



*The Abdus Salam*  
*International Centre for Theoretical Physics*



**2052-59**

**Summer College on Plasma Physics**

***10 - 28 August 2009***

**Momentum Transport and Zonal Flow Generation in Magnetized Plasmas**

Jens Juul Rasmussen  
*Risø National Laboratory for Sustainable Energy*  
*Denmark*

# Momentum Transport and Zonal Flow Generation in Magnetized Plasmas

Jens Juul Rasmussen

V. Naulin, J. Madsen, A.H. Nielsen, J. Kastrup, M. Hoffmann,  
O.E. Garcia\*

Association EURATOM - Risø DTU, Roskilde, Denmark

\*Dept. Physics and Technology, University of Tromsø, Tromsø, Norway

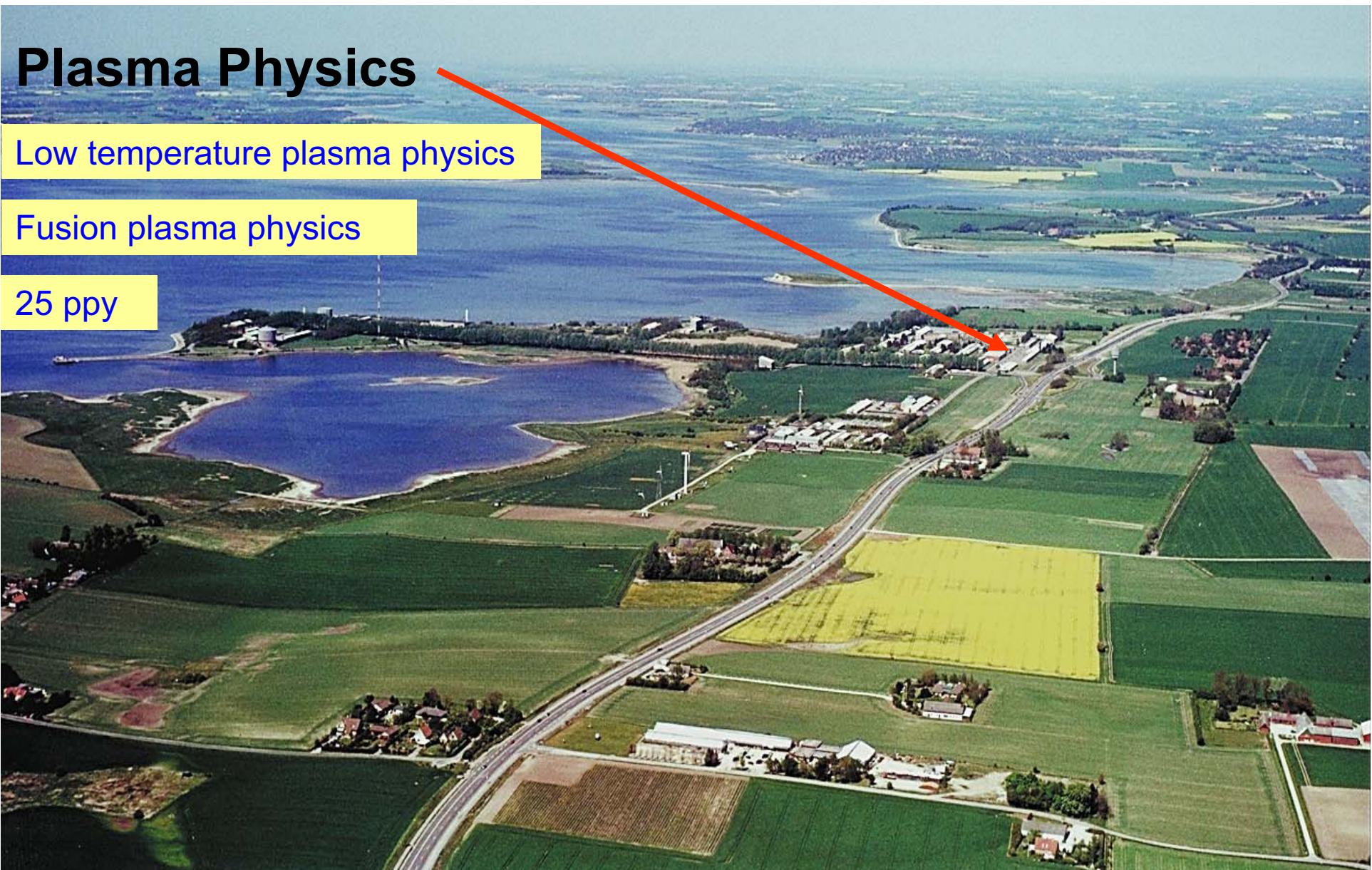
[jjra@risoe.dtu.dk](mailto:jjra@risoe.dtu.dk)

Risø DTU  
National Laboratory for Sustainable Energy

$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$

$\Theta^{\sqrt{17}} + \Omega \int \delta e^{i\pi} =$   
 $\mathcal{E}_{\infty} = \{2.7182818284$   
 $\chi^2 \Sigma \gg ,$   
 $\sum!$

# Risø National Laboratory for Sustainable Energy, Technical University of Denmark



**Plasma Physics**

Low temperature plasma physics

Fusion plasma physics

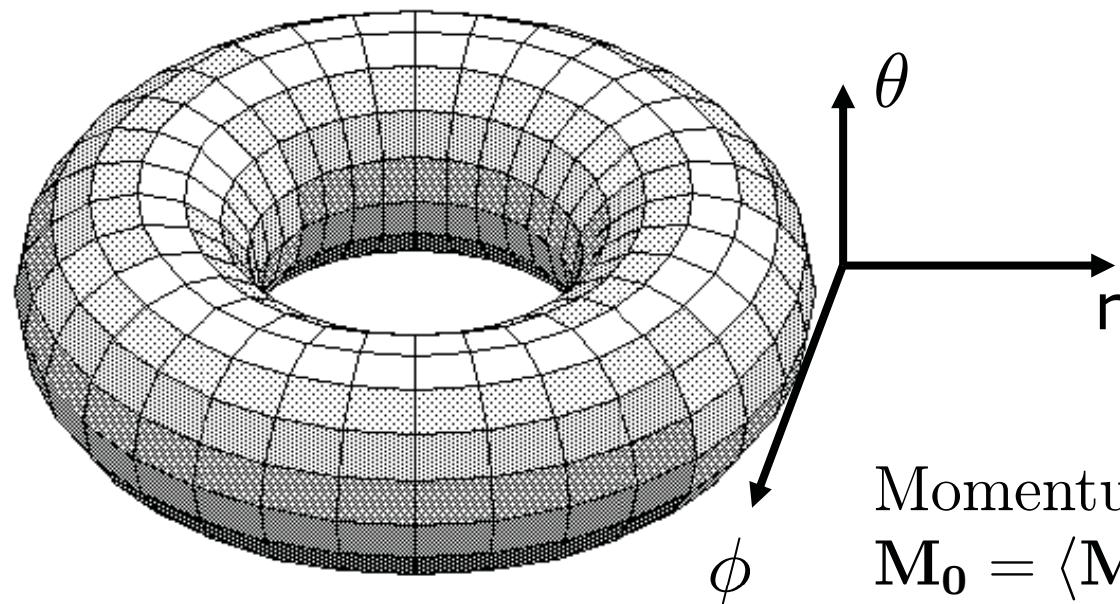
25 ppy

# Motivation

- Turbulence and the associated transport is known to be the most important transport channel for degrading confinement of hot plasmas
- Increasing interest in understanding the momentum generation and transport in magnetized plasmas : importance for confinement scenarios
- Momentum is now measurable in Tokamak devices and results on momentum transport are becoming available, understanding is still lacking
- Discussion of momentum transport in simple models, with illustrations from simulations and experiments

# Momentum density

Toroidal magnetized plasma



Momentum density (mass = 1) :  
 $\mathbf{M}_0 = \langle \mathbf{M} \rangle = \langle n\mathbf{v} \rangle$

$\langle \cdot \rangle$  averaging over  $^{\circ}$  ux surface

Toroidal momentum density :  $M_{\phi 0} = \langle M_\phi \rangle = \langle nv_\phi \rangle$

Poloidal momentum density :  $M_{\theta 0} = \langle M_\theta \rangle = \langle nv_\theta \rangle \rightarrow$

Consider here the B-perp momentum density and flux

# Anomalous transport in turbulent plasmas

Radial transport across confining magnetic field

Particle density flux :

$$\Gamma_0 = \langle \Gamma \rangle = \langle n v_r \rangle = \langle \tilde{n} \tilde{v}_r \rangle$$

Energy density flux :

$$Q_0 = \langle Q \rangle = \langle n T v_r \rangle = n_0 \langle \tilde{T} \tilde{v}_r \rangle + T_0 \langle \tilde{n} \tilde{v}_r \rangle + \langle \tilde{n} \tilde{T} \tilde{v}_r \rangle$$

Momentum density flux (poloidal momentum):

$$\Pi_0 = \langle \Pi \rangle = \langle n v_\theta v_r \rangle = n_0 \langle \tilde{v}_\theta \tilde{v}_r \rangle + v_{\theta 0} \langle \tilde{n} \tilde{v}_r \rangle + \langle \tilde{n} \tilde{v}_\theta \tilde{v}_r \rangle$$

Reynolds stress term:  
turbulence  $\rightarrow$  flows
↑  
**Passive convection**

$$n = n_0 + \tilde{n}, T = T_0 + \tilde{T}, v = v_0 + \tilde{v}, (v_{r0} = 0)$$

**Myra et al Phys. Plasma 15, 032304 (2008)**

# Impact of fluxes

- Particle and energy flux degrade confinement and provide hazards to plasma facing components
- Momentum flux is not directly harmful in the same manner, but momentum flux sets up plasma rotation and zonal flows that control turbulence and transport
- Momentum flux is thought to be instrumental in the L-H transition in Tokamak plasmas
- In absence of sources and sinks:  
Global momentum is conserved :  $\partial_t M_0 + v_r \partial_r M_0 = 0$

No global spin up! Bi-polar flows excited

# Example: Turbulence – flow interplay



## Turbulence model: ESEL – 2D interchange dynamics

A self-consistent description of fluctuations and intermittent transport in the edge/SOL by employing the RISØ ESEL (Edge SOL Electrostatic) model for interchange dynamics that:

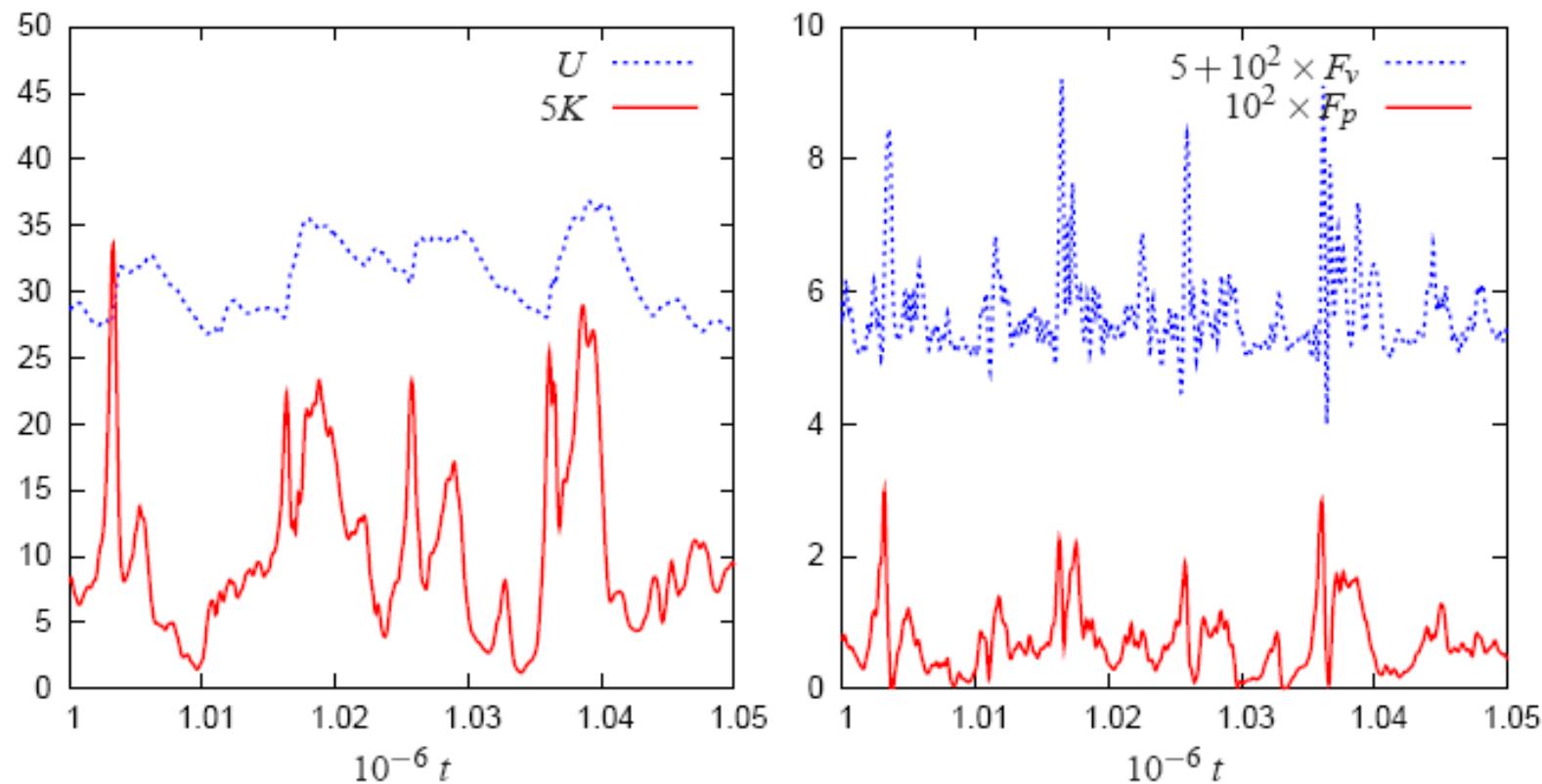
- include separate plasma production ``edge" and loss region ``SOL",
- allow self-consistent flows and profile relaxations,
- profiles and fluctuations are **NOT** separated,
- conserve particles and energy in collective dynamics.

Results agree well with experimental observations!

E.g., TCV, Lausanne (Garcia *et al.* PPCF **48**, L1 (2006)) and JET (Naulin *et al* IAEA-2006))

Garcia, Naulin, Nielsen, Rasmussen, PRL **92** 165003 (2004); Phys. Plasmas **12**, 090701 (2005); Physica Scripta **T122**, 89 (2006); Fundamenski *et al.*, Nucl. Fusion **47**, 417 (2007).

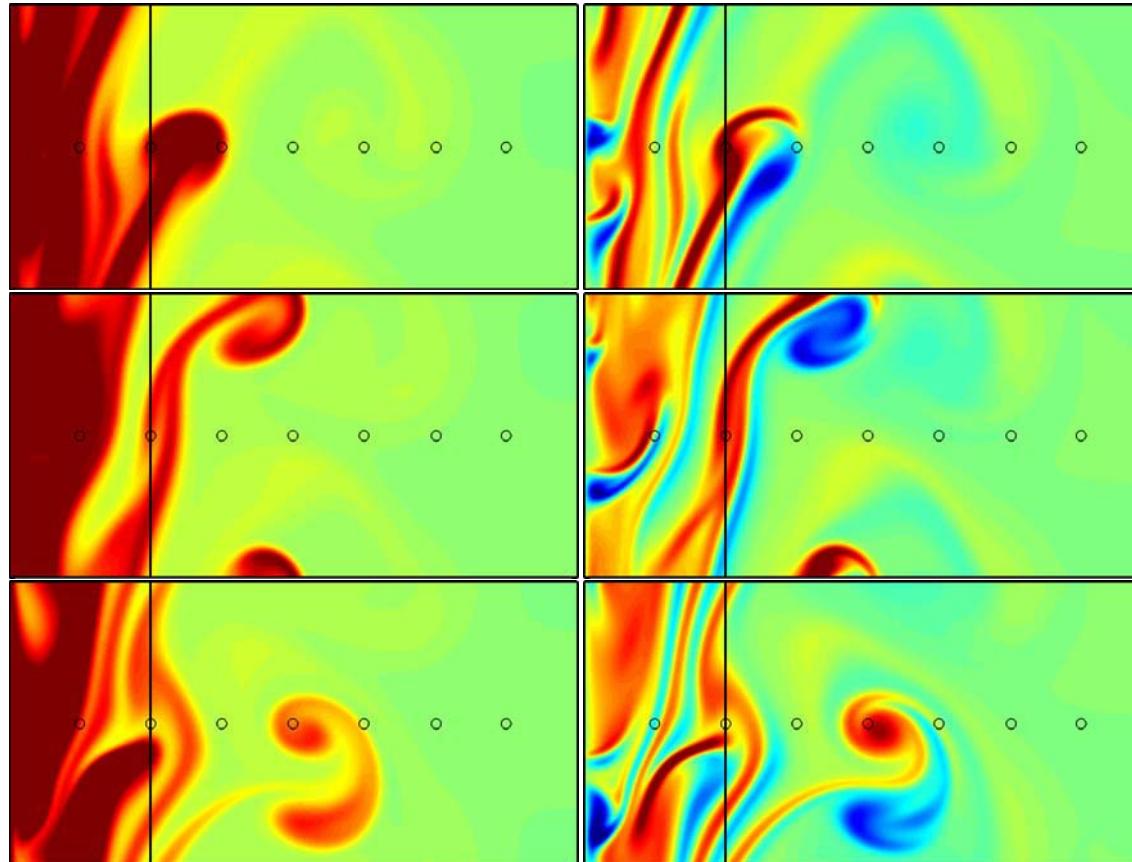
# Energetics and Energy Transfer



*Bursting : Kinetic energy contained in the mean ("zonal flow"),  $U$ , and in fluctuating motions  $K$ .*

*The collective energy transfer terms  $F_p$  (from potential energy in gradients to fluctuations) and  $F_v$  (from fluctuation to flows)*

# Spatial structure during a burst



*Particle density (left) and vorticity (right) during a burst ( $\Delta t = 500$ )  
Blob like-structure in plasma density and dipole structure in vorticity*

# Potential Vorticity, PV



PV – originating from geophysical fluid dynamics – is a quantity conserved on a fluid element along the Lagrangian trajectory.

**Rhines Annual Rev. Fluid Mech 11 401 (1979)**

Plasma case: ion momentum equation for cold ions: PV similar to fluid PV.

Intrinsically 2-Dim equation, ion vorticity  $\omega$  ( $\nabla \times \mathbf{v}$ )

$$\Omega = \frac{\omega + \omega_c}{n}$$

$$\frac{d\Omega}{dt} = \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \Omega = 0$$

Conservation of PV dictates the dynamics

Effective mixing homogenisation of  $\Omega$ ; provide zonal flow  $\langle \omega \rangle$

**Rasmussen et al Physica Scripta T122, 44, (2006);  
Basu et al. Phys Plasma 10, 2696 (2003)**

Recently, applied to momentum transport in magnetized plasma by Diamond et al (**PPCF 50, 124018 (2008)**).

# Potential Enstrophy Evolution I

Applying the drift wave scaling: fluctuations around a background density, PV expands:

$$\Omega \approx \omega - n, \quad \Omega = \Omega_0 + \tilde{\Omega} \quad \begin{aligned} \mathbf{v} \text{ is the } E \times B \text{ velocity} \\ (\text{convection velocity}) \end{aligned}$$

$$\frac{d\Omega}{dt} = \frac{\partial \tilde{\Omega}}{\partial t} + v_r \frac{\partial \Omega_0}{\partial r} + \mathbf{v} \cdot \nabla \tilde{\Omega} = 0$$

Conservation equation for potential enstrophy,  $\langle \tilde{\Omega}^2 \rangle$ :

$$\partial_t \langle \tilde{\Omega}^2 \rangle + \partial_r \langle v_r \tilde{\Omega}^2 \rangle + \langle v_r \tilde{\Omega} \rangle 2 \partial_r \Omega_0 = 0$$

The poloidally averaged flow

$$\partial_t v_{\theta 0} + \partial_r \langle v_r \tilde{v}_\theta \rangle = \mu \partial_r^2 v_{\theta 0} - \nu v_{\theta 0}$$

**Diamond et al PPCF 50, 124018 (2008)**

## Potential Enstrophy Evolution II

Applying:  $\partial_r \langle v_r \tilde{v}_\theta \rangle = \langle v_r \tilde{\omega} \rangle$ ,  $\langle v_r \tilde{\Omega} \rangle = \langle v_r \tilde{\omega} \rangle - \langle v_r \tilde{n} \rangle$

to obtain the zonal momentum conservation:

$$\frac{\partial}{\partial t} \left( v_{\theta 0} - \frac{\langle \tilde{\Omega}^2 \rangle}{2\partial_r \Omega_0} \right) = \frac{\partial_r \langle v_r \tilde{\Omega}^2 \rangle}{2\partial_r \Omega_0} - \langle v_r \tilde{n} \rangle + \mu \partial_r^2 v_{\theta 0} - \nu v_{\theta 0}$$

zonal velocity
enstrophy transport
particle flux
dissipation

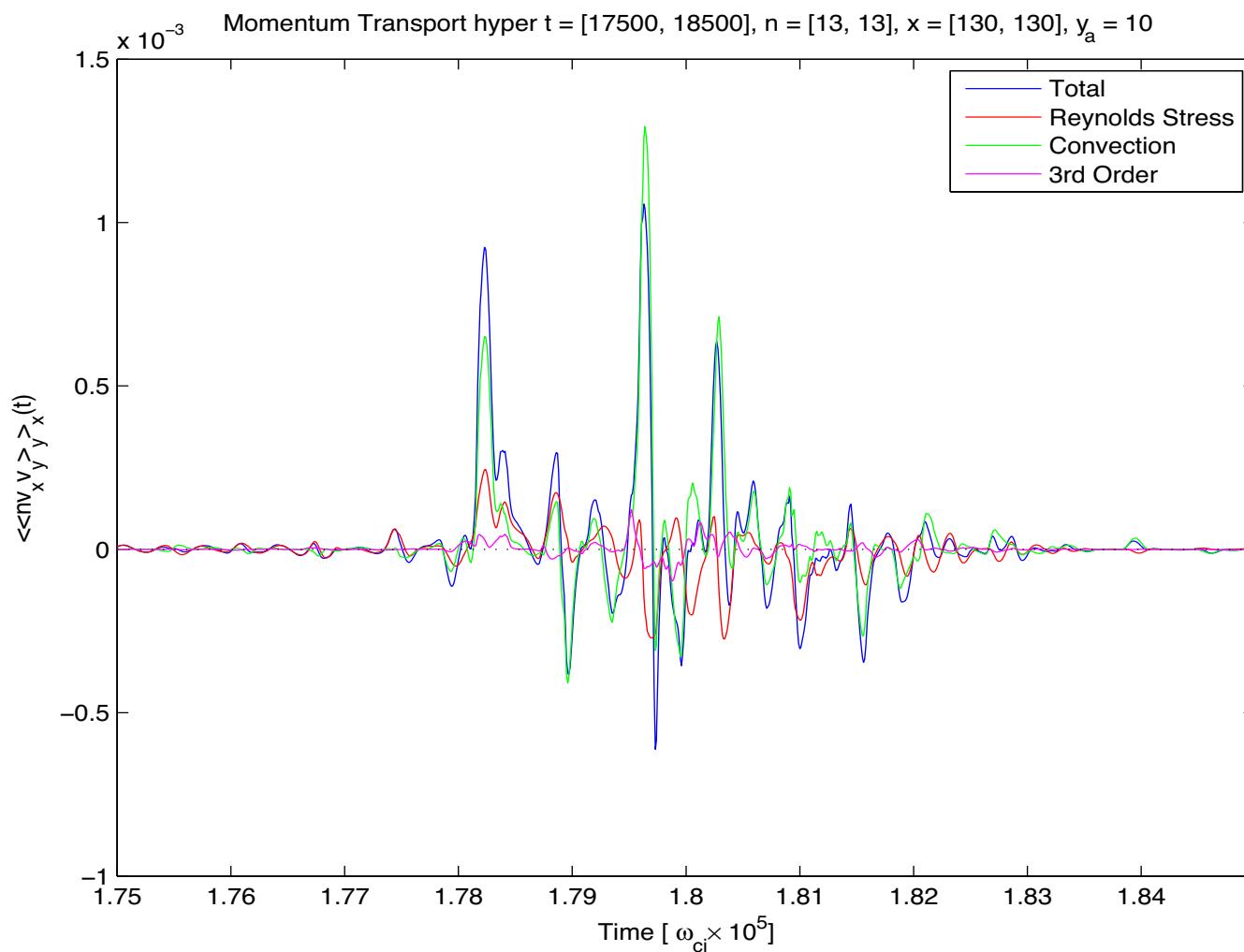
↑      ↑      ↑      ↑  
wave momentum density

Poloidal flow connected to particle flux

Flow structures regulated by dissipation profiles

Stationary turbulence cannot excite zonal flow in absence of "RHS"

# Momentum Transport by Blob



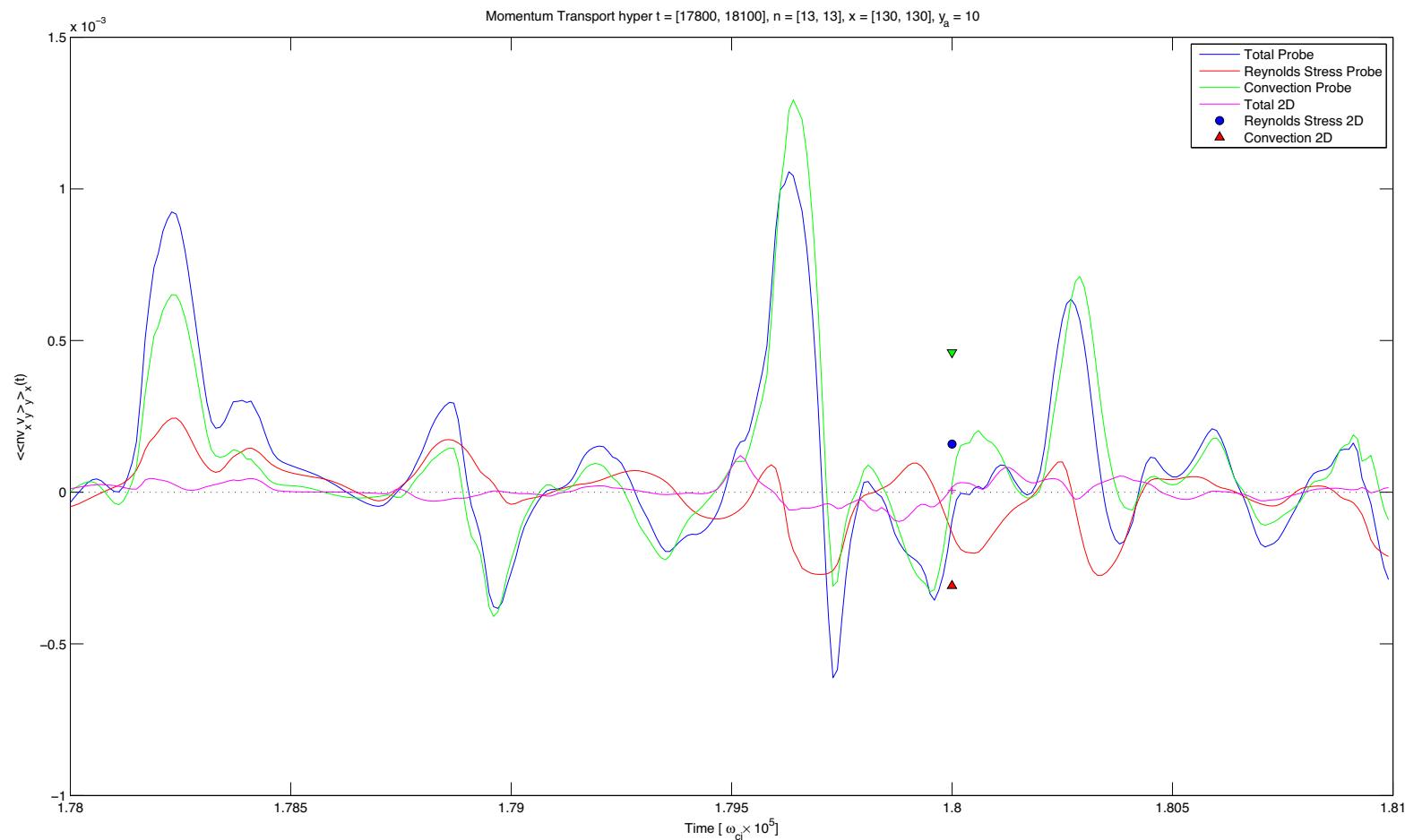
ESEL  
simulations,  
modelling  
probe  
measurements  
in the SOL

Components of momentum flux “measured” by probes in the SOL

# Momentum Transport by Blobs



Expanded scale.

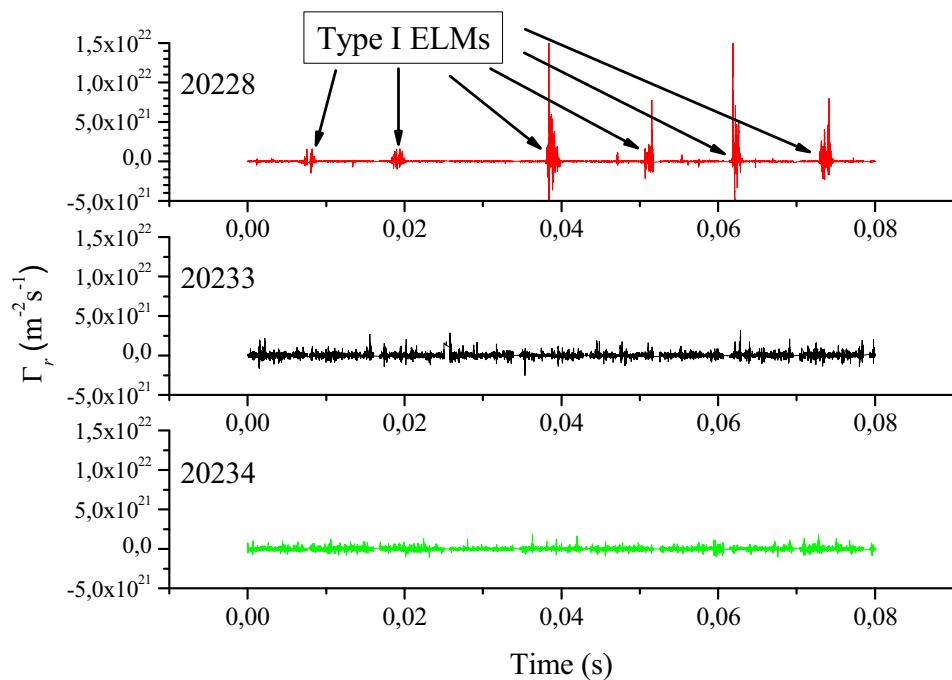


Blob carries its momentum along

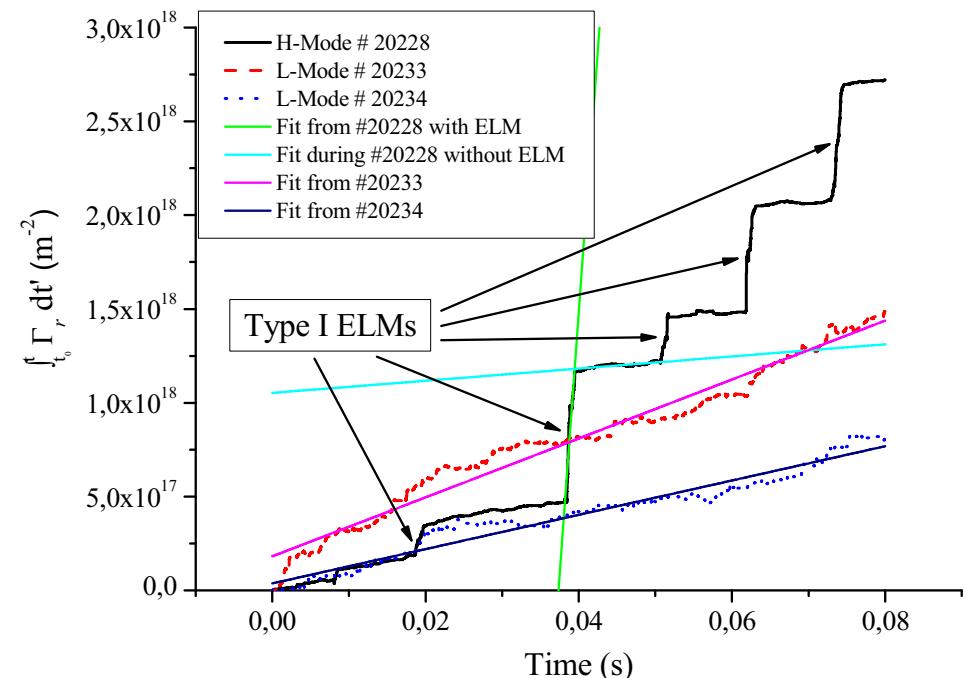
# Particle density flux in H- and L-mode in ASDEX UG



Particle density flux  $\Gamma$



Integrated  $\Gamma$



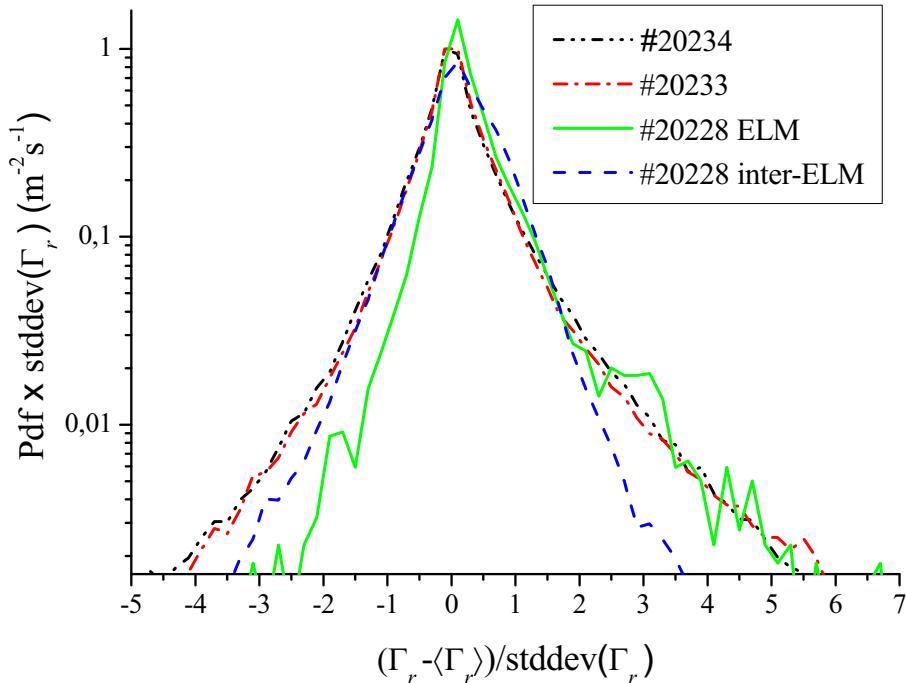
Probe measurements in the SOL of Asdex UG during ELMy H-mode and L-mode.

Ionita, Schrittwieser et al EPS 2008

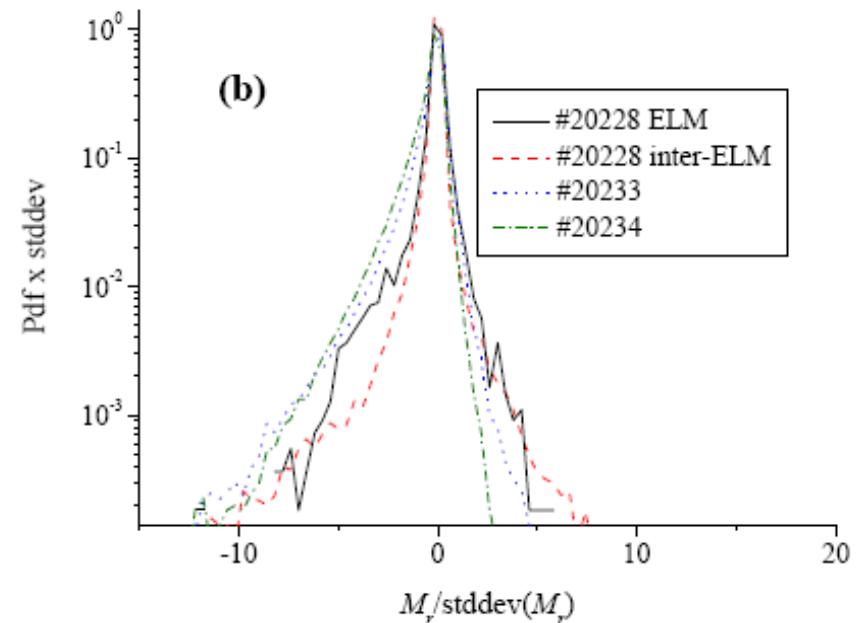
# PDF of the particle and momemtum density flux in the SOL

Similar statistics!

Ionita et al , ICPP - 2008  
and to be published

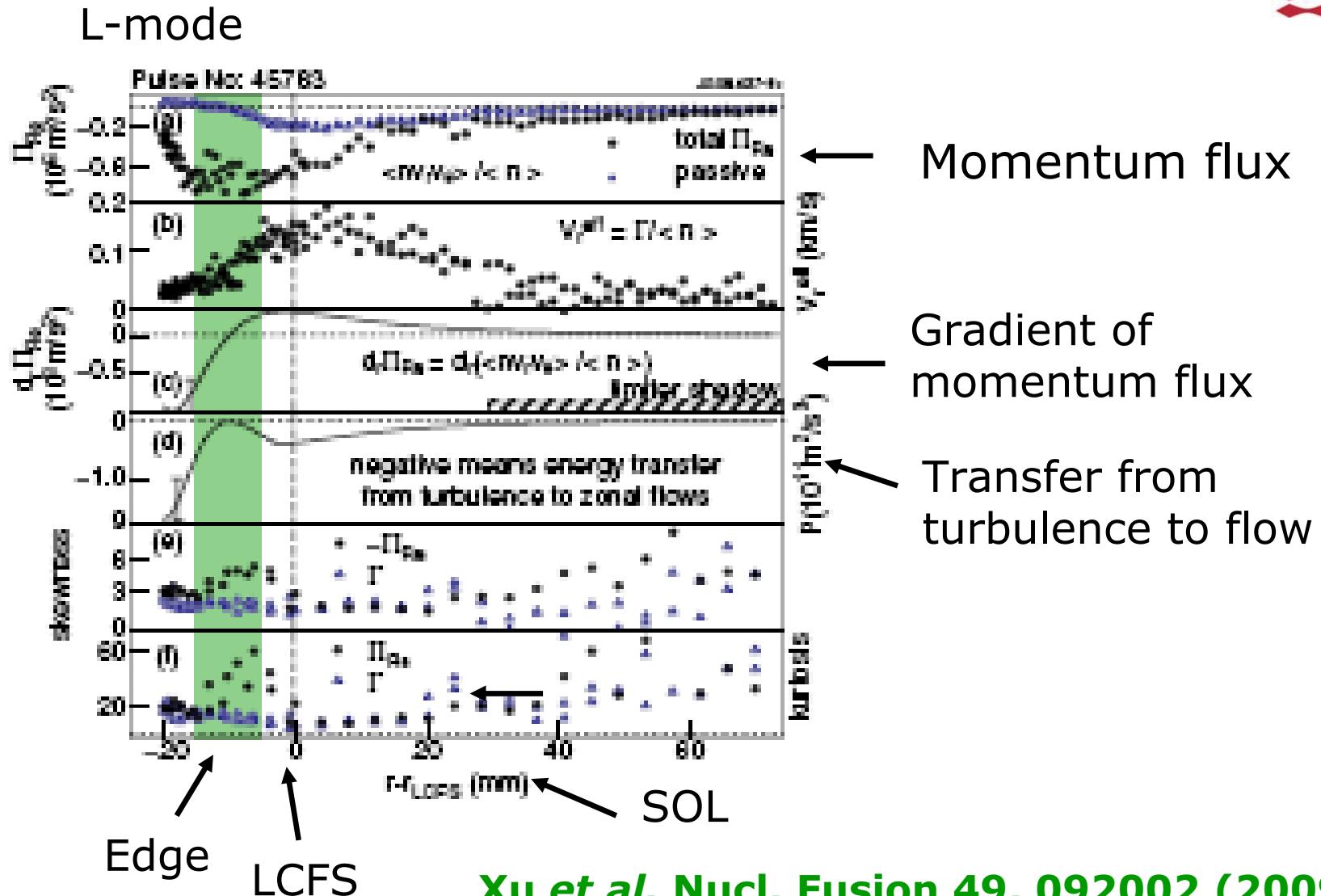


Renormalized PDF of  $\Gamma$  in H-mode during ELM activities, in between ELMs and in two L-mode cases.



Renormalized PDF of momentum density flux,  $\pi$ , in H-mode during ELM activities, in between ELMs and in two L-mode cases.

# Momentum Flux in the JET Edge Plasma



# Summary

- Momentum transport and balance in magnetized plasmas are governed by the Potential Vorticity “conservation”
- Momentum transport is strongly related to particle and energy transport
- Transport events, blobs and ELM structures carry momentum along
- Momentum loss in the SOL important for flow generation in the plasma edge
- Full 3-D description needed for poloidal and toroidal momentum balance

# Thank you for your attention



# some details

# Model Equations: ESEL

2D model for cold ions and quasi-neutrality  $n_i \approx n_e = n$ .

$$\text{Vorticity } \Omega = \nabla \times \vec{u}_E \cdot \hat{z} = \nabla_{\perp}^2 \phi$$

2D interchange dynamics

$$\frac{dn}{dt} + n\mathcal{C}(\phi) - \mathcal{C}(nT) = \nabla \cdot \left( \nu_e \rho_e^2 (\nabla n - \frac{n}{2T} \nabla T) \right) - \frac{n}{\tau_{\parallel n}}$$

$$\frac{dp}{dt} - \frac{5}{3}p\mathcal{C}(\phi) + \frac{5}{3}\mathcal{C}(pT) = \frac{3}{2} \nabla \cdot (\kappa_{\perp} \nabla T + \nu_e \rho_e^2 \nabla p) - \frac{p}{\tau_{\parallel p}}$$

$$\frac{d\Omega}{dt} - \mathcal{C}(nT) = \nu_{\Omega} \nabla_{\perp}^2 \Omega - \frac{\Omega}{\tau_{\parallel \Omega}} \quad (-\sigma\phi) \leftarrow \text{Sheath dissipation}$$

Advection derivative and curvature operators defined by

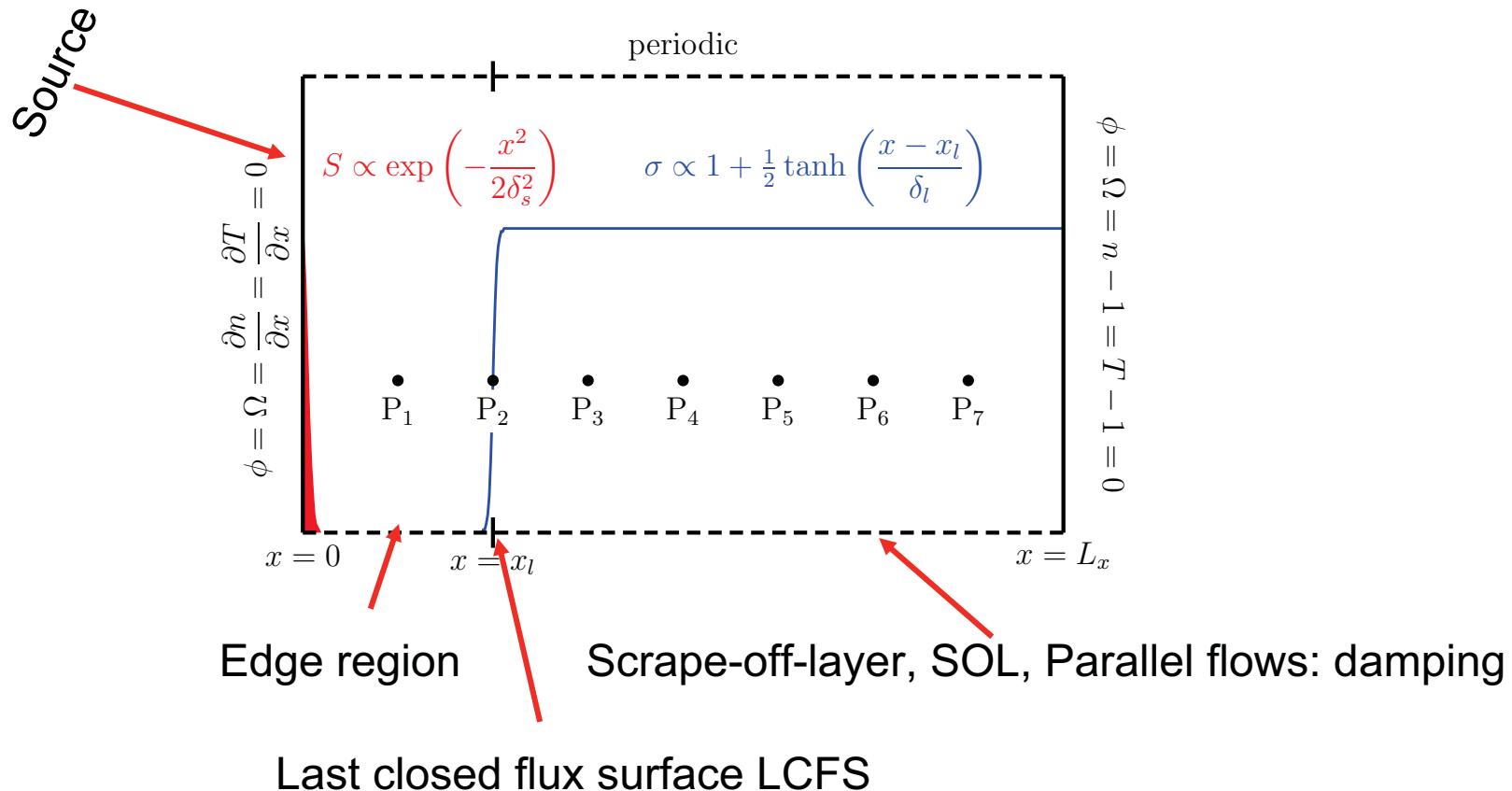
$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B} \hat{z} \times \nabla \phi \cdot \nabla, \quad \mathcal{C} = \nabla \left( \frac{1}{B} \right) \cdot \hat{z} \times \nabla, \quad B(x) = \frac{1}{1 + \epsilon + \zeta x}.$$

In SOL particle transport along open field lines: modelled by linear damping

# Geometry in the Simulations



Domain  $L_x = 2L_y = 200$ , resolution  $512 \times 256$ ,  $x_{LCFS} = 50$ . SOL damping rates  $\sigma_n = \sigma_\Omega = \sigma_T/5 = 3\zeta/2\pi q$  with  $q = 3$ ; magnetic curvature  $\epsilon = 0.25$ ,  $\zeta = 5 \times 10^{-4}$ ; collisional diffusion  $\nu = 10^{-2}$ ; timespan  $4 \times 10^6$



# Instability, Energy Integrals



Interchange instability:  $N = -B'(p'_0 - \frac{5}{3}B') \leq 0$  instability at low field side.

Naulin et al.; PoP 10, 1075 (2003)

Define the kinetic energy of the °uctuating and poloidal mean motions,

$$v_0(x, t) = \frac{1}{L_y} \int_0^{L_y} v_y(\vec{x}, t) dy = \partial\phi_0/\partial x:$$

$$K(t) = \int \frac{1}{2} \left( \nabla_{\perp} \tilde{\phi}' \right)^2 d\vec{x}, \quad U(t) = \int \frac{1}{2} v_0^2 d\vec{x}.$$

Energy transfer rates from thermal energy to the °uctuating motions, and from the °uctuating to the poloidal mean °ow:

$$F_p(t) = \int p\mathcal{C}(\phi) d\vec{x}, \quad F_v(t) = \int \tilde{v}_x \tilde{v}_y \frac{\partial v_0}{\partial x} d\vec{x}.$$

$F_p$  is also a measure of the turbulent energy transport.