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2050-2

Targeted Training Activity: Predictability of Weather and Climate: Theory and Applications to Intraseasonal Variability

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Modeling the Weather and Climate

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International Centre for Theoretical Physics

Targeted Training Activity:

Predictability of Weather and Climate

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(1) The Global Atmospheric Circulation: Observations

(2) **Modeling the Weather and Climate**

(3) Errors in Forecasts: Roles of Initial States, Model Errors, and Chaos

(4) Climate Predictability on Seasonal Time Scale: Role of Boundary Forcing

(5) Seasonal Mean Predictability over the Pacific - North American Region

Atmospheric Numerical Weather Prediction and Climate Models

Atmospheric Components of Coupled Models

(1) Dynamics. 2 choices:

- Primitive equations: hydrostatic relationship $\partial p/\partial z = -\rho g$
- Non-hydrostatic equations

(2) Horizontal domain usually global, with spherical geometry. 2 choices:

- Variables represented at grid points for both dynamical and physical processes
- Variables represented in *spectral coefficients (spherical harmonics)* for dynamics, and in grid point space for physical processes

(3) Vertical domain includes troposphere and stratosphere

(4) Some physical processes explicitly represented

- Latent heat release due to resolved (large-scale) moist saturation
- Gravity waves that are resolved (large-scale)

(5) Parameterizations for many physical processes not explicitly resolved

Primitive Equations

(1) Filtered version of the fundamental equations of fluid dynamics

(2) Assumption made that vertical domain is much smaller than horizontal domain, so that the vertical velocity is much smaller than the horizontal velocity

(3) Assumption (2) is consistent with the hydrostatic equation, which relates the mass to the vertical derivative of pressure. This filters out sound waves from the set of equations.

(4) Often solved with the use of pressure (or a related quantity) as the vertical coordinate.
In these "pressure" coordinates, the fundamental dynamical equations consist of:

Momentum equations for horizontal flow - Newton's Second Law in a rotating frame of reference. ($F = ma = m\, dv/dt$)

Thermodynamic equation ($T\, dS/dt = Q$) where S=entropy, Q=heating

Conservation of mass

Conservation of water (vapor + liquid + ice)

The fundamental dynamical non-linear nature follows from the distinction between the *Lagrangian* derivative, which is the rate of change of a parcel or air, that is the rate of change "following the flow" and the *Eulerian* derivative, which is the rate of change at a fixed position (x, y, p, t).

As an example, consider the thermodynamic equation:

$$T\frac{dS}{dt} = Q \tag{1}$$

where S is the entropy per unit mass, and Q the rate of thermodynamic heating.

The entropy is given (for an ideal gas) by

$$S = C_p \ln(\Theta) = C_p \ln\left(T\left(\frac{p_0}{p}\right)^{\kappa}\right) \tag{2}$$

where $p_0 = 1000$ hPa, C_p is the specific heat at constant pressure, and $\kappa = R/C_p$, where R is the ideal gas constant in the atmospheric equation of state $p = \rho R T$.

Using equation 1 in equation 2 we get:

$$\begin{aligned} C_p \left(\frac{T}{\Theta}\right) \frac{d\Theta}{dt} &= Q \\ \frac{d\Theta}{dt} &= Q \left(\frac{p_0}{p}\right)^{\kappa} \frac{1}{C_p} \end{aligned} \tag{3}$$

The derivative $\frac{d\Theta}{dt}$ is the *Lagrangian* derivative, and is written in the *Eulerian* framework as:

$$\frac{d\Theta}{dt} = \frac{\partial\Theta}{\partial t} + \frac{u}{a\cos(\phi)}\frac{\partial\Theta}{\partial\lambda} + \frac{v}{a}\frac{\partial\Theta}{\partial\phi} + \omega\frac{\partial\Theta}{\partial p} = Q\left(\frac{p_0}{p}\right)^{\kappa}\frac{1}{C_p} \tag{4}$$

Here (u, v) is the horizontal velocity at constant pressure, $\omega = \frac{dp}{dt}$ is the Lagrangian change in pressure, (λ, ϕ) are longitude and latitude, and a is the earth's radius.

Important Physical Processes

Explicitly Resolved **Parameterized**

Solar Radiation - both incoming and reflected

Thermal radiation upward from the ground

Thermal radiation, both upward and downward, from gases in the troposphere, and from clouds.

Latent heat release from condensation of water vapor due to resolved motions.

Latent heat release and motion due to motions not resolved - convection (includes both deep convection and shallow convection).

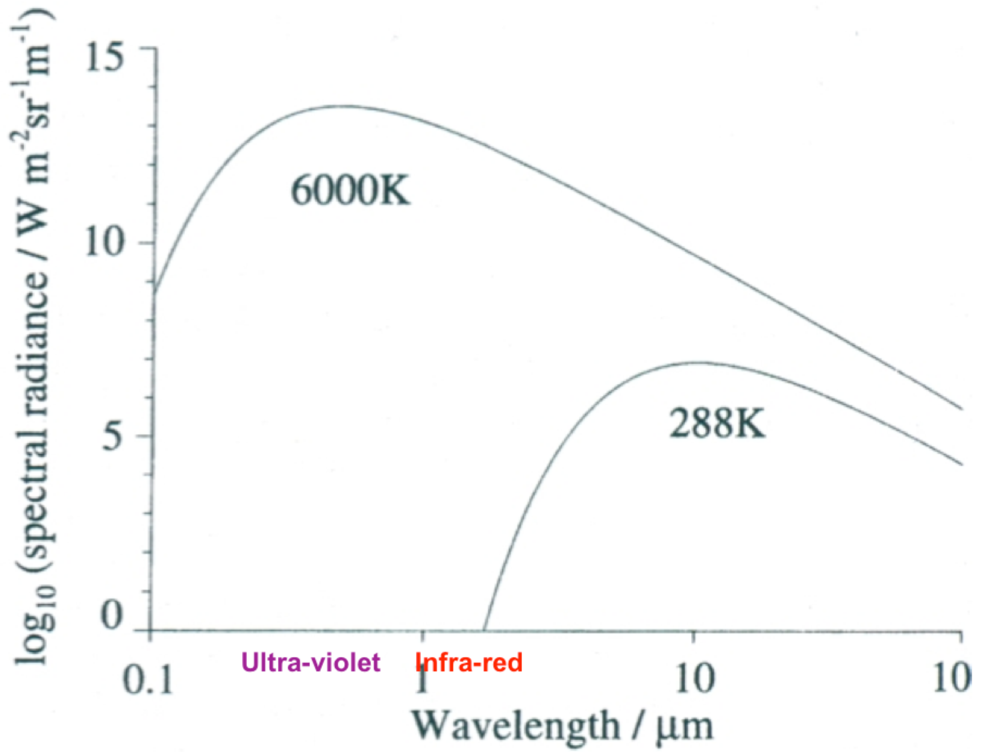
Planetary Boundary Layer diffusion and turbulence.

These processes can be very non-linear, in fact not even analytic!

From Andrews

3 ATMOSPHERIC RADIATION

Figure 3.1 Logarithm of the black-body spectral radiance $B_\lambda(T)$, plotted against the logarithm of wavelength λ, for $T =$ 6000 K, a typical temperature of the solar photosphere, and 288 K, the Earth's mean surface temperature.



B_λ for T = 6000 K (characteristic of the sun) and for T = 288 K (characteristic of the Earth). Note the use of log scale on both axes. Note also that the "sun" curve lies above the "earth curve" for all wavelengths

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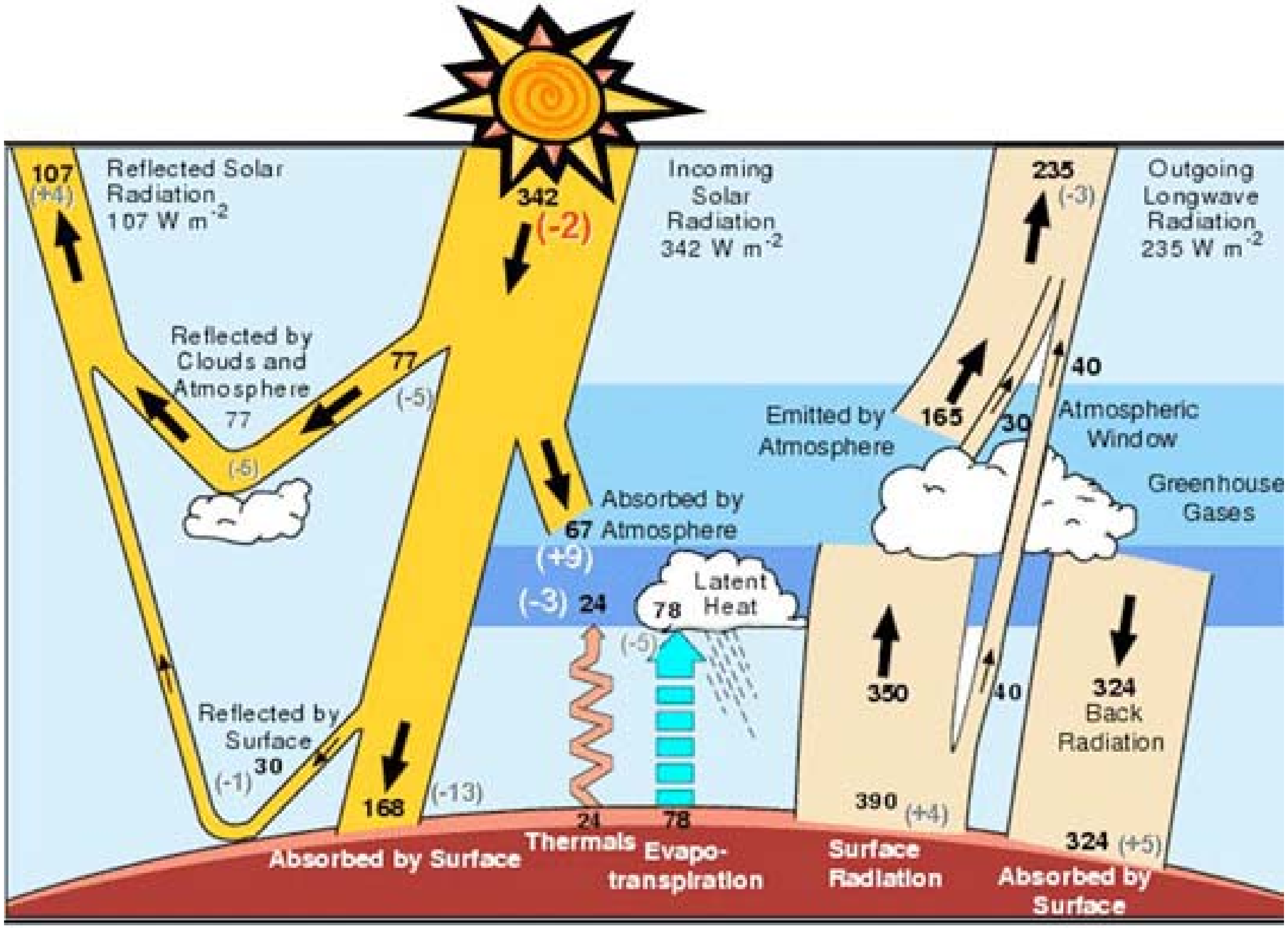
http://www.global-greenhouse-warming.com/images/SolarRadiationDiagram.jpg

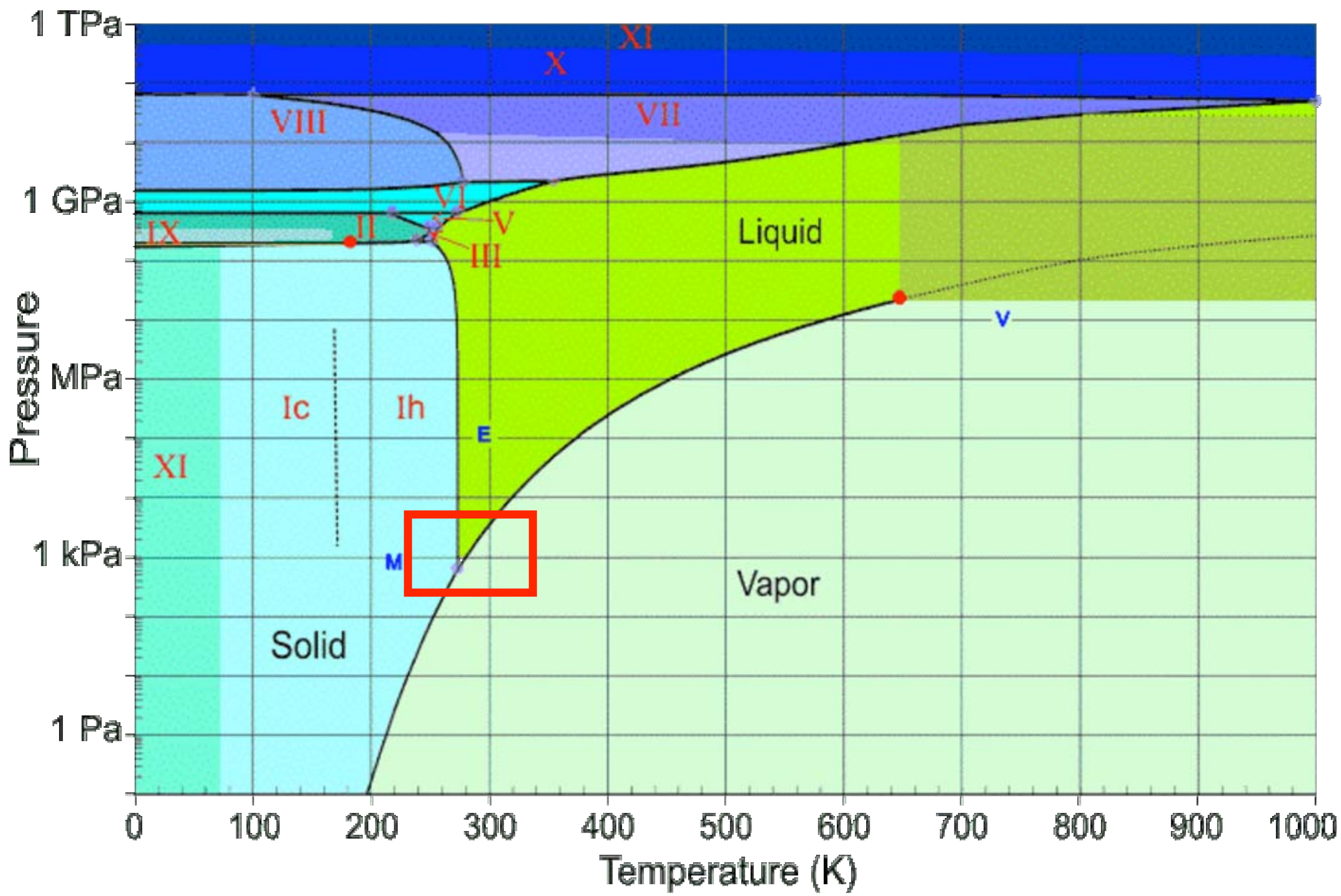


Basic Radiation Balance Forcing the Atmosphere-Ocean-Earth

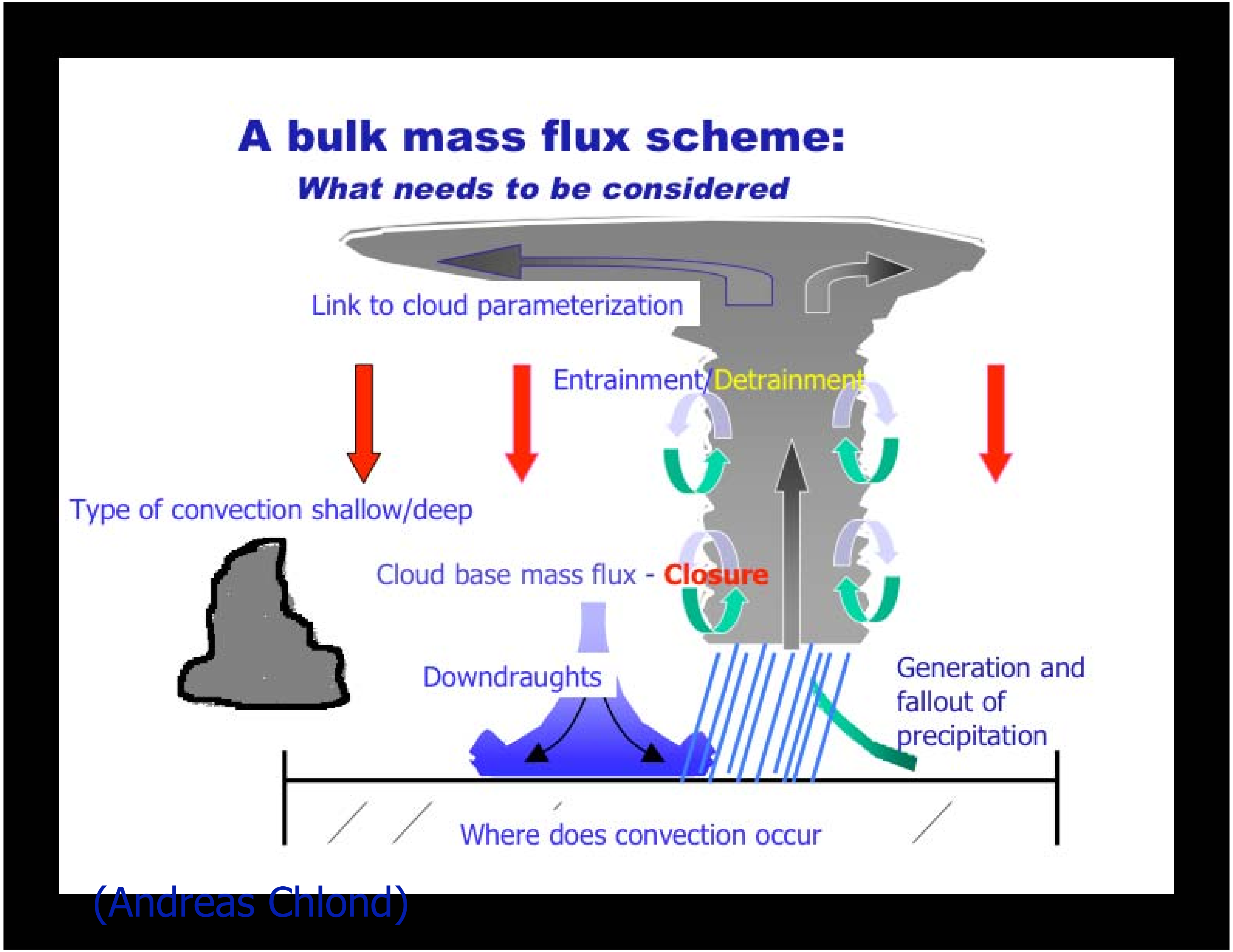
http://www.lsbu.ac.uk/water/images/phase.gif

Phase Diagram of Water

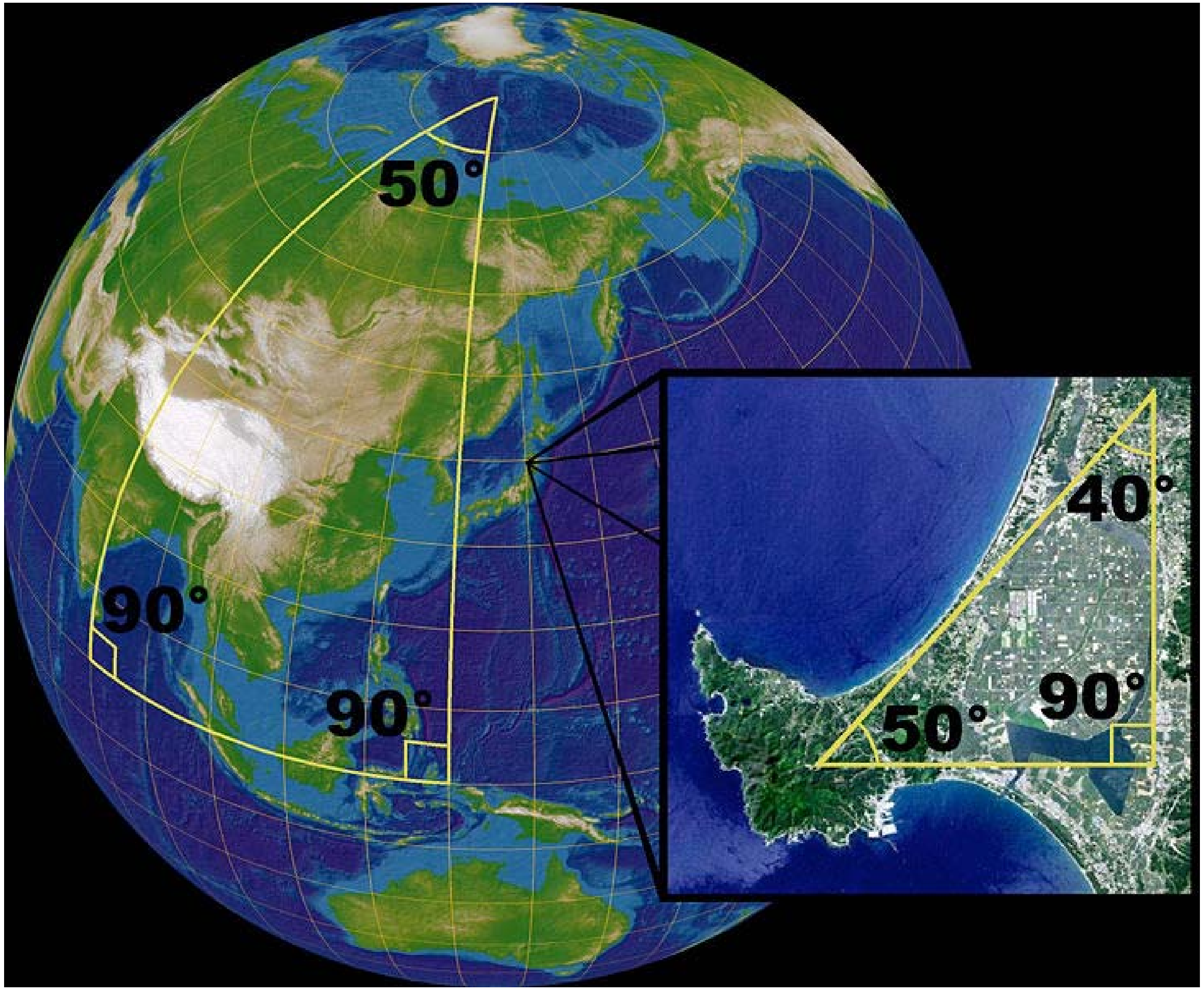
(Typical Atmosopheric water vapor conditions shown in red box)



Thermodynamic Importance of Water:Energy Released or Absorbed by Changes of Phase: Vapor <--> Liquid <--> Solid


A bulk mass flux scheme:
What needs to be considered
Link to cloud parameterization
Entrainment/Detrainment
Type of convection shallow/deep
Cloud base mass flux - Closure
Downdraughts
Generation and fallout of precipitation
Where does convection occur
(Andreas Chlond)

Challenges in Representing Fields at Discrete Points on the Sphere



http://en.wikipedia.org/wiki/File:Triangles_(spherical_geometry).jpg

1 Spectral Models: How Fields are Represented

1.1 Longitude Variation: Zonal Harmonics

Any field F at a fixed latitude, fixed pressure level, and fixed time, is a periodic function of longitude λ. It can be represented as a sum of harmonic functions as:

$$\begin{aligned} F &= F_0 + \sum_{m=1}^{m=\infty} \left(C_m \cos(m\lambda) + S_m \sin(m\lambda) \right) && (1) \\ &= F_0 + \sum_{m=1}^{m=M} \left(C_m \cos(m\lambda) + S_m \sin(m\lambda) \right) && (2) \end{aligned}$$

where F_0 is just the zonal mean, or average over all longitudes. Equation 1 is always a valid way to write any field, no matter how rapid the variation in longitude.

On the other hand, equation 2 is *truncated*, representing only larger scales, that is, wavelengths corresponding to $\Delta\lambda = 2\pi/M$ or longer.

The coefficients C_m and S_m are real. The functions $\cos(m\lambda)$ and $\sin(m\lambda)$ are called *basis functions.* The integers m are referred to as the *zonal wave numbers.*

1.2 Latitude and Longitude Variations: Spherical Harmonics

One way to address the challenges in representing a continuous field $G(\lambda, \phi)$ as a function of longitude λ and latitude ϕ on the sphere is to use basis functions which are already defined for the sphere. These are called *spherical harmonics.*

Any field $G(\lambda, \phi)$ can always be written, in truncated form, as:

$$\begin{aligned} G(\lambda, \phi) &= \sum_{n=1}^{n=N} \sum_{m=-N}^{m=N} F_{n,m} \, P_{n,m}(\mu) e^{im\lambda} \\ &= \sum_{n=1}^{n=N} \sum_{m=0}^{m=N} P_{n,m}(\mu) \left(2F^R_{n,m} \cos(m\lambda) - 2F^I_{n,m} \sin(m\lambda) \right) \end{aligned} \tag{3}$$

- Here we have a product of harmonic functions in longitude $(\cos(m\lambda)$

and $\sin(m\lambda)$) and polynomials in the variable $\mu = \cos(\phi)$, the *Legendre polynomials* $P_{n,m}(\mu)$

- The index m is just the zonal wavenumber as before. The index n is called the *total wave number*.
- The Legendre polynomial $P_{n,m}(\mu)$ depends on both n and m.
- The magnitude of m can not exceed that of n.
- The coefficients $F_{n,m}$ are complex, and can be written in terms of real and imaginary parts as: $F_{n,m} = F^R_{n,m} + F^I_{n,m}$

While the zonal wave number m is familiar from before, it is the em total wave number n that gives the "scale" of the wave. To see this, remember that in ordinary wave theory, the function $e^{\vec{k}\cdot\vec{x}}$ is an eignfunction of the operator ∇^2 with eigenvalue $-k^2$:

$$\nabla^2 e^{\vec{k}\cdot\vec{x}} = -k^2 e^{\vec{k}\cdot\vec{x}} \tag{4}$$

which identifies k as the dimensional wave number, and $2\pi/k$ as the physical wavelength.

In a similar way, the function $P_{n,m}(\mu)e^{im\lambda}$ (also called a *spherical harmonic* is an eigenfunction of ∇^2:

$$\nabla^2 P_{n,m}(\mu)e^{im\lambda} = -\frac{n(n+1)}{a^2} P_{n,m}(\mu)e^{im\lambda} \tag{5}$$

where a is the radius of the sphere (the earth). The associated physical wavelength is given by:

$$\lambda = 2\pi \sqrt{\left(a^2 \frac{1}{n(n+1)}\right)} \tag{6}$$

2 Spectral Models

Spectral models represent the scalar variables (temperature, vorticity, divergence and at all levels, and surface pressure) as a sum over spherical harmonics as in equation 3. It is the complex coefficients $F_{n,m}$ that change in time as the fields evolve.

A mode truncation identified as TN (for example, T85), means that $N = 85$ is the upper limit for total wave number n in the sum of equation 3.

What is the physical resolution of a spectral model with Triangular truncation?

Definition 1: The wavelength λ of the smallest wave retained in the spherical harmonic basis set: $\lambda = 2\pi/k = 2\pi a \, / \, [\, N(N+1) \,]^{1/2}$ (**N** is highest total wavenumber in truncated basis set, k is the wavenumber).

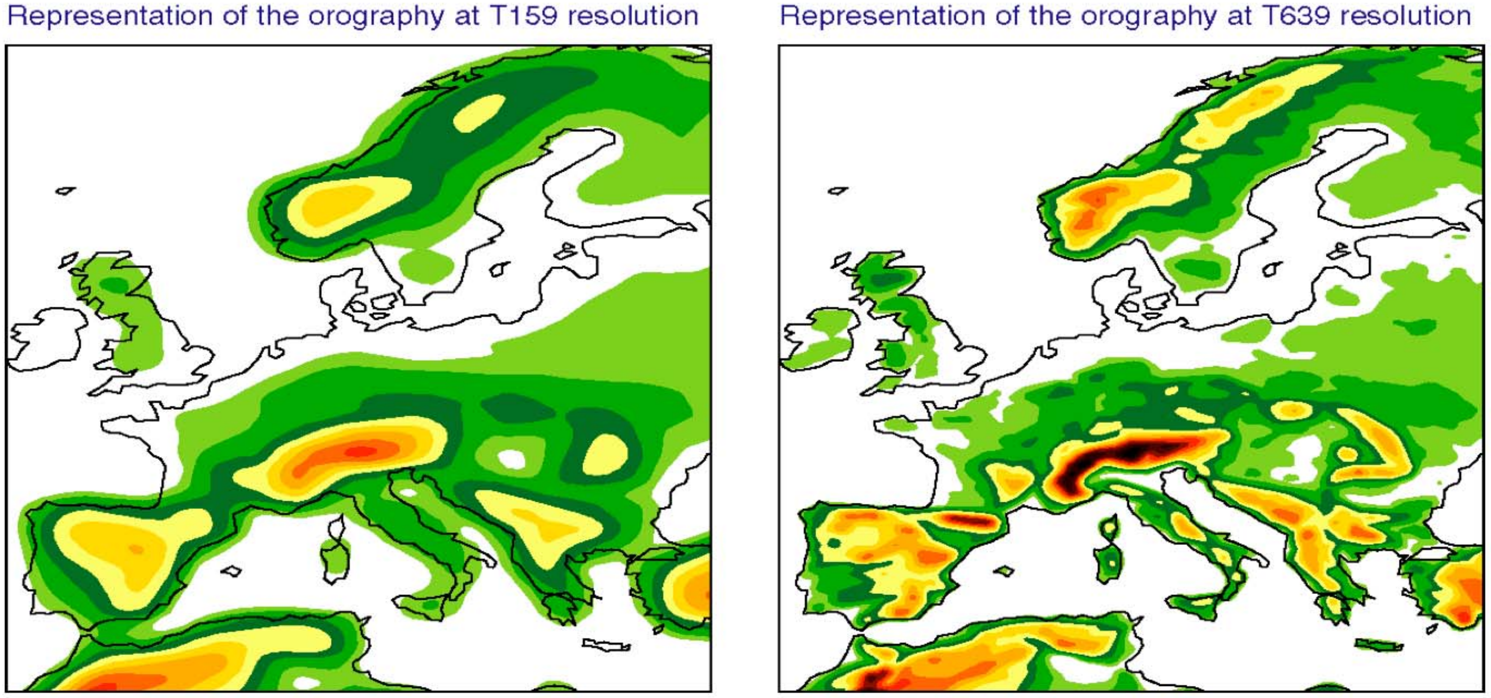
Definition 2: The size **d** of a very sharply defined local feature (delta function) when expressed in terms of the spherical harmonic basis set.

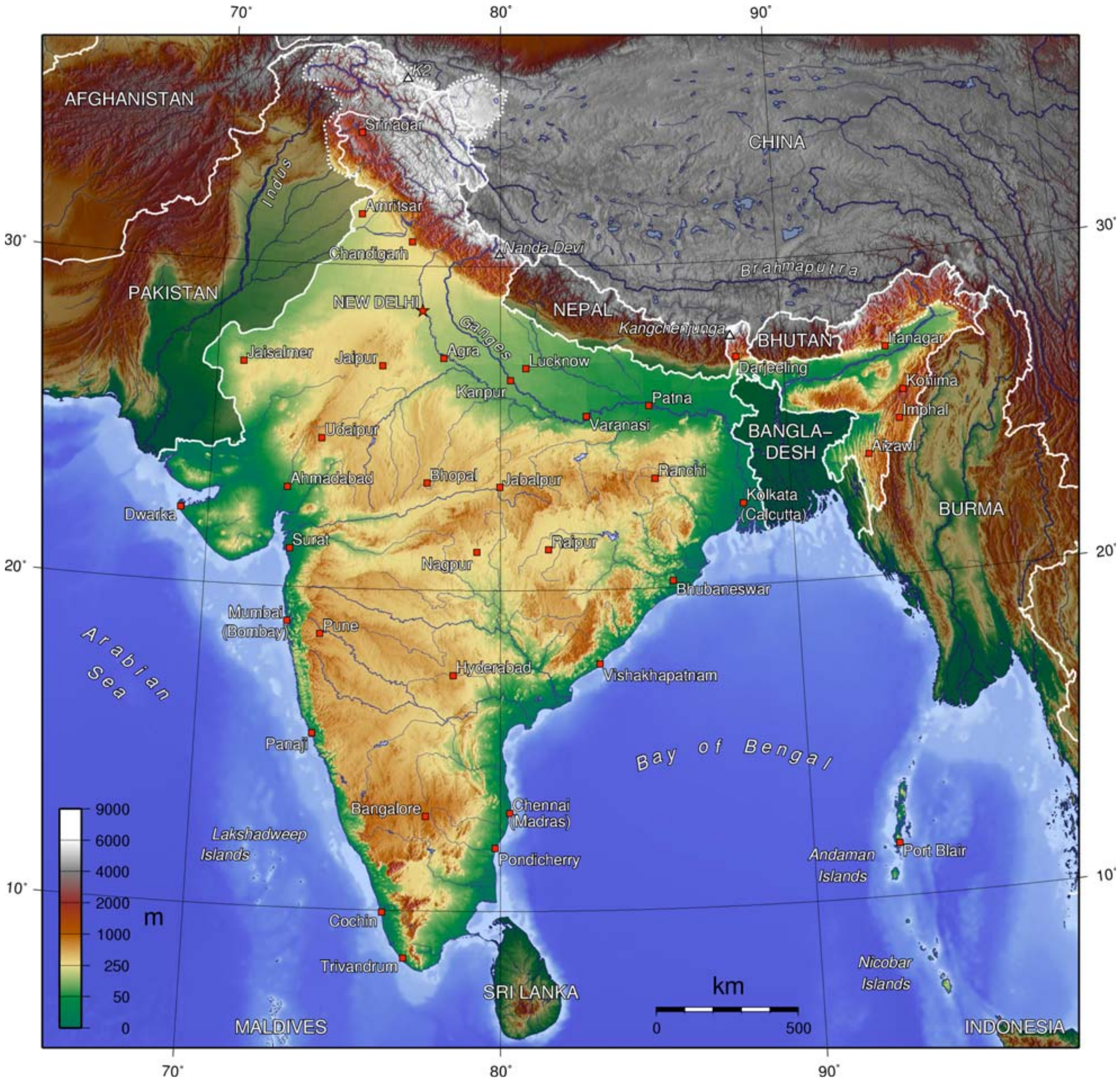
N	21	42	63	106	213
λ	1859	942	628	377	188 km
d	2979	1507	1009	601	300 km

J. Lander and B. J. Hoskins, 1997

1. The importance of spatial resolution **for representing mountains**

A high spatial resolution is needed to achieve an accurate representation of the system physical processes. Similarly, the representation of the orography becomes more realistic with increased horizontal resolution.




70˚
80˚
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30˚
20˚
10˚
AFGHANISTAN
PAKISTAN
CHINA
NEPAL
BHUTAN
BANGLA-
DESH
BURMA
SRI LANKA
MALDIVES
INDONESIA
K2
Srinagar
Indus
Amritsar
Chandigarh
Nanda Devi
Brahmaputra
NEW DELHI
Ganges
Kangchenjunga
Itanagar
Darjeeling
Kohima
Imphal
Aizawl
Jaisalmer
Jaipur
Agra
Lucknow
Kanpur
Patna
Varanasi
Udaipur
Ahmadabad
Bhopal
Jabalpur
Ranchi
Kolkata
(Calcutta)
Dwarka
Surat
Raipur
Nagpur
Bhubaneswar
Mumbai
(Bombay)
Pune
Arabian
Sea
Hyderabad
Vishakhapatnam
Panaji
Bay of Bengal
Bangalore
Chennai
(Madras)
Pondicherry
Lakshadweep
Islands
Andaman
Islands
Port Blair
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Nicobar
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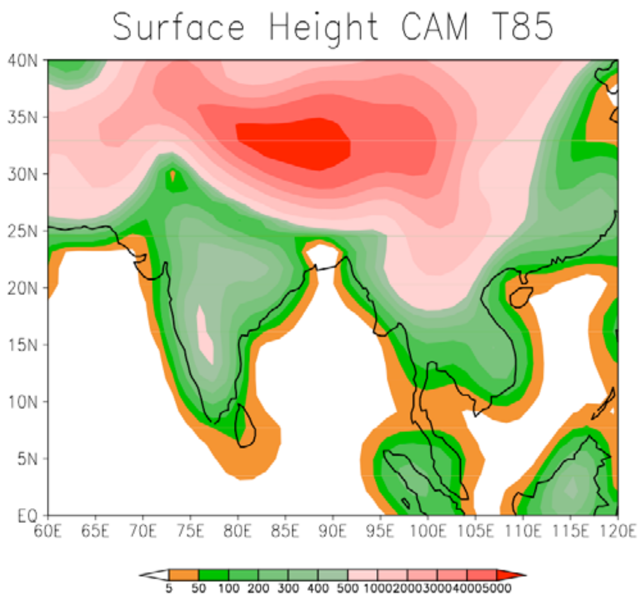
Surface Height CAM T85
40N
35N
30N
25N
20N
15N
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60E 65E 70E 75E 80E 85E 90E 95E 100E 105E 110E 115E 120E
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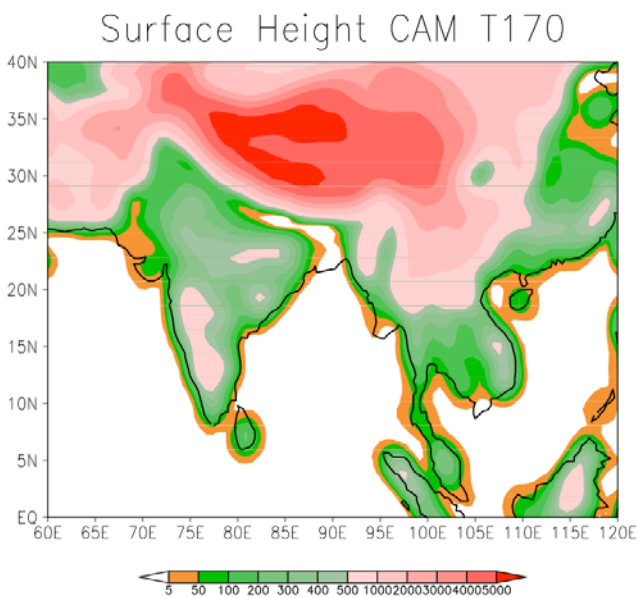
Surface Height CAM T170
40N
35N
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5N
EQ
60E 65E 70E 75E 80E 85E 90E 95E 100E 105E 110E 115E 120E
5 50 100 200 300 400 500 1000 2000 3000 4000 5000

Atmospheric Forecast and Climate Models

Variables which predicted by explicit evolution equations ("prognostic"):

At every model vertical level:
(1) Horizontal flow (zonal and meridional winds)
(2) Temperature
(3) Water Vapor
(4) Liquid and Solid water (clouds)

Surface Fields:
(1) surface pressure

Weather forecasting AGCM resolution: T 799 (ECMWF); T 382 (NCEP)
Climate change coupled model simulation resolution: T85 (NCAR CCSM)