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Recoillness resonant emission and detection of antineutrinos: some basic questions

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Recoilless Resonant Emission and Detection of Antineutrinos: Some basic Questions

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TOWARDS NEUTRINO TECHNOLOGIES

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Outline

I) Bound-state β-decay: resonant character

^3H – ^3He system

II) Mössbauer $\overline{\nu}_e$: Basic questions

1) Phononless transition: Recoilfree fraction; lattice expansion and contraction
2) Linewidth: homogeneous and inhomogeneous broadening
3) Relativistic effects: Second-order Doppler shift
   a) temperature
   b) zero-point motion

III) Answers to basic questions

IV) Interesting experiments

V) Conclusions

I) β-decay

I) Bound-state β-decay

J. N. Bahcall, Phys. Rev. 124, 495 (1961)

$$A(Z-1) \rightarrow \boxed{A(Z) + e^-} + \bar{\nu}_e$$

$\bar{\nu}_e$ – source
mono-energetic

Bound-state atomic orbit.
Not a capture of e^- initially created in a continuum state (less probable).

2-body process, $\bar{\nu}_e$ has a fixed energy:

$$\boxed{E_{\bar{\nu}_e} = Q + B_z - E_R}$$ where

$Q = (M_{Z-1} - M_Z)c^2$ end-point energy

B_z binding energy of electron

E_R recoil energy $\boxed{^3He + e^-}$ recoils

Example:

$$^3H \rightarrow \boxed{^3He + e^-} + \bar{\nu}_e$$

Atomic orbit in ^3He

I) β-decay

Reverse process (absorption):

$$\bar{\nu}_e + \text{A(Z)} + \text{e}^- \rightarrow A(Z-1)$$

in atomic orbit

target for $\bar{\nu}_e$

Example:

$$\bar{\nu}_e + {}^3He + \text{e}^- \rightarrow {}^3H$$

in atomic orbit of ^{3}He

energy required for $\bar{\nu}_e$:

$$E_{\bar{\nu}_e}' = Q + B_z + E_R'$$

^3H recoils

Bound-state β-decay has a resonant character which is (partially) destroyed by the recoil in source and target.

Candidates for recoilless neutrino emission and absorption

TABLE I. Candidates for recoilless neutrino absorption.

Nuclide	Q (keV)	τ (yr)	f_R[a] Recoilless fraction	α (10^{-4})	γ (10^{-16}) Line broadening	σ_{eff} (10^{-36} cm^2)	σ_{eff}/τ[b]
^{3}H	18.6	12.3	0.40	200[c]	8	0.1	1.0
^{63}Ni	68	92	0.07	1	1	10^{-9}	10^{-9}
^{93}Zr	60	1.5×10^{6}	0.18	1	7×10^{-5}	10^{-12}	10^{-16}
^{107}Pd	33	6×10^{6}	0.62	1	2×10^{-5}	10^{-11}	10^{-16}
^{151}Sm	76	90	0.11	1	1	10^{-9}	2×10^{-9}
^{171}Tm	97	1.9	0.04	1	50	5×10^{-9}	3×10^{-7}
^{187}Re	2.6	4×10^{10}	1.0	1000[d]	10^{-9}	2×10^{-7}	10^{-15}
^{193}Pt	61	50	0.29	1	2	3×10^{-8}	8×10^{-8}
^{157}Tb	58	150	0.29	0.4[d]	0.7	2×10^{-9}	10^{-9}
^{163}Ho	2.6	7000	1	73[d]	0.01	7×10^{-3}	1×10^{-4}
^{179}Ta	115	1.7	10^{-2}	0.5[d]	60	10^{-10}	6×10^{-9}
^{205}Pb	60	1.4×10^{7}	0.3	8[d]	10^{-5}	10^{-11}	10^{-16}

[a] Recoilless fraction calculated for effective Debye temperatures assuming that the nuclei are imbedded in W, and that the simple approximations in the text are valid.
[b] Normalized to 1.0 for ^{3}H.
[c] From Ref. 4.
[d] Estimated from atomic wave function calculations of the relevant shells.

W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

^{3}H-^{3}He system

$Decay$	$E_{\bar{\nu}_e}^{res}$	$ft_{1/2}$	$B\beta / C\beta$
$^{3}H \rightarrow {}^{3}He$	18.60 keV	1132 sec	6.9x10^{-3} (80% ground state, 20% excited states)

Resonance cross section (without Mössbauer effect): $\sigma \approx 1\text{x}10^{-42}$ cm^{2}

To observe bound-state β-decay: 100-MCi sources (^{3}H) and kg-targets (^{3}He) would be necessary

Thermal motion:

Doppler energy profile,
width: 0.16 eV

Recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2} \approx 0.06eV$$

II) Basic Questions

1) Phononless transition:

a) Recoilfree fraction: $f = e^{-\left(\frac{E}{\hbar c}\right)^2 \cdot \langle x^2 \rangle} \longrightarrow f < 1$

Conventional Mössbauer effect (with photons):
Source and absorber (target) involve the same type of atoms, e.g., the isotope ^{57}Fe.

recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2}$$

Debye model:

$T \rightarrow 0$: $f(T \rightarrow 0) = \exp\left\{-\frac{E^2}{2Mc^2} \cdot \frac{3}{2k_B \Theta}\right\}$

recoil energy

f depends on: transition energy E
mass M of the atom
Debye temperature Θ

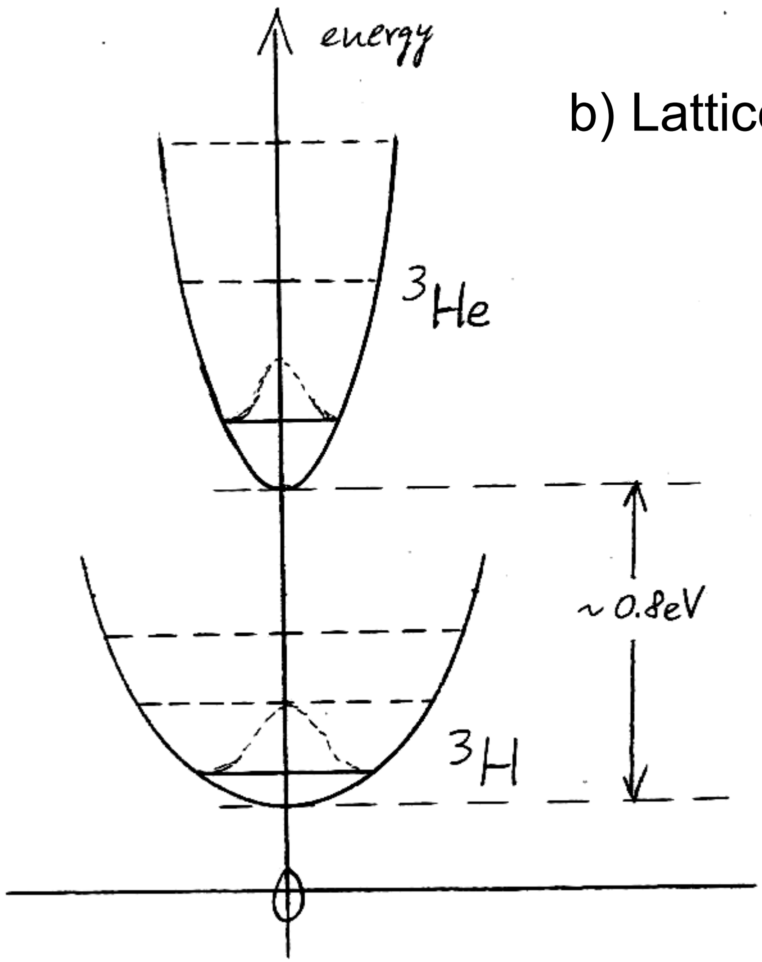
Example: ^{3}H – ^{3}He

typically: $f(0) \approx 0.27$ for Θ≈800K

Emission and absorption:

$f^{^3H} \cdot f^{^3He} \approx 0.07$ for T→0

II) Basic Questions



M. J. Puska et al., PRB10, 5382 (1984)

^{3}H as well as ^{3}He in metallic lattices:

Nb metal, tetrahedral interstitial sites

b) Lattice expansion and contraction: in addition to recoil

Nuclear transformations occur when $\overline{\nu}_e$ is emitted or captured. ^{3}He and ^{3}H use different amounts of lattice space. Will this cause lattice excitations (phonons)?

Lattice-deformation energies of ^{3}H and ^{3}He in Nb metal:

$$E_L(^3H) = 0.099\,eV\,;\,E_L(^3He) = 0.551\,eV$$

$$f^L(T \to 0) \le \exp\left\{-\frac{E_L(^3He) - E_L(^3H)}{k_B\Theta}\right\} \approx 1\cdot 10^{-3}$$

$f^L(0)^2 \approx 1\cdot 10^{-6}$ and $f(0)^2 \cdot f^L(0)^2 \approx 7\cdot 10^{-8}$

→ Theoretical calculations:

David Ceperley, University of Illinois

Mark G. Raizen, University of Texas at Austin

II) Basic Questions

2) Linewidth

minimal width (natural width): $\Delta E^{nat} = \Gamma = \hbar / \tau$ τ: lifetime

^3H: τ = 17.81 y ⟶ $\Delta E^{nat} = \Gamma = 1.17 \cdot 10^{-24} eV$ (extremely narrow)

Two types of line broadening:

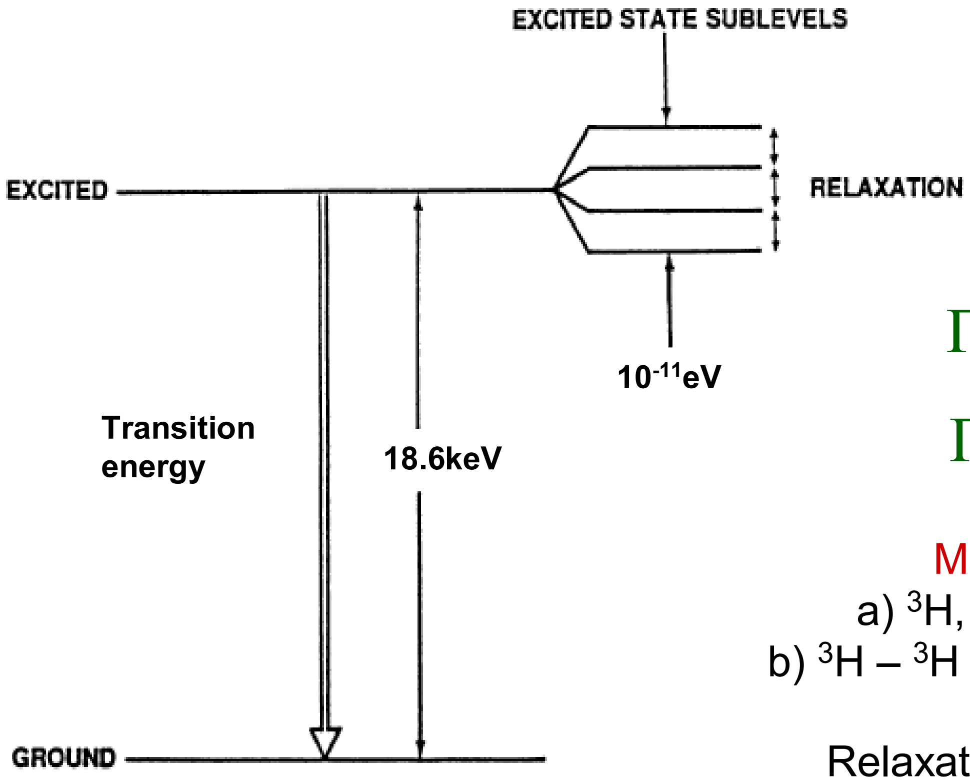
a) homogeneous broadening ↓ due to fluctuations, e. g. of magnetic fields, **stochastic processes**

b) inhomogeneous broadening ↓ due to stationary effects, e.g. impurities, **lattice defects which cause variations of line shifts**

How big are these broadening effects?

II) Basic Questions

a) homogeneous broadening: stochastic processes, **not** connected to lattice vibrations, present at all temperatures



Measurements: ^3H (Pd), ^3H (Ti-H), NbH

Typical relaxation times: T_2~2ms, 79µs

$$\Gamma_{exp} \sim 5\times10^{-11}eV \sim 4\times10^{13}\,\Gamma$$

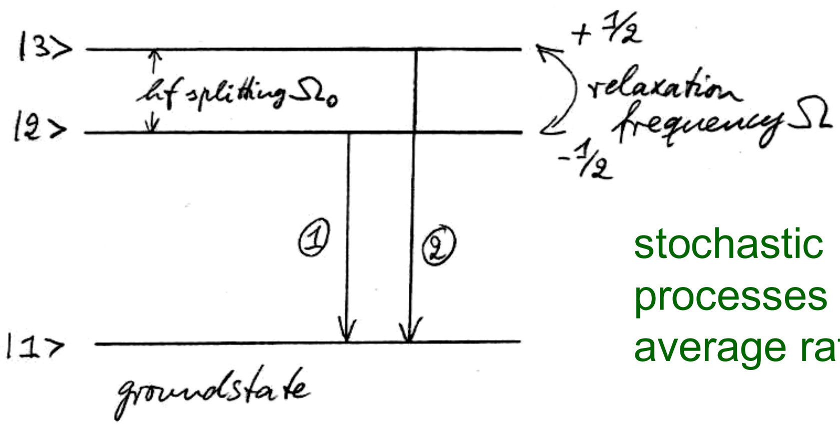
$$\Gamma_{exp} \sim \Gamma\ (^{67}Zn)$$

Magnetic interactions:
a) ^3H, ^3He with nuclei of metallic lattice
b) ^3H – ^3H magnetic dipolar spin-spin interaction

Relaxation between the sublevels affects the lineshape and the total linewidth.

The linewidth is determined by the relaxation rate.

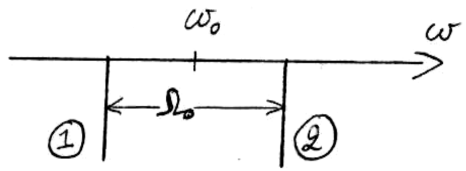
Homogeneous Broadening: Magnetic Relaxation



stochastic processes average rate: Ω

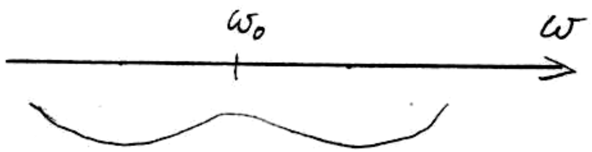
Simplest magnetic relaxation model

Sudden, irregular transitions (relaxation) between hyperfine-split states
→irregular (random) phase changes of transitions to ground state, no correlation to original phase
→it might take a long time to come back to the original frequency



$\Omega << \Omega_0$

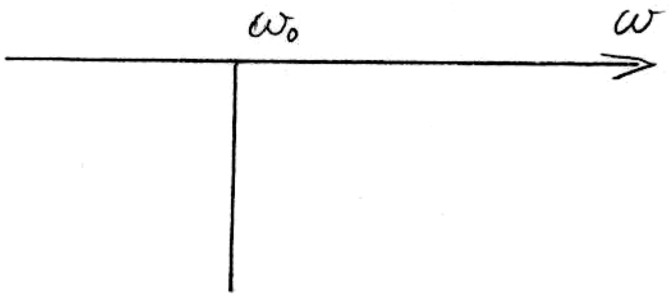
Two lines of (almost) natural width:
With increasing Ω, the lines broaden
→effective lifetime (time-uncertainty principle)



$\Omega \approx \Omega_0$

Intensity is distributed over a broad pattern, which extends over the total hf splitting Ω_0 as suggested by the time-energy uncertainty principle

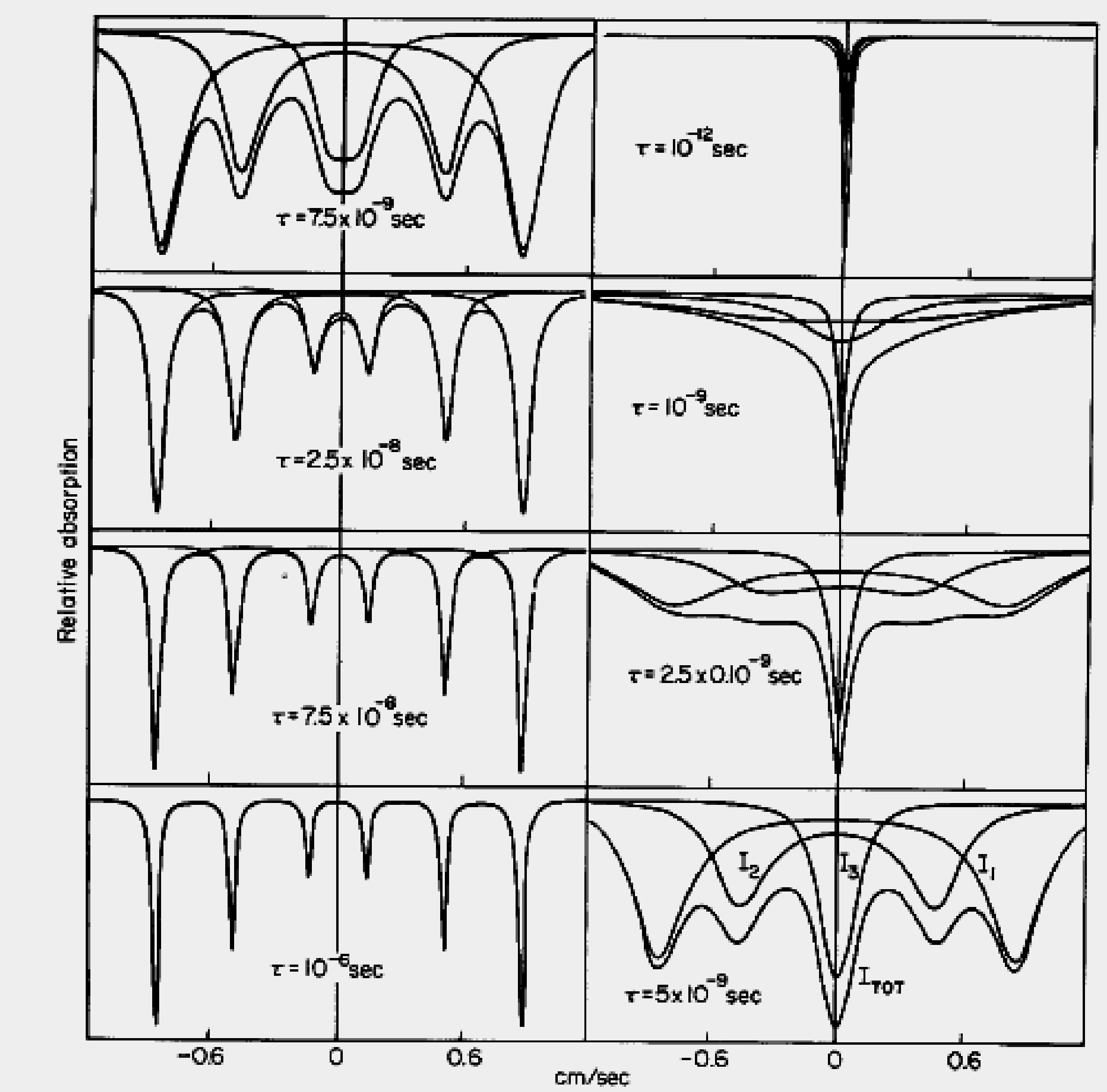
$^3H/^3He$ system in Nb metal: $\Omega_0 \sim 10^5\ s^{-1}$ and $\Omega \sim 8\text{x}10^4\ s^{-1}$. → $\Gamma_{exp} \sim 5\text{x}10^{-11}$eV ~$4\text{x}10^{13}\ \Gamma$.



$\Omega >> \Omega_0$

Motional narrowing: one line at the center of the hf splitting of practical natural width. Stochastic frequency changes: between lines 1 and 2. Averaging process over many short parts of the lifetime. **Not the case for $^3H/^3He$.**

II) Basic Questions



Relaxation spectra for ^{57}Fe

Superposition of 3 doublets

Average relaxation rate: $\Omega = 2\pi / \tau$

H.H. Wickman and G.K. Wertheim in: *Chemical Applications of Mössbauer Spectroscopy,* V.I. Goldanskii and R. Herber, editors; pp. 548 (New York: Academic Press, 1968)

II) Basic Questions

b) inhomogeneous broadening: Stationary effects: lattice defects, impurities

Conventional Mössbauer spectroscopy: Different binding strengths due to inhomogeneities affect the energy of the photons in the **same type** of nucleus in source and target.

⟶ Shift of photon energy by typically 10^{-7} – 10^{-9} eV (hyperfine interaction)

In the best single crystals: inhomogeneities cause shifts of 10^{-13} to 10^{-12} eV.
For $\bar{\nu}_e$: corresp. to $10^{11}\,\Gamma$ or $10^{12}\,\Gamma$ or even larger.

Binding energies of ^3H and ^3He in an inhomogeneous metallic lattice **directly** influence the $\bar{\nu}_e$ energy.

Binding energies per atom: ~eV range (Coulomb interaction).

⟶ Variation of the $\bar{\nu}_e$ energy much larger than neV, typically: meV range.

⟶ Variation of the $\bar{\nu}_e$ energy by only 10^{-6} eV → $10^{18}\,\Gamma$.

II) Basic Questions

3) Relativistic effects

Second-order Doppler shift due to mean-square atomic velocity <V²>

moving system

Time-dilatation effect: $\Delta t = \dfrac{\Delta t'}{\sqrt{1-(V/c)^2}}$

stationary system

Frequencies: $\omega = \omega' \sqrt{1-(V/c)^2} \approx \omega' \left(1 - \dfrac{V^2}{2c^2}\right)$

Second-order Doppler shift: $\Delta\omega = \omega - \omega' = -\omega' \dfrac{V^2}{2c^2}$

Reduction of frequency (energy)

II) Basic Questions

Within the Debye model:

$$\frac{\Delta E}{E} = \frac{9k_B}{16Mc^2}(\Theta_s - \Theta_t) + \frac{3k_B}{2Mc^2}\left[T_s \cdot f\left(\frac{T_s}{\Theta_s}\right) - T_t \cdot f\left(\frac{T_t}{\Theta_t}\right)\right]$$

Zero-point energy

where

$$f\left(\frac{T}{\Theta}\right) = 3\left(\frac{T}{\Theta}\right)^3 \cdot \int_0^{\Theta/T} \frac{x^3}{\exp x - 1} dx$$

If $|T_s - T_t| = 1$ degree $\longrightarrow$ $\Delta E = 1.9 \cdot 10^{-9}\, eV \approx 1.6 \cdot 10^{15}\, \Gamma \approx 35 \cdot \Gamma_{\exp}$

Heat generation in the source: 1 kCi ~ 0.1 W; causes temperature gradient ΔT.
Nb metal: supercond. below 9.2K; → poor heat conductor → stay above 9.2K.
J. Schiffer: 1Γ≈0.05 nK, throughout the sample.

Low temperatures: $T_s \approx T_t \approx 0 \longrightarrow [....] \approx 0$

However, zero-point energy remains!

If $|\Theta_s - \Theta_t| = 1$ degree (0.08meV) → $\Delta E / E \approx 2 \cdot 10^{-14} \longrightarrow \Delta E \approx 3 \cdot 10^{14}\, \Gamma \approx 7 \cdot \Gamma_{\exp}$

Similar conclusion as reached from different binding energies.

III) Answers to Basic Questions

A) Principle difficulties

1) Probability for phononless emission and detection: ~ $7x10^{-8}$;
due to lattice expansion and contraction (not present with conventional Mössbauer effect) and due to recoil.

2) Homogeneous line broadening: $\Gamma_{exp} > 4x10^{13}\,\Gamma \approx 5x10^{-11}$ eV.

3) Inhomogeneous line broadening due to random distribution of ^3H and ^3He in metal lattice (entropy) → **direct** influence of binding energies.

$\Gamma_{exp} \gg 10^{12}\,\Gamma$, with much luck $\Gamma_{exp} \approx 10^{18}\,\Gamma \approx 10^{-6}$ eV
(200 times broader than the ^{57}Fe Mössbauer resonance).

4) Relativistic effects (zero point energy) due to different binding energies:

$$\Gamma_{exp} > 3x10^{14}\,\Gamma \approx 3x10^{-10}\text{ eV}.$$

III) Answers to Basic Questions

B) Technological difficulties (to name only two)

1) Heat production in source of 1kCi is 0.1 W, → temperature gradients ΔT.
For natural width Γ, $\Delta T \ll 10^{-11}$K (relativistic effect).

2) Stability of apparatus for continuous measurement:
e.g., mechanical and temperature variations must be negligible for time comparable to lifetime, i.e. for ~ 20 years.

^3H-^3He system

A) Preparation of source and target

Source:

^3H chemically loaded into metals to form hydrides (tritides), e.g., Nb: in tetrahedral interstitial sites (IS).

Target:

^3He accumulates with time due to the tritium trick:

$$Nb^3H_x \xrightarrow{\text{time=200d}} Nb^3H_{x\text{-}y}{}^3He_y \xrightarrow{\text{remove}} Nb^3He_y$$

Remove ^3H by isotopic exchange $^3H \rightarrow D$

R. S. Raghavan, hep-ph/0601079 v3, 2006

^3H-^3He system

How much metal for source and target?

Source:

1 kCi of ^3H (~100mg ^3H): ~3g of Nb^3H

for NMR studies: 0.5 kCi ^3H in 2.4g $PdH_{0.6}$

Target:

100mg of ^3He implies ~100g of Nb^3H aged for 200 d

^3H-^3He system

B) Event rates for ^3H – ^3He recoilless resonant capture of antineutrinos

Base line	^3H	^3He	Antineutrino capture per day	$R\beta(\Delta t=65d)$ per day
5 cm	1 kCi	100 mg	$\sim 40\times10^3$	~40
10 m	1 MCi	1 g	$\sim 10^3$	~10

1) only homogeneous broadening, assuming
$\rightarrow \Gamma_{exp}=9\times10^{-12}eV\approx 8\times10^{12}\Gamma$
$\sigma_{res}\approx 3\times10^{-33}$ cm^2

2) **no** lattice expansion and contraction

$R\beta(\Delta t)$/day: Reverse β-activity rate after growth period $\Delta t=65d=0.01\tau$

R. S. Raghavan, hep-ph/0601079 v3, 2006

IV) Interesting experiments

1) Do Mössbauer (anti)neutrinos oscillate?

2) Determination of mass hierarchy and oscillation parameters
Δm^2_{32} and Δm^2_{12}: 0.6% and $\sin^2 2\theta_{13}$: 0.002

3) Search for sterile neutrinos

4) Gravitational redshift experiments (Earth).

IV) Interesting experiments

1) Do Mössbauer neutrinos oscillate?

S.M. Bilenky et al., Phys. Part. Nucl. **38**, 117 (2007)
S.M. Bilenky, arXiv: 0708.0260
S.M. Bilenky et al., J. Phys. G **34**, 987 (2007)
E.Kh. Akhmedov et al., arXiv: 0802.2513; JHEP **05** (2008) 005
S.M. Bilenky et al., arXiv: 0803.0527 v2; J. Phys. G **35** (2008) 095003
E.Kh. Akhmedov et al., arXiv: 0803.1424 v2; J. Phys. G **36** (2009) 078001
S.M. Bilenky et al., arXiv: 0804.3409; J. Phys. G **36** (2009) 078002
S.M. Bilenky et al., arXiv: 0903.5234
J. Kopp, arXiv: 0904.4346

IV) Interesting experiments

1) Do Mössbauer neutrinos oscillate?
Different approaches to neutrino oscillations

A) Evolution of the neutrino state Ψ(t) in time:

Neutrino oscillations occur if Ψ(t) is a superposition of states of neutrinos with different energies
→ non-stationary phenomenon
→ No oscillations for Mössbauer neutrinos

B) Evolution of the neutrino wave function in space and time:

→ Oscillations are possible in both the non-stationary and also in the stationary case (Mössbauer neutrinos)

IV) Interesting experiments

2) If Mössbauer neutrinos do oscillate:

Ultra-short base lines for neutrino-oscillation experiments

For only two flavors: $P(\nu_a \to \nu_b) = \sin^2 2\Theta \cdot \sin^2(\pi L / L_0)$

Amplitude: $\sin^2 2\Theta$ Oscillatory term: $\sin^2(\pi L / L_0)$

Oscillation length: $L_0 = 4\pi\hbar c \dfrac{E}{|\Delta m^2|} \approx 2.480 \dfrac{E/MeV}{|\Delta m^2|/eV}$ [m]

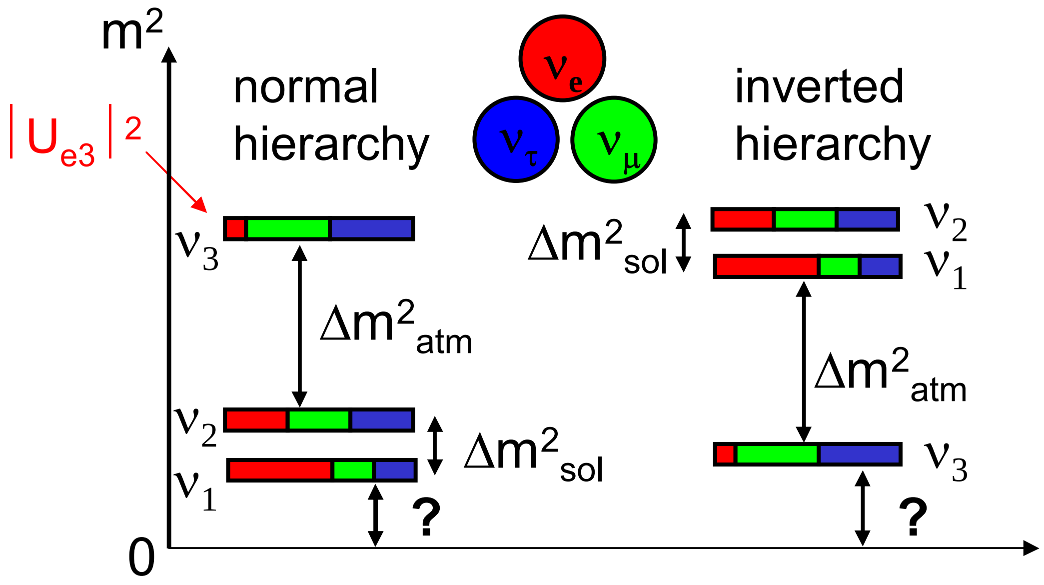
A) Determination of Θ_{13}: *E=18.6 keV* instead of *3 MeV.*

Δm^2_{23} observed with *atmospheric* neutrinos

Chooz experiment: $\sin^2 2\Theta_{13} \leq 2 \cdot 10^{-1}$ Oscillation base line: $L_0/2 \sim 9.3$ m

→ Base line *L* of *9.3 m* instead of *1500 m*

B) Mass hierarchy and oscillation parameters



H. Nunokawa et al., hep-ph/0503283

To determine mass hierarchy:
Measure Δm^2 in reactor-neutrino and muon-neutrino (accelerator long-baseline) disappearance channels to better than a fraction of 1%

H. Minakata et al., hep-ph/0602046

For $\sin^2 2\theta_{13}=0.05$ and 10 different detector locations one can reach uncertainties:
in Δm^2_{31} and Δm^2_{12}: 0.6%,
in $\sin^2 2\theta_{13}$: 0.002

H. Minakata et al.: hep-ph/0701151
S. Parke et al.: 0812.1879 (hep-ph)
Phase changes of atmosph. w.r. to solar oscill.

$|\Delta_{31}| > |\Delta_{32}|$ Phase advances

$|\Delta_{31}| < |\Delta_{32}|$ Phase retarded

IV) Interesting experiments

3) Search for conversion of $\bar{\nu}_e \longrightarrow \nu_{sterile}$

LSND experiment: $\Delta m^2 \approx 1eV^2$ and $\sin^2 2\theta \sim 0.1$ to 0.001

(largely excluded by MiniBooNE)

Possibility: $\bar{\nu}_e \longrightarrow \nu_{sterile}$

V. Kopeikin et al. : hep-ph/0310246

Test: Disappearance experiment with 18.6 keV antineutrinos

→ Oscillation length L_0 ~5cm!

→ Ultra-short base line, difficult to reach otherwise

IV) Interesting experiments

4) Gravitational redshift experiments (Earth)

Gravitational redshift: $\frac{\delta E}{E} = \frac{gh}{c^2}$

Experimental linewidth: $\Gamma_{\exp} = \Delta = 5 \cdot 10^{-11} eV$

$\Delta = \frac{\hbar\omega}{c^2} g h_\Delta$ where h_Δ is height corresponding to 1 experimental linewidth

$\longrightarrow h_\Delta \approx 25m$

For $\Gamma_{\exp} \approx 10^{-6} eV \longrightarrow$ unrealistic

Can **not** be used to determine the neutrino mass

Gravitational spectrometer

V) Conclusions

A) Phononless resonant emission and detection of antineutrinos:

$^{3}H - {}^{3}He$ system.

B) Experiment is very difficult, if not impossible.

Not possible to reach natural width.

1) Principle difficulties:

a) Probability for phononless emission and detection might be smaller than expected due to lattice expansion and contraction after the transformation of the nucleus:

Additional reduction factor of 1×10^{6}.

b) Homogeneous broadening (stochastic processes) and inhomogeneous broadening (variation of binding energies and of zero-point energies).

$$\longrightarrow \quad \Gamma_{exp} \approx 10^{18}\, \Gamma \approx 10^{-6}\ \text{eV}$$

V) Conclusions

2) Many technological difficulties, for example:

a) Temperature differences within the source and between source and target (temperature shift)
b) Stability (mechanical and temperature) of apparatus for continuous measuring times of ~20 years.

C) Interesting experiments:

a) Do Mössbauer neutrinos oscillate?
b) Mass hierarchy and accurate determination of oscillation parameters
c) Search for sterile neutrinos (LSND experiment, MiniBooNE?)
d) Gravitational redshift experiments (Earth).

Extra slides

Papers

Earlier papers:

W. M. Visscher, Phys. Rev. 116, 1581 (1959)

W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

More recent papers:

R. S. Raghavan, hep-ph/0601079 v3, 2006

W. Potzel, Phys. Scrip. T127, 85 (2006);

S. M. Bilenky, F. von Feilitzsch, and W. Potzel,
J. Phys. G: Nucl. Part. Phys. **34**, 987 (2007);

E. Kh. Akhmedov, J. Kopp, and M. Lindner, 0802.2513 (hep-ph)

I) β-decay

Resonance cross section

$$\sigma = 4.18 \cdot 10^{-41} g_0^2 \cdot \frac{\rho(E_{\bar{\nu}_e}^{res})}{ft_{1/2}} \left[cm^2\right]$$

L.A. Mikaélyan, et al.: Sov. J. Nucl. Phys. 6, 254 (1968)

$$g_0 = 4\pi \left(\frac{\hbar}{mc}\right)^3 |\Psi|^2 \approx 4 \left(\frac{Z}{137}\right)^3$$

for low Z, hydrogen-like ψ
m: electron mass
$|\psi|^2$: probability density of e in A(Z)

$\rho(E_{\bar{\nu}_e}^{res})$: resonant spectral density, i.e., number of $\bar{\nu}_e$ in an energy interval of 1MeV around $E_{\bar{\nu}_e}^{res}$

$ft_{1/2}$ value: reduced half-life of decay

$ft_{1/2} \approx 1000$: super-allowed transition

II) Recoilless antineutrino emission and detection: Mössbauer neutrinos

1) Recoilfree fraction

Stop thermal motion!
Make E_R negligibly small!

recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2}$$

^3H as well as ^3He in metallic lattices:
freeze their motion →no Doppler broadening.
$M \rightarrow M_{lattice} >> M$
Leave lattice unchanged, leave phonons unchanged.

zero-point energy

Energy of lattice with N particles: $E_L = \sum_{s=1}^{3N} (n_s + 1/2)\hbar\omega_s$ $\quad (n_s = 0,1,2,\ldots)$

3N normal modes

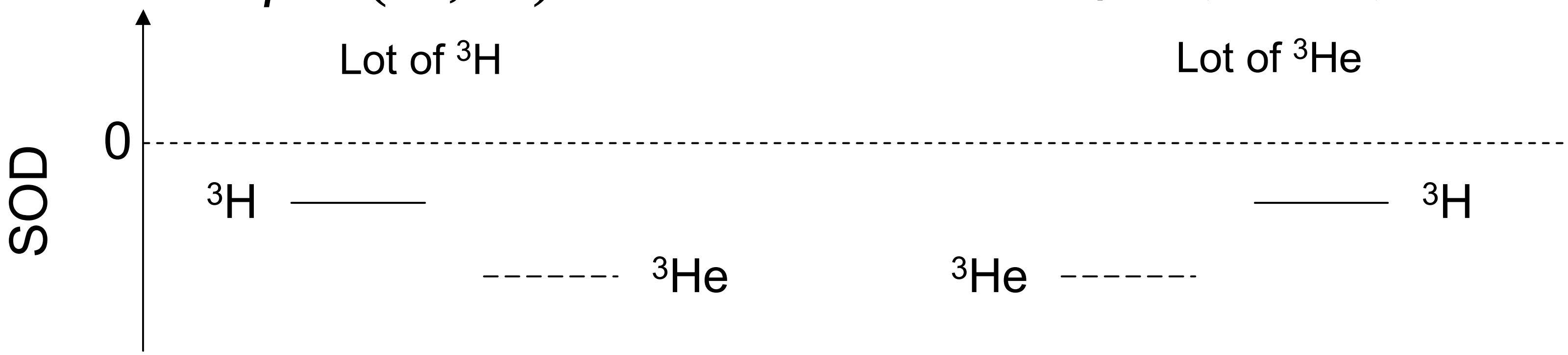
$$E_L = \int_0^{\omega_{max}} \left(\overline{n(\omega)} + 1/2\right)\omega \cdot Z(\omega)\, d\omega \quad \text{with} \quad \overline{n(\omega)} = 1/\left(\exp(\hbar\omega/k_B T) - 1\right)$$

$Z(\omega)\cdot d\omega$: number of oscillators with frequency *ω* between *ω* and *ω* + *dω*

II) Recoilless antineutrino …

What does this mean for the effective values Θ_s and Θ_t ?

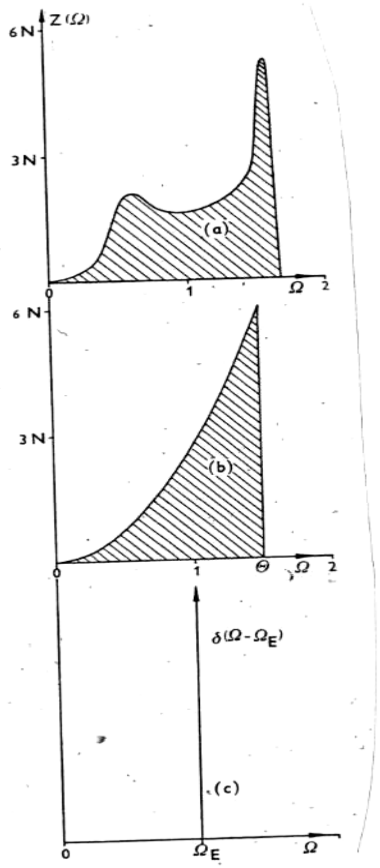
Source	Target
$^3H \longrightarrow {}^3He$	$^3He \longrightarrow {}^3H$
$f^{\,^3He}(M,{}^3H)$	$f^{\,^3H}(M,{}^3He)$

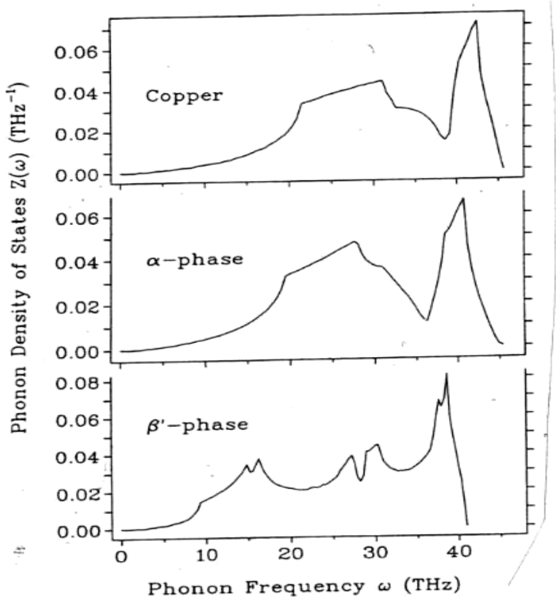


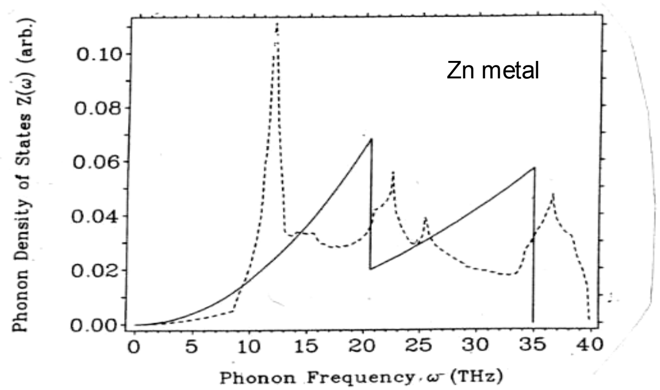
The differences of these SOD values in source and target have to be the same. In a practical experiment this means:

The Debye temperature for ^3H has to be the same in source and target. The same holds for ^3He. The Debye temperatures of ^3H and ^3He in the metal matrix do <u>not</u> have to be equal.

Phonon density of states







2) Linewidth

^{109m}Ag: gravitational spectrometer

$$\Gamma \approx 1.2 \cdot 10^{-17}\, eV \qquad \tau \approx 40s$$

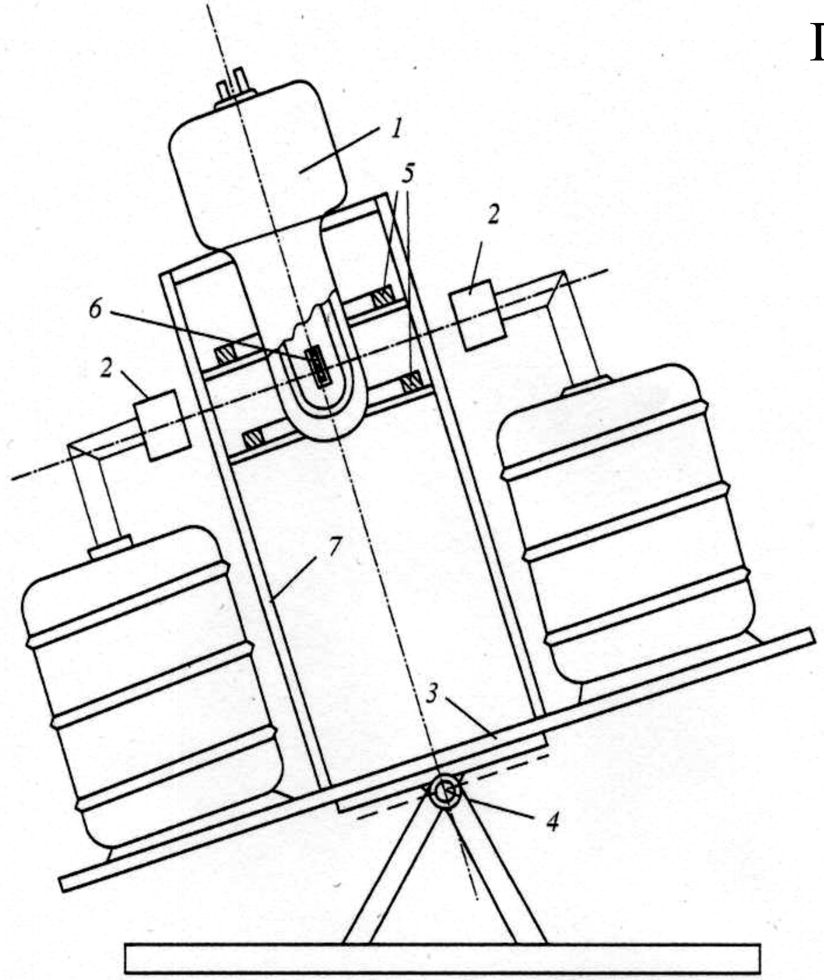
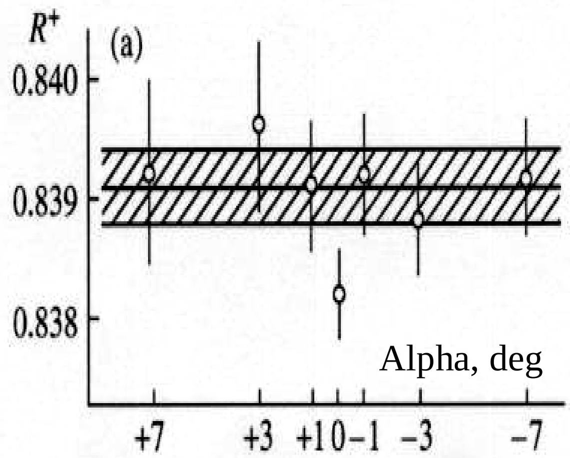


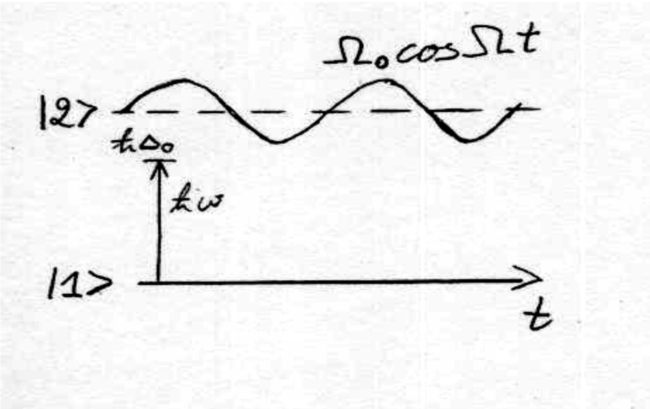
Fig. 1. Scheme of the gravitational gamma spectrometer: (*1*) cryostat, (*2*) germanium gamma detectors, (*3*) rotating platform, (*4*) support of cryostat and Helmholtz coils, (*5*) Helmholtz coils, (*6*) gamma sources, and (*7*) rotation axic of the platform.



V.G. Alpatov et al., Laser Physics 17 (2007) 1067

Homogeneous Broadening: Frequency Modulation

M. Salkola and S. Stenholm, Phys. Rev. A **41**, 3838 (1990)



$$A \propto \sum_{k=-\infty}^{k=+\infty} J_k^2(\eta) \frac{1}{\left[(\Delta_0/\Gamma) - k\xi\right]^2 + 1}$$

sum of Lorentzians, located at $\omega = \omega_0 \pm k\Omega$

Ω_0 : max. freq. deviation

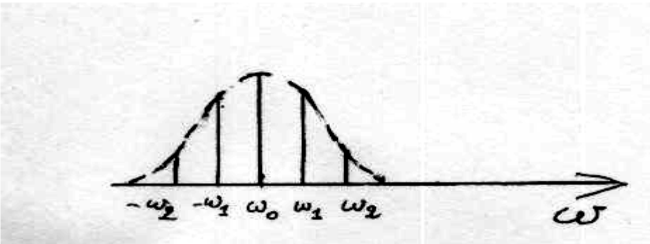
Ω : relaxation frequency

$\eta = \dfrac{\Omega_0}{\Omega}$: modulation index

$$\Delta_0 = \omega_0 - \omega$$

Γ : linewidth

$$\xi = \frac{\Omega}{\Gamma}$$

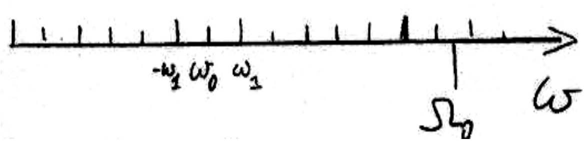


$$\eta \approx 1 \Rightarrow \Omega \approx \Omega_0$$

$$\Gamma << \Omega$$

motional narrowing: $\Omega >> \Omega_0 \Rightarrow \eta \approx 0$

only center line at ω_0 survives



$$\Omega_0 >> \Omega \Rightarrow \eta >> 1$$

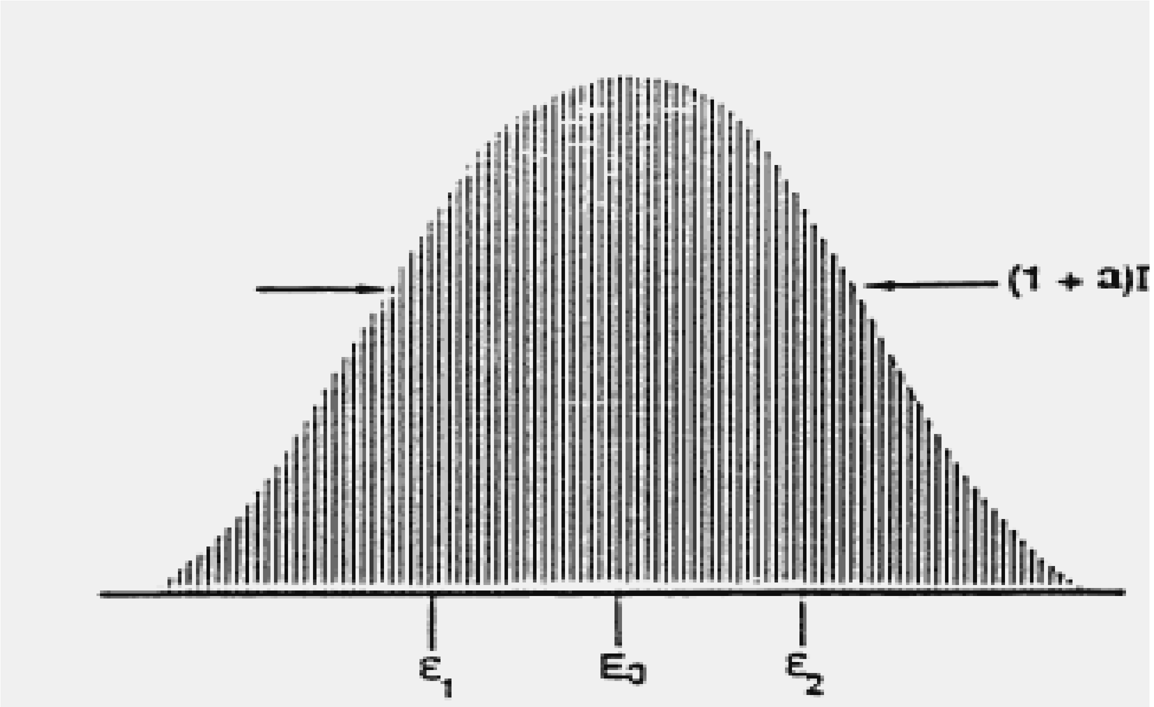
many sidebands→at ω_0 very little intensity

motional narrowing: not possible

Typical for resonaces in Ag and for the $^3H/^3He$ system. For Ag: $\Omega_0 \sim 10^5\ s^{-1}$ and $\Omega \sim 10\ s^{-1}$

II) Recoilless antineutrino …

b) inhomogeneous broadening

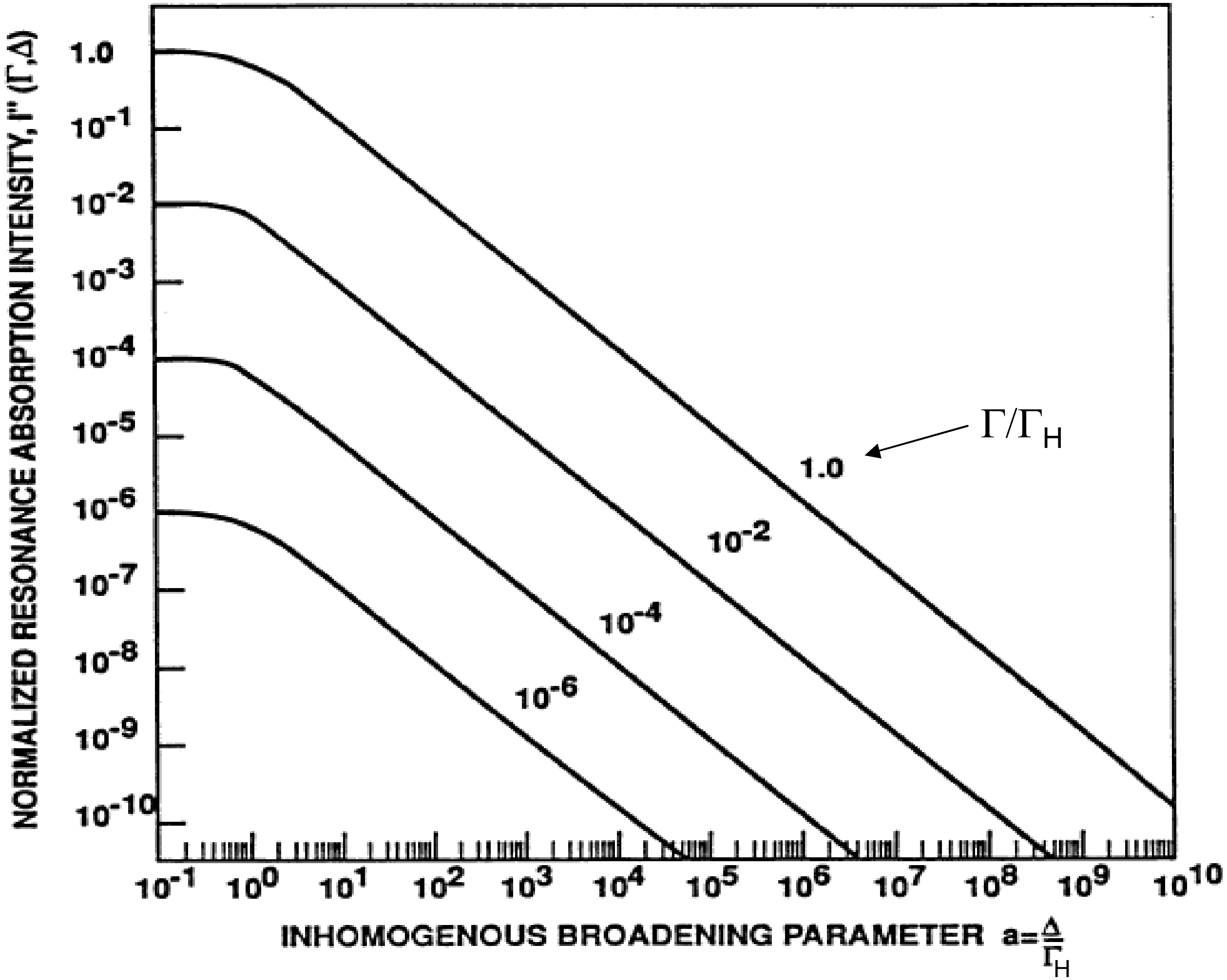


Many individual resonances displaced from the nonperturbed resonance energy E_0

In the best single crystals: $(1 + a)\Gamma \sim 10^{-13}$ eV corresp. to $10^{11}\,\Gamma$ or larger

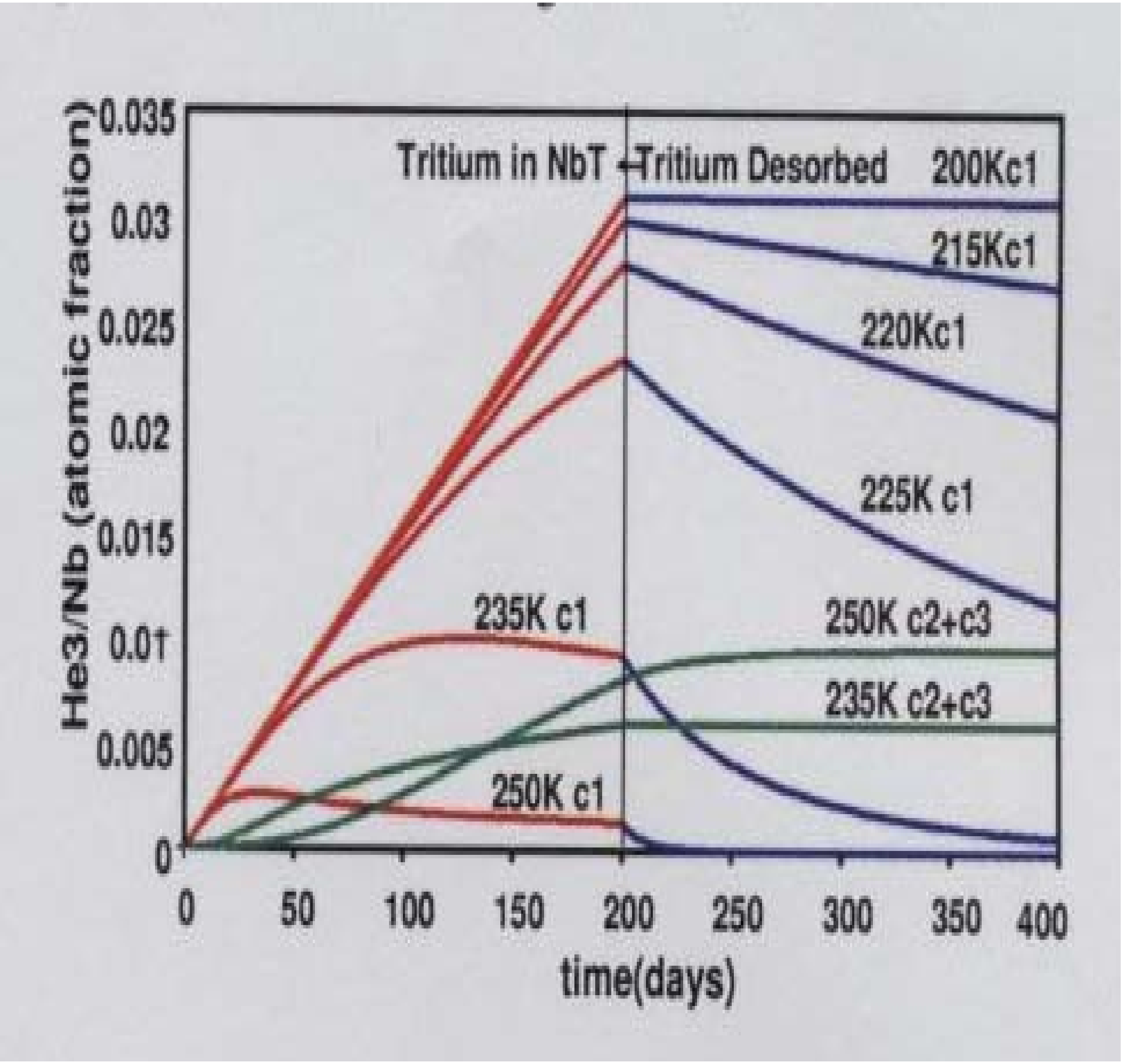
Both types of broadening reduce the resonant reaction intensity

II) Recoilless antineutrino …



B. Balko, I. W. Kay, J. Nicoll, J. D. Silk, and G. Herling, Hyperfine Interactions **107**, 283 (1997).

IV) Consequences ...



^3He generated in Nb: c1: concentration in interstitial sites for different temperatures and times. The He in the T-free absorber below 200K is almost all interstitial.

R.S. Raghavan: hep-ph/0601079 revised v3; calculations: Sandia Natl. Lab., USA

IV) Consequences …

Table 1 He transport parameters in NbT at 200K

M_1T_1	E1 eV	E2 eV	E3 eV	D/cm^2 s
M=Nb	0.9[a]	0.13[b]	0.43[b]	1.1E-26[c]

[a] Ref. 7; [b] Ref. 9; [c] Assumes *tritium* pre-exponential D_0 (ref. 6)

Table 2. Theoretical (Ref. 7) EST & ZPE for T and ^{3}He in Nb interstitial sites (IS)

Site	EST (eV)		ZPE (eV)	
	T	He	T	He
TIS	-0.133	-0.906	0.071	0.093
OIS	-0.113	-0.903	0.063	0.082

6 TIS
3 OIS

EST: self-trapping energy
ZPE: zero-point energy

Table 3. Nearest neighbor (NN) Displacements(%) and measured[6] activation energies Eac(eV) in NbIS (Ref. 7)

	1st NN Displacement			2nd NN Displacement.		
	H	D	T	H	D	T
TIS	4.1	3.9	3.9	-0.37	-0.36	-0.35
OIS	7.7	7.5	7.4	0.2	0.19	0.19
Eac[6]	0.106	0.127	0.135			

Little difference between Deuterium and Tritium

theoretical

experimental activation energies

III) Interesting experiments

Question: What will be the state of the neutrino after some time (at some distance L)?

A) Evolution in time

Schrödinger equation for evolution of any quantum system:

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \longrightarrow |\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

$$P(\nu_l \to \nu_{l'}) = \left|\sum_{k=1}^{3} U_{l'k} e^{-i(E_k - E_i)t} U^*_{lk}\right|^2$$

No matter what the neutrino momenta are !

If E_k=E_i, there will be no neutrino oscillations: $P(\nu_l \to \nu_{l'}) = \delta_{l'l}$
The neutrino state is stationary

If E_k are different, neutrino state is non-stationary.
→time-energy uncertainty relation holds: $\Delta E \cdot \Delta t \geq 1$

Δt is the time interval during which the state of the system is significantly changed

If E_k≠E_i, the uncertainty relation takes the form: $(E_k - E_1)\cdot t \approx \frac{\Delta m^2_{1k}}{2E} t$

III) Interesting experiments

B) Evolution in time and space

Mixed neutrino state at space-time point $x = (t, \vec{x})$:

$$\left|\nu_l\right\rangle_x = \sum_{k=1}^{3} e^{-ip_k x} U_{lk}^* \left|\nu_k\right\rangle \longrightarrow P(\nu_l \to \nu_{l'}) = \left|\sum_{k=1}^{3} U_{l'k} e^{-i(p_k - p_1)x} U_{lk}^*\right|^2$$

with $(p_k - p_1) = \dfrac{E_k^2 - E_1^2}{E_k + E_1} t - (p_k - p_1)L$ and $E_i^2 = p_i^2 + m_i^2$

a) $t \approx L \longrightarrow (p_k - p_1)x \approx \dfrac{\Delta m_{1k}^2}{2E} L$ oscillatory phase

b) neutrinos: different masses have the *same* energy

⟶ neutrino state is stationary

$$\longrightarrow p_k \neq p_i : \quad (p_k - p_i)x = \frac{\Delta m_{1k}^2}{2E} L \qquad P(\nu_l \to \nu_{l'}) = \left|\sum_{k=1}^{3} U_{l'k} e^{-i\Delta m_{1k}^2 \frac{L}{2E}} U_{lk}^*\right|^2$$

III) Interesting experiments

Mössbauer neutrinos:

Energy width: $\Gamma_{exp} = 8.6 \cdot 10^{-12} eV$

a) $(E_3 - E_2) \approx \frac{\Delta m^2_{23}}{2E} \approx 6.5 \cdot 10^{-8} eV$ Δm^2_{23} observed with *atmospheric* neutrinos

⟶ Mössbauer neutrinos take a long time to change significantly

⟶ Time-energy uncertainty: Extremely long “oscillation “ length

Determination of Θ_{13}: *E=18.6 keV* instead of *3 MeV.*

Chooz experiment: $\sin^2 2\Theta_{13} \leq 2 \cdot 10^{-1}$ Oscillation base line: $L_0/2$~9.3 m

b) Δm^2_{12} observed with *solar* neutrinos

$$(E_2 - E_1) \approx \frac{\Delta m^2_{12}}{2E} \approx 2.1 \cdot 10^{-9} eV$$

Amplitude: $\sin^2 2\Theta_{12} \approx 0.82$

Oscillation base line: $L_0/2$~300 m

Oscillation length: $L_0 = 4\pi\hbar c \frac{E}{|\Delta m^2|} \approx 2.480 \frac{E / MeV}{|\Delta m^2| / eV}$ [m]

III) Interesting experiments

5) Real-time, ^{3}H-specific signal of $\bar{\nu}_e$ resonance

a) sudden change of the magnetic moment from
-2.1nm (^{3}He)→+2.79nm (^{3}H)

→ transient (~0.1ms) magnetic field which couples to electron moment of ^{3}H via hyperfine interaction

→ Read-out by SQUID

b) new electrons appear in the Nb bands when ^{3}H is formed.
These electrons cause additional specific heat that grows linearly with ^{3}H concentration.

→ detectable by ultra-sensitive (micro)-calorimeters ?

III) Answers to Basic Questions

B) Technological difficulties (to name only two)

1) Heat production in source of 1kCi is 0.1 W, → temperature gradients ΔT.
For natural width Γ, $\Delta T \ll 10^{-11}$K (relativistic effect).

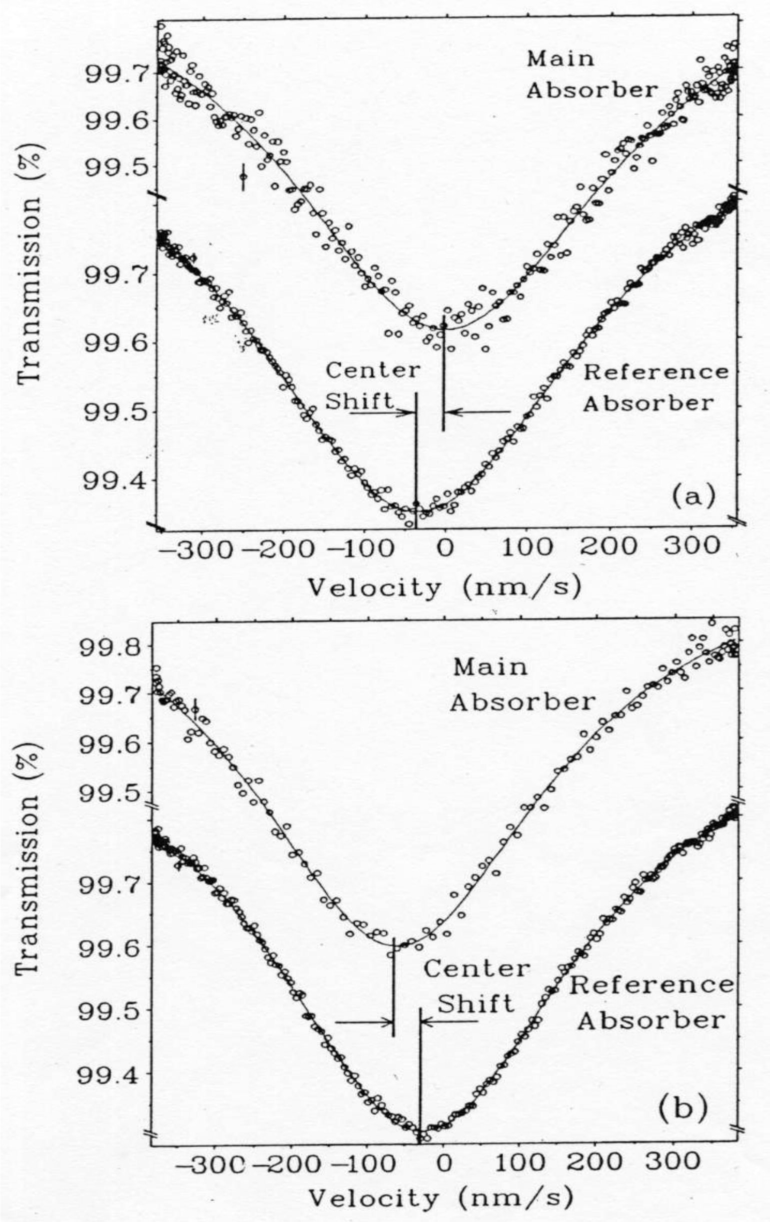
2) Stability of apparatus for continuous measurement:
e.g., mechanical and temperature variations must be negligible for time comparable to lifetime, i.e. for ~ 20 years.

C) Age of the source

To use ^3H sources produced at different times, i.e., the age itself of the ^3H source does not influence the linewidth.

In an $\bar{\nu}_e$ experiment, the clock is started together with the measurement when source and target are arranged in their fixed positions.

Red(blue)shift ^{67}ZnO-Mössbauer exp.



gravitational redshift

difference in height: 1m
in gravitational field of Earth

gravitational blueshift

accuracy: $(\Delta E/E) \leq 1\text{x}10^{-18}$

W. Potzel et al., Hyp. Interact. 72, 197 (1992)

Gravitational Redshift Experiment

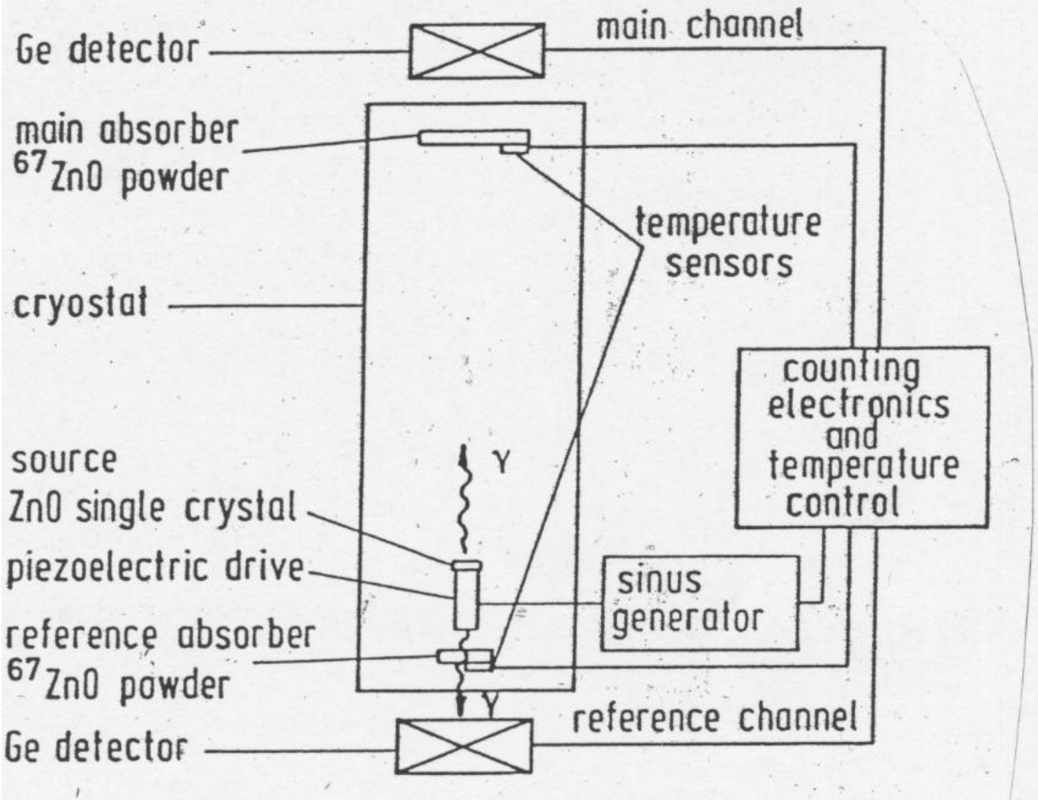


Fig. 3. Basic set-up of the Mössbauer gravitational redshift experiment. Two transmission experiments are carried out simultaneously: through the reference absorber and through the main absorber. A piezoelectric drive moves the source sinusoidally with respect to both absorbers.