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How well can we determine the reactor neutrino spectra?

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How well can we determine the reactor antineutrino spectrum?

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Outline:

1) Motivation and the early history of the subject.
2) Two methods of determining the spectrum:
   Counting the contribution of individual fission fragments or Converting the measured electron spectra.
3) Assessing the uncertainties of the conversion procedure.
4) Small corrections - towards few percent accuracy.
5) How changing fuel composition affects the spectra.
6) Summary and conclusions

Motivation:

Whether one wants to monitor nuclear reactors using their $\overline{\nu}_e$ emission or to study neutrino properties (oscillations, magnetic moment, etc.) the knowledge of the $\overline{\nu}_e$ flux and its energy and time structure , and of the associated uncertainties is an essential ingredient. This will be so also for future applications, even to a large extent for the attempts to determine θ_{13} at nuclear reactors using two detector arrangements.

Brief history of the reactor neutrino spectrum determination:

1. First `modern' evaluations were done in late 1970 and early 1980 (Davis et al. 1979, Vogel et al. 1981, Klapdor & Metzinger 1982). Earlier evaluations assumed too much feeding of low-lying states and thus resulted in considerably harder $\overline{\nu}_e$ spectra than what is known today.
2. During the 1980-1990 a series of measurements of the **electron** spectra associated with the fission of ^{235}U, ^{239}Pu and ^{241}Pu were performed at ILL Grenoble by Schreckenbach *et al.* These were **converted** into the electron antineutrino spectra by the authors.
3. This is basically what is used as of now, even though some effort was made to measure the β decay of various short lived fission fragments (Tengblad et al, 1989) and new calculations were performed (see e.g. Kopeikin et al, hep-ph/0308186).
4. The missing electron spectrum of the fast neutron fission of ^{238}U is being measured by Schreckenbach *et al.* now.

Historical development of the reactor neutrino flux measurements

Some Expts	ν_e / t·day	Bkg / t·day	S/B	Sys %
Savannah River	~600	~3,000	0.2	~24
Rovno	1700	220	7.7	~10
Bugey	370	160	2.3	~5.0
Chooz	2.2	0.24	9.1	2.7
KamLAND	~0.001	~ 0.00016	5.8	2 for Δm^2 4 for θ_{12}

Clearly, systematic errors of the past experiments were the limiting factor. To overcome this for future precision experiments one will need a `monitor detector' and/or reduce the systematic errors.

As a first step we must know the number of fissions of the four fuels (^{235}U,^{239}Pu,^{241}Pu, and ^{238}U) as function of time. The reactor power is measured by the operators with ~0.6-0.7% accuracy, and the fuel composition is known from simulation to better than 1%. The energy per fission is well known, but needs explanations.

Table from Huber & Schwetz hep-ph /0407026

ℓ	N_ℓ^ν	E_ℓ [MeV]
^{235}U	$1.92(1 \pm 0.019)$	201.7 ± 0.6
^{238}U	$2.38(1 \pm 0.020)$	205.0 ± 0.9
^{239}Pu	$1.45(1 \pm 0.021)$	210.0 ± 0.9
^{241}Pu	$1.83(1 \pm 0.019)$	212.4 ± 1.0

192.9 ± 0.5	202.7 ± 0.1
193.9 ± 0.8	205.9 ± 0.3
198.5 ± 0.8	207.2 ± 0.3
200.3 ± 0.8	210.6 ± 0.3

Number of $\bar{\nu}_e$ per fission above 1.8 MeV,

Energy per fission without neutrinos and long lived fragments but including the energy associated with the neutron captures

Energy per fission without neutrinos and long lived fragments

Energy per fission from the mass excesses

There are two basic ways to determine the $\overline{\nu}_e$ spectrum of the individual fuels (^{235}U, ^{239}Pu, ^{241}Pu, and ^{238}U):

a) Summation method; adding the spectra of individual fission fragments, weighed by their fission yields.
b) Conversion method; converting the measured electron spectrum of a given $\overline{n}$ fuel into the ν_e spectrum.

Reactor spectrum by summation:

1) Fission yields Y(Z,A,t), essentially all known with sufficient accuracy. (Note the indicated time dependence, t is the time when the fission begins)

2) β decay branching ratios $b_{n,i}(E_0^i)$ for decay branch *i*, with endpoint E_0^i , Some are known but some (particularly for the very short-lived and hence high Q-value) are **unknown**.

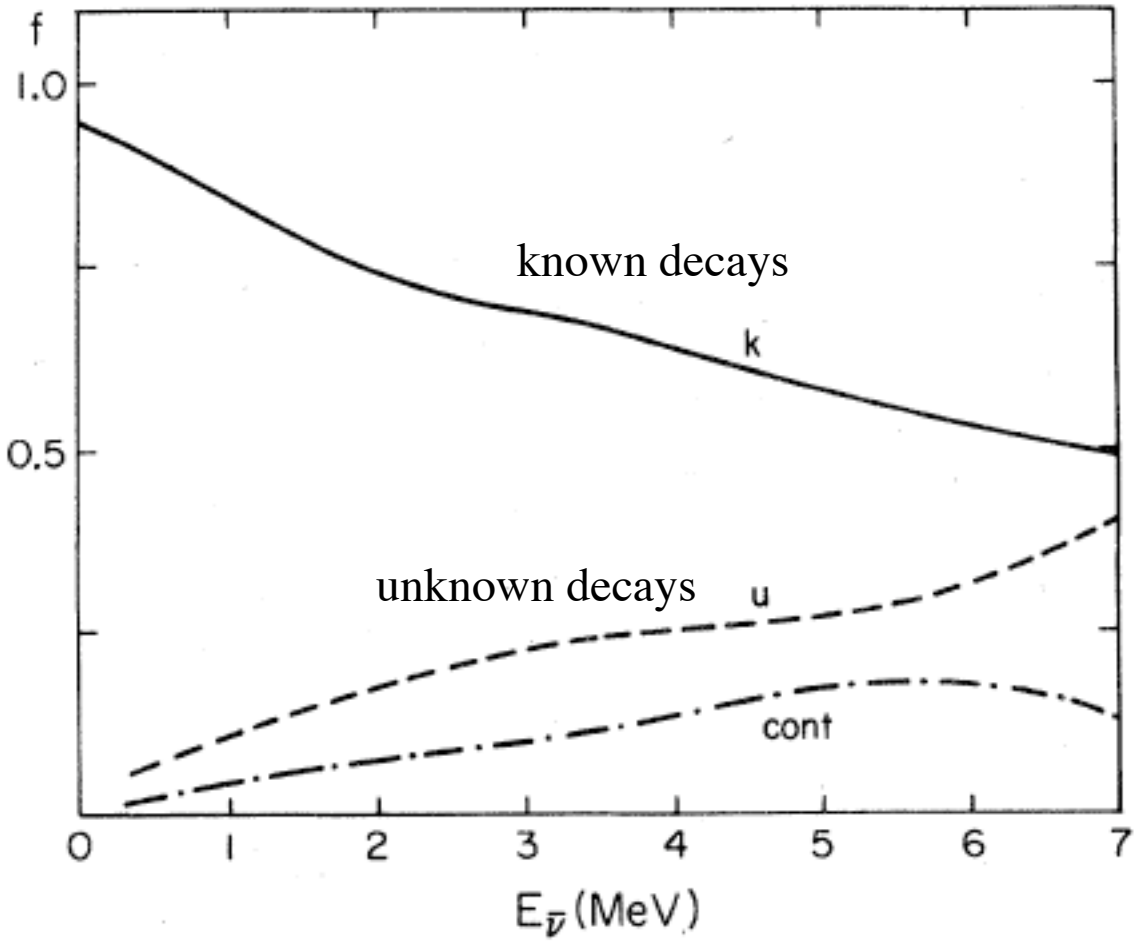
3) β decay shape, usually assumed allowed shape, known $P(E_\nu,E_0^i,Z)$ or for electrons $E_e = E_0 - E_\nu$.

Then: $\mathbf{dN/dE = \Sigma_n\, Y_n(Z,A,t)\, \Sigma_i\, b_{n,i}(E_0^i)\, P(E_\nu,E_0^i,Z)}$

and a similar formula for electrons. That needs to be slightly corrected (recoil terms, qed corrections, forbidden decays)

Contribution from β decay of fission fragments with different characteristics

(information somewhat dated, some more data available now)



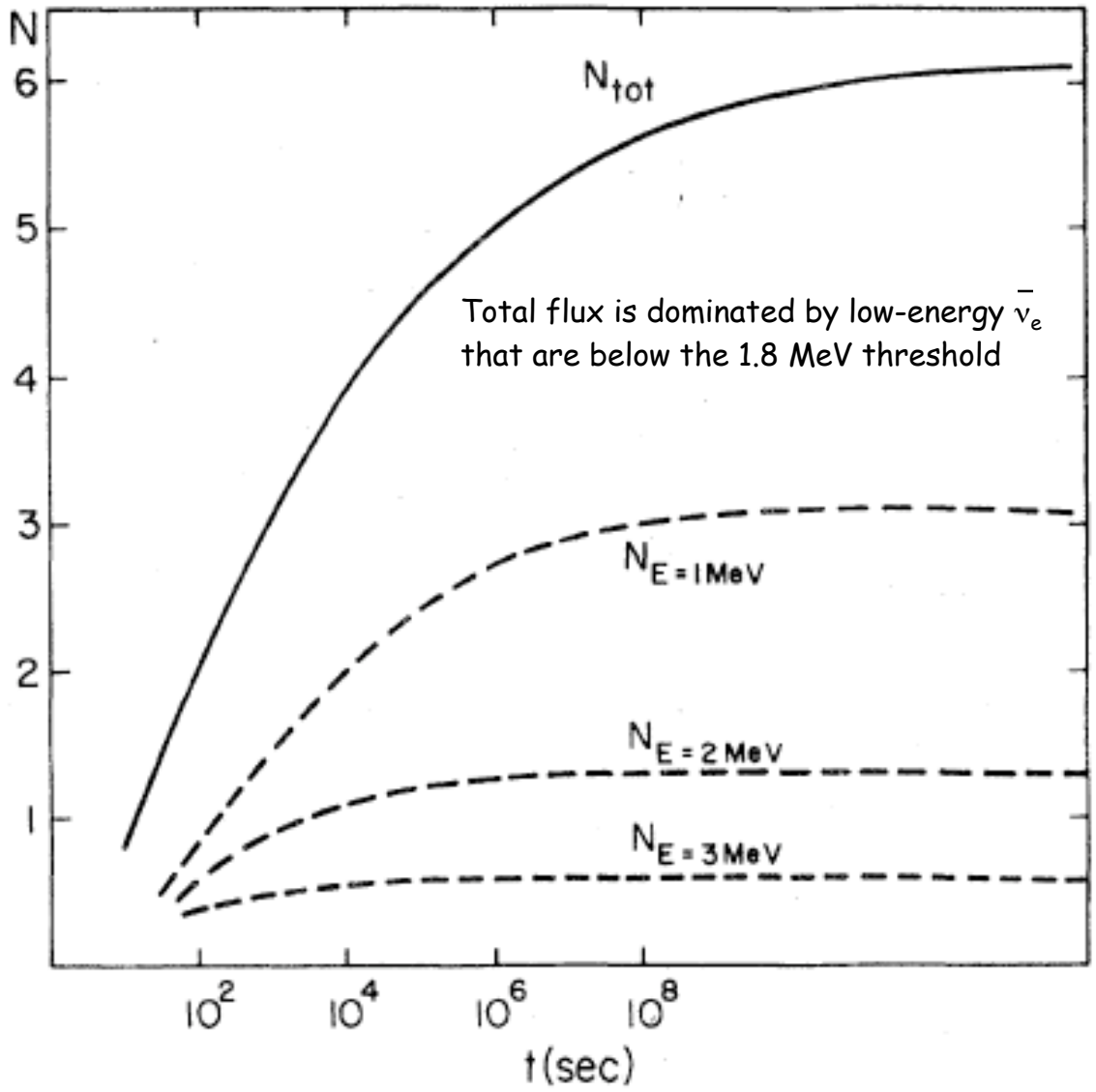
cont = summed γ spectrum measured in coincidence with an electron

Straightforward calculations must rely on nuclear models for the β decay of the `unknown' fission fragments

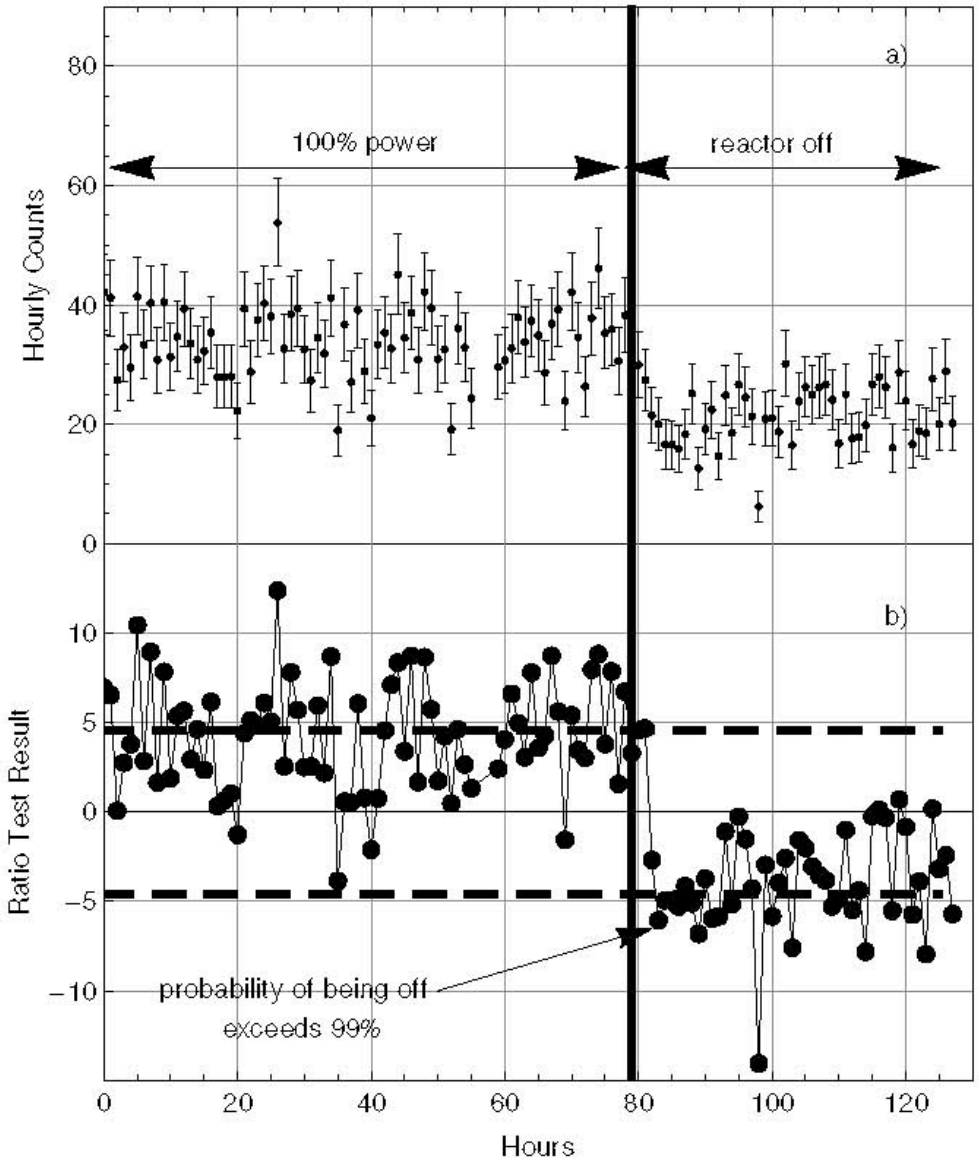
Explanation of the `continuum' method:

- The nuclear spectrum becomes very dense at higher excitation energies. Hence the γ spectra associated with the high Q-value β-decay contain many weak lines. The traditional method of decay scheme reconstruction becomes difficult or impossible
- Instead one can add the energies of all γ-rays that are in coincidence with an electron. By doing that we can determine the excitation energy of the state that was populated. Counting the frequency with which a given state is populated gives the corresponding branching ratio.
- The method was originally employed for ~60 short lived and high Q-value fission fragments that represent ~20% of the higher energy electron and ν_e spectrum. It has been extended later to other short lived fission fragments, reducing the `unknown' category in the previous slide.

Electron and $\bar{\nu}_e$ spectra do not saturate immediately. The low energy part reaches equilibrium only after a long time, while for E > 2 MeV the equilibrium is reached in about a day. Calculated for ^{235}U.



At 1 MeV the spectrum changes by ~20% between 1 day and 2 years.
At 2 MeV it changes by ~3%, and at 3 MeV by ~0.3%.



Plot from arXiv:0804.4723
by A. Bernstein *et al.*
This prototype reactor monitor was installed 25 m from the core of the San Onofre reactor. It can detect reactor shut down in ~5 hours.
The gradual decrease of the neutrino flux is clearly visible.

The summation method is straightforward.

a) One can use the most complete set of experimental data (ENSDF) plus the set of data obtained by calorimetric method (Tengblad).
b) Resulting electron spectrum can be compared with the data of Schreckenbach.
c) Correction can be applied to reduce the discrepancy (Saclay group) or (original papers) the effect of unknown decays can be added using appropriate nuclear models.
d) Once the set of endpoints and branching ratios is given, calculation of the ν_e spectrum is straightforward.
e) The only complication are the deviations from the allowed shapes of individual branches (small corrections, see later)

How the conversion of the electron spectrum into the $\bar{\nu}$ works?

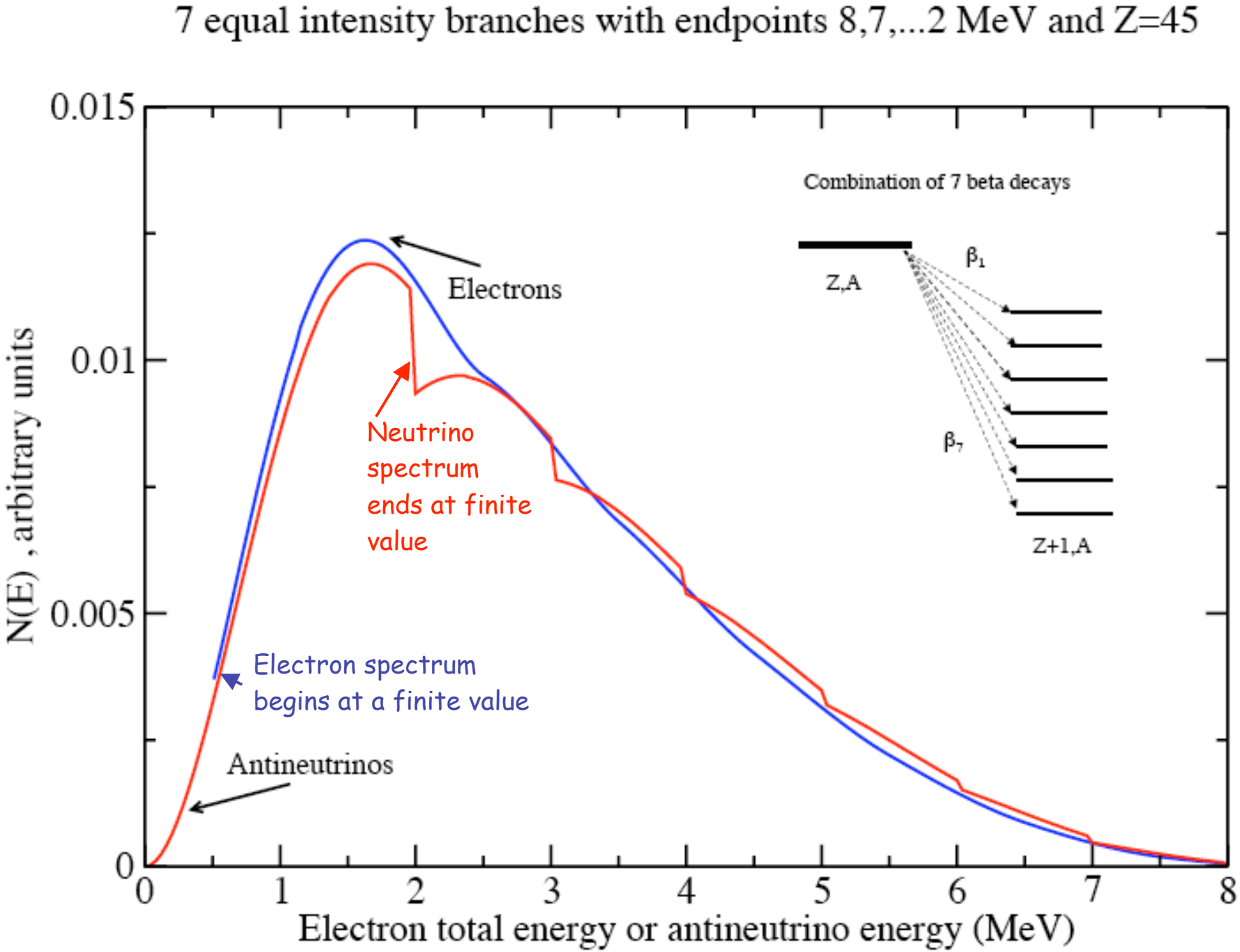
a) For a single β decay it is trivial: $E_\nu = E_0 - E_e$ where E_0 is the decay energy

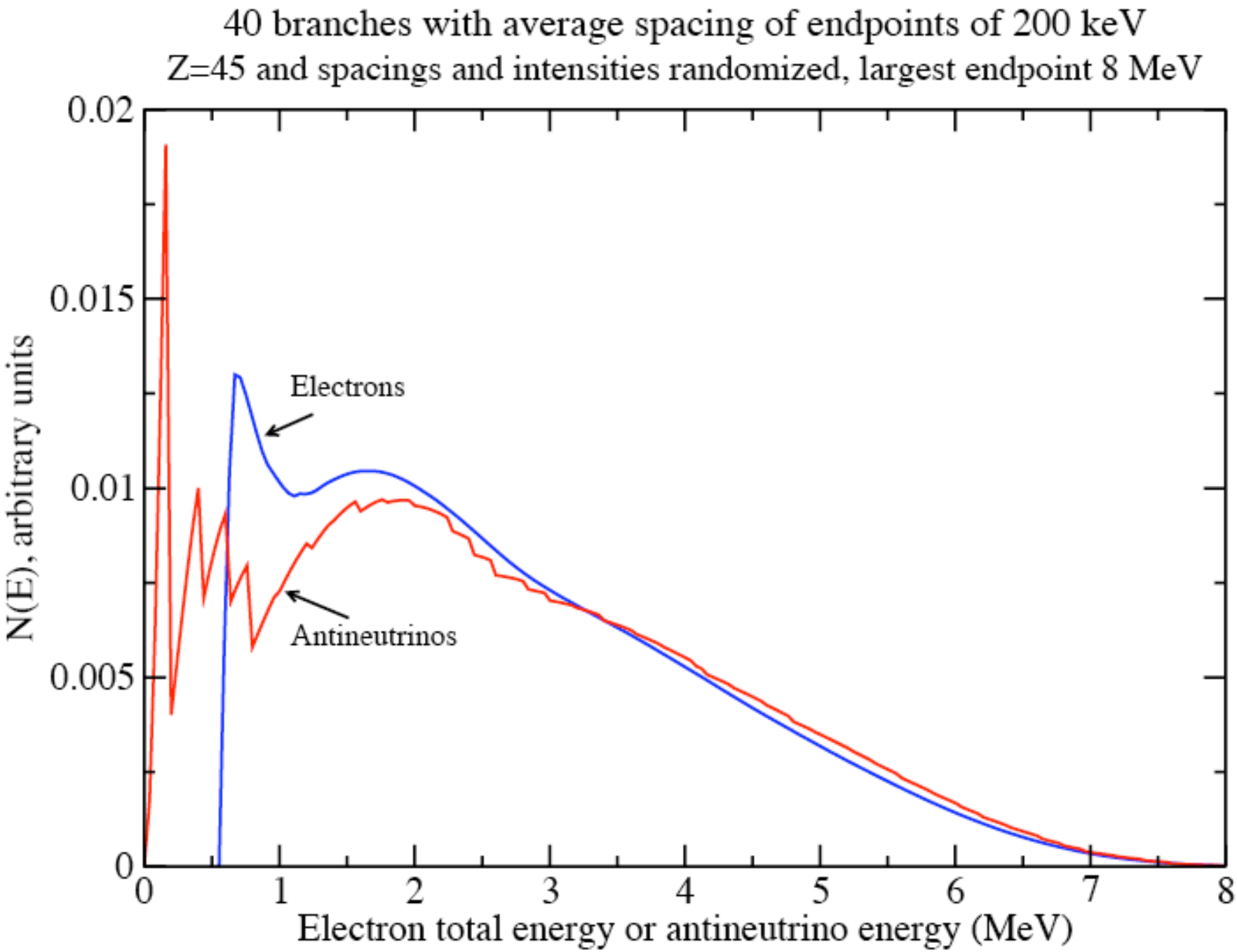
b) For a decay with many known branches
$Y(E_e) = \Sigma_i\, b_i\, k(E_0^i,Z)\, p_e\, E_e\, (E_0^i - E_e)^2\, F(E_e,Z)$ where $k(E_0^i,Z)$ is a normalization and b_i are the branching ratios
Once b_i and E_0^i are known, $Y(E_\nu)$ can be easily calculated

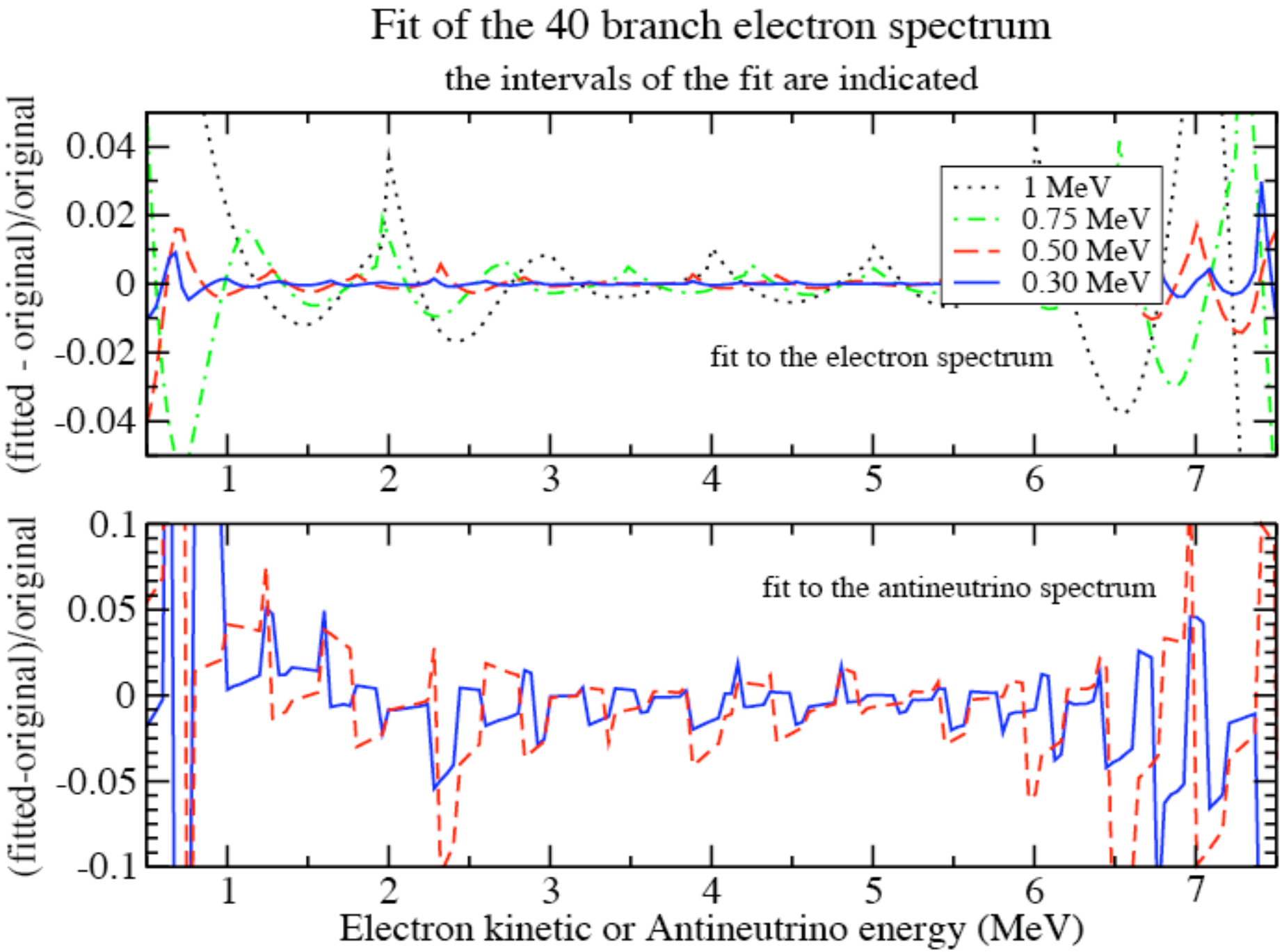
c) Now suppose that b_i and E_0^i are unknown, but $Y(E_e)$ is measured. One then can ***assume*** that E_0^i are e.g. equidistantly distributed, and fit for b_i.
By varying the number of branches, one can check that the result is convergent. (30 branches were used in Schreckenbach et al.)

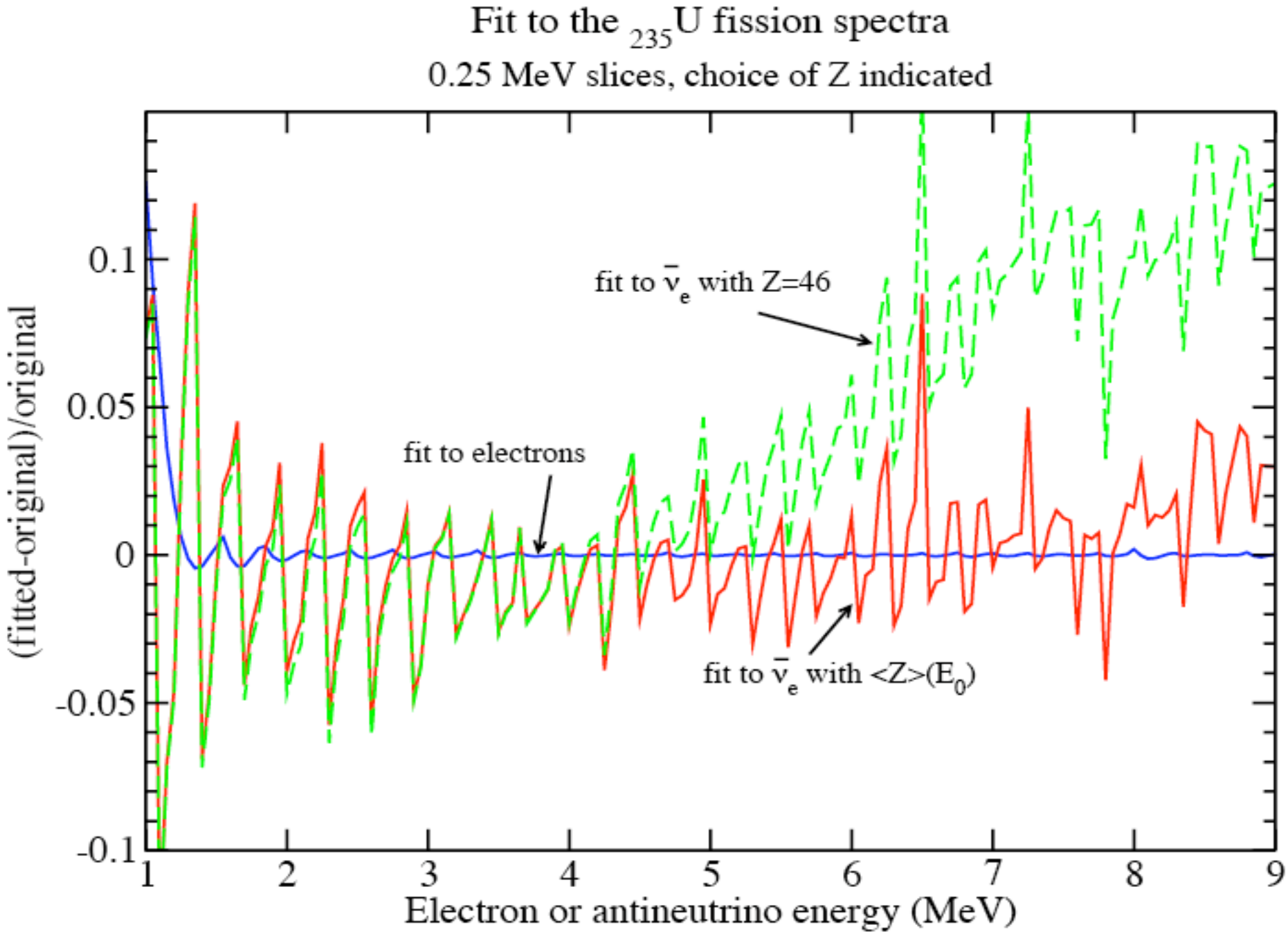
d) In the actual case Z is also unknown. Some procedure for choosing Z, or Z(E) must be chosen and tested.

e) The error associated with the procedure must be determined.

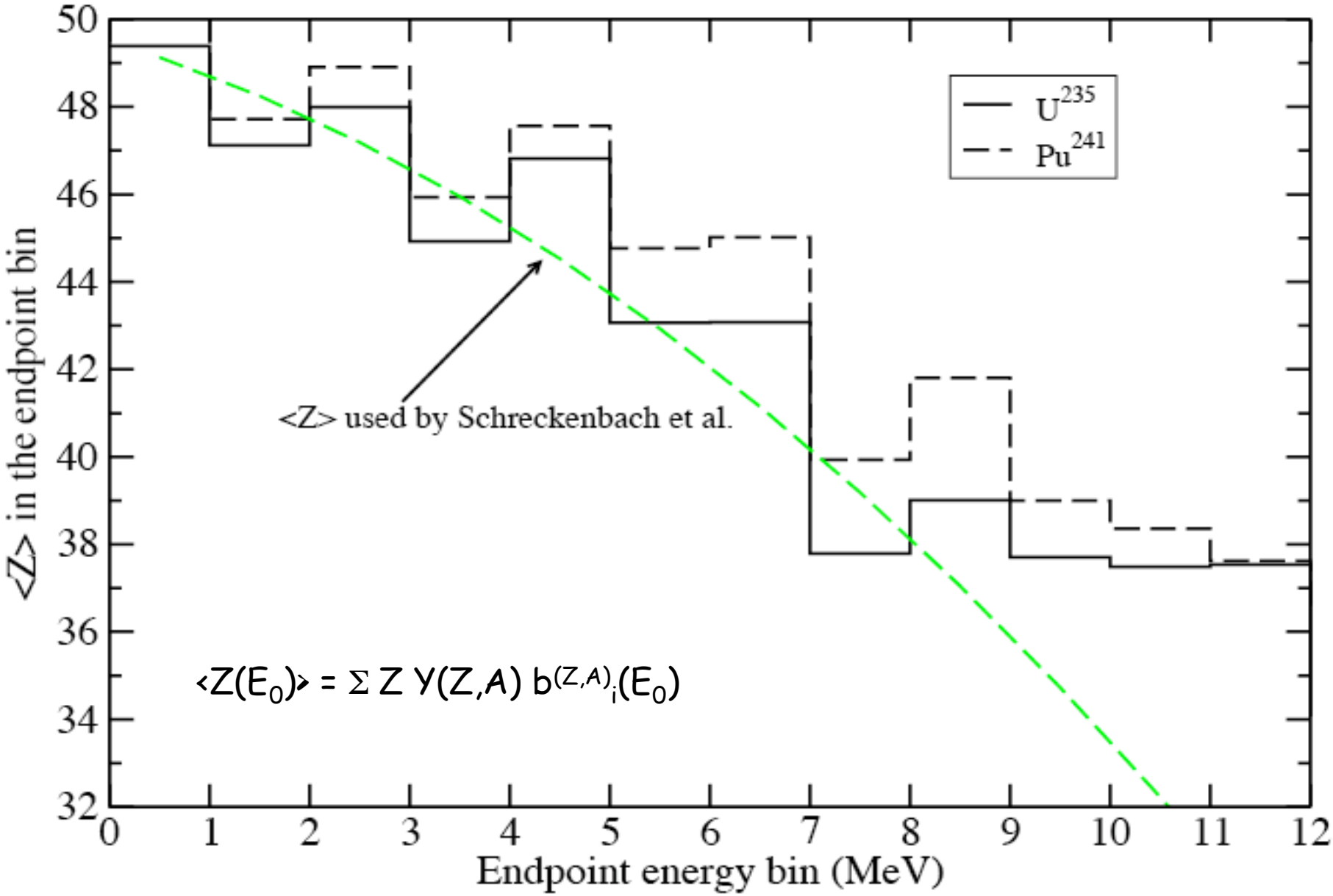

7 equal intensity branches with endpoints 8,7,...2 MeV and Z=45
Combination of 7 beta decays
Z,A
β1
β7
Z+1,A
Electrons
Neutrino spectrum ends at finite value
Electron spectrum begins at a finite value
Antineutrinos
N(E) , arbitrary units
0.015
0.01
0.005
0
0
1
2
3
4
5
6
7
8
Electron total energy or antineutrino energy (MeV)

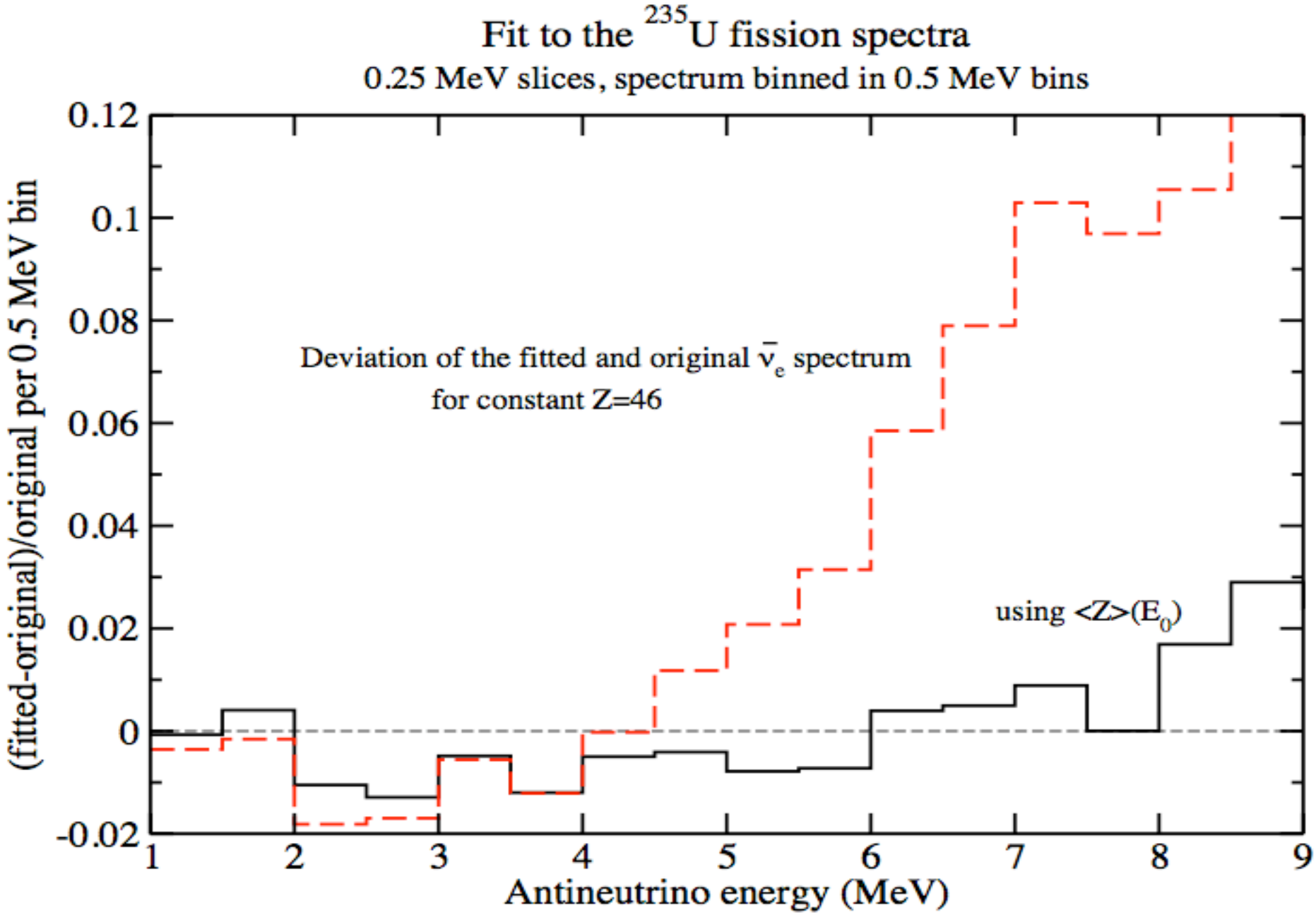

40 branches with average spacing of endpoints of 200 keV
Z=45 and spacings and intensities randomized, largest endpoint 8 MeV
N(E), arbitrary units
0.02
0.015
0.01
0.005
0
Electrons
Antineutrinos
0
1
2
3
4
5
6
7
8
Electron total energy or antineutrino energy (MeV)


Fit of the 40 branch electron spectrum
the intervals of the fit are indicated
(fitted - original)/original
0.04
0.02
0
-0.02
-0.04
1 MeV
0.75 MeV
0.50 MeV
0.30 MeV
fit to the electron spectrum
1
2
3
4
5
6
7
(fitted-original)/original
0.1
0.05
0
-0.05
-0.1
fit to the antineutrino spectrum
1
2
3
4
5
6
7
Electron kinetic or Antineutrino energy (MeV)


Fit to the $_{235}$U fission spectra
0.25 MeV slices, choice of Z indicated
fit to $\bar{\nu}_e$ with Z=46
fit to electrons
fit to $\bar{\nu}_e$ with <Z>(E$_0$)
(fitted-original)/original
Electron or antineutrino energy (MeV)
0.1
0.05
0
-0.05
-0.1
1
2
3
4
5
6
7
8
9

For indicated fuel the average charge of the β decay with a given endpoint interval




Fit to the ^{235}U fission spectra
0.25 MeV slices, spectrum binned in 0.5 MeV bins
(fitted-original)/original per 0.5 MeV bin
Antineutrino energy (MeV)
Deviation of the fitted and original $\bar{\nu}_e$ spectrum
for constant Z=46
using <Z>(E$_0$)
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
1
2
3
4
5
6
7
8
9

Summary on conversion:

a) The procedure has been tested using available experimental (and small amount of nuclear model) data. With sufficiently small spacings of the fictitious levels, followed by binning into large bins, one can achieve **~1%** accuracy.

b) However, this can be achieved only if one uses a correct prescription for the dependence of the nuclear charge Z (or <Z>) on the endpoint energy. **Thus, the conversion procedure need be combined with a reasonable summation method.**

The previous slides were calculated assuming that all β-decay branches have the allowed shape ~ $F(Z,E)p_eE_e(E_0-E_e)^2$

This is not exactly true. There are some unique first forbidden β decays, there are qed effects of order α, there are effects of weak magnetism (linear slope in electron energy) and there are higher order Coulomb effects (these are E_e/M_n or α effects).

These effects need to be included for accurate simulation and conversion.(Schreckebach *et al.* included these effects in an approximation. It is difficult to judge the error associated with those corrections, 100% was assumed)

For a single branch beta decay

$$N(E_e,E_0) = N^{allowed}(E_e,E_0)\,[1 + (A_W + A_C)E_e + \alpha\delta_{qed}(E_e,E_0)]$$

weak magnetism

axial-Coulomb interference

Hadronic current expressed in terms of nucleon fields Ψ:

$$j^{\rho\dagger} = \overline{\Psi}\tau^{+}\Big[g_V(q^2)\gamma^{\rho} + ig_M(q^2)\frac{\sigma^{\rho\nu}}{2m_p}q_\nu - g_A(q^2)\gamma^{\rho}\gamma_5 - g_P(q^2)q^{\rho}\gamma_5\Big]\Psi,$$

Vector $g_V(q^2) = g_V/(1 + q^2/M_V^2)^2$, $g_V = 1$, $M_V = 0.85$ GeV
Axial vector $g_A(q^2) = g_A/(1 + q^2/M_A^2)^2$, $g_A = 1.26$, $M_A = 1.09$ GeV
Weak Magnetism $g_M(q^2) = (\mu_p - \mu_n)\, g_V(q^2)$
Induced pseudoscalar $g_P(q^2) = 2m_p g_A(q^2)/(q^2 + m_\pi^2)$

This slide for accuracy enthusiasts:

Weak magnetism is a part of the hadronic weak current present because nucleons are composite objects. Its magnitude is based on the relation between the isovector electromagnetic current and the weak charged current. It has been tested in several light nuclei. The main effect is presence of a `slope' in the β spectrum.

Treating nucleus as a collection of independent bound nucleons one obtains (Konopinski 1966) the slope parameter:

$$A_W \approx \frac{4}{3}\frac{\langle p|\vec{l}+(\mu_p-\mu_n)\vec{\sigma}|n\rangle}{C_A M_n \langle p|\vec{\sigma}|n\rangle}$$

This can be further simplified in the most common case of $|j_p - j_n| = 1$ when $\langle p|\vec{l}|n\rangle = -1/2\langle p|\vec{\sigma}|n\rangle$

In that approximation $A_W \sim 0.005$/MeV (used by Schreckenbach)

Another slide for accuracy enthusiasts:

The Coulomb slope factor arises from the interference between the GT matrix element $\langle\sigma\tau_+\rangle$ and the second forbidden $\langle\sigma r^2\tau_+\rangle/R^2$.
Its magnitude can be estimated as (Behrens-Janecke 1971)

$A_C = -10\, Z\alpha R/9\hbar c \sim -0.011/\mathrm{MeV}$ (for Z=46)

This is for a surface charge distribution, for uniform distribution
$10/9 \rightarrow 2/3$

However, others obtain a somewhat different factor, e.g.
(Holstein 1974) $10/9 \rightarrow 48/35$ and $2/3 \rightarrow 8/9$

Thus, the order of magnitude is always $Z\alpha R/\hbar c$, but the coefficient in front, while O(1), remains uncertain. Note that this correction has a similar magnitude but opposite sign than the weak magnetism.

The last slide for accuracy enthusiasts:

The QED correction $\alpha\delta_{qed}(E_e, E_0)$ is well known for the allowed β decays (Sirlin 1967 or Yokoo 1973).

The difficulty there is that a part of that involves the emission of bremsstrahlung, thus a three-body decay invalidating the relation $E_\nu = E_0 - E_e$. In practice both the electron and antineutrino spectra should be evaluated by integrating over the bremsstrahlung spectrum (see Vogel 1984).

In an early evaluation (Vogel 1984) the QED correction was globally characterized as

$$N(E_\nu) = N^{conv}(E_\nu)[1 + 0.005(E_\nu(MeV) - 3]$$

This is $\alpha\delta_{qed}(E_\nu)$ fitted for the whole spectrum

Lets turn now to the important question of determining the **fuel composition** from the measured $\bar{\nu}_e$ spectrum.

There are several things to keep in mind:

a) The fuel composition changes during the refueling cycle.
b) The spectra of individual fuels are slightly different, because the fission yields are different.
c) These effects need be combined with the cross section of the detecting reaction.

During the refueling cycle, the fuel composition, i.e., the contribution of different isotopes to the reactor power, changes. This burn-up process will cause also changes in the neutrino spectrum.

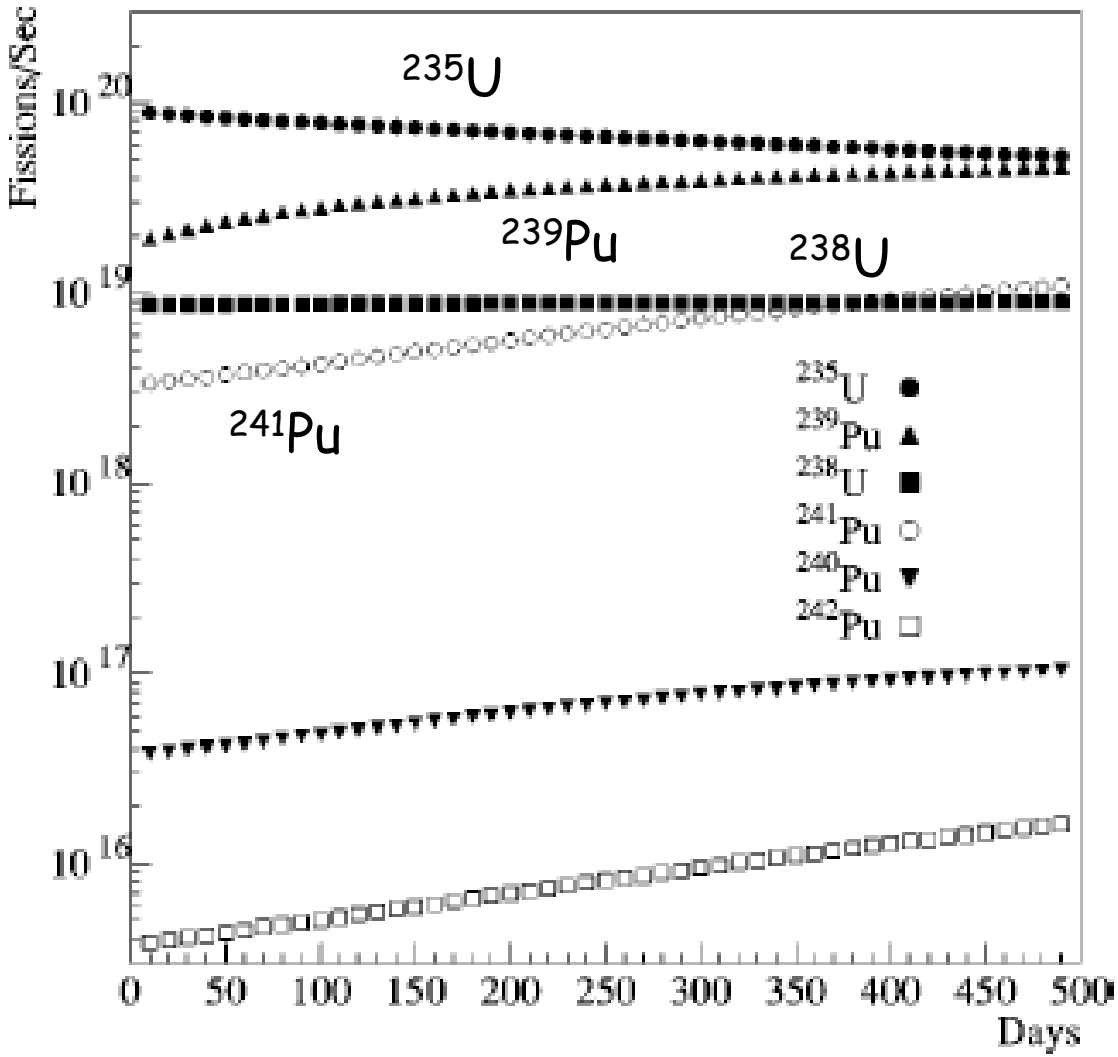
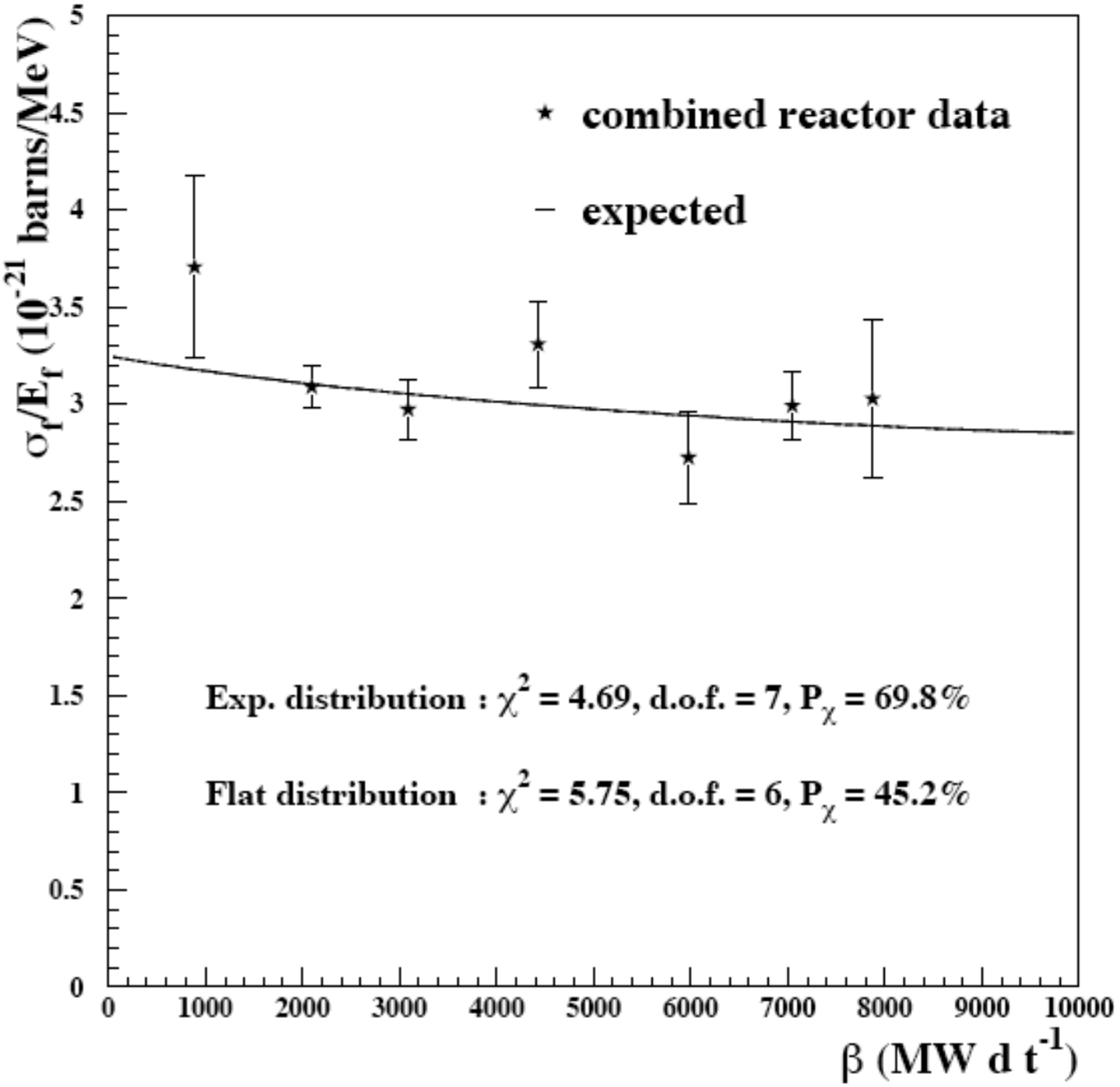


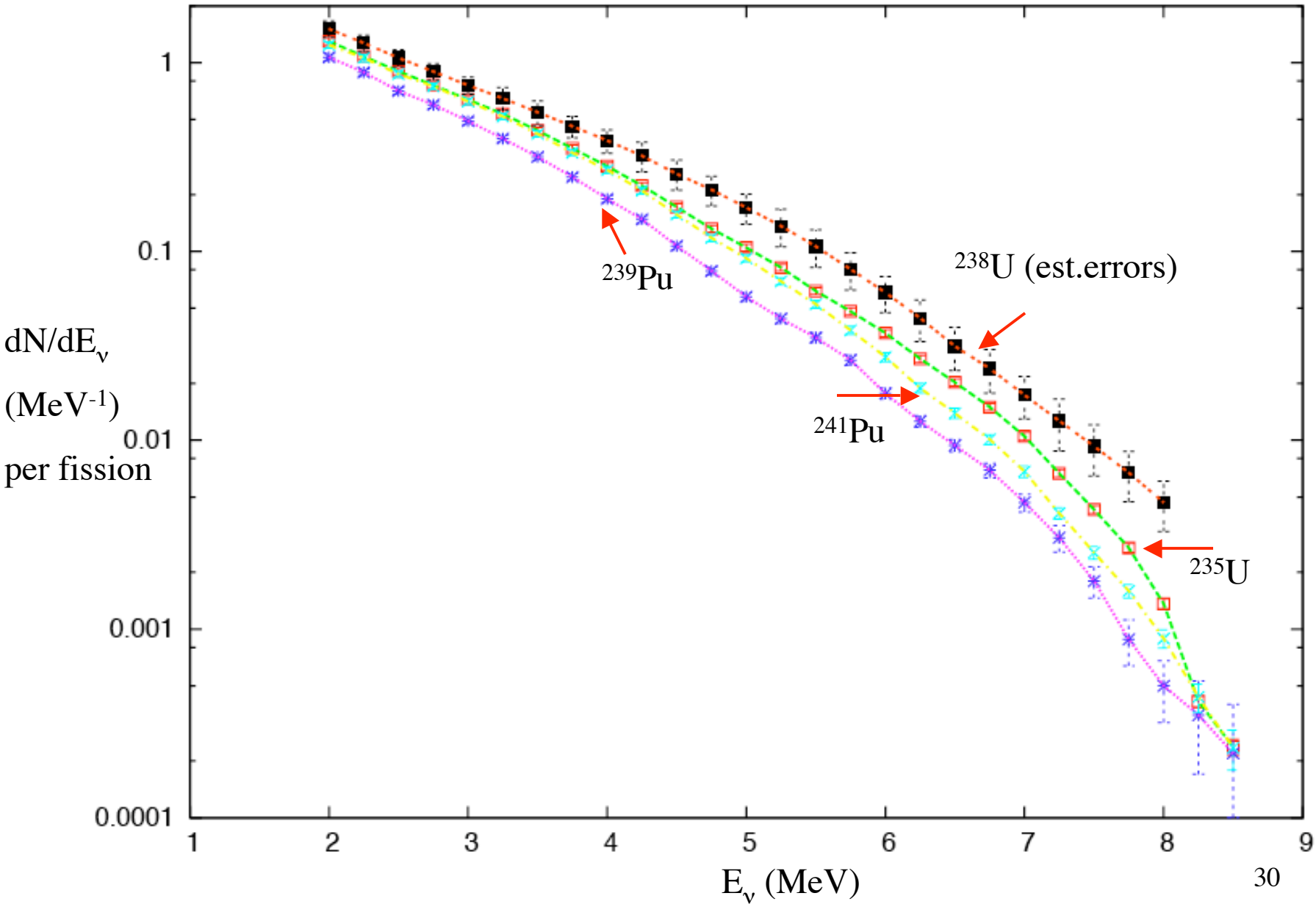
FIG. 6. Time evolution of fission rates for each of the six most important isotopes in one of the Palo Verde reactor cores. The horizontal scale covers a full fuel cycle, at the end of which about 1/3 of the core is replaced with fresh fuel. Only the four most important isotopes are normally used to predict $\bar{\nu}_e$ yields.

Testing the burn-up time dependence in the Chooz experiment

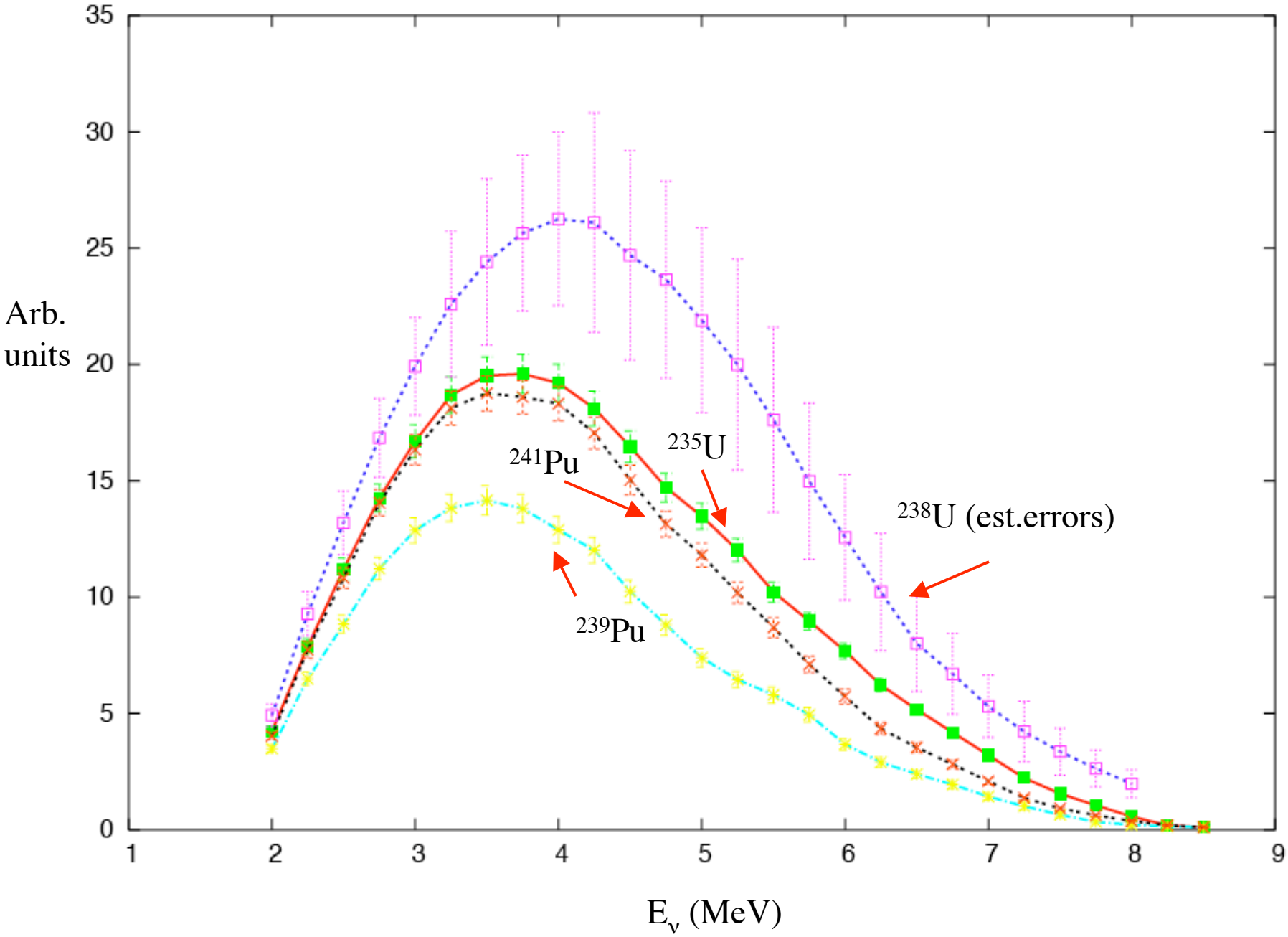


Even though the expected time dependence gives a better fit, a flat distribution is also compatible with the experimental data

Spectra of ^{235}U, ^{239}Pu, ^{241}Pu derived from electron spectra, and ^{238}U calculated

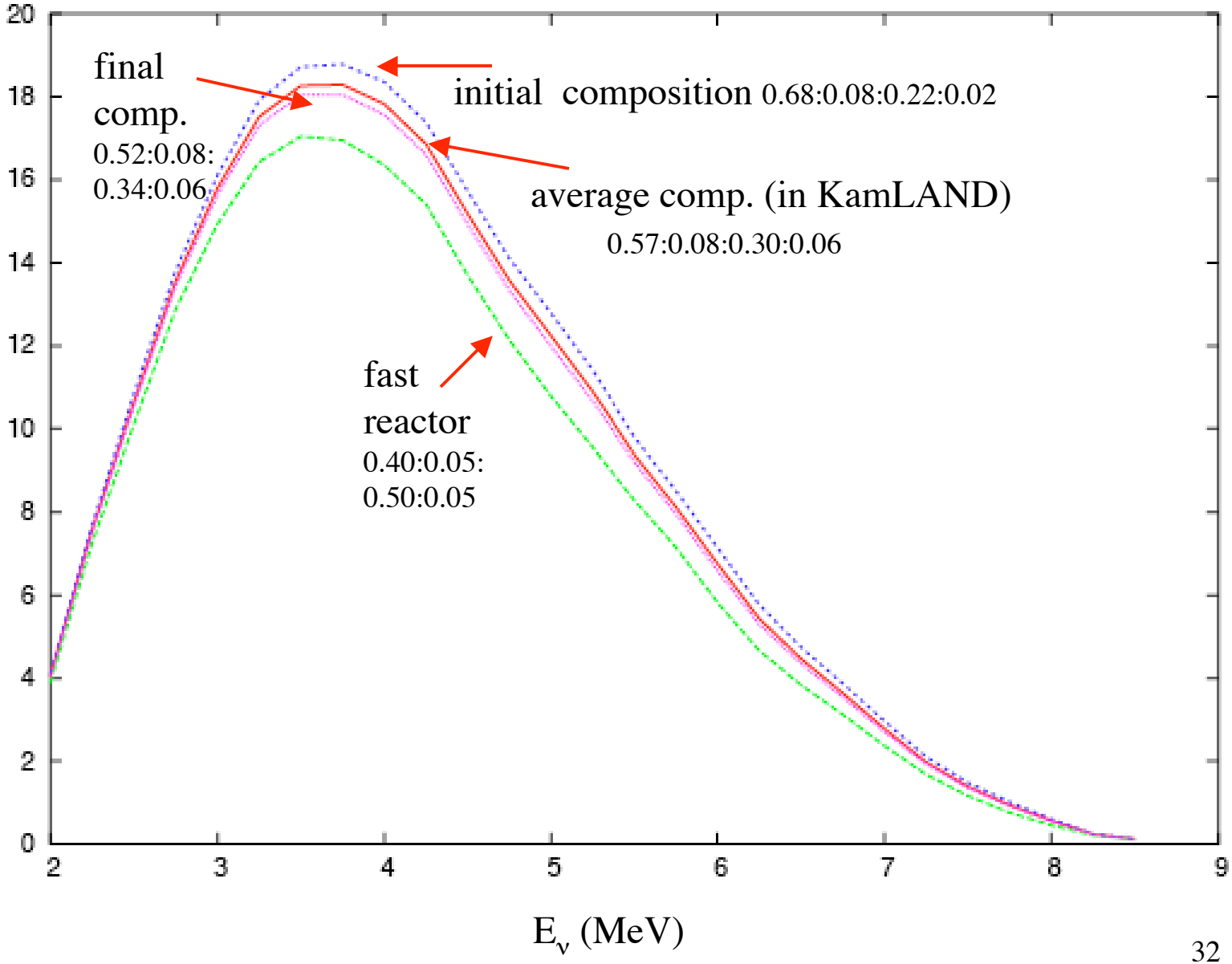


Positron yields for different fuels



Positron yields for different fuel compositions

$^{235}U:^{238}U:^{239}Pu:^{241}Pu$



Summary,conclusions,challenge:

1) Build high statistics, low background, close detectors that convincingly and accurately observe the effect of the reactor burn-up. (This will be achieved, presumably, with the `close detectors' in the θ_{13} experiments.)
2) Verify that the observations agree with expectations based on the separate ^{235}U, ^{239}Pu, ^{241}Pu, and ^{238}U spectra.
3) By doing all of that, form a basis for application of the ν_e monitoring of reactors not only for the study of neutrino oscillations but for any other purpose as well.
4) As far as reactor monitoring is concerned, based on these finding we can decide the optimal strategy; either a relatively simple detector that measures well the total rate, or a more sophisticated detector that tries to see changes in the spectrum shape.