Computing matrix roots: algorithms and related problems

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Let p be a positive integer. A primary pth root of a square matrix A is a solution of the matrix equation $X^p - A = 0$ which can be written as a polynomial of A.

If A has no nonpositive real eigenvalues then there exists only one primary pth root whose eigenvalues lie in the sector

$$\mathcal{S}_p = \{z \in \mathbb{C} \setminus \{0\} \, : \, |\arg(z)| < \pi/p\}, \tag{1}$$

which is called principal pth root and denoted by $A^{1/p}$.

The main numerical problem is to compute $(A^{1/p})^r$, for $0 < r < p$ integer. This problem is encountered in certain applications, among which financial models, and in the numerical computation of other matrix functions.

We present some reliable algorithms based either on matrix iterations or on the Schur normal form of A and we discuss their computational issues and some open problems.