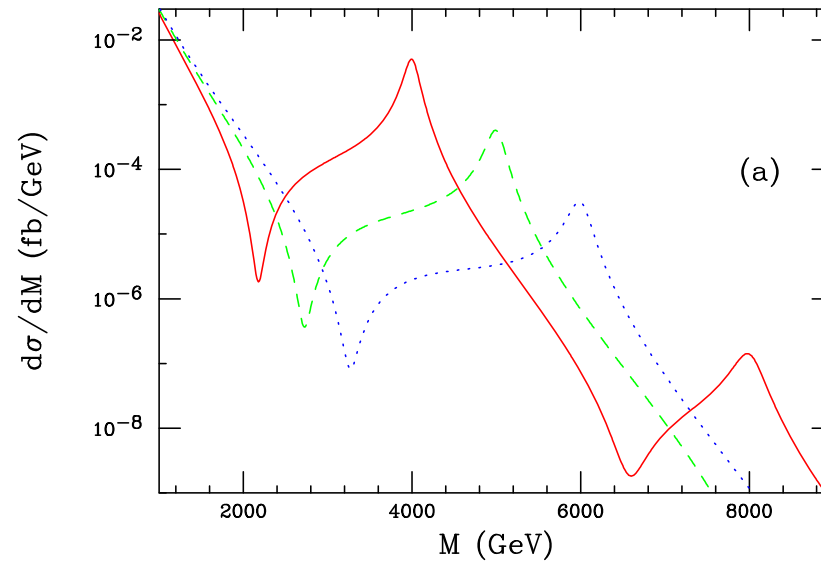
### C. KK Resonant States at Colliders: (RS)

If the SM fields (photons, electrons,  $Z, W, H^0 \dots$ ) also propagate in extra dimensions, then they have KK excitations.

Direct search bounds:

$$M_{\gamma, Z, W}^* \sim \frac{1}{R} > 4 \text{ TeV}.$$

Resonant production at the LHC:

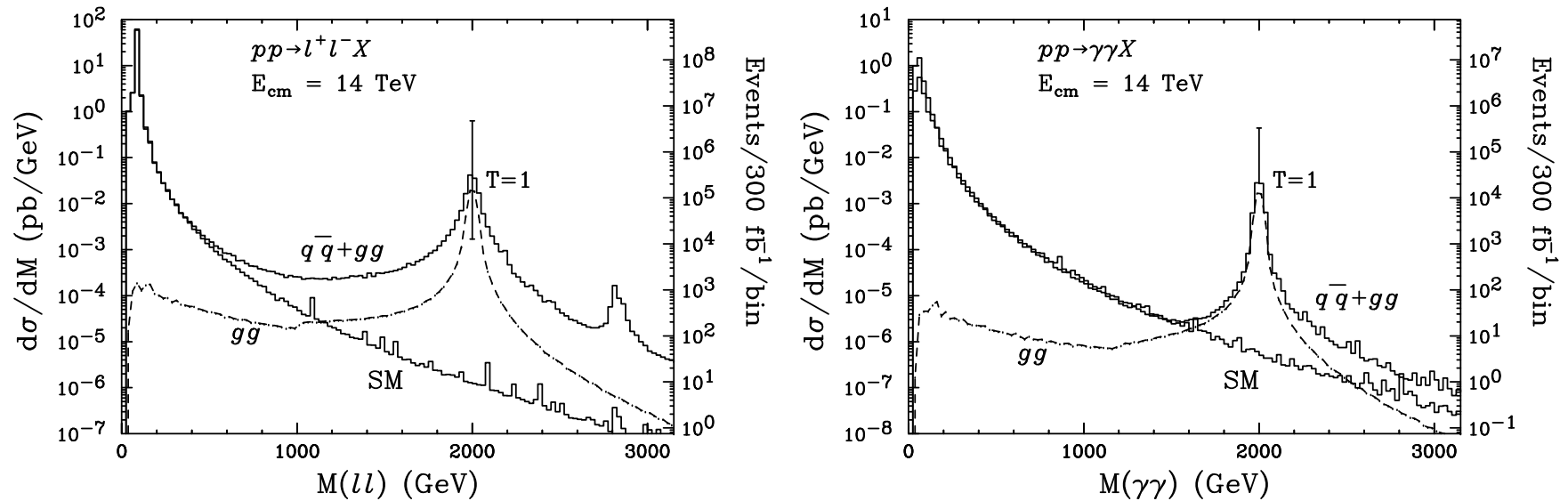


## D. Stringy States at Colliders

Future colliders may reach the TeV string threshold  
thus directly produce the “stringy” resonant states.

Amplitude factor near the resonance

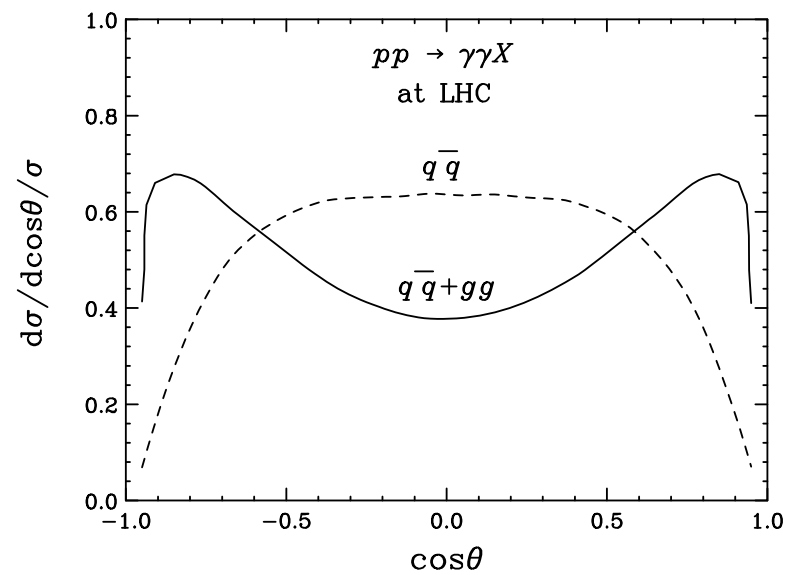
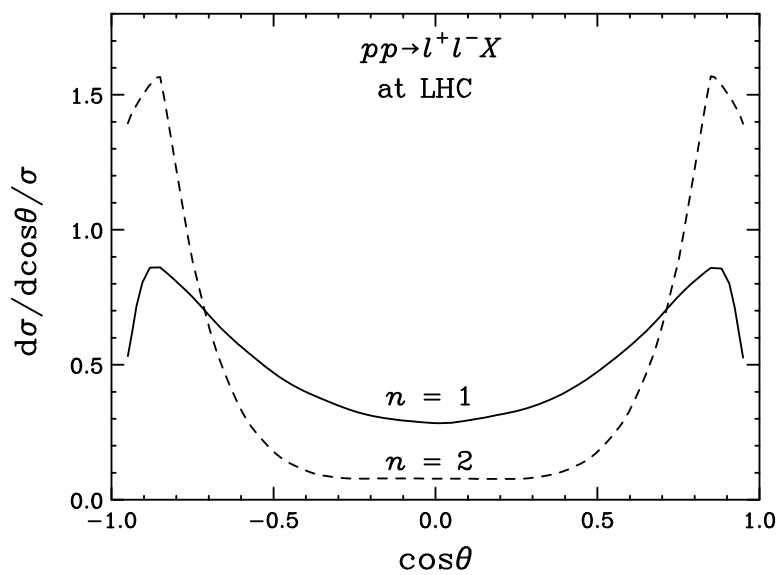
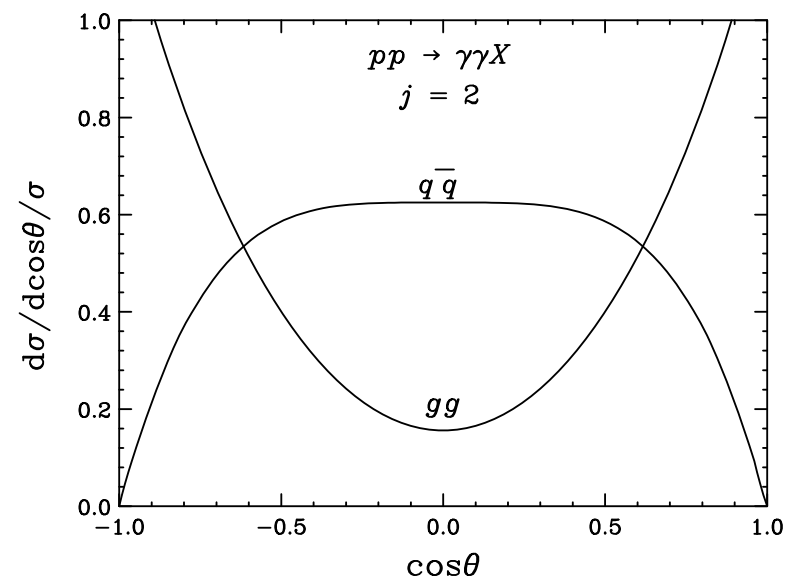
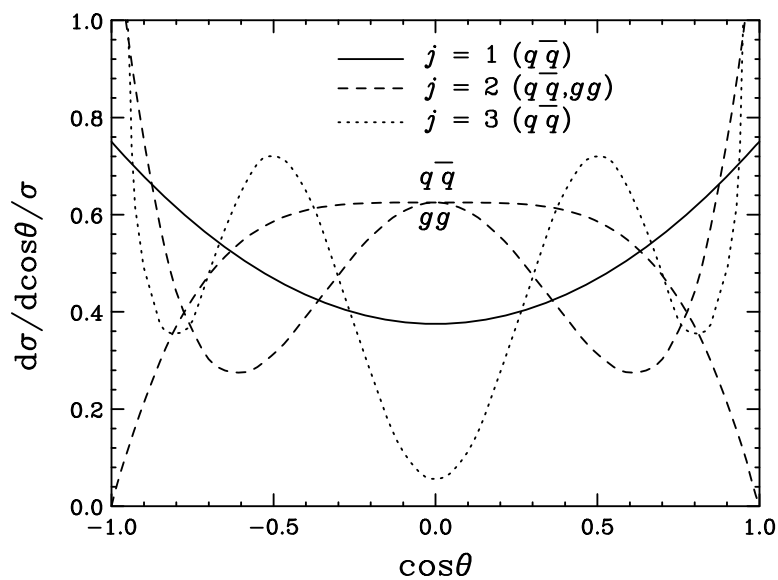
$$\mathcal{M}(s, t) \sim \frac{t}{s - nM_S^2}, \quad \text{its mass } M_n = \sqrt{n}M_S.$$



where  $T$  is an unknown gauge factor (Chan-Simon factor), typically  $1 - 4$ .  
With  $300 \text{ fb}^{-1}$ , if no signal seen, we expect to reach bounds for

$$M_S > 8 \text{ (10) TeV for } T = 1 - 4.$$

Very rich structure of angular distributions:



## E. Black Hole Production at Colliders

For a black hole of mass  $M_{BH}$ , its size is

$$r_{bh} \approx \frac{1}{M_D} \left( \frac{M_{BH}}{M_D} \right)^{\frac{1}{n+1}} \rightarrow \frac{M_{BH}}{M_{pl}^2} \text{ in 4d.}$$

At higher energies and shorter distances (impact parameter)

$$E_{cm} > M_{BH} > M_D, \quad b_{impact} < r_{bh},$$

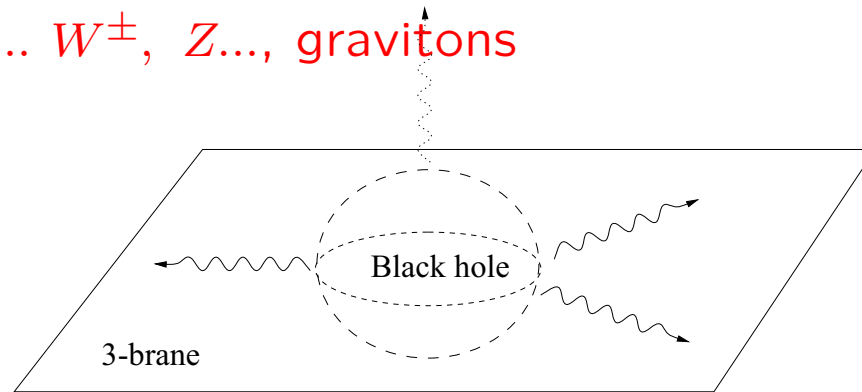
black holes formation is the dominant quantum gravity phenomena.

Black holes copiously produced at the LHC energies:

$M_{BH}$	$n = 4$	$n = 6$
5 TeV	$1.6 \times 10^5 \text{ fb}$	$2.4 \times 10^5 \text{ fb}$
7 TeV	$6.1 \times 10^3 \text{ fb}$	$8.9 \times 10^3 \text{ fb}$
10 TeV	6.9 fb	10 fb

Black holes “decay” via Hawking radiation:

$\gamma$ ,  $\nu$ ,  $e^\pm$ , *hadrons*, ...  $W^\pm$ ,  $Z$ ..., gravitons



Spectacular events:

- very luminous in the detector!
- lepton-number/baryon-number violation (?)
- spherical/angular momentum orientation (?) ... ..
- to the least, LHC is a “safe machine”. †

†S.Giddings and M.Mangano, arXiv:0806.3381

## (F). Final remarks:

### (a.) Kinematics can help a lot!

Basic techniques/considerations seeking for new particles and interactions. are applicable to many new physics searches.

Prominent examples include:

- Drell-Yan type of new particle production in  $s$ -channel:

$$Z' \rightarrow \ell^+ \ell^-, W^+ W^-; \quad W' \rightarrow \ell \nu, W^\pm Z;$$

$$Z_H \rightarrow ZH; \quad W_H \rightarrow W^\pm H;$$

$$V^{0,\pm} \rightarrow t\bar{t}, W^+ W^-; \quad t\bar{b}, W^\pm Z;$$

$$\text{heavy KK/stringy states} \rightarrow \ell^+ \ell^-, \gamma\gamma, \dots;$$

$$\text{single } \tilde{q}, \tilde{\ell} \text{ via R parity violation.}$$

- $t$ -channel gauge boson fusion processes:

$$W^+ W^-, ZZ, W^\pm Z \rightarrow H, V^{0,\pm}, \text{ light SUSY partners};$$

$$W^+ W^+ \rightarrow H^{++};$$

$$W^+ b \rightarrow T.$$

- Heavy flavor enrichment is another important feature for new physics:

$$H \rightarrow, b\bar{b}, \tau^+\tau^-; H^+ \rightarrow t\bar{b}, \tau^+\nu; \tilde{H} \rightarrow \tilde{\chi}H; \tilde{t} \rightarrow \tilde{\chi}^+b, \tilde{\chi}^0t; V_8, \eta_t \rightarrow t\bar{t} \text{ etc.}$$

- Large missing energies are important indication for beyond SM physics!  
See Konstantin's lectures.

However, at hadron collider environments, certain class of experimental signals may be way more complex than the simple examples above.

Major discoveries highly anticipated at the LHC,  
but very challenging!

Final Recap: