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Summer School on Particle Physics in the LHC Era

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Beyond the Standard Model

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Behind and Beyond the Standard Model

Riccardo Rattazzi



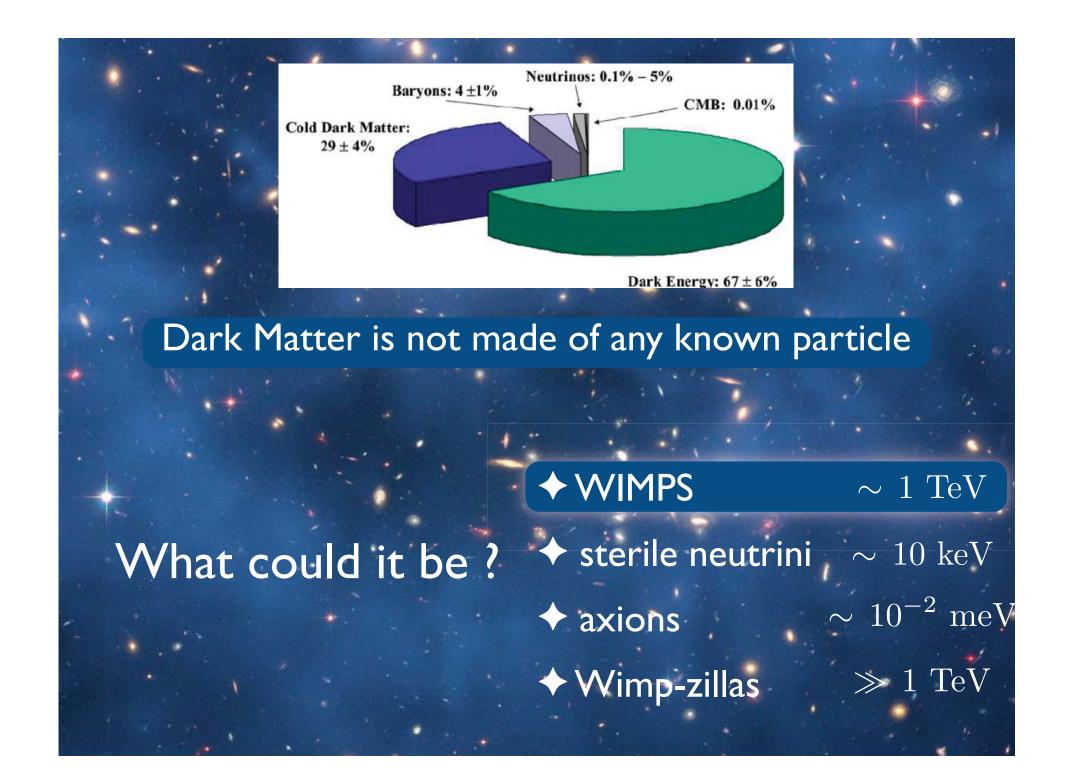
- I. A critical view on the Standard Model
 - Obvious limitations of the Standard Model
 - Effective Quantum Field Theories: couplings, mass scales and accidental symmetries
 - The Standard Model as an effective theory (baryon & lepton number, flavor, precision EW tests)
 - Naturally light particles & generation of mass hierarchies in field theory
 - Strong CP problem and the axion
- 2. Supersymmetry
- 3. Grand Unification
- 4. Overview

Standard Model: defined by gauge symmetry & multiplet content

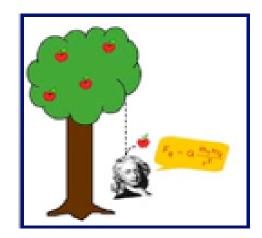
gauge group
$$SU(3) \times SU(2) \times U(1)_Y \times \text{gravity}$$

 $\begin{pmatrix} G_{\mu}^A & W_{\mu}^I & B_{\mu} & g_{\mu\nu} \end{pmatrix}$
 $\begin{pmatrix} H^{\alpha} & & & \\ H^{\alpha} & & & \\ & & & & \end{pmatrix}$
fermions $q_L, u_R, d_R, \ell_L, e_R$

$$\begin{split} \mathcal{L} &= -\frac{1}{4g_3^2} G_{\mu\nu}^2 - \frac{1}{4g_2^2} W_{\mu\nu}^2 - \frac{1}{4g_Y^2} B_{\mu\nu}^2 + |D_{\mu}H|^2 + V(H) \\ &\bar{q}_L \ \mathcal{D}q_L + \bar{u}_R \ \mathcal{D}u_R + \bar{d}_R \ \mathcal{D}d_R + \bar{\ell}_L \ \mathcal{D}\ell_L + \bar{e}_R \ \mathcal{D}e_R \\ &+ Y_u^{ij} \bar{q}_L^i H^{\dagger} u_R + Y_d^{ij} \bar{q}_L^i H d_R^j + Y_e^{ij} \bar{\ell}_L^i H e_R + \frac{\lambda^{ij}}{M} (H\ell^i) (H\ell^j) + \cdots \\ &+ \sqrt{g} M_P^2 \left(R(g) - \lambda + \cdots \right) \end{split}$$



Gravity



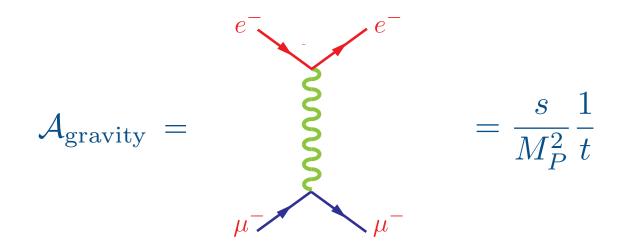
General Relativity at the quantum level only makes sense as an **Effective** Quantum Field Theory

There is an absolute upper bound on the energy scale at which General Relativity makes sense

Gravity couples to all other particles



absolute upper bound on energy scale up to which the SM can be valid



quantum effects untractable at $E \sim M_P \simeq 10^{19} \text{ GeV}$

M_P is huge and thus gravity is not necessarily of urgent concern for the LHC

But previous argument only sets an upper bound on relevant gravity scale. In the scenario of large extra dimensions gravity becomes indeed strong at around a TeV

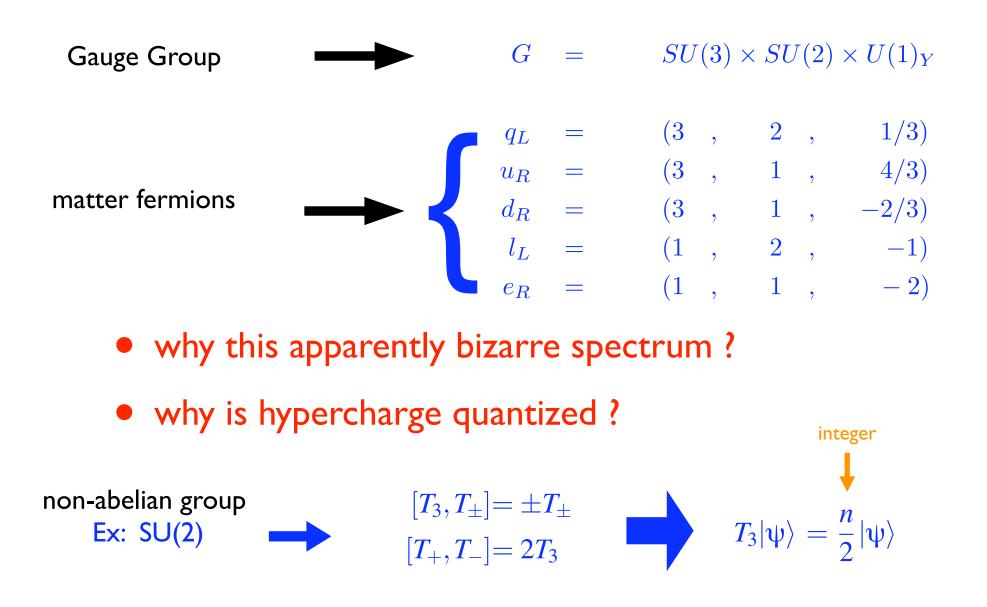
The fate of gravity is of crucial importance to develop a theory of the very early universe

The other 3 forces...





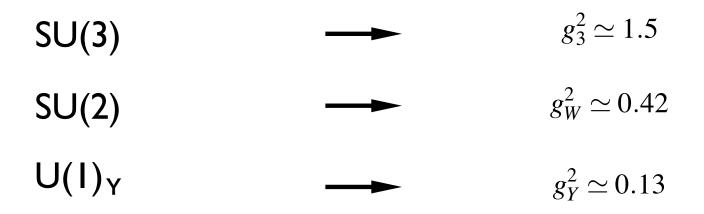




abelian group: no quantization condition

Can one build new theory with non-abelian hypercharge ?

Strength of forces at $E \approx M_z$



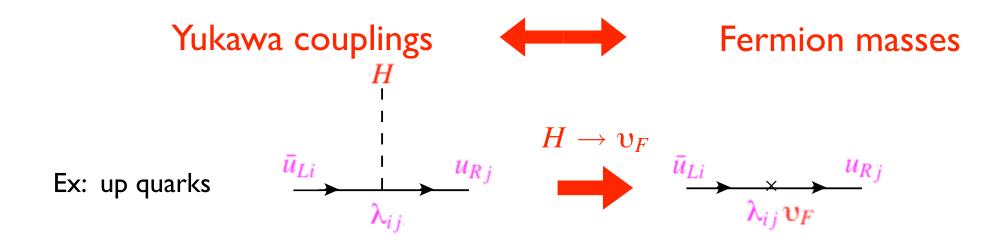
• they differ, but not wildly

• strength of gravity at $E \approx M_z$

$$G_N M_Z^2 \equiv \frac{M_Z^2}{M_P^2} \sim 10^{-34}$$

Matter

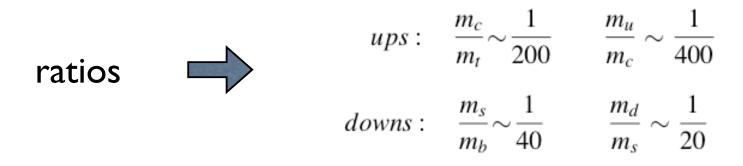




• mass eigenvalues: $m_i = \lambda_i \langle H \rangle \equiv \lambda_i \upsilon_F = \lambda_i \times (174 \text{GeV})$

 fermion masses are inputs ... but the observed spectrum begs for an explanation

family type	ups	downs	leptons	
3rd	$m_t = 175$	$m_b = 4.2$	$m_{\tau} = 1.7$	masses in GeV
2nd	$m_{c} = 1.2$	$m_{s} = 0.1$	$m_{\mu} = 0.1$	
lst	$m_u = 3 \times 10^{-3}$	$m_d = 5 \times 10^{-3}$	$m_e = 5 \times 10^{-4}$	



analogy with the spectrum of hydrogen lines before Bohr



explained by Bohr
$$E_n = -\frac{2\pi^2 e^4 m_e}{h^2 n^2}$$



what is the analogue of Bohr atom in the case of fermion masses ?

Neutrino masses

 $\Delta m_{atm}^2 \simeq 2 \times 10^{-3} \,\mathrm{eV}^2 \qquad \Delta m_{sol}^2 \simeq 0.8 \times 10^{-4} \,\mathrm{eV}^2$

 $\sin^2 2\theta_{atm} = 0.9 - 1.0 \qquad \tan^2 \theta_{sol} = 0.3 - 0.6$

we were hoping to get illuminated on the structure of quarks and charged lepton spectrum, but we weren't

overall neutrino mass scale points to existence of new dynamics at a scale around 10¹⁴ GeV

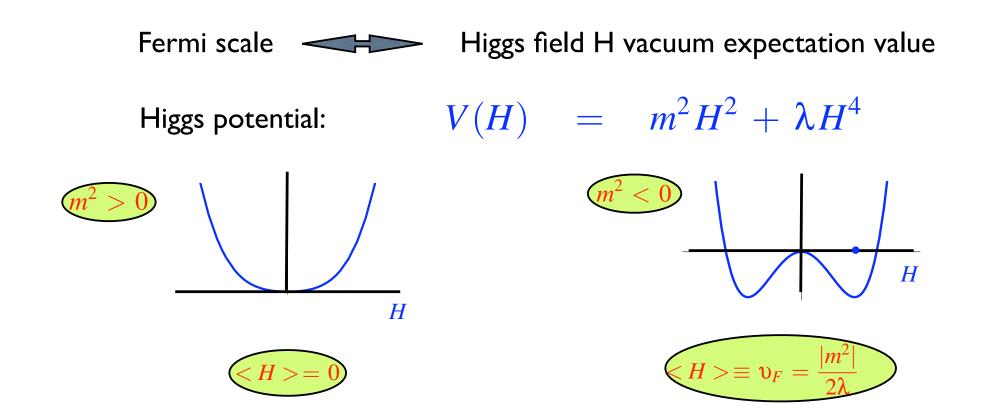
the smallness of neutrino masses can be viewed as yet another success of SM

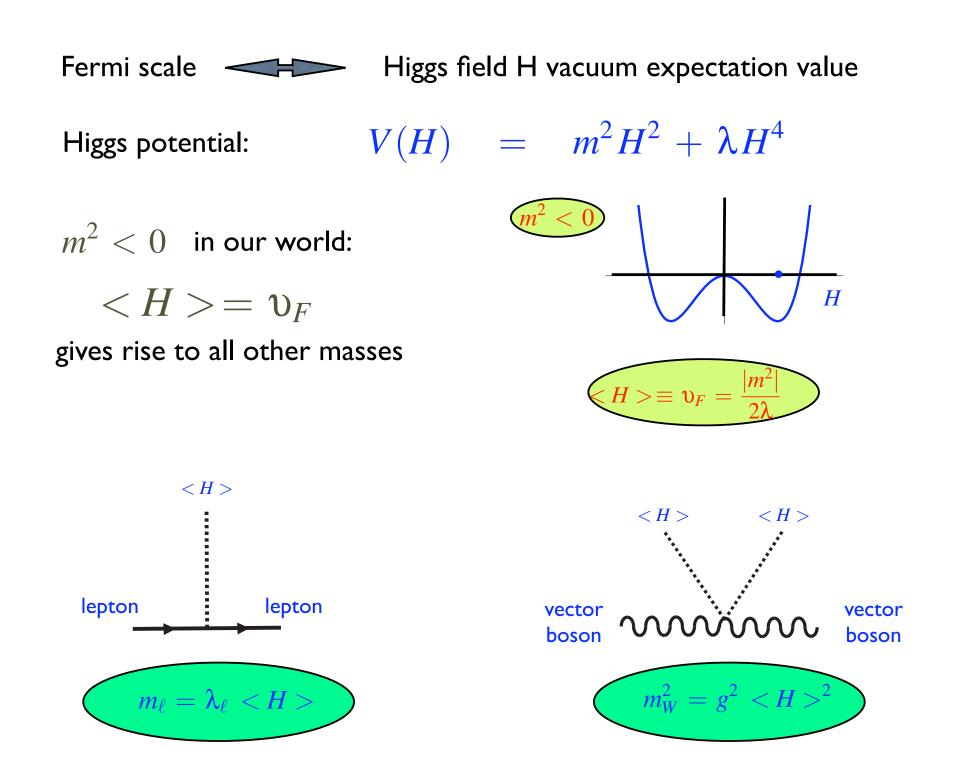
Is this simple picture correct? Are neutrini Majorana particles?

The fifth force

or

how weak interactions became weak

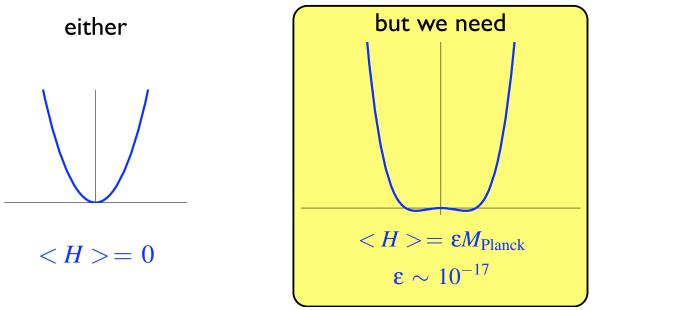


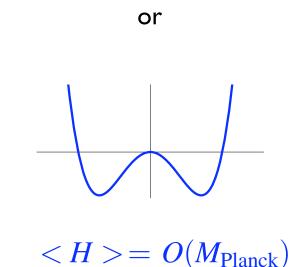


$$V(H) = m^2 H^2 + \lambda H^4$$
perturbativity $\lambda \lesssim 16\pi^2$

$$V(H) = \sqrt{\frac{-m^2}{2\lambda}} \gtrsim O(\frac{|m|}{4\pi})$$

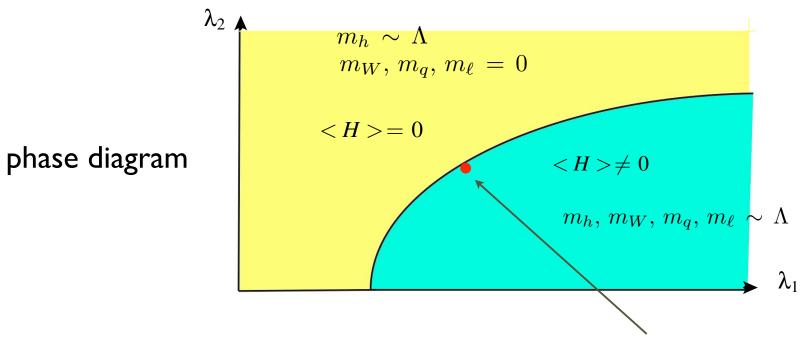
• m^2 picks up all sorts of additive quantum corrections if SM valid up to Planck scale then it is natural to expect $|m^2| \sim O(M_{\text{Planck}}^2)$





Graphical picture of hierarchy puzzle

$$\mathcal{L}_{fund} = \mathcal{L}(g_1, g_2, \Lambda, \dots; H, W^I_{\mu}, q, \ell, \dots)$$



SM lives extremely close to the critical line is

One way to phrase the hierarchy problem is more simply

Why
$$v_F \ll M_{Planck}$$
 ?

 $(G_F \gg G_N)$

 \diamond In order to better appreciate this question, we must understand why $m_{proton} \ll M_{Planck}$ is not considered to be a problem

As we shall see, the problem is in a sense deeper than stated above: bringing Planck scale down to TeV is not fully satisfactory Not enough to generically ask **why**

In order to infere **where*** do we expect new physics to show up

We need to better understand **what** is the Standard Model

* at what energy scale

Effective field theory approach to particle physics

working at tree level first

Physical scales & couplings

Ex: most general Lagrangian for scalar field

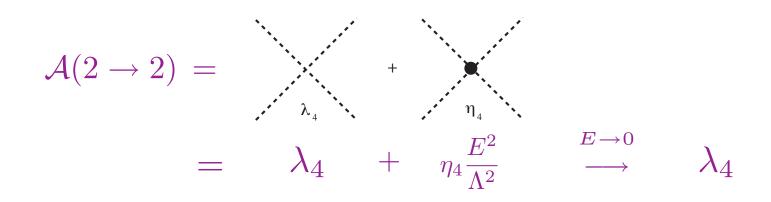
$$\mathcal{L} = \partial_{\mu} \phi \partial^{\mu} \phi - m^{2} \phi^{2} + \lambda_{4} \phi^{4} + \frac{\lambda_{6}}{\Lambda^{2}} \phi^{6} + \frac{\lambda_{8}}{\Lambda^{4}} \phi^{8} + \cdots$$
$$+ \frac{\eta_{4}}{\Lambda^{2}} \phi^{2} \partial_{\mu} \phi \partial^{\mu} \phi + \frac{\eta_{6}}{\Lambda^{4}} \phi^{4} \partial_{\mu} \phi \partial^{\mu} \phi \cdots$$

dimensions
$$\rightarrow$$

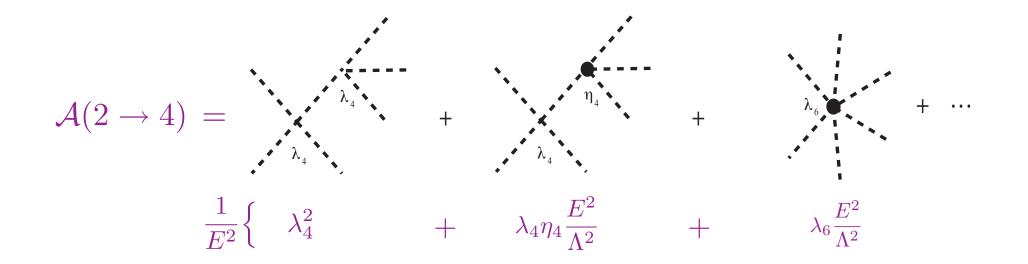
$$\begin{cases} [\mathcal{L}] = \frac{Energy}{(Length)^3} = E^4 \\ [\partial_{\mu}] = \frac{1}{Length} = E \end{cases} \qquad [\phi] = E \end{cases}$$

 $\lambda_i, \eta_i = \text{dimensionless}$

(assume O(1))



$$\mathcal{L} = \partial_{\mu} \phi \partial^{\mu} \phi - m^{2} \phi^{2} + \lambda_{4} \phi^{4} + \frac{\lambda_{6}}{\Lambda^{2}} \phi^{6} + \frac{\lambda_{8}}{\Lambda^{4}} \phi^{8} + \cdots$$
$$+ \frac{\eta_{4}}{\Lambda^{2}} \phi^{2} \partial_{\mu} \phi \partial^{\mu} \phi + \frac{\eta_{6}}{\Lambda^{4}} \phi^{4} \partial_{\mu} \phi \partial^{\mu} \phi \cdots$$



 $E \ll \Lambda$ only a finite number of terms in the lagrangian are important the infinite set of couplings with negative mass dimensions is *irrelevant*

coupling
$$g$$
 with dimension $[g] = d$ $\overline{g} \equiv g E^{-d}$

dimensionless quantity controlling strength of interaction

weak coupling
$$\checkmark$$
 $\bar{g} \ll 1$

★ d > 0 : relevant at small E

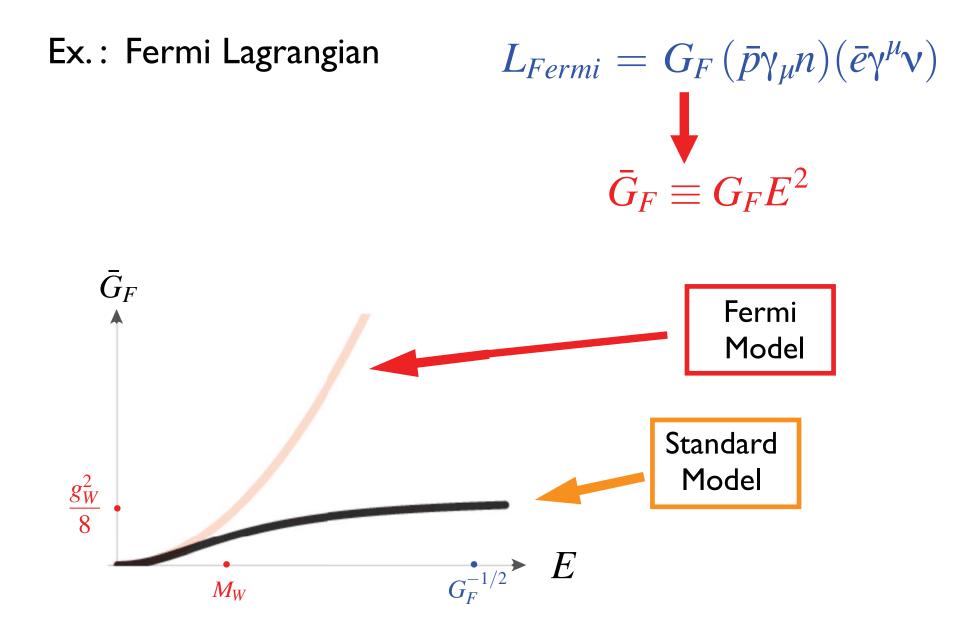
• *Ex: can treat mass as perturbation at E>> m* $\left(\bar{m}^2 = \frac{m^2}{F^2}\right)$

★ d = 0 : relevant at all energies (marginal)

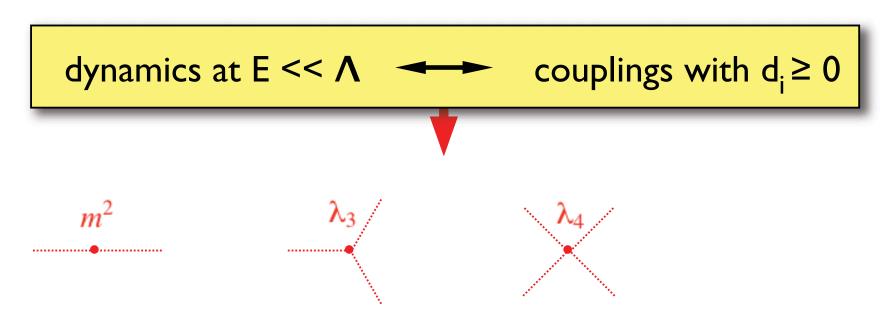
• gauge and Yukawa couplings

★ d < 0 : irrelevant al small E

• perturbative expansion breaks down at high enough E



Imagine all couplings with d_i < 0 scale like inverse powers of a single scale Λ



- $(m^2, \lambda_3, \lambda_4)$ fully describe an elementary (pointlike) particle
- $(\lambda_5, \lambda_6, \dots)$ correspond to inner structure
- to probe structure, $E \approx \Lambda$ is needed \rightarrow wavelength $\approx \frac{1}{\Lambda}$

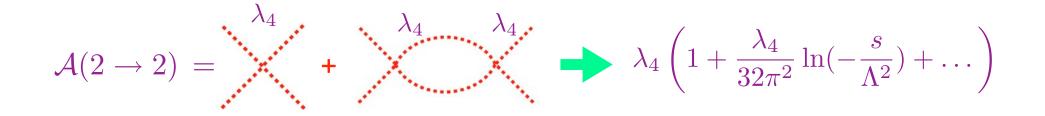
Now at the quantum level.....

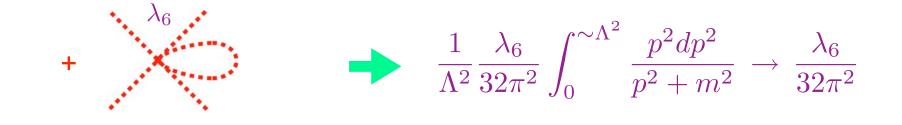
(a more physical picture of renormalizability)

Problem: internal momentum of loops is not fixed by external momentum

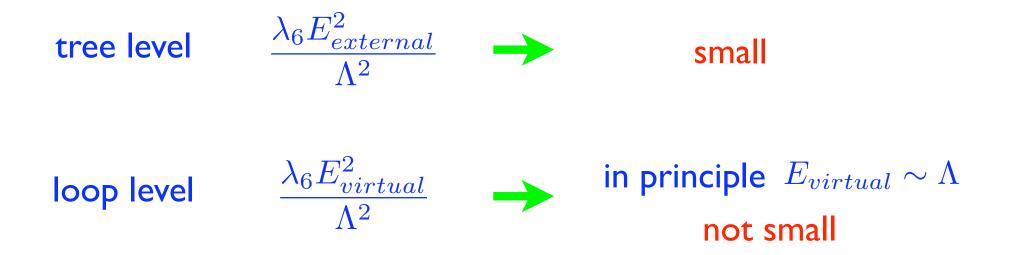
contributions enhanced by powers of cut-off

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{4!} \lambda_4 \phi^4 - \frac{1}{6!} \frac{\lambda_6}{\Lambda^2} \phi^6 - \frac{\lambda_8}{\Lambda^4} \phi^8 + \cdots$$





does not vanish when $\Lambda o \infty$



Apparently operators of arbitrarily high dimension matter!

But notice that UV enhanced contribution is **local**



UV enhanced contribution is just a renormalization of quartic term

Result generalizes to all orders

Power divergent effects can be reabsorbed by renormalization of coefficient of lower dimension operators

must exist a scheme where these effects are absent ab initio

Dimensional Regularization

$$\mathcal{L}_{eff} = \mathcal{L}(g,\lambda)^{d \leq 4} + \frac{1}{\Lambda} \mathcal{L}^{d=5} + \frac{1}{\Lambda^2} \mathcal{L}^{d=6} + \dots$$

at $E \ll \Lambda$, neglecting effects $\left(rac{E}{\Lambda}
ight)^{\#}$, \mathcal{L}_{eff} is equivalent to

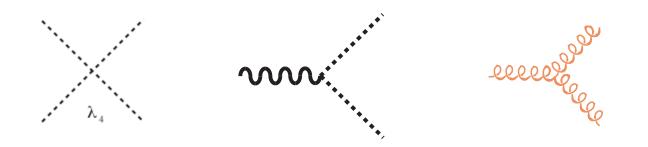
$$\mathcal{L}'_{eff} = \mathcal{L}(g', \lambda')^{d \le 4} + 0$$

virtual effects of $\mathcal{L}^{d=5}, \mathcal{L}^{d=6}, \dots$ accounted just by *renormalization*

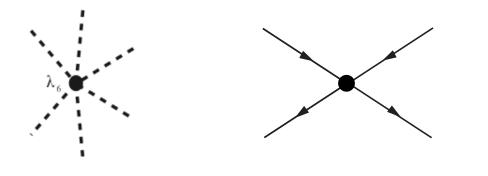
$$g, \lambda \longrightarrow g', \lambda'$$

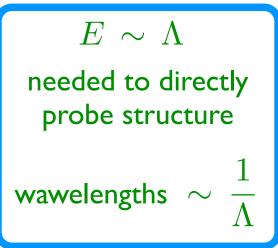
$E \ll \Lambda$ physics is described by *renormalizable* Lagrangian $\mathcal{L}^{d \leq 4}$

the 'renormalizable' terms (dimension 4 or less) fully describe elementary (pointlike) particles



'non-renormalizable' terms (dimension 5 or more) describe inner structure of particles





Analogy with multipole expansion in electrodynamics

$$\rho(x)$$

$$\rho(x)$$
 = charge density
 $\Phi(x)$ = clostric potential

$$\Psi(x)$$
 – electric potential

$$E_{\text{int}} = \int \Phi(x) \rho(x) d^3 x = \Phi(0) \int \rho(x) + \partial_i \Phi(0) \int x^i \rho(x) + \frac{1}{2} \partial_i \partial_j \Phi(0) \int x^i x^j \rho(x) + \dots$$
$$= \Phi(0) Q_0 + \partial_i \Phi(0) Q_1^i + \frac{1}{2} \partial_i \partial_j \Phi(0) Q_2^{ij} + \dots$$
$$\sim Q \qquad \sim QR \qquad \sim QR^2$$

At wavelengths > R light emission is dominated by dipole term

Accidental symmetries

$E \ll \Lambda$

dynamics determined by a **few** `renormalizable' couplings

extra (accidental) symmetries

Example: parity in QED is respected by `renormalizable' interactions

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \gamma^{\mu} D_{\mu} \psi + \bar{\psi} (m_1 + i \gamma_5 m_2) \psi + \frac{a}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

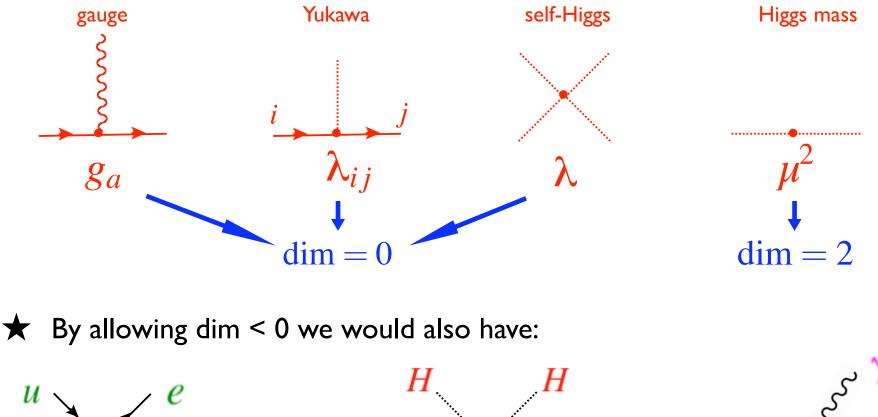
$$(m_1 + i\gamma_5 m_2) \rightarrow m = \sqrt{m_1^2 + m_2^2}$$
 by chiral rotation $\psi \rightarrow e^{i\beta\gamma_5}\psi$
 $F_{\mu\nu}\tilde{F}^{\mu\nu}$ = total derivative

dim 6 operator
violates parity
$$O_{F} = \frac{1}{\Lambda^2} (\bar{\psi} \gamma_\mu \psi) (\bar{\psi} \gamma_\mu \gamma_5 \psi)$$

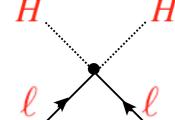
generated in SM by Z-exchange

$$\frac{1}{\Lambda^2} \sim G_F = \frac{1}{v^2}$$

Standard Model interactions







 $\frac{1}{\Lambda_{R}^{2}}\left(u_{\alpha}C\gamma_{\mu}u_{\beta}\right)\left(eC\gamma^{\mu}d_{\delta}\right)\varepsilon^{\alpha\beta\gamma}$

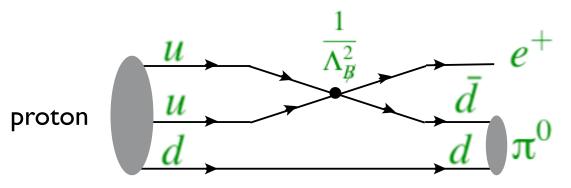
 $\left(\ell^a C \,\ell^b\right) H_a H_b$

Baryon number violation

Lepton number violation

Flavor violation

I) B+L violation: proton decay



$$p
ightarrow e^+ \pi^0$$

Superkamiokande: $\tau_p > 8.2 \times 10^{33}$ years

2) L violation: neutrino masses

3) Flavor violation

$$\mathcal{L} = \bar{q}_L \hat{Y}_d H^{\dagger} d_R + \bar{q}_L V_{CKM} \hat{Y}_u H u_R + \bar{\ell} \hat{Y}_{\ell} H^{\dagger} e_R$$

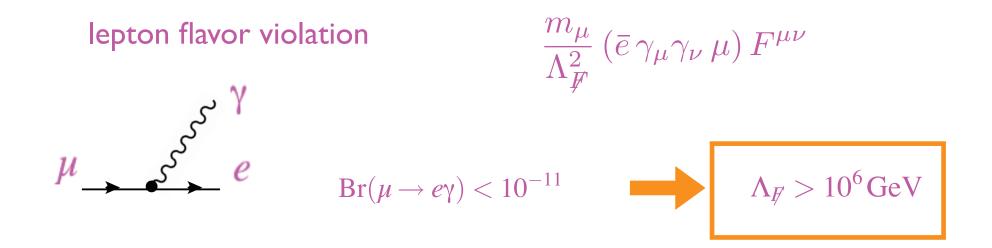
$$\hat{Y}_d = \begin{pmatrix} \lambda_d & & \\ & \lambda_s & \\ & & \lambda_b \end{pmatrix} \qquad \hat{Y}_u = \begin{pmatrix} \lambda_u & & \\ & \lambda_c & \\ & & \lambda_t \end{pmatrix} \qquad \hat{Y}_{\ell} = \begin{pmatrix} \lambda_e & & \\ & \lambda_\mu & \\ & & \lambda_\tau \end{pmatrix}$$

• absence of $\nu_R \longrightarrow L_e, L_\mu, L_\tau$ are conserved

lacksquark very special quark Flavor violation all due to V_{CKM} \clubsuit

Glashow-Iliopoulos-Maiani (GIM) suppression mechanism

$$K - \bar{K} \text{ mixing} \qquad \begin{cases} s & \mathsf{form} d \\ \bar{d} & \mathsf{form} d \\ \bar{d} & \mathsf{form} d \end{cases} \sim \quad \frac{G_F \alpha_W}{4\pi} \left(\sin \theta_C \cos \theta_C \right)^2 \left(\frac{m_c}{M_W} \right)^2 \quad \left[\bar{d}_L \gamma^\mu s_L \right]^2 \end{cases}$$



We are tempted to conclude that the scale of "compositeness" Λ in the S.M. is extremely high



The hierarchy problem

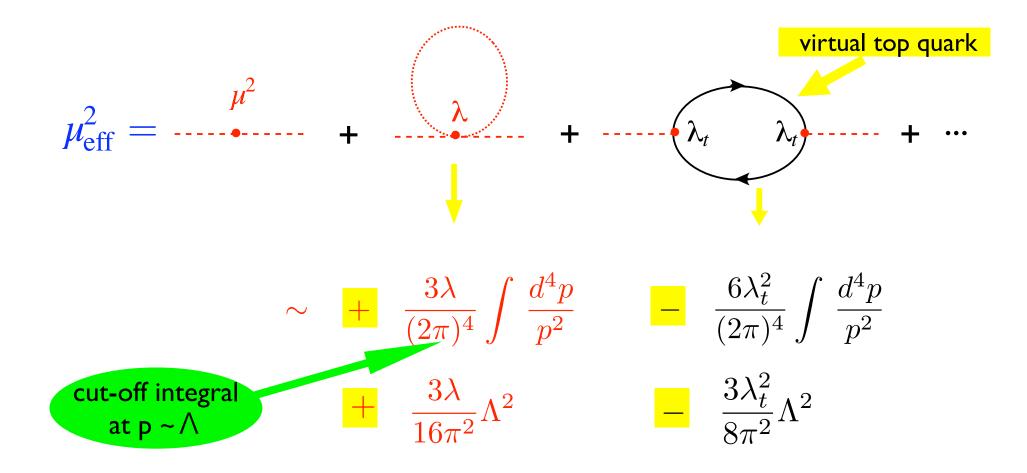
$$\mathcal{L}_{SM} = \mathcal{L}^{d=2} + \mathcal{L}(g,\lambda)^{d=4} + \frac{1}{\Lambda} \mathcal{L}^{d=5} + \dots$$
$$\overset{\clubsuit}{\mu^2 H^{\dagger} H}$$

is it reasonable to expect $|\mu^2| \ll \Lambda^2$?

one way to try and answer is to assume a hierarchy exists at tree level:

$$|\mu_{\rm tree}^2| \ll \Lambda^2$$

and estimate quantum effects to see if they mantain this hierarchy



 $\mu_{\rm eff}^2$ does not like to stay small when $\Lambda \rightarrow \infty$!!

quantum correction to the vacuum energy: top quark contribution

$$\Delta E = -\frac{1}{2} \sum_{i,k} \omega(k) = -\frac{12}{2} \int \sqrt{k^2 + m_t^2} \frac{d^3k}{(2\pi)^3} =$$

$$= -6 \int \left\{ k + \frac{m_t^2}{2k} + \cdots \right\} \frac{d^3k}{(2\pi)^3}$$

$$m_t^2 = \lambda_t^2 H^{\dagger} H$$

$$= -\frac{3}{2\pi^2} \int k^2 dk^2 - H^{\dagger} H \times \left(\frac{3}{4\pi^2} \lambda_t^2 \int dk^2\right)$$

$$\Delta \mu^2$$

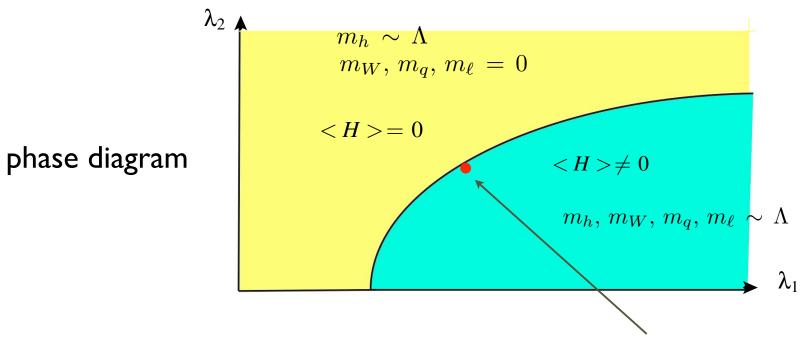
$$\Delta \mu^2$$

$$\mu_{eff}^2 = \mu^2 + c\Lambda^2$$
large $\Lambda \longrightarrow \mu^2$ must be tuned to make μ_{eff}^2 small
fine-tuning: $\frac{\mu^2 + c\Lambda^2}{\Lambda^2} \sim \frac{\upsilon_F^2}{\Lambda^2} \stackrel{\Lambda=10^{15}\text{GeV}}{=} 10^{-30}$

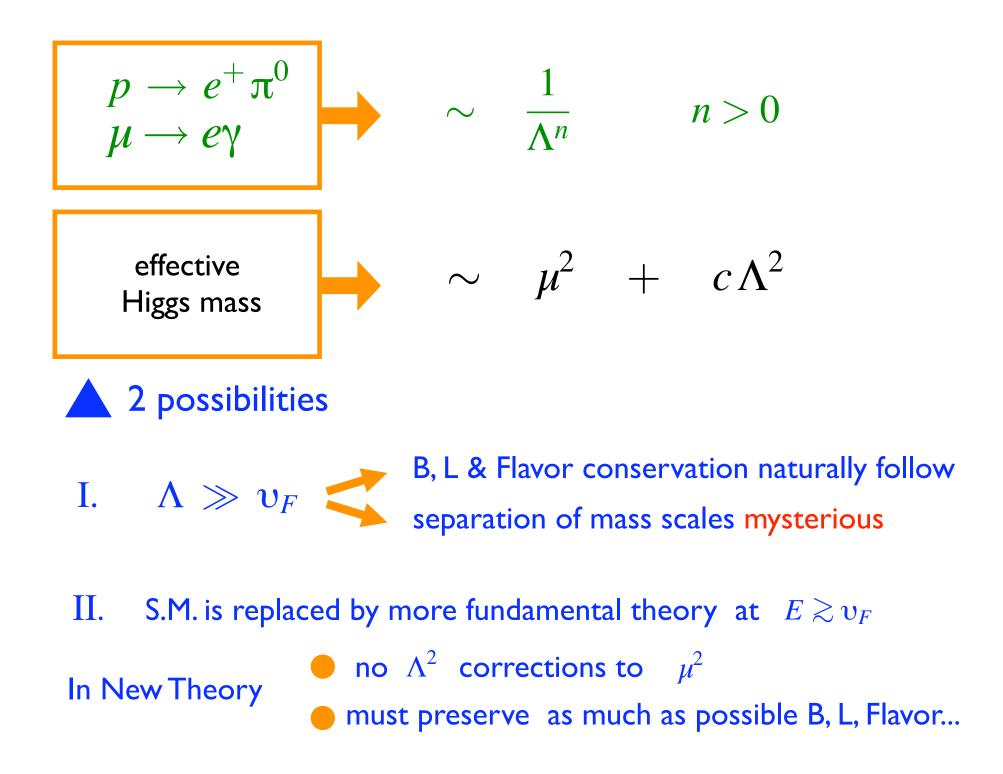
This is the hierarchy problem

Graphical picture of hierarchy puzzle

$$\mathcal{L}_{fund} = \mathcal{L}(g_1, g_2, \Lambda, \dots; H, W^I_{\mu}, q, \ell, \dots)$$



SM lives extremely close to the critical line is



The possibility of having $~~\Lambda \sim v_F$

makes LHC very exciting