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Moessbauer neutrinos in quantum mechanics and quantum field theory

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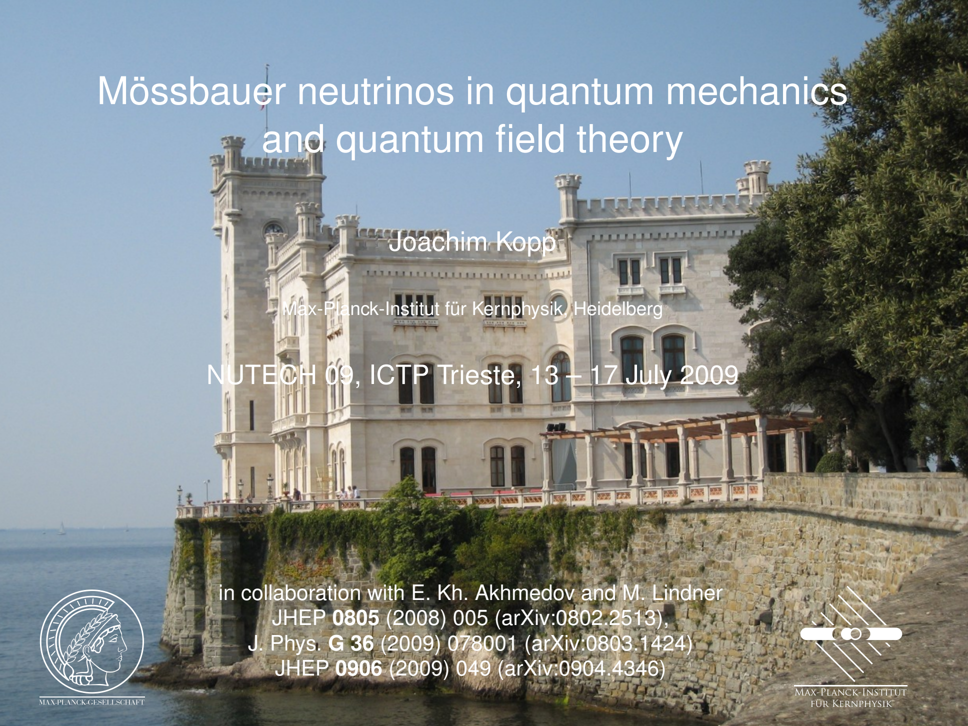

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NUTECH 09, ICTP Trieste, 13 – 17 July 2009

in collaboration with E. Kh. Akhmedov and M. Lindner
JHEP **0805** (2008) 005 (arXiv:0802.2513),
J. Phys. **G 36** (2009) 078001 (arXiv:0803.1424)
JHEP **0906** (2009) 049 (arXiv:0904.4346)

MAX-PLANCK-GESELLSCHAFT

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Outline

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Mössbauer neutrinos

Proposed experiment:

$$\begin{aligned} \text{Production:} \quad & {}^3\text{H} && \to {}^3\text{He}^+ + \bar{\nu}_e + e^-(\text{bound}) \\ \text{Detection:} \quad & {}^3\text{He}^+ + e^-(\text{bound}) + \bar{\nu}_e && \to {}^3\text{H} \end{aligned}$$

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Experimental challenges: W. Potzel

- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
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- Can the resonance condition be fulfilled?

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Theoretical questions:

- Can Mössbauer neutrinos oscillate? S. M. Bilenky, F. v. Feilitzsch, W. Potzel
- What is the effect of line broadening on oscilltions?
    - Inhomogeneous line broadening $\longrightarrow$ E. Akhmedov's talk
    - Homogeneous line broadening $\longrightarrow$ this talk
    - Special case: Natural line broadening $\longrightarrow$ this talk
- Comparison of different formalisms (QM $\leftrightarrow$ QFT)

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Mössbauer neutrinos in QFT

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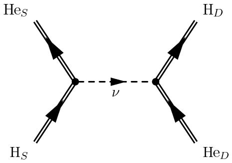

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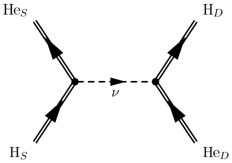

External particles reside in harmonic oscillator potentials.
E.g. for ^3H atoms in the source:

$$\psi_{H,S}(\vec{x}, t) = \left[\frac{m_H \omega_{H,S}}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_H \omega_{H,S} |\vec{x} - \vec{x}_S|^2\right] \cdot e^{-iE_{H,S} t}$$

Homogeneous line broadening by solid state effects

- Fluctuating electromagnetic fields in solid state crystal
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- Classical Mössbauer effect: Homogeneous and inhomogeneous broadening both lead to Lorentzian line shapes
  - Experimentally indistinguishable
  - We expect a result similar to that for the case of inhomogeneous broadening
- Ansatz: Introduce modulation factors of the form

$$f_{A,B}(t) = \exp\left[-i \int_0^t dt' \left[E_{A,B}(t') - E_{A,B,0} \right] \right]$$

in the ^{3}H and ^{3}He wave functions ($A = \mathrm{H}, \mathrm{He}$, $B = S, D$).

J. Odeurs, Phys. Rev. **B52** (1995) 6166

Transition amplitude for homogeneous line broadening

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 \left(\frac{m_{\mathrm{H}}\omega_{\mathrm{H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\mathrm{H}}\omega_{\mathrm{H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{\mathrm{H},S}t_1} \\
\cdot \left(\frac{m_{\mathrm{He}}\omega_{\mathrm{He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\mathrm{He}}\omega_{\mathrm{He},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{+iE_{\mathrm{He},S}t_1} \\
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\cdot \sum_j \mathcal{M}_S^{\mu}\mathcal{M}_D^{\nu *}|U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} \exp\left[-ip_0(t_2 - t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)\right] \\
\cdot \bar{u}_{e,S}\gamma_\mu(1-\gamma^5)\,\frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon}\,(1+\gamma^5)\gamma_\nu u_{e,D}
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$$\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm He}\omega_{{\rm He},S}|\vec{x}_1 - \vec{x}_S|^2\right] f^*_{{\rm He},S}(t_1)\, e^{+iE_{{\rm He},S}t_1}$$

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Evaluation:

- $d^3x_1\, d^3x_2$-integrals are Gaussian
- d^3p-integral: Use Grimus-Stockinger theorem (limit of propagator for large $L = |\vec{x}_D - \vec{x}_S|$).

W. Grimus, P. Stockinger, Phys. Rev. **D54** (1996) 3414, hep-ph/9603430

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Transition rate $\Gamma \propto \langle \mathcal{A}\mathcal{A}^* \rangle$
(statistical average of $\mathcal{A}\mathcal{A}^*$ over all possible ^{3}H and ^{3}He states).

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Transition rate for homogeneous line broadening (2)

Result:

$$\Gamma \propto \underbrace{\exp\left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2}\right]}_{\text{Lamb-Mössbauer factor}} \underbrace{\exp\left[-\frac{|\Delta m^2|}{2\sigma_p^2}\right]}_{\text{Localization}} \underbrace{\frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}}}_{\text{Resonance}}$$

$$\cdot \underbrace{\left\{1 - 2s^2c^2\left[1 - \frac{1}{2}(e^{-L/L_S^{\mathrm{coh}}} + e^{-L/L_D^{\mathrm{coh}}}) \cos\left(\pi \frac{L}{L^{\mathrm{osc}}}\right)\right]\right\}}_{\text{Oscillation/Coherence}}$$

$$L_{S,D}^{\mathrm{coh}} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}\,, \qquad L_{jk}^{\mathrm{osc}} = \frac{4\pi E}{\Delta m_{jk}^2}$$

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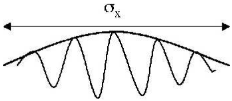
Reason: It is impossible to distinguish an ensemble of neutrino wave packets with identical momentum distributions (but different spacetime positions) from an ensemble of plane waves whose individual momenta follow the same distribution.

Kiers Nussinov Weiss

Interpretation in the wave packet picture



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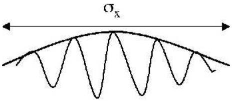
- Decoherence in production and detection processes

  If an experiment can distinguish different mass eigenstates, oscillations will vanish.

  $\Rightarrow$ Localization condition $\Delta m^2 \ll \sigma_p$.



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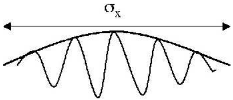

- Decoherence during propagation

  Decoherence caused by wave packet separation due to different group velocities if $L \gtrsim L^{\mathrm{coh}}$.

Amplitude for broadening by natural line width

Take into account the instability of ^{3}H in the source and the detector.

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left(\frac{m_{\mathrm{H}}\omega_{\mathrm{H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\mathrm{H}}\omega_{\mathrm{H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{\mathrm{H},S}t_1} \\
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\end{aligned}
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(correctness of this formula can be verified in the Wigner-Weisskopf approach)

E. Akhmedov, J. Kopp, M. Lindner, JHEP **0805** (2008) 005 (arXiv:0802.2513)

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Probability for broadening by natural line width

$$\mathcal{P} \propto \sum_{j,k} \theta(T_{jk})\, |U_{ej}|^2 |U_{ek}|^2$$

$$\cdot \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right] e^{i\left(\sqrt{\bar{E}^2 - m_j^2} - \sqrt{\bar{E}^2 - m_k^2}\right)L}$$

$$\cdot\, e^{-\gamma T_{jk}}\, e^{-L/L_{jk}^{\text{coh}}}\, \frac{\sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j})\right] \sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k})\right]}{(E_S - E_D)^2}$$

where $T_{jk} = \min\left(T - \frac{L}{v_j},\, T - \frac{L}{v_k}\right)$ and $L_{jk}^{\text{coh}} = \frac{4\bar{E}^2}{\gamma |\Delta m_{jk}^2|}$

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- Oscillation term: $e^{i\left(\sqrt{\bar{E}^2 - m_j^2} - \sqrt{\bar{E}^2 - m_k^2}\right) L}$
- Lamb-Mössbauer factor: $\exp\left[-(p_{jk}^{\rm min})^2/\sigma_p^2\right]$
- Localization term: $\exp\left[-|\Delta m_{jk}^2|/2\sigma_p^2\right]$
- Coherence term: $e^{-L/L_{jk}^{\rm coh}}$

Probability for broadening by natural line width (2)

- Resonance term

$$\frac{\sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j})\right] \sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k})\right]}{(E_S - E_D)^2}$$

does *not* depend on γ, but *only* on the total measurement time T (Heisenberg principle).

Probability for broadening by natural line width (2)

- Resonance term

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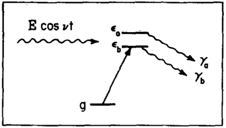
  does *not* depend on γ, but *only* on the total measurement time T (Heisenberg principle).

- Analogy: subnatural spectroscopy in quantum optics
  - Atom is excited instantaneously to state $|b\rangle$.
  - Continuous irradition with frequency ν.
  - Probability for exiciting state $|a\rangle$ is proportional to $[(\nu - \nu_{\text{res}})^2 + (\gamma_a - \gamma_b)^2/4]^{-1}$, not $[(\nu - \nu_{\text{res}})^2 + (\gamma_a + \gamma_b)^2/4]^{-1}$.



P. Meystre, O. Scully, H. Walther, Optics Communications **33** (1980) 153

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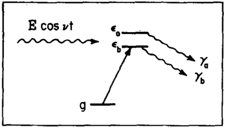

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- Here:
  - $|b\rangle \Leftrightarrow$ ^{3}H atom in the source, ^{3}He atom in the detector
  - $|a\rangle \Leftrightarrow$ ^{3}He atom in the source, ^{3}H atom in the detector
  - Excitation of $|b\rangle \Leftrightarrow$ Production of source
  - Transition $|b\rangle \to |a\rangle \Leftrightarrow$ neutrino production, propagation and absorption

Probability for broadening by natural line width (3)

- Note: If the source is produced at $t = 0$, but the experiment is started at a later time $t = t_0$, the width of the resonance term will depend not on T, but on $T - t_0$.

Outline

Lorentzian wave packets

Describe Mössbauer neutrino as a Lorentzian wave packet:

$$\langle p|\bar{\nu}_{eS}(t)\rangle = \frac{1}{N_S} \sum_j U_{ej}\, f_{jS}\, \frac{\sqrt{\gamma_S/2\pi}}{p - p_{jS} + i\gamma_S/2}\, \exp\big[-iE_j t\big]\, |\nu_j\rangle$$

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Detection process: Projection onto

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Fudge factors

$$f_{jS} \equiv \exp\left[\frac{\bar{E}^2 - m_j^2}{2\sigma_{pS}^2}\right], \qquad f_{jD} \equiv \exp\left[\frac{\bar{E}^2 - m_j^2}{2\sigma_{pD}^2}\right]$$

describe dependence of production/detection amplitudes on neutrino mass (cannot be computed in QM, but only in QFT).

Transition amplitude, probability, and rate in QM

Transition amplitude:

$$\mathcal{A}(t, L) = \int dp \, \langle \bar{\nu}_{eD} | p \rangle \, \langle p | \bar{\nu}_{eS}(t) \rangle$$

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Transition rate:

$$\Gamma = \frac{1}{4\pi L^2} \, \Gamma_0^{\mathrm{MB}} \, \mathcal{P}(L) \, \sigma^{\mathrm{MB}}$$

Approximations and definitions

Use that momentum space wave packets are very narrow

$$E_j = \sqrt{p^2 + m_j^2}\, t \simeq \bar{E}_j\, t + \bar{v}_j t (p - \bar{p}_j)\,,$$

$$\bar{p}_j = (p_{jS} + p_{jD})/2\,, \qquad \bar{E}_j = \sqrt{\bar{p}_j^2 + m_j^2}\,, \qquad \bar{v}_j = \frac{\bar{p}_j}{\sqrt{\bar{p}_j^2 + m_j^2}}$$

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Use that neutrinos are ultra-relativistic ($m_j \ll \bar{E}_j$):

$$p_{jS} \simeq E_{S,0} - (1 - \xi_S)\frac{m_j^2}{2E_{S,0}}\,, \qquad p_{jD} \simeq E_{D,0} - (1 - \xi_D)\frac{m_j^2}{2E_{D,0}}$$

$$\bar{E}_j \simeq \bar{E} + \bar{\xi}\frac{m_j^2}{2\bar{E}}\,, \qquad \bar{p}_j \simeq \bar{E} - (1 - \bar{\xi})\frac{m_j^2}{\bar{E}}\,, \qquad \bar{v}_j \simeq 1 - \frac{m_j^2}{2\bar{E}^2}\,,$$

$$\bar{E} \equiv \frac{1}{2}(E_{S,0} + E_{D,0})\,, \qquad 1 - \bar{\xi} \equiv \frac{\bar{E}}{2}\left(\frac{1 - \xi_S}{E_{S,0}} + \frac{1 - \xi_D}{E_{D,0}}\right)$$

Transition rate in QM

$$\Gamma \propto \exp\left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m^2|}{2\sigma_p^2}\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}}$$
$$\cdot \left\{1 - 2s^2c^2\left[1 - \frac{1}{2}(e^{-L/L_S^{\rm coh}} + e^{-L/L_D^{\rm coh}}) \cos\left(\pi \frac{L}{L^{\rm osc}}\right)\right]\right\}$$

. . . identical to QFT result.

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Few input parameters:	Many input parameters:
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Abstract formalism	More transparent formalism

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  - QM is the less abstract, but also the less complete formalism


Thank you!