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From Core to Crust: Towards an Integrated Vision of Earth's Interior

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Some recent advances in dynamo and geodynamo theory

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Some recent advances in dynamo and geodynamo theory

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Trieste. July 21, 2009

Dynamical processes in the Earth's core

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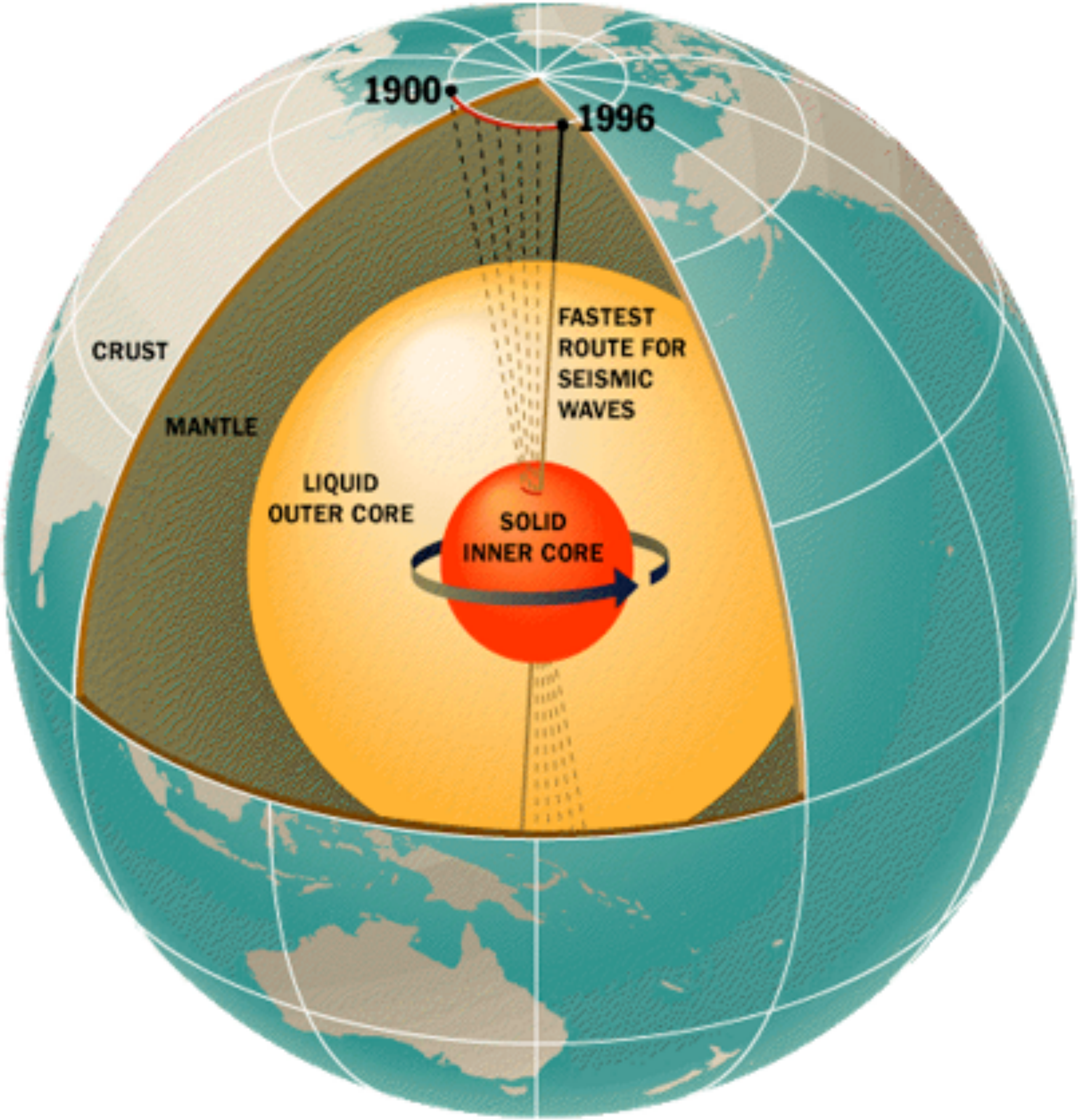
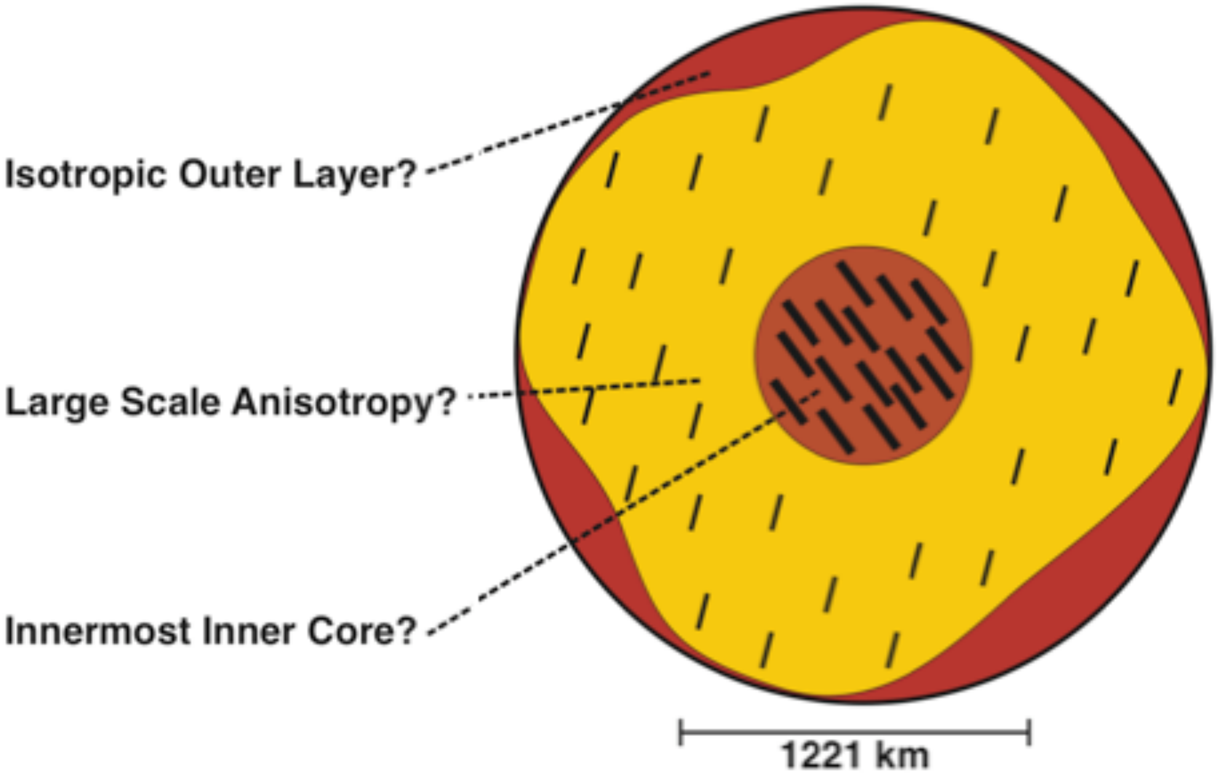

1900
1996
FASTEST
ROUTE FOR
SEISMIC
WAVES
CRUST
MANTLE
LIQUID
OUTER CORE
SOLID
INNER CORE



Cartoon of inner core (as seen today)

Innermost inner core first proposed by Miaki Ishii & Adam Dziewonsky; seems to be in increasing favor.

I wish to focus on "isotropic" outer layer.

(Cartoon courtesy of John Hernlund)

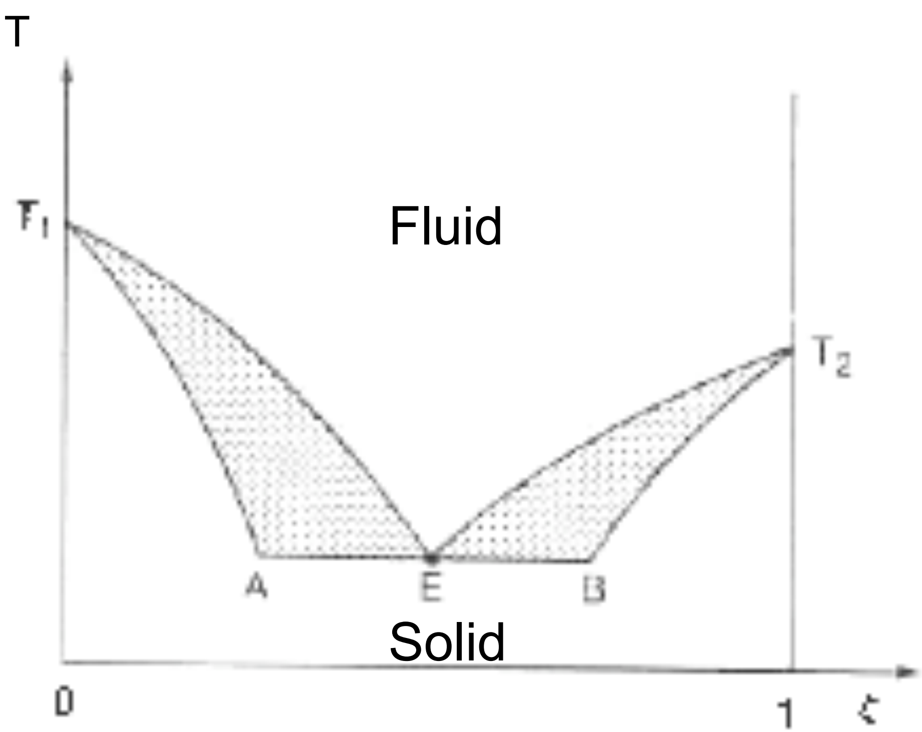
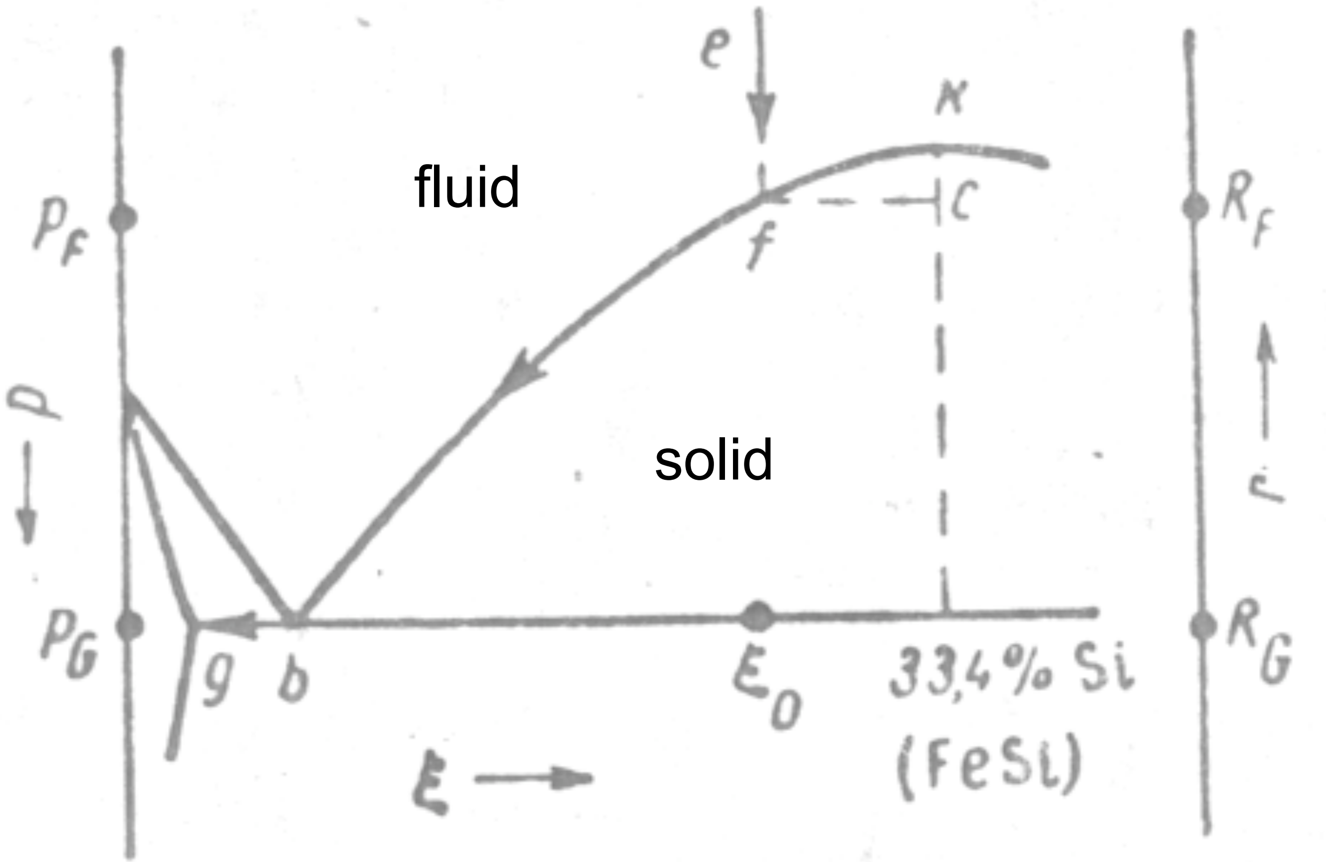


Fig. 1. A phase diagram for a binary alloy having liquidus T_1ET_2 and solidus T_1A and BT_2. The stippled region shows states where solid and liquid phase can co-exist in thermal equilibrium. The figure really shows a constant p cross-section of three dimensional surfaces in pTξ-space where ξ is the mass fraction of one constituent. Thus for example, the eutectic point, E, is but one point on a eutectic line.

Phase diagram for laboratory work

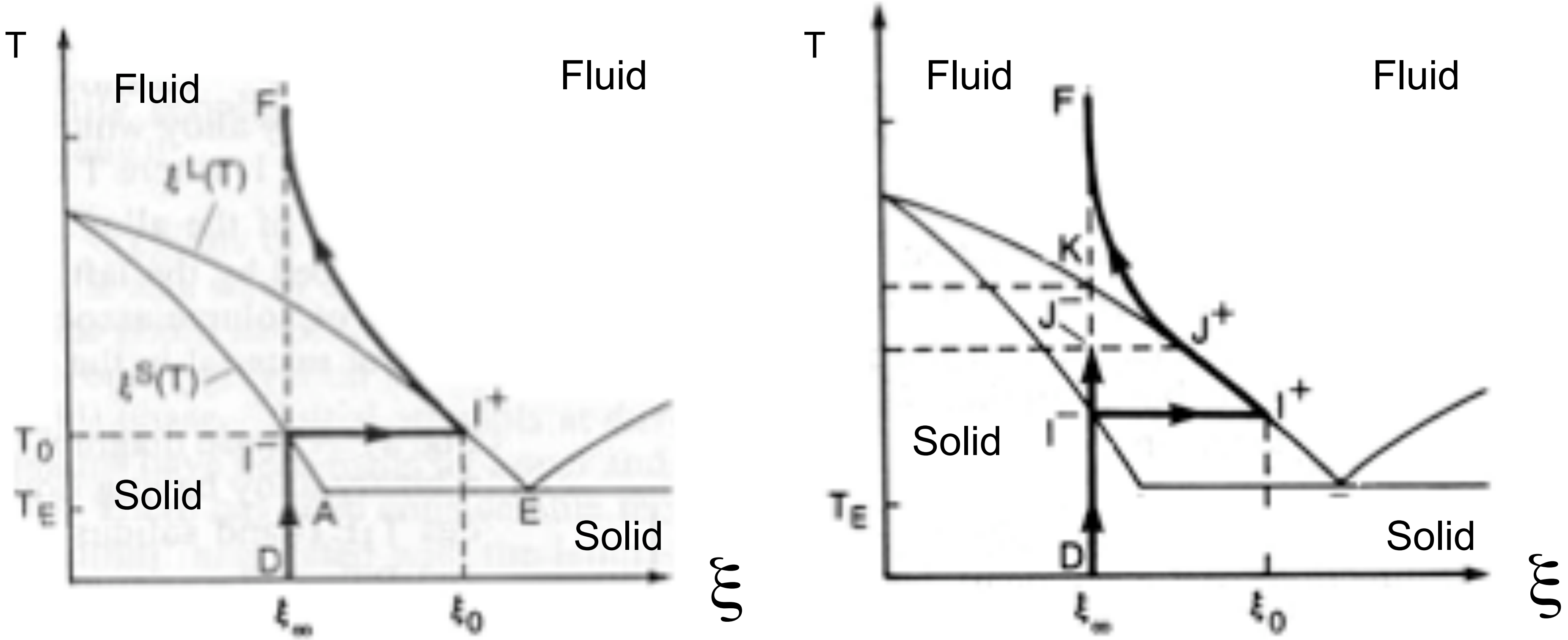
This “butterfly diagram” is a section of a ξ(T,p) surface



Braginsky’s 1963 picture

Note: unlike the phase diagrams for laboratory experiments, this is a ξp plot, not a ξT plot

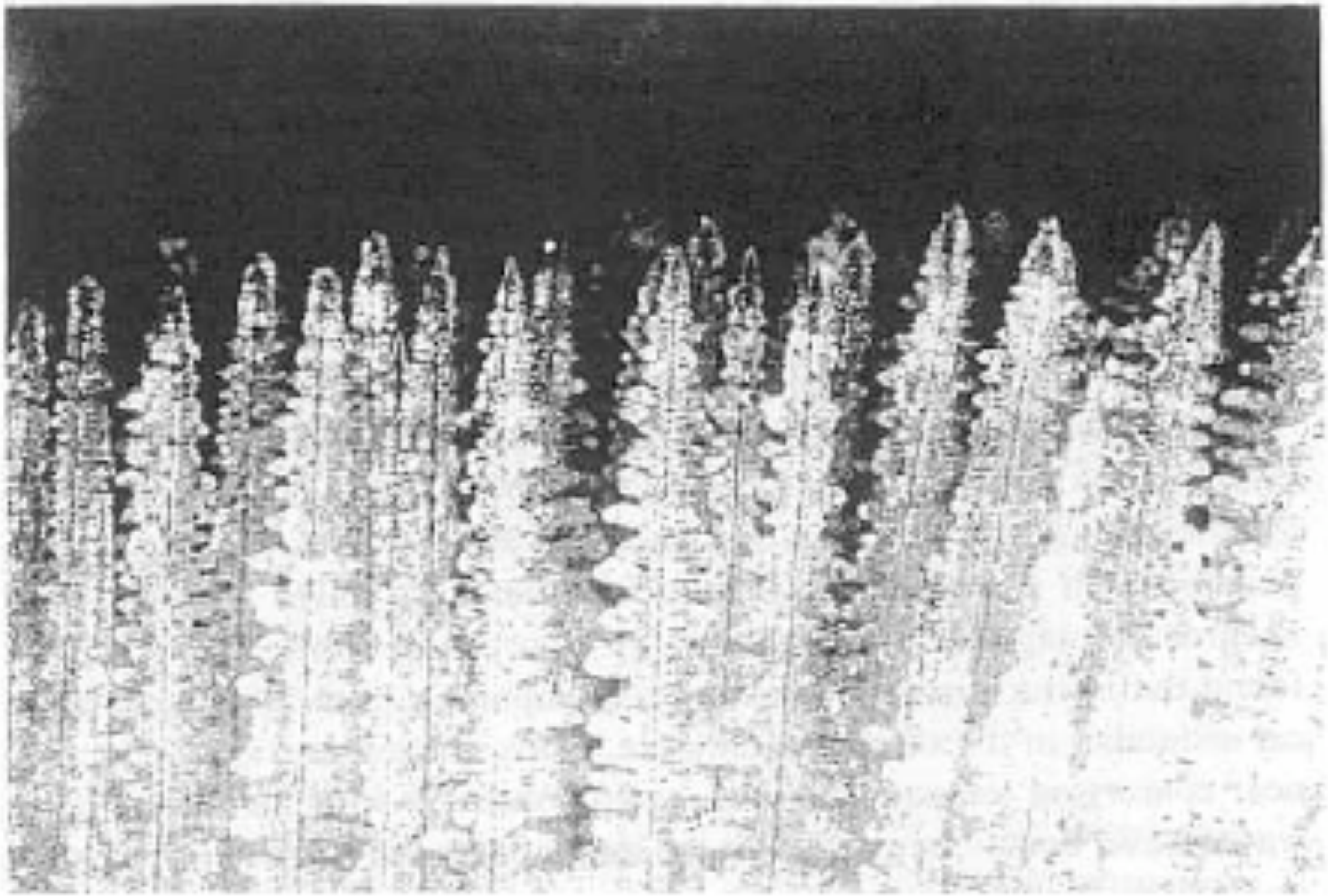
Note also that core composition ξ_0 is taken to be to the right of eutectic (b).



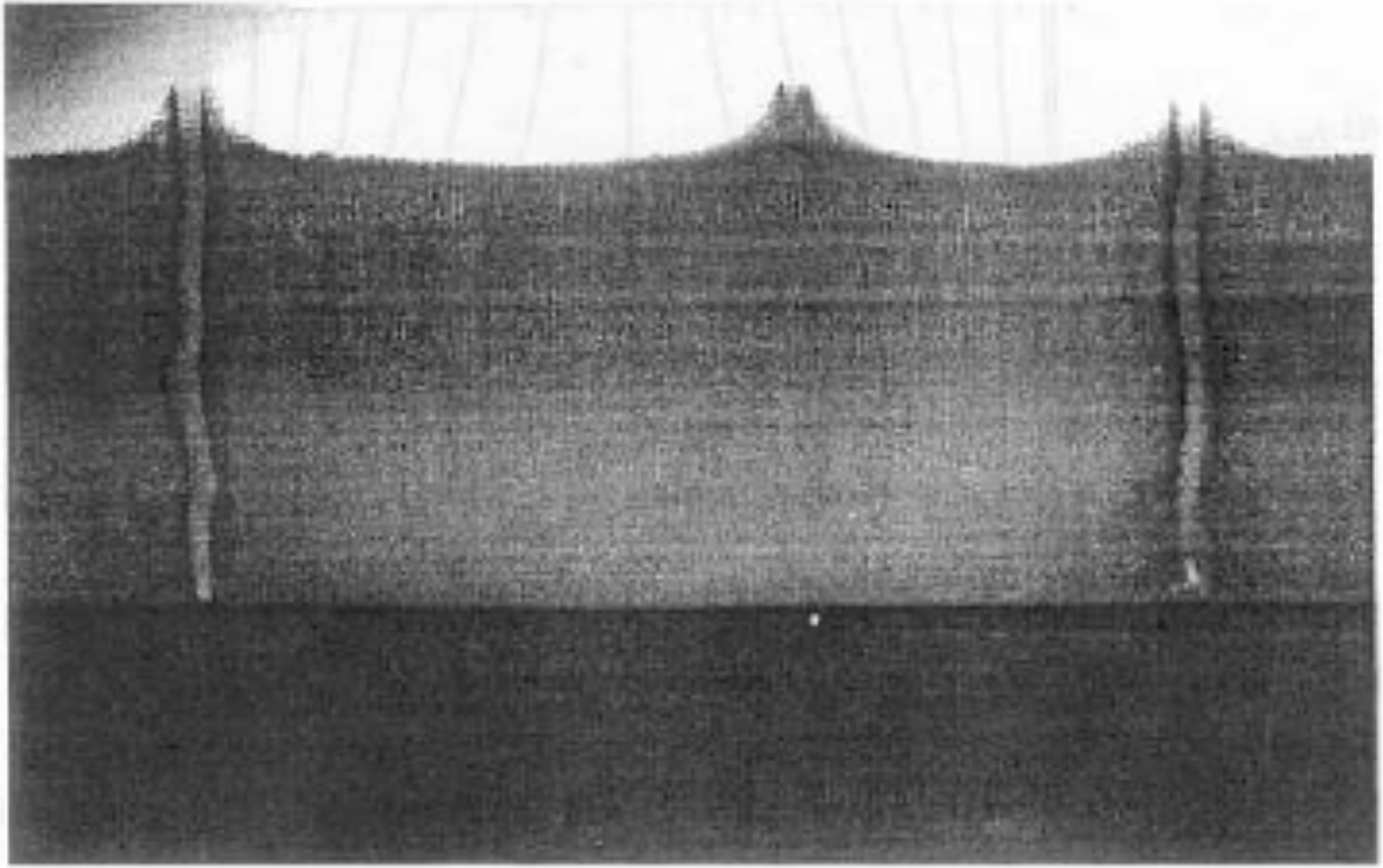
What happens when the alloy is moved through a chill.

Left: slow cooling.

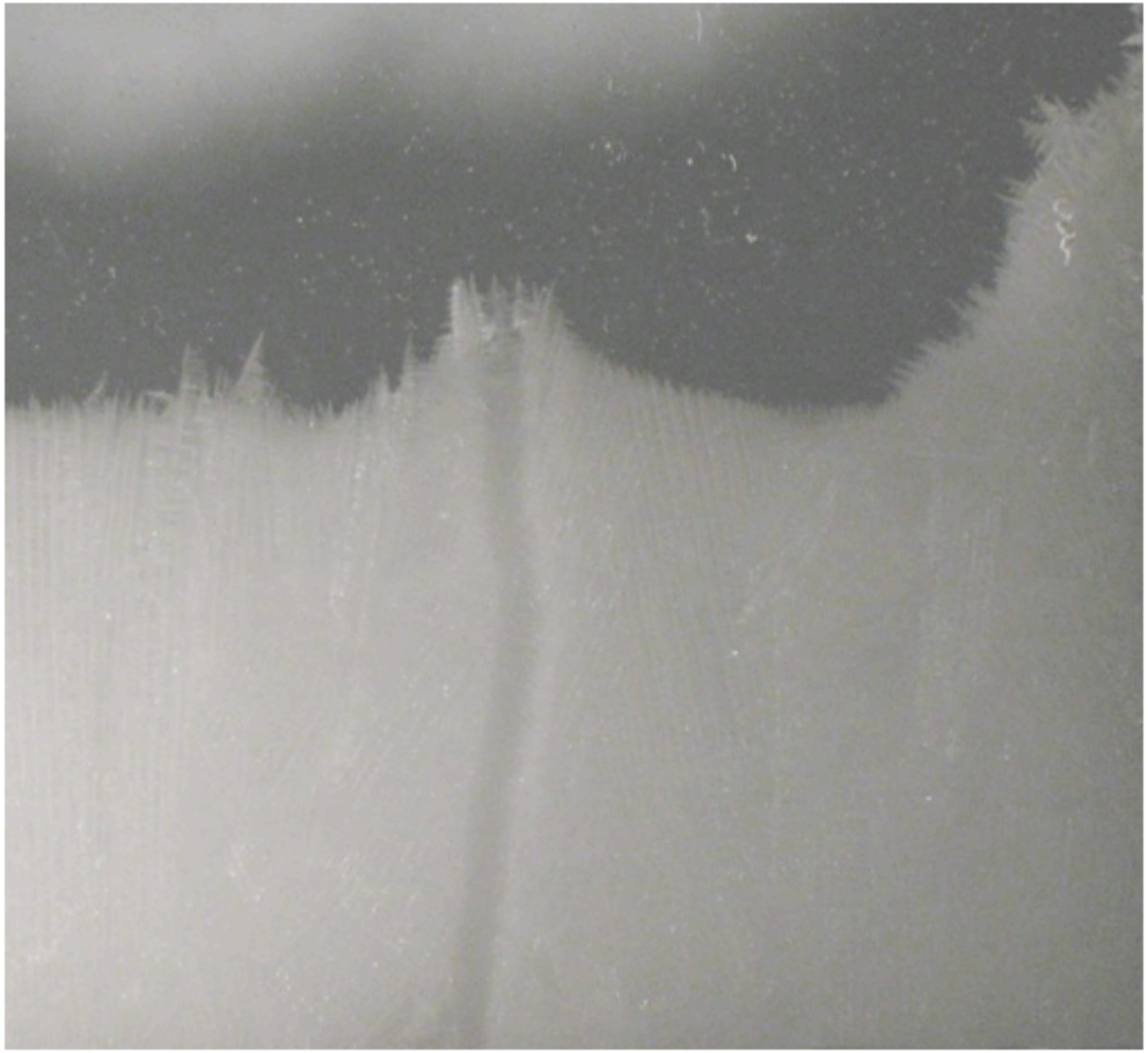
Right: fast cooling, constitutional supercooling, formation of a mush.



Dendrite tips at the top of the mushy layer



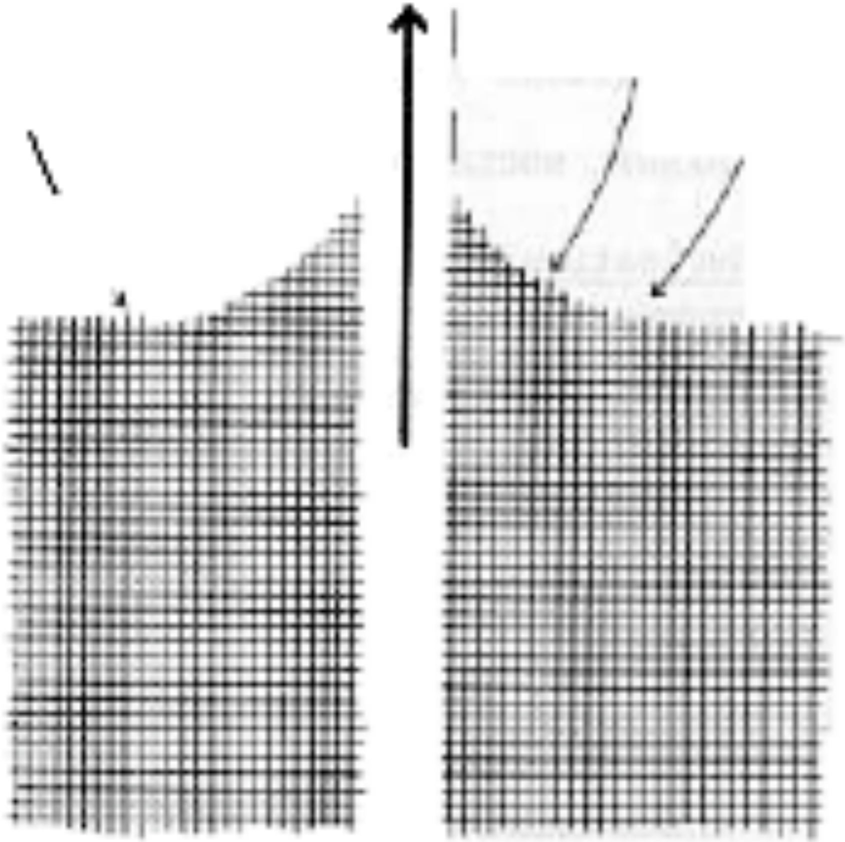
Chimneys (channels) within mushy layer



A chimney

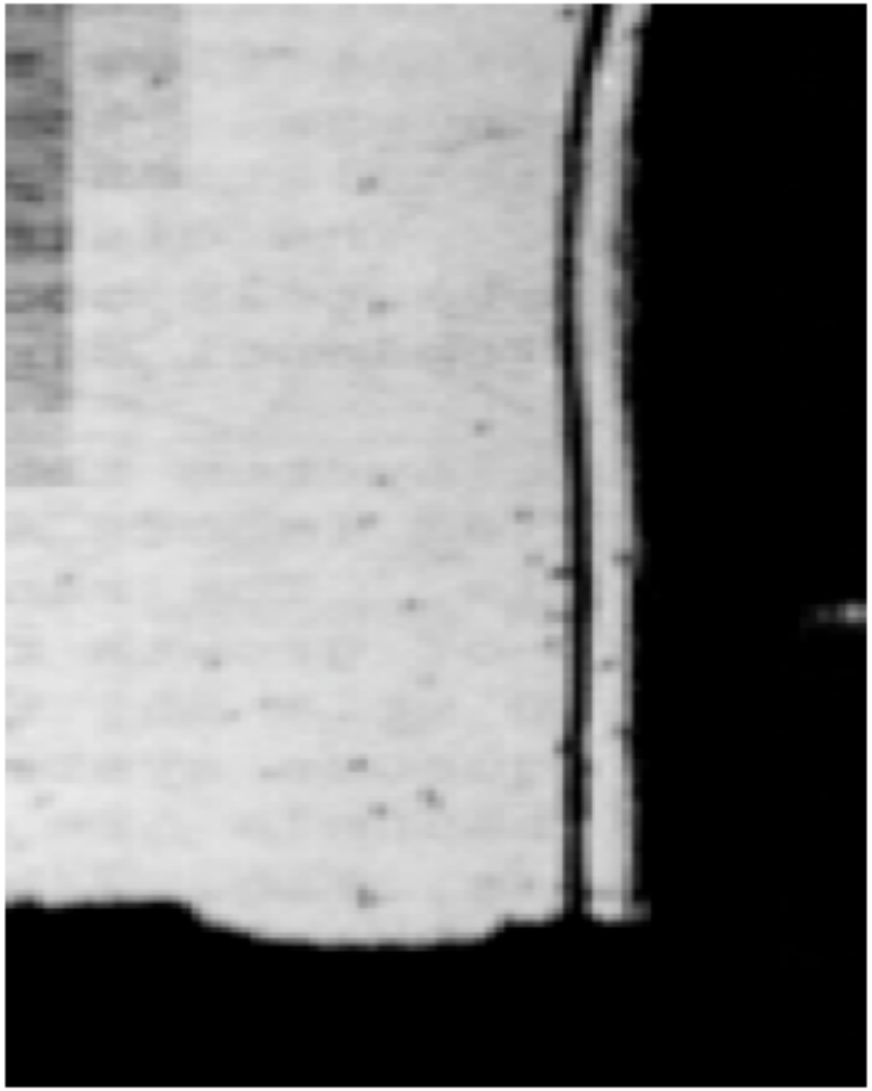
From experiment of Nathanael Machicoane, Gilles Montagnac and Stephane Labrosse

Photograph by John Hernlund

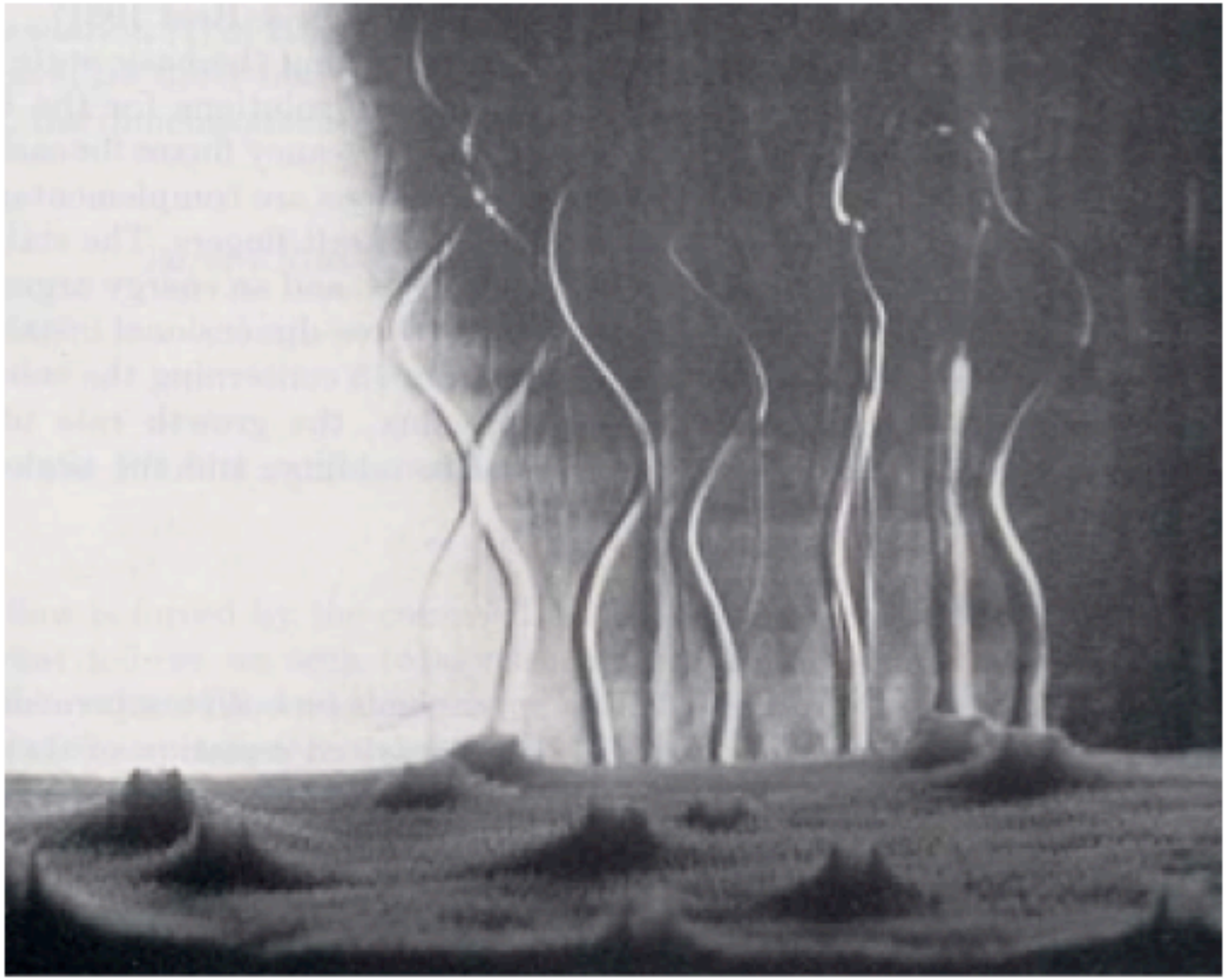


Modeling a chimney

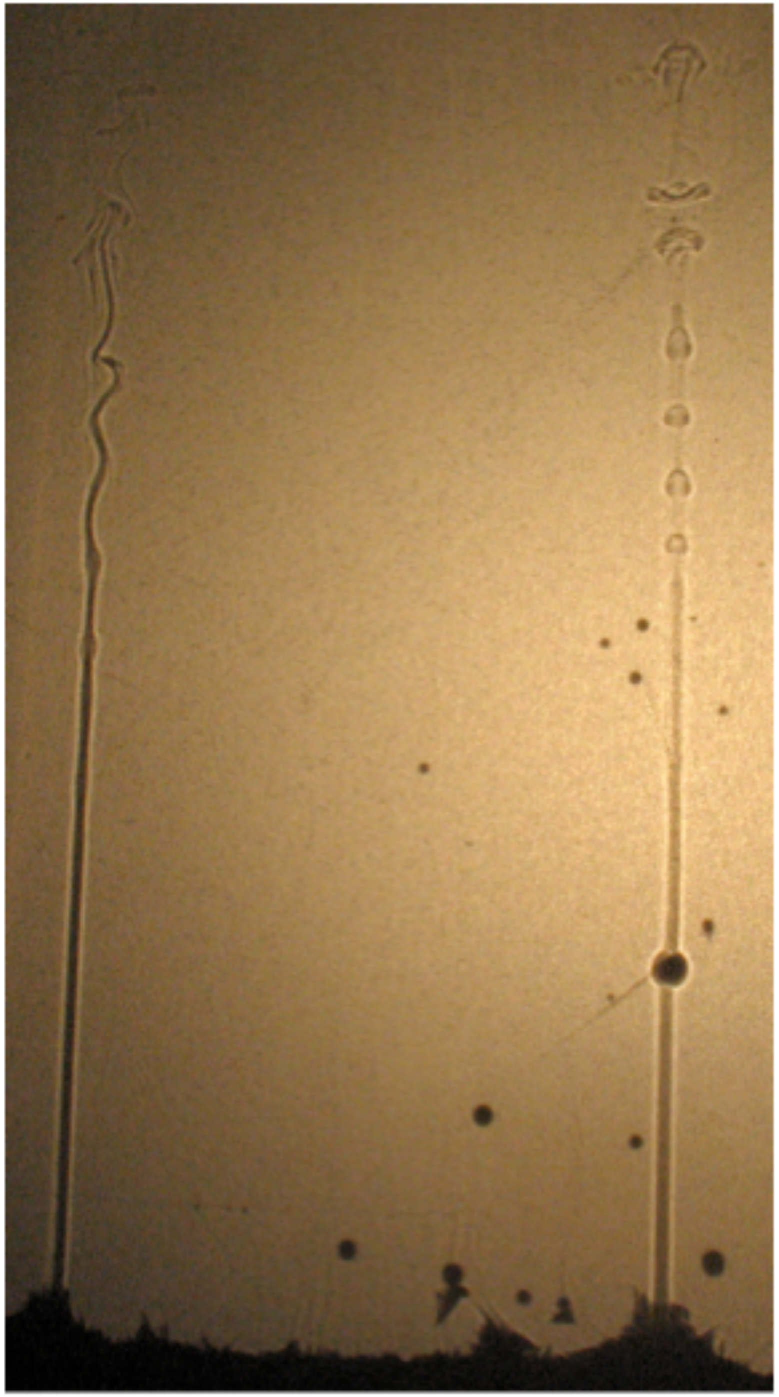
Cf. Loper & Roberts, *Studies in Appl Math.* (2001)



A plume rising from a chimney



Many plumes rising from a mushy layer
(Eltayeb & Loper, JFM, 1991)



Two plumes from chimneys

Note break up into blobs

Note also solitary wave

(From experiment of Nathanael Machicoane, Gilles Montagnac and Stephane Labrosse Photograph by John Hernlund}

The cooling Earth

The Earth is cooling by $\sim$ 10K/1Gyr. The core is cooling so fast that it is vigorously convecting and therefore is in a nearly homogeneous and homentropic state. Today's prevailing opinion: the core was entirely fluid early in its history but about 1Gyr ago it started to freeze *at the bottom*. (This is because the melting point gradient exceeds the adiabatic gradient.) The freezing front is the Inner Core Boundary (ICB). It is currently advancing at $\sim$1mm/year as it freezes.

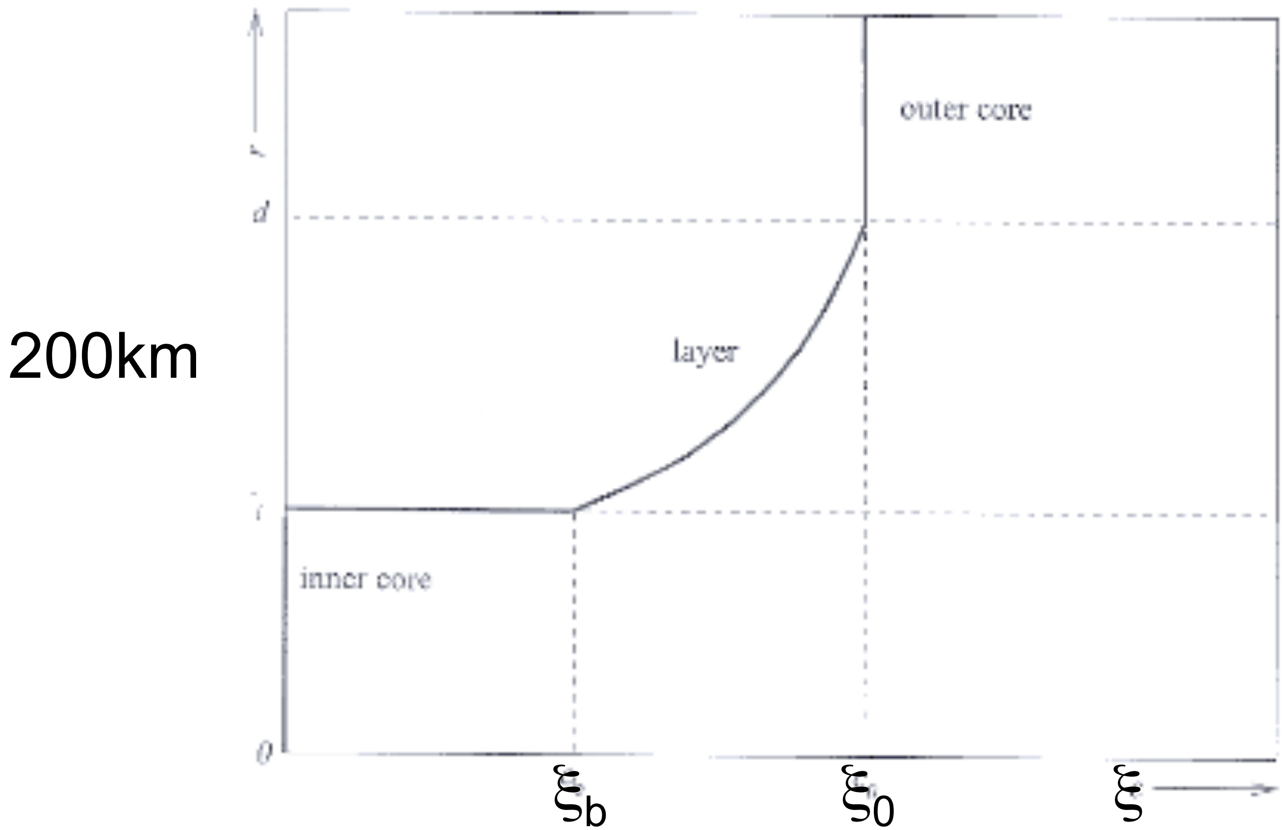
Question: *Is this fast enough for constitutional supercooling and the formation of a mush at the ICB?* Answer (Loper & Roberts, 1981): **Yes**. Since then seismological evidence has lent support.

Deviations from classical picture

Earth models such as PREM (the *Preliminary Reference Earth Model* of Anderson and Dziewonski) rely on homogeneity of ξ and S, but seismology is beginning to detect small deviations from homogeneity of ξ and S both at the *top* and at the *bottom* of the core. In addition to inhomogeneity of the inner core (e.g., the innermost core):

- Souriau & Poupinet (1991) argue for a layer of inhomogeneity $\sim$150km thick at the *bottom* of the fluid core. Other recent findings are tending to confirm this. Gubbins *et al.*(2008) have a model in which the layer is on the liquidus, not on an adiabat.

- Braginsky (1984,...,2006) proposed, from study of the geomagnetic secular variation, that there is a "hidden (inverted) ocean" at the *top* of the core. $\sim$10km thick. Recent high pressure experiments support this idea (Ozawa *et al.*, 2009).



Layer at bottom of fluid core

Gubbins, Masters and Nimmo, GJI (2008)

Theory of geomagnetism (background)

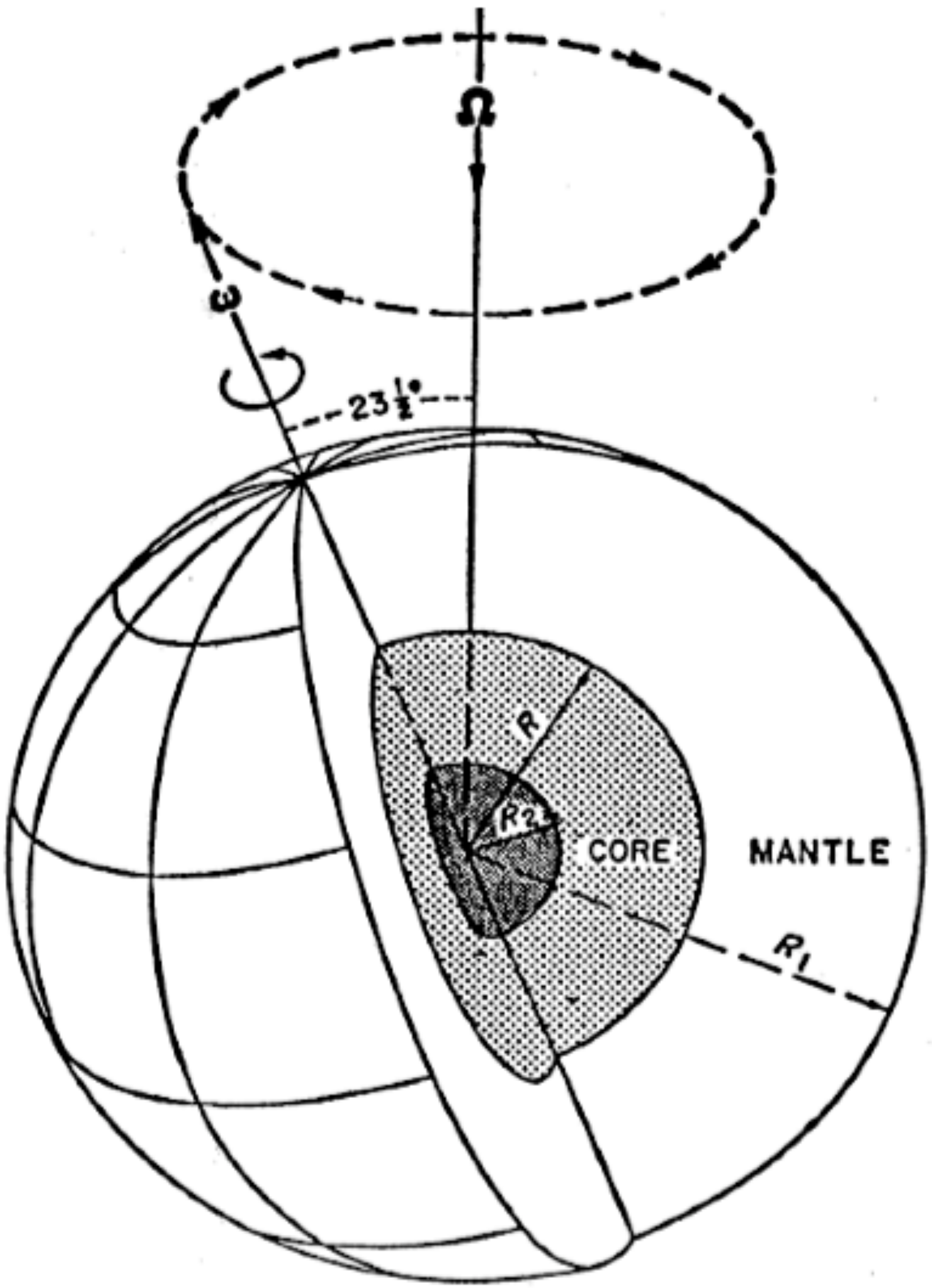
The *geomagnetic field* is created by electric currents flowing in the Earth's core, which is thought to be an iron alloy, that is a moderately good electrical conductor. The electric currents are created by motions of this conducting medium, as in a self-excited dynamo. This dynamo is called the *geodynamo*. The motions of the fluid are driven either by

I. thermal and compositional buoyancy, or by

II. the luni-solar precession, or by both.

Geodynamo theory is therefore a branch of *magnetohydrodynamics* (MHD) or, in case I, of *magnetoconvection* theory.

The favored theory is I, but II has never been properly explored.



Ω
ω
23½°
R
R2
CORE
MANTLE
R1

I. Buoyancy-driven geodynamos

All current theory starts with a *reference state* of uniform composition ξ_a and uniform specific entropy S_a. This implies that an adiabatic gradient dT_a/dr of about -0.5^oK/km at the core mantle boundary (CMB) diminishing with depth.

Convection is driven by deviations, ξ_c and S_c from the adiabatic state which create the buoyancy force per unit mass of

$$C\rho\mathbf{g}, \quad \text{where} \quad C = -\alpha^S S_c - \alpha^\xi \xi_c = \text{codensity}.$$

where

$$\alpha^S = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial S}\right)_{P,\xi} = \frac{\alpha T}{C_p} = \text{entropic expansion coefficient}$$
$$\alpha^\xi = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial \xi}\right)_{P,S} = \text{compositional expansion coefficient}$$

The amount of radioactivity (K^{40}?) in the core is unknown but usually assumed negligible compared with latent heat release through freezing at the inner core boundary (ICB).

I: Buoyancy-driven geodynamos

Solve magntoconvection equations, i.e., MHD equations with an added buoyancy force, $\rho C\mathbf{g}$. Also Coriolis force, $-2\rho\boldsymbol{\omega}\times\mathbf{u}$, must be included. Seek three-dimensional solutions.

Main numerical difficulties: computationally

$$E \equiv \frac{\nu}{2\Omega c^2} \gtrsim 10^{-7} \quad \text{but} \quad E \quad \text{for core} \sim 10^{-15}$$

$$P_m \equiv \frac{\nu}{\eta} \gtrsim 0.1 \qquad \text{but } P_m \text{ for core} \sim 10^{-6}$$

$$Ra \equiv \frac{g\alpha\beta c^4}{\nu\kappa} \lesssim 10^{11} \text{ but } Ra \text{ for core} > 10^{20}$$

Scaling laws (Christensen *et al.*). Other points:

- Early simulations (Glatzmaier & Roberts 1995, 1996) remarkably successful despite ν being too large
- Recent simulations (e.g., Kagayama *et al.*) with smaller ν less Earth-like. Perpelexing!
- Possible explanation (Sakuraba & Roberts, 2009?): see below

The role of the Earth's mantle

Typical flow velocities in core and mantle are

$$u_{\text{core}} \approx 10^{-4}\text{m/s}\,, \qquad u_{\text{mantle}} \approx 10^{-10}\text{m/s}\,.$$

This wide disparity makes it impractical to treat core and mantle as one dynamical system, but allows an excellent approximate way of treating them separately:

The temperature, T_{cmb}, of the core-mantle boundary (CMB) provides the boundary condition for studying mantle convection, and the resulting outward heat flux, q_{cmb}, provides the boundary condition for studying core convection.

The mantle is the "valve" that controls core dynamics.

Most simulators have assigned a uniform T_{cmb} in studying core convection, e.g., Kageyama *et al.*(2008); Takahashi *et al.*(2008). Ataru and I believe that a uniform q_{cmb} is more realistic and that the resulting differences in behavior are substantial for small E.

Question: Does the thermal boundary condition on the CMB make much of a difference?

To seek an answer, consider 2 models:

USTM = Uniform Surface Temperature Model
(i.e., $T_{\rm cmb}$ independent of θ and ϕ)

UHFM = Uniform Heat Flux Model
(i.e., $q_{\rm cmb}$ independent of θ and ϕ)

Aim: To compare the USTM with the UHFM.
(The USTM is very like Kageyama *et al.*'s model.)

The models are thermally driven using Boussinesq approximation.

$$E=5\times10^{-7}, \quad p_m=\frac{\nu}{\eta}=0.2, \quad Ra=\frac{g\alpha\beta c^4}{\nu\kappa}=3.2\times10^{10}$$

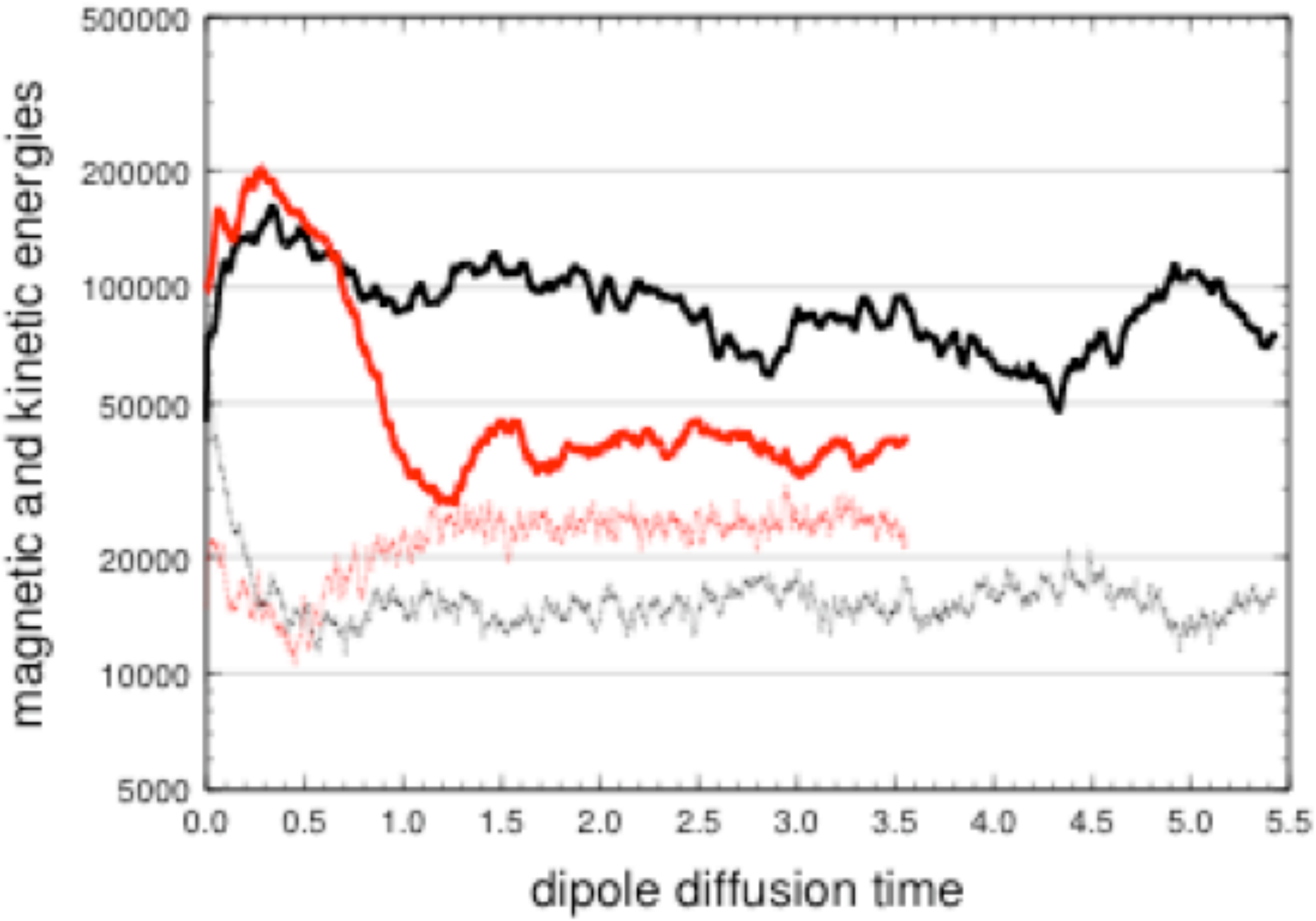
They are in every respect identical except for the thermal boundary condition at $r=c$.

Character of UHFM

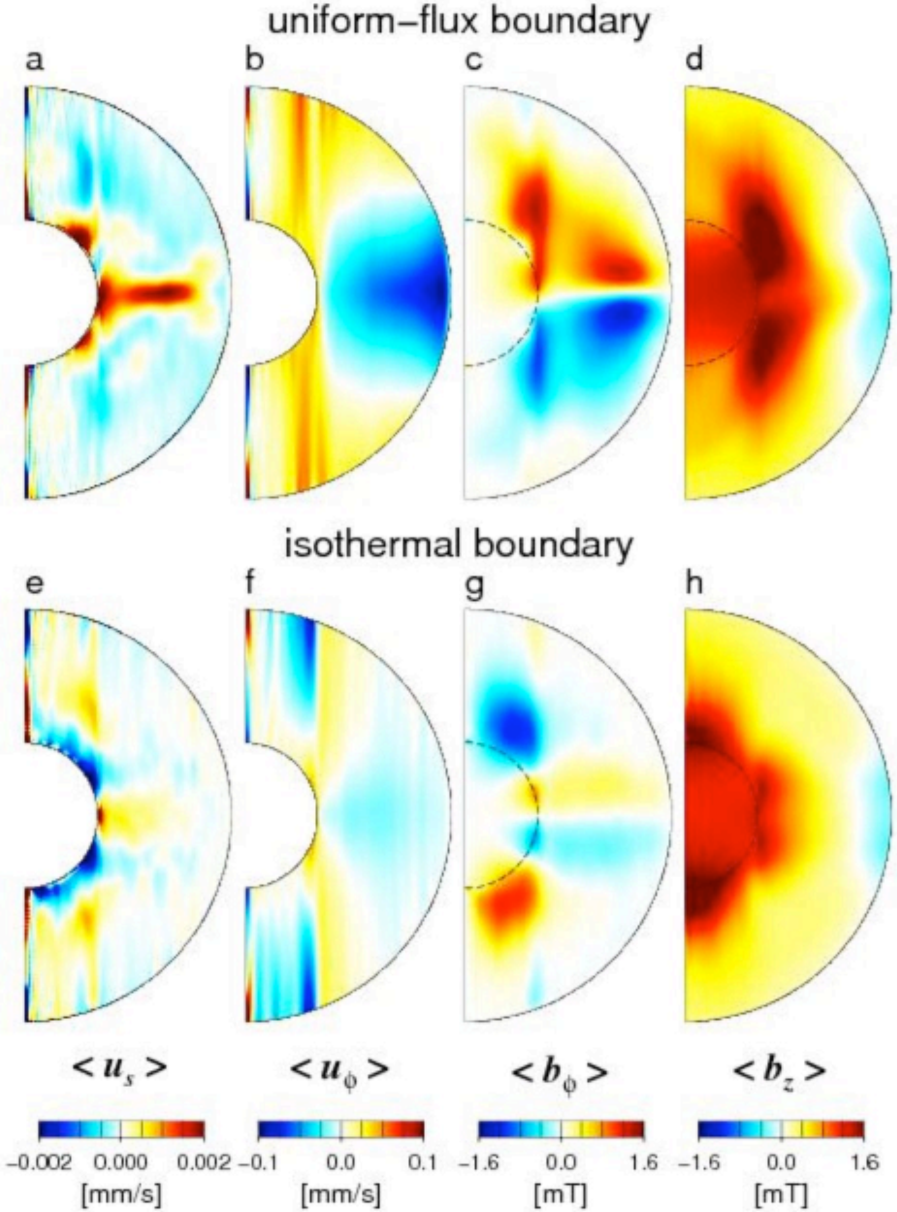
- $E_m/E_k = 5.2$, i.e., large (but not as large as for Earth)
- **b** is strongly dipolar
- Several flux patches at low and mid latitudes, caused by eruption of strong zonal subsurface field
- Westward drift (a little weaker than for Earth)
- Toroidal field and velocity show $m \approx 6$ periodicity in ϕ
- Velocity field rather 2D (as in PT theorem)
- approximately an $\alpha\omega-$dynamo

Character of USTM

- $E_m/E_k < 2$
- **b** is dipolar, though not as strongly dipolar as the UHFM
- Virtually no flux patches
- Westward drift virtually nonexistent
- Small scale sheel-like flow and field structures of high wave number, as found by Kageyama *et al.*(2008)
- No large-scale flow and field structure.
- Velocity field rather 2D (as in PT theorem)
- Inertial forces significant in dynamical balance



magnetic and kinetic energies
500000
200000
100000
50000
20000
10000
5000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
dipole diffusion time



uniform−flux boundary
a
b
c
d
isothermal boundary
e
f
g
h
< u_s >
< u_φ >
< b_φ >
< b_z >
−0.002 0.000 0.002
[mm/s]
−0.1 0.0 0.1
[mm/s]
−1.6 0.0 1.6
[mT]
−1.6 0.0 1.6
[mT]

Some interpretation

$$2\rho\Omega\langle u_s\rangle = \langle j_z b_s - j_s b_z\rangle$$

where $\langle Q\rangle$=average of Q over t and ϕ.
On $z = 0$ (for both models)

$$2\rho\Omega\langle u_s\rangle_E \approx -\langle j_s\rangle_E\langle b_z\rangle_E$$

Some interpretation (continued)

On $z = 0$ (for both models)

$$2\rho\Omega\langle u_s\rangle_E \approx -\langle j_s\rangle_E\langle b_z\rangle_E \, .$$

In the UHFM there is a significant $\langle u_s\rangle_E$, but not in the USTM.

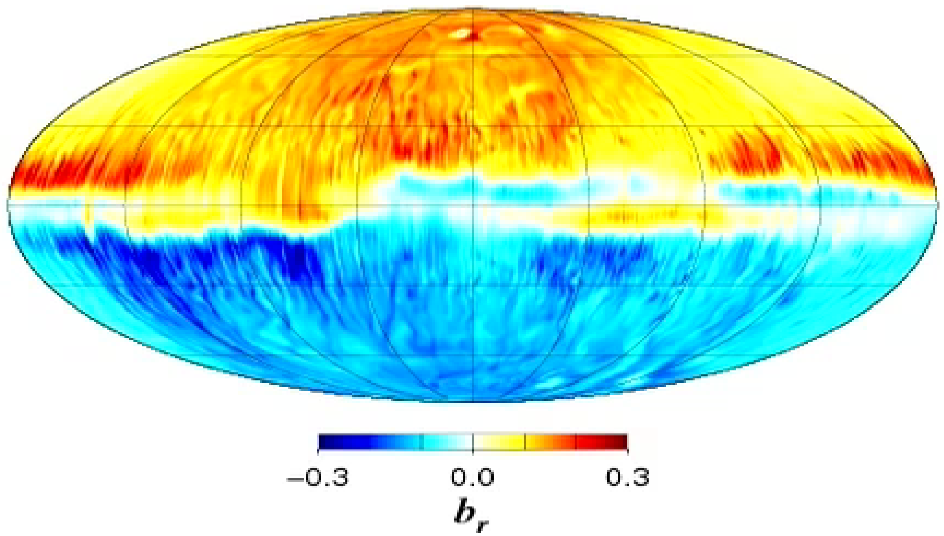
Therefore in the USTM, either $\langle j_s\rangle_E$ or $\langle b_z\rangle_E$ are small; in fact, both are small, but in the UHFM both are significant.

The larger $\langle j_s\rangle_E$ creates the larger b_ϕ on each side of $z = 0$, leading to the flux patches of the UHFM.

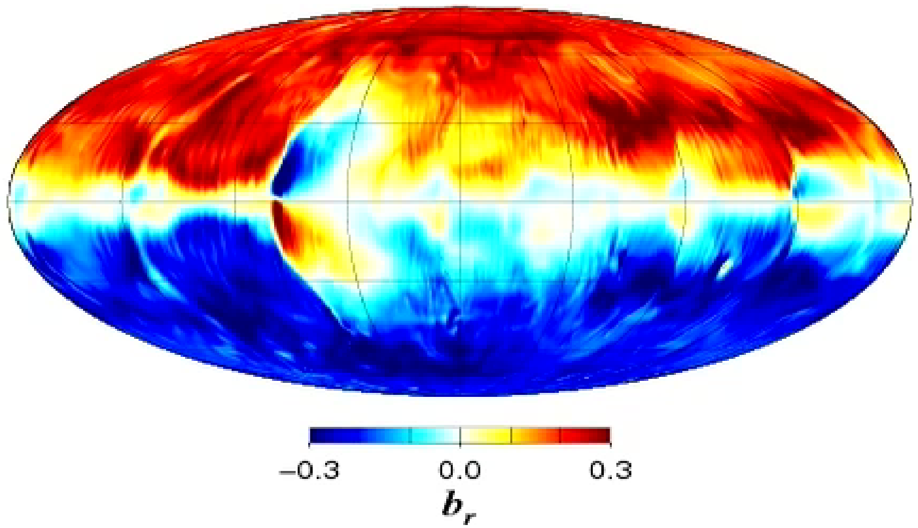
The larger $\langle b_z\rangle_E$ leads to the greater dipole moment of the UHFM.

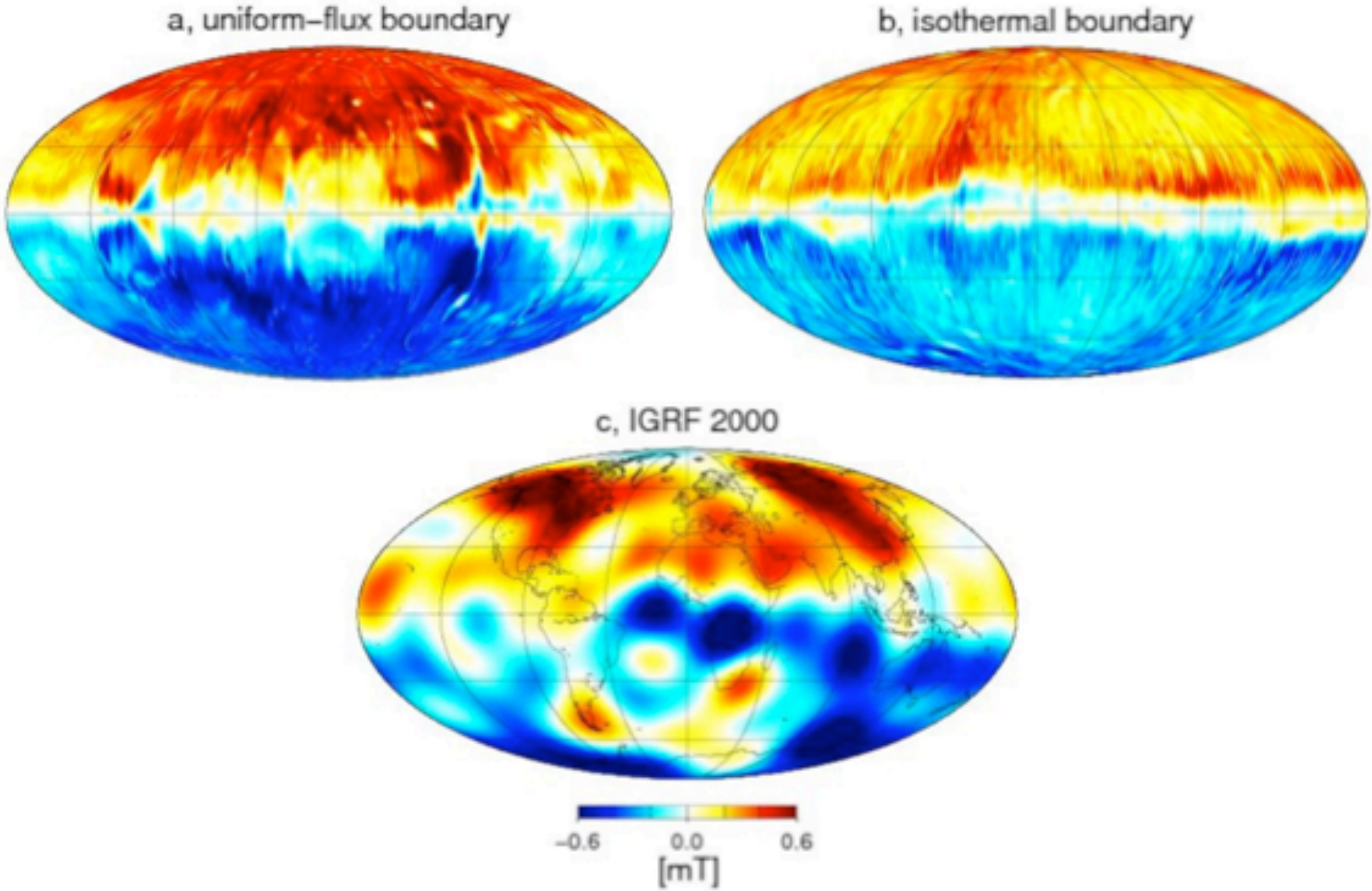
The larger $\langle u_s\rangle_E$ of the UHFM demands a conterflow u_s at high latitudes and a significant meridional circulation, which the USTM lacks.

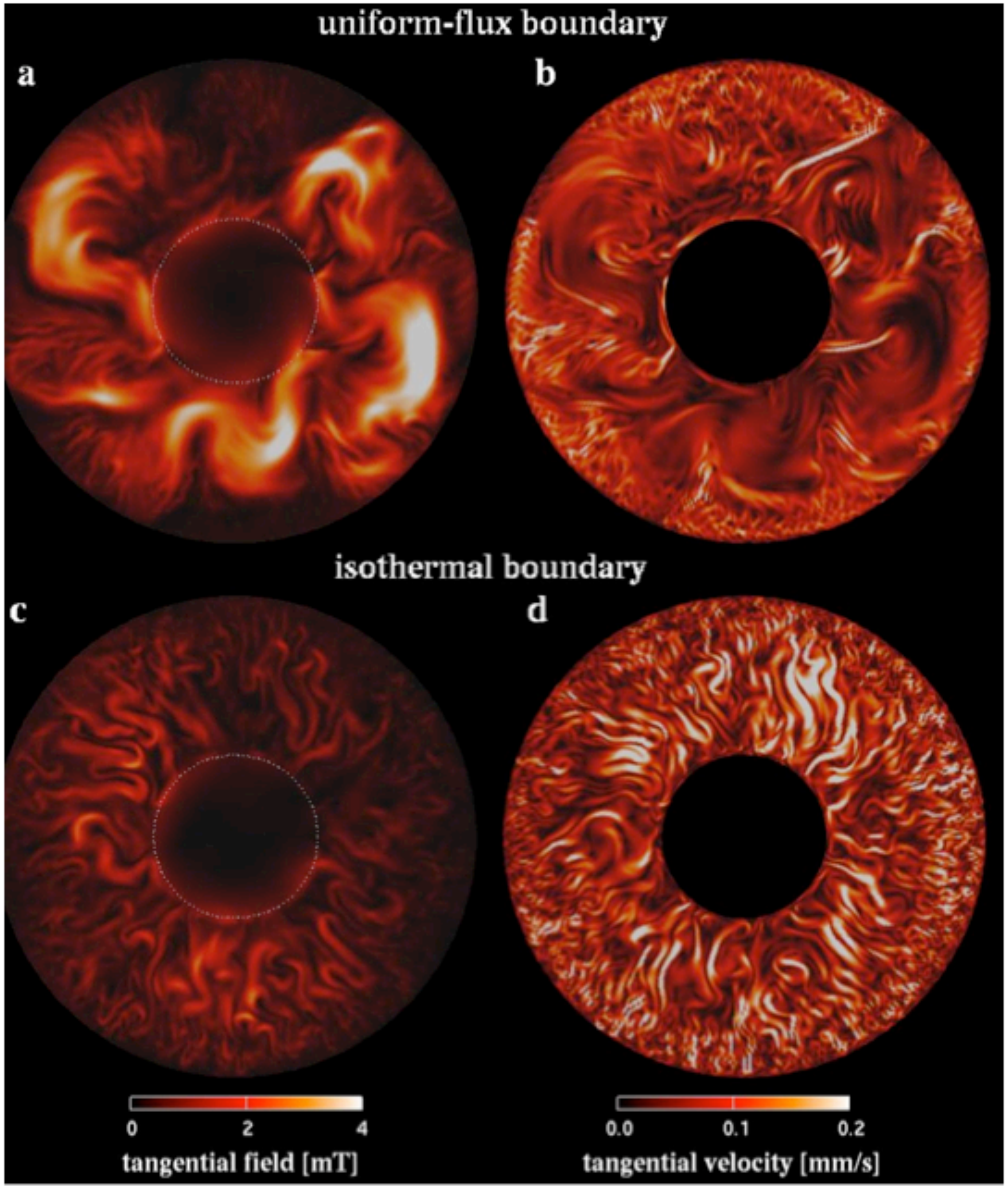
t = 0.19500 (00000 year)

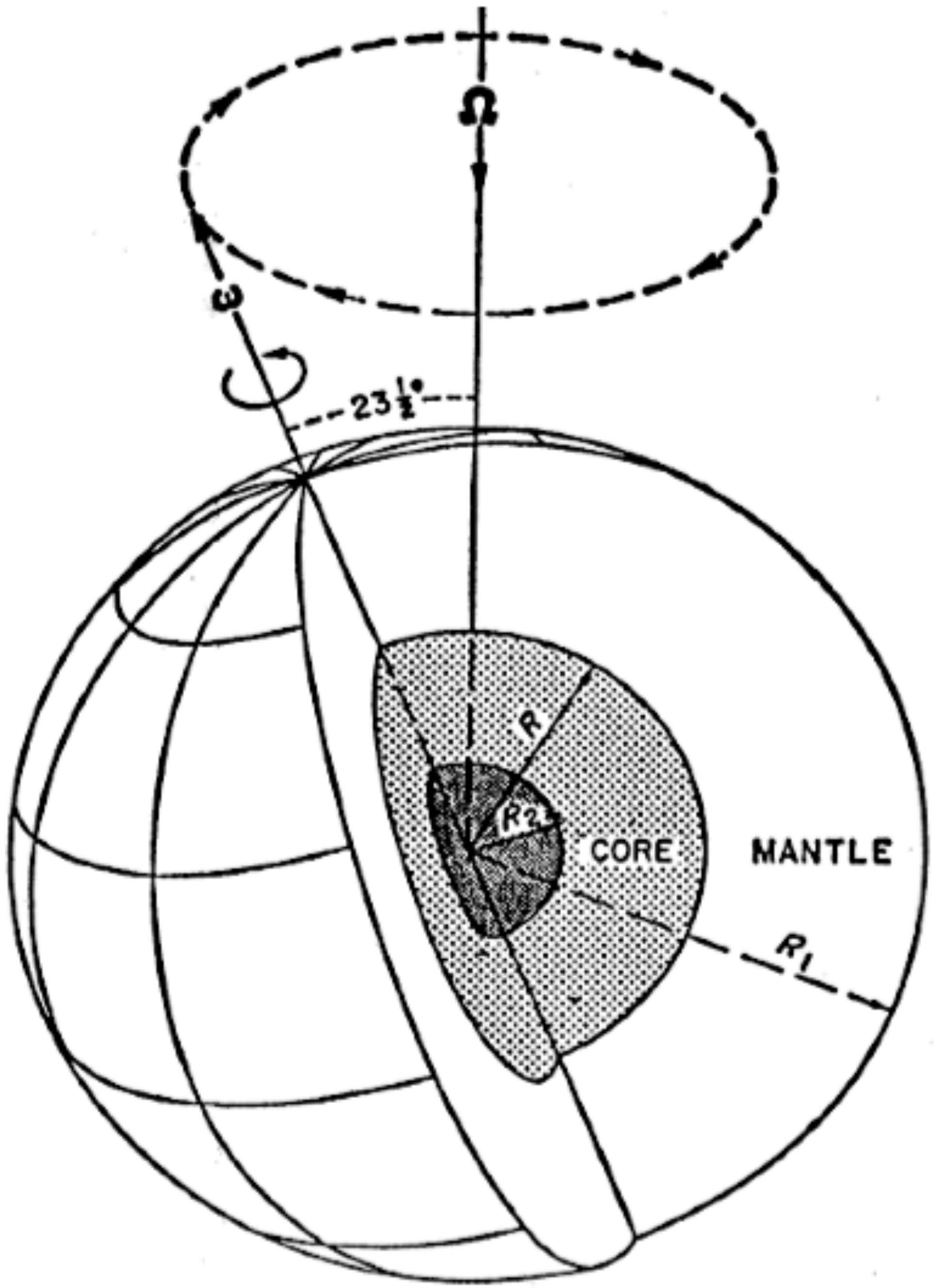


t = 0.48000 (00000 year)

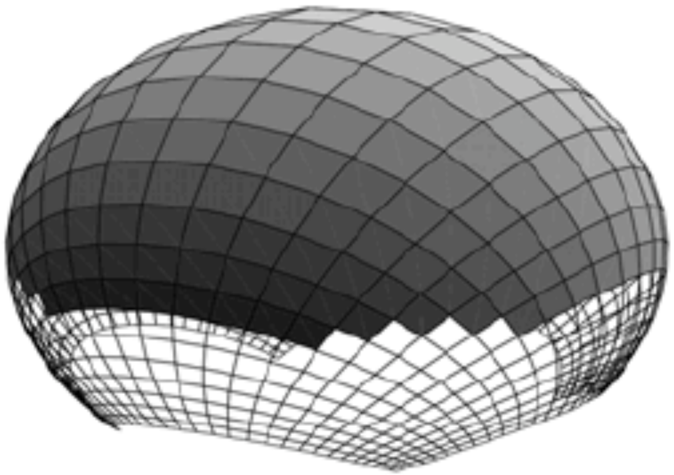
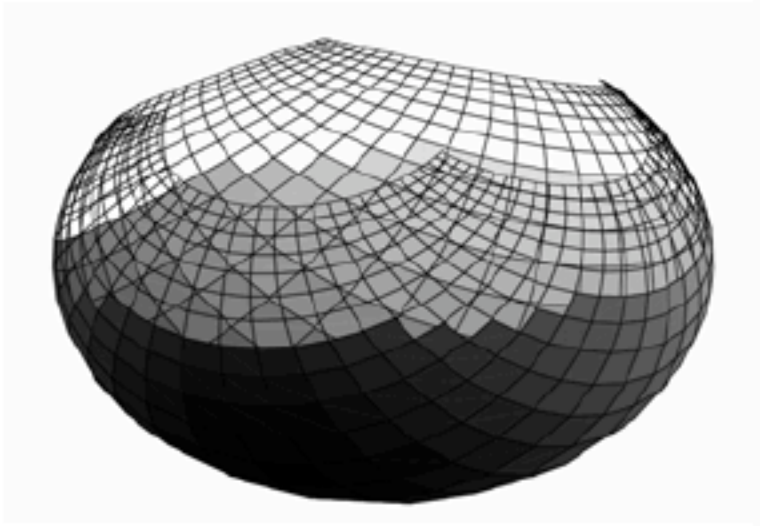
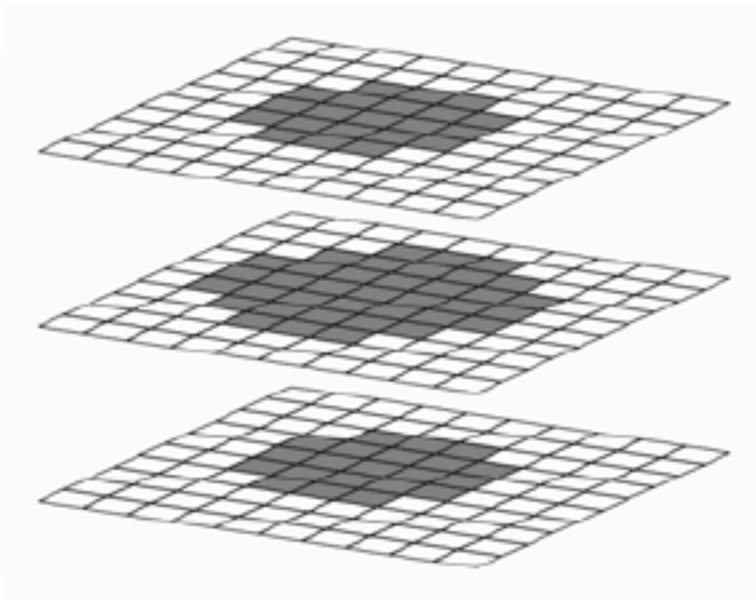


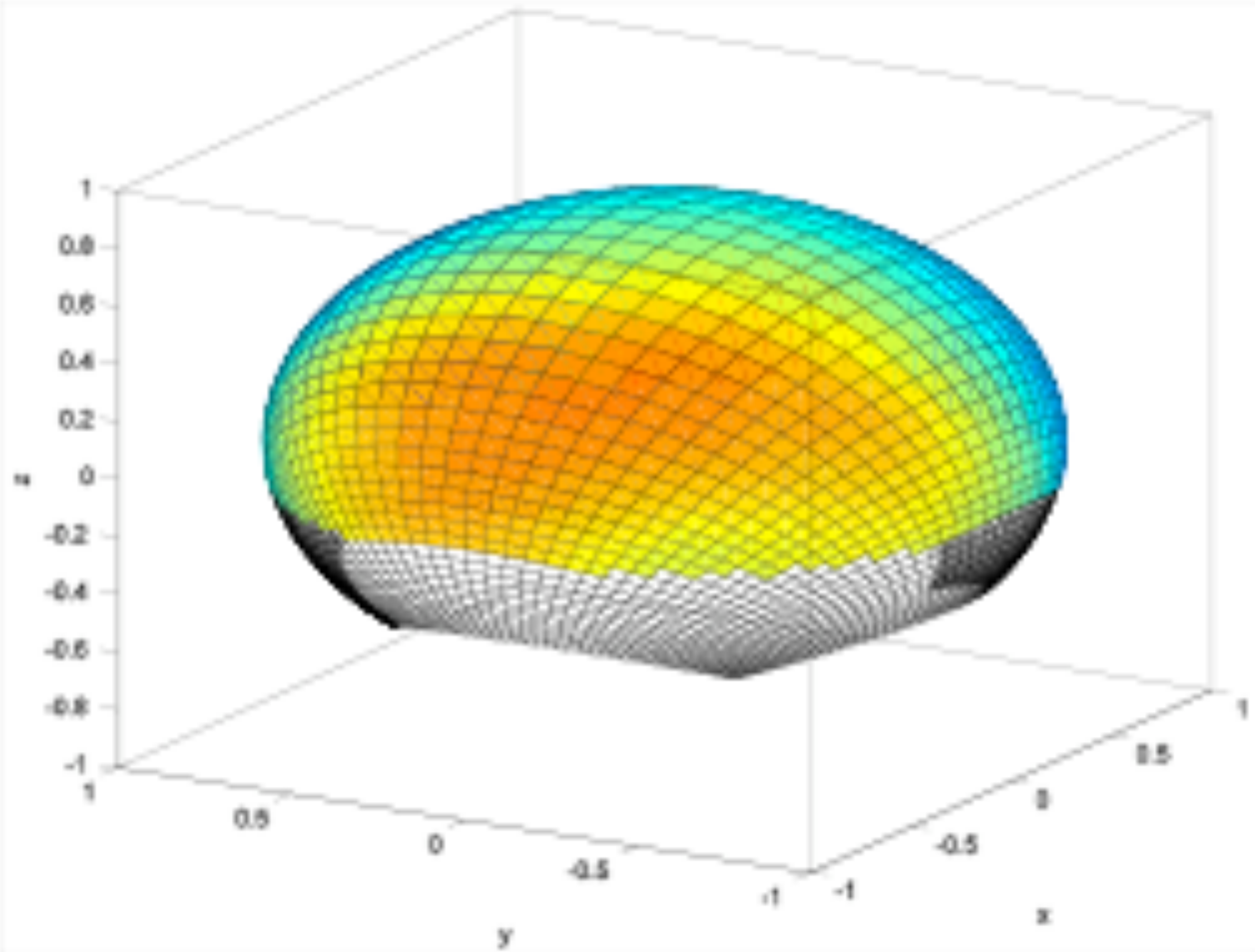

a, uniform–flux boundary
b, isothermal boundary
c, IGRF 2000
–0.6
0.0
0.6
[mT]

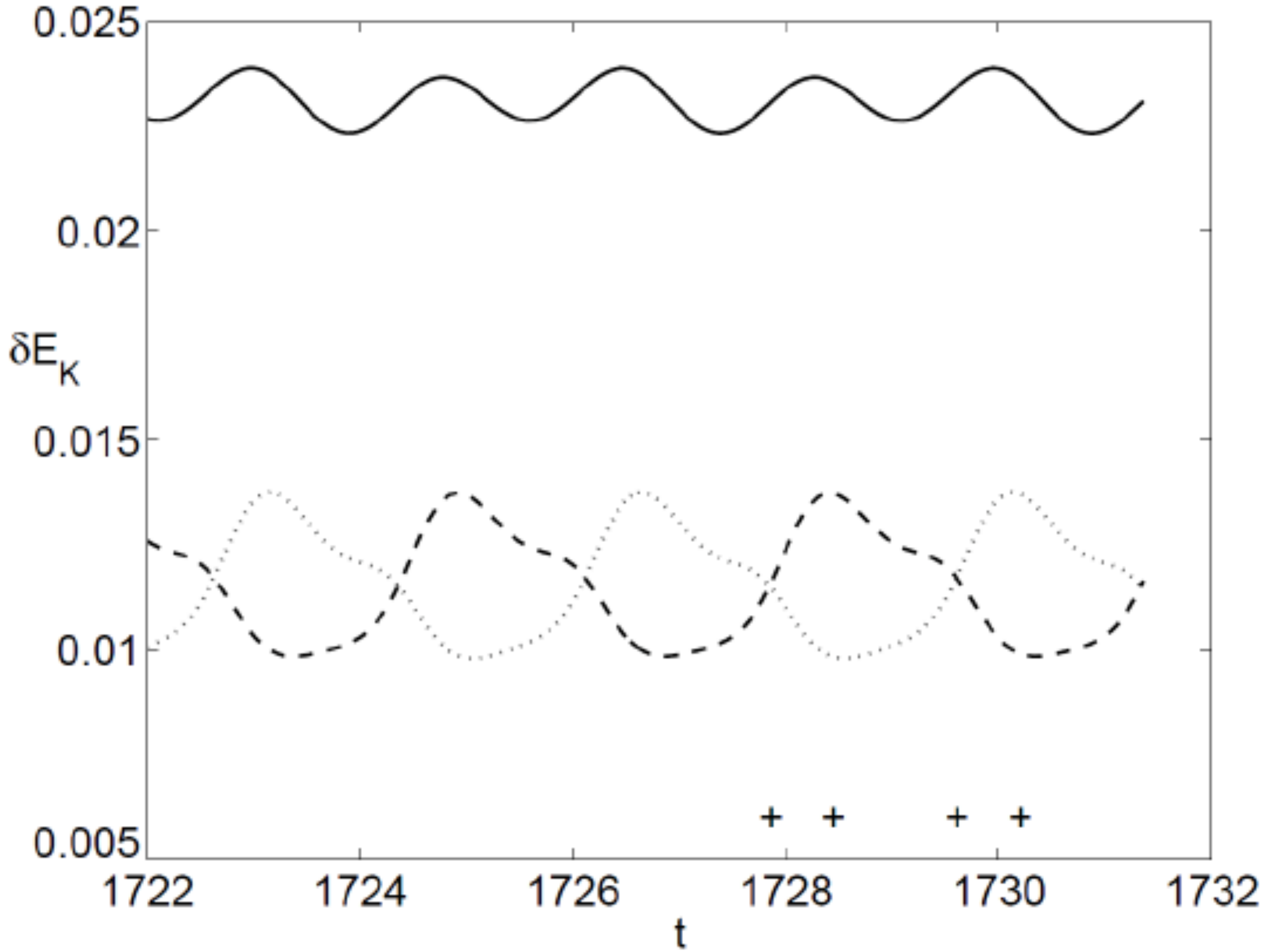

uniform-flux boundary
a
b
isothermal boundary
c
d
0
2
4
tangential field [mT]
0.0
0.1
0.2
tangential velocity [mm/s]



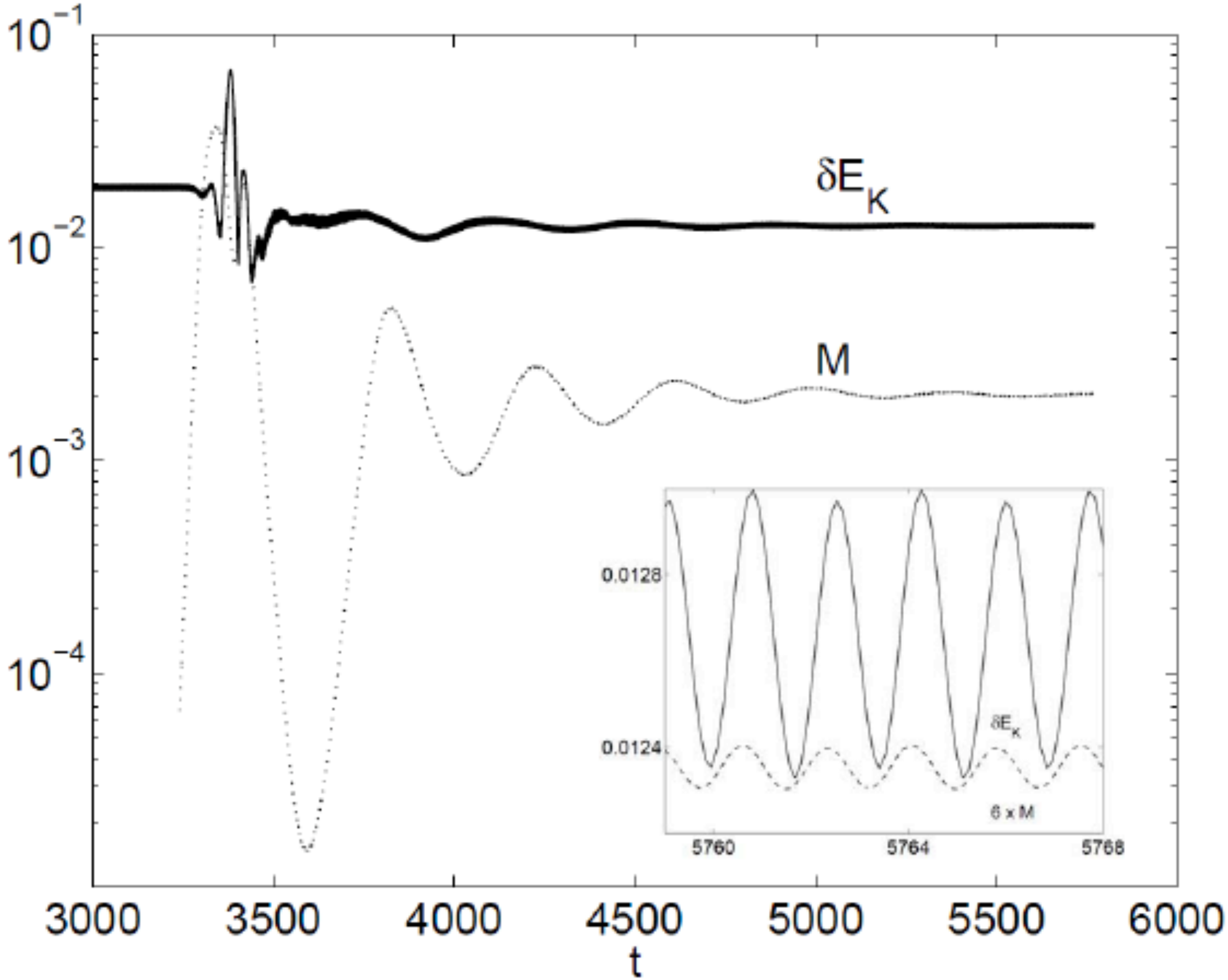
Ω
ω
23½°
R
R2
CORE
MANTLE
R1

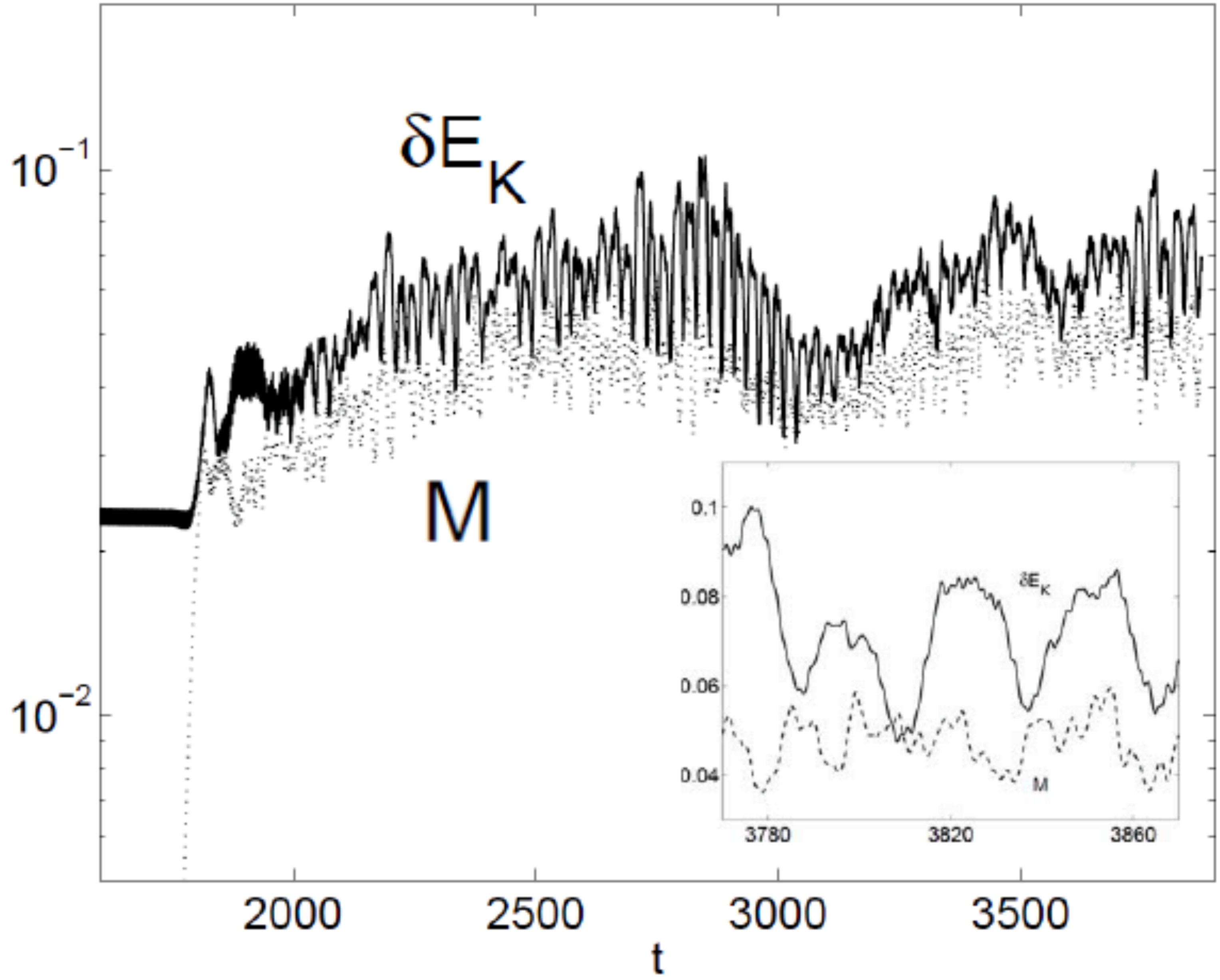

1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
z
1
0.5
0
-0.5
-1
y
1
0.5
0
-0.5
-1
x



Kinetic energy in non-magnetic precession



10^{-1}
10^{-2}
10^{-3}
10^{-4}
δE_K
M
0.0128
0.0124
δE_K
6 x M
5760
5764
5768
3000
3500
4000
4500
5000
5500
6000
t



δE_K
M
10^-1
10^-2
2000
2500
3000
3500
t
0.1
0.08
0.06
0.04
δE_K
M
3780
3820
3860

The End