The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2052-53

Summer College on Plasma Physics

10 - 28 August 2009

Wave breaking of electrostatic waves in warm plasma

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Acknowledgments:

P. Norreys, D. Burton, A. Noble, ASTEC

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Motivation

Wave breaking is a poorly understood phenomenon in plasma physics… but why?

- Too many conflicting definitions
- Too many models
- No effort to understand consequences of one's own model
- No effort to compare different models

Definition of wave breaking

"Wave breaking occurs when the forward fluid or plasma motion is larger than the wave phase speed"

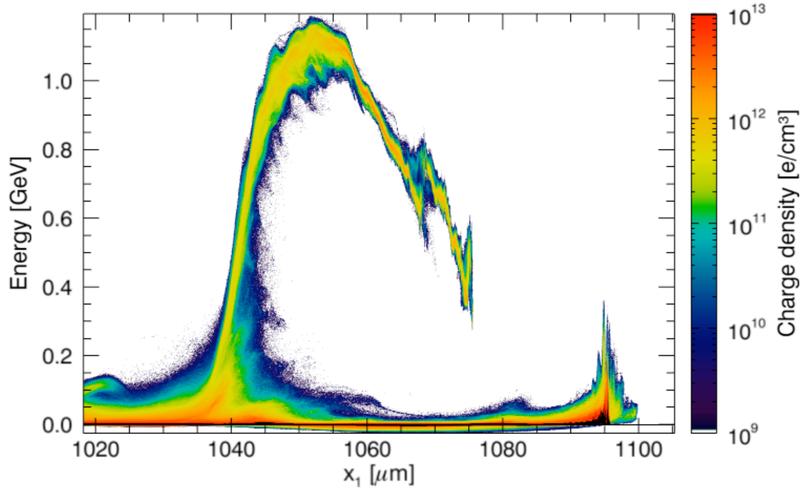


Image: L.O. Silva

Wave breaking in cold plasma

Wave breaking for an oscillation in cold plasma with wave number k and amplitude A, according to Dawson:

- Neighbouring plasma fluid elements "collide" during oscillations
- $k*A = 1$
- $v = \omega*A = (\omega/k)*k*A = v_\varphi$
- Plasma fluid elements overtake (and get trapped in) the wave

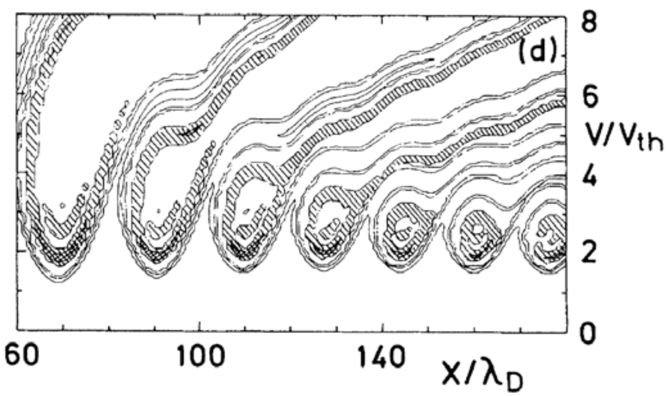
J.M. Dawson, Phys. Rev. **113**, 383 (1959)

Wave breaking in warm plasma

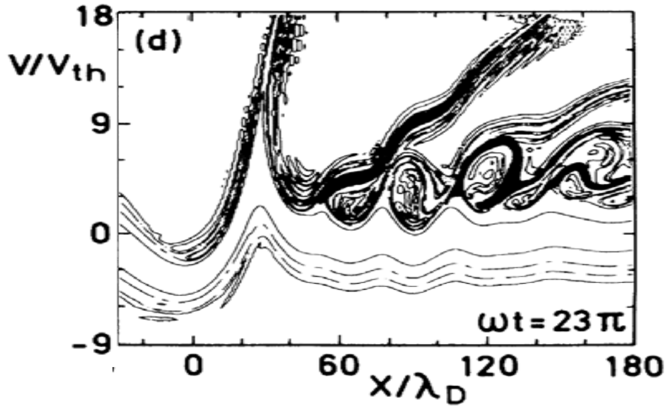
There is always some trapping in a warm plasma

Wave breaking implies heavy particle trapping

trapping only



trapping and wave breaking



Definition: a wave breaks when it traps particles at the electron sound speed: $s_0^2 = 3k_BT/m_e$

A. Bergmann and P. Mulser,
Phys. Rev. E **47**, 3585 (1993)

Wave breaking in practice

Most theoretical papers use the following approach:

- Derive warm-fluid model from Vlasov equation
- Push model until it breaks down
- Confuse model breakdown with wave breaking
- If model does not break down, wave breaking does not exist ???

Problem with standard approach

- Breakdown is a mathematical property of a model
- One must prove separately (e.g. by relating breakdown to particle trapping) that this corresponds to a physical phenomenon like wave breaking
- Few authors bother to do this
- Result: different models, different breakdown limits, different “wave breaking limits”

Case 1: quasi-static waves

Wave in a homogeneous plasma, only depends on coordinate $\zeta = z - v_\varphi * t$

Cold, non-relativistic: $E_{WB} = v_\varphi$

J.M. Dawson, Phys. Rev. **113**, 383 (1959)

Cold, relativistic: $E_{WB} = \sqrt{2(\gamma_\varphi - 1)}$

Akhiezer and Polovin, Sov. Phys. JETP **3**, 696 (1956)

Warm, non-relativistic:

$$E_{WB} = v_\varphi \left[1 - 8/3 \left(\beta / v_\varphi^2\right)^{1/4}\right]^{1/2}, \quad \beta = 3k_B T / \left(mc^2\right)$$

T.P. Coffey, Phys. Fluids **14**, 1402 (1971)

Warm, relativistic: needs more attention

Case 1: warm, (ultra-)relativistic

Many models for this case, most are invalid.

Relativistic waterbag: $E_{WB} \propto \sqrt{\ln\left(\gamma_\varphi^2 \beta\right)} \Big/ \beta^{1/4}$

- Proper relativistic plasma pressure
- Good correspondence between model breakdown and particle trapping
- Particle trapping handled properly
  - Use of warm-plasma potential to study trapping in warm plasma
  - No separate plasma pressure term in particle Hamiltonian, as it should be
- $E_{WB} \to \infty$ for $\gamma_\varphi \to \infty$, as it should be

Katsouleas and Mori, PRL **61**, 90 (1988)

Trines and Norreys, Phys. Plasmas **13**, 123102 (2006)

Case 1: alternative models

Warm-plasma approximation: $E_{WB} \propto 1/\beta^{1/4}$
misbehaves for $\gamma_\varphi \to \infty$ (E_{WB} fails to diverge)

J.B. Rosenzweig, Phys. Rev. A **38**, 3634 (1988)
Z.M. Sheng and J. Meyer-ter-Vehn, Phys. Plasmas **4**, 493 (1997)
Schroeder, Esarey and Shadwick, PRE **72**, 055401(R) (2005)
Schroeder *et al.*, Phys. Plasmas **13**, 033103 (2006)
Esarey *et al.*, Phys. Plasmas **14**, 056707 (2006)

Three-fluid model: $E_{WB} \propto \sqrt{\gamma_\varphi/3}$
behaves too much like cold plasma

J.B. Rosenzweig, Phys. Rev. A **40**, 5249 (1989)

Method of characteristics: $E_{WB} = \infty$
wave never breaks in this model

Aleshin *et al.*, Plasma Phys. Rep. **19**, 523 (1993)
A. Khachatryan, Phys. Plasmas **5**, 112 (1998)

Case 2: non-quasi-static waves

Non-quasi-static plasma waves exhibit a spectrum of "weird" phenomena, such as:

- position-dependent frequency
- amplitude-dependent frequency
- secular behaviour
- wave number advection in thermal plasma
- curbing of wave breaking by thermal effects (unheard of for quasi-static waves)

Non-constant frequency

Relativistic cold-plasma oscillations according to Polovin:

$$\frac{\partial^2 p}{\partial \tau^2} + \frac{\omega_p^2(\underline{x})}{\sqrt{1+p^2}}\, p = 0$$

Position-dependent frequency can result from:

- non-constant density
- position-dependent amplitude

R.V. Polovin, Sov. Phys. JETP **4**, 290 (1957)

Secular behaviour

Consider the following plasma oscillation:

$$\delta = A\sin[\psi(\underline{x},\tau)], \quad k \equiv \partial\psi/\partial\underline{x}, \quad \omega \equiv -\partial\psi/\partial\tau$$

It follows (Whitham): $$\frac{\partial k}{\partial \tau} + \frac{\partial \omega}{\partial \underline{x}} = 0$$

If ω depends on x, then k depends on τ !

Example: k will grow linearly on a downward density ramp

Significant consequences for wave breaking

J.M. Dawson, Phys. Rev. **113**, 383 (1959)

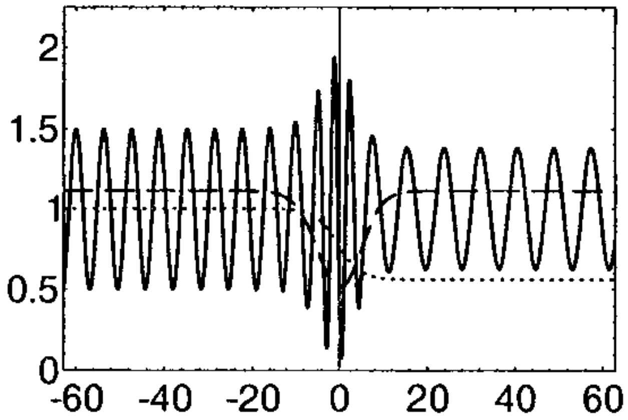
J.F. Drake *et al.*, Phys. Rev. Lett. **36**, 196 (1976)

Secular behaviour II

Wave breaking condition (Dawson, Coffey):

$$kA = 1 - \left(\beta / v_\varphi^2\right)^{1/4}$$

If k grows, then v_φ will decrease (Bulanov) and wave breaking amplitude will decrease



Dotted: plasma density
Solid: plasma oscillations
Dashed: local phase speed

Bulanov *et al.*, PRE **58**, R5257 (1998)

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Wave number advection

Add thermal effects (Bohm-Gross):

$$\omega^2 = \omega_p^2 + 3v_T^2 k^2$$

This leads to:

$$\frac{\partial k}{\partial \tau} + 3v_T^2 \frac{k}{\omega} \frac{\partial k}{\partial \underline{x}} = -\frac{\omega_p}{\omega} \frac{\partial \omega_p}{\partial \underline{x}}$$

Thermal effects make k advect away from the "hot spot" !

Which will win, secular behaviour or thermal effects?

R. Trines, PRE **79**, 056406 (2009)

Curbing of wave breaking

Wave on density ramp, total density drop Δn:

- In cold plasma, k will grow indefinitely and wave will always break
- In warm plasma, growth of k is curbed by thermal effects
- For small amplitude, small Δn and large plasma temperature, wave will not break
- Compare with quasi-static waves, where thermal effects facilitate wave breaking

R. Trines, PRE **79**, 056406 (2009)

Case 3: driven waves (resonance absorption)

- Related to previous case (density ramp), but waves are driven by long laser beam
- Found 4 different models for warm plasma:
  - Ginzburg, Propagation of EM waves in plasma (1960)
  - Kruer, Phys. Fluids 22, 1111 (1979)
  - Bezzerides and Gitomer, Phys. Fluids 26, 1359 (1983)
  - Bergmann and Mulser, PRE 47, 3585 (1993)
- Will have to pull all these together to solve this problem: not easy

Conclusions

- Wave breaking: an important phenomenon, often misunderstood because of bad definitions
- With good definitions and careful analysis, one can make significant headway
- Thermal effects and secular behaviour together lead to fascinating behaviour
- Resonance absorption will be next
- Always some other aspect to explore, e.g. using more than 1 dimension!