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Plasma physics of cosmic collisionless explosions: The role of the relativistic filamentation and electrostatic instabilities

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Contents:

1. Introduction

- Observations of the photon radiation at TeV energies from cosmic particle accelerators by the H.E.S.S. and MAGIC air Cherenkov telescopes have detected more than 70 sources:

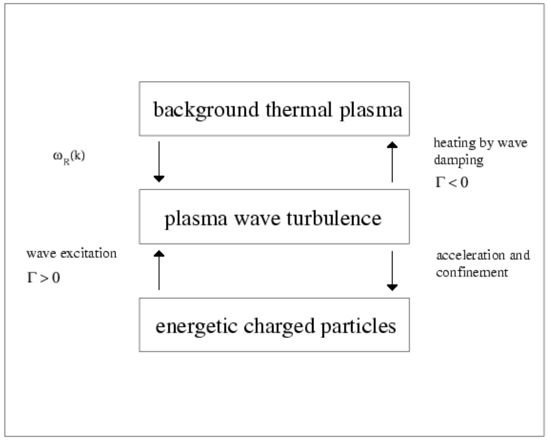
  7 supernova remnants, 18 pulsar wind nebulae, 2 extended sources, 4 binary systems, 20 extragalactic active galactic nuclei, 21 unidentified sources

- These objects therefore are cosmic charged particle accelerators ("usual suspects"): in order to produce photons with energies of 1 TeV= 10^{12} eV, the radiating charged particles must have even higher energies (non-thermal radiation processes).

- **Striking**: "Extreme" (compact, high photon luminosities, high magnetic field strength) objects (Pulsars=neutron stars, pulsar wind nebulae, supernova shock waves, microquasare, active galactic nuclei (AGNs)) with established high (often relativistic) velocity outflows (particle beams, jets, shock waves)

- AUGER experiment finds close correlation of arrival directions of UHE cosmic rays with location of known AGNs

- **Fundamental physics issue**: Conversion of directed kinetic outflow energy into high energy cosmic charged particles (=cosmic rays) in interactions with ambient target matter and photon radiation fields



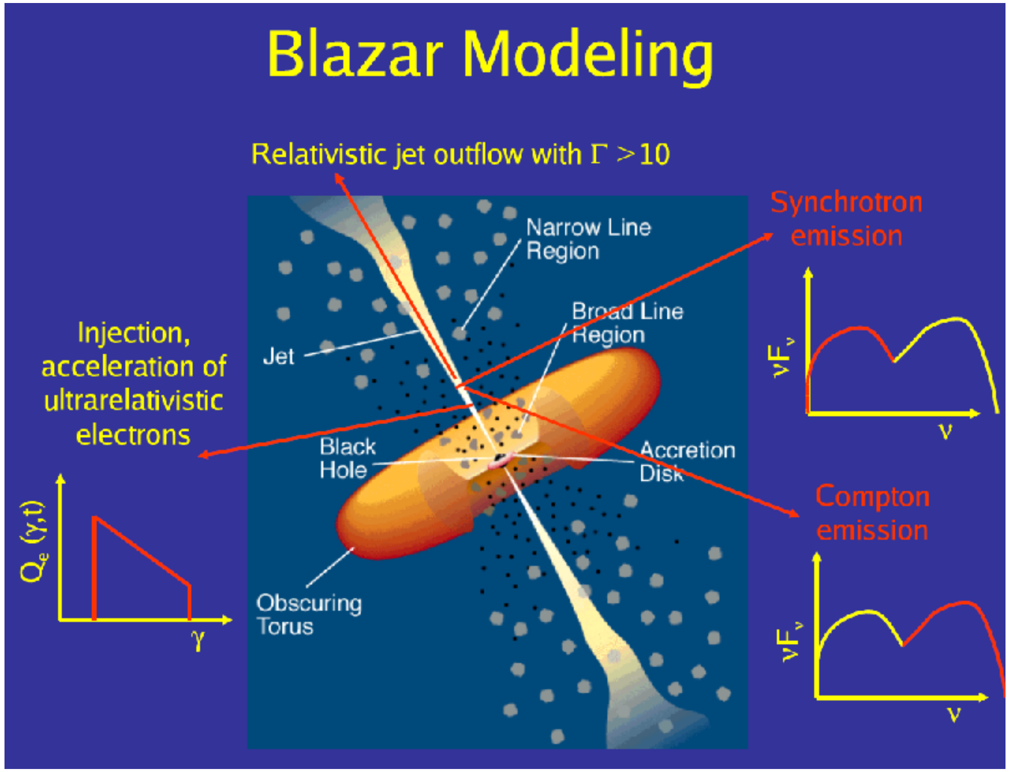
- However: all nonstellar cosmic plasmas are collision-free: plasma parameter $g = \nu_{e-e}/\omega_{p,e} \ll 1$, i.e. electromagnetic interactions dominate elastic collisions.

  Consequences: (a) particle distribution functions are not Maxwellians , (b) ideal MHD is not applicable, (c) anomalous MHD requires determination of nonideal viscosities, heat conduction coefficient etc., (d) if shock waves form their properties are different from classical hydrodynamical shocks, (e) full kinetic theory is required

- Have to understand plasma physics of cosmic explosions in collision-free environment. How is explosion energy dissipated ?

2. Nonthermal radiation processes

Active Galactic Nuclei: Doppler boosted γ-ray emission from the jet.



Are radiating electrons primary accelerated particles or secondaries from hadron interactions?

2.1. Electron and positron radiation processes

– Synchrotron radiation
– Inverse Compton scattering of transverse target photons
– Electrostatic bremsstrahlung=inverse Compton scattering of longitudinal electrostatic plasma waves into transverse photons
– Nonthermal bremsstrahlung
– Pair annihilation

2.2. Hadronic radiation processes

– Pion production in inelastic hadron-hadron collisions
– Pion production in inelastic hadron-photon collisions
– Suprathermal proton bremsstrahlung (difference from ordinary bremsstrahlung: here electrons at rest and heavy proton is moving)

Photon production spectra from all production processes strongly depend on equilibrium energy spectrum $N(\vec{r}, \gamma, t)$ of radiating particles!
Equilibrium spectrum results from competition af all acceleration and energy loss processes and particle transport processes. For more information: Schlickeiser 2002, Cosmic Ray Astrophysics, Springer

2.3. Hadronic processes contain leptonic processes

Pion production in inelastic hadron-hadron- and hadron-photon-collisions:

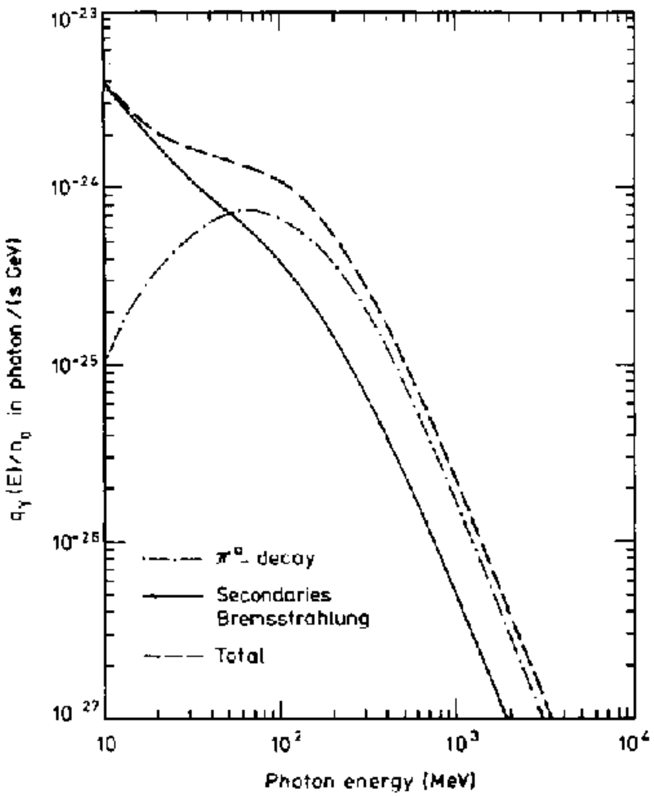
neutral pions $\pi^0 \to 2\gamma$ directly decay into two photons. Detection of characteristic (symmetry at 70 MeV$\simeq 0.5 m_\pi c^2$) photon energy spectrum discriminates between hadron-produced and lepton-produced photons (Morrison 1958, Ginzburg 1964).

Charged pions decay via myons into secondary electrons and neutrinos $\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$, $\mu^\pm \to e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$. Relation to high energy neutrino astronomy (ICECUBE, KM3NET).

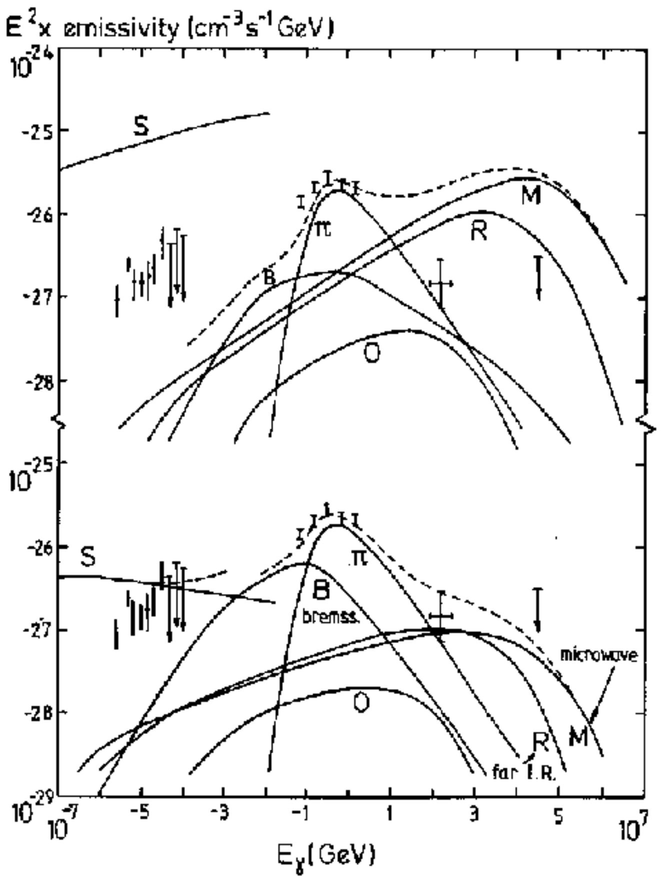
Good templates by Kelner et al. (2007) and Kelner and Aharonian (2008) for implementing hadron-hadron and hadron-photon collisions.

Secondary electrons undergo all leptonic radiation processes (2.1). Contamination of pure π^0-decay spectrum by secondary electron radiation contribution.

Problem: bremsstrahlung-contamination by secondary $e^{\pm}$ from $\pi^{\pm} \rightarrow e^{\pm} + ..$ in same inelastic hadron-collisions (e.g. RS 1982)



Electron-contamination minimal at photon energies of 1-100 GeV (Protheroe und Wolfendale 1980)



3. Acceleration of cosmic ray particles

3.1. Role of electric fields

Acceleration of charged particles requires **electric fields**:
from equations of motion

$$\dot{\vec{p}} = \vec{F} = q\left[\vec{E}_T(\vec{x},t) + \frac{\vec{v}\times\vec{B}_T(\vec{x},t)}{c}\right], \quad \dot{\vec{x}} = \vec{v} = \frac{\vec{p}}{\gamma m_a}$$

$\longrightarrow$ particle spectra are controlled by particle rigidity $R = p/q$

$\longrightarrow$ $\vec{p}\cdot\frac{d\vec{p}}{dt} = \frac{1}{2}\frac{dp^2}{dt} = q\,\vec{p}\cdot\vec{E}$ or

$$\frac{dp^2}{dt} = 2q\,\vec{p}\cdot\vec{E}$$

$\longrightarrow$ change of energy $W = mc^2\sqrt{1+\frac{p^2}{m^2c^2}}$

$$\frac{dW}{dt} = \frac{1}{2W}\frac{dp^2}{dt} = \frac{q}{W}\vec{p}\cdot\vec{E}$$

but cosmic plasmas have very large conductivities
$\rightarrow$ difficult to sustain steady, constant electric fields
Electric fields appear as
(a) transient phenomena (magnetic reconnection, aperiodic fluctuations in Weibel instability)
(b) fluctuating wave fields (plasma turbulence) in magnetized plasmas

$$\vec{E} = \vec{0} + \delta\vec{E},\ \vec{B} = \vec{B}_0 + \delta\vec{B}$$

with averages $< \delta\vec{E} >= 0,\ \ < \delta\vec{B} >= 0$

3.2. 4 fundamental types of acceleration processes:

(1) Magnetic reconnection (generation of particle beams by reconnecting ordered magnetic fields, then (4))
(2) (Diffusive) shock acceleration (but kinetic description of shock structure required)
(3) Stochastic acceleration ($\delta\vec{E}$)
(4) Conversion of bulk motion (=particle beams) to energetic particles by $\delta\vec{B}$ alone (relativistic pick-up process)

Understanding acceleration/conversion processes (1)-(4) $\leftrightarrow$ understanding electric and magnetic fluctuations

3.3. Equations used

Competition between injection, escape, energy gain (acceleration) and energy loss (catastrophic and continous) processes

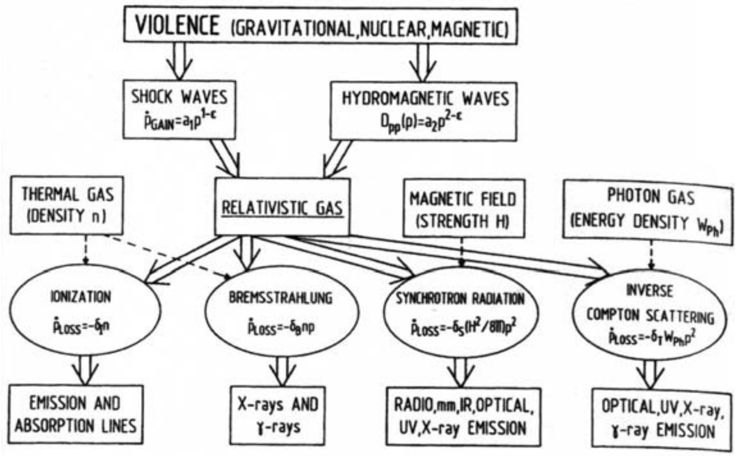


Diagram showing schematically the basic physical model for the formation of cosmic ray momentum spectra. The **arrows** represent the direction of energy flow

Balance equation in 5-dim. phase space $(\vec{r}, p, t)$:

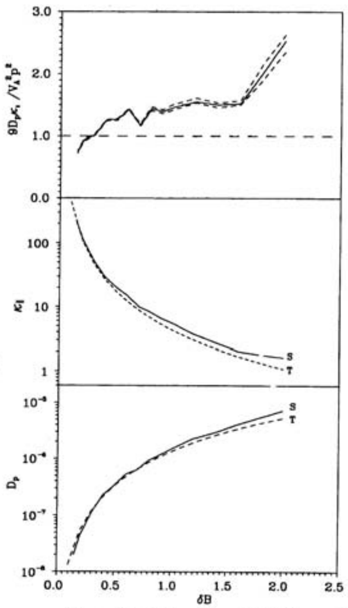
$$\frac{dN}{dt} = \text{sources(injection, acceleration)} - \text{losses}$$

$$\frac{dN}{dt} = \frac{\partial N}{\partial t} + \frac{\partial N}{\partial t}_{B_0+\delta B, \delta E} + \frac{\partial}{\partial p}(\dot{p}N) + \frac{N}{T_c} = q(p, \vec{r}, t)$$

3.4. Theoretical description of particle acceleration

Two methods:
– numerical simulations
– quasilinear theory (QLT) with nonlinear improvements e.g. SOQLT (Shalchi 2009)

QLT approach is perturbation calculation of charged particle orbits in partially random field $(\vec{B} = \vec{B}_0 + \delta\vec{B}, \vec{0} + \delta\vec{E})$ to first order in ratio $q_L = (\delta B/B_0)^2 < 1$.



The simulated values of the spatial diffusion coefficient ($\kappa_\parallel$), the momentum diffusion coefficient (D_p) and their product in comparison with the quasi-linear values versus the wave amplitude δB (in units of B_0) for linearly polarized plane Alfvén waves. From Michalek and Ostrowski (1996 [351])

QLT: reduction of collisionless Boltzmann equation for phase space density $f(\vec{x}, p, \mu, \phi, t)$ to Fokker-Planck-equation for gyrotropic part $f_0(\vec{x}, p, \mu, t)$. Correlation tensors of electric and magnetic field fluctuations determine 25 Fokker-Planck-coefficients.

In the presence of low-frequency MHD plasma wave turbulence (shear Alfven waves, fast and slow magnetosonic waves) with $\delta B \ll \delta E \rightarrow$ reduction of Fokker-Planck-equation to *diffusion-convection-equation* for isotropic, gyrotropic part of phase space density $F(\vec{x}, p, t) = N(\vec{x}, p, t)/4\pi p^2$ ($z \parallel B_0$)

$$\frac{\partial N}{\partial t}_{B_0+\delta B, \delta E} = -\mathrm{div}\left[\kappa \mathrm{grad N} - \tilde{\mathrm{V}}\mathrm{N}\right] - \frac{\partial}{\partial \mathrm{p}}\left[\mathrm{p}^2 \mathrm{A} \frac{\partial(\mathrm{Np}^{-2})}{\partial \mathrm{p}} + \frac{\mathrm{p}}{3}(\mathrm{div}\tilde{\mathrm{V}})\mathrm{N}\right]$$

with spatial diffusion tensor, spatial convection, momentum diffusion (Fermi II), momentum convection (Fermi I) terms calculated as pitch-angle ($\mu = \cos\theta$) averages of 5 Fokker-Planck coefficients:

parallel spatial diffusion coefficient

$$\kappa_{\parallel} = \frac{v^2}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}$$

perpendicular spatial diffusion coefficients

$$\kappa_{xx,yy} = \frac{1}{2}\int_{-1}^{+1} d\mu D_{xx,yy}(\mu)$$

momentum diffusion coefficient

$$A = \frac{1}{2}\int_{-1}^{1} d\mu\, [D_{pp} - \frac{D^2_{\mu p}}{D_{\mu\mu}}]$$

cosmic ray bulk speed

$$V = U + \frac{1}{3p^2}\frac{\partial}{\partial p}(p^3 D), \quad D = \frac{3v}{4p}\int_{-1}^{1} d\mu (1-\mu^2)\frac{D_{\mu p}}{D_{\mu\mu}}$$

calculated as pitch-angle ($\mu = \cos\theta$) averages of 5 Fokker-Planck coefficients e.g.

$$D_{pp} = \Re \int_0^\infty d\tau < \dot{p}(t)\dot{p}^*(t+\tau) >$$

where

$$\dot{p} = \frac{\Omega pc}{vB_0}[\mu\delta E_\parallel + \frac{\sqrt{1-\mu^2}}{2}(\delta E_L e^{-\imath\phi} + \delta E_R e^{\imath\phi})]$$

Relation to correlation tensors of electric and magnetic field fluctuations

$$(\delta B)^2 = \int d^3k P(\vec{k}), \quad (\delta E)^2 = \int d^3k R(\vec{k})$$

specifying nature and geometry of plasma turbulence (e.g. superposition of different plasma modes)

Introduction

Nonthermal . . .

Acceleration of . . .

Dissipation of . . .

Any cosmic outflow . . .

Summary and . . .

For **spatially unresolved** sources $\rightarrow$ theory for volume-integrated particle spectrum

$$\eta(p,t) = \int_{\text{source}} d^3r N(p,\vec{r},t)$$

simpler balance equation in 2-dim. phase space (p,t)

$$\frac{\partial \eta}{\partial t} - \frac{\partial}{\partial p}\left[p^2 A \frac{\partial(\frac{\eta}{p^2})}{\partial p} + (a_1 p + \dot{p})\eta)\right] + \frac{\eta}{T} = Q(p,t)$$

where $T^{-1} = T_c^{-1} + T_{\text{esc}}^{-1}$

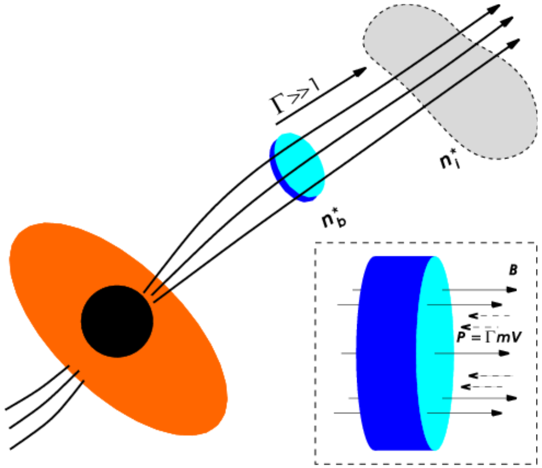
1) determine "equilibrium" energy spectrum $\eta(p,t)$ of radiating particles

2) then calculate resulting radiation products (photons, neutrinos)

4. Dissipation of cosmic outflows in unmagnetized/magnetized environment

Interaction of outflow jet plasma with ambient interstellar medium:

Free energy = kinetic relativistic outflow energy



What are the relevant dissipation processes?

5. Any cosmic outflow is counterstream plasma

Look at unmagnetized case here (Michno and RS 2009).
Outflow distribution function in laboratory system: 4 cold sreams

$$f^*(p^*_\perp, p^*_\parallel) = \frac{\delta(p^*_\perp)}{2\pi p^*_\perp}\Big[n_1\delta(p^*_\parallel - \gamma_1 m_+ v_1) + n_1\delta(p^*_\parallel - \gamma_1 m_- v_1)$$

$$+n_2\delta(p^*_\parallel - \gamma_2 m_+ v_2) + n_2\delta(p^*_\parallel - \gamma_2 m_- v_2)\Big]$$

Two ($i = 1, 2$) overall-neutral mono-energetic plasma beams of densities n_i propagating with different positive velocities $\vec{v}_i = v_i\vec{e}_z$, $v_i = \beta_i c$ along the z-axis. Each individual particle beam consists of the same number of positively ($+$) and negatively ($-$) charged particles, so one has charge neutrality in each stream. $m_\pm$ denote the masses of the charged particles, Lorentz factors $\Gamma_i = [1 - (v_i/c)^2]^{-1/2}$.
Because of the assumption of an equal number of positively and negatively charged particles, no restrictions apply to the values of n_1, n_2, v_1 and v_2 in order to avoid large-scale charge and current densities. Difference to 3-stream (plasma electrons, plasma ions, beam electrons) analysis of Bret (2008).

5.1. Lorentz transformation to counterstream frame

Transform the lab-distribution function to a different inertial system $(p_\perp, p_\parallel, E)$ propagating with velocity $\vec{V} = V\vec{e}_z$, $V = Bc$, $\Gamma = (1 - B^2)^{-1/2}$ along the z-axis. Using the invariance of the phase space function $f = f^*$ we obtain for the particle distribution function in the new coordinate system

$$f(p_\perp, p_\parallel) = \frac{\delta(p_\perp)}{2\pi p_\perp}\Big[N_1\delta(p_\parallel - \Gamma_1 m_+ u_1) + N_1\delta(p_\parallel - \Gamma_1 m_- u_1)$$

$$+N_2\delta(p_\parallel - \Gamma_2 m_+ u_2) + n_2\delta(p_\parallel - \Gamma_2 m_- u_2)\Big]$$

with

$$N_1 = n_1\Gamma(1 - \beta_1 B), \quad N_2 = n_2\Gamma(1 - \beta_2 B),$$

$$u_1 = \frac{\beta_1 - B}{1 - \beta_1 B}c, \;\; u_2 = \frac{\beta_2 - B}{1 - \beta_2 B}c,$$

$$\Gamma_1 = \Gamma\gamma_1(1 - \beta_1 B), \;\; \Gamma_2 = \Gamma\gamma_2(1 - \beta_2 B)$$

Counterstream frame of reference defined by $u_1 = -u_2 = u$ yielding for the transformation velocity B

$$B = \frac{1}{\beta_1 + \beta_2}\left[1 + \beta_1\beta_2 - \sqrt{(1 - \beta_1^2)(1 - \beta_2^2)}\right] = \frac{1}{\beta_1 + \beta_2}\left[1 + \beta_1\beta_2 - \gamma_1^{-1}\gamma_2^{-1}\right]$$

5.2. Plasma dispersion relation

Calculate the Maxwell operator of linear fluctuations in counterstream coordinate system adopting fluctuations of the form $\exp[\imath\vec{k}\cdot\vec{x}-\imath\omega t]$ with wave vector $\vec{k}=k_\parallel\vec{e}_z+k_\perp\vec{e}_x$. Solution of dispersion relation besides electromagnetic waves

$$\left[\frac{c^2k^2}{\omega^2}-1+\frac{\Omega^2(1+r)}{\omega^2}\right]\left[1-\frac{g\Omega^2}{\Gamma_0^2}\right]=\frac{4r\Omega^4u^2k_\perp^2}{\omega^2(\omega^2-k_\parallel^2u^2)^2} \tag{11}$$

with abbreviation

$$g(k_\parallel)\equiv\frac{1}{(\omega-k_\parallel u)^2}+\frac{r}{(\omega+k_\parallel u)^2},$$

the plasma frequency of the first component

$$\Omega^2\equiv\frac{4\pi e^2(1+\chi)n_1}{m_-\gamma_1}$$

and density ratio

$$r=\frac{A_2}{A_1}=\frac{N_2}{N_1}=\frac{n_2(1-\beta_2B)}{n_1(1-\beta_1B)}=\frac{n_2\gamma_1}{n_1\gamma_2}\in[0,\infty]$$

Introduction
Nonthermal ...
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Any cosmic outflow ...
Summary and ...

5.3. Electrostatic instability (EI) for parallel propagation

For fluctuations parallel to the flow speed the dispersion relation (11) yields electrostatic waves

$$1 = \frac{g\Omega^2}{\Gamma_0^2} = \frac{\Omega^2}{\Gamma_0^2}\left[\frac{1}{(\omega - ku)^2} + \frac{r}{(\omega + ku)^2}\right]$$

with the maximum growth rate shown

$$(\Im\omega)_{\max}(r) = \frac{3^{1/2}ku\alpha}{1+\alpha+\alpha^2} = \frac{3^{1/2}\omega_2\alpha}{2(1+\alpha+\alpha^2)}\left[1 + \frac{1+2\alpha+3\alpha^2}{\alpha^3(2+\alpha)}\right]^{1/2}$$

$$= \frac{3^{1/2}\Omega\alpha}{2\Gamma_0(1+\alpha+\alpha^2)}\left[1 + \frac{1+2\alpha+3\alpha^2}{\alpha^3(2+\alpha)}\right]^{1/2}$$

where

$$\frac{1+2\alpha}{\alpha^3(2+\alpha)} = r$$

Interpolation formula

$$(\Im\omega)_{\max}(r) \simeq \frac{3^{1/2}}{2^{4/3}}\omega_2\frac{r^{1/3}}{(1+r)^{1/6}} = \frac{3^{1/2}}{2^{4/3}}\frac{\Omega}{\Gamma_0}\frac{r^{1/3}}{(1+r)^{1/6}}$$

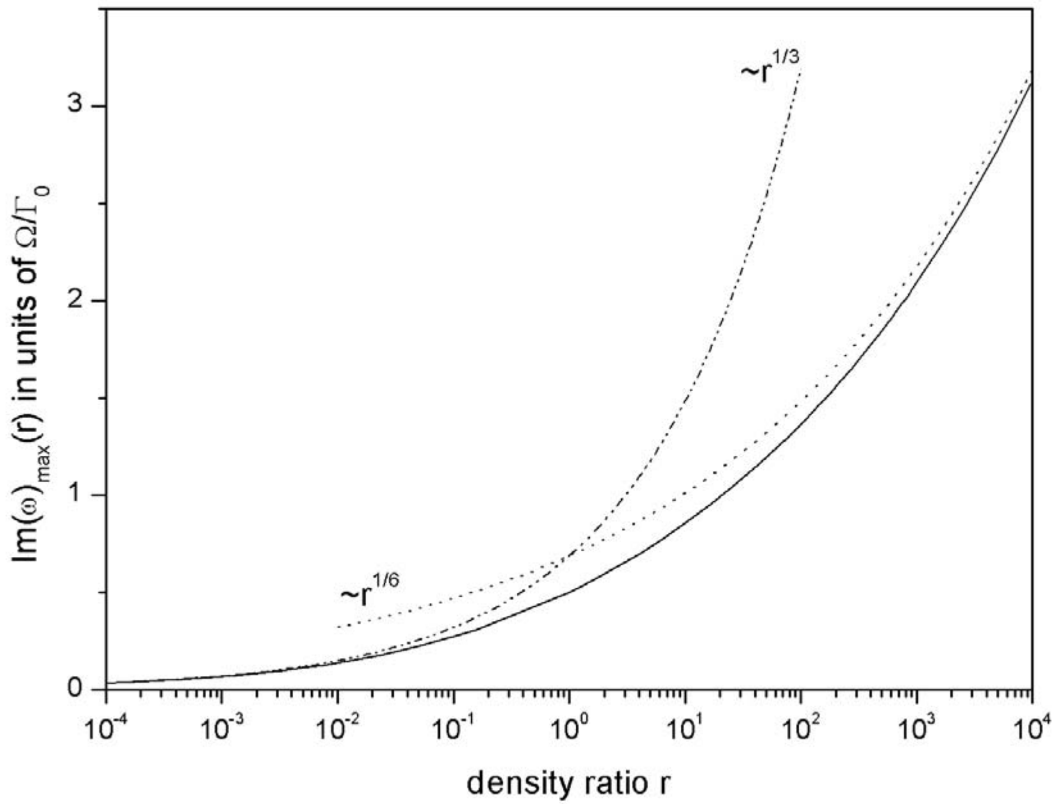


Figure 1: Maximum growth rate of the electrostatic instability in units of Ω/Γ_0 as a function of the density ratio r.

5.4. Filamentation instability (FI) for perpendicular propagation

For fluctuations perpendicular to the flow speed the dispersion relation (11) becomes with $x \equiv \frac{\omega^2}{\Omega^2(1+r)}$

$$\frac{c^2k^2}{\Omega^2(1+r)} = F(x),\ F(x) = \frac{x(x-1)(x-\Gamma_0^{-2})}{(x-x_1)(x+x_2)}$$

where

$$x_{1,2} = \frac{1}{2\Gamma_0^2}\left[\sqrt{1+\frac{16r}{(1+r)^2}\Gamma_0^2(\Gamma_0^2-1)} \pm 1\right] > 0$$

Aperiodic fluctuation solutions occur for negative $x = -y$, $y \geq 0$, corresponding to purely imaginary frequencies $\omega = \pm\imath\Omega(1+r)^{1/2}y^{1/2}$ so that one root is purely growing in time.

$$\frac{c^2k^2}{\Omega^2(1+r)} = F(y),\ F(y) = \frac{y(y+1)(y+\Gamma_0^{-2})}{(x_2-y)(y+x_1)}$$

The function $F(y)$ is shown for $\Gamma_0 = 2$ and $r = 0.5$.
Solutions with real y require positive values of the function $F(y) \geq 0$ which is only possible for values $y \leq x_2$.

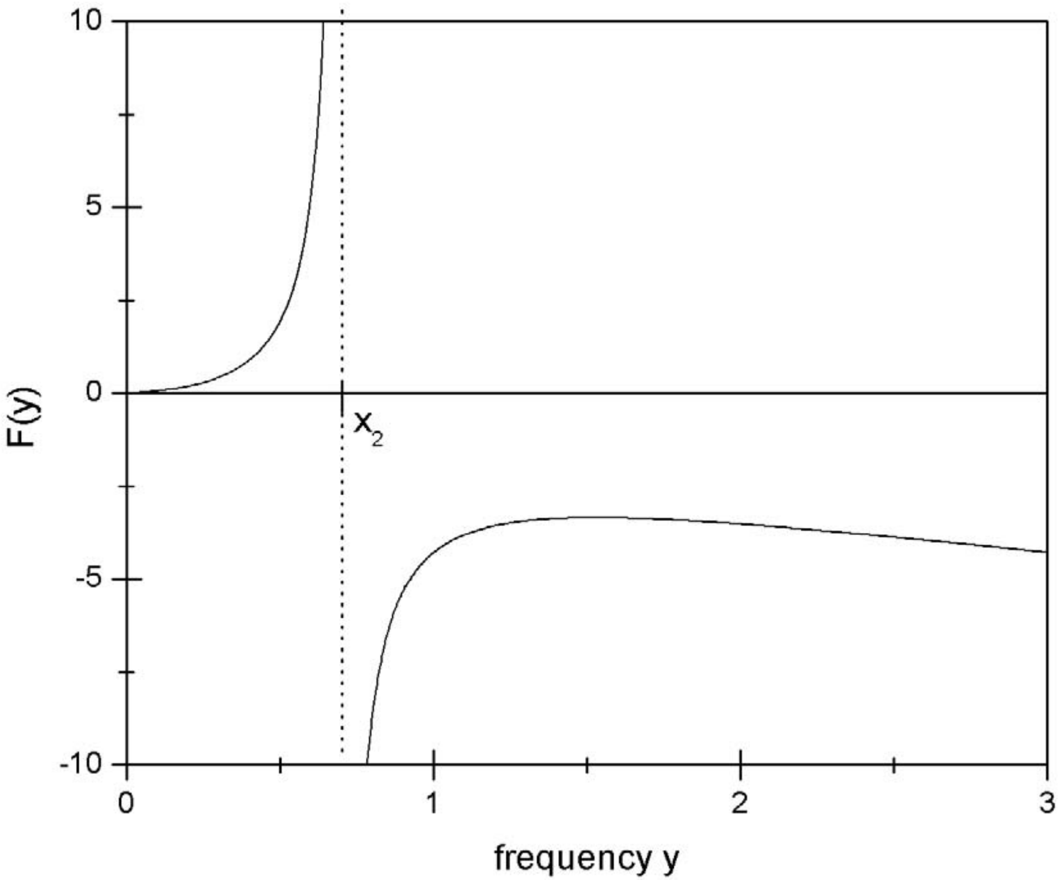


Figure 2: Sketch of the function $F(y)$ for $\Gamma_0 = 2$ and $r = 0.5$.

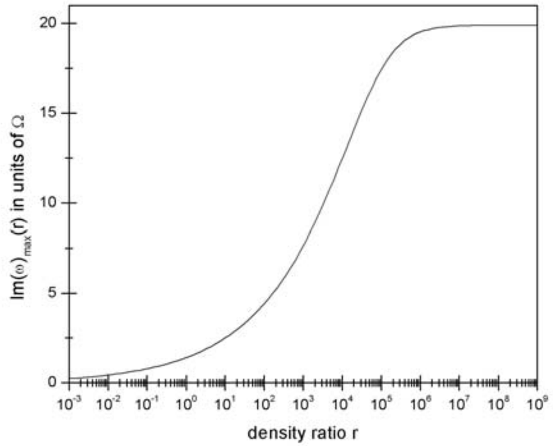


Figure 3: Maximum growth rate of the filamentation instability as a function of density ratio r calculated for $\Gamma_0 = 10$.

The maximum growth rate of the filamentation instability occurs at large wavenumbers and as a function of the density ratio r is given by

$$(\Im\omega)_{\max}(r) = \Omega(1+r)^{1/2}x_2^{1/2} = \frac{\Omega(1+r)^{1/2}}{2^{1/2}\Gamma_0}\left[\sqrt{1+\frac{16r}{(1+r)^2}\Gamma_0^2(\Gamma_0^2-1)}-1\right]^{1/2}$$

5.5. Comparison of maximum growth rates for EI and FI

5.5.1. Nonrelativistic flows

For nonrelativistic flows $\mathtt{u} \ll c$ the EI has the maximum growth rate

$$(\Im\omega)_{\max}(r) \simeq \frac{3^{1/2}}{2^{4/3}}\Omega\frac{r^{1/3}}{(1+r)^{1/6}}$$

at parallel wavenumbers

$$k_{\parallel,\max} \simeq \frac{\Omega(1+r)^{1/2}}{2\mathtt{u}}$$

The FI attains its maximum growth rate

$$(\Im\omega)_{\max}(r) \simeq 2\Omega\frac{\mathtt{u}}{c}\left(\frac{r}{r+1}\right)^{1/2}$$

at perpendicular wavenumbers much larger than

$$k_{\perp} \gg \frac{\Omega(1+r)^{1/2}}{c}$$

For nonrelativistic flows the EI has a much larger (factor $c/\mathtt{u} \gg 1$)) maximum growth rate than the FI. We conclude that for nonrelativistic flows the EI is the fastest instability. Hence nonrelativistic outflows preferentially generate **electric** fluctuations $\rightarrow$ stochastic acceleration.

5.5.2. Relativistic flows

At relativistic flows ($\Gamma_0 \gg 1$) FI growth rate factor Γ_0^2 larger than EI growth rate for density ratios $r \leq \Gamma_0^{12}$. For relativistic flows the FI is the fastest instability, generating $\mathbf{magnetic}$ fluctuations $\rightarrow$ conversion but not stochastic acceleration.

5.6. Open questions

– Role of oblique fluctuations (Bret 2009)?
– Effect of finite plasma temperatures, guide magnetic field
– Nonlinear evolution of fluctuations ?

5.7. Relativistic pick-up

In relativistic outflows filamentation instabilities are the most important dissipation processes generating aperiodic magnetic fluctuations $\delta B \propto \exp(\imath \vec{k} \cdot \vec{x} + \Gamma t), \quad \omega_R = 0$. Because $\omega_R = 0$ these fluctuations are not group-propagating (zero phase and group speed), so they stand with the exciter.

These instabilities are best known in unmagnetized ($B_0 = 0$) plasmas but also occur in weakly magnetized ($B_0 < B_c$) plasmas. The instabilities are suppressed in strongly magnetized plasmas ($B_0 > B_c$).

Interstellar ions, electrons and neutrals enter the outflow plasma as weak beam, beam excites aperiodic magnetic fluctuations (δB) propagating perpendicular to counterstream direction that then isotropise incoming p^+, e^- after time t_s, i.e. **pick-up of interstellar ions and electrons in outflow**

consequence 1: outflow picks up interstellar p^+ and e^- with

$$E_{p,\max} = \Gamma m_p c^2 = 100\Gamma_{100} \text{ GeV}$$

$$E_{e,\max} = \Gamma m_e c^2 = 0.05\Gamma_{100} \text{ GeV}$$

primary energy output at TeV energies in lab frame:

in $p^+ - e^-$ outflows $E^*_{\gamma,\max} = 20\Gamma^2_{100}$ TeV

in pair outflows $E^*_{\gamma,\max} = 0.01\Gamma^2_{100}$ TeV

6. Summary and conclusions

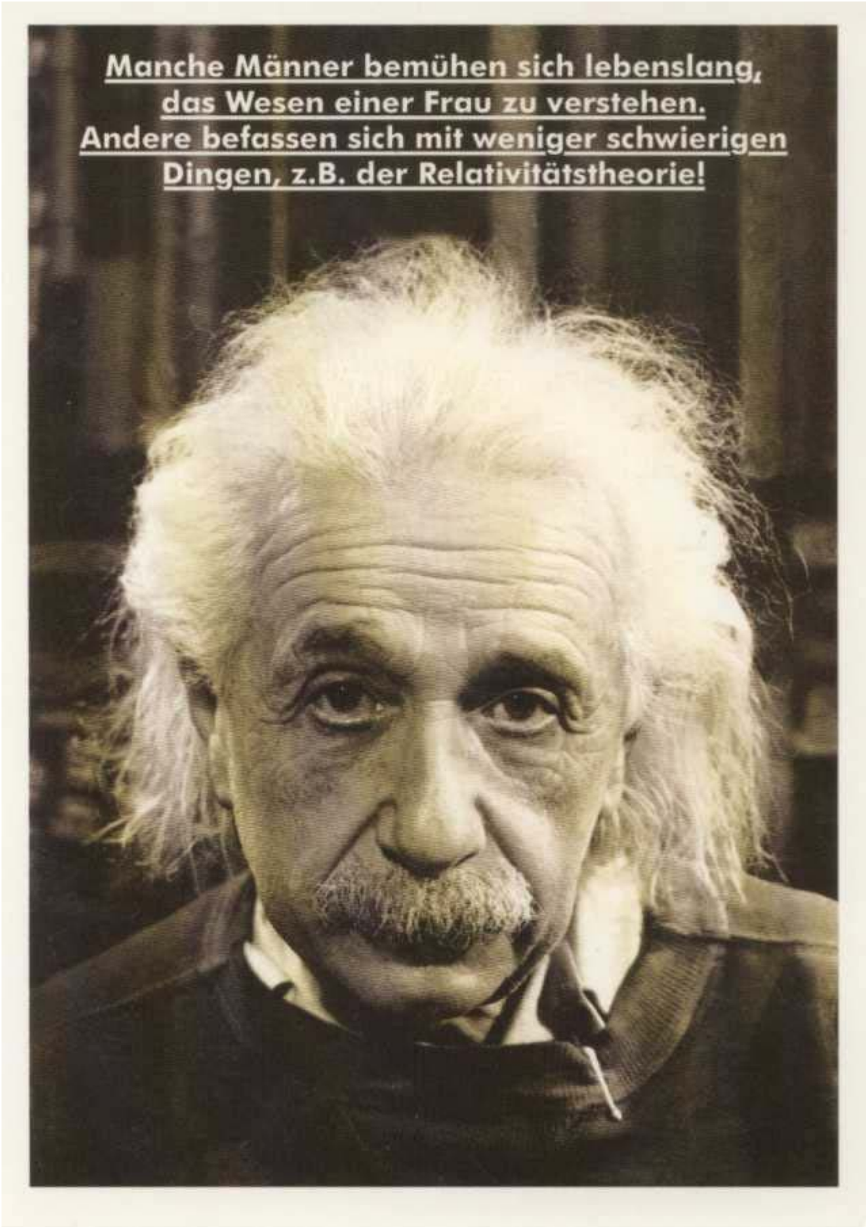
- We have identified the fundamental dissipation processes of cosmic explosions in collisionless palsma environments.

- In relativistic outflows central role played by filamentation instabilities that generate **magnetic** fluctuations. In nonrelativistic outflows electrostatic instabilities are quicker generating **electric** fluctuations.

- For **magnetic** fluctuations energization by conversion processes (relativistic pick-up) most relevant. For **electric** fluctuations stochastic acceleration processes most important.

- We begin to understand the formation of particle momentum spectra from competition of acceleration/energization, loss and escape processes.

- Ongoing efforts to improve theory in all details. New discipline: **particle-photon-plasma astrophysics**


Manche Männer bemühen sich lebenslang,
das Wesen einer Frau zu verstehen.
Andere befassen sich mit weniger schwierigen
Dingen, z.B. der Relativitätstheorie!