The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2048-15

From Core to Crust: Towards an Integrated Vision of Earth's Interior

20 - 24 July 2009

Elastic properties of Earth materials

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Measuring Elastic Properties of Earth Materials

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Deutsches GeoForschungsZentrum Potsdam

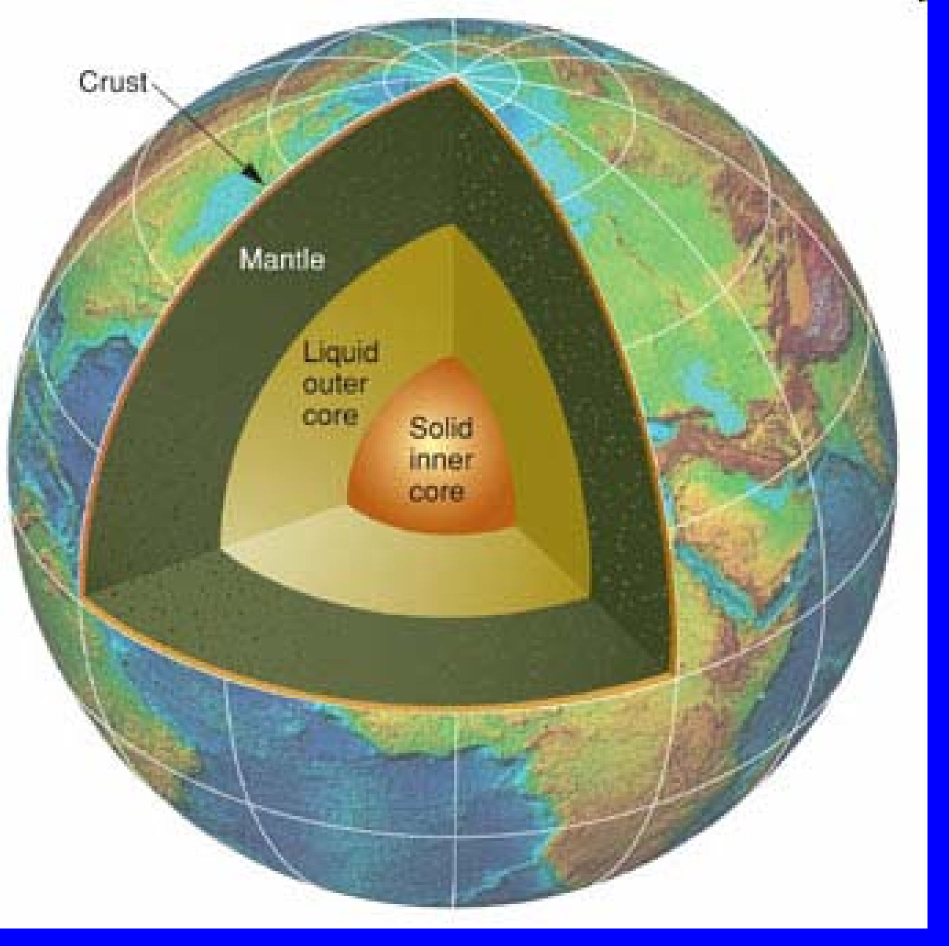
GFZ

Helmholtz Centre

POTSDAM

The Earth:

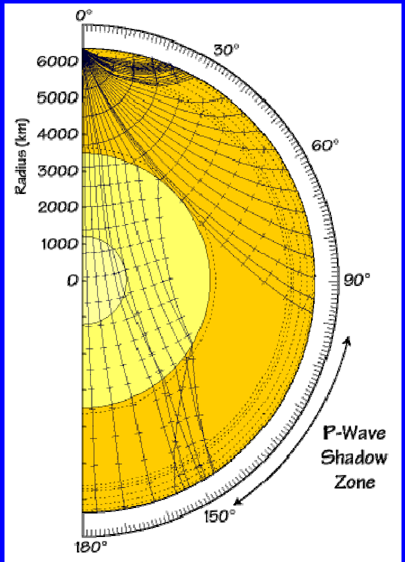
Internal structure and composition



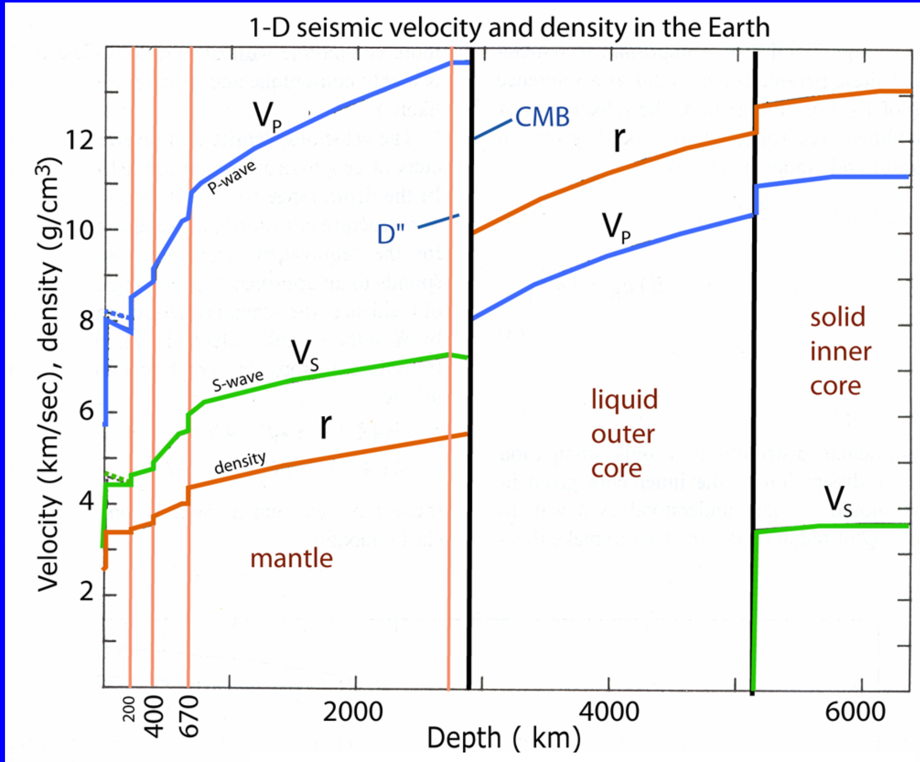
from the web

Seismology:

1D structure



PREM



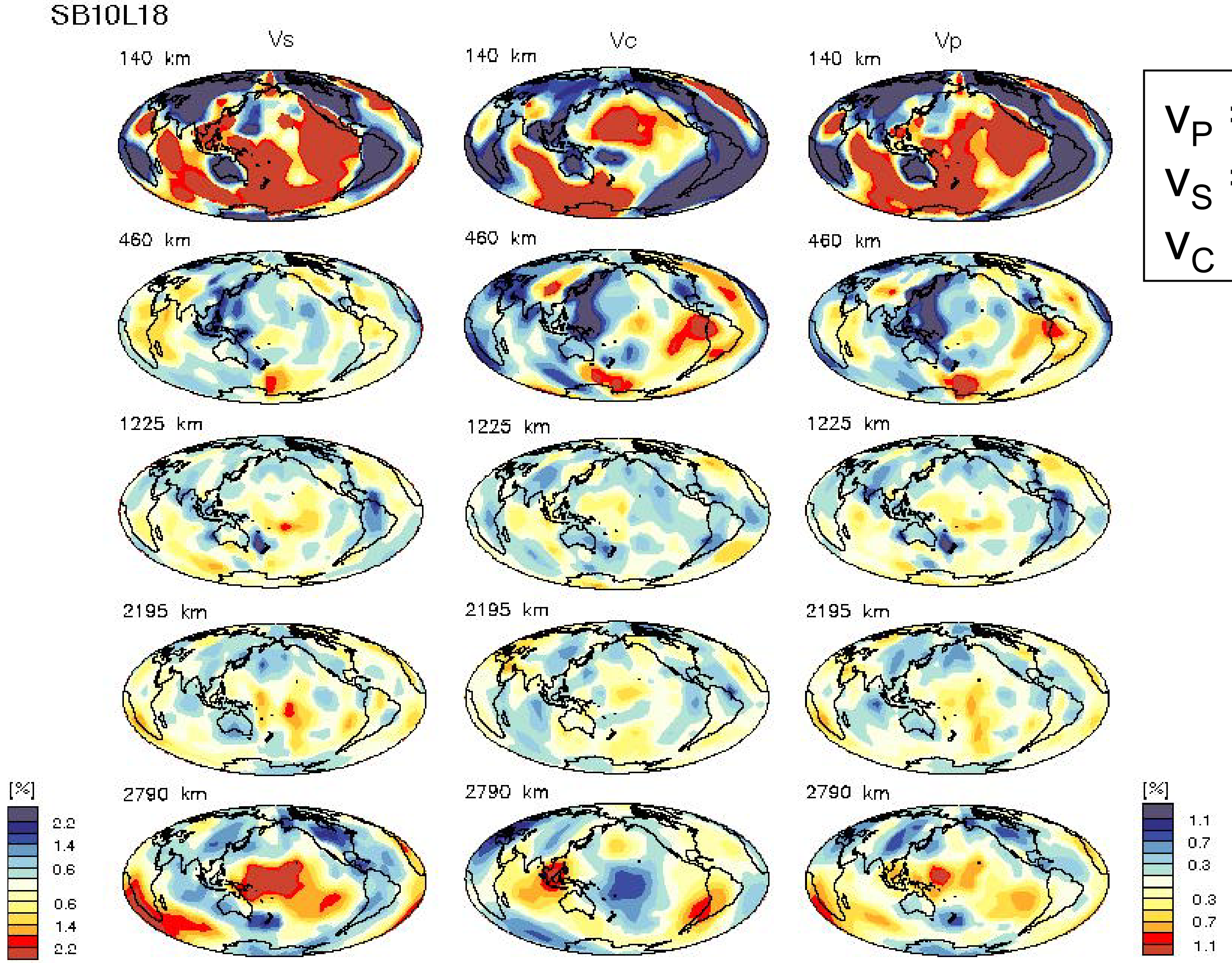
Dziewonski and Anderson (1981)

$$v_P = [(K + 4/3G)/\rho]^{1/2},\ v_S = (G/\rho)^{1/2}$$

from the web

Seismology:

3D structure, heterogeneity

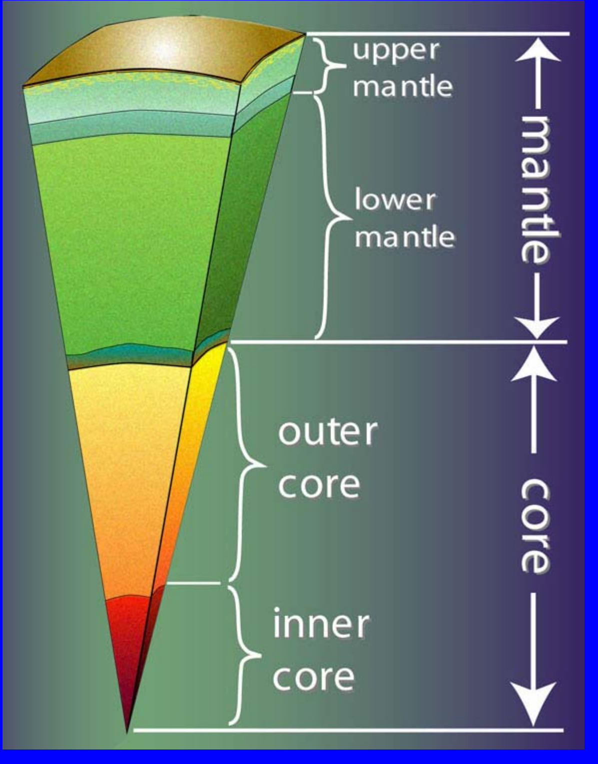


Note: the scale for Vc is the same as for Vp

$v_P = [(K + 4/3G/\rho)]^{1/2}$
$v_S = (G/\rho)^{1/2}$
$v_C = (K/\rho)^{1/2}$

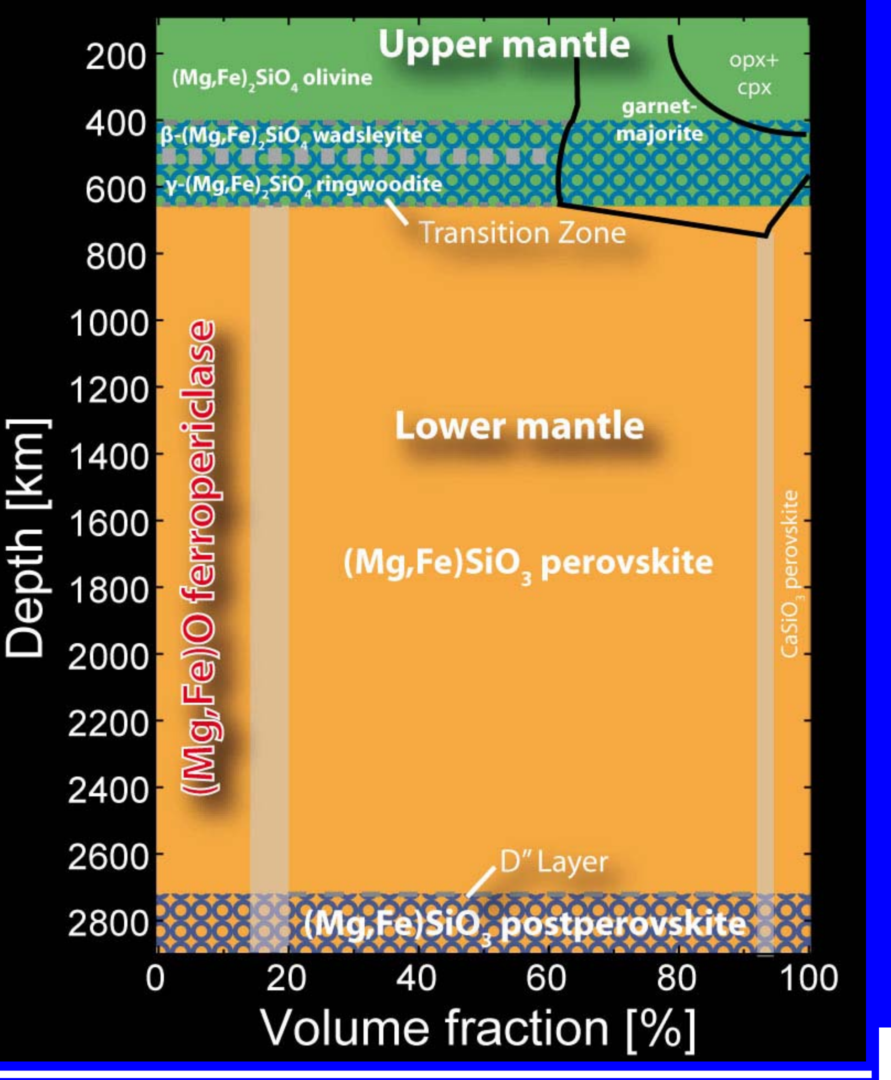
Mineralogical model of the deep Earth:

The role of mineral physics



from E.J. Garnero

$$<v> = \sum_i v_i \cdot F(\mathcal{V}_i)$$



courtesy of H. Marquardt

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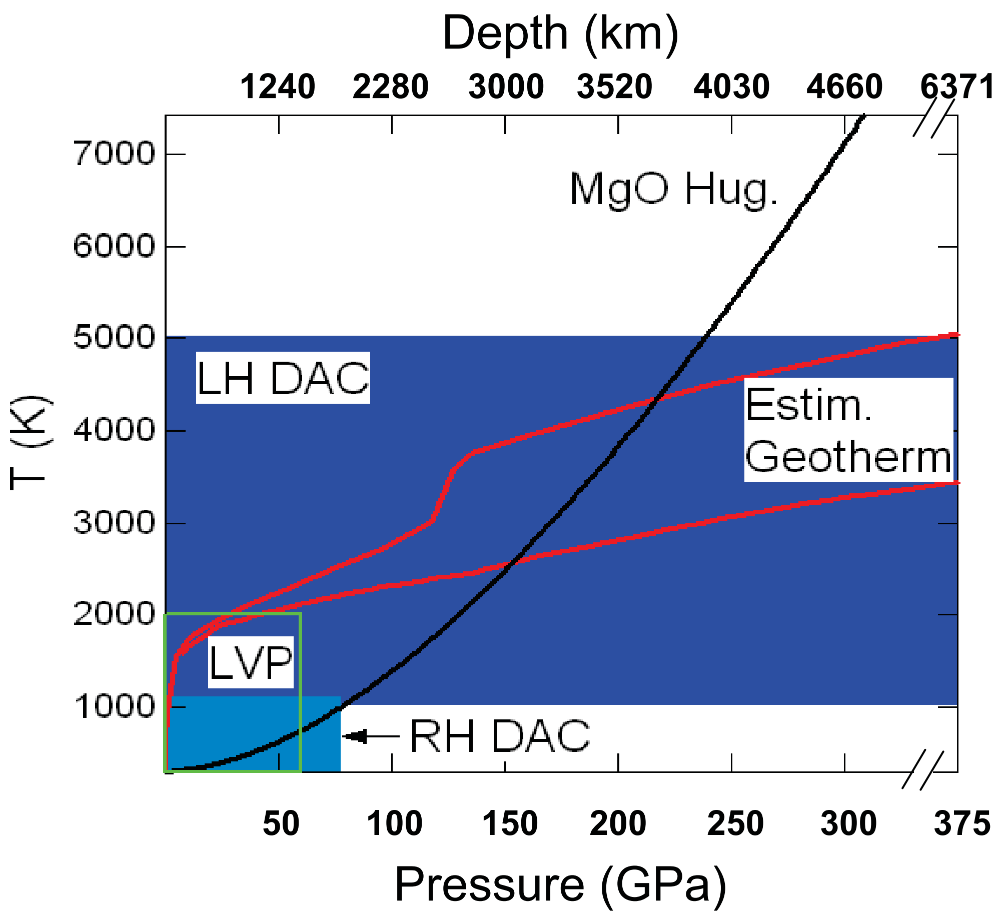
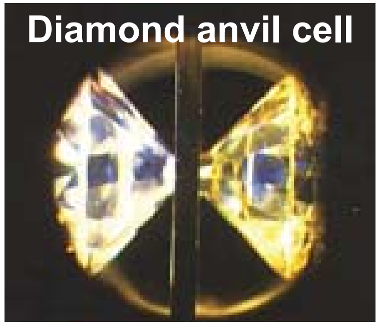
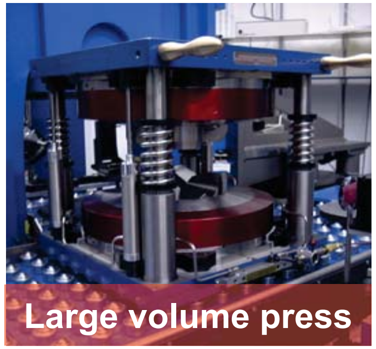
Accessing extreme conditions:

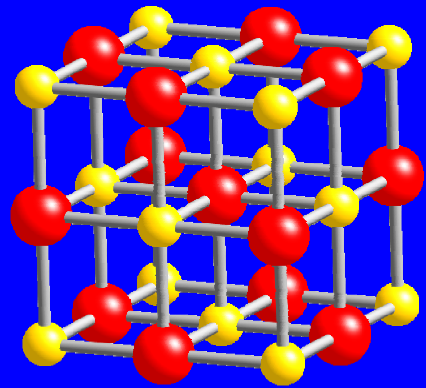
Experimental methods







Linear elastic solid



Hooke's law

$$\sigma_{ij} = c_{ijkl} \in_{kl}$$
$$\in_{ij} = s_{ijkl} \sigma_{kl}$$
$$c_{ijkl} s_{klmn} = \delta_{im} \delta_{jn}$$

$$\in_{ij} = 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$$

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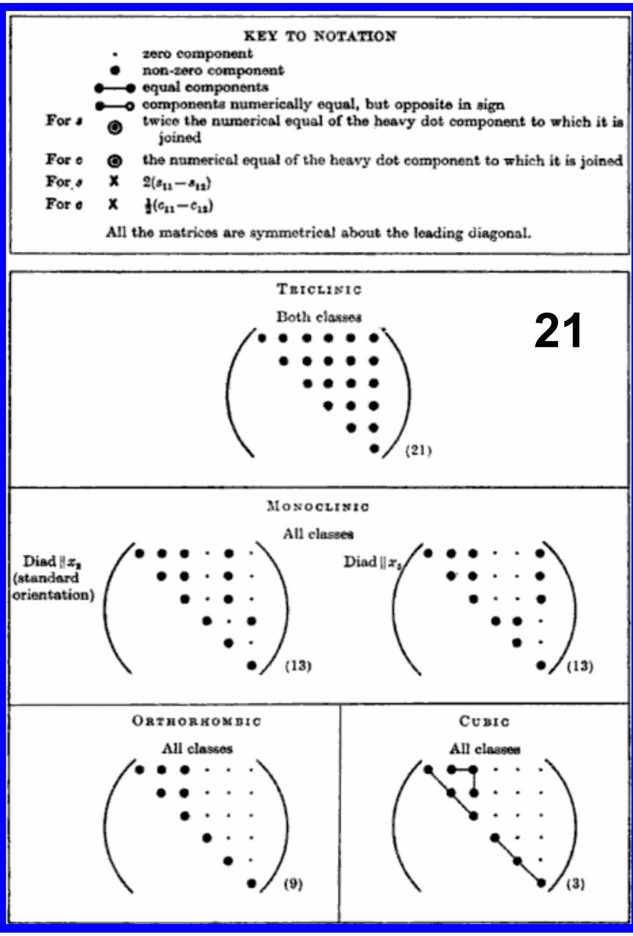
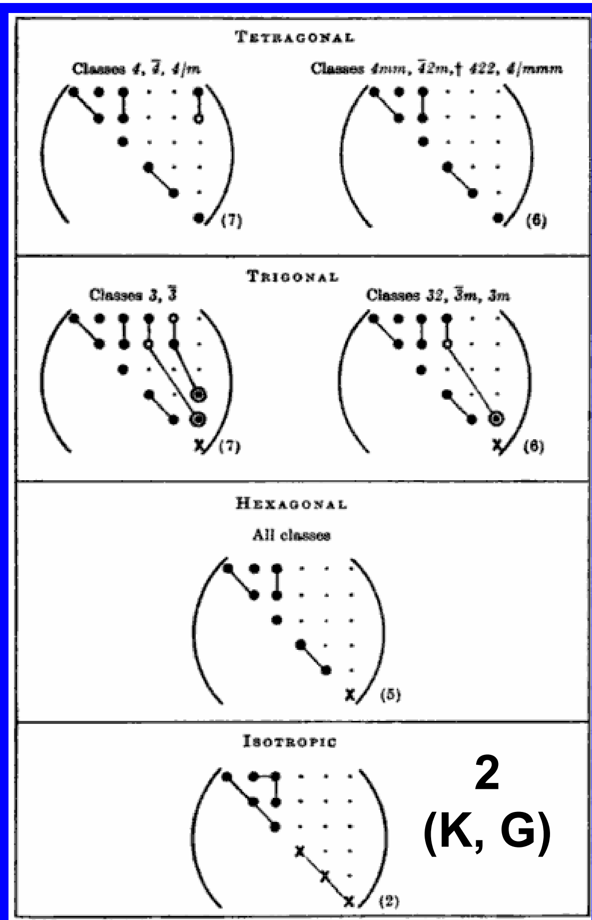
The symmetry of the elastic tensor

σ_{ij}, ϵ_{ij} symmetric $\rightarrow$ 6 independent components

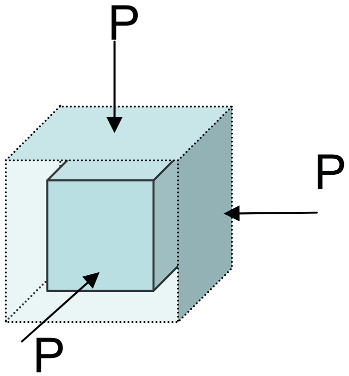
c_{ij}, s_{ij} (in matrix form) $\rightarrow$ 6×6 components





from Nye (1958)

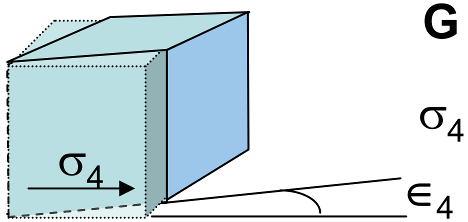
Elastic moduli of isotropic media



K (Bulk modulus) $= (c_{11} + 2c_{12})/3$

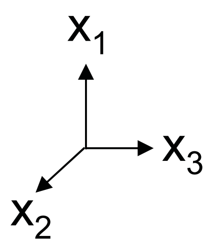
$P = -3K \epsilon_1$

$$\boxed{\epsilon_1 = \epsilon_2 = \epsilon_3}$$



G (Shear modulus) $= (c_{11} - c_{12})/2$

$\sigma_4 = 2G \epsilon_4$



$\epsilon_{ij} = 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$

$ij \to \alpha$

Dynamic behavior of elastic media:

Equations of motion

$$\rho \, \partial^2 u_i / \partial t^2 = c_{ijkl} \partial^2 u_k / \partial x_j \partial x_l$$

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Eulerian finite strain

Anisotropic case

$$E_{ij} = 1/2[\delta_{ij} - (\partial X_i/\partial x_j)(\partial X_j/\partial x_i)]$$

Isotropic case

$$E_{ij} = -f\ \delta_{ij}$$

$$f = 1/2[(V_0/V)^{2/3} - 1] = 1/2[(\rho/\rho_0)^{2/3} - 1]$$

Strain dependence of C_{ijkl}:

Free energy

$$F_{T0} = F_0 + 1/2 a_{ij} E_i E_j + 1/6 a_{ijk} E_i E_j E_k + 1/24 a_{ijkl} E_i E_j E_k E_l + \ldots$$

Elastic constants

$$c_{ijkl,T0} = (1 + 2f)^{7/2} \{ c_{ijkl}^0 + b_1 f + 1/2 b_2 f^2 + \ldots \} - P\Delta_{ijkl}$$

from Stixrude and Lithgow-Bertelloni (2005)

Temperature dependence of C_{ijkl}:

Quasi-harmonic approximation

$$F(V,T) = F_0 + F_{T0}(V,T_0) + [F_q(V,T) - F_q(V,T_0)]$$

$$F_q(V,T) = \rho_0 kT \sum_{\lambda} \ln\{1 - \exp[-h\nu_\lambda(E_i)/kT]\}$$

from Stixrude and Lithgow-Bertelloni (2005)

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Temperature dependence of C_{ijkl}

$$c_{ijkl} = c_{ijkl,T0} + \rho[c_1 \Delta U_q - c_2 \Delta(C_V T)]$$

$$c_1 = F(\gamma_{ij}, \eta_{ijkl}), \quad c_2 = F(\gamma_{ij})$$

$$\gamma_{ij} = -\partial \ln \nu_\lambda / \partial E_{ij} \qquad \eta_{ijkl} = \partial \gamma_{ij} / \partial E_{kl}$$

from Stixrude and Lithgow-Bertelloni (2005)

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Aggregate properties

Voigt: uniform ϵ_{ij} $\quad \sigma_{ij} = \langle c_{ijkl} \rangle \, \epsilon_{kl}$

Reuss: uniform σ_{ij} $\quad \epsilon_{ij} = \langle s_{ijkl} \rangle \, \sigma_{kl}$

Aggregate properties

Voigt:	$M_V = \sum f_i M_i$
Reuss:	$1/M_R = \sum f_i / M_i$

f: volume fraction
M: elastic modulus

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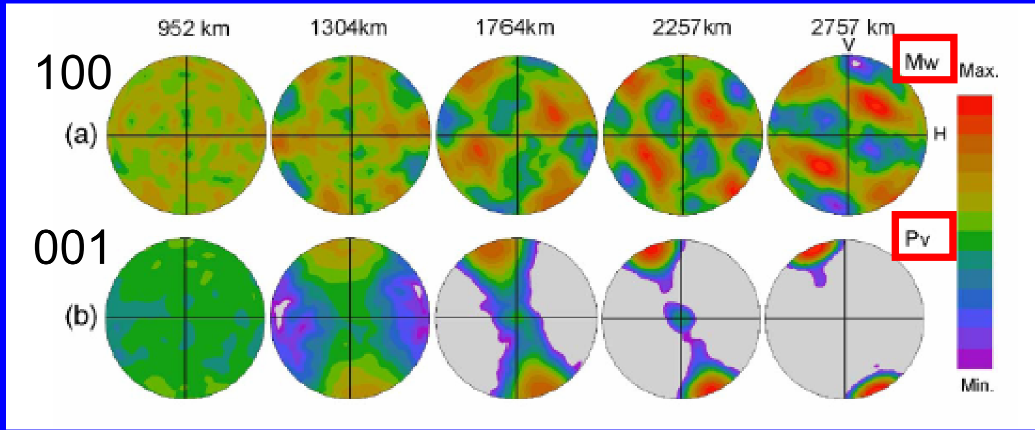
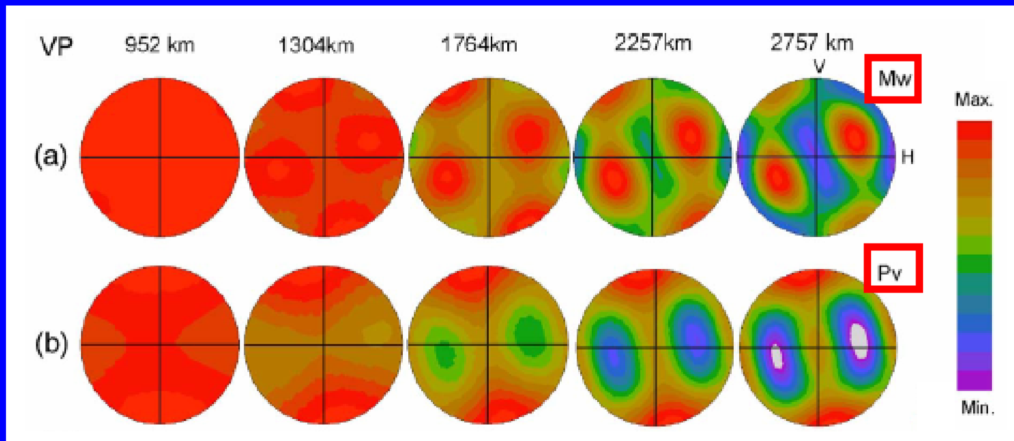
Aggregate properties



ODF $f(g)dg = dV_g/V$



$$\langle c_{ijkl} \rangle = \int g_{ip} g_{jq} g_{kr} g_{ls} \, c'_{pqrs} \, f(g)dg$$

from Wenk et al. (2005)

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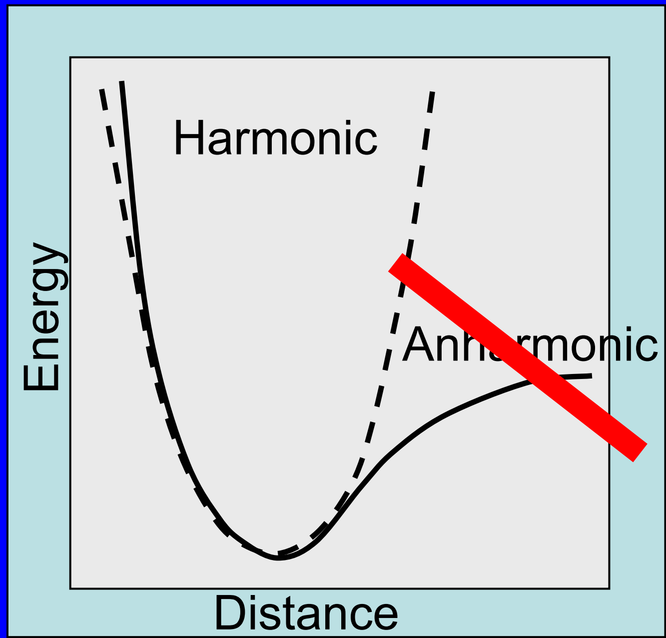
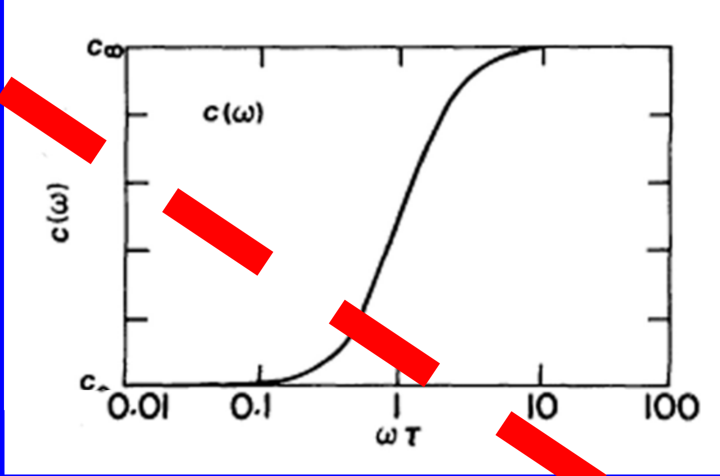
Limitations

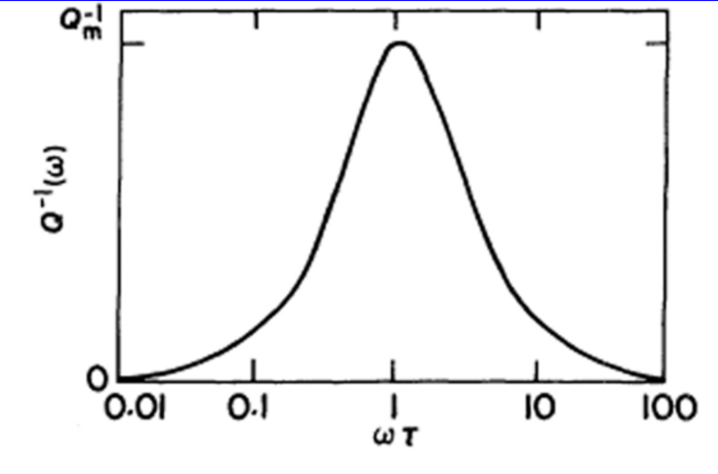
Anharmonicity



$$(\partial M/\partial T)_P = (\partial M/\partial T)_V + (\partial M/\partial V)_T(\partial V/\partial T)_P$$

Anelasticity



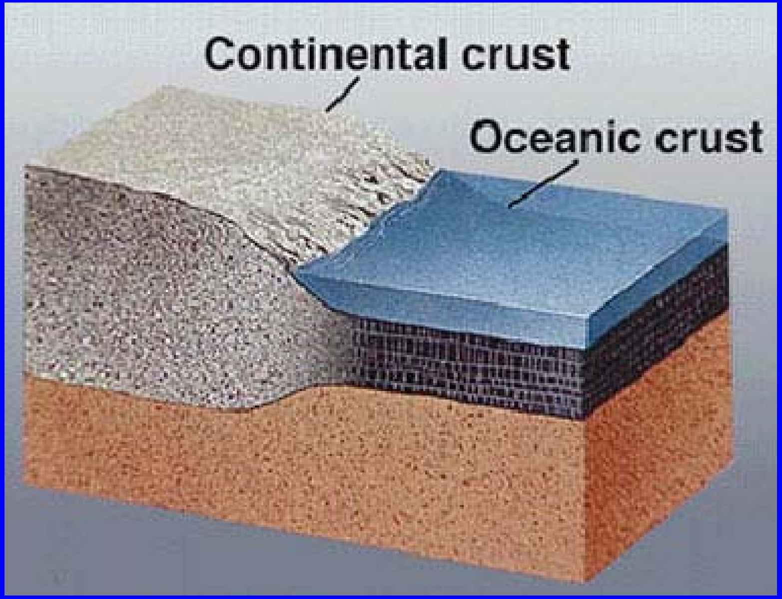


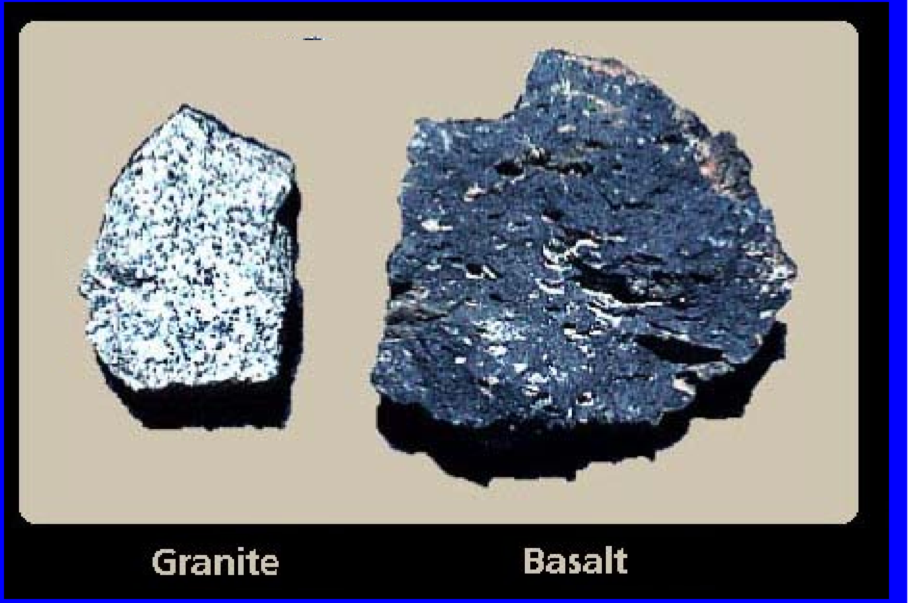
$$M(\omega) = M_1(\omega) + iM_2(\omega)$$

$$Q^{-1}(\omega,\tau) = |\Delta E|/(2\pi E) = M_2(\omega)/M_1(\omega)$$

$$\tau_i = \tau_{i0} \exp[E^*/(kT)]$$

Minerals of the crust

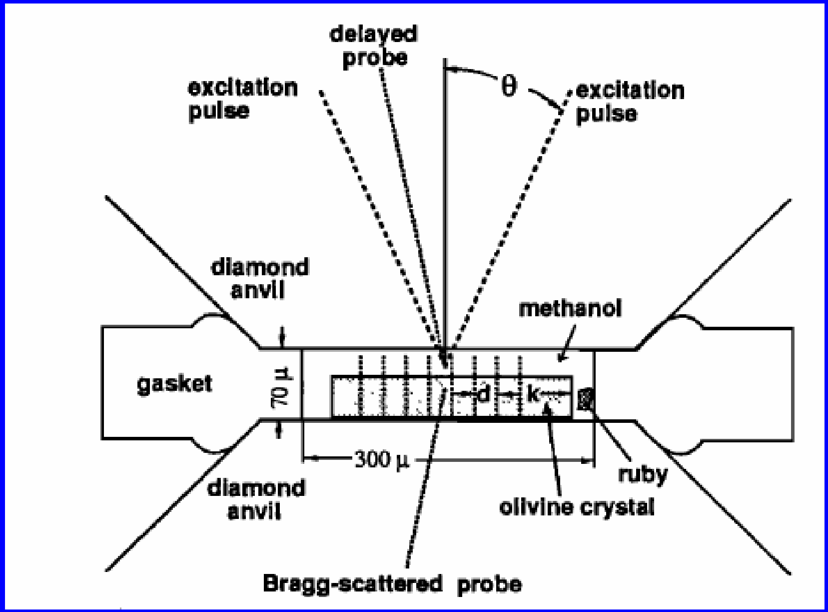




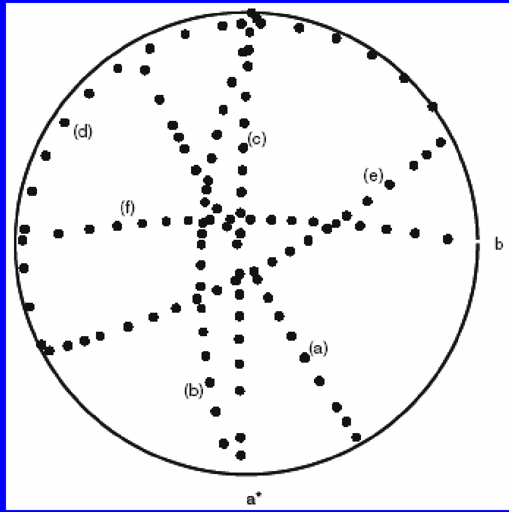
from the web

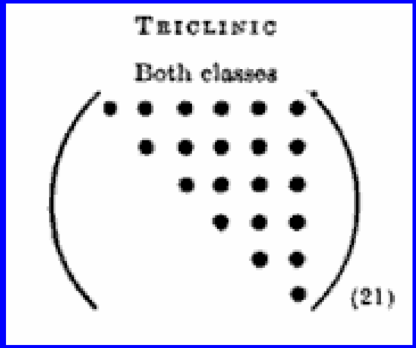
Elasticity of crustal minerals:

Impulsively stimulated light scattering



Albite ($NaAlSi_3O_8$)

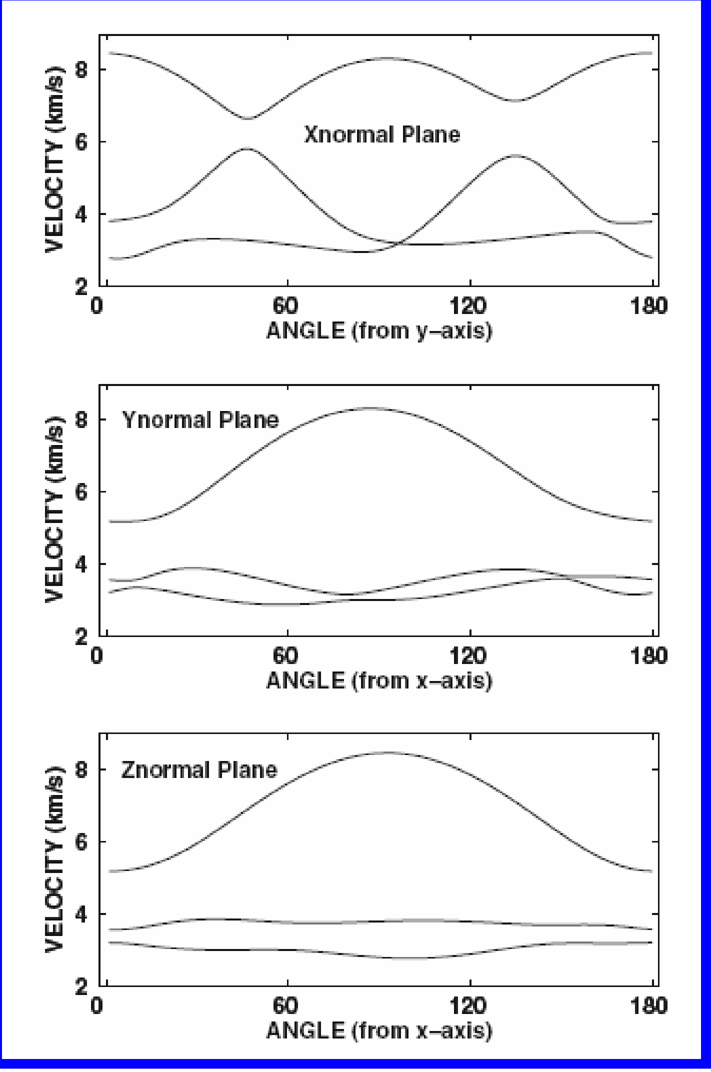


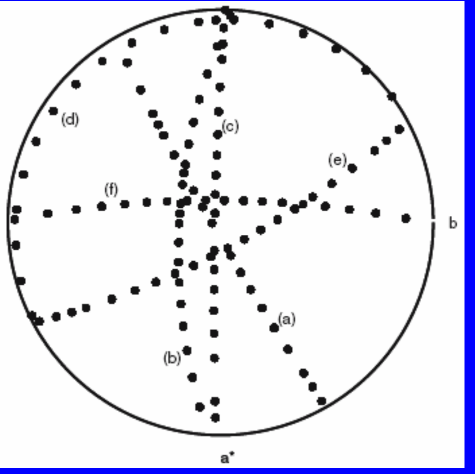


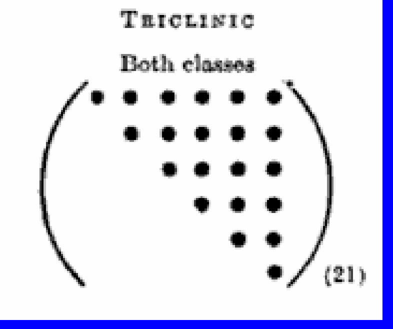
from Brown et al. (1997, 2006)

Elasticity of crustal minerals:

Albite ($NaAlSi_3O_8$)





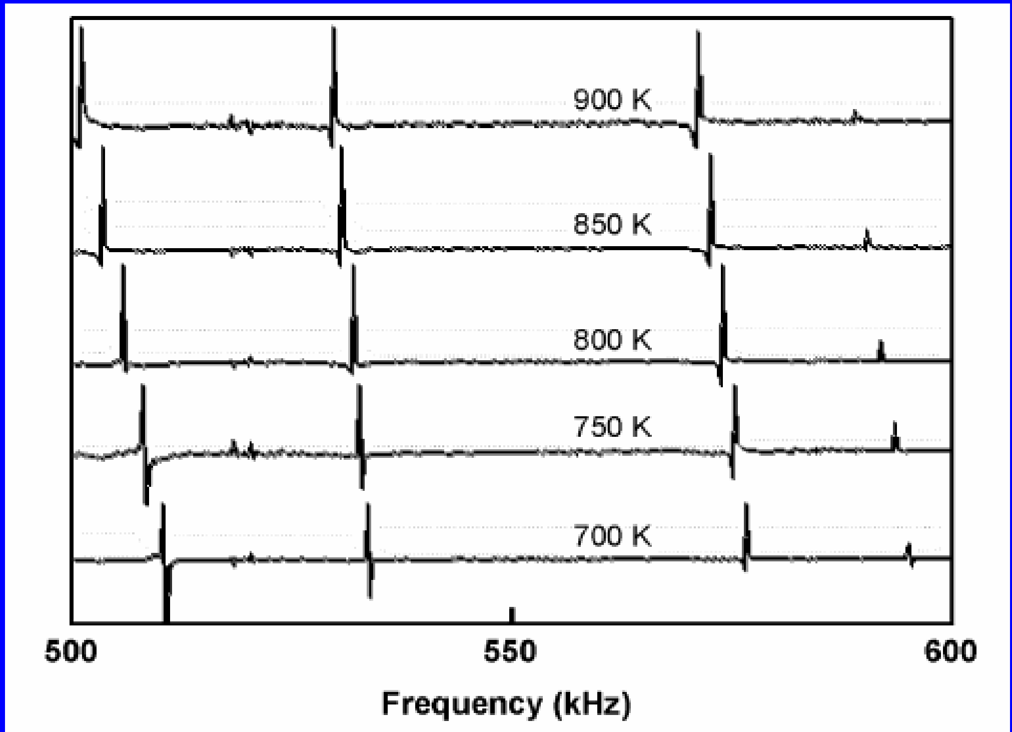


from Brown et al. (2006)

Elasticity of crustal minerals:

Diopside [$CaMgSiO_3$] at high temperature

Resonant ultrasound spectroscopy



from Isaak et al. (2006)

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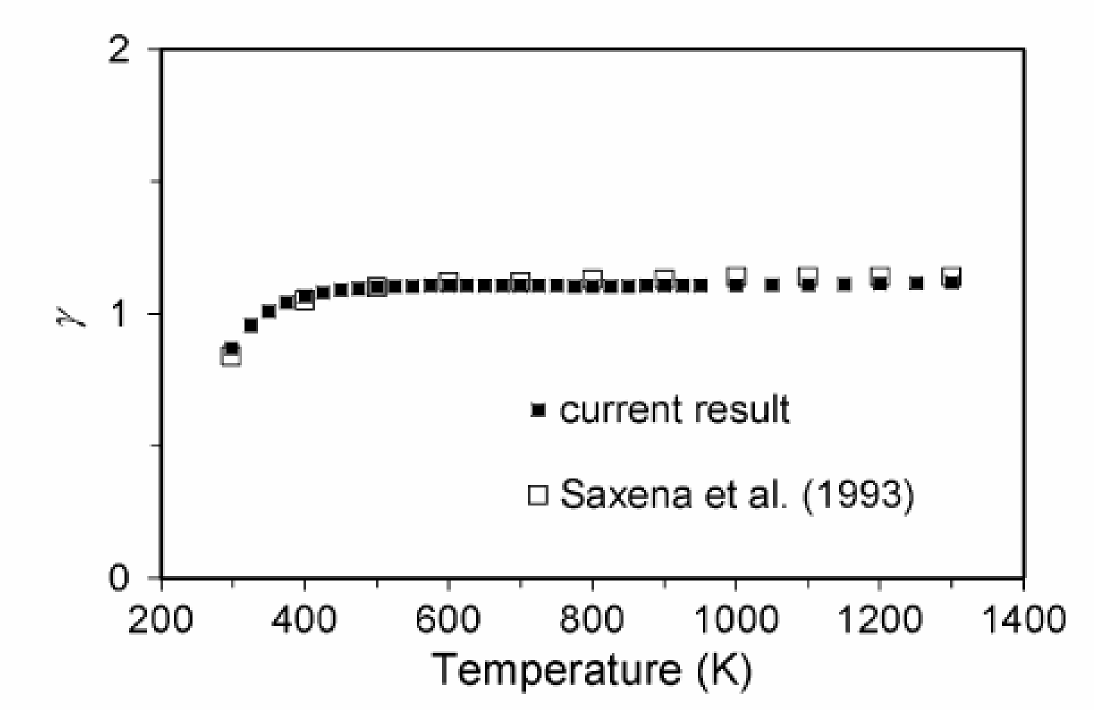
Elasticity of crustal minerals:

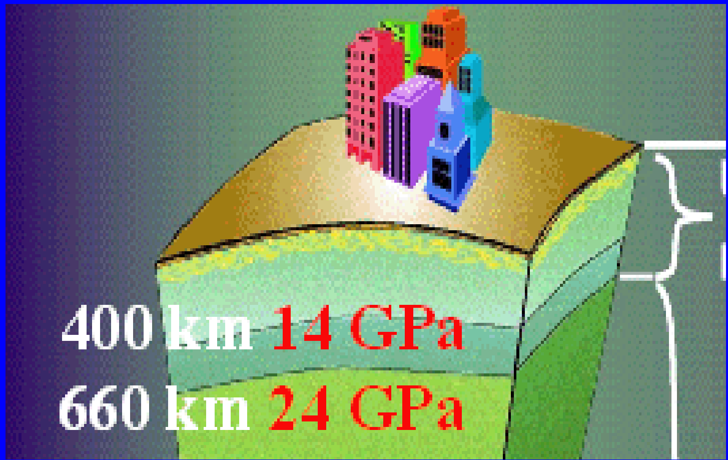
Diopside [$CaMgSiO_3$] at high temperature



$$\gamma_{Th} = VK_S\alpha_V/C_P$$

from Isaak et al. (2006)

The upper mantle



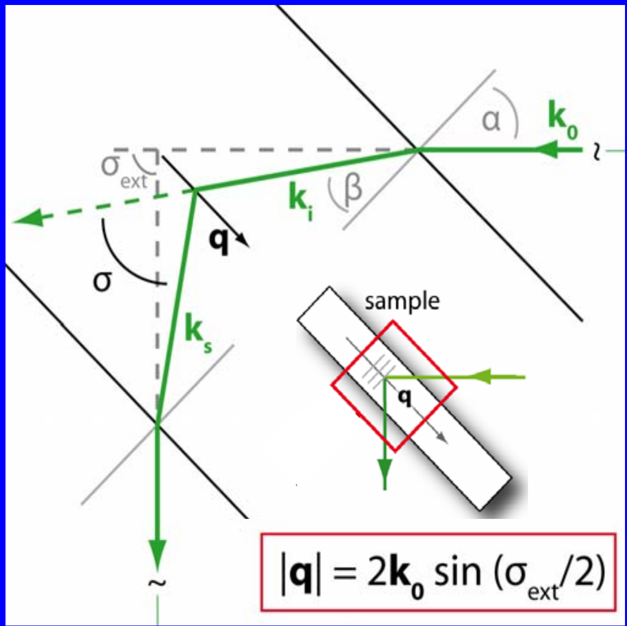
Ol. + Gar.+Pyr.

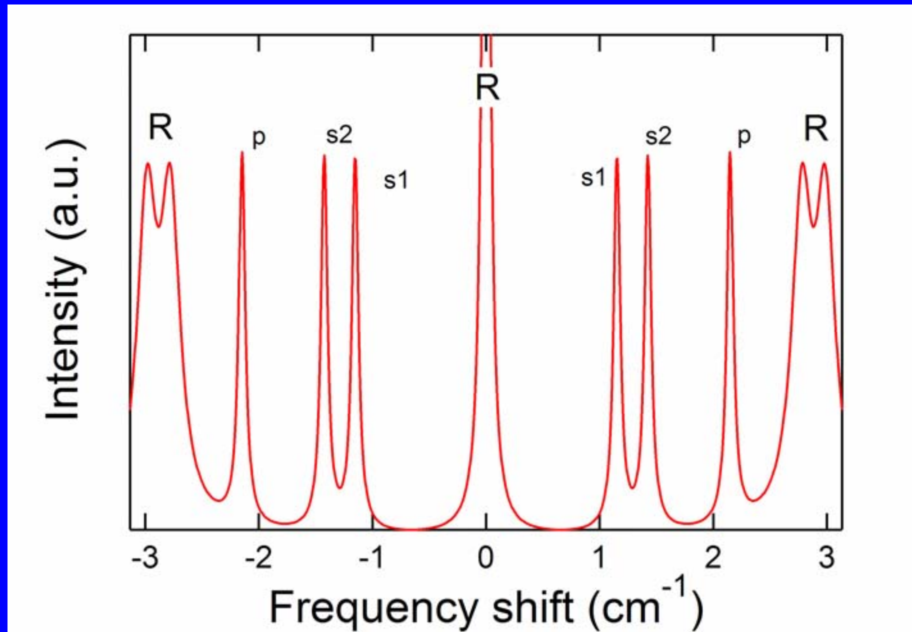
Cations substitutions

from the web

Elasticity of upper mantle minerals:

Brillouin scattering



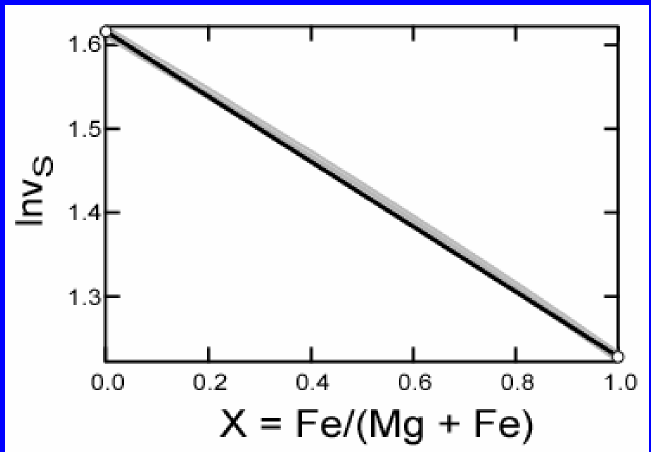


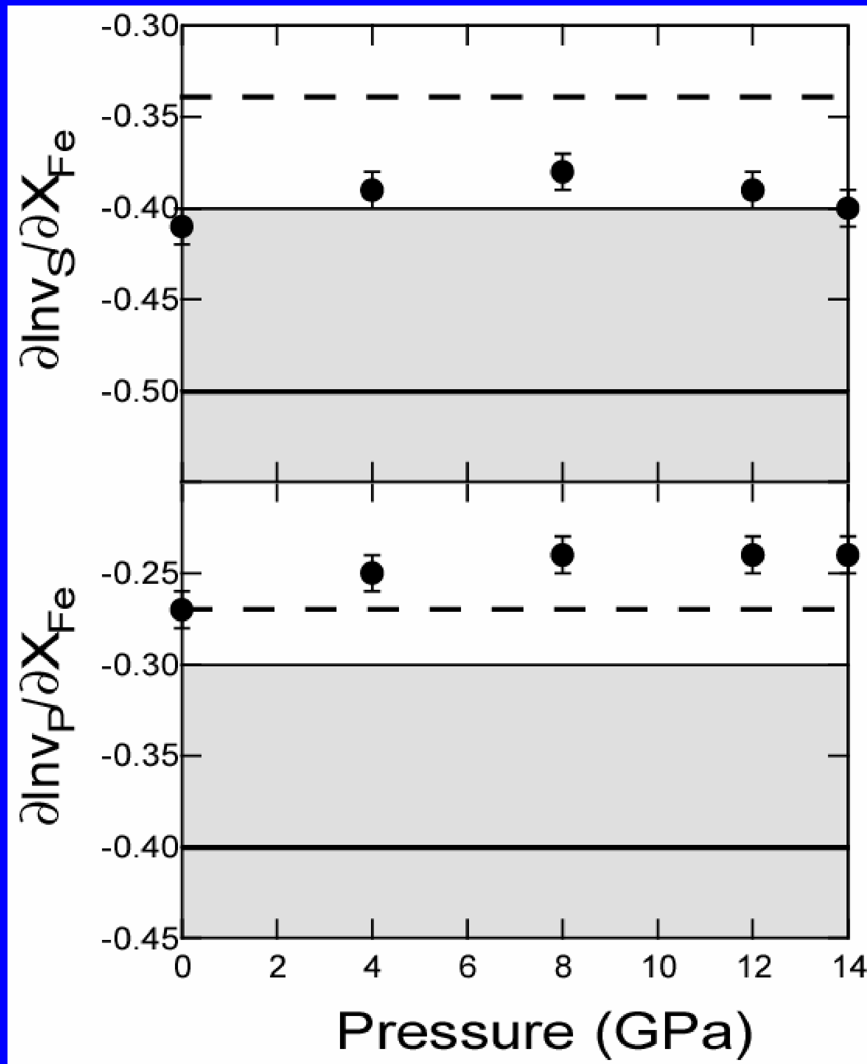
from Speziale et al. (2005)

Elasticity of upper mantle minerals:

Effect of Mg-Fe substitution in olivine [$(Mg,Fe)_2SiO_4$]

Brillouin scattering

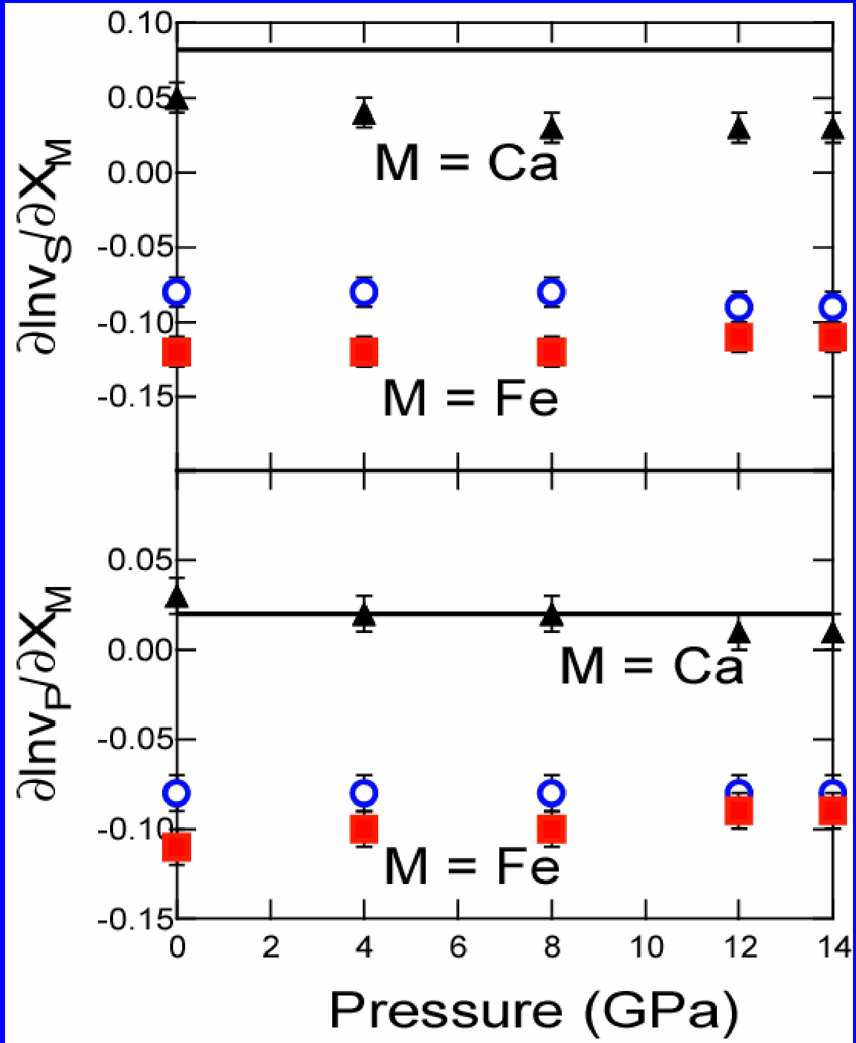




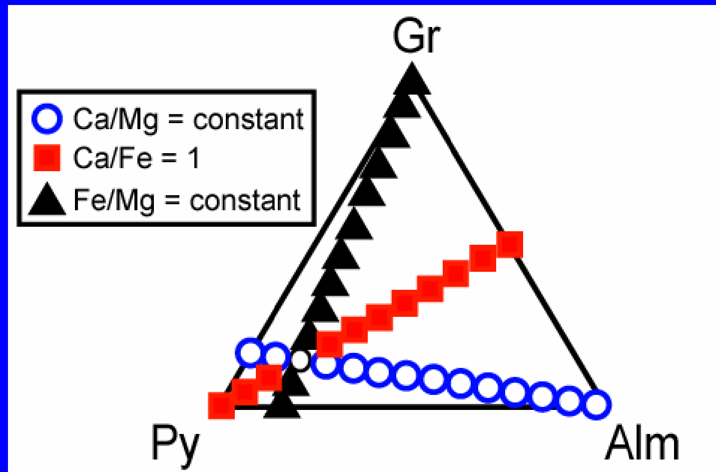
from Speziale et al. (2005)

Elasticity of upper mantle minerals:

Effect of Ca-Mg-Fe substitution in garnet [$(Mg,Fe,Ca)_3Al_2Si_3O_{12}$]

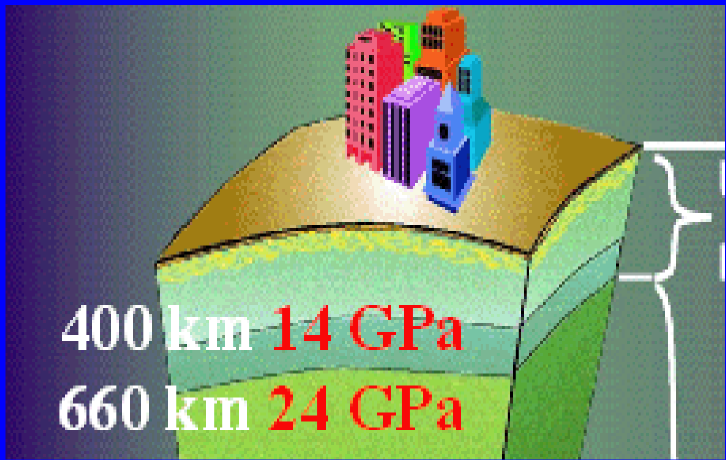


Brillouin scattering



from Speziale et al. (2005)

Transition zone



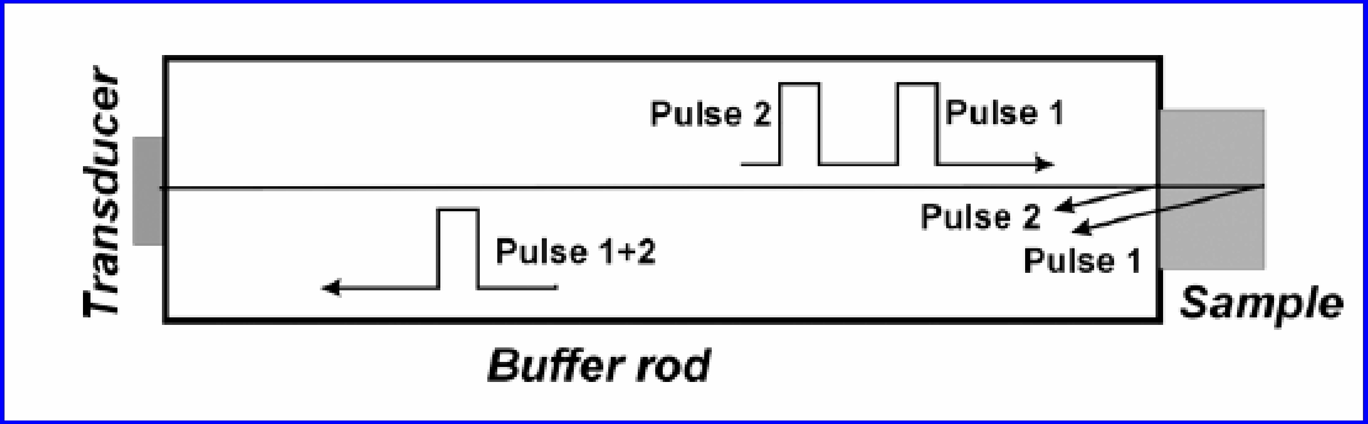
Ol. polymorphs
+ Garnet-
Majorite

- Pressure/Temperature effects
- H incorporation

from the web

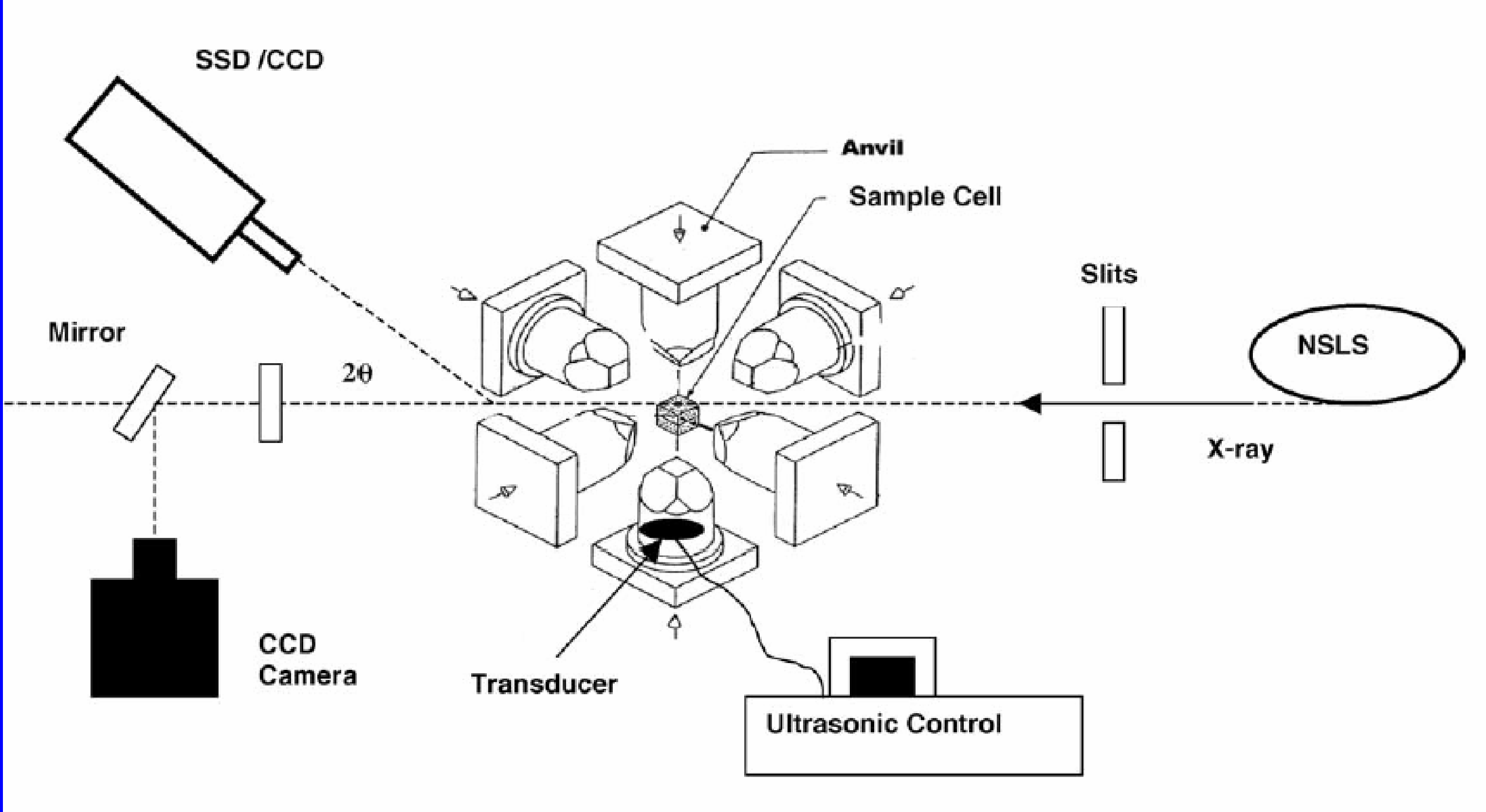
Elasticity of transition zone minerals:

Ultrasonic interferometry



from Angel et al. (2009)

Elasticity of transition zone minerals:



Ultrasonic interferometry in
the Large Volume Press

from Li et al. (2004)

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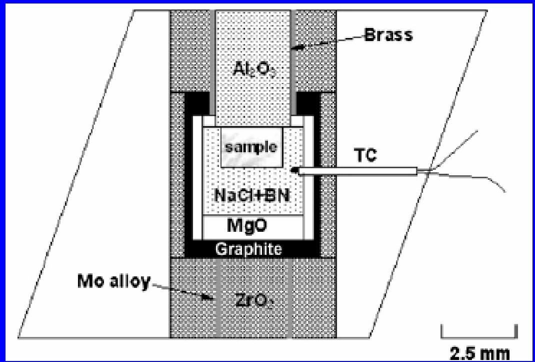
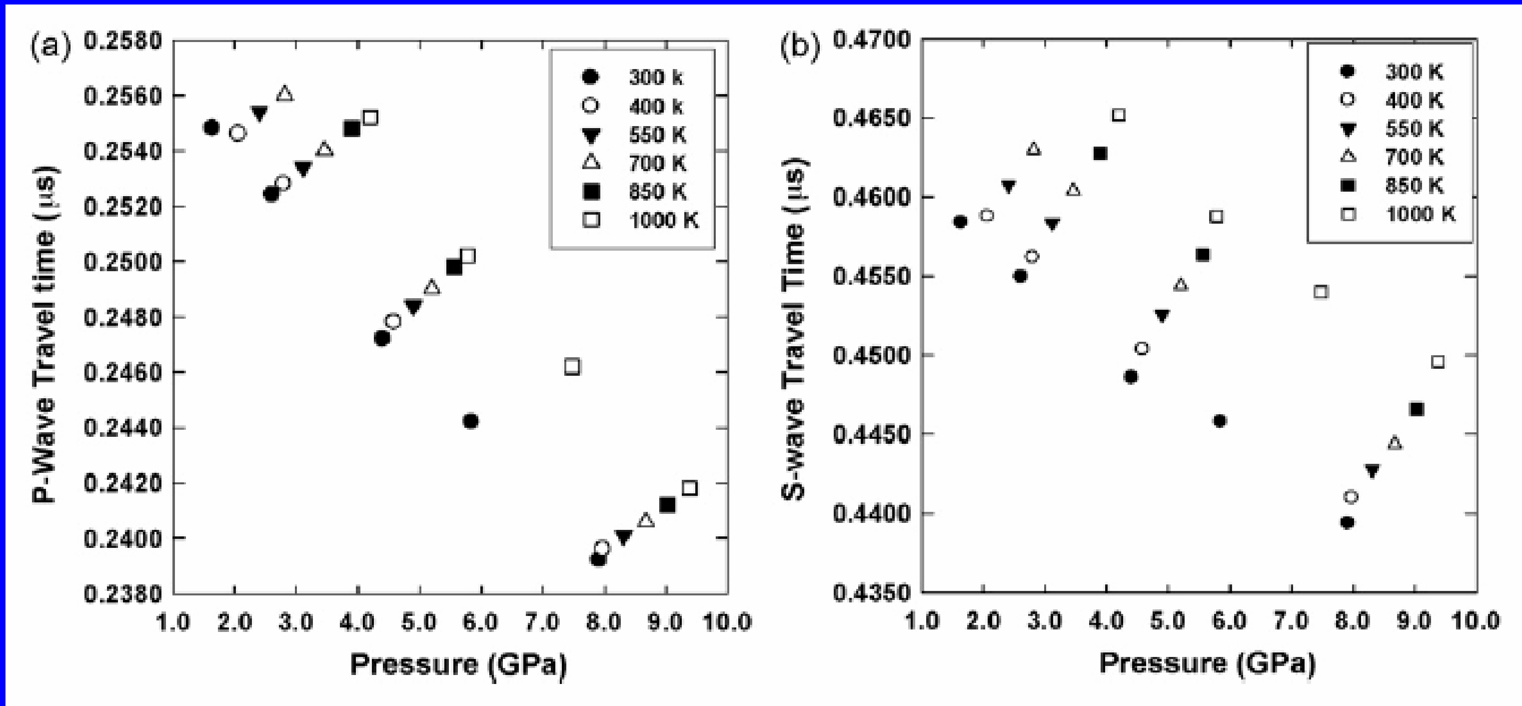
Elasticity of transition zone minerals:

Majorite garnets [$(Mg,Fe)_3Al_2Si_3O_{12}$ – $(Mg,Fe)_4Si_4O_{12}$]



Ultrasonic interferometry
In the Large Volume Press



from Gwanmesia et al. (2009)

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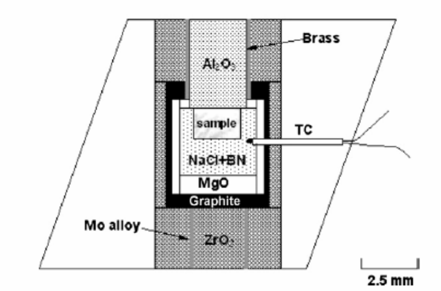
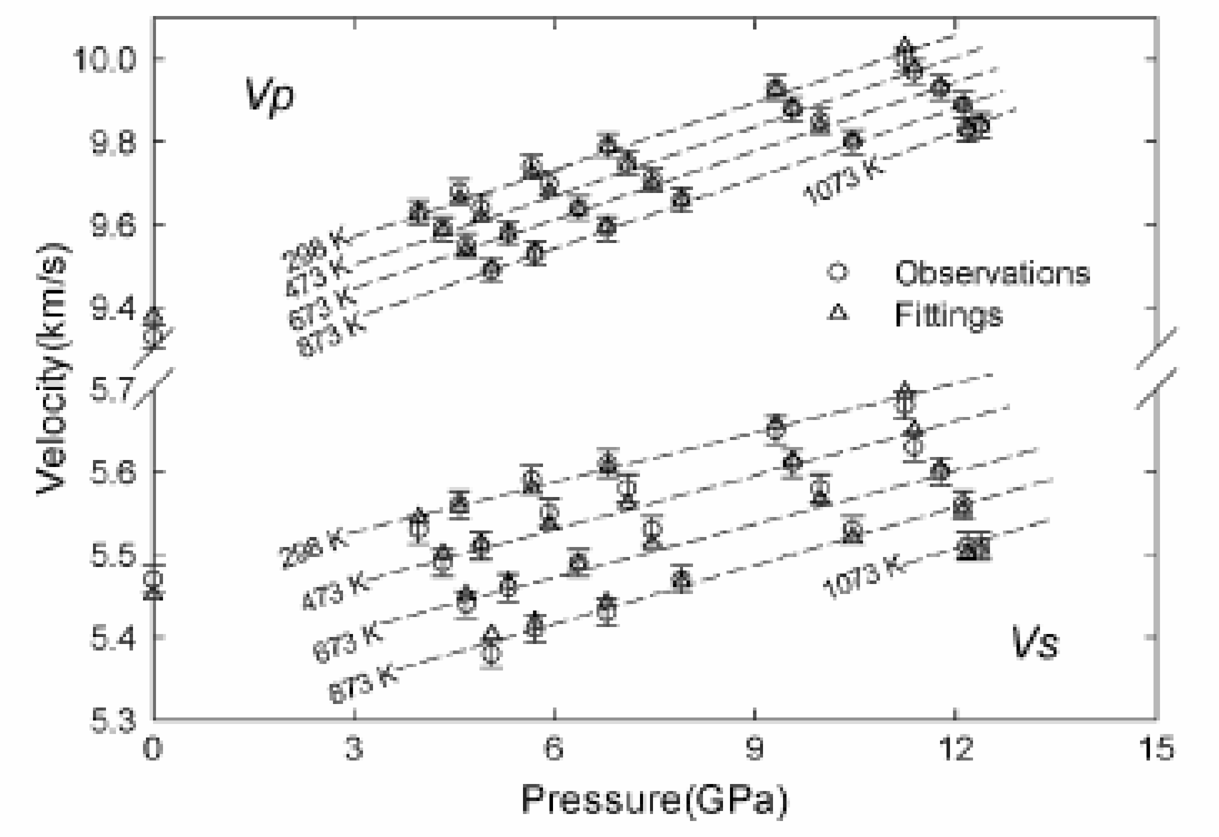
Elasticity of transition zone minerals:

Wadsleyite [β-$(Mg,Fe)_2SiO_4$]

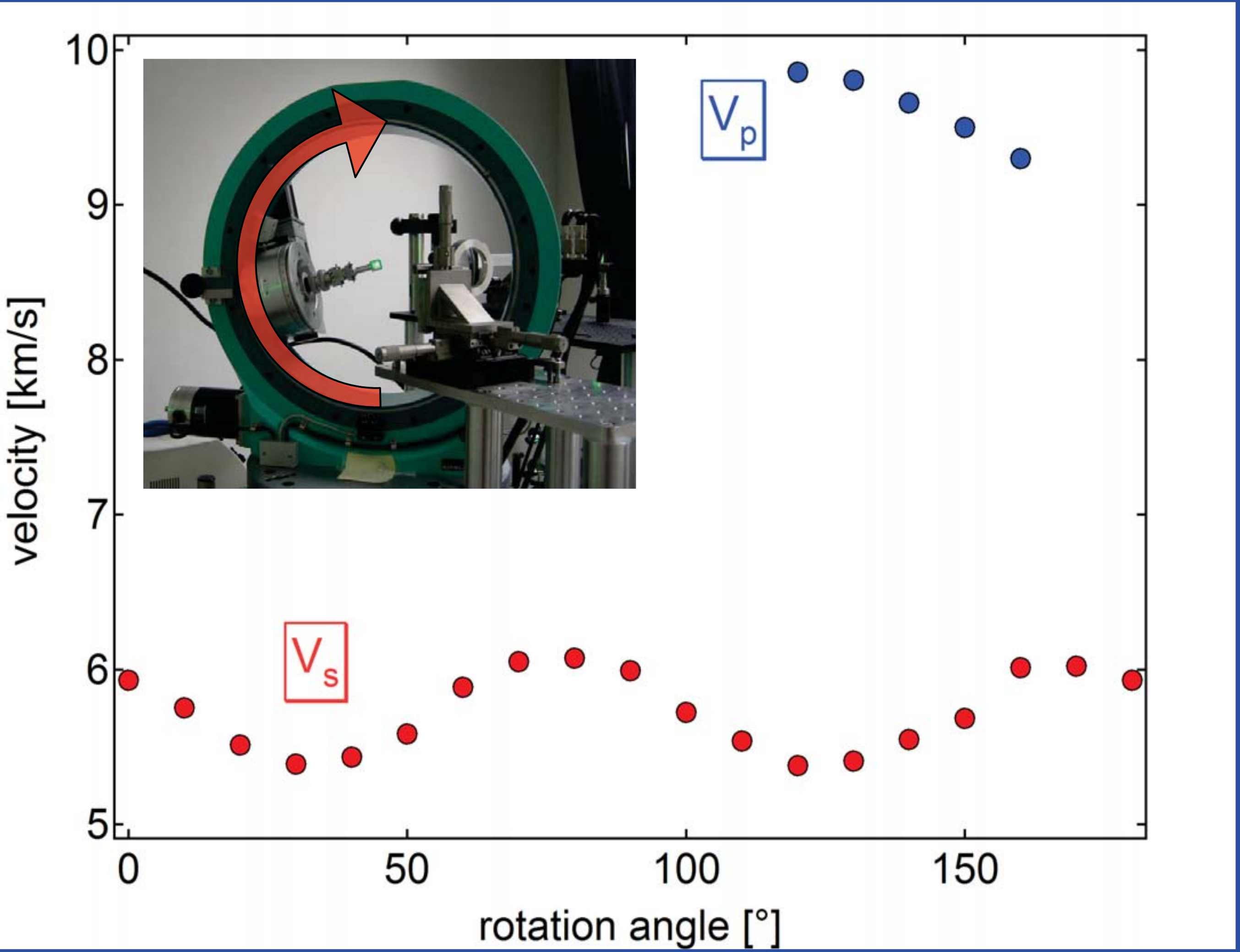
Ultrasonic interferometry
In the Large Volume Press





from Liu et al. (2009)

Elasticity of transition zone minerals:



Single-crystal elastic anisotropy by Brillouin scattering

from Marquardt et al. (2009)

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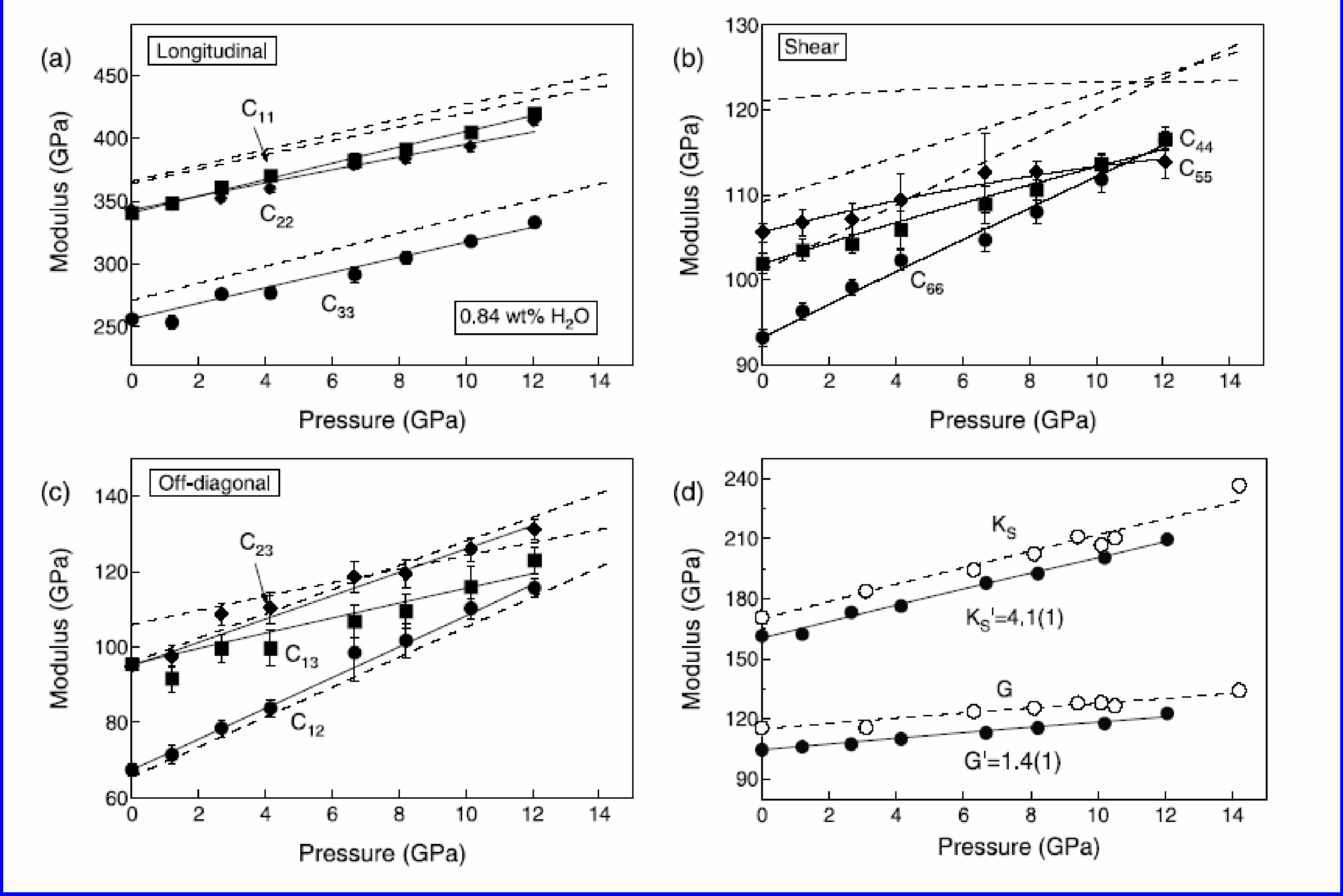
Elasticity of transition zone minerals:

OH in wadsleyite [β-Mg_2SiO_4]

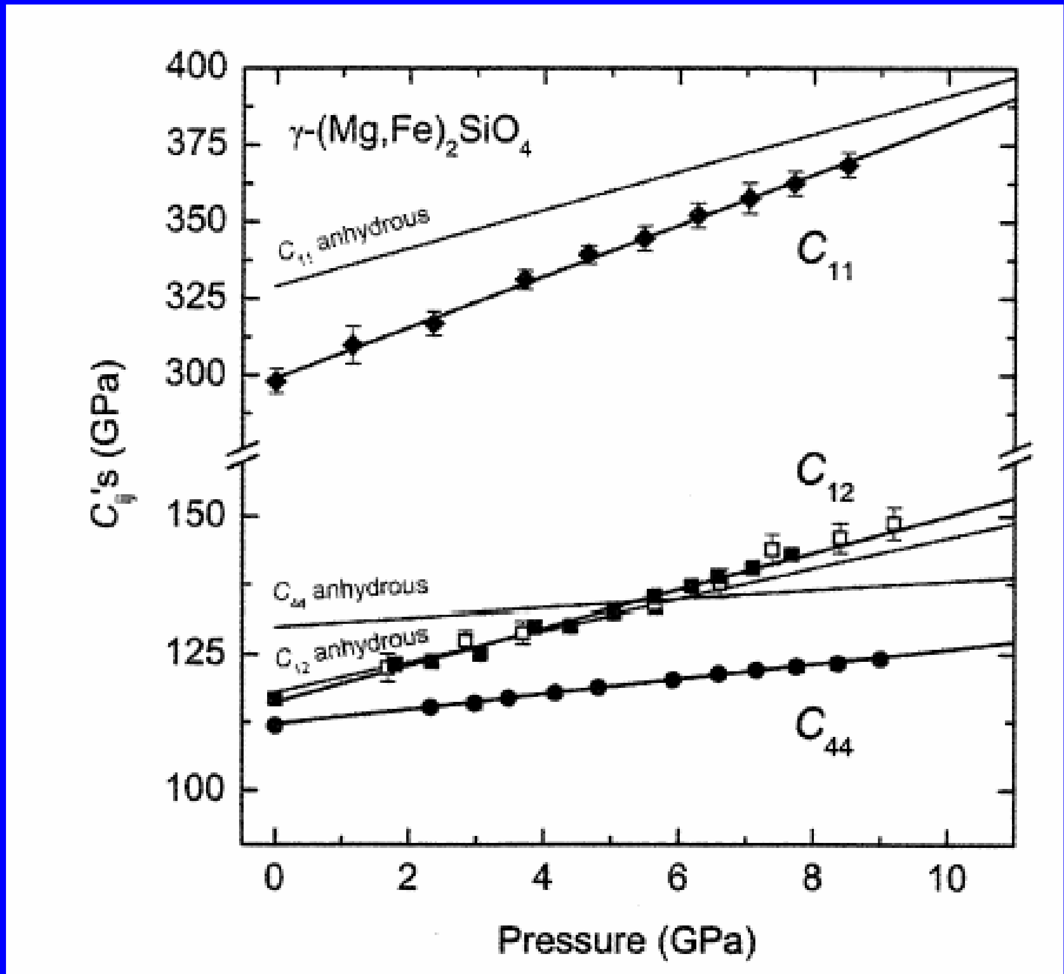


Brillouin scattering

from Mao et al. (2008)

Elasticity of transition zone minerals:

OH in ringwoodite [γ-$(Mg,Fe)_2SiO_4$]



Brillouin scattering

from Jacobsen and Smyth (2006)

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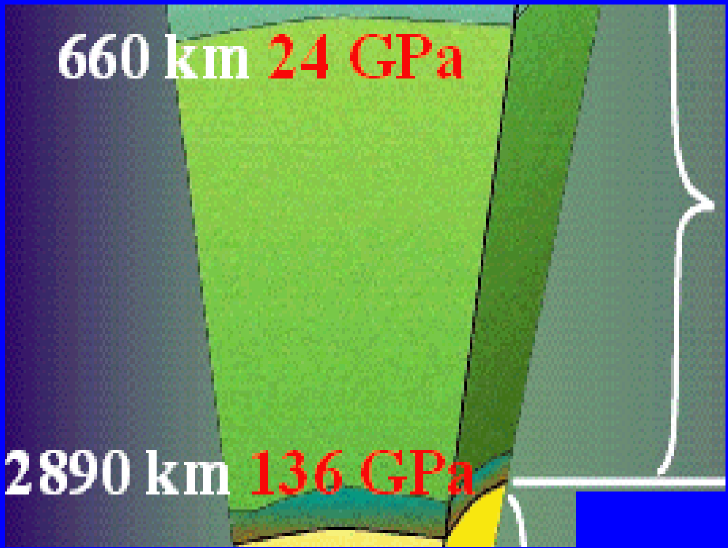
Lower mantle



Perovskites + (Mg,Fe)O

PostPerovskite + (Mg,Fe)O

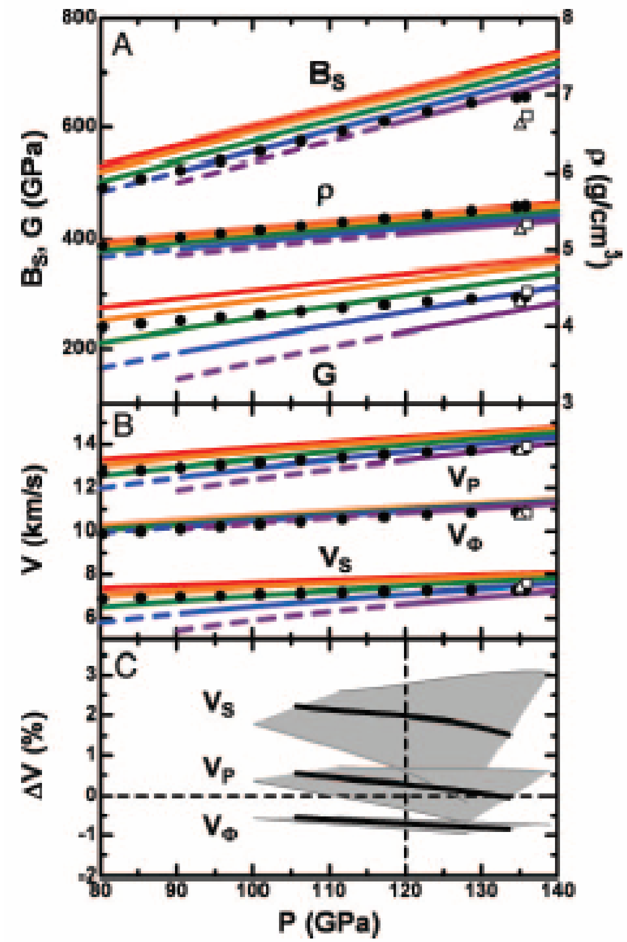
- Perovskite / Postperovskite
- Effect of Spin transition of Fe in (Mg,Fe)O

from the web

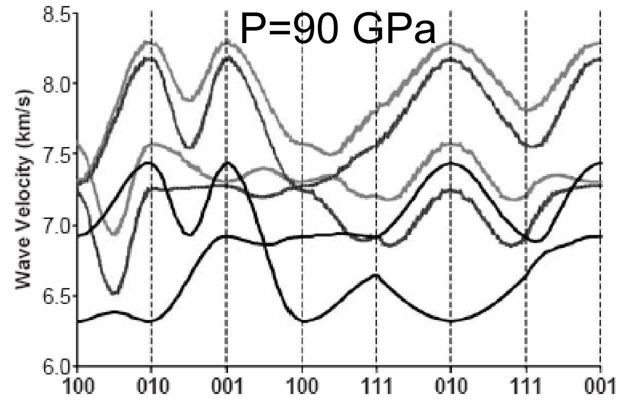
Elasticity of lower mantle minerals:

Perovskite-postperovskite [$(Mg,Fe)SiO_3$]

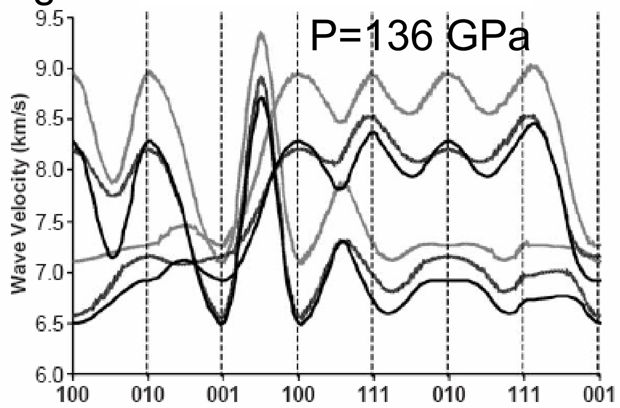
Post-perosvskite



Vs Perosvskite



Vs Post-perosvskite



Ab initio calculations

Wentzcovitch et al. (2006)

Stackhouse et al. (2005)

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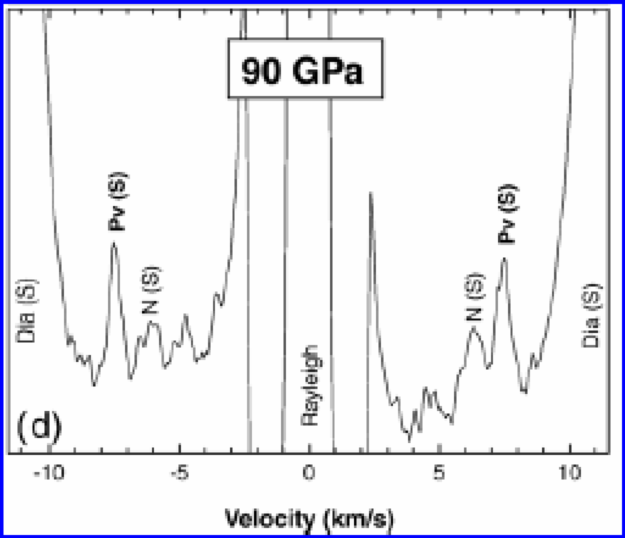
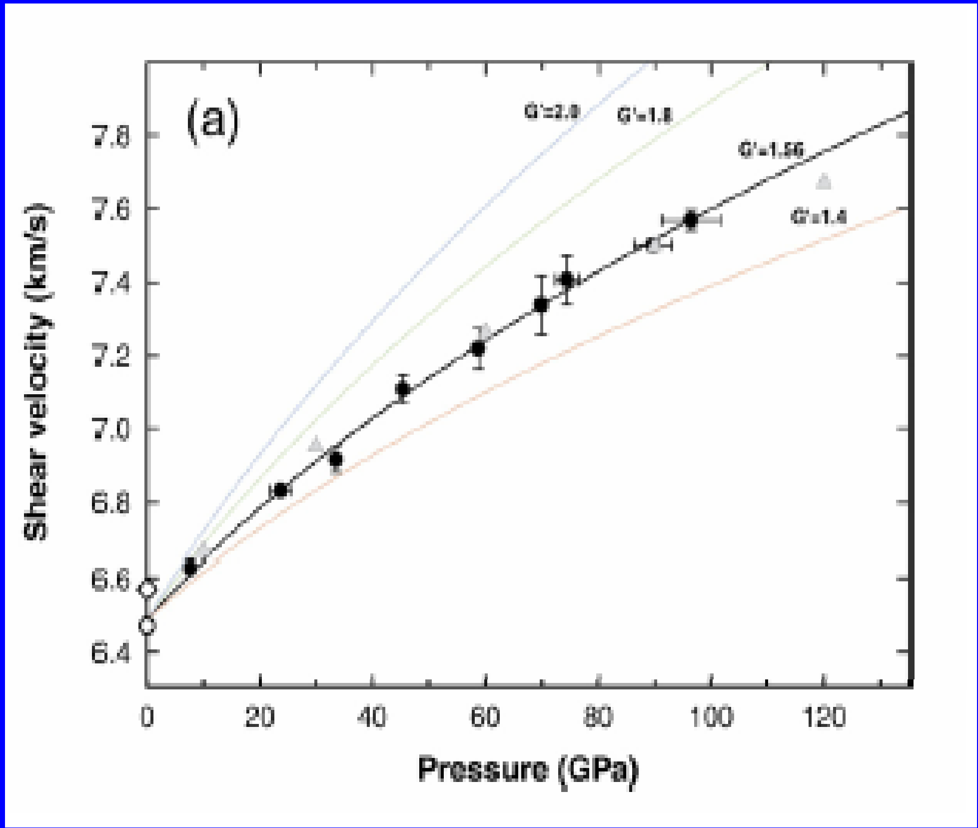
Elasticity of lower mantle minerals:

Sound velocity of perovskite ($MgSiO_3$)

Brillouin scattering



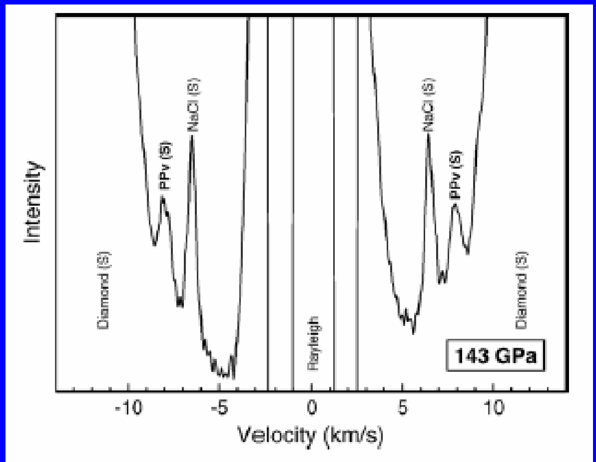


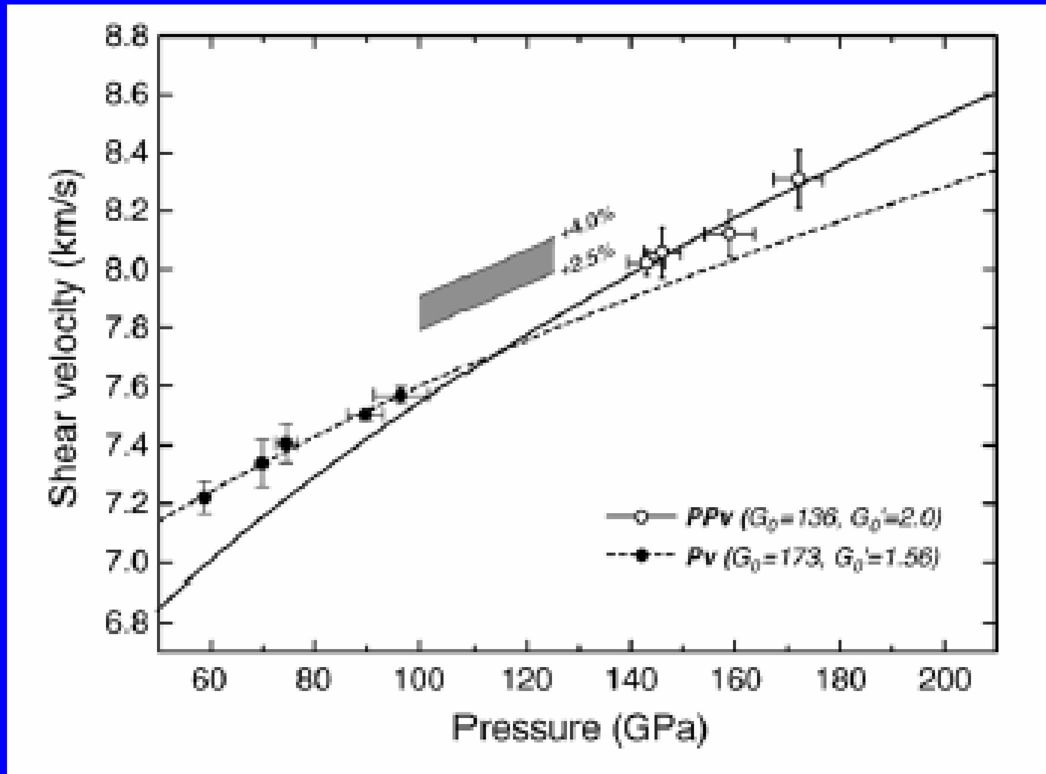
from Murakami et al. (2007)

Elasticity of lower mantle minerals:

Sound velocity of postperovskite ($MgSiO_3$)

Brillouin scattering



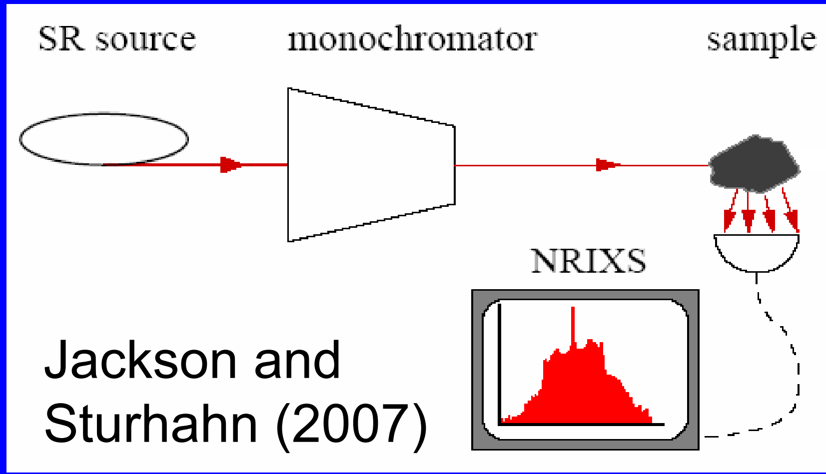


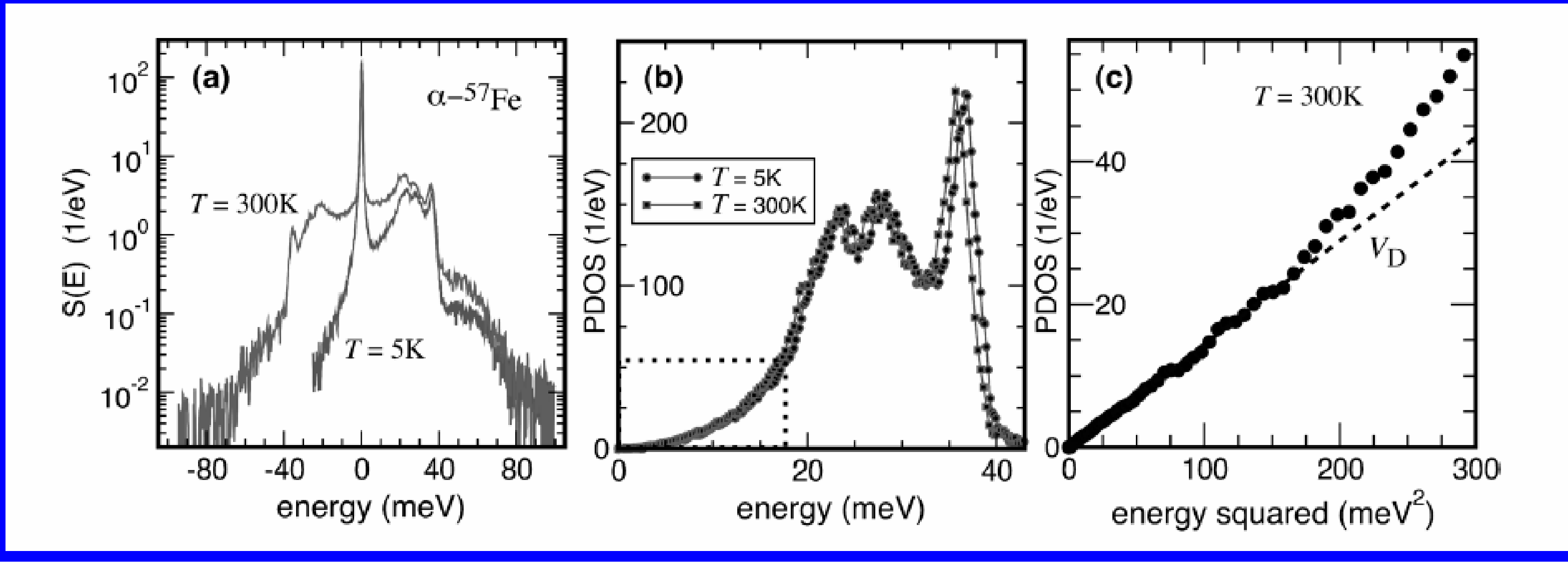
from Murakami et al. (2007)

Elasticity of lower mantle minerals:

Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Nuclear resonant X-ray inelastic scattering



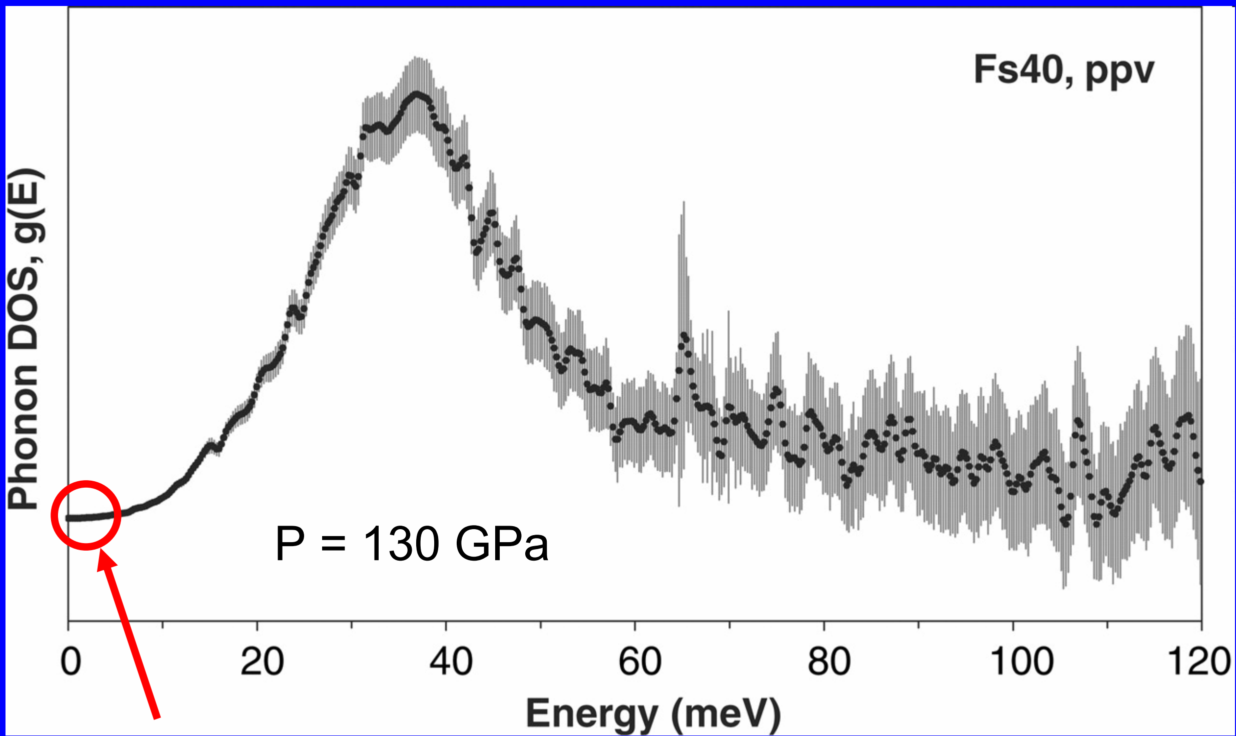


from Sturhahn et al. (2009)

Elasticity of lower mantle minerals:

Low sound velocity in Fe-rich postperovskite [$(Mg_{0.6}Fe_{0.4})SiO_3$]

Nuclear resonant inelastic X-ray scattering

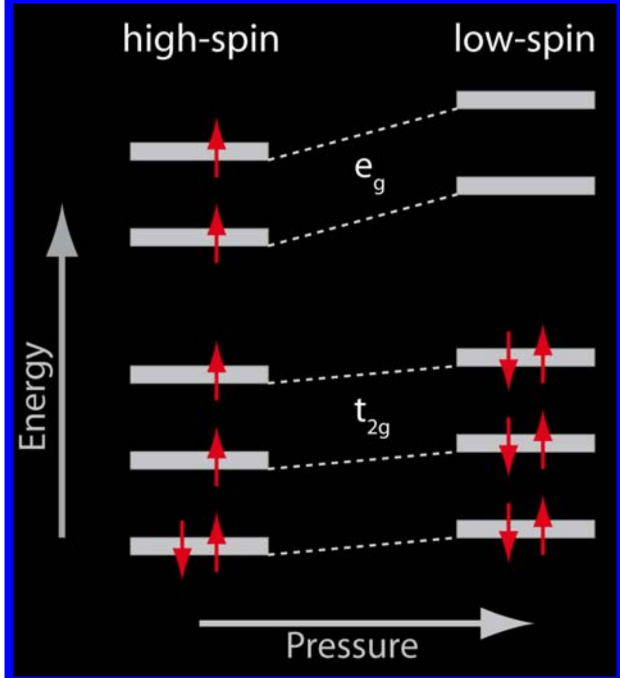


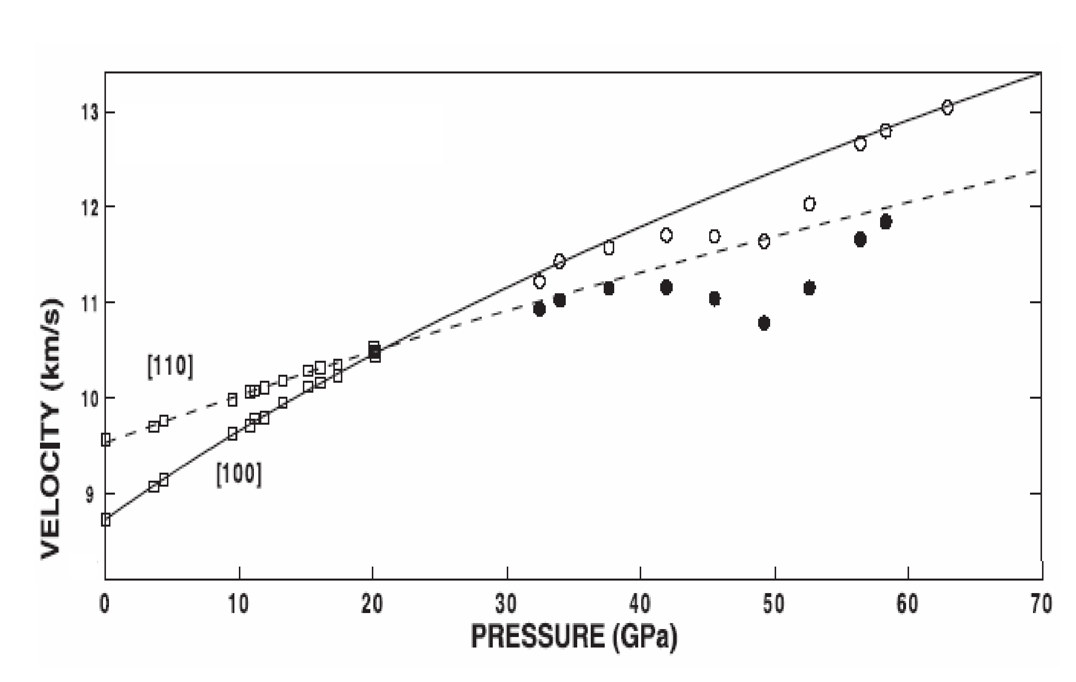
$3/v_D^3 = 1/v_P^3 + 2/v_S^3$

from Mao et al. (2006)

Elasticity of lower mantle minerals:

Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]





Impulsively stimulated light scattering

from Crowhurst et al. (2008)

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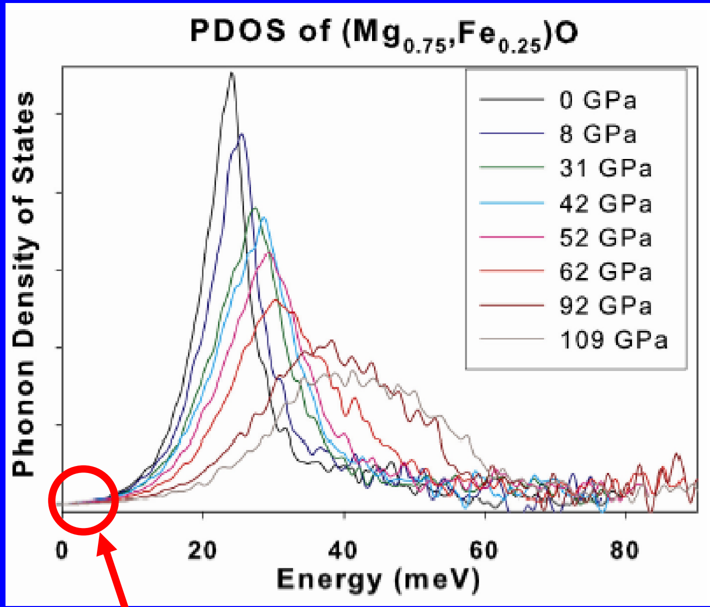
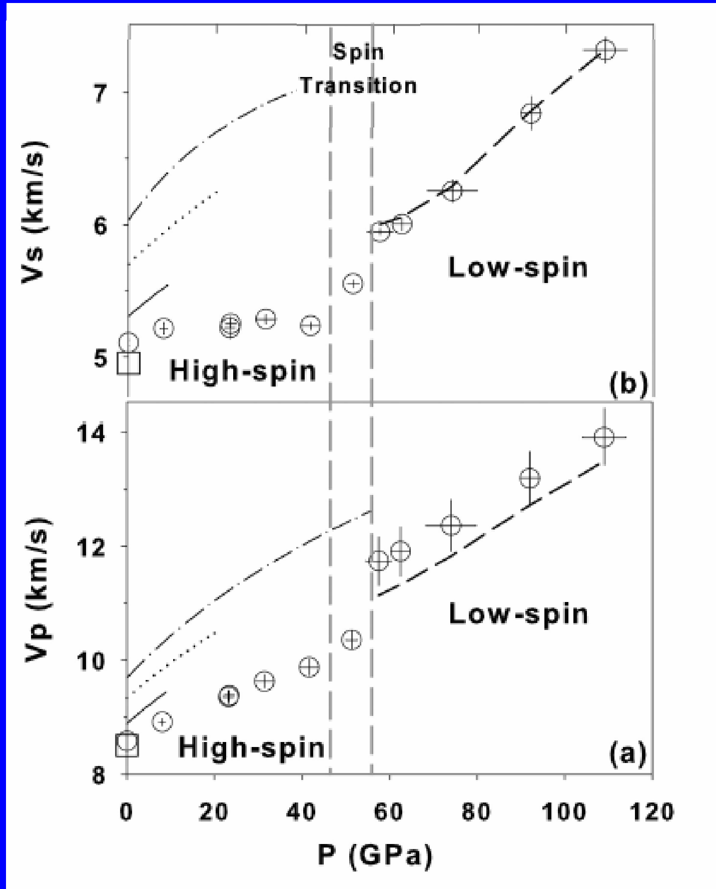
Elasticity of lower mantle minerals:

Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

Nuclear resonant X-ray inelastic scattering



$$3/v_D^3 = 1/v_P^3 + 2/v_S^3$$



from Lin et al. (2006)

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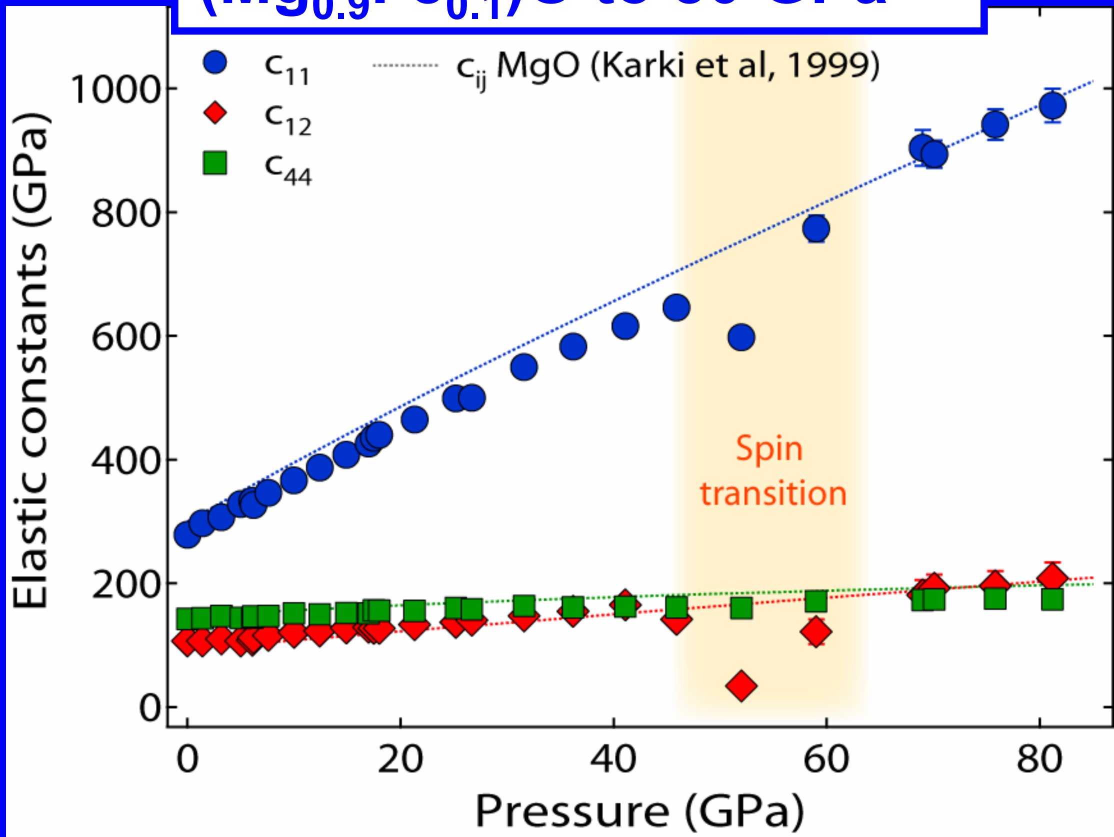
Elasticity of lower mantle minerals:

Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

$(Mg_{0.9}Fe_{0.1})O$ to 80 GPa



Brillouin scattering

from Marquardt et al. (2009)

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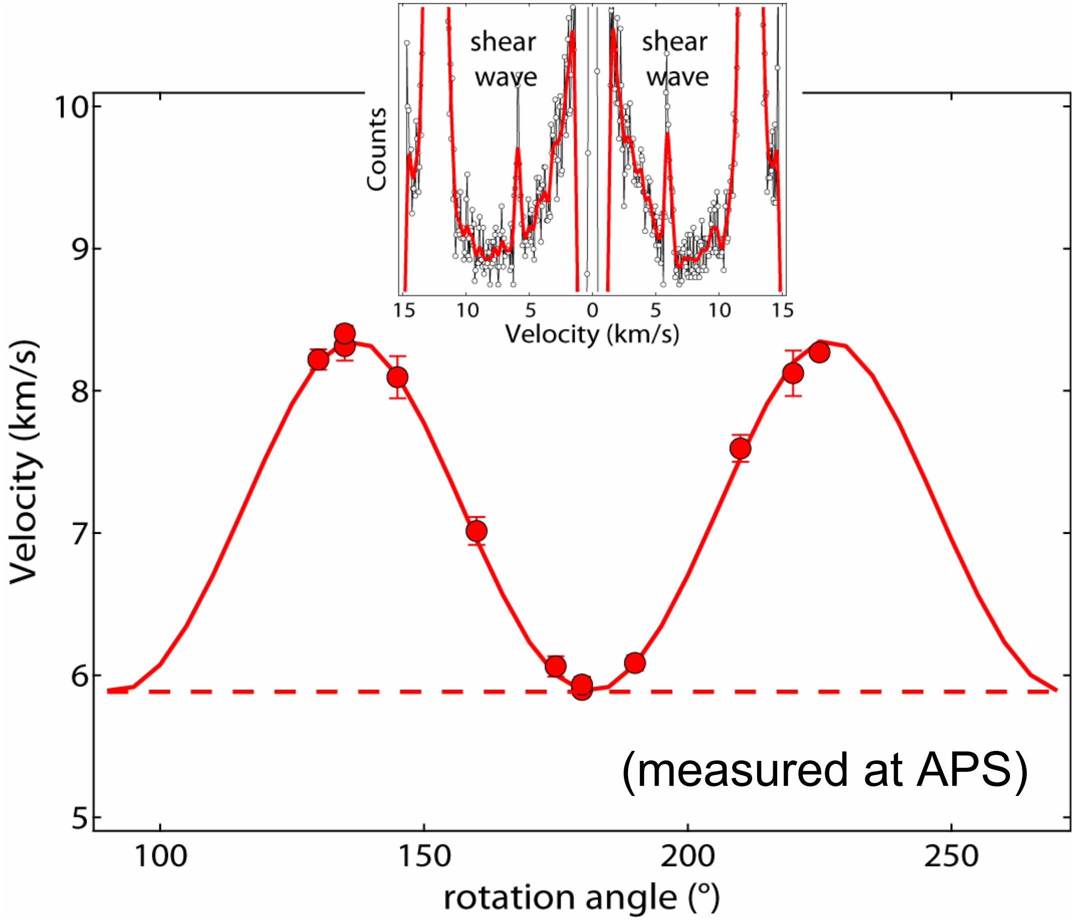
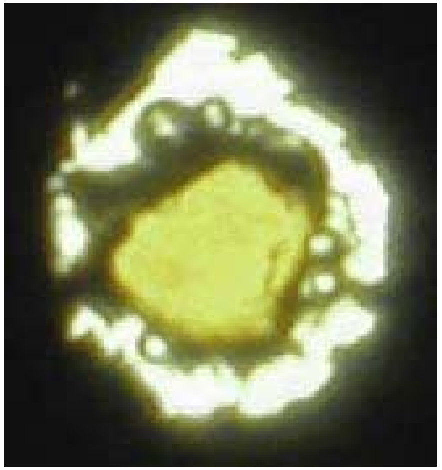
Single-crystal MgO at 81 GPa:

Shear elastic anisotropy





Marquardt et al. (subm.)

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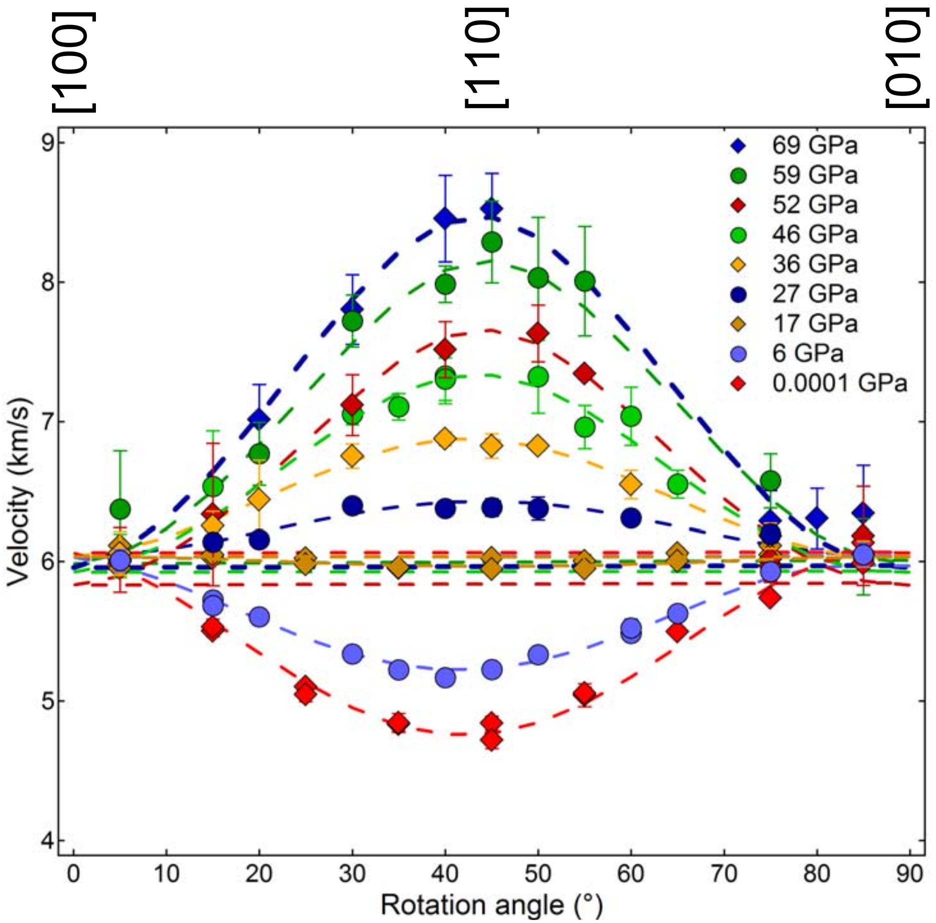
Velocity anisotropy (measurements):

$(Mg_{0.9}Fe_{0.1})O$ to 81 GPa



Marquardt et al. (2009)

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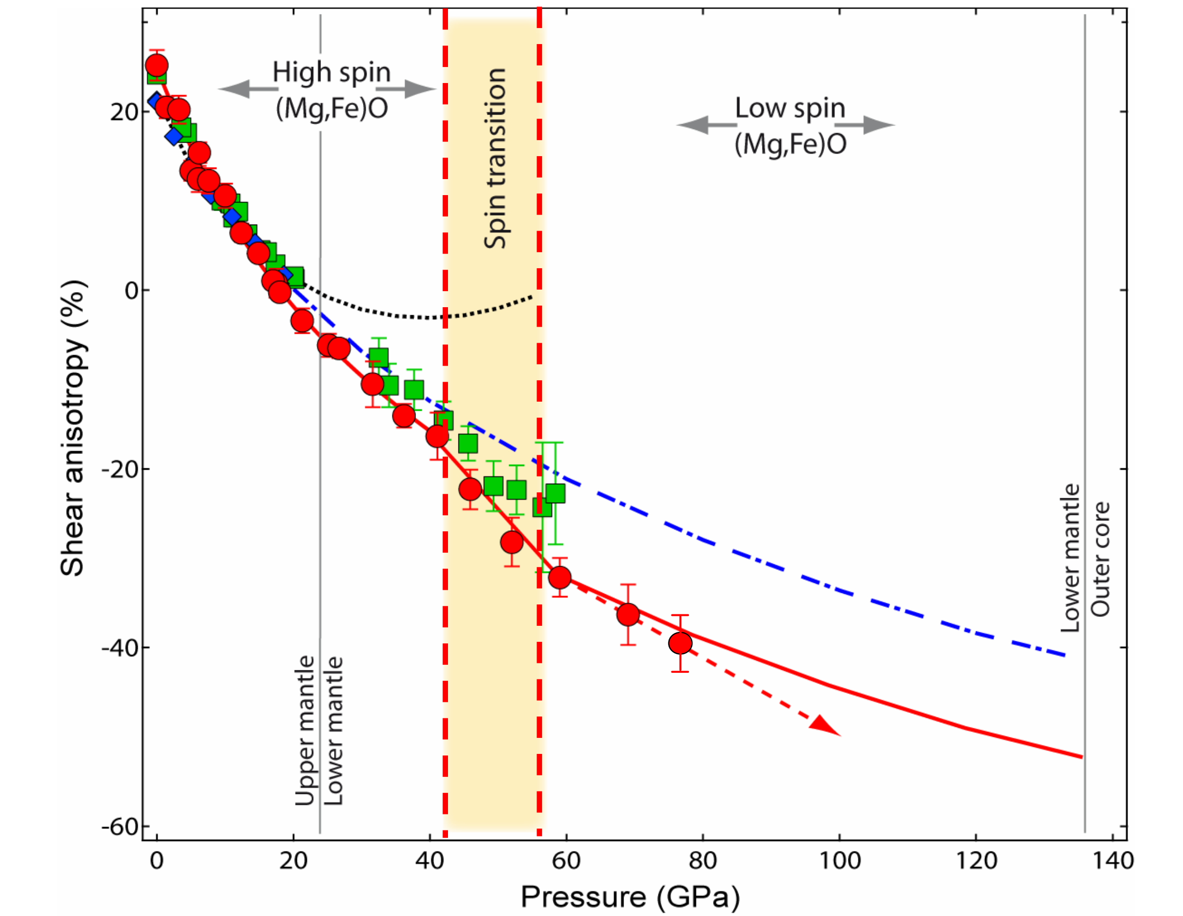
Elasticity of lower mantle minerals:

Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]

$(Mg_{0.9}Fe_{0.1})O$ to 80 GPa



from Marquardt et al. (2009)

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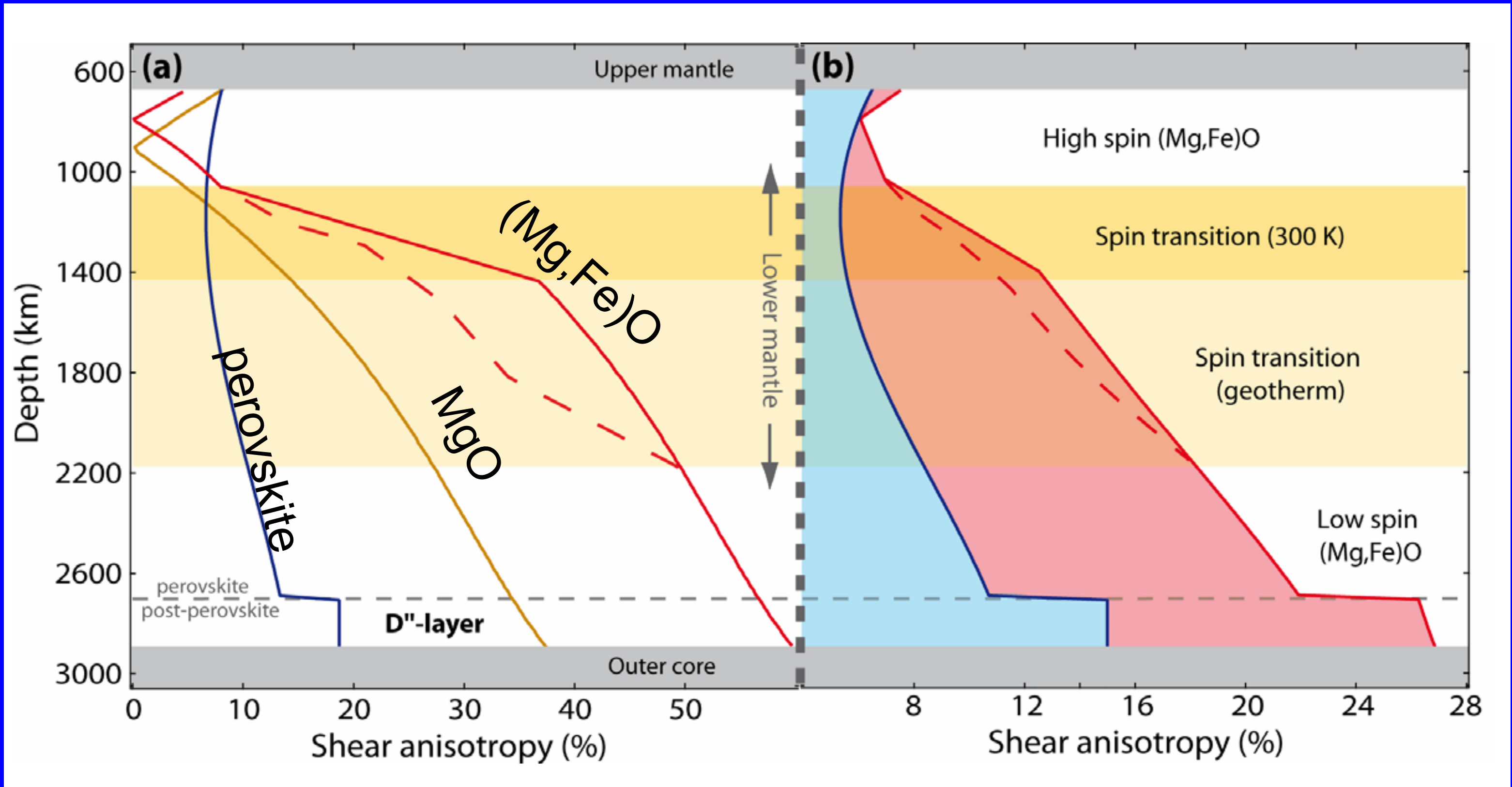
Elasticity of lower mantle minerals:

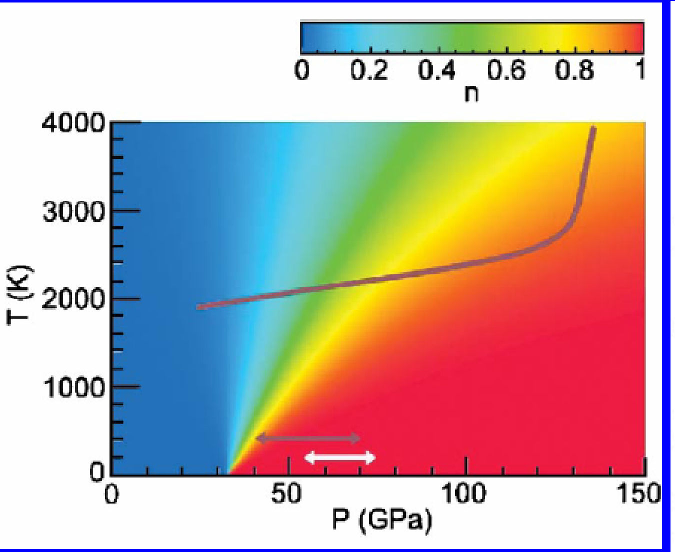
Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]



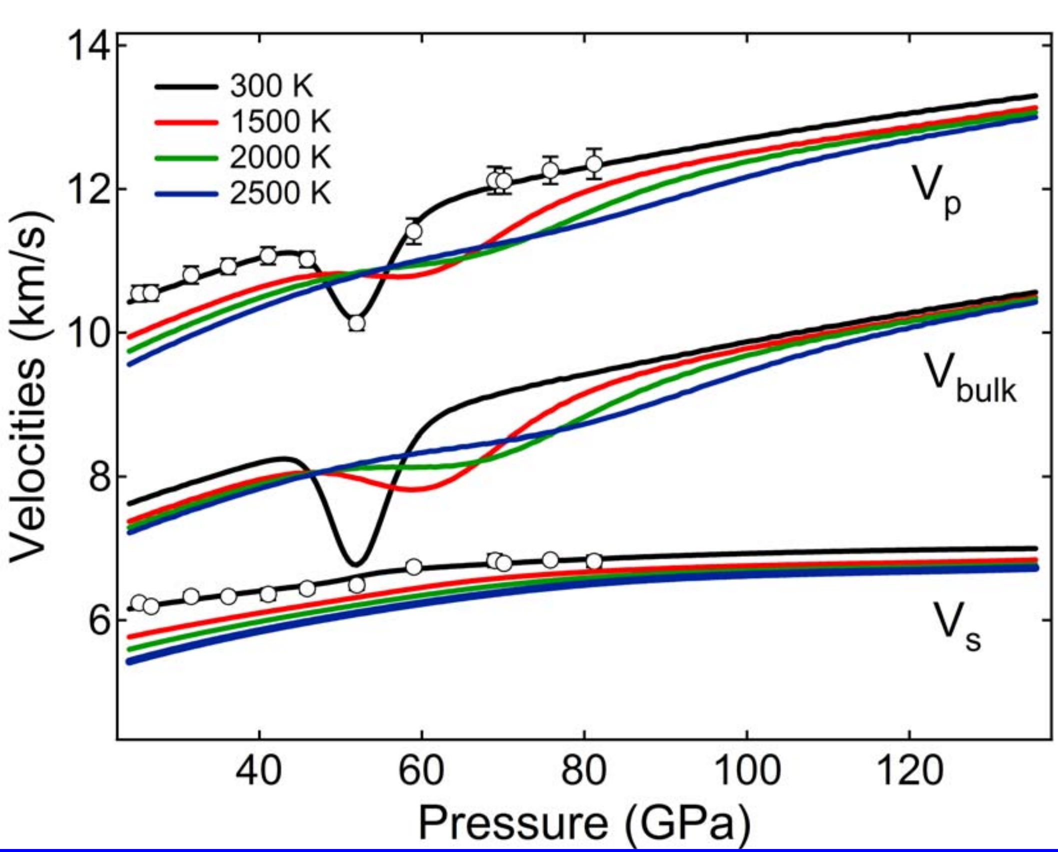
from Marquardt et al. (2009)

Elasticity of lower mantle minerals:

Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]



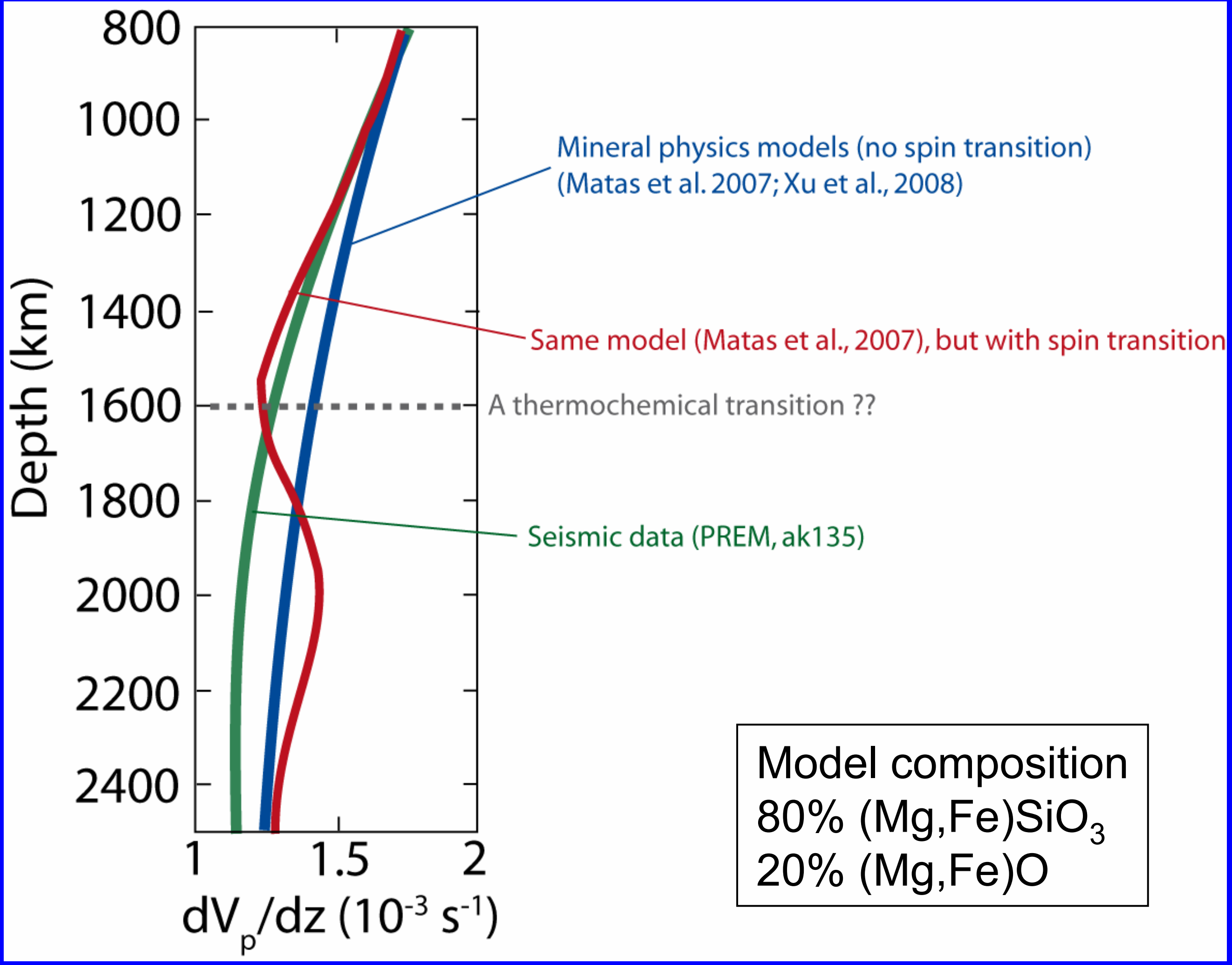
Tsuchiya et al. (2006)



from Marquardt et al. (2009)

Elasticity of lower mantle minerals:

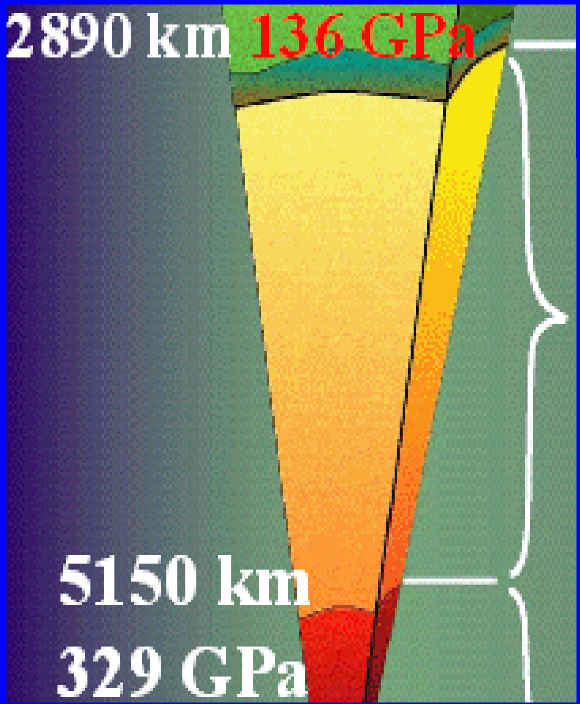
Effects of Fe spin-transition on ferropericlase [(Mg,Fe)O]



Model composition
80% $(Mg,Fe)SiO_3$
20% $(Mg,Fe)O$

from Cammarano et al. (submitted 2009)

Outer core



Liquid Fe + alloying elements

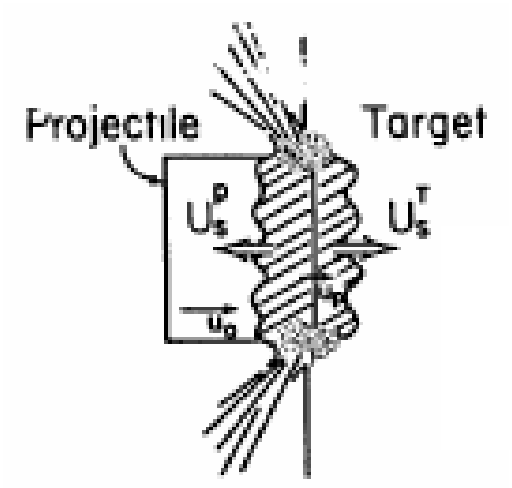
- Sound velocity of liquid Fe at core conditions

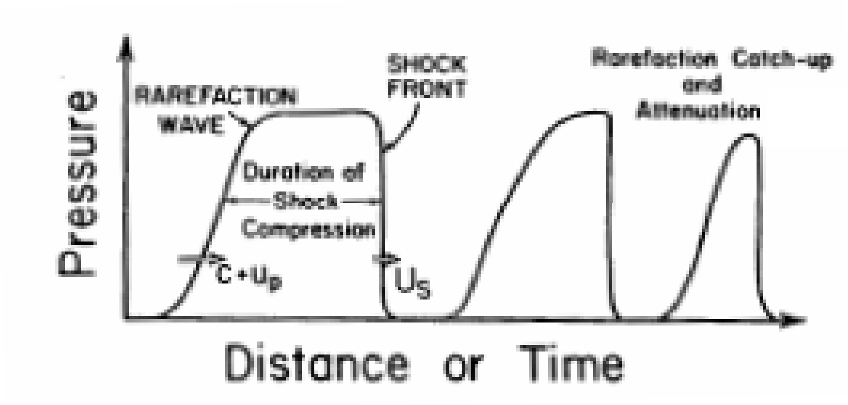
from the web

Helmholtz Centre Potsdam

Elasticity of the outer core

Shock wave velocity measurements



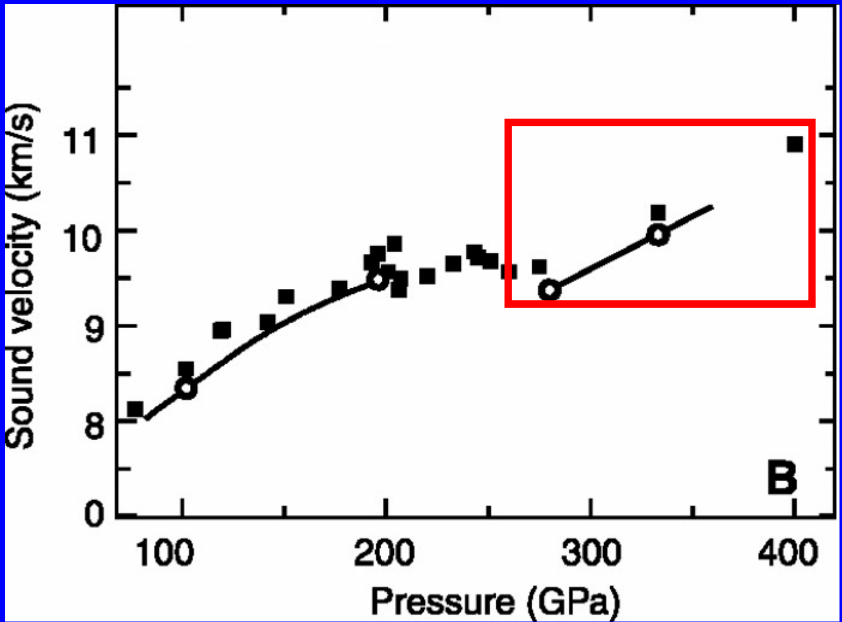


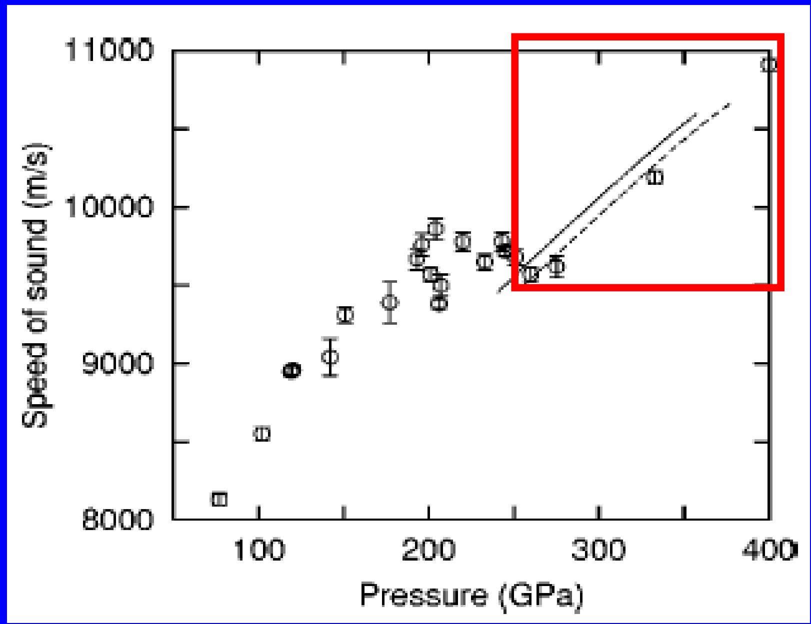
$\rho_0 U_S = \rho_1 (U_S - u_P)$
$P = \rho_0 U_S u_P$
$E_1 - E_0 = 1/2 \rho_0 U_S u_P^2$

from Jeanloz (1989)

Elasticity of the outer core

Computational and experimental constraints on sound velocity of liquid Fe





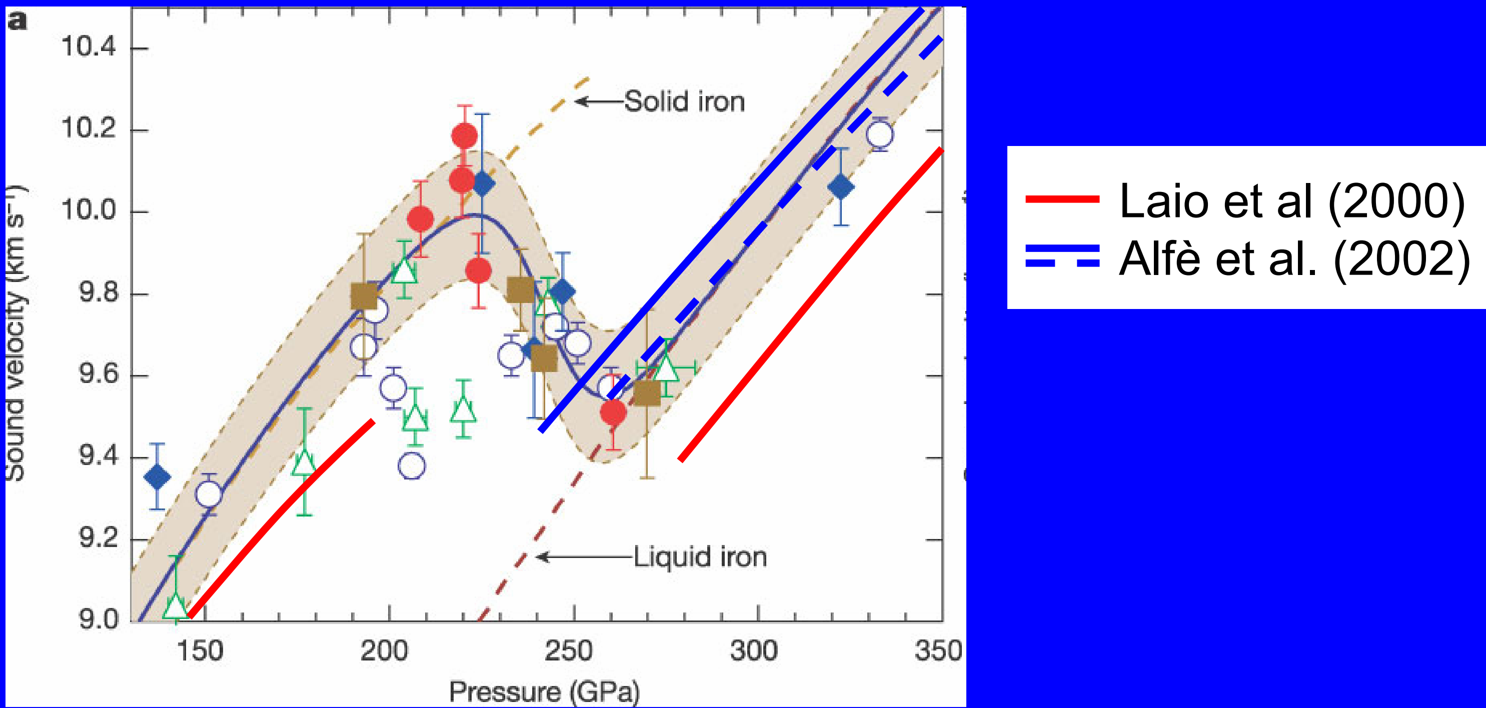
from Laio et al. (2000)

from Alfè et al. (2002)

Elasticity of the outer core

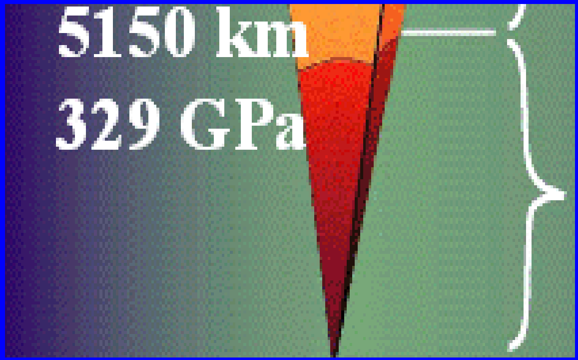
New experimental results on sound velocity of liquid Fe

Shock wave velocity measurements



from Nguyen et al. (2004)

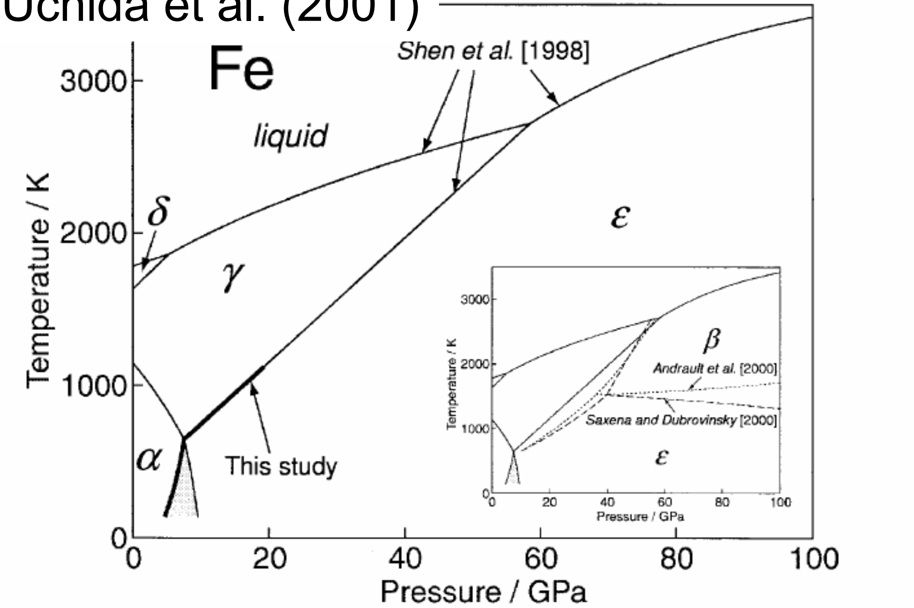
Inner core



Solid Fe + alloying elements

- Sound velocity of ε-Fe
- Elastic anisotropy of ε-Fe

Uchida et al. (2001)



from the web

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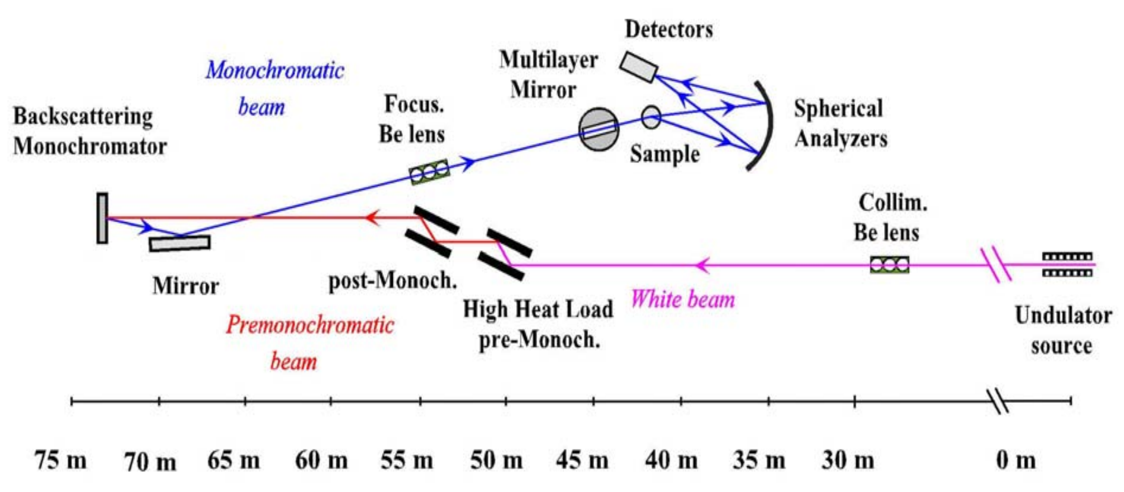
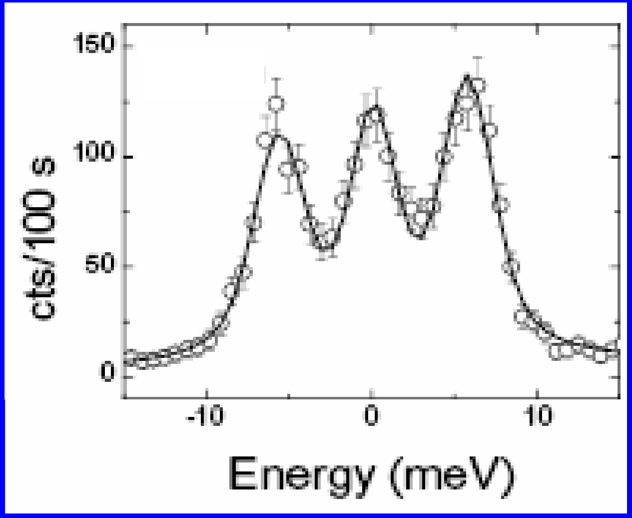
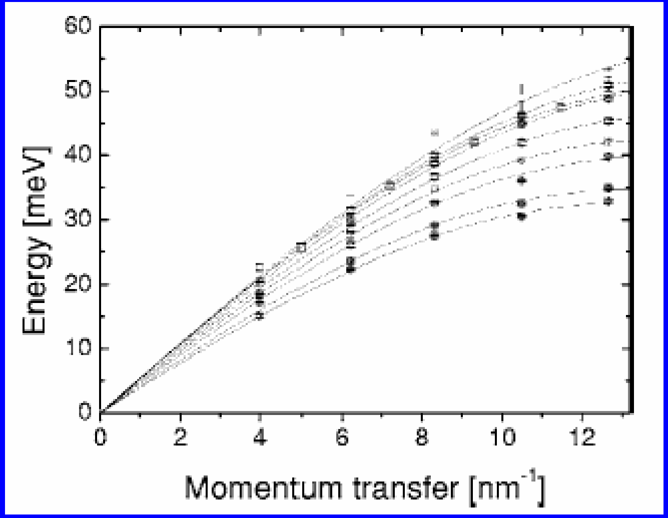
Elasticity of Fe in the inner core

Momentum resolved inelastic X-ray scattering



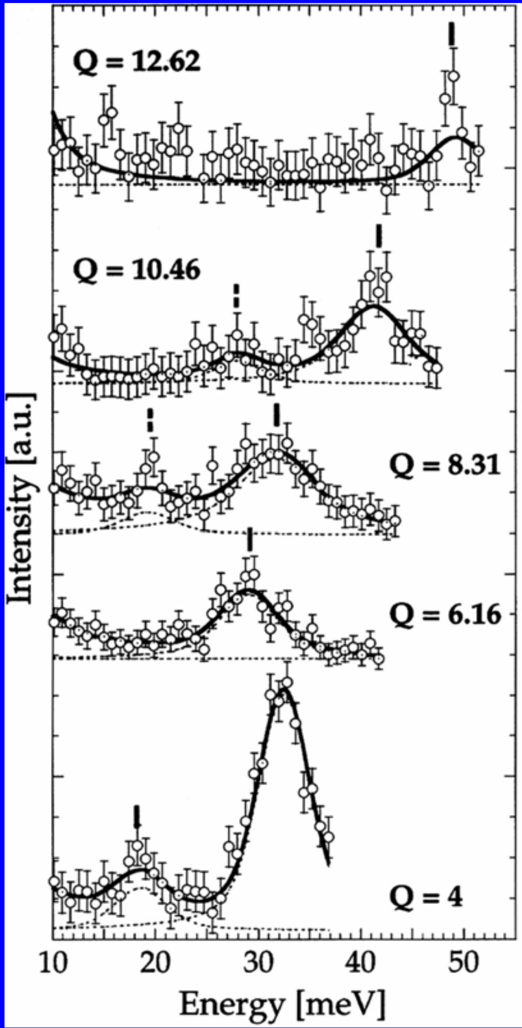


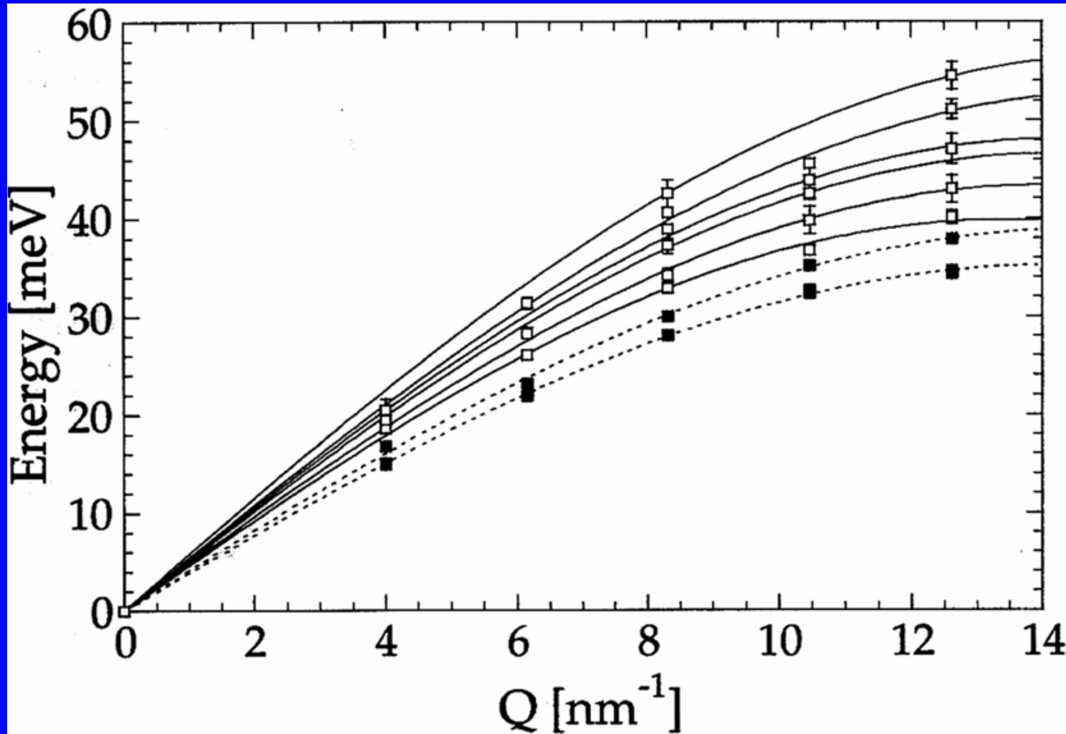


from Antonangeli et al. (2004)

Elasticity of Fe in the inner core

Momentum resolved inelastic X-ray scattering

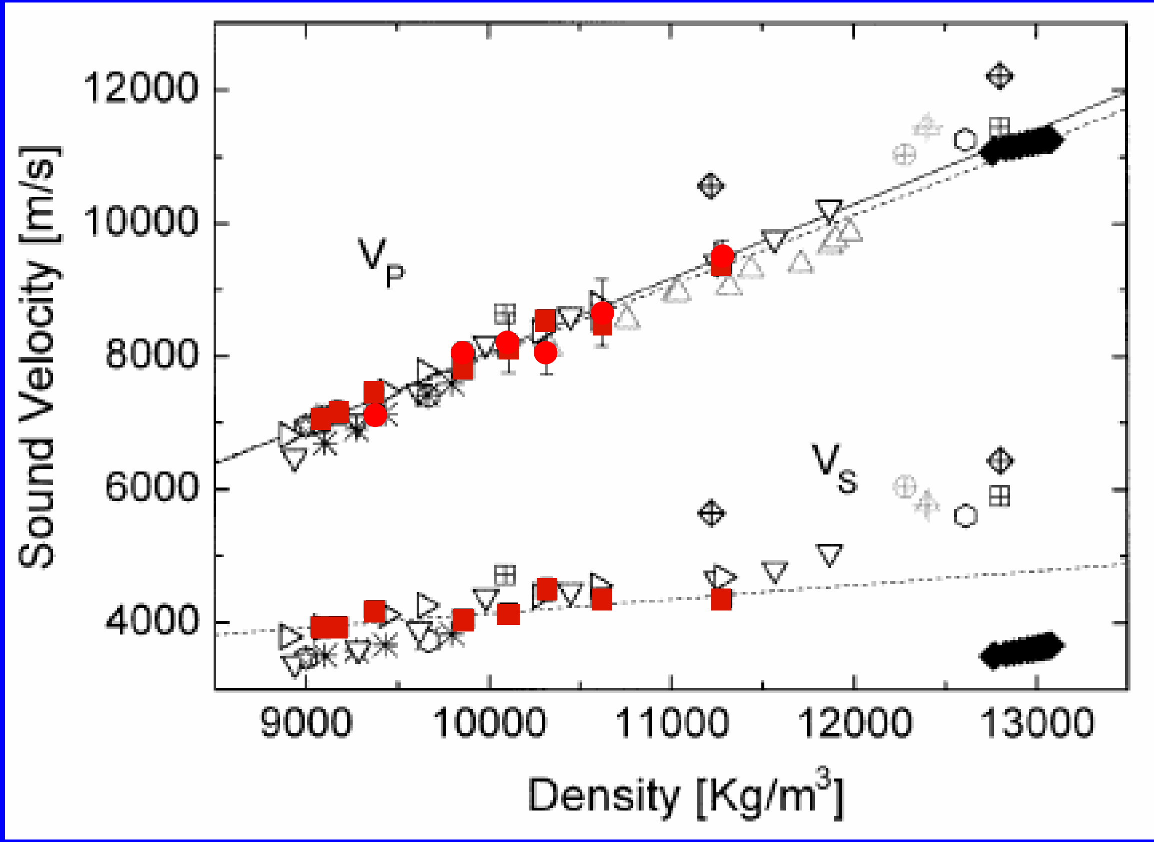




from Fiquet et al. (2001)

Elasticity of Fe in the inner core

Momentum resolved inelastic X-ray scattering

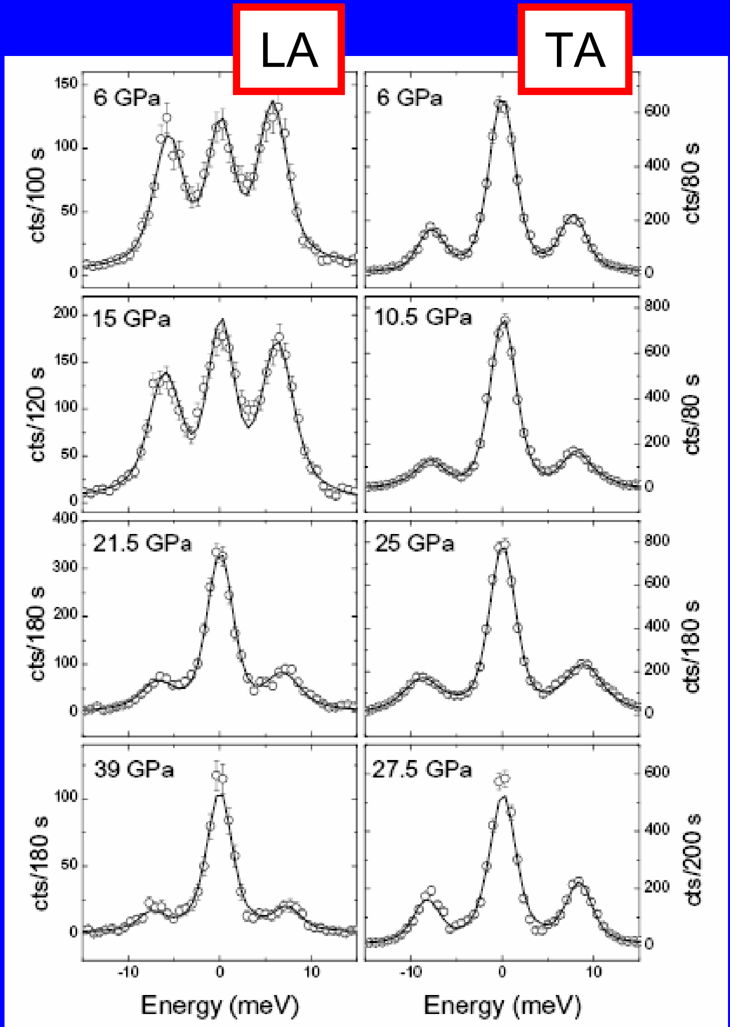


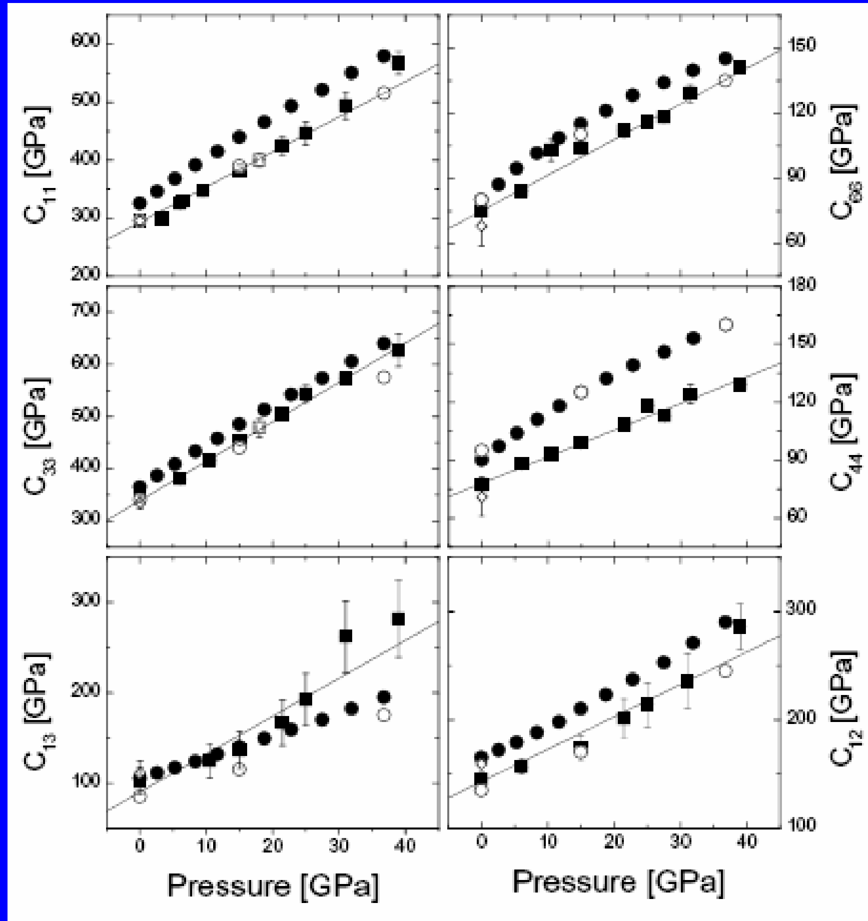
from Fiquet et al. (2001); Antonangeli et al. (2004)

Elasticity of single-crystal hcp Co:

Structural proxy for ε-Fe

Momentum resolved inelastic X-ray scattering





from Antonangeli et al. (2004)

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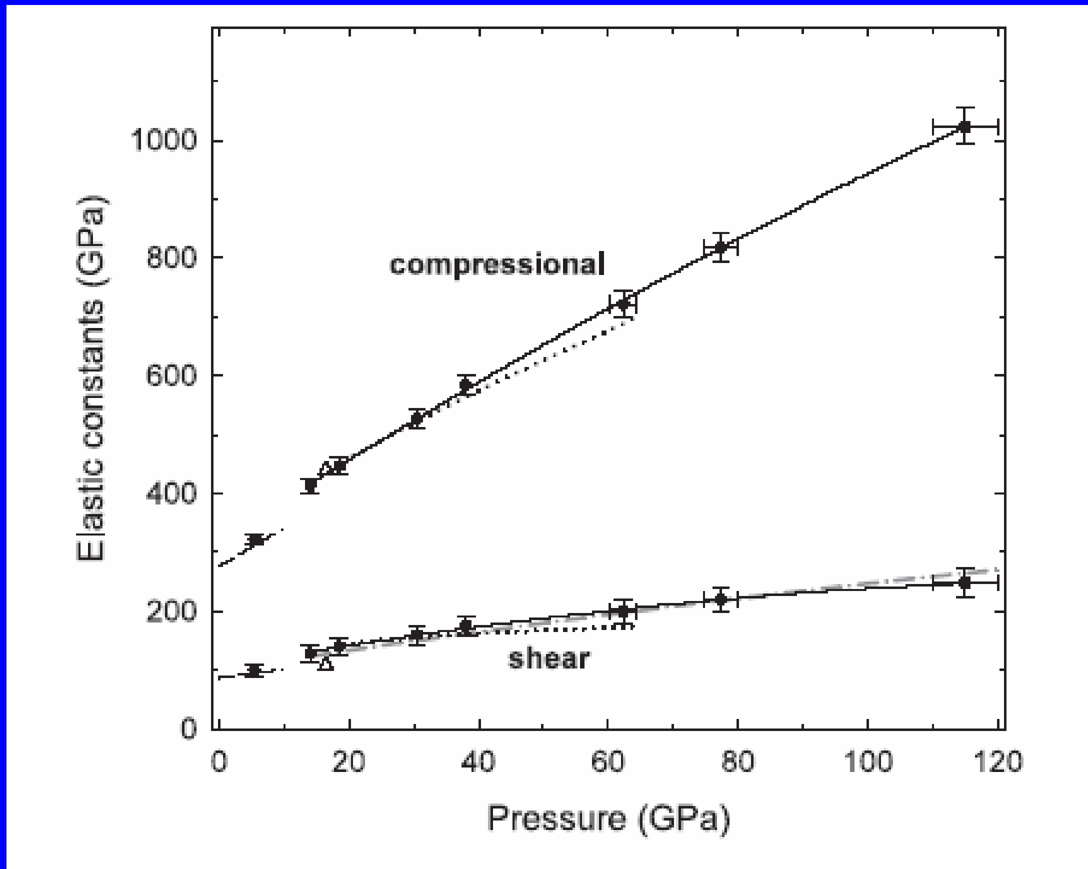
Elasticity of Fe in the inner core

Impulsively stimulated light scattering

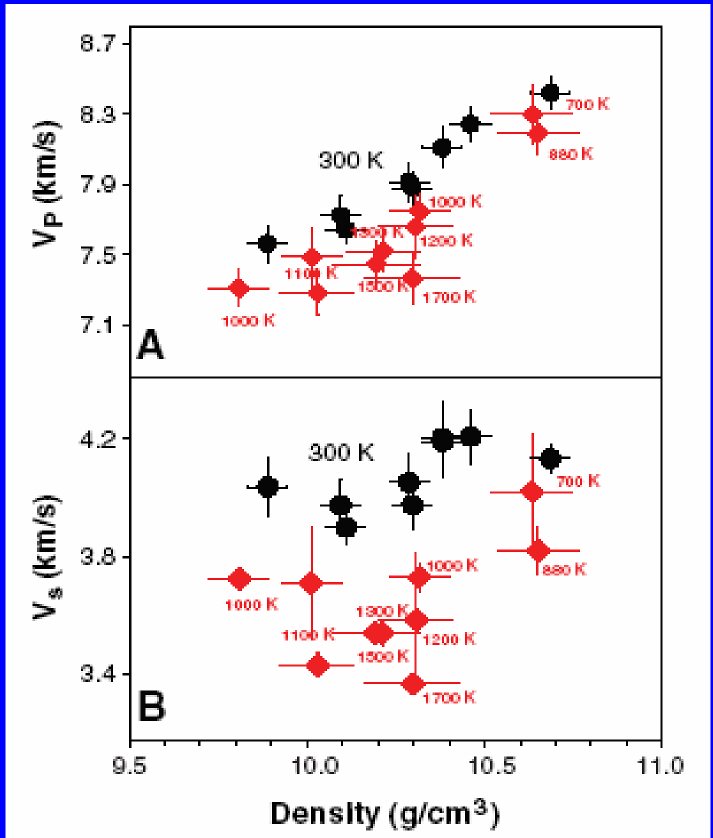


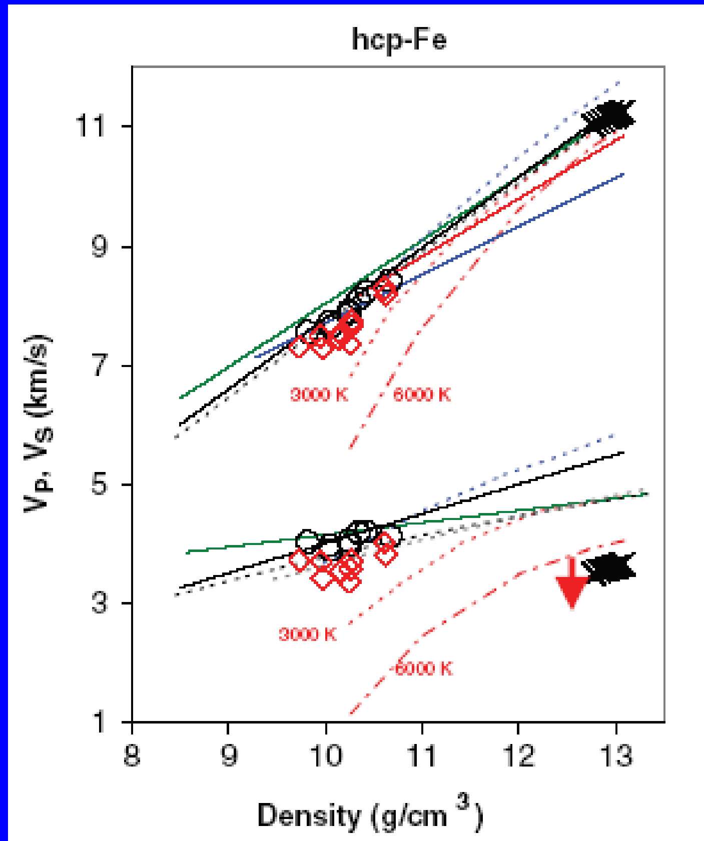
from Crowhurst et al. (2004)

Elasticity of Fe in the inner core

High P-T average velocity for isotropic aggregates

Nuclear resonant X-ray inelastic scattering

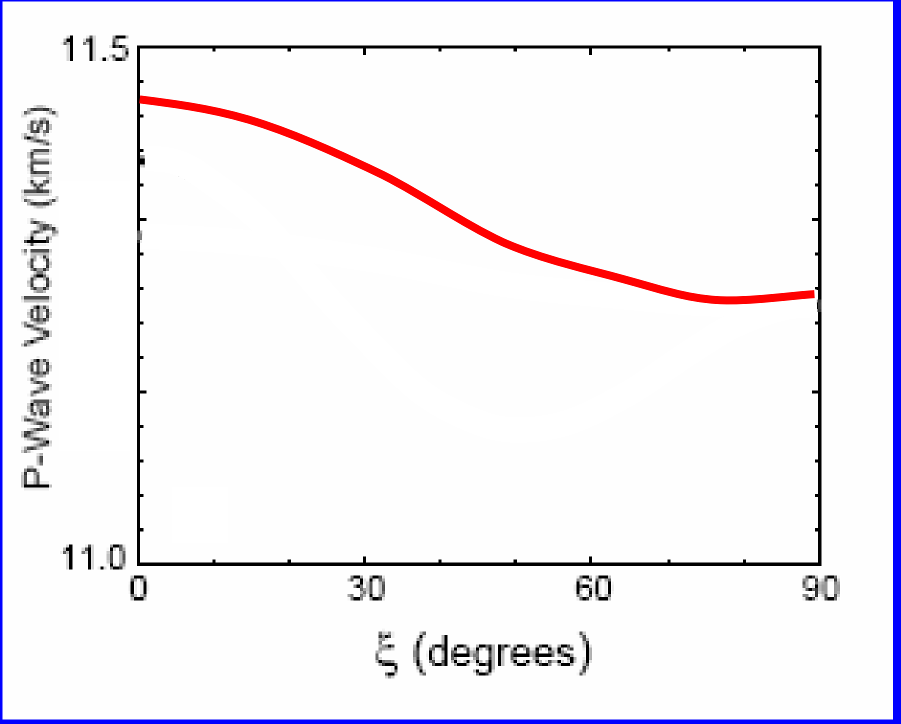




from Lin et al. (2005)

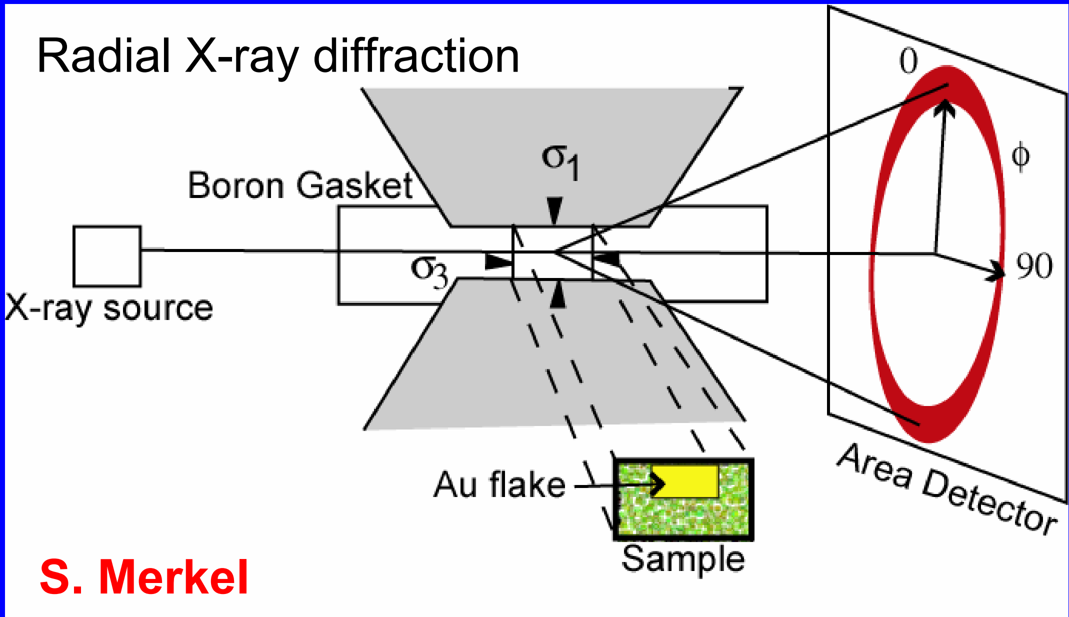
Elasticity of Fe in the inner core

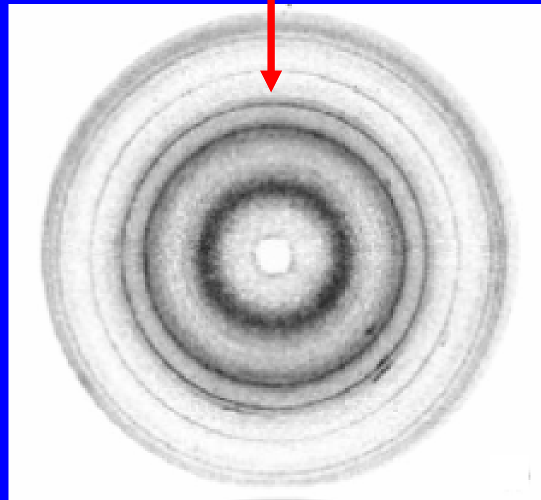
Seismic anisotropy of the inner core

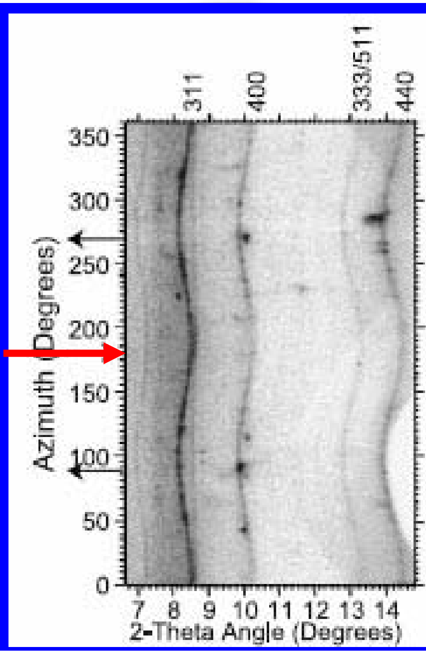


from Ishii et al. (2002)

Elasticity of Fe in the inner core







$$d_m(hkl) + d_P(hkl)\,[1 + (1-\cos^2\phi)\,Q(hkl)]$$
$$Q(hkl) = F(\sigma_{11} - \sigma_{33}, s_{ijkl})$$

from Wenk et al. (2004)

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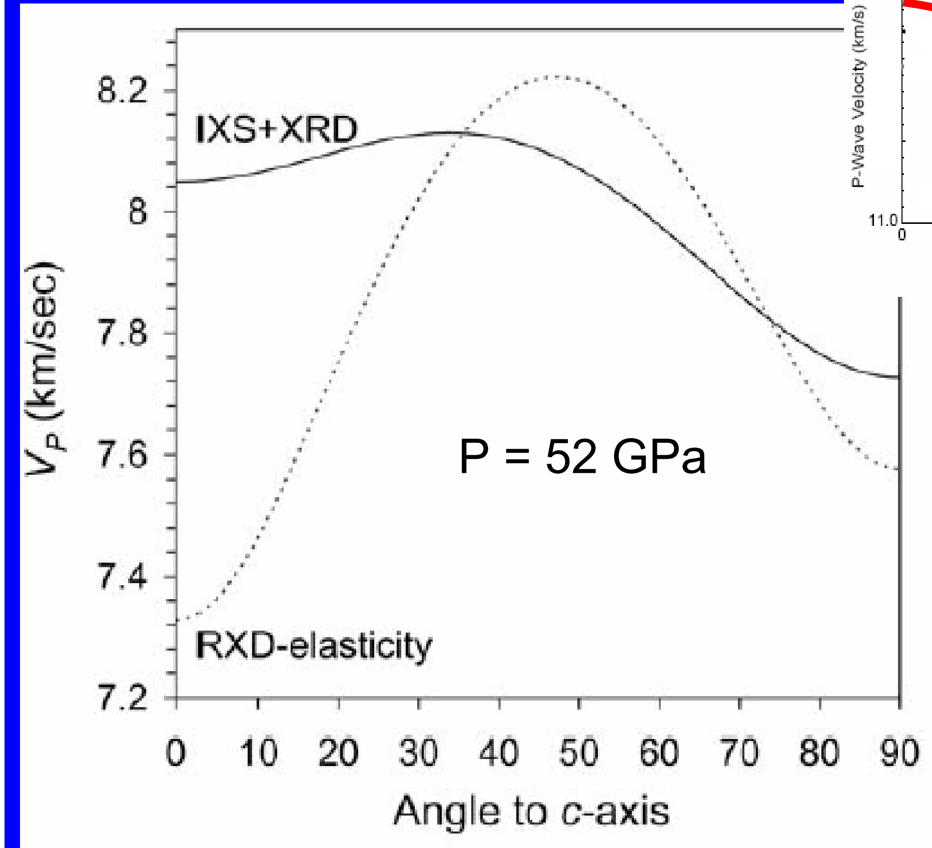
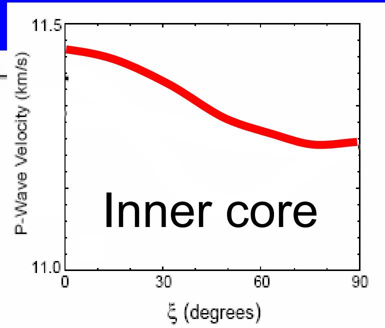
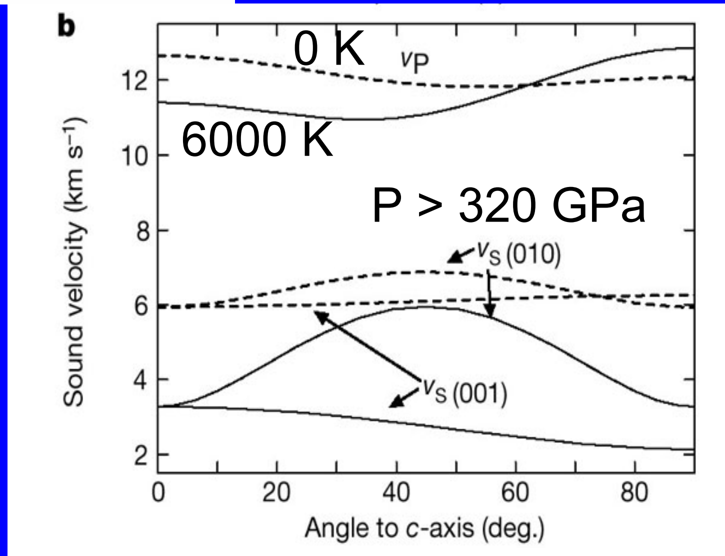
Elasticity of Fe in the inner core

Single-crystal anisotropy of ε-Fe: a combined approach







Radial X-ray diffraction
Momentum resolved inelastic X-ray scattering
Nuclear resonant X-ray inelastic scattering

Ab initio

Mao et al. (2008)

Steinle-Neumann et al. (2001)

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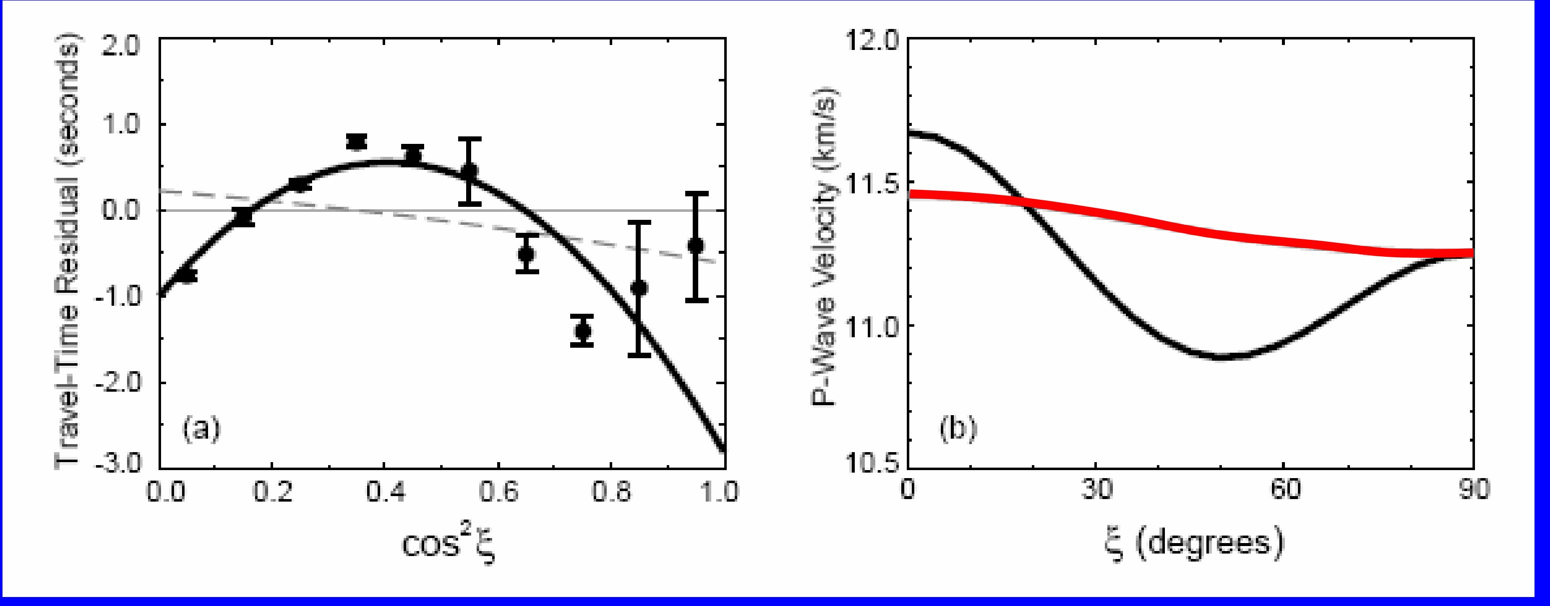
Elasticity of the inner core

More complex picture from seismology



from Ishii and Dziewonski (2003)

New perspectives
(my personal view)

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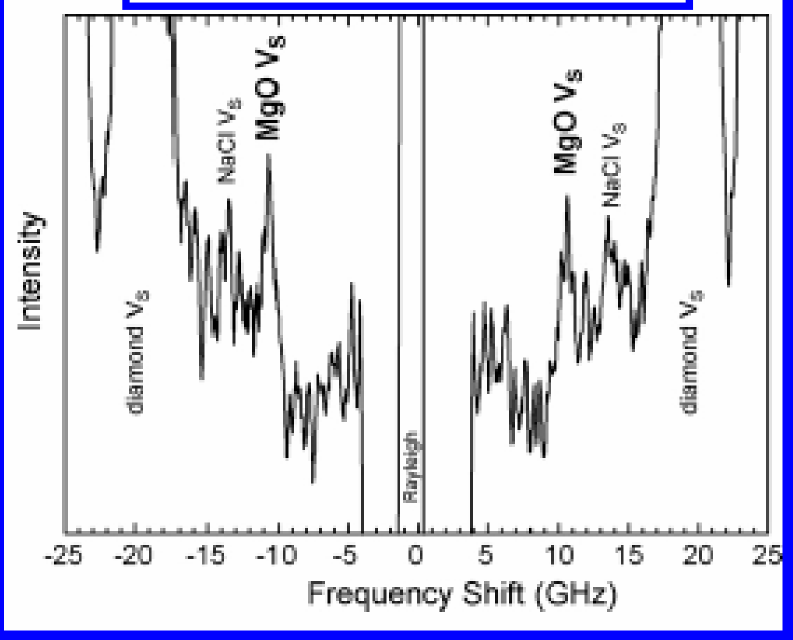
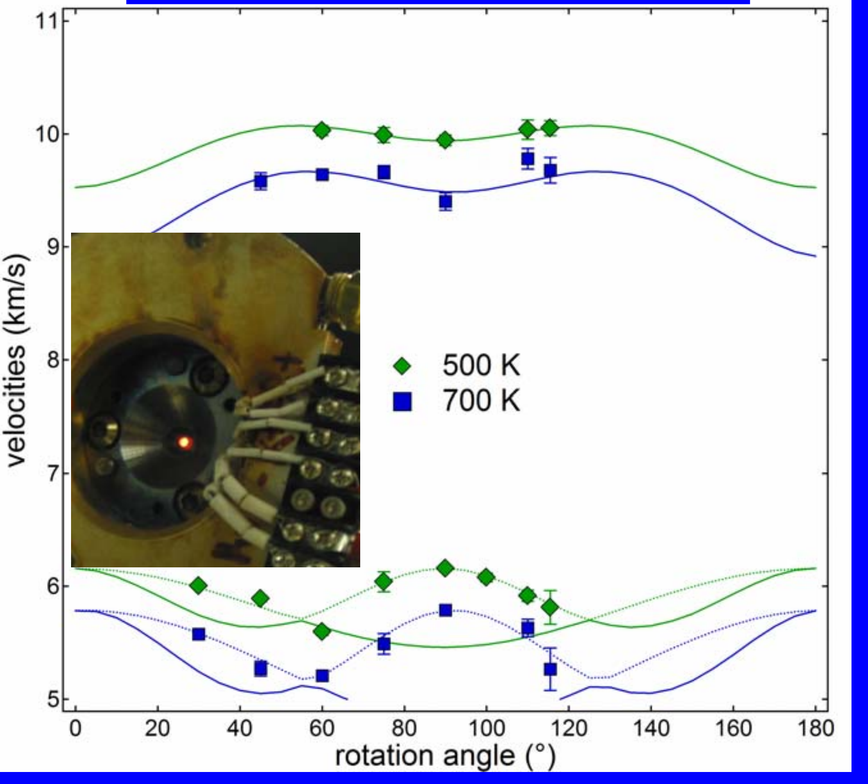
Experiments at simultaneous high-pressure and high-temperature

P = 49 GPa; T = 2000 C



$(Mg_{0.9}Fe_{0.1})O$ at 12 GPa



from Murakami et al. (2009)

Marquardt et al. (unpubl.)

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A enormous “thank you” to Hauke (my first student and good friend)