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Summer College on Nonequilibrium Physics from Classical to Quantum Low Dimensional Systems

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Quantum Hall interferometers far from equilibrium

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Quantum Hall Interferometers

Far from Equilibrium

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Collaborators:

Y. Gefen and M. Veillette: Phys. Rev. B 76, 085320 (2007)

D. Kovrizhin: arXiv:0903.3387

Outline

- Quantum Hall edge states

  and edge state interferometers

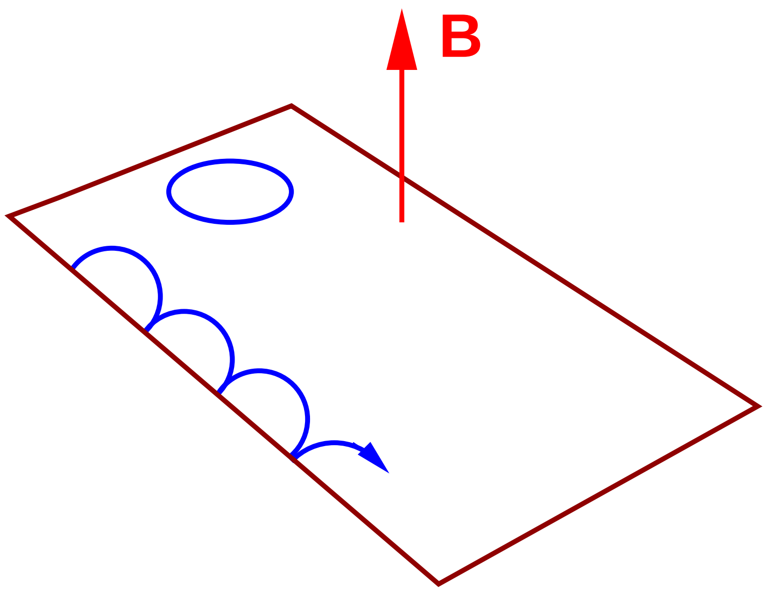
- Interferometer experiments far from equilibrium

- The two particle problem

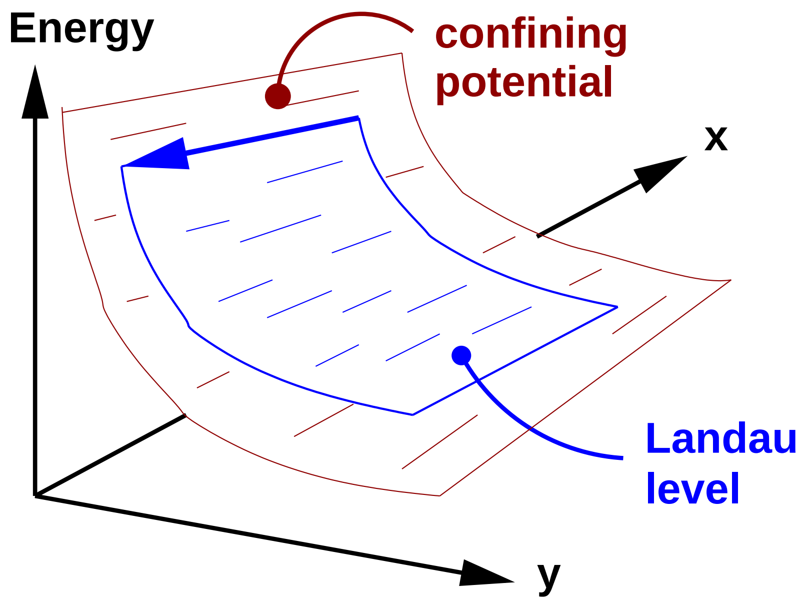
  and the many-body problem

Quantum Hall Edge States

Classical skipping orbits



Quantum edge states

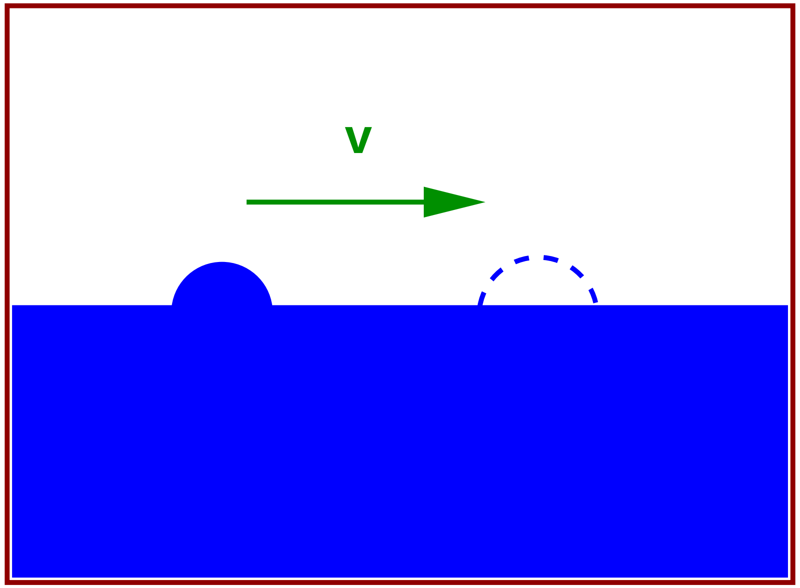


Two-dimensional electron gas in magnetic field

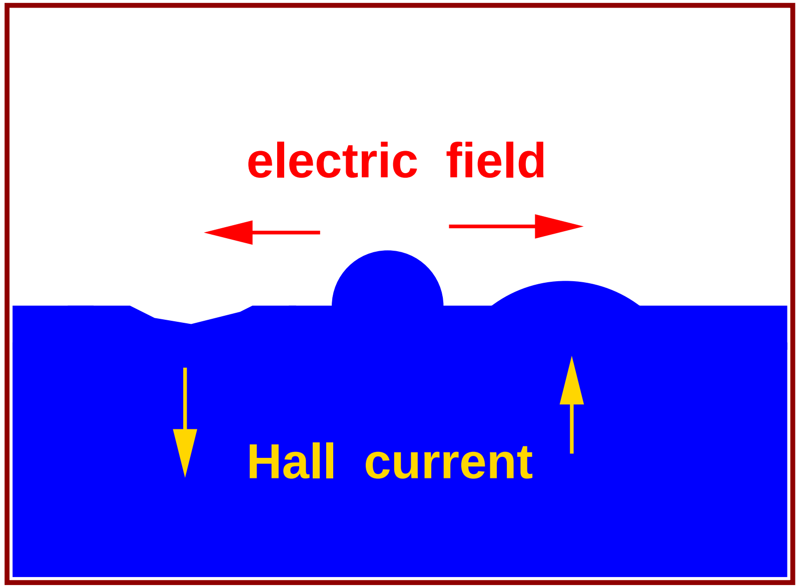
Edge state Hamiltonian: $\mathcal{H} = \int \psi^\dagger(x)(-i\hbar v \partial_x)\psi(x)dx$

Edge state dynamics with interactions

Free propagation



Charge flow in and out of bulk

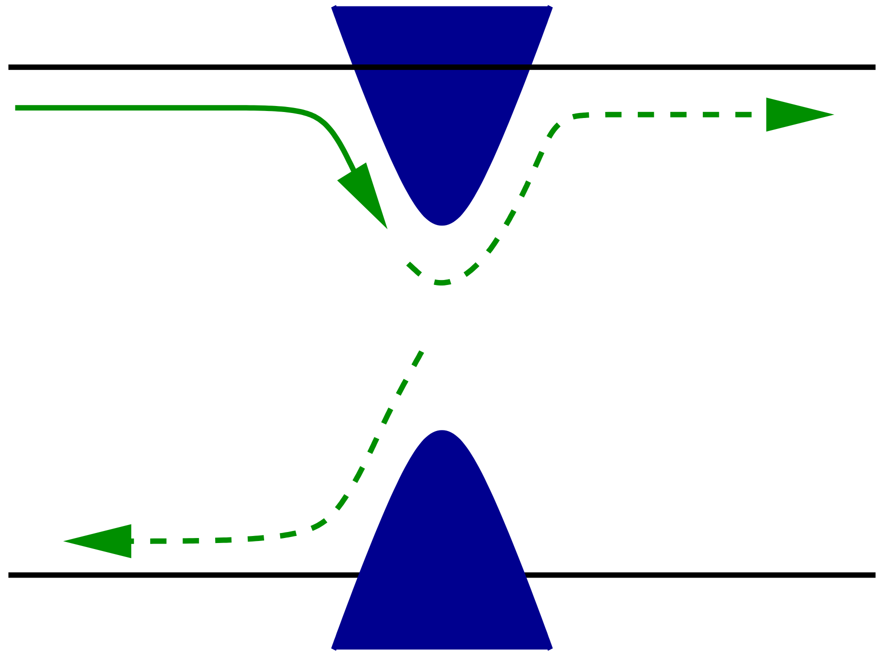


Interactions $\longrightarrow$ edge magnetoplasmon dispersion

$$\omega(q) = vq[1 + \kappa\ell \log(1/q\ell)]$$

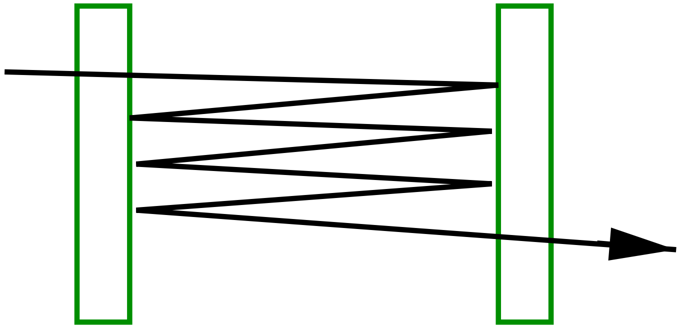
Manipulating Edge States

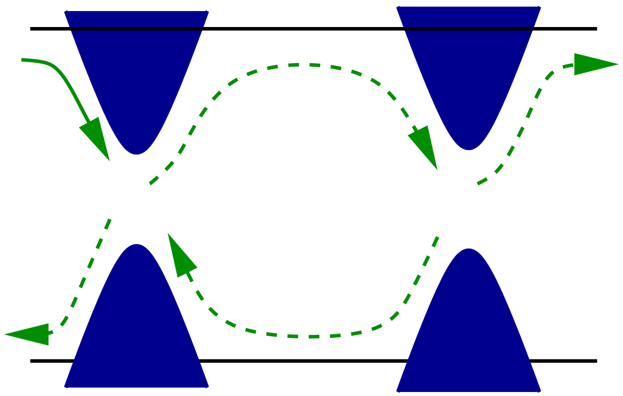
Quantum point contacts as beam splitters



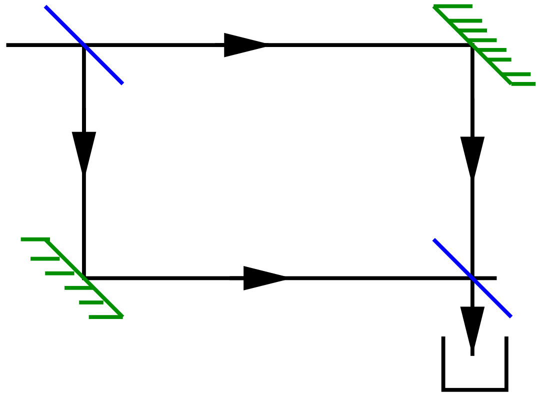
Edge State Interferometer Design

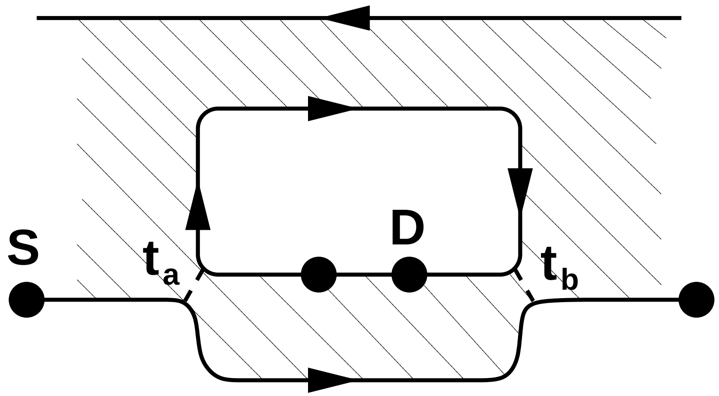
Fabry-Perot



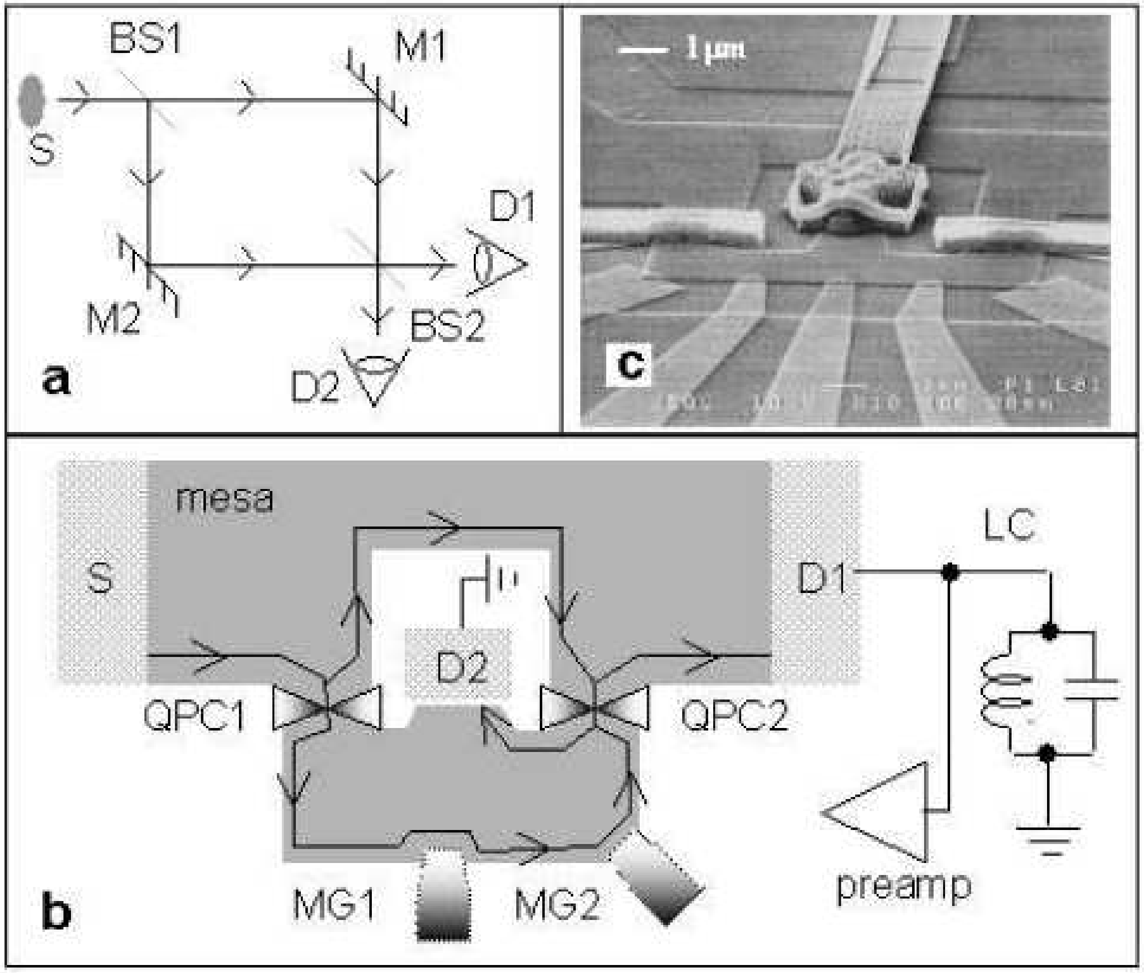


Mach-Zehnder





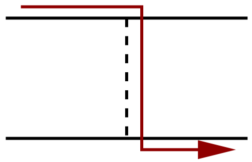
Experimental system



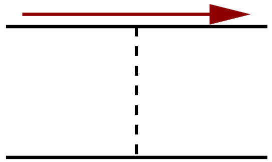
Heiblum Group, Weizmann Institute

Elementary theory

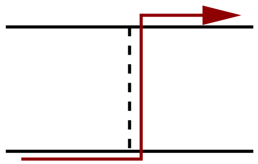
Scattering amplitudes



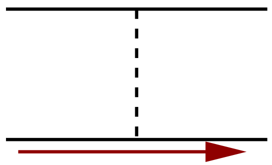
$$\frac{1}{\sqrt{2}}$$



$$\frac{1}{\sqrt{2}}$$

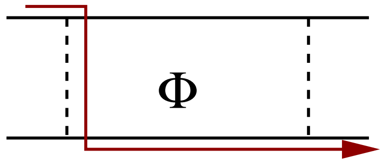


$$-\frac{1}{\sqrt{2}}$$

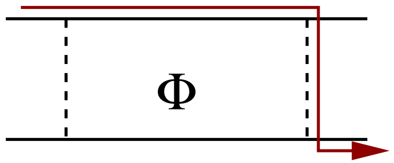


$$\frac{1}{\sqrt{2}}$$

Paths through interferometer



$$\frac{1}{2}\mathrm{e}^{\mathrm{i}\Phi/2}$$

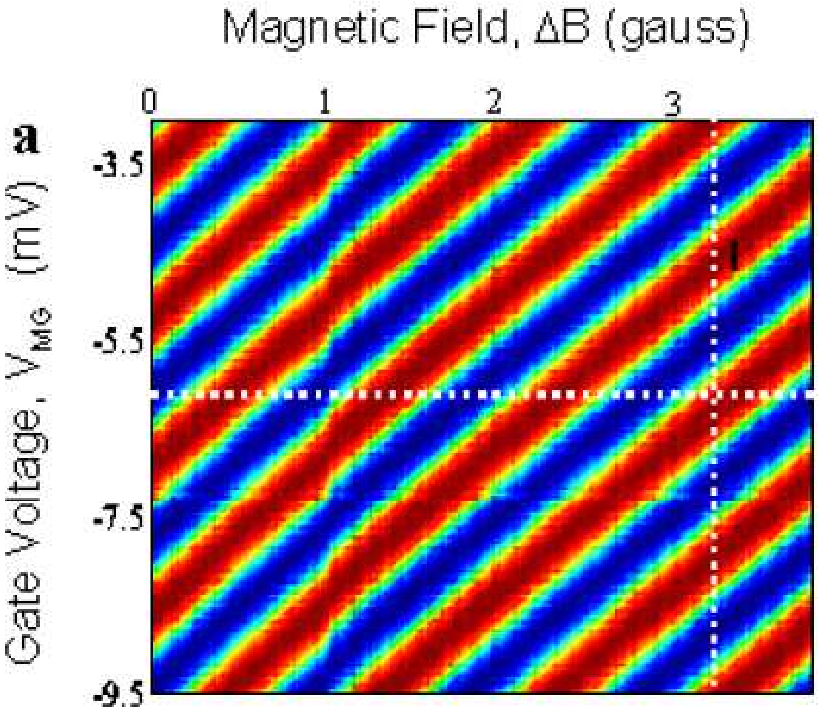


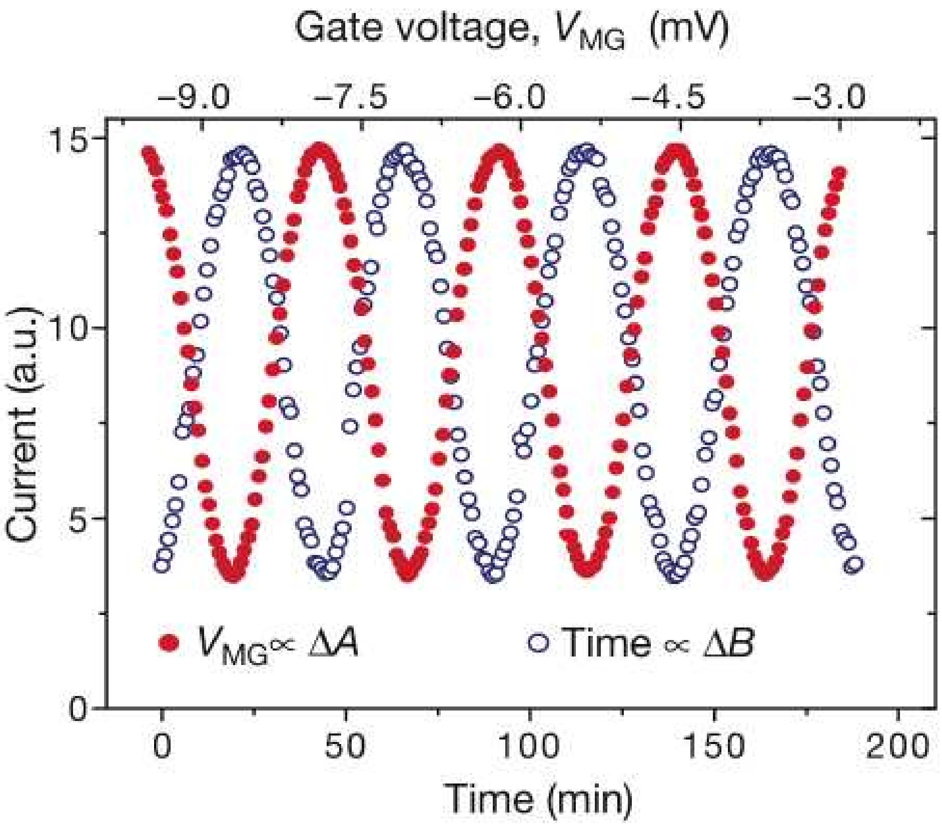
$$\frac{1}{2}\mathrm{e}^{-\mathrm{i}\Phi/2}$$

Combined amplitude $A = \cos(\Phi/2)$

Current $I \propto |A|^2 = \frac{1}{2}[1+\cos(\Phi)]$

Fringes in Edge State Interferometer

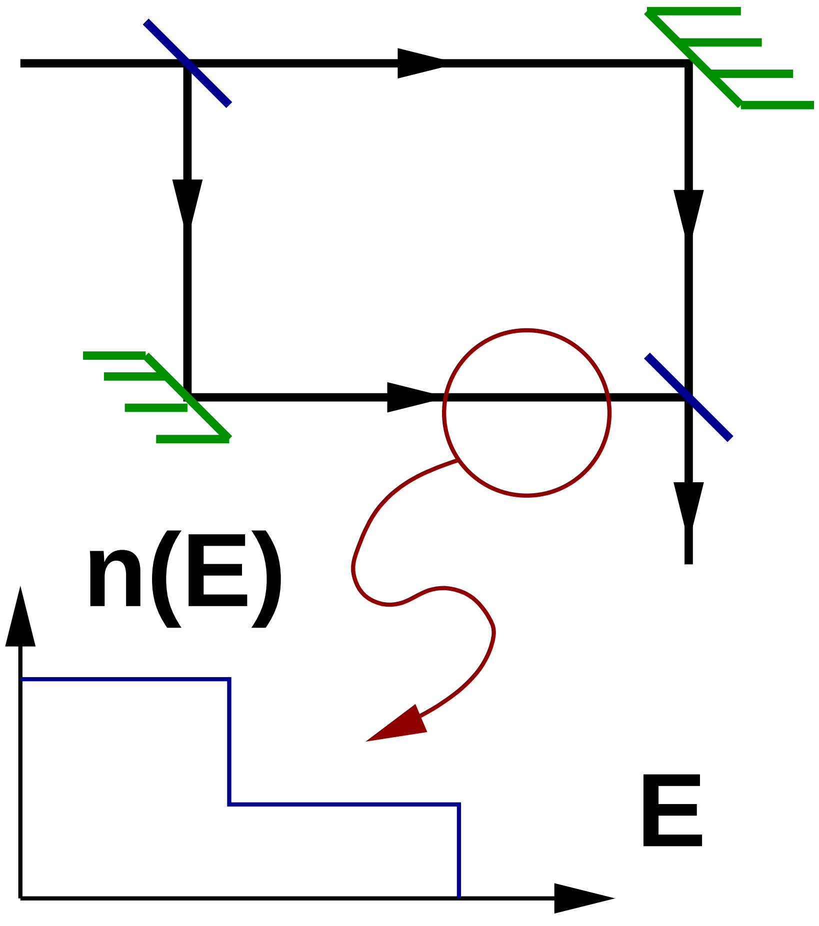




G_{SD} **vs** $Flux\ density$ **and** $Area$

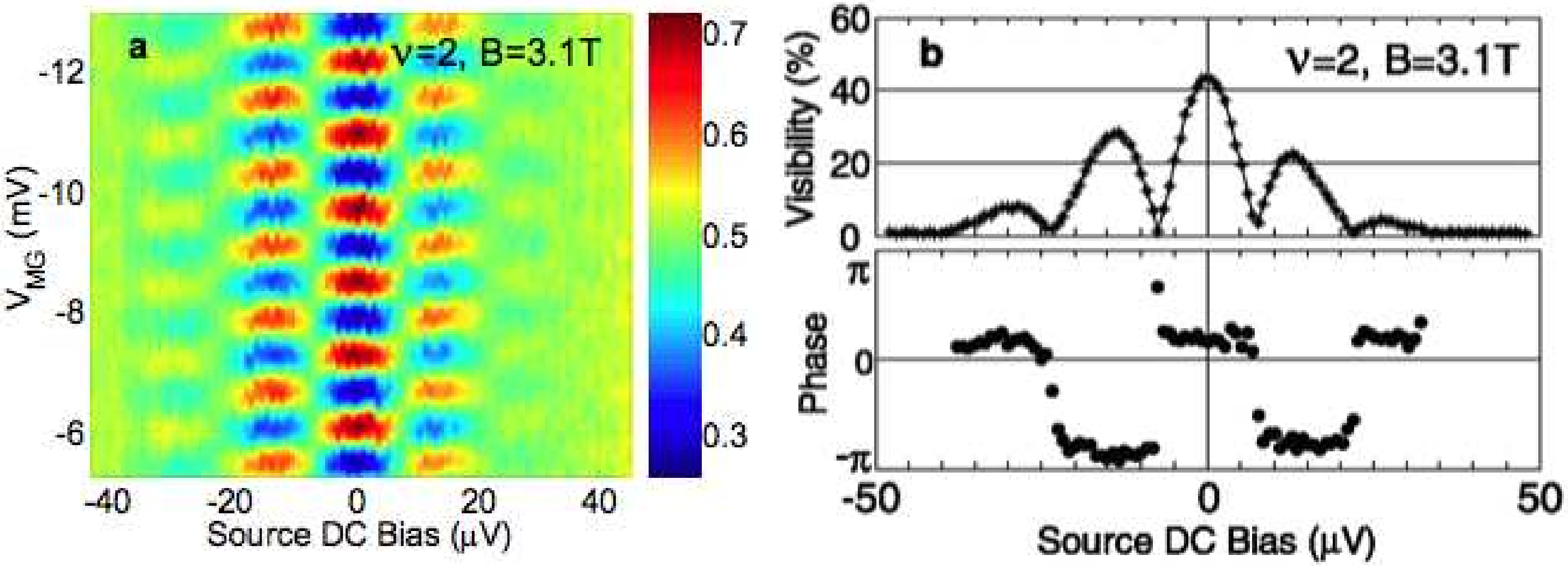
Interferometer out of equilibrium

Decoherence from inelastic scattering



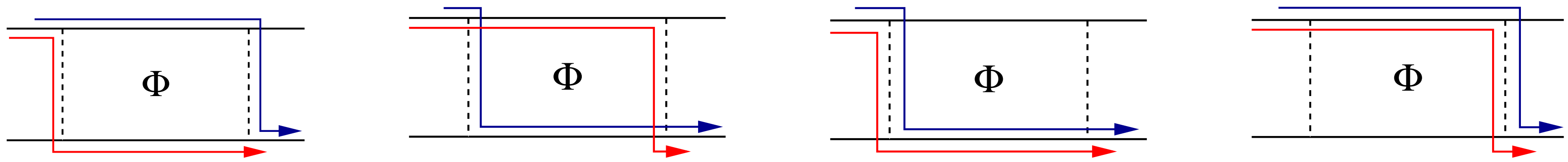
Surprises from experiment

Oscillatory dependence of visibility on bias



Clues from two-particle problem

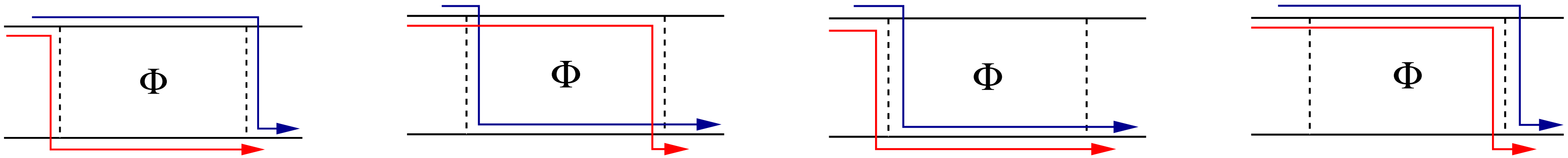
Two-particle paths: both transmitted $A_2 = \frac{1}{2}(1 + \mathrm{e}^{-\mathrm{i}U\tau/\hbar}\cos\Phi)$



$$A_2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\mathrm{e}^{\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} + \frac{1}{4}\mathrm{e}^{-\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}$$

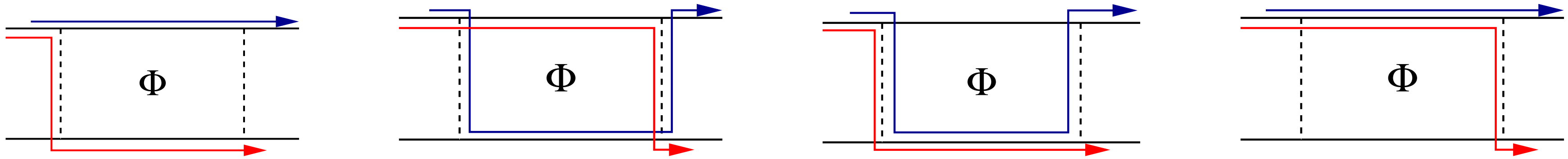
Clues from two-particle problem

Two-particle paths: both transmitted $A_2 = \frac{1}{2}(1 + \mathrm{e}^{-\mathrm{i}U\tau/\hbar}\cos\Phi)$



$$A_2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\mathrm{e}^{\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} + \frac{1}{4}\mathrm{e}^{-\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}$$

Two-particle paths: first transmitted $A_1 = -\frac{\mathrm{i}}{2}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}\sin\Phi$



$$A_1 = \frac{1}{4} - \frac{1}{4} - \frac{1}{4}\mathrm{e}^{\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} + \frac{1}{4}\mathrm{e}^{-\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}$$

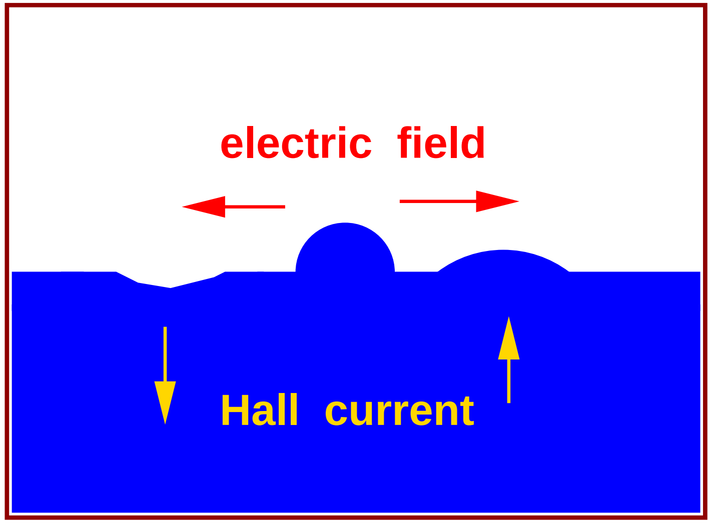
Current

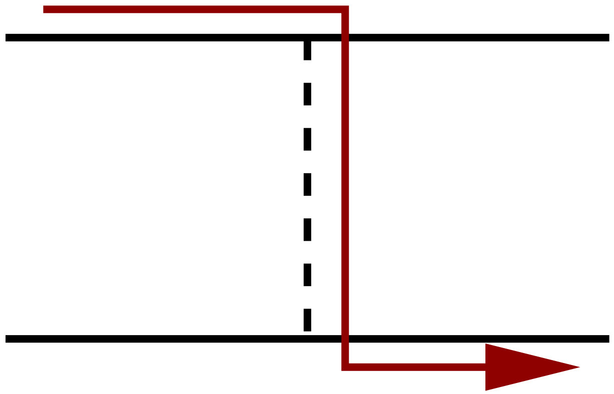
$$\boxed{I \propto 2|A_2|^2 + 2|A_1|^2 = 1 + \cos\Phi\cos(U\tau/\hbar)}$$

Many electron system

Must combine theories of

collective modes **and** **single-particle tunneling**

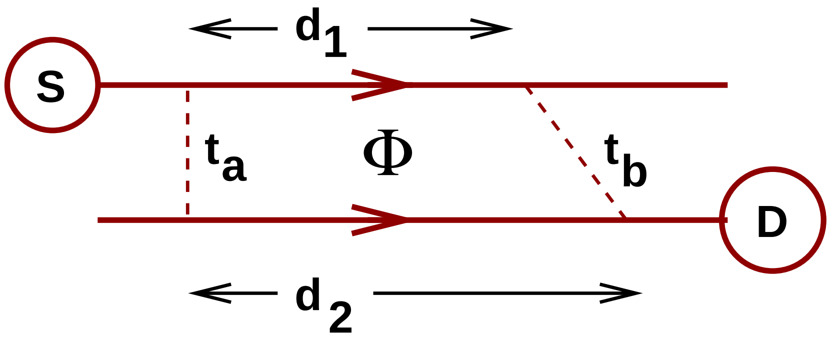




Solvable problem: interactions only within interferometer

Model

$$\mathcal{H} = \mathcal{H}_{\mathrm{edge}} + \mathcal{H}_{\mathrm{int}} + \mathcal{H}_{\mathrm{tun}}$$



$$\mathcal{H}_{\mathrm{edge}} = -\mathrm{i}\hbar v \int \mathrm{d}x \, [\psi_1^\dagger(x)\partial_x\psi_1(x) + \psi_2^\dagger(x)\partial_x\psi_2(x)]$$

$$\mathcal{H}_{\mathrm{tun}} = \left[t_a\psi_1^\dagger(0)\psi_2(0) + t_b\psi_1^\dagger(d_1)\psi_2(d_2) + \mathrm{H.c.}\right]$$

$$\mathcal{H}_{\mathrm{int}} = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}y\, U(x,y)[\rho_1(x)\rho_1(y) + \rho_2(x)\rho_2(y)]$$

$$\rho_i(x) = \psi_i^\dagger(x)\psi_i(x)$$

Simplification: include interactions only within interferometer

$U(x,y) = g$ **for** $0 \leq x, y \leq d$, $U(x,y) = 0$ **otherwise**

Outline of solution - I

Want current $\langle\Psi(0)|\mathrm{e}^{\mathrm{i}\mathcal{H}t}\hat{I}\mathrm{e}^{-\mathrm{i}\mathcal{H}t}|\Psi(0)\rangle$ **at large** t

for initial state $|\Psi(0)\rangle$ **with** $\mu_1 - \mu_2 = eV$

Handle interactions with bosonization

but perturbative treatment of tunneling insufficient

Exact soln possible if electrons interact only within MZI

Use 'interaction' representation

$$\mathcal{H}_0 = \mathcal{H}_{\mathrm{edge}} + \mathcal{H}_{int} \qquad \mathcal{H}_1 = \mathcal{H}_{\mathrm{tun}}$$

$$\hat{I}(t) = \mathrm{e}^{\mathrm{i}\mathcal{H}_0 t}\hat{I}\mathrm{e}^{-\mathrm{i}\mathcal{H}_0 t} \qquad |\Psi(t)\rangle = \mathrm{T}_{t'}\left[\mathrm{e}^{-\mathrm{i}\int_0^t \mathcal{H}_1(t')\mathrm{d}t'}\right]|\Psi(0)\rangle$$

Outline of solution - II

Bosonization

$$\Psi(x,t) = (2\pi a)^{-1/2} F(t) \mathrm{e}^{2\pi \mathrm{i} N(t) x/L} \mathrm{e}^{-\mathrm{i}\phi(x,t)}$$

Find $\phi(x,t)$ **by solving scattering problem**

$$\partial_t \phi(x,t) = -v\partial_x \phi(x,t) - \frac{1}{2\pi\hbar}\int \mathrm{d}y\, U(x,y)\partial_y \phi(y,t)$$

Re-express $\phi(x,t)$ **in terms of** $\psi^\dagger(y)$ **and** $\psi(y)$

Invert bosonization identity to get

$$(2\pi a)^{-1/2} F(t) \mathrm{e}^{2\pi \mathrm{i} N(t) x/L} = \psi(x - vt)\mathrm{e}^{\mathrm{i}\phi(x-vt)}$$

Outcome

Electrons accumulate interaction phase in MZI

$$\psi(d,t) = \mathrm{e}^{-\mathrm{i}\hat{Q}}\psi(d - vt)$$

$$\hat{Q} = \int Q(x - [d - vt])\rho(x)\mathrm{d}x$$

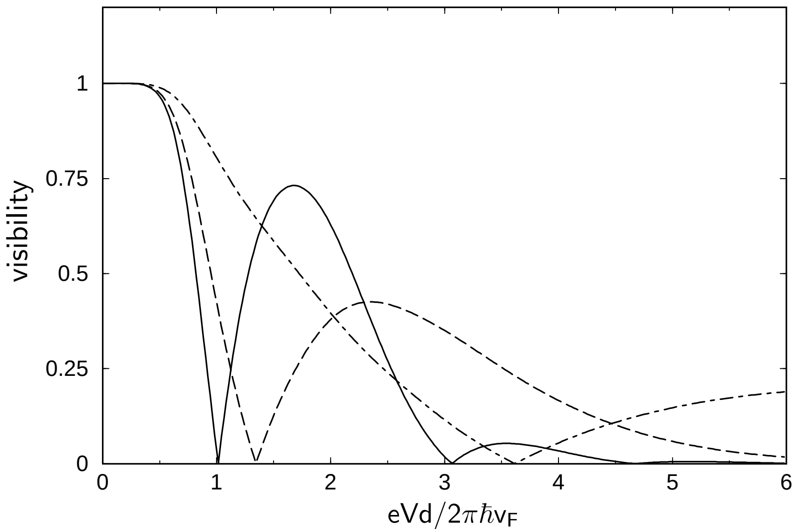
Interaction effects entirely contained in kernel $Q(x)$

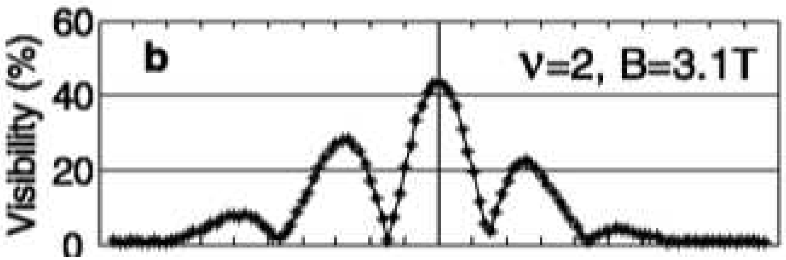
– phase accumulated by electron at d, t

from interactions with electrons x **distance ahead**

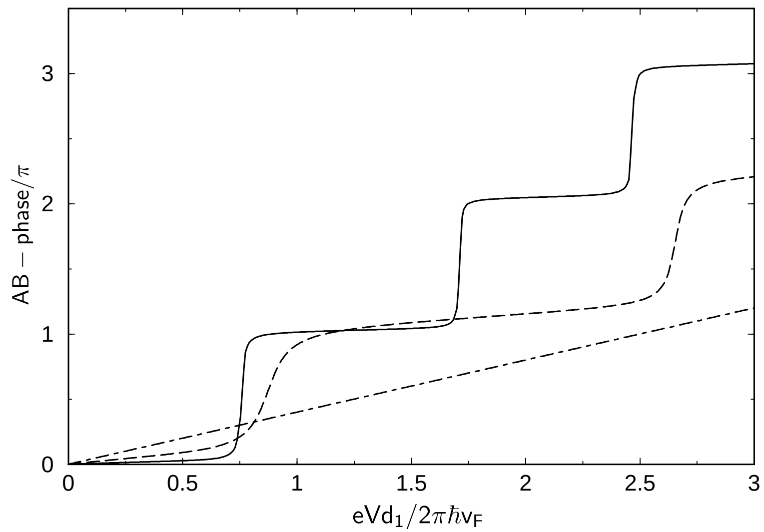
Results: bias dependence of fringes

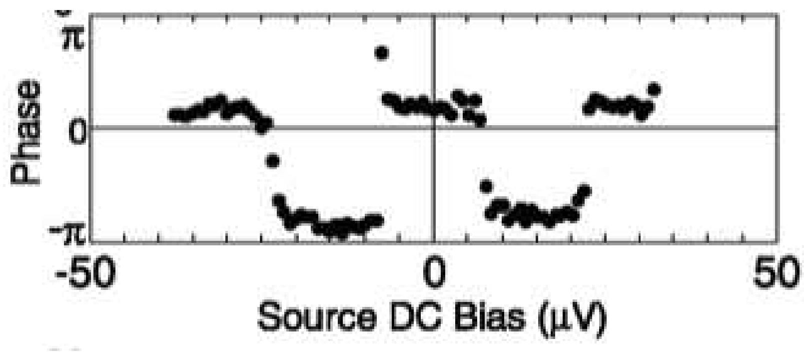
Visibility





Phase





Summary

Electronic MZI far from equilibrium

Striking interaction effects:

- Evidence of multiparticle interference
- Cannot be understood via mean field picture
- Exact solution for minimal model

Some References

Experiment

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