The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2046-3

Summer College on Nonequilibrium Physics from Classical to Quantum Low Dimensional Systems

6 - 24 July 2009

Electron Cooling in Normal Metal-Superconductor Tunnel Junctions

Frank Hekking
Laboratoire de Physique et Modélisation

des Milieux Condensés

Maison des Magistères Jean Perrin

Electron Cooling
in Normal Metal -Superconductor
Tunnel Junctions

Frank Hekking

Université Joseph Fourier

Laboratoire de Physique et Modélisation
des Milieux Condensés
Maison des Magistères Jean Perrin
CNRS-Grenoble, France

Nonequilibrium Physics from Classical to Quantum Low dimensional Systems
ICTP Summer College, Trieste Italy, 06.7.2009 - 24.7.2009

Collaborators

A. Vasenko
LPMMC, Grenoble, France

H. Courtois, B. Pannetier, L. Pascal, S. Rajauria
Néel Institute, Grenoble, France

J. Pekola
Low Temperature Laboratory, TKK Helsinki, Finland

NANOSciERA consortium « NANOFRIDGE »

Outline

1. Introduction & motivation
- Electronic refrigeration with NIS junctions

2. Effects of Andreev reflection
- Enhanced Joule heat production due to coherence

3. Quasiparticle relaxation in S
- A note on the interest of quasiparticle traps

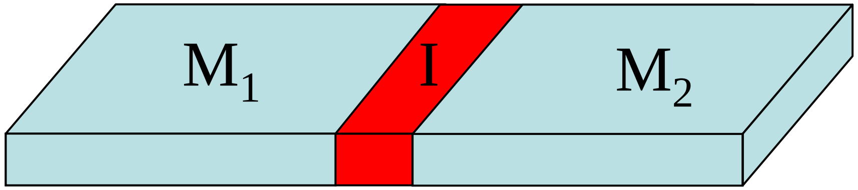
4. Towards a Brownian refrigerator
- Cooling out of noise?

Conclusions

1. Introduction & motivation

Electronic refrigeration with NIS junctions

Physics of tunnel junctions



Hamiltonian

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_T$$

$$H_T = \sum_{kq\sigma} T_{kq} c^\dagger_{q\sigma} c_{k\sigma} + \text{H.c.}$$

Golden Rule: tunnel rate

$$\Gamma_{1\to 2} = \frac{1}{e^2 R_T} \int dE n_1(E) n_2(E) f_1(E)[1 - f_2(E)]$$

tunnel resistance   density of states   electronic distribution

Nature of metal: dimensionless density of states

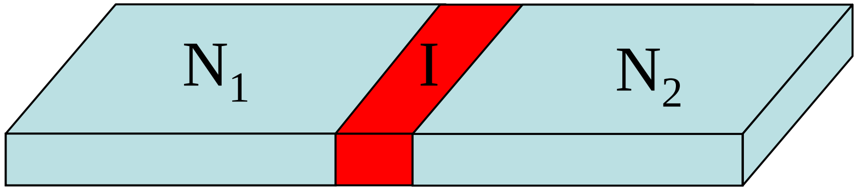
Normal metal

$$n_N(E) = 1$$

Superconducting metal

$$n_S(E) = \begin{cases} \dfrac{|E|}{\sqrt{E^2 - \Delta^2}} & |E| > \Delta \\ 0 & \textit{otherwise} \end{cases}$$

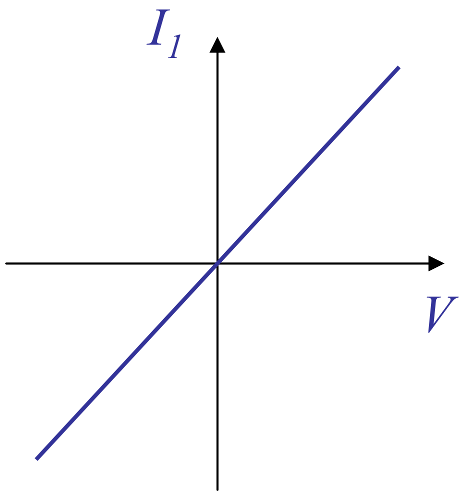
Thermo-electric transport in normal tunnel junctions



Voltage bias:

$$\mu_1 - \mu_2 = eV$$

Charge current out of electrode 1



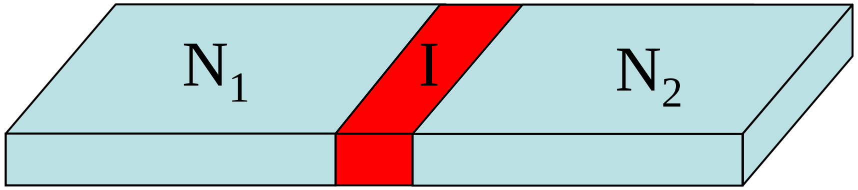
$$\begin{aligned} I_1(V) &= e[\Gamma_{1\to2}(V) - \Gamma_{2\to1}] \\ &= \frac{1}{eR_T}\int dE[f(E) - f(E+eV)] \\ &= V/R_T \\ &= -I_2(V) \end{aligned}$$

Fermi distribution

$$f(E) = [e^{\beta E} + 1]^{-1}$$

Electric current is conserved: $I_1 + I_2 = 0$

Thermo-electric transport in normal tunnel junctions



Voltage bias:

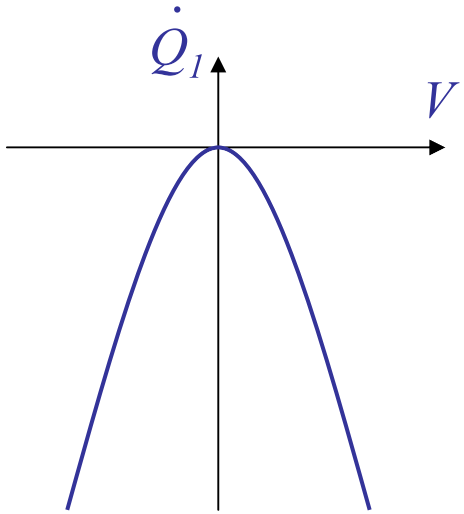
$$\mu_1 - \mu_2 = eV$$

Thermodynamics:

$$dQ = TdS = dU - \mu dN \longrightarrow E_k = \epsilon_k - \mu$$ *per particle*

Heat (entropy) current out of electrode 1

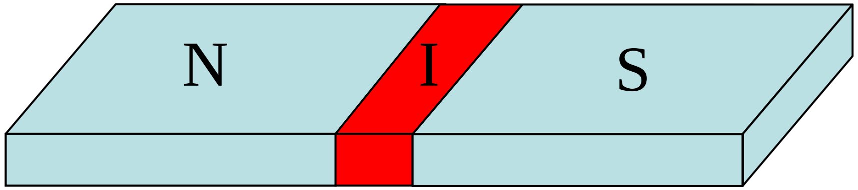
$$\begin{aligned}\dot{Q}_1(V) &= \frac{1}{e^2 R_T}\int dE E[f(E) - f(E+eV)] \\ &= -V^2/2R_T \\ &= \dot{Q}_2(V)\end{aligned}$$



Total entropy production: Joule heat $Q_1 + Q_2 = -V^2/R_T = -IV < 0$

(regardless of polarity: even function of V)

Thermo-electric transport in superconducting junctions



Voltage bias:

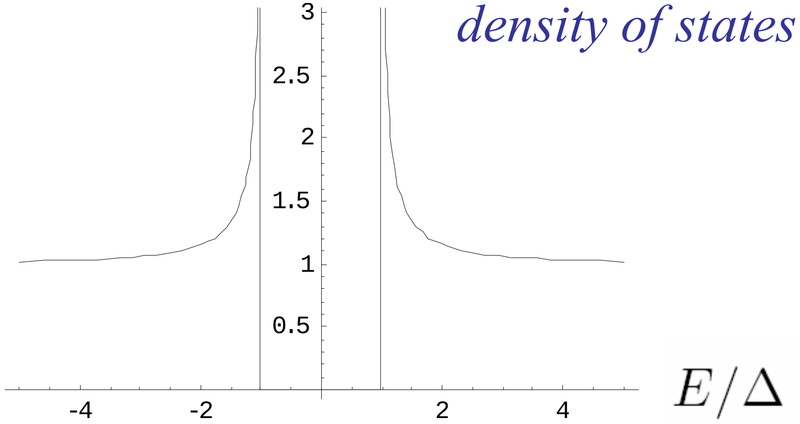
$$\mu_1 - \mu_2 = eV$$

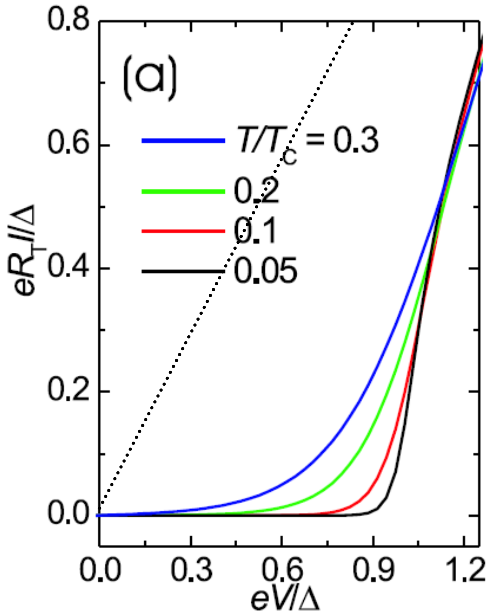
Charge current out of normal electrode

$$\begin{aligned} I_N(V) &= \frac{1}{eR_T}\int dE\, n_S(E+eV)[f(E) - f(E+eV)] \\ &= -I_S(V) \end{aligned}$$

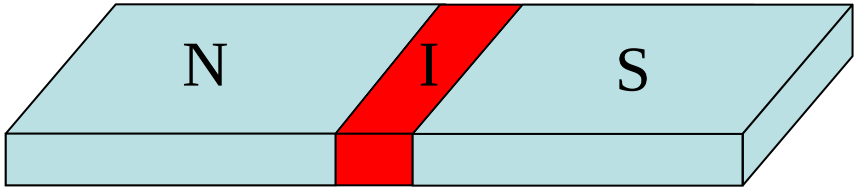
$n_S(E)$

density of states





Thermo-electric transport in superconducting junctions



Voltage bias:

$$\mu_1 - \mu_2 = eV$$

Heat (entropy) current out of normal electrode

$$\begin{aligned} \dot{Q}_N(V) &= \frac{1}{e^2 R_T} \int dE n_S(E+eV)E[f(E) - f(E+eV)] \\ &= -IV - \dot{Q}_S(V) \\ &\neq \dot{Q}_S(V) \end{aligned}$$

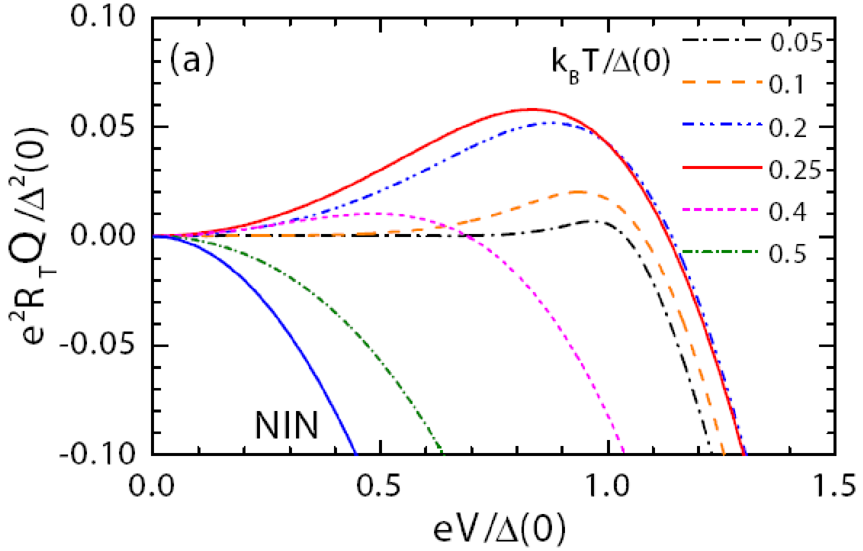
(Nahum, Eiles and Martinis, APL 94
Giazotto et al. RMP 06)

At optimum:

$$T \approx 0.25 \ \Delta/k_B$$

$$\dot{Q} \simeq 6 \times 10^{-2} \Delta^2/e^2 R_T$$

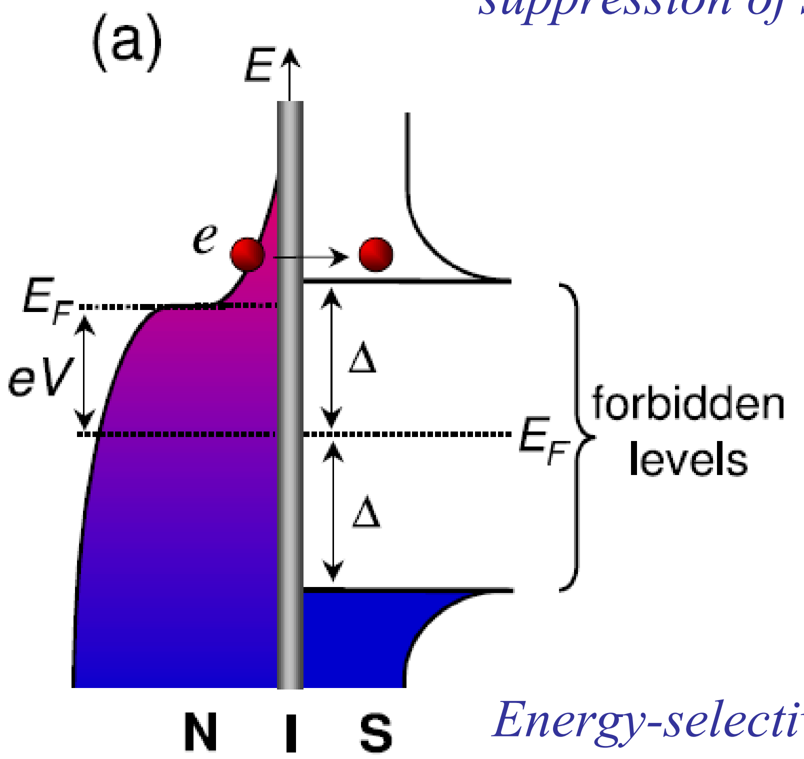
η ~ 20%



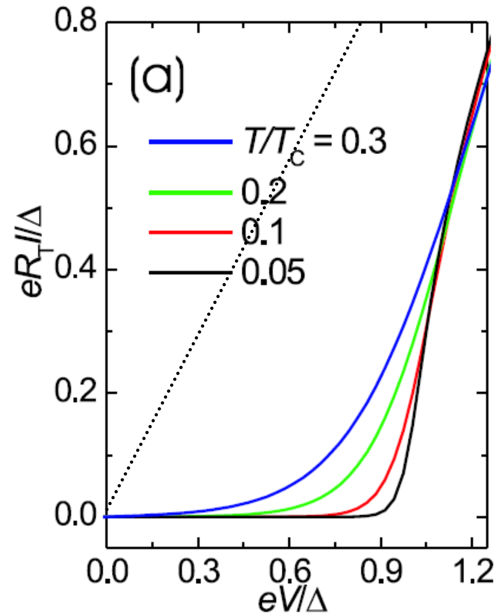
cooling

heating

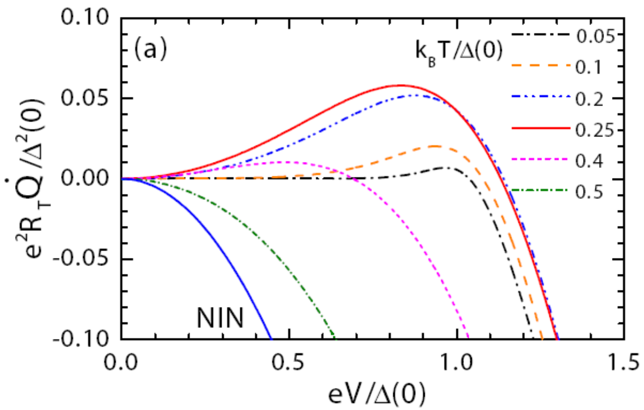
Density of states effect



Removal of states in the gap:
suppression of subgap current

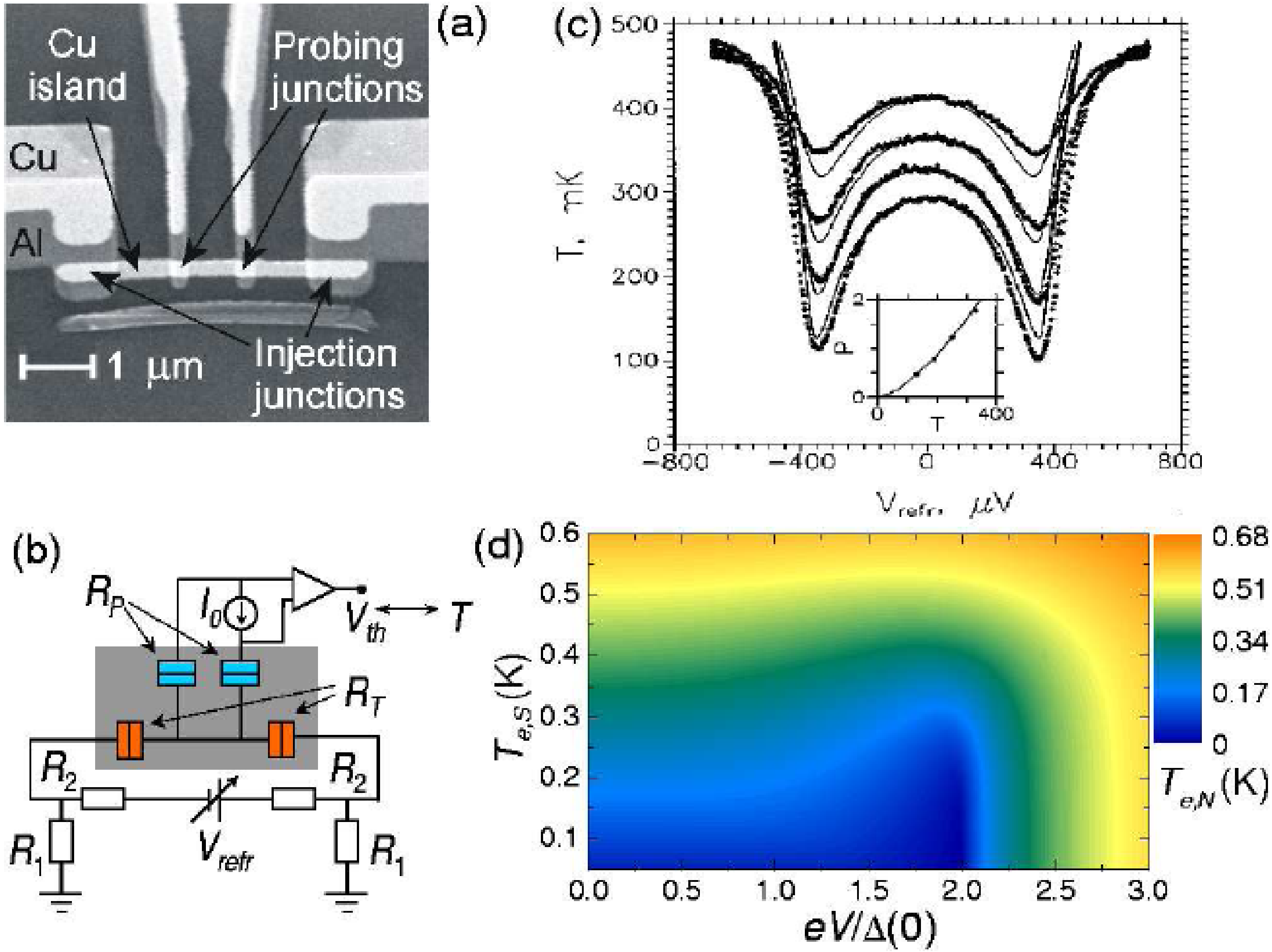


Energy-selective
quasiparticle tunnelling:



(Nahum, Eiles and Martinis, APL 94
Giazotto et al. RMP 06)

Experiments on superconducting junctions

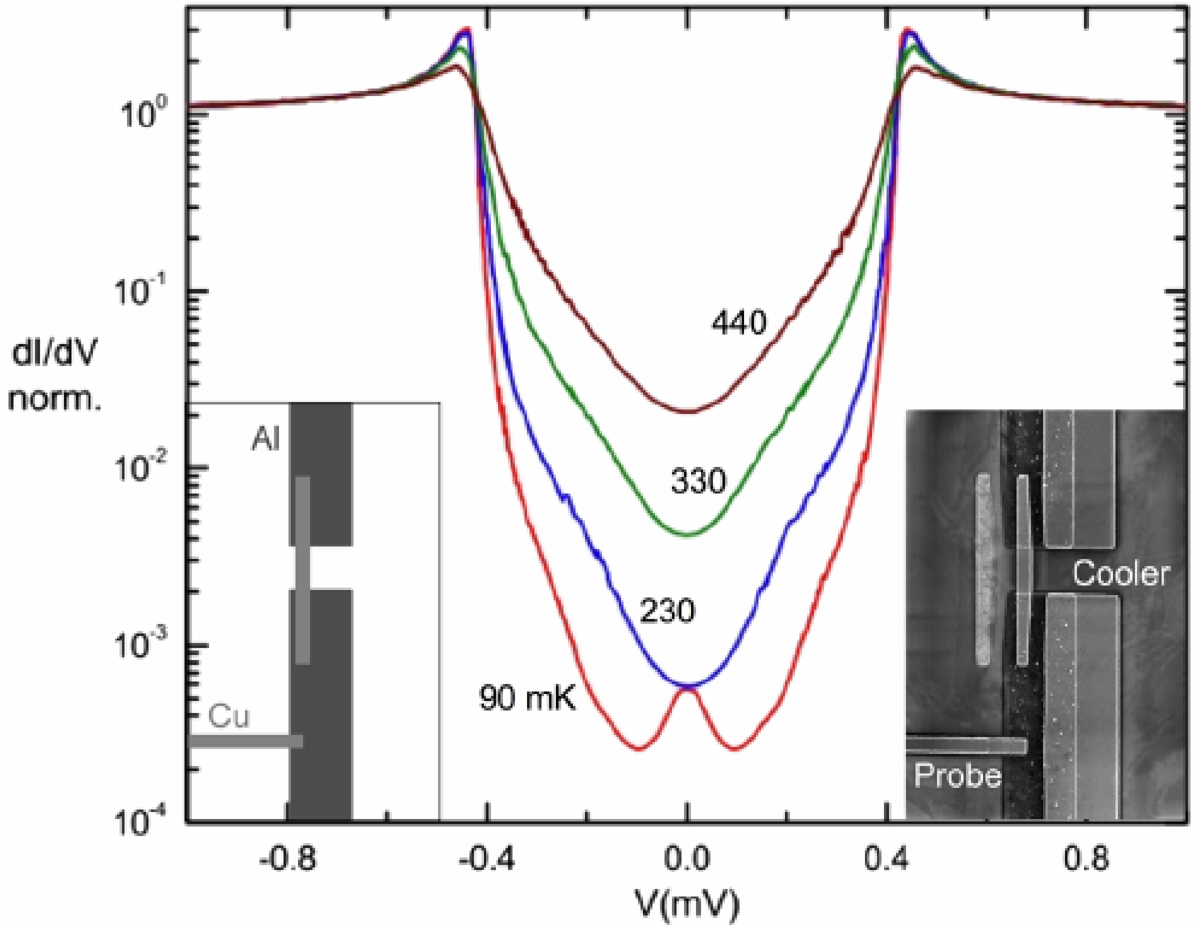


(Leivo et al. APL 96, Pekola et al., PRL 04)

2. Effects of Andreev reflection

Enhanced Joule heat production due to coherence

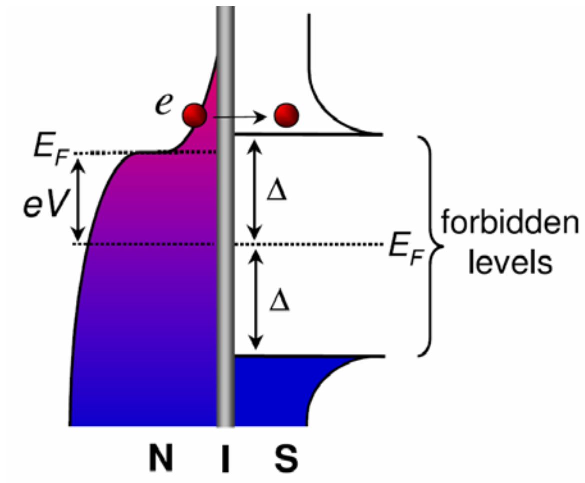
Zero-bias anomaly in NIS coolers

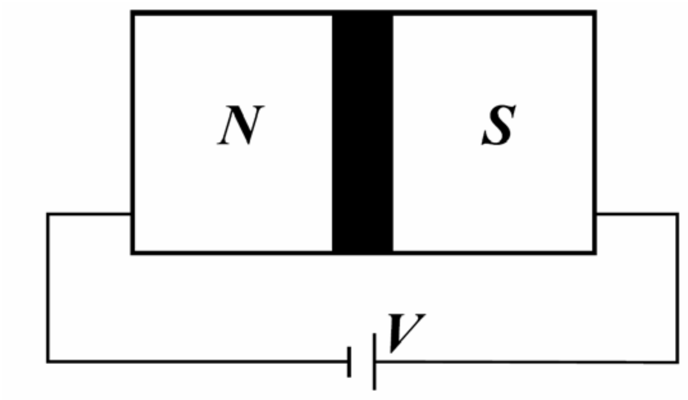


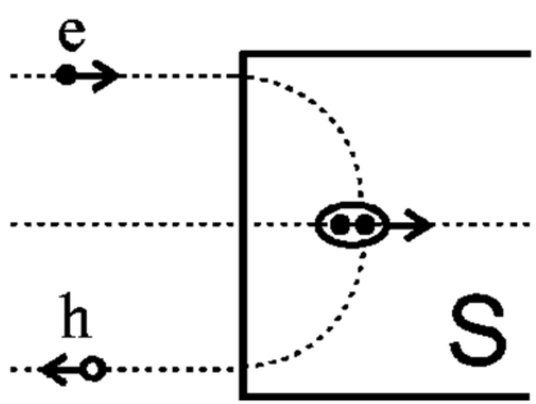
(Rajauria et al. PRL 08)

Single-particle & Andreev currents

Single particle current
E > Δ





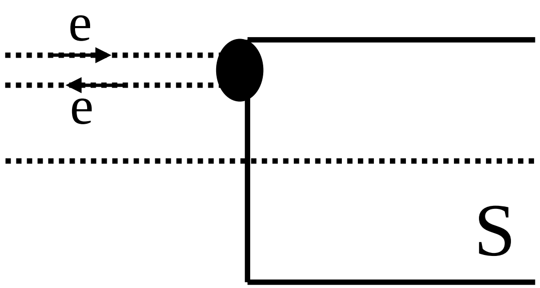


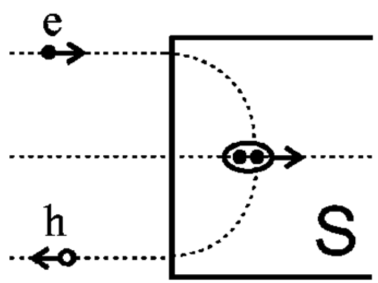
Andreev current
E < Δ

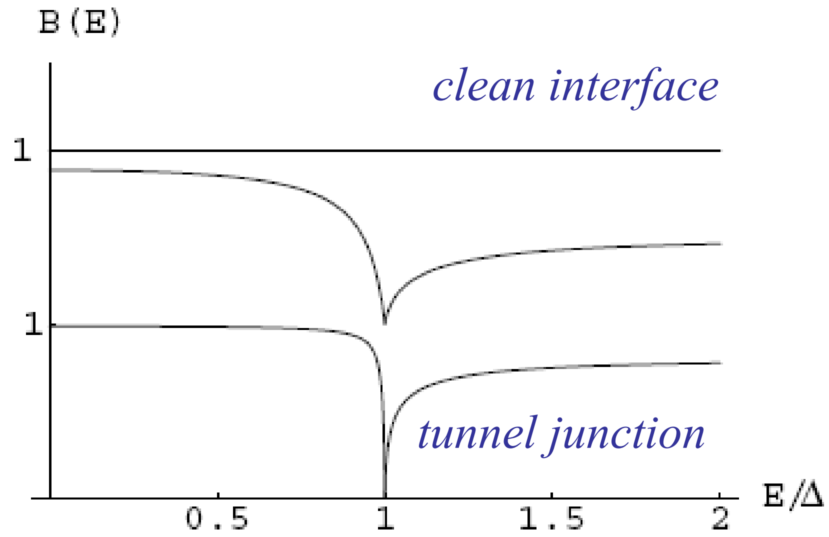
Ballistic junctions: Bogolubov-de Gennes theory (1)

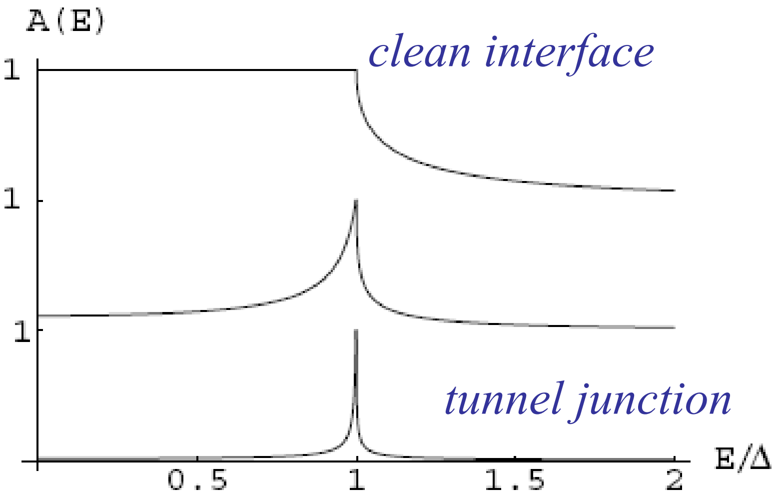
(Blonder, Tinkham & Klapwijk, PRB 82)

$$I_{NS} = \frac{e}{\pi\hbar}\int dE[f(E) - f(E+eV)][1 - B(E) + A(E)]$$



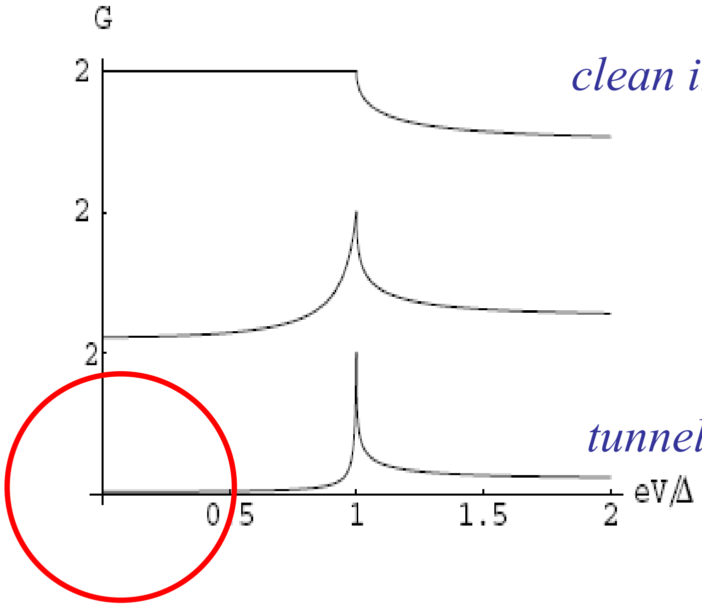


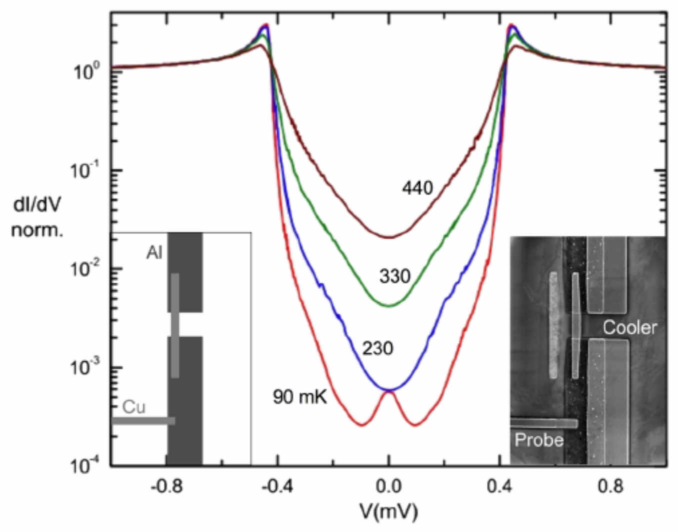




Ballistic junctions: Bogolubov-de Gennes theory (2)

Current-voltage characteristic

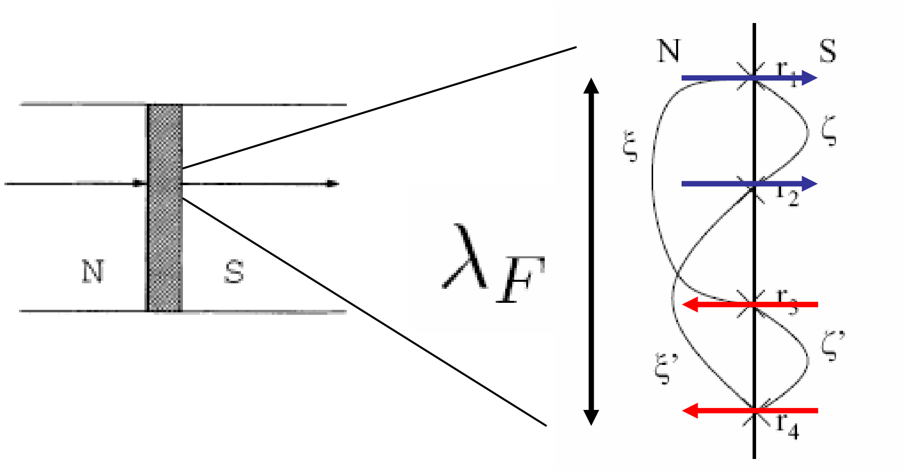




No anomaly…

… cf. experiment.

Thermo-electric subgap transport in ballistic tunnel junctions



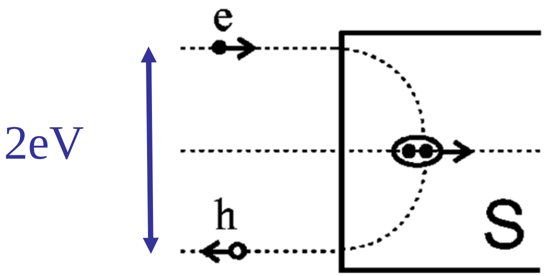
Subgap current

$$I = V/R_A$$

Subgap resistance

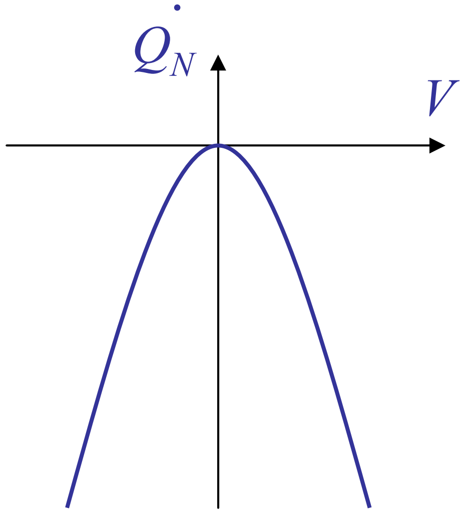
$$R_A = G_Q R_T^2 (k_F^2 S) \gg R_T$$

Heat current out of electrode N

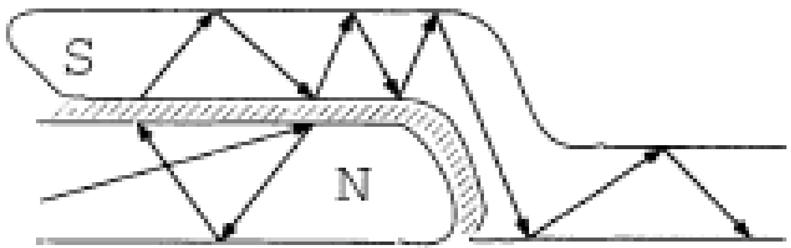


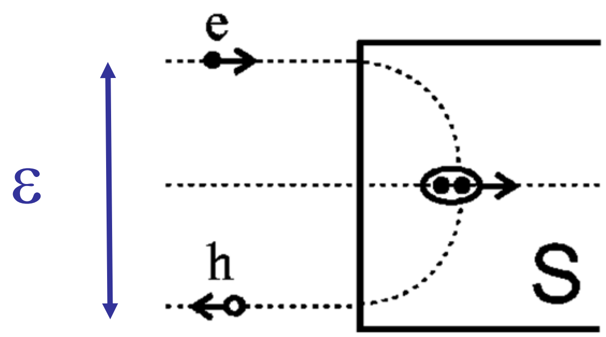
$$\dot{Q}_N = -V^2/R_A = -IV$$

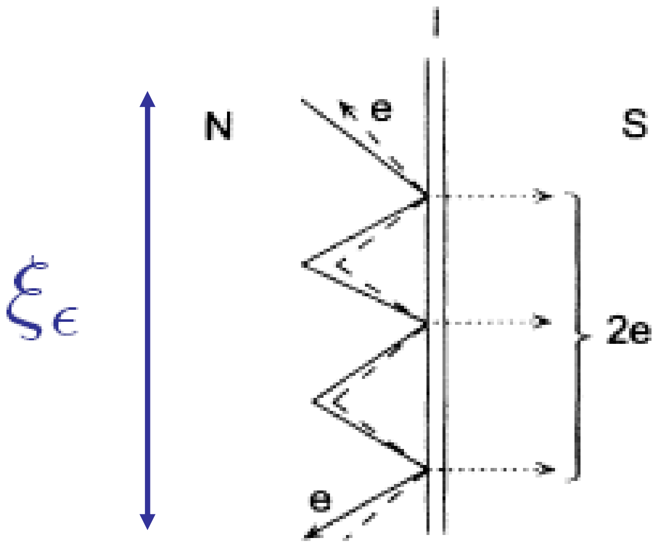
Total entropy production fully efficient:
Joule heat entirely deposited in N!



Subgap current in diffusive tunnel junctions (1)







Electron-hole dephase

$$\delta\phi_{eh} \propto \epsilon t$$

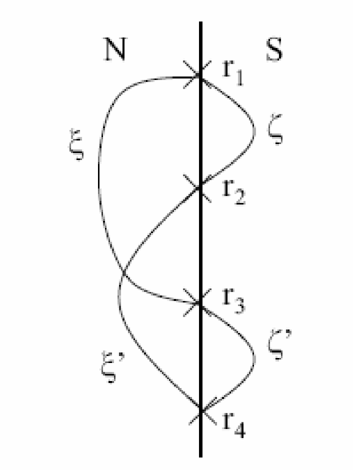
Relative phase lost after distance

$$\xi_\epsilon = \sqrt{\hbar D/\epsilon}$$

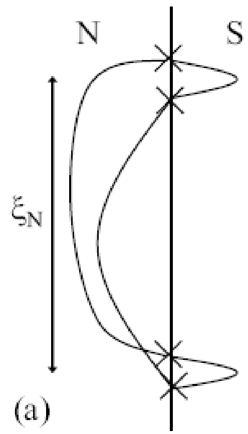
Amplitudes to be summed coherently over length ξ !

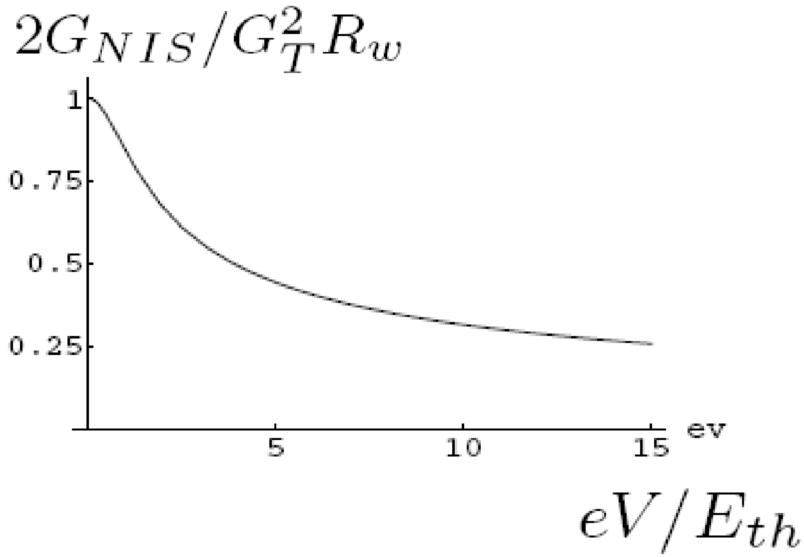
(Hekking & Nazarov, PRL 93; PRB 94)

Subgap current in diffusive tunnel junctions (2)



coherent sum





$$\xi_N = \sqrt{\hbar D/\max\{eV, k_B T\}}$$

Spectral current

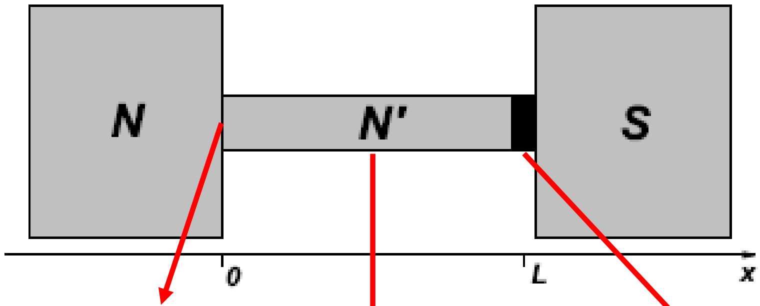
$$I(\epsilon) = \frac{\hbar}{8e^3\nu_0} \int_{barrier} d^2r_1 d^2r_2 \cos\left[\phi(\vec{r}_1) - \phi(\vec{r}_2)\right] g_T(\vec{r}_1) g_T(\vec{r}_2) \times$$
$$\left[P_\epsilon(\vec{r}_1 - \vec{r}_2) + P_{-\epsilon}(\vec{r}_1, \vec{r}_2)\right].$$

Subgap current

$$I_{NIS} = 2e\Gamma_{NIS} = \int d\epsilon I(\epsilon)\left[f(\epsilon/2 - eV) - f(\epsilon/2 + eV)\right]$$

(Hekking & Nazarov, PRL 93; PRB 94)

Beyond perturbation theory: Keldysh-Usadel theory



$$\check{G} = \begin{pmatrix} \hat{g}^R & \hat{G}^K \\ 0 & \hat{g}^A \end{pmatrix}, \quad \hat{G}^K = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A$$

continuity

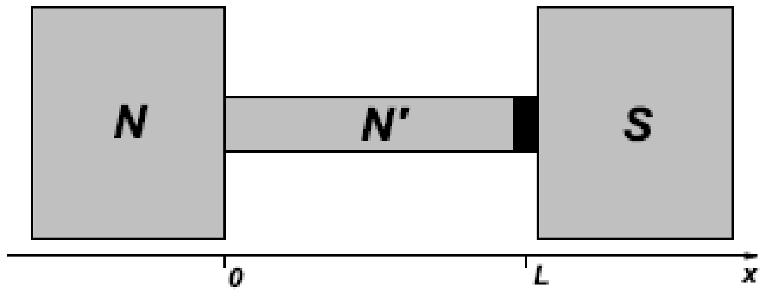
$$\check{J}_{N'} = (g_S/g_N)\check{J}_S = (2g_N R)^{-1}\left[\check{G}_{N'}, \check{G}_S\right]$$

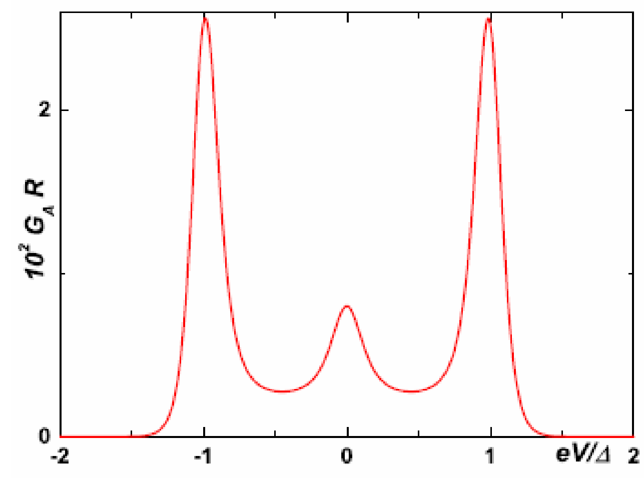
$$\left[\sigma_z E, \check{G}\right] = i\mathcal{D}\partial_x \check{J}, \quad \check{J} = \check{G}\partial_x \check{G}, \quad \check{G}^2 = 1$$

(Vasenko et al., unpublished)

Beyond perturbation theory: Keldysh-Usadel theory

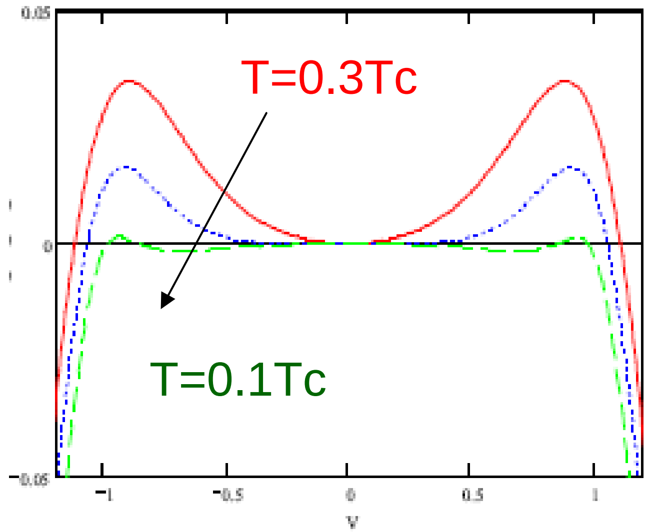
dI/dV vs. bias

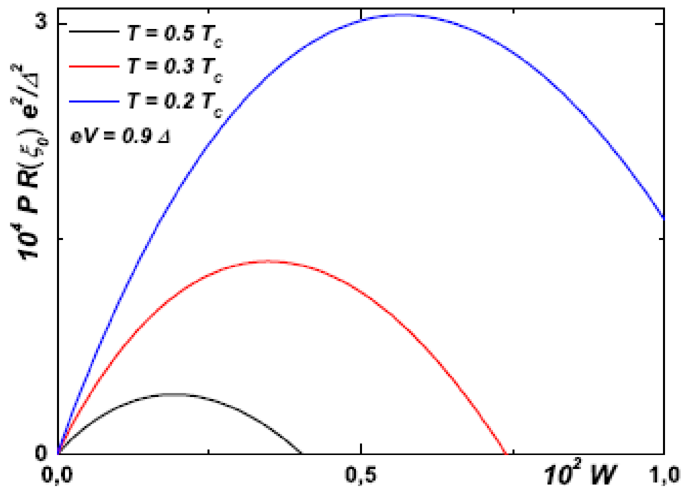




Cooling power vs. bias

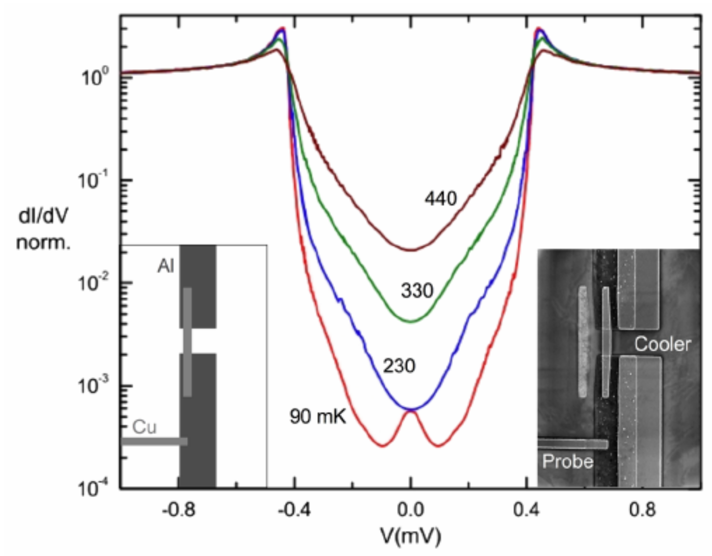
Cooling power vs. transparency

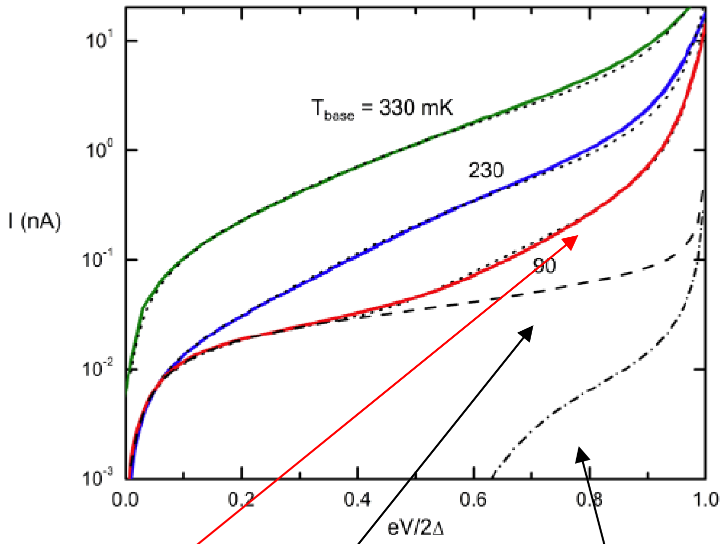


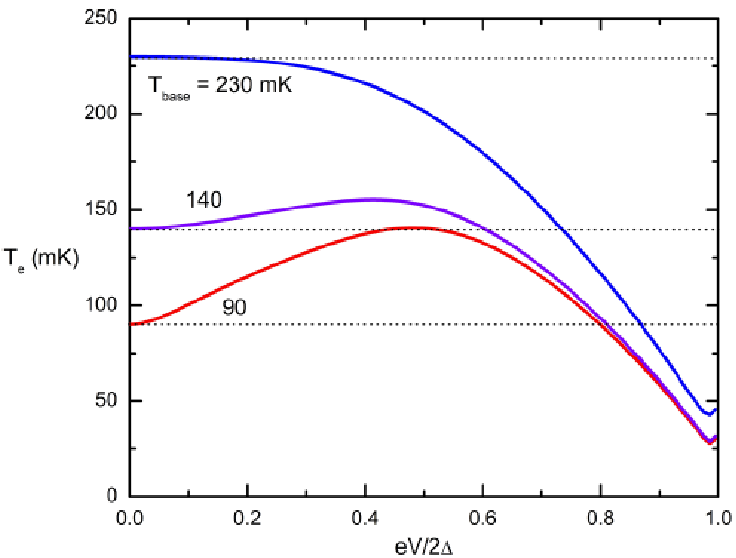


(Vasenko et al., unpublished)

Andreev reflection effects in SINIS coolers







Full theory

Two-particle tunneling

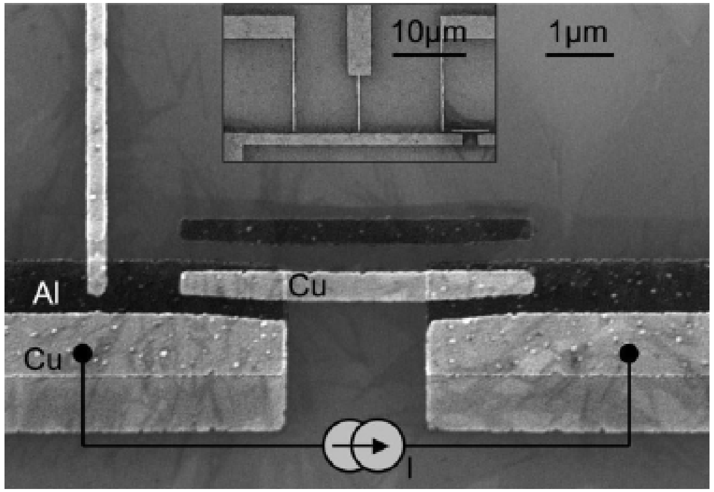
Single particle effects

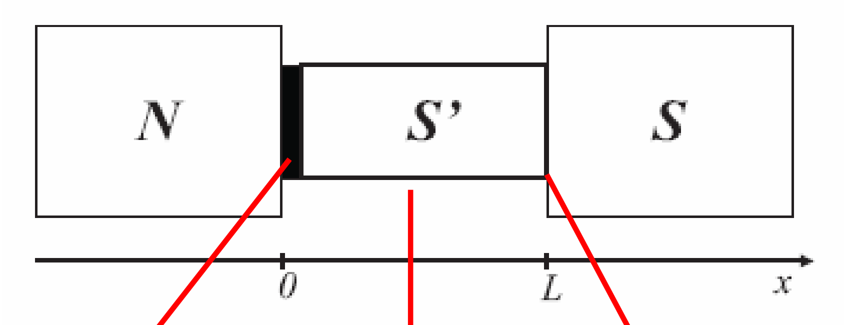
(Rajauria et al. PRL 08)

3. Quasiparticle relaxation in S

A note on the interest of quasiparticle traps

Nonequilibrium quasiparticle effects





continuity

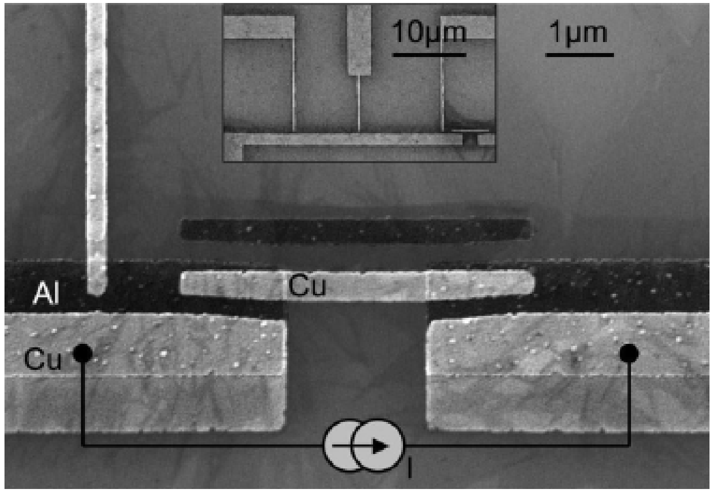
$$(\sigma_N/g_N)\check{J}_{-0} = \check{J}_{+0} = (W/\xi_0)\left[\check{G}_{-0}, \check{G}_{+0}\right]$$

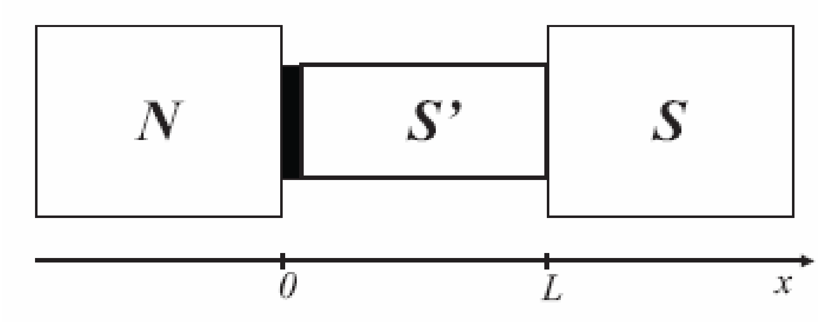
$$\left[\check{\sigma}_z E + \check{\Delta},\ \check{G}\right] = i\mathcal{D}\partial\check{J}, \quad \check{J} = \check{G}\partial\check{G}, \quad \check{G}^2 = \check{1}$$

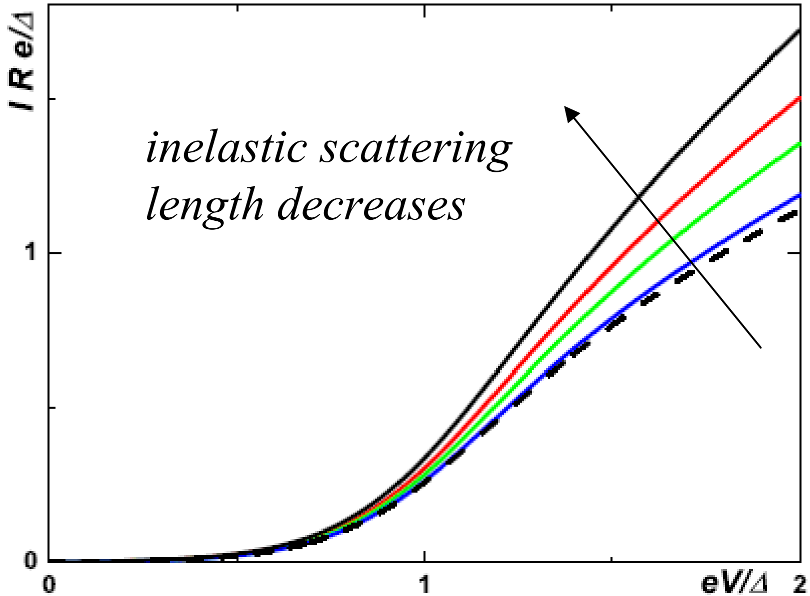
+ *inelastic scattering in relaxation time approach*

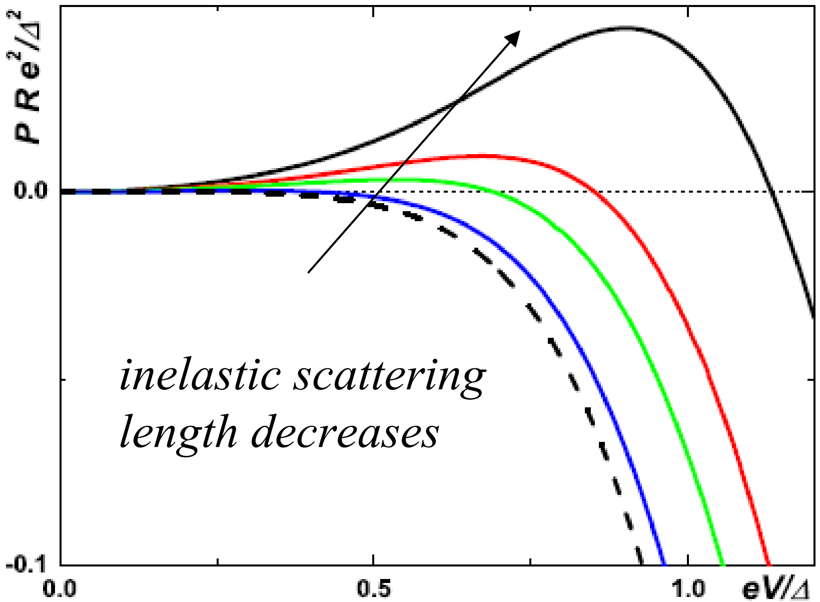
(Vasenko & Hekking, JLTP 08)

Nonequilibrium quasiparticle effects









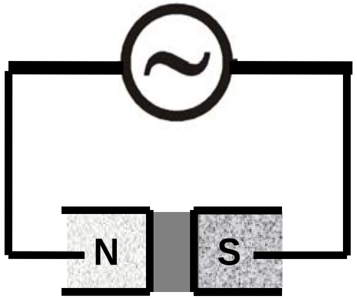
(Vasenko & Hekking, JLTP 08)

4. Towards a Brownian refrigerator

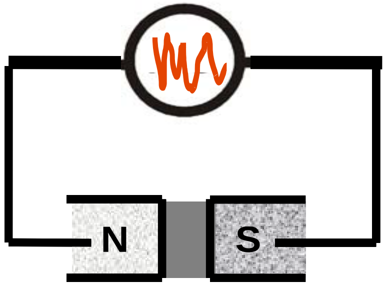
Cooling out of noise?

What is the effect of a time-dependent voltage?

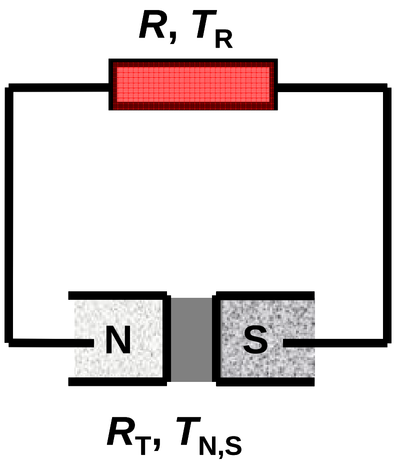
DC bias refrigerates regardless of polarity...



AC drive *– refrigerates N if frequency and amplitude are not too high*

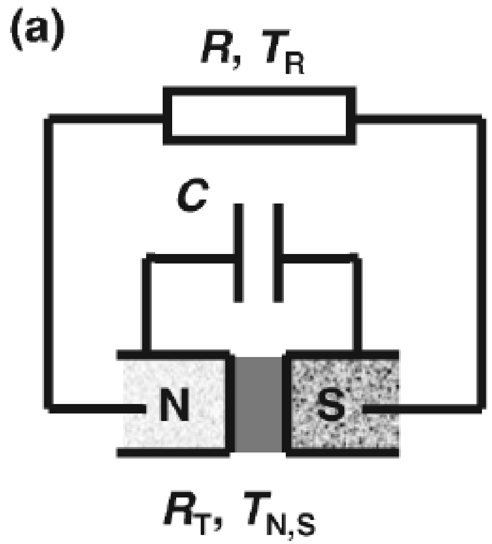


Stochastic drive *– refrigerates N if spectrum is ”suitable”*



Cooling with a hot resistor?

Fluctuations in NIS junction



Hamiltonian

$$\hat{H} = \hat{H}_N + \hat{H}_S + \hat{H}_T + \hat{H}_R$$

Tunnelling in the presence of fluctuations

$$H_T = \sum_{kq\sigma} T_{kq} c^{\dagger}_{q\sigma} c_{k\sigma} e^{-i\varphi} + \mathrm{H.c.}$$

phase related to voltage $\varphi(t) = eVt/\hbar$

(Ingold and Nazarov, Les Houches 92)

Voltage fluctuations induce phase fluctuations: fluctuation-dissipation theorem

$$\langle \varphi(t)\varphi(0)\rangle_R = 2\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{Re}Z_t(\omega)}{R_K} \frac{e^{-i\omega t}}{1-e^{-\beta\hbar\omega}}$$

Real part of resistance

$$\frac{\mathrm{Re}Z_t(\omega)}{R_K} = \frac{1}{R_K}\mathrm{Re}\left[\frac{1}{i\omega C + 1/R}\right] = \frac{1}{g}\frac{1}{1+(\omega/\omega_R)^2}$$

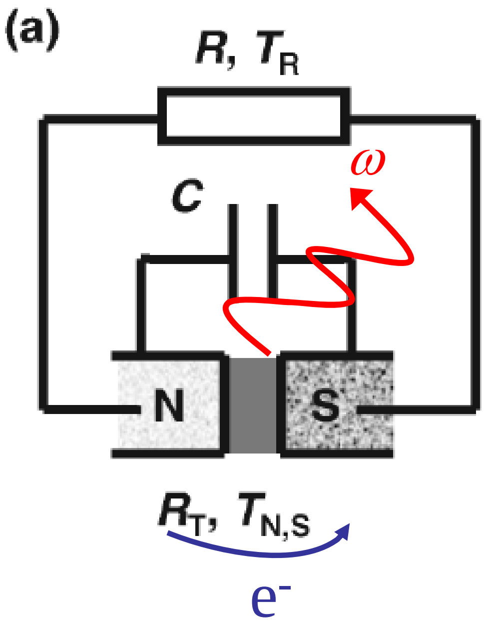
cut-off frequency $\omega_R = \dfrac{1}{RC} = \dfrac{g}{\pi}\dfrac{E_c}{\hbar}$

charging energy

$E_c = e^2/2C$

dimensionless conductance $g = \dfrac{R_K}{R}$

Heat current in NIS junction with fluctuations



Tunnelling in the presence of fluctuations yields photon exchange with probability

$$P(E) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \mathrm{d}t \exp\left[J(t) + \frac{i}{\hbar}Et\right]$$

$$J(t) = \langle[\varphi(t) - \varphi(0)]\varphi(0)\rangle$$

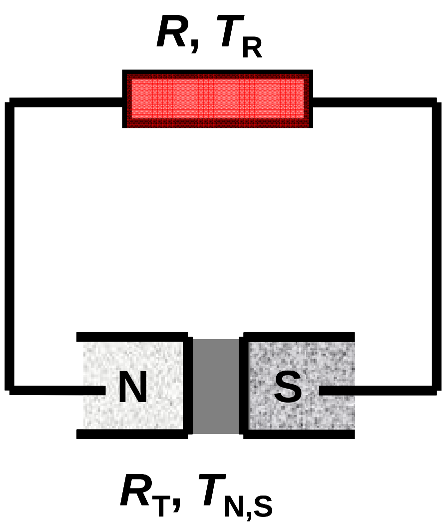
(Ingold and Nazarov, Les Houches 92)

Heat current in the presence of photon exchange:

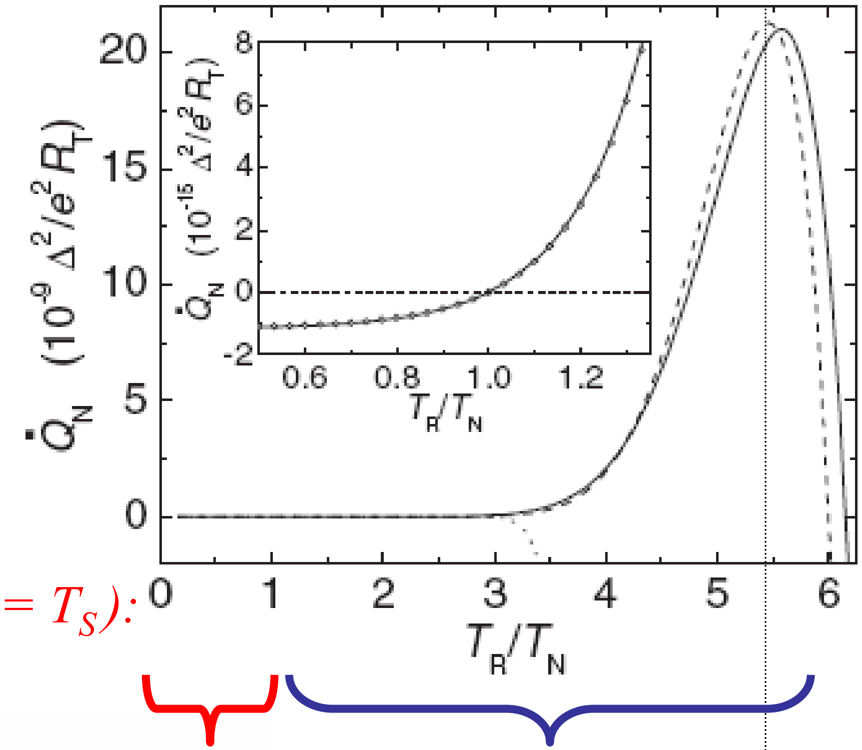
$$\dot{Q}_{\mathrm{N}\to\mathrm{S}} = \frac{1}{e^2 R_T}\int dE\,dE'\,\underbrace{E' f(E')}_{\textit{in N}}\underbrace{[1 - f(E)]n_S(E)}_{\textit{in S}}\underbrace{P(E' - E)}_{\textit{probability to exchange photon } \omega = E' - E}$$

(Pekola & Hekking, PRL 07)

Cooling out of noise: case *g << 1*



Numerical integration:



If g << 1: $P(E) = \delta(E - E_c)$

Approximate form for heat current ($T_N = T_S$):

$$\dot{Q}_N \simeq \frac{\sqrt{2\pi\Delta k_B T_N}}{e^2 R_T}\Delta e^{-\Delta/k_B T_N} \times \left\{\left[1 - \left(2\frac{T_R}{T_N} - 1\right)\frac{E_C}{\Delta}\right] e^{(T_R/T_N - 1)E_C/k_B T_N} + \frac{E_C}{\Delta} - 1\right\}.$$

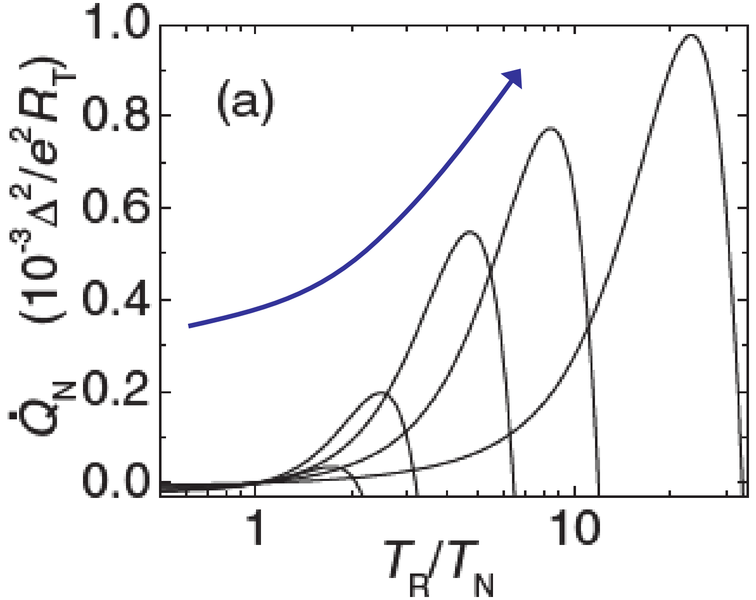
$$T_R^*/T_N \sim \Delta/E_C$$

optimum temperature

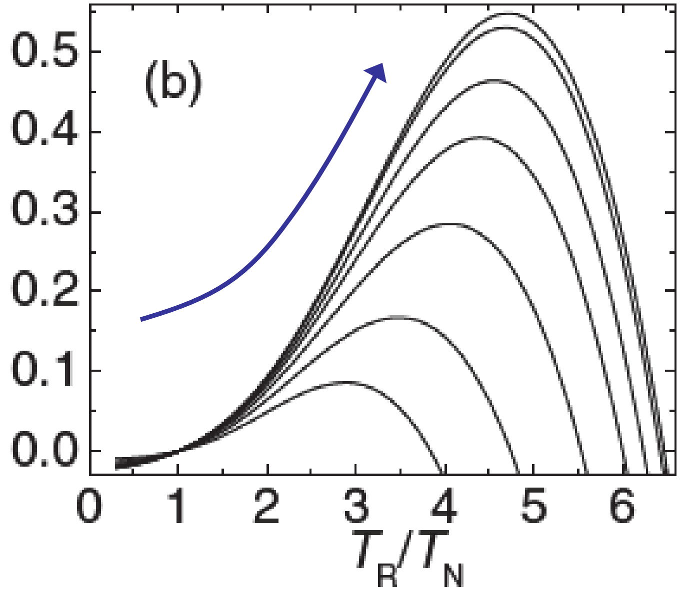
(Pekola & Hekking, PRL 07)

Cooling out of noise: general case

Increasing Δ/E_c



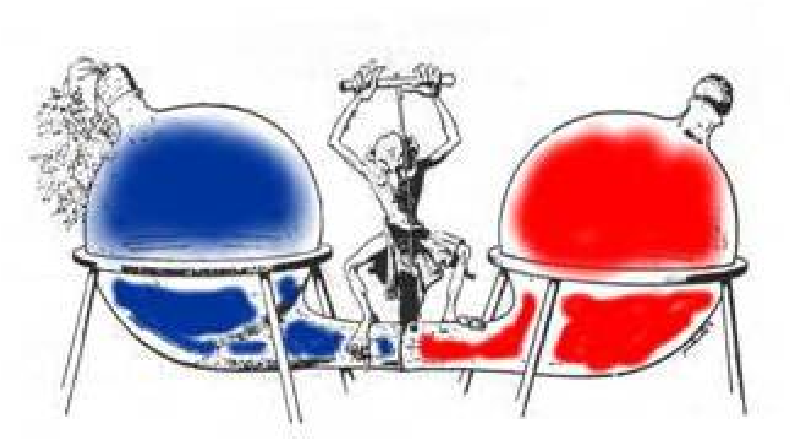
Decreasing g



(Pekola & Hekking, PRL 07)

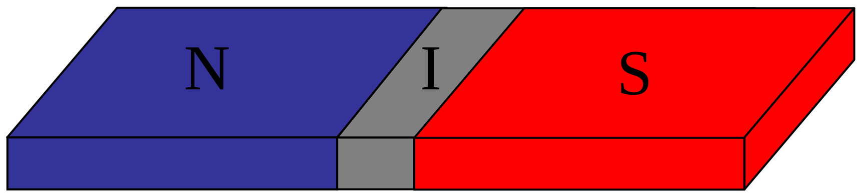
Is the Brownian fridge a Maxwell demon?

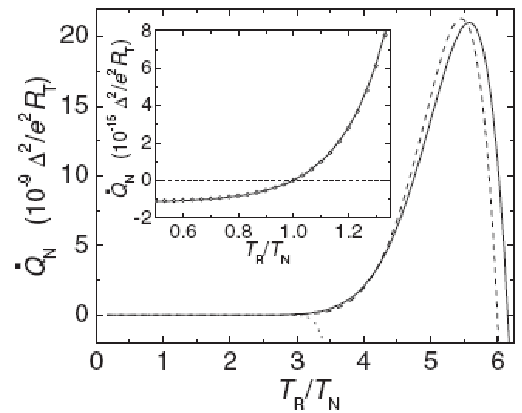
The demon allows hot particles to pass from left to right and cold ones from right to left...



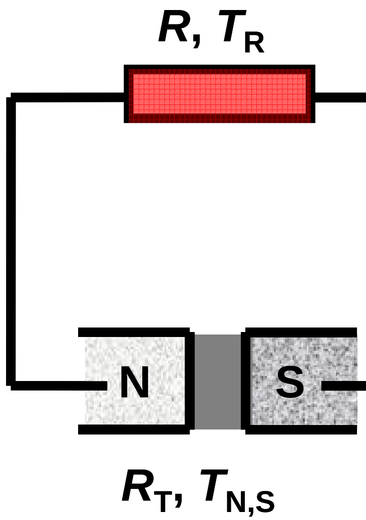
... and decreases the entropy of the system.

NIS junction: where is the demon?

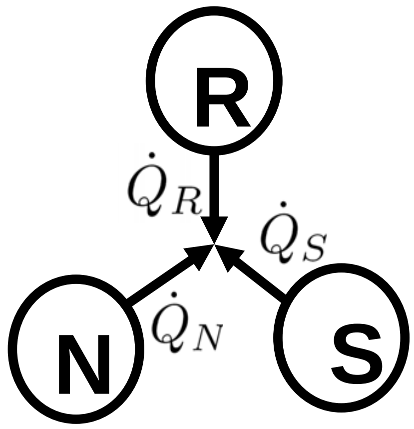




Entropy production in the Brownian refrigerator



$\longleftrightarrow$



Rate for entropy production

$$\dot{S} = -\dot{Q}_N/T_N - \dot{Q}_S/T_S - \dot{Q}_R/T_R$$

Sum of heat currents is zero

$$\dot{Q}_N + \dot{Q}_S + \dot{Q}_R = 0$$

Quantity to analyze

$$\dot{S}/k_B = (\beta_R - \beta_N)\dot{Q}_N + (\beta_R - \beta_S)\dot{Q}_S$$

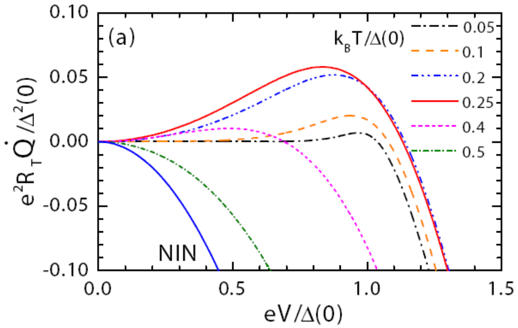
Special cases: $\beta_R < \beta_N < \beta_S$, $\beta_R > \beta_N > \beta_S$ analytically

Other cases: numerically

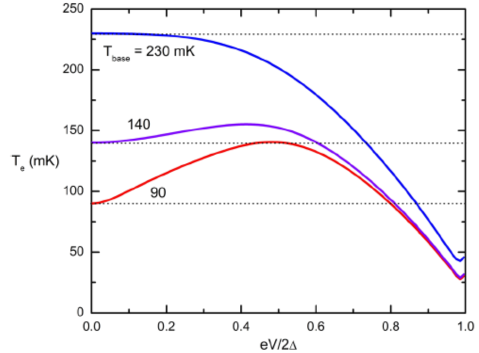
$\dot{S}$ ***ALWAYS*** $\boldsymbol{\geq 0}$

Conclusions

- DC-biased NIS junctions can be used for electronic cooling



- Refrigeration performance decreased by Andreev reflection process and lack of qp relaxation



- NIS junction as a rectifier: cooling out of noise, but leaving thermodynamics intact