2052-23

Summer College on Plasma Physics

10 - 28 August 2009

Gyrokinetic simulations of fusion plasmas

Frank Jenko
Max-Planck-Institut für Plasmaphysik
Germany

Frank Jenko

F. Merz, T. Görler, P. Xanthopoulos,
T. Dannert, D. Told, T. Hauff

Gyrokinetic simulations of fusion plasmas

Max-Planck-Institut für Plasmaphysik, Garching
Universität Ulm

Summer College on Plasma Physics
ICTP, Trieste – Italy, 13 August 2009

The nonlinear gyrokinetic equations

$$f = f(\mathbf{X}, v_\|, \mu; t)$$

Advection/Conservation equation

$$\boxed{\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_\| \frac{\partial f}{\partial v_\|} = 0}$$

X = gyrocenter position

v$_{\|}$ = parallel velocity

μ = magnetic moment

$$\dot{\mathbf{X}} = v_\| \mathbf{b} + \frac{B}{B_\|^*} \left(\frac{v_\|}{B} \bar{\mathbf{B}}_{1\perp} + \mathbf{v}_\perp \right)$$

$$\mathbf{v}_\perp \equiv \frac{c}{B^2} \bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega} \mathbf{b} \times \nabla (B + \bar{B}_{1\|}) + \frac{v_\|^2}{\Omega} (\nabla \times \mathbf{b})_\perp$$

$$\dot{v}_\| = \frac{\dot{\mathbf{X}}}{m v_\|} \cdot \left(e \bar{\mathbf{E}}_1 - \mu \nabla (B + \bar{B}_{1\|}) \right)$$

Appropriate field equations

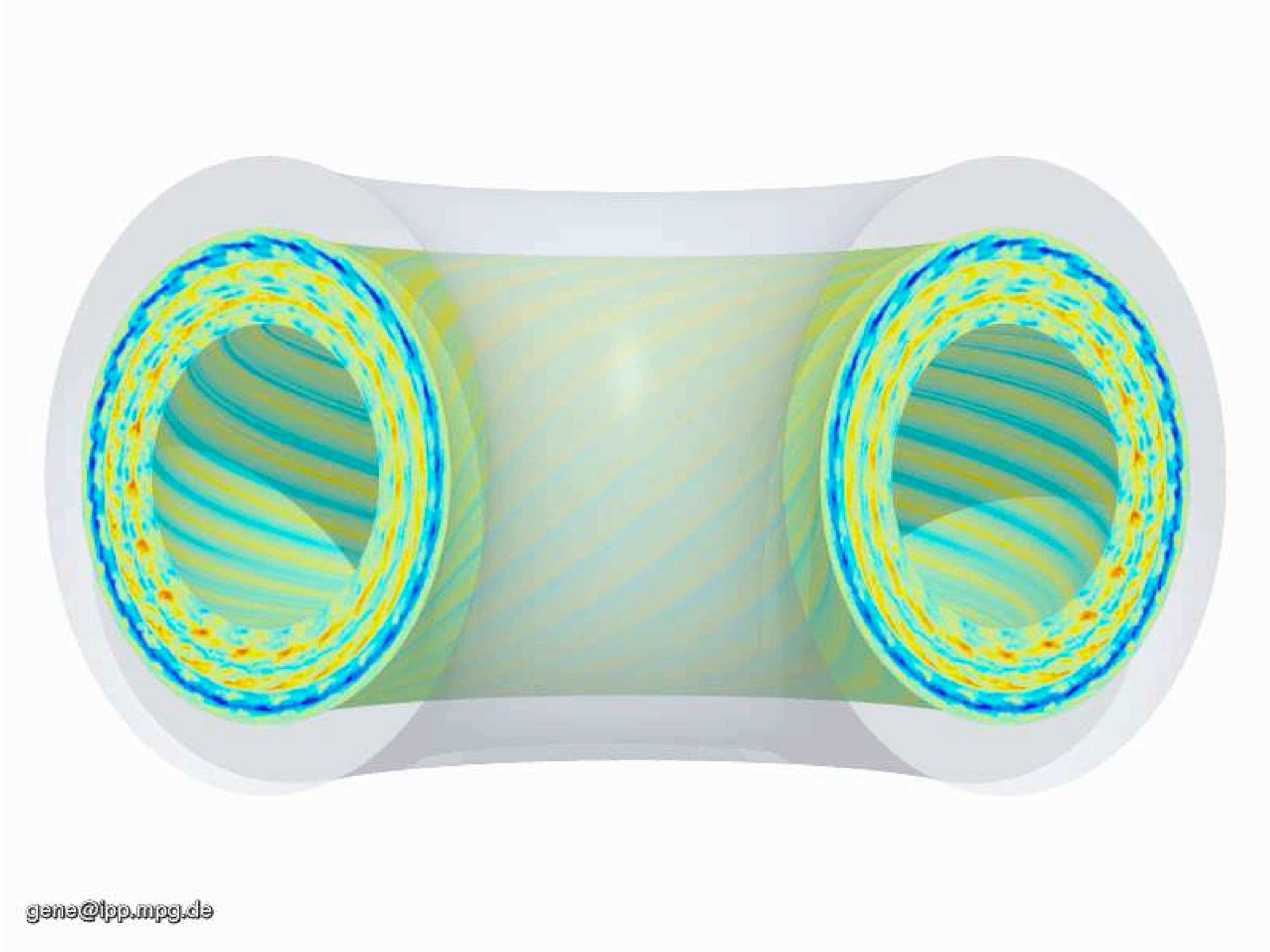
$$\frac{n_1}{n_0} = \frac{\enclose{circle}{\bar{n}_1}}{n_0} - \left(1 - \|I_0^2\|\right) \frac{e\phi_1}{T} + \|x I_0 I_1\| \frac{B_{1\|}}{B}$$

$$\nabla_\perp^2 A_{1\|} = -\frac{4\pi}{c} \sum \enclose{circle}{\bar{J}_{1\|}}$$

$$\frac{B_{1\|}}{B} = -\sum \epsilon_\beta \left(\frac{\enclose{circle}{\bar{p}_{1\perp}}}{n_0 T} + \|x I_1 I_0\| \frac{e\phi_1}{T} + \|x^2 I_1^2\| \frac{B_{1\|}}{B} \right)$$

Nonlinear integro-differential equations in **5 dimensions**...

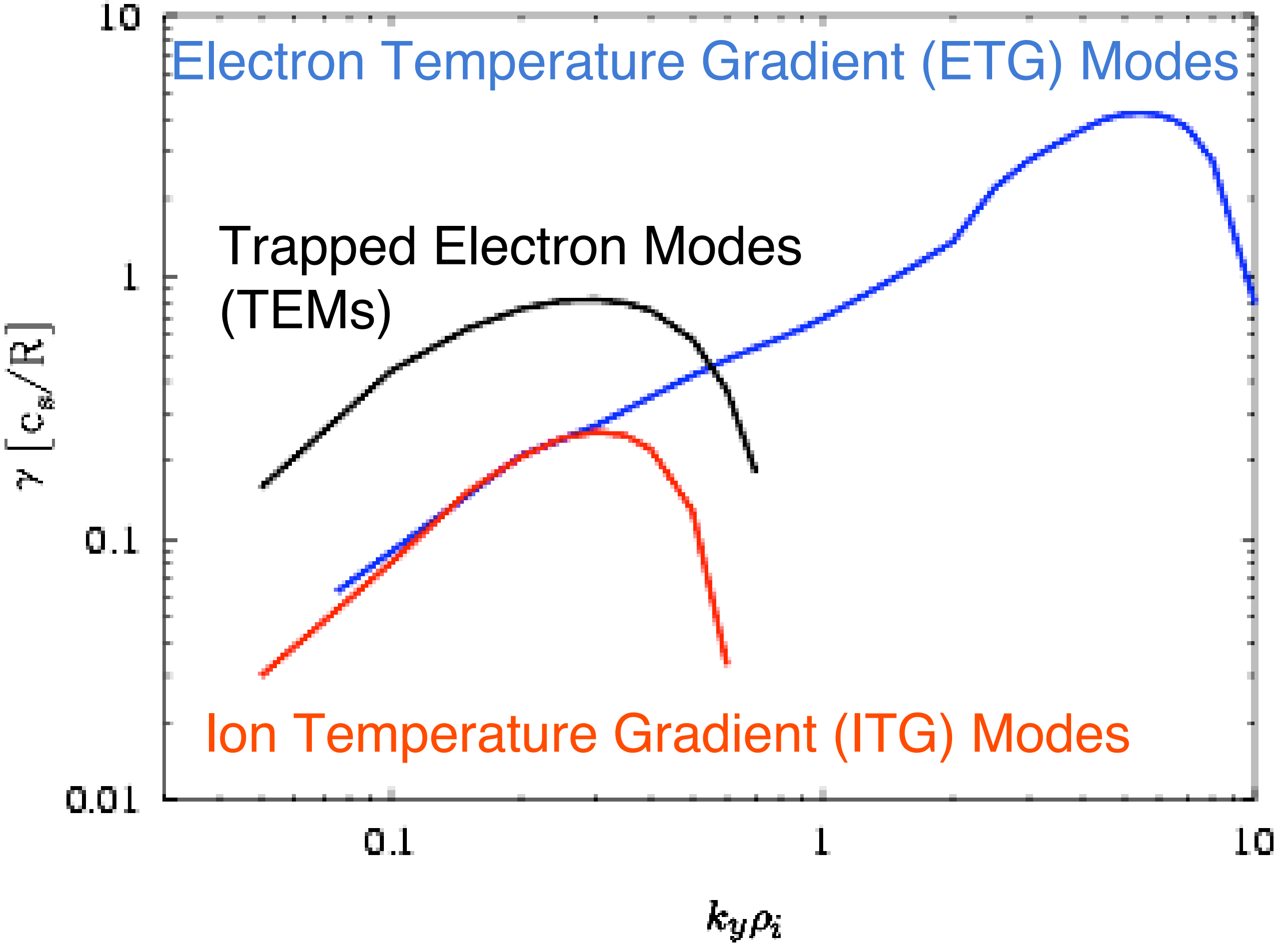
Computational GK reveals surprises



www.ipp.mpg.de/~fsj/gene

Plasma microturbulence:
Linear drive

Different kinds of microinstabilities drive different kinds of plasma turbulence



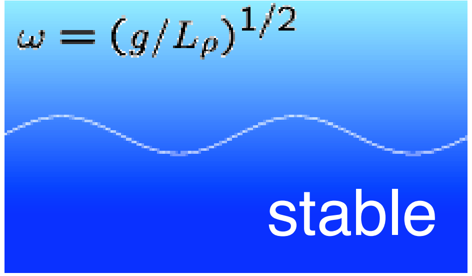
Drive (= transport) range dynamics is not universal.

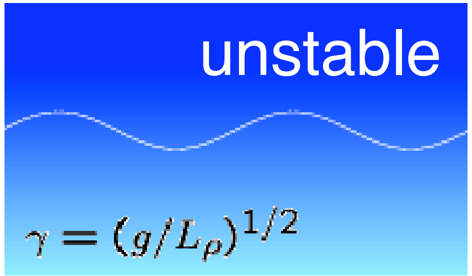
Drive range physics itself is a multi-scale problem.

In the adiabatic limit, ITG and ETG modes are isomorphic.

Gradient-driven microinstabilities

Perpendicular dynamics: de-/stabilization in out-/inboard regions



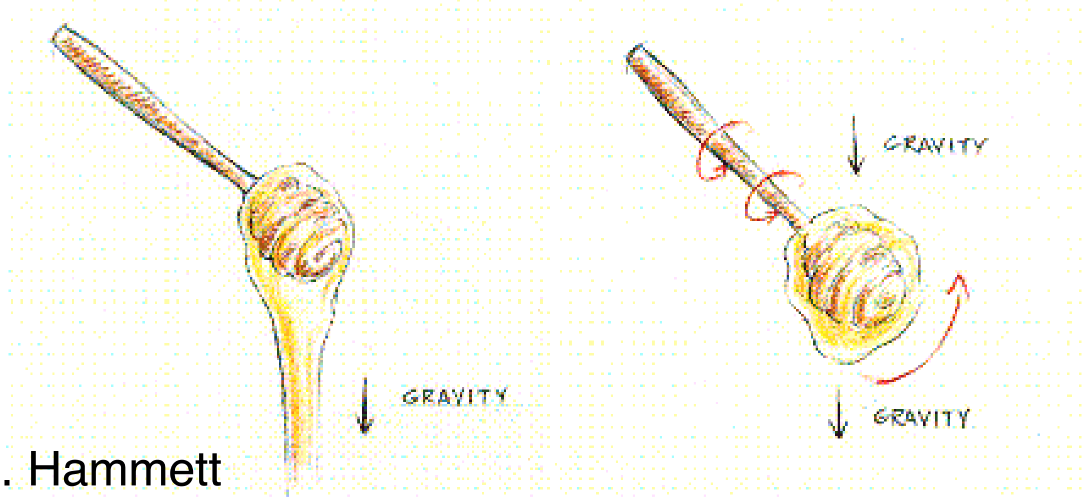


Rayleigh-Taylor instability

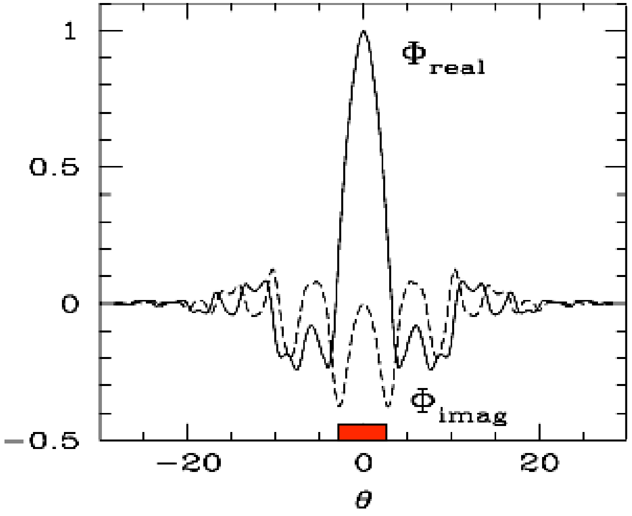
Analogy in a plasma:

$$g_{\text{eff}} = v_t^2/R$$

Parallel dynamics: localization in outboard regions



G. Hammett



Basic properties of ITG modes

$$\boxed{\gamma_{\text{eff}} \approx (k_\perp \rho_i)\,\frac{v_t}{L_T} - \mathcal{C}\,\frac{v_t}{R}}$$

Existence of critical temperature gradients

$$k_\perp \rho_i \approx 1 \;: \qquad \gamma_{\text{eff}} > 0 \qquad \leftrightarrow \qquad \frac{R}{L_T} > \left(\frac{R}{L_T}\right)_{\text{crit}}$$

Temperature profiles tend to be ‘stiff’ (cp. solar convection zone).

Typical space scales: several ion gyroradii (not system size)

$$\frac{R}{L_T} \sim \left(\frac{R}{L_T}\right)_{\text{crit}} \;: \qquad \gamma_{\text{eff}} > 0 \qquad \leftrightarrow \qquad (k_\perp \rho_i) > (k_\perp \rho_i)_{\text{crit}}$$

ETG/ITG modes: Critical gradients

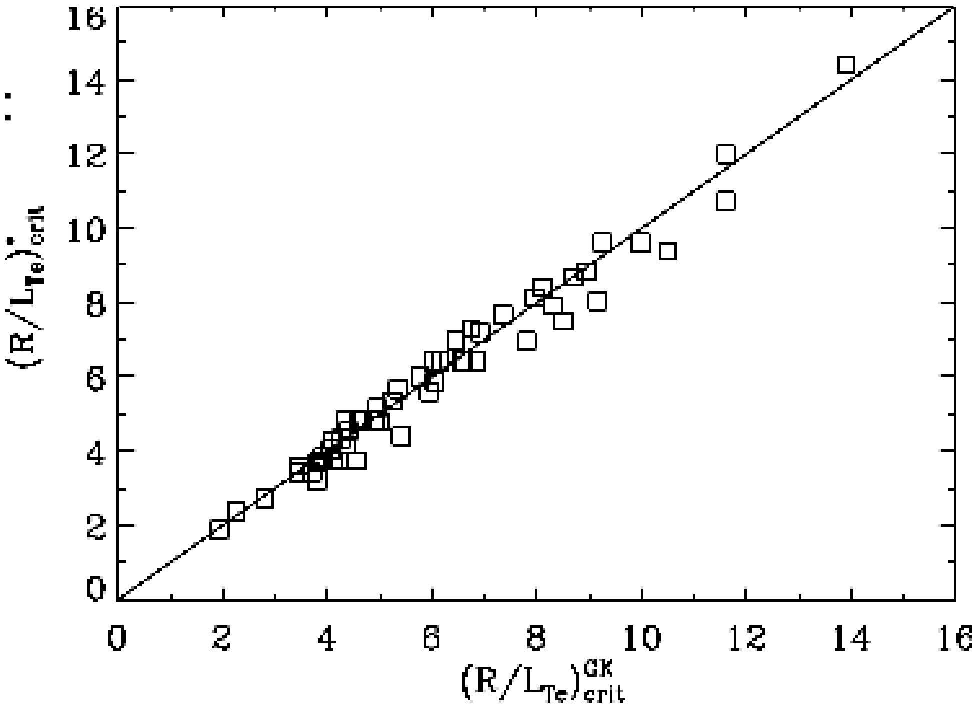
Linear stability of ETG/ITG modes [Jenko, Dorland & Hammett 2001]

Linear gyrokinetic simulations (GS2):

$$\boxed{(R/L_{T_j})_{\text{crit}} \approx (1+\tau_j)\,(1.33+1.91\,\hat{s}/q)}$$

$$\tau_e \equiv T_e/T_i \equiv 1/\tau_i$$

Limiting cases (analytical results):
- Hahm & Tang 1989 (for high s/q)
- Romanelli 1989 (for low s/q)



Thousands of linear GK simulations condense into one simple formula…

Plasma microturbulence: Nonlinear saturation?

Saturation of ITG modes: zonal flows

Gyrokinetic Simulations of Plasma Microinstabilities

simulation by

Zhihong Lin et al.

Science 281, 1835 (1998)

Structure formation

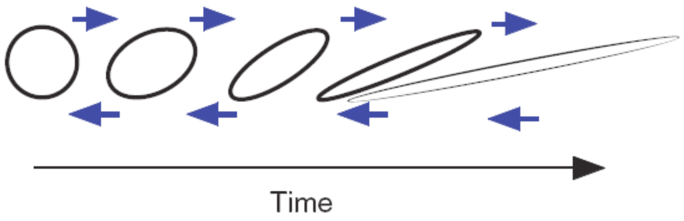
Emergence of **zonal ExB flows** (due to symmetry breaking!)

They are linearly neutrally stable but excited nonlinearly

Zonal flows in geo-/astrophysics

Effect on turbulent transport

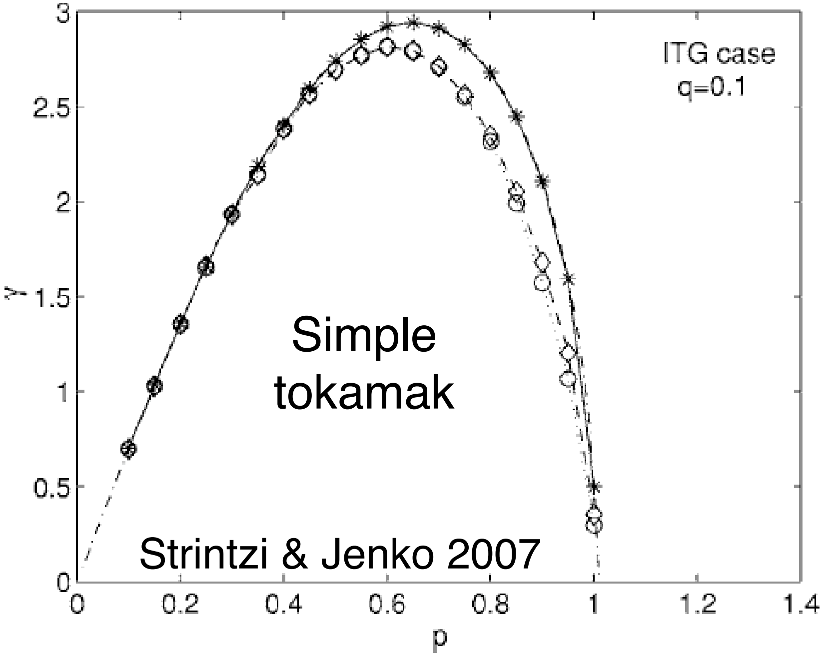
Zonal flows may reduce or even suppress the turbulent transport by means of vortex shearing



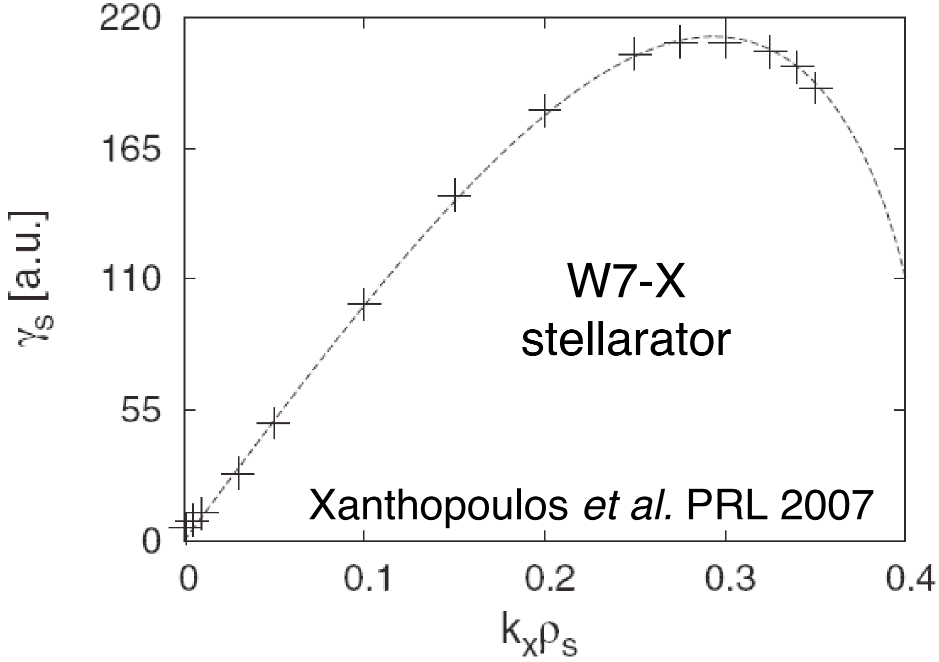
Secondary instabilities & ZF generation

- Large-amplitude streamers are Kelvin-Helmholtz unstable

  [Cowley at al. 1991; Dorland & Jenko PRL 2000]
- This secondary instability contains a zonal-flow component
- Near-equivalence to 4-mode and wave-kinetic approaches



Strintzi & Jenko 2007



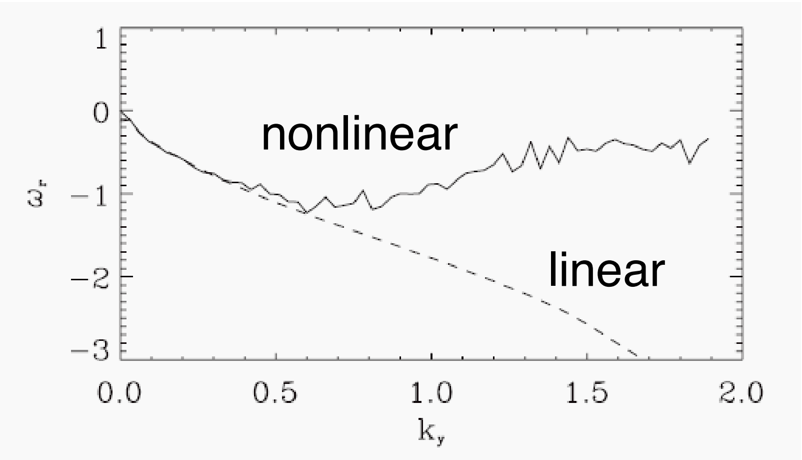
Xanthopoulos *et al.* PRL 2007

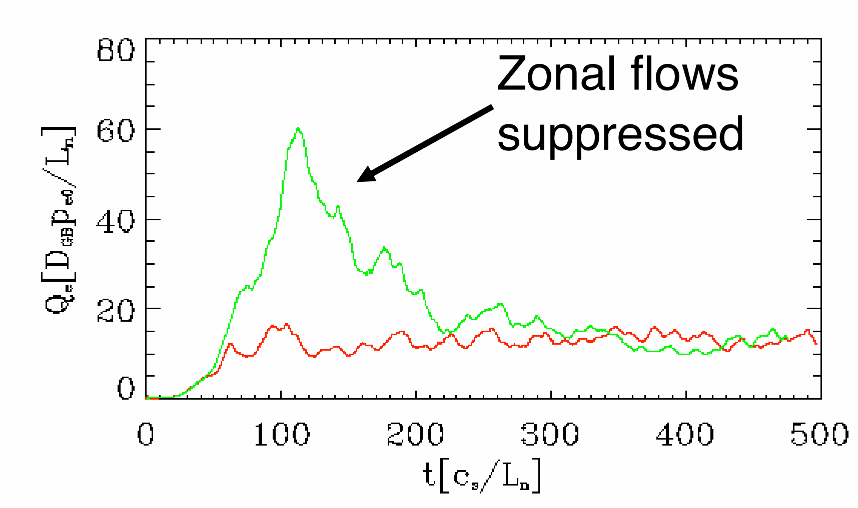
Features of TEM turbulence

Saturated phase of TEM turbulence simulations:

- In the drive range, nonlinear and linear frequencies are identical

- In the drive range, there is no significant shift of cross phases w.r.t. linear ones

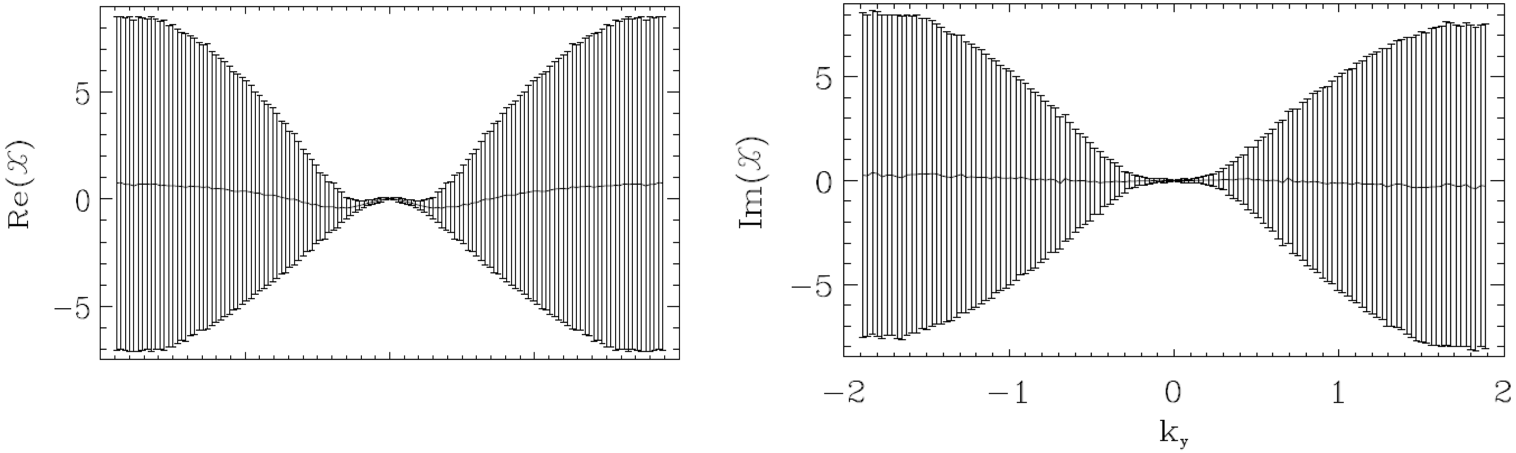



Zonal flows suppressed

– No dependence of transport level on zonal flows [Dannert & Jenko 2005]

Theory-motivated statistical analysis

- Both weak and strong turbulence theories suggest that the ExB nonlinearity can be represented by a coherent part $\mathcal{N}l[g] \sim g$ and a random noise part
- $\mathcal{N}l[g]$ and g are fluctuating quantities; minimizing the model error $\langle|\mathcal{N}[g] - \mathcal{X}g|^2\rangle$, we obtain $\mathcal{X} = \langle g^*\mathcal{N}[g]\rangle/\langle|g|^2\rangle$

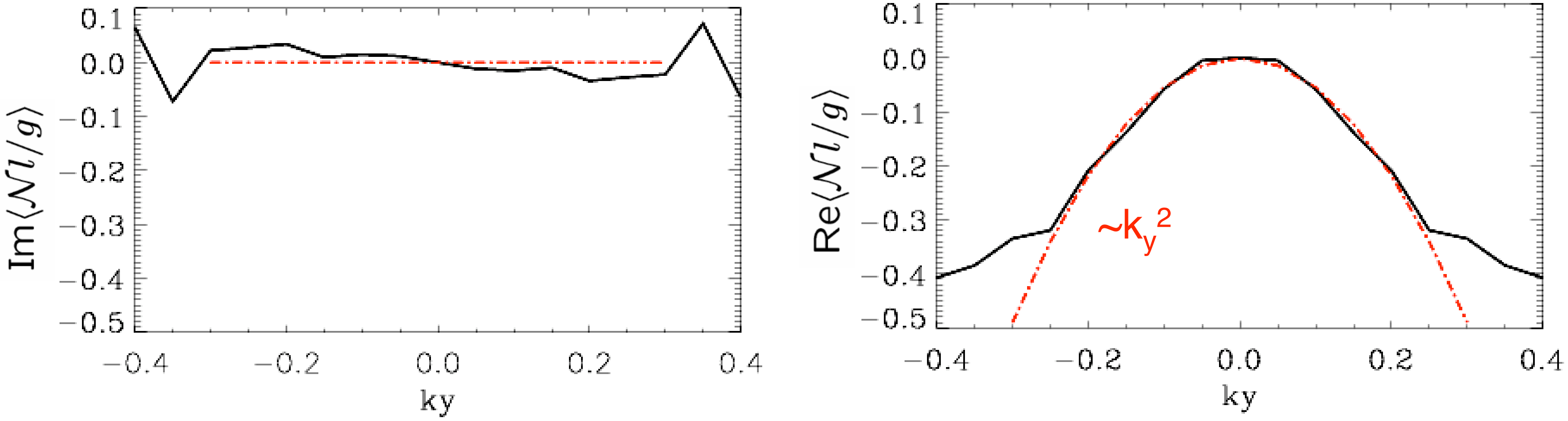


Saturation of TEMs: “eddy damping”

Merz & Jenko, PRL 2008

Low-k_y drive range: large transport contributions, but small random noise; here, one finds:

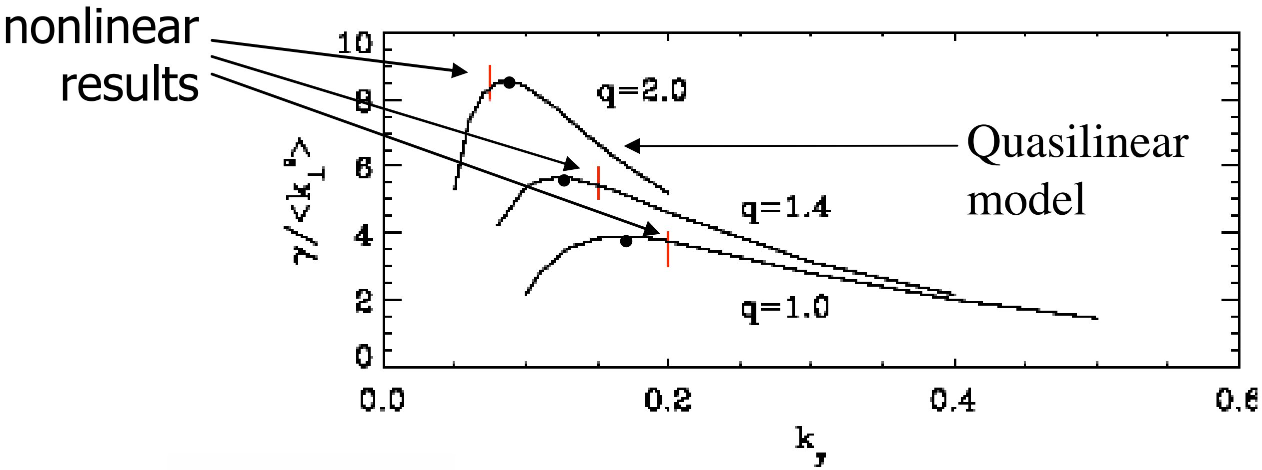
$$\boxed{\mathcal{N}l[g] \simeq D(-k_\perp^2)g = D\nabla_\perp^2 g}$$



This is in line with various theories, including Resonance Broadening Theory (Dupree), MSR formalism (Krommes), Dressed Test Mode Approach (Itoh).

Quasilinear transport model

- Fick‘s law $Q \sim D_0 \frac{R}{L_{Te}}$ gives $\boxed{Q_e \propto \max_{k_y} \left[\frac{\gamma}{\langle k_\perp^2 \rangle} \right] \frac{R}{L_{T_e}}}$
- Application: q dependence of TEM-induced transport

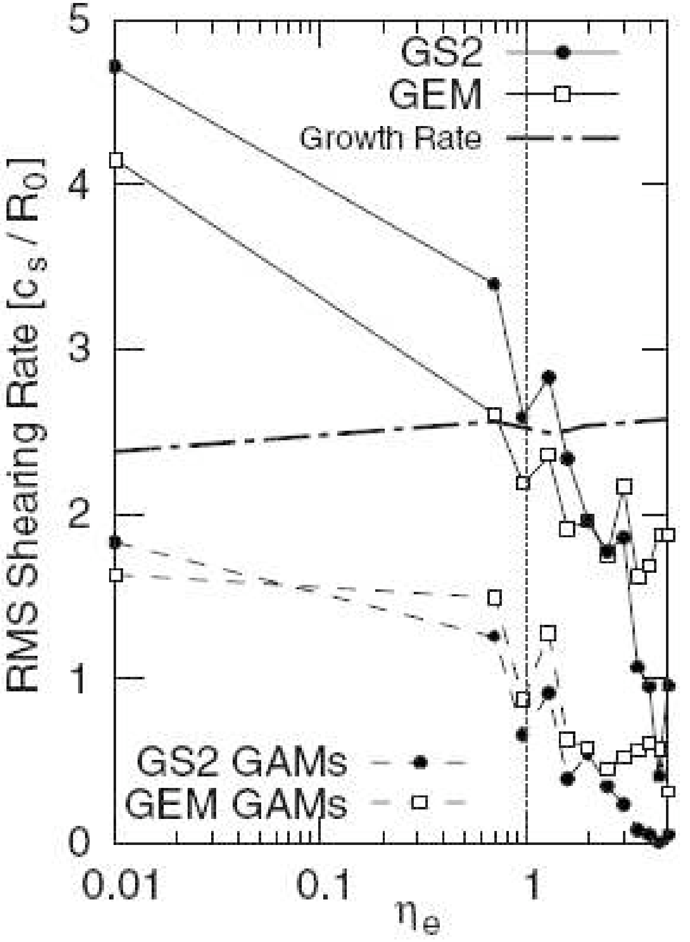


[Jenko & Dannert '05]

- Scaling: $Q_e \propto q^\nu$
- The quasilinear model captures the q-dependence seen in nonlinear simulations (here $\nu \approx 1.7$) and in experiments ($\nu = 1 - 2$)

ZF / Non-ZF regimes

Ernst et al., PoP 2009



ExB shearing rates exceed the growth rate *only* for $\eta_e < 1$

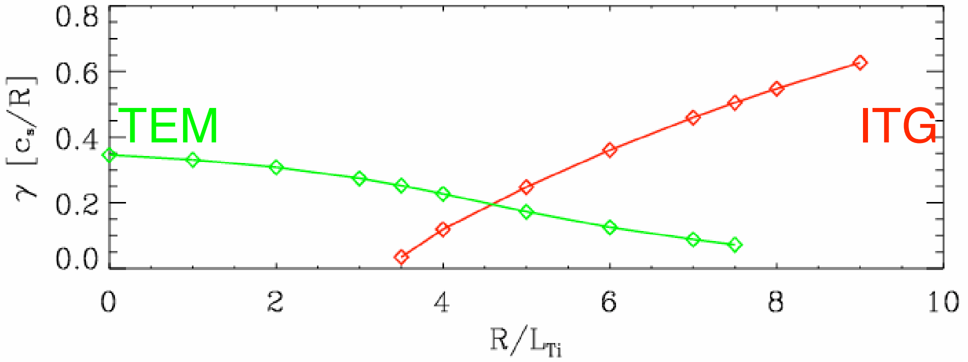
For mainly temperature gradient driven TEM turbulence, ZFs (and GAMs) are “unimportant”

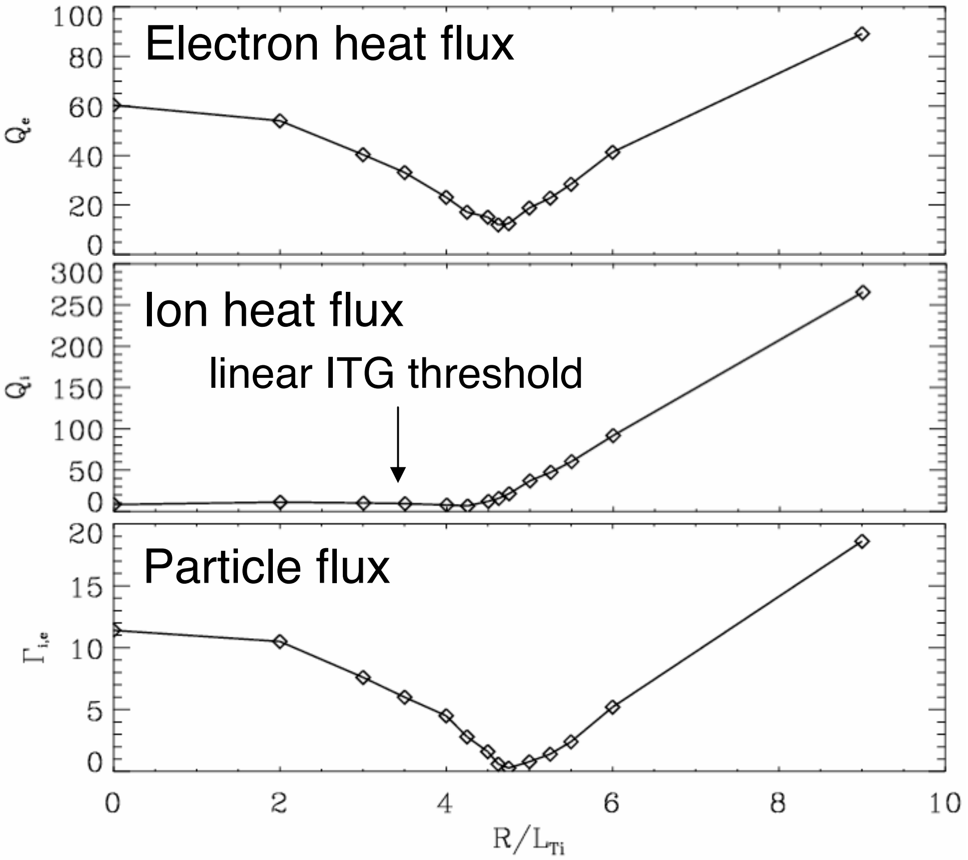
Thus, in a wide region of parameter space, the standard drift-wave / ZF paradigm does not hold

Nonlinear ITG/TEM interactions

[F. Merz, PhD Thesis 2008]

- Linear growth rates (k_y=0.25), using GENE as an EV solver
- TEM regime: Electron heat flux is suppressed, not increased
- ITG regime: Nonlinear upshift of critical R/L_{Ti}
- Nonlinear ITG/TEM coexistence

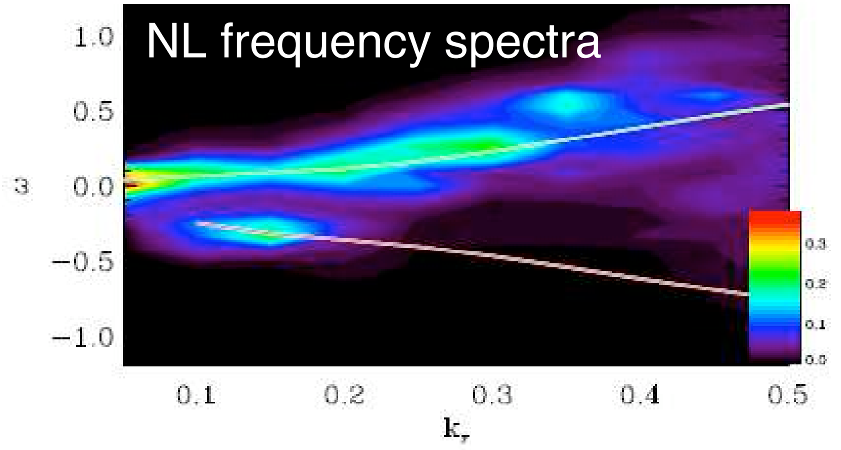



Electron heat flux

Ion heat flux

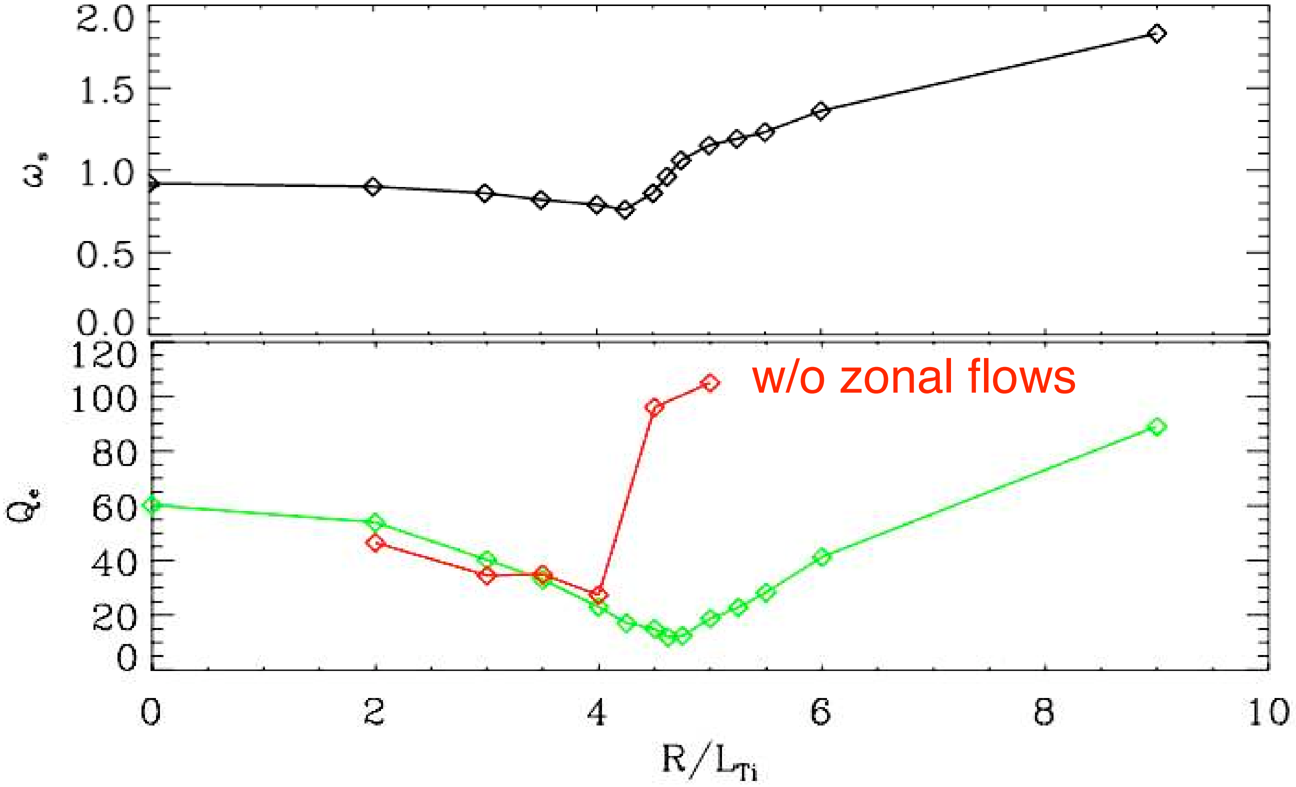
linear ITG threshold

Particle flux


NL frequency spectra

Zonal flow behavior

- TEM-ITG transition changes the role of zonal flows
- Relatively sharp transition seen in the value of the ExB shearing rate ω_s and in simulations where zonal flows have been artificially suppressed



Plasma microturbulence: Sub-ion-gyroradius scales?

High-k turbulence simulations

(Pure) ETG turbulence can induce significant electron heat transport:

$\boxed{\chi_{\mathrm{e}}^{\mathrm{ETG}} \gg \frac{\rho_e^2 v_{te}}{L_{T_e}} \text{ is possible}}$ (Jenko, Dorland, Rogers & Kotschenreuther, PoP 2000)

For comparison: $\chi_{\mathrm{i}}^{\mathrm{ITG}} \approx 0.7\,\frac{\rho_s^2 c_s}{L_{T_i}}$ (Cyclone base case)

Confirmed, e.g., by (Idomura *et al*., NF 2005),
(Nevins *et al.,* PoP 2006), and (Bottino *et al.,* PoP 2007)

ETG turbulence in concert with longer wavelengths (ITG, TEM, etc.):

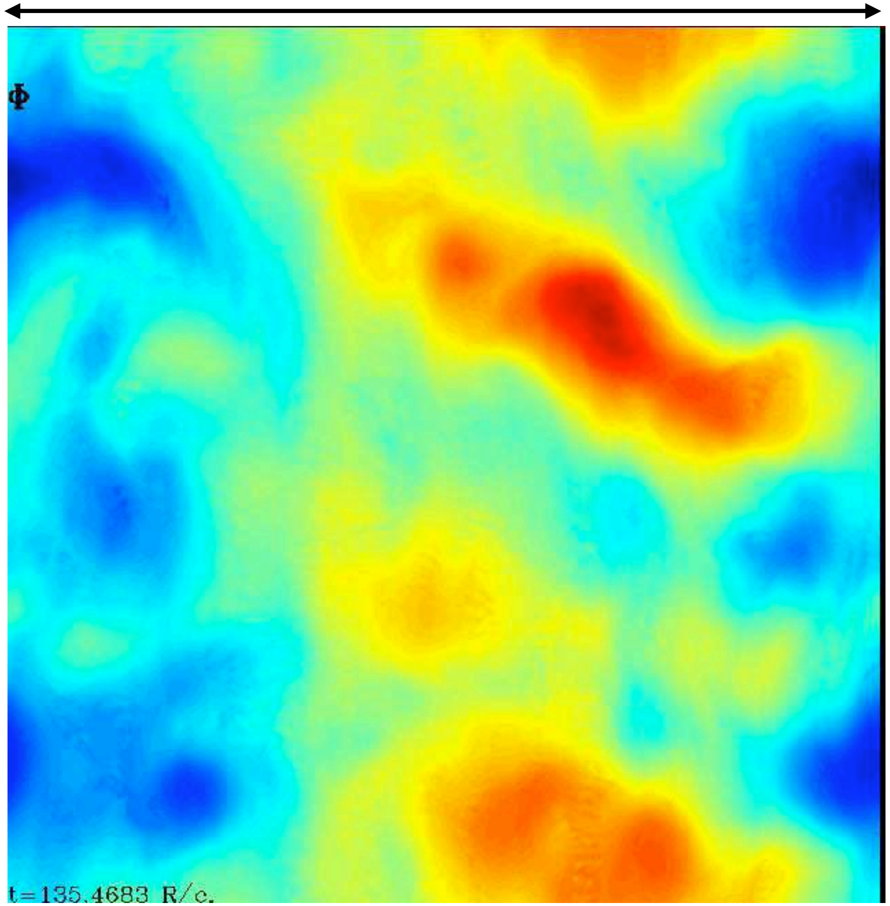
First gyrokinetic multiscale simulations:

Transport in the tokamak edge (Jenko, J Plasma Fus Res 2004)

Recently: Similar work for core parameters by Candy and Waltz

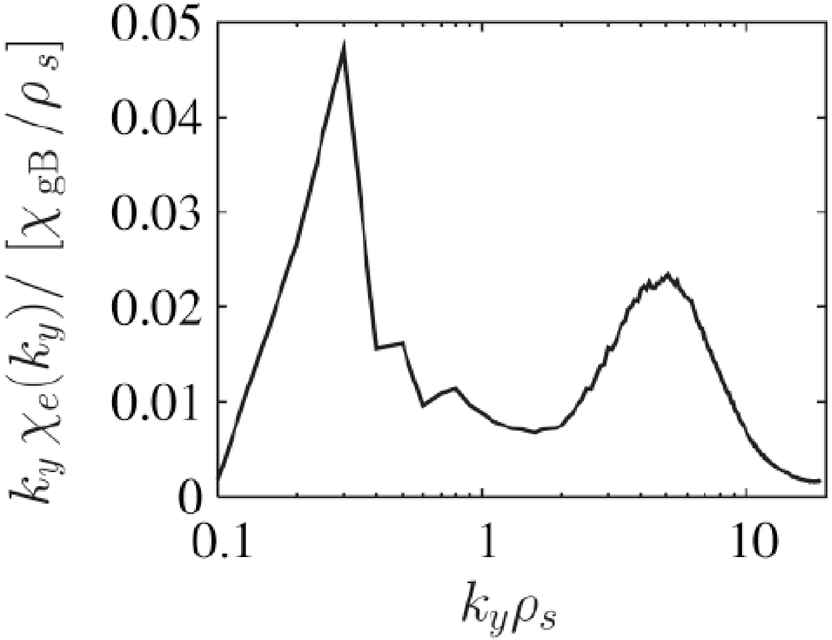
Coexistence of ITG and ETG modes

box size: ~64 ρ_i resolution: ~2ρ_e



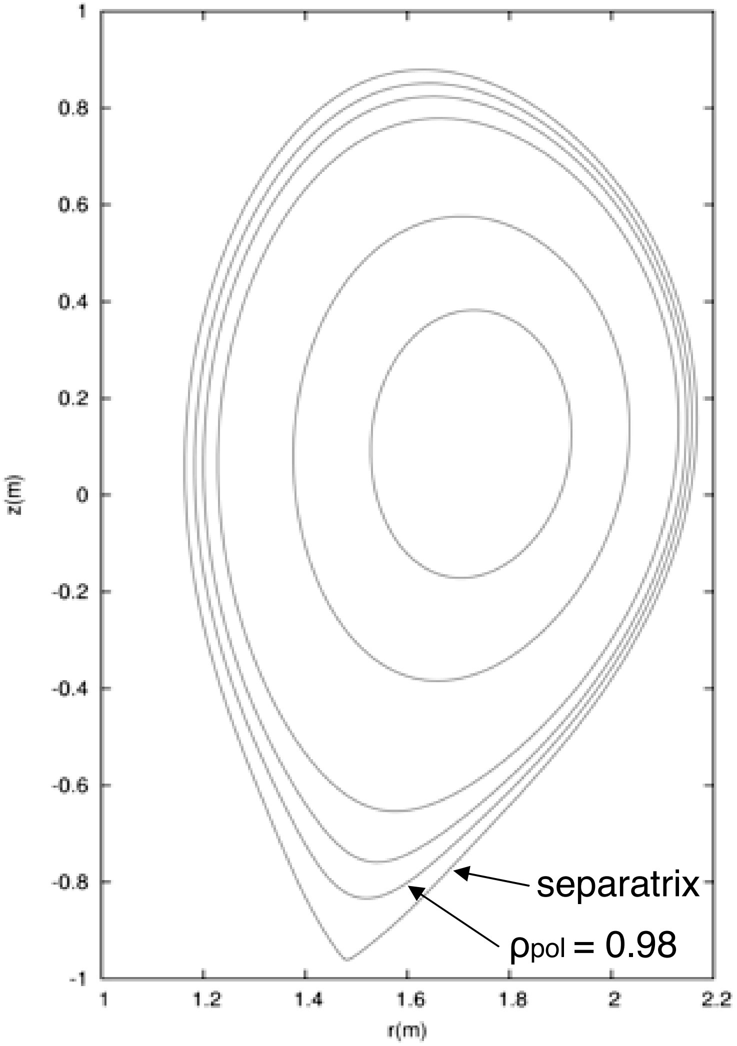
Reduced mass ratio (400), but still > 100,000 CPU-h.

[Görler & Jenko, PRL 2008]

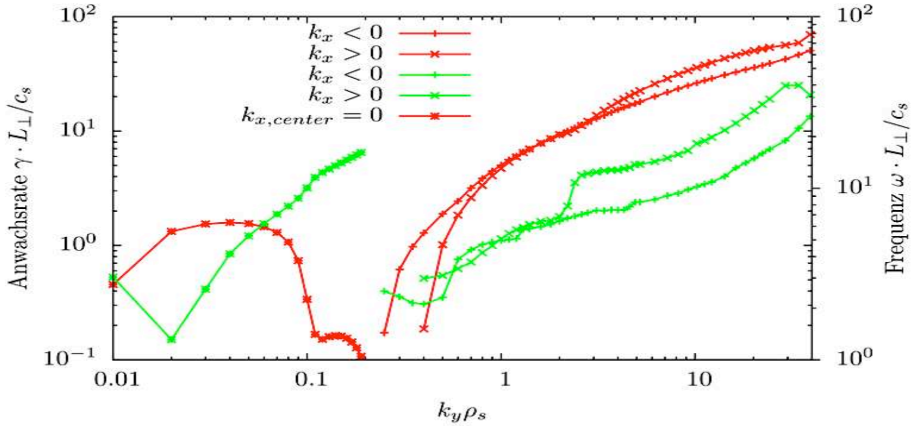


<u>ITG/TEM/ETG turbulence</u>: Large fraction of electron heat transport is carried by electron scales (cmp. recent experiments).

ASDEX Upgrade #20431 (ρ_{pol} = 0.98)



Jenko et al., PoP 2009

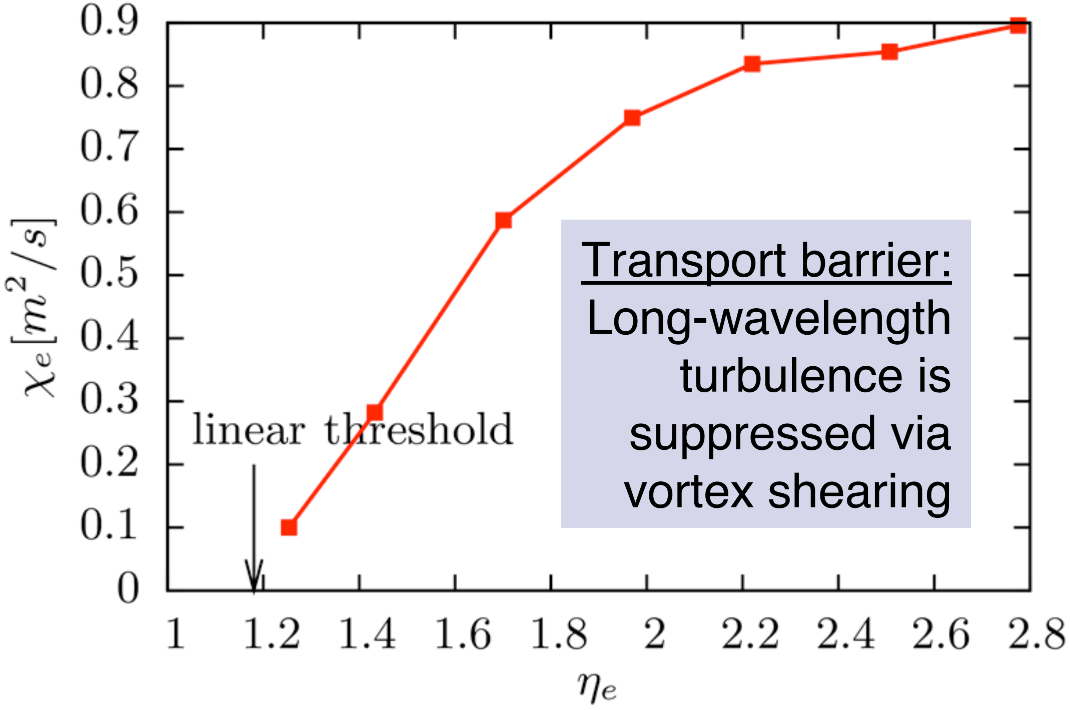


Edge transport barrier region:

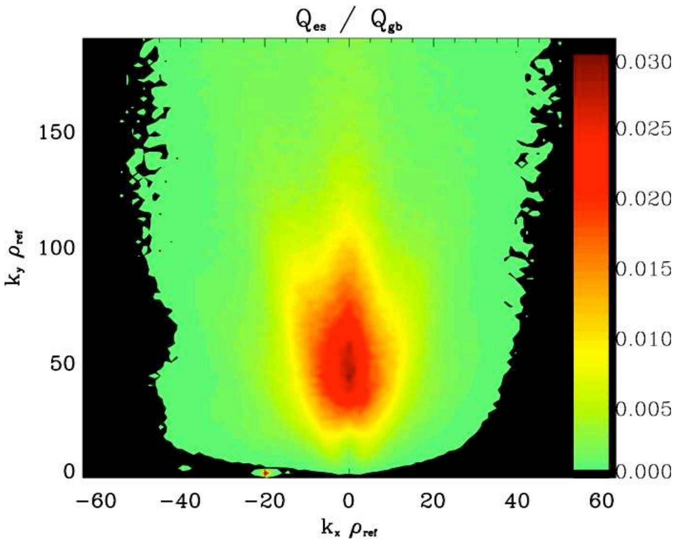
- $k_y\rho_s < 0.1$ → ITG mode
- $k_y\rho_s \sim 0.15$ → microtearing mode
- $k_y\rho_s > 0.2$ → ETG mode

H-mode edge: Electron heat diffusivity

Jenko et al., PoP 2009

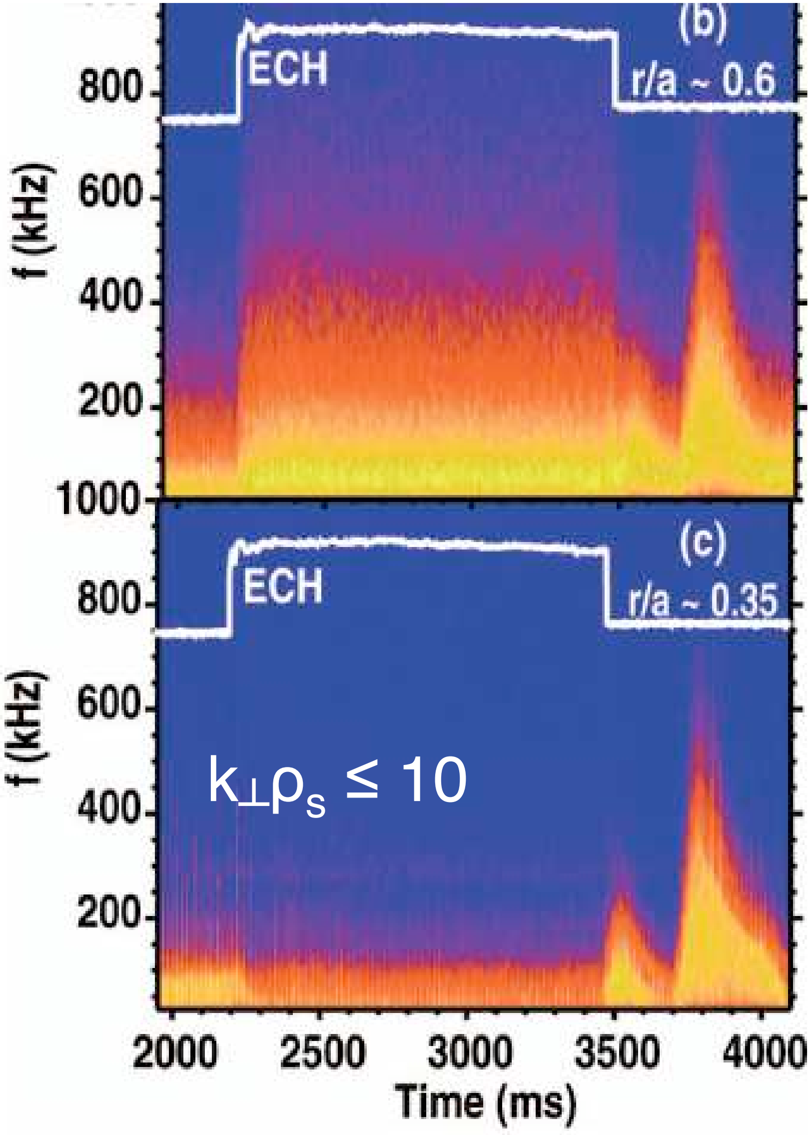


Transport barrier:
Long-wavelength turbulence is suppressed via vortex shearing



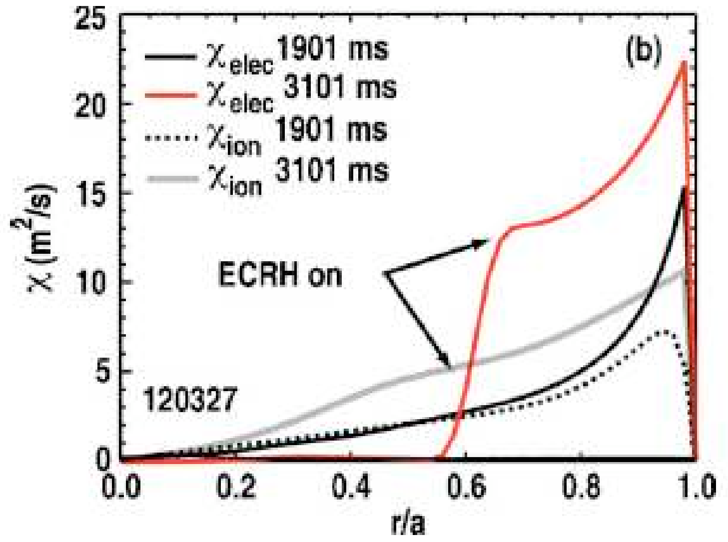
ETG turbulence is able to explain the residual electron heat transport in H-mode edge plasmas.

High-k turbulence in DIII-D



$k_\perp \rho_s \leq 10$

Rhodes et al., PoP 2007



High-k density fluctuations and electron heat fluxes are enhanced via ECH (no significant change w.r.t. low/medium-k fluctuations)

High-k turbulence in NSTX

Mazzucato et al., PRL 2008
Smith et al., PRL 2009

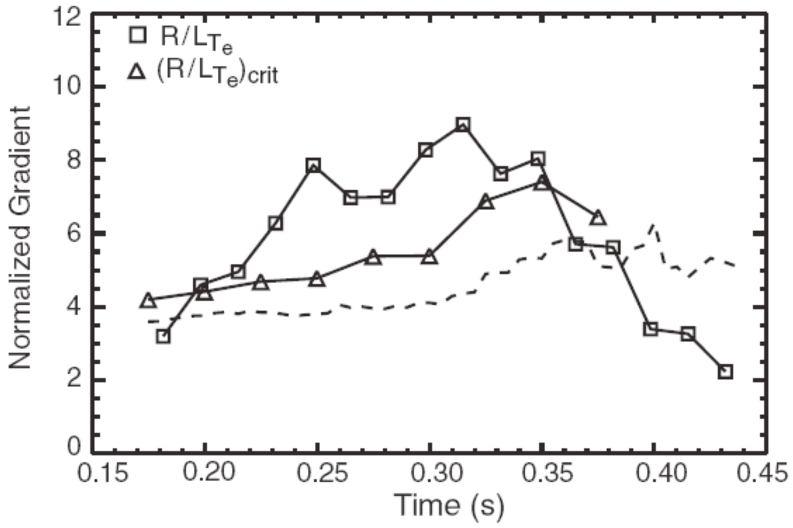


FIG. 7. Time evolution of measured gradient R/L_{T_e} (squares) and GS2 critical gradient $(R/L_{T_e})_{\text{crit}}$ for the onset of the ETG mode (triangles). The dashed line is the critical gradient from Ref. [19].

ETG modes are linearly unstable

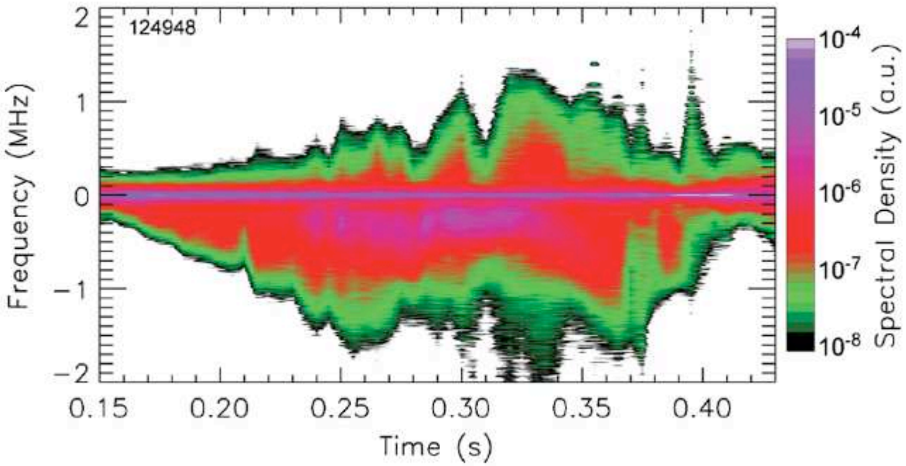


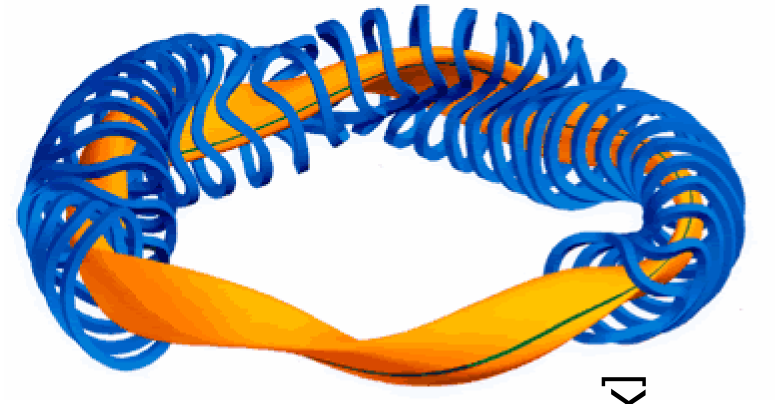
FIG. 3 (color). Logarithmic contour plot of spectral density of fluctuations with $k_\perp \rho_e = 0.2$–0.4 at $R = 1.2$ m. Negative frequencies correspond to wave propagation in the electron diamagnetic direction.

High-frequency density fluctuations are detected; their amplitude is correlated with the threshold distance

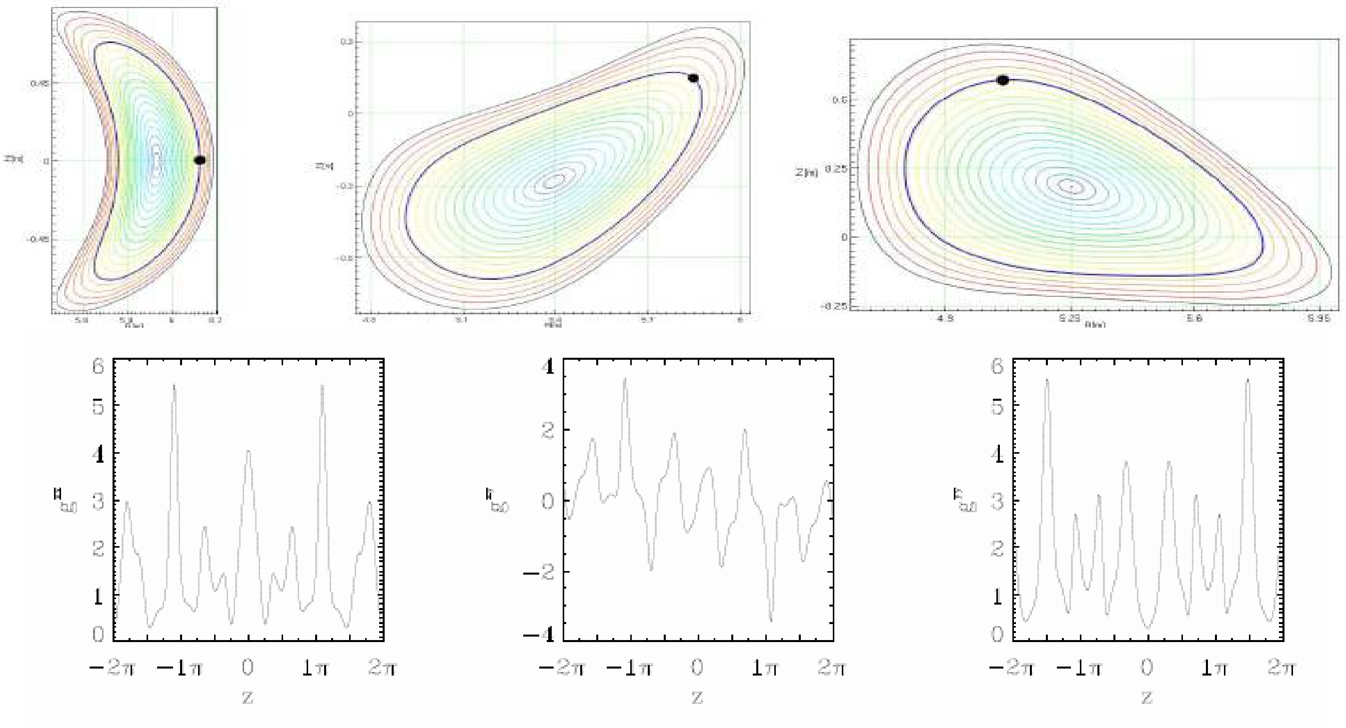
Turbulent transport in optimized stellarators

GENE flux-tube simulations for W7-X

Wendelstein 7-X stellarator:
optimized with respect to
neoclassical transport



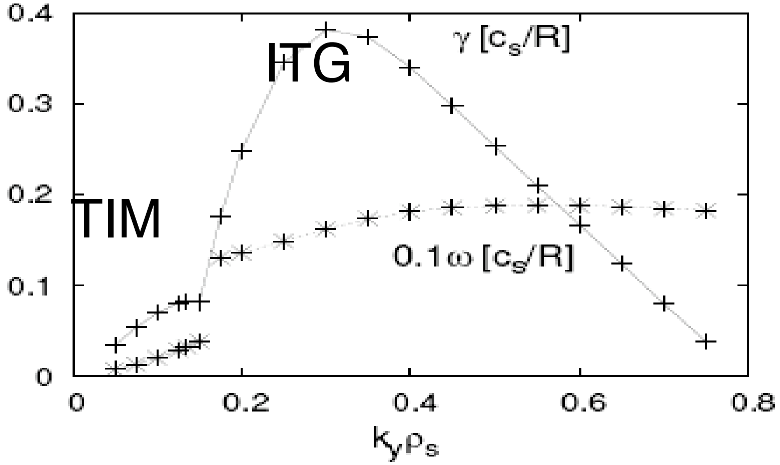
[Xanthopoulos & Jenko, PoP 2006]



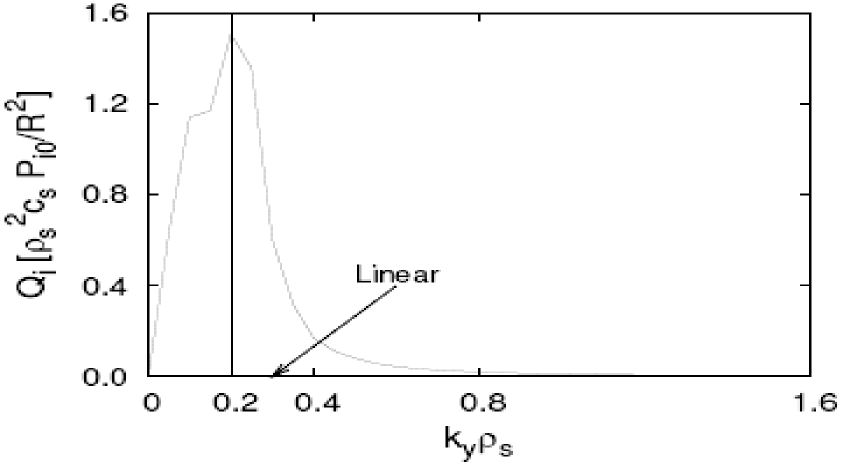
Nonlinear ITG/TIM coexistence

GENE simulations for W7-X (close to the magnetic axis; adiabatic electrons): Trapped ion modes and ITG modes coexist linearly and nonlinearly.

Linear growth rate spectrum



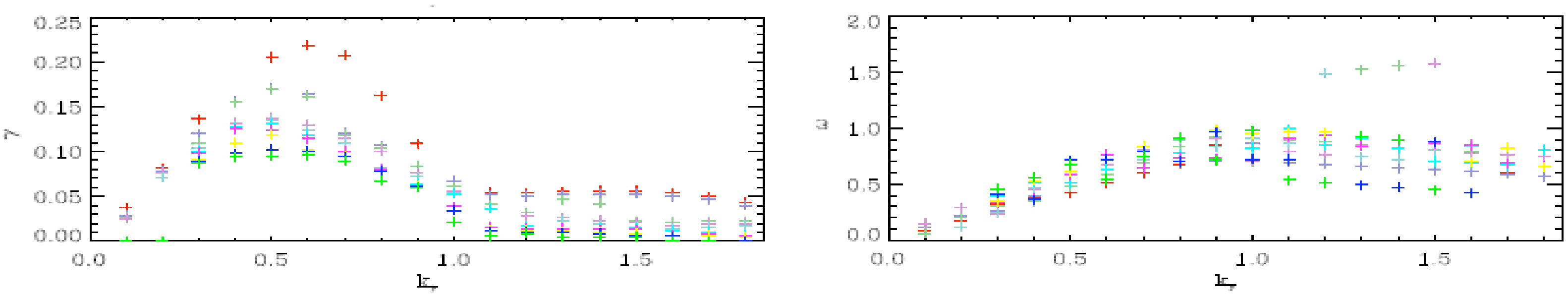
Nonlinear transport spectrum



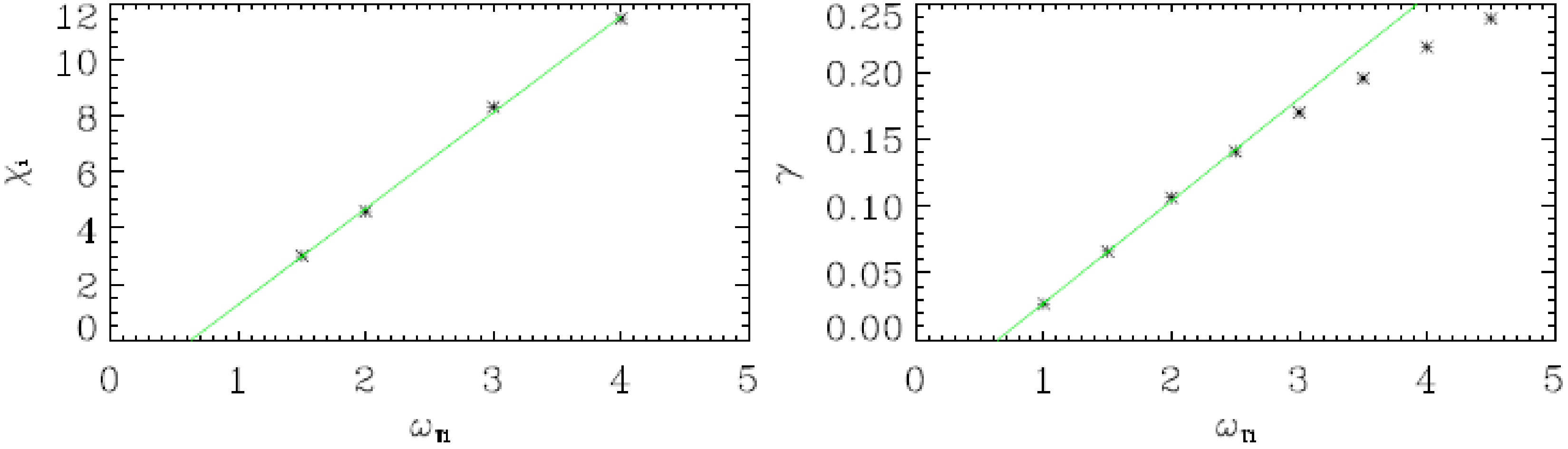
[Xanthopoulos, Merz, Görler & Jenko, PRL 2007]

ITG turbulence in W7-X (kinetic electrons)

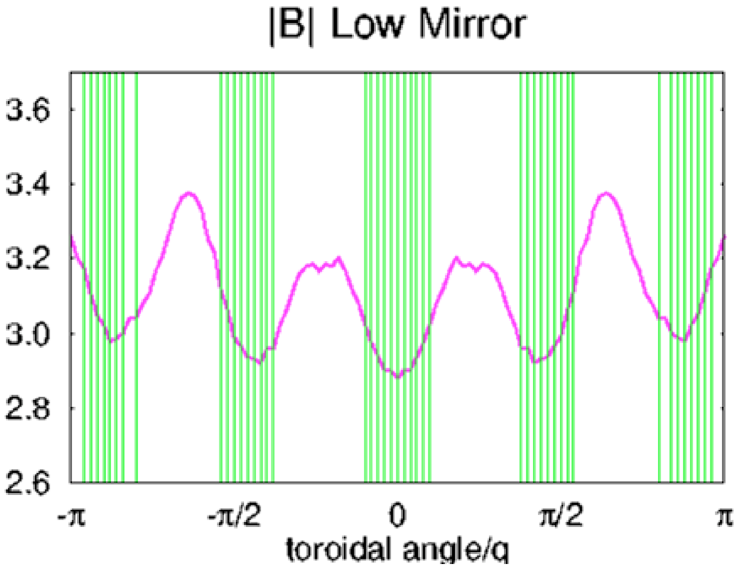
Many modes are unstable, with similar growth rates.



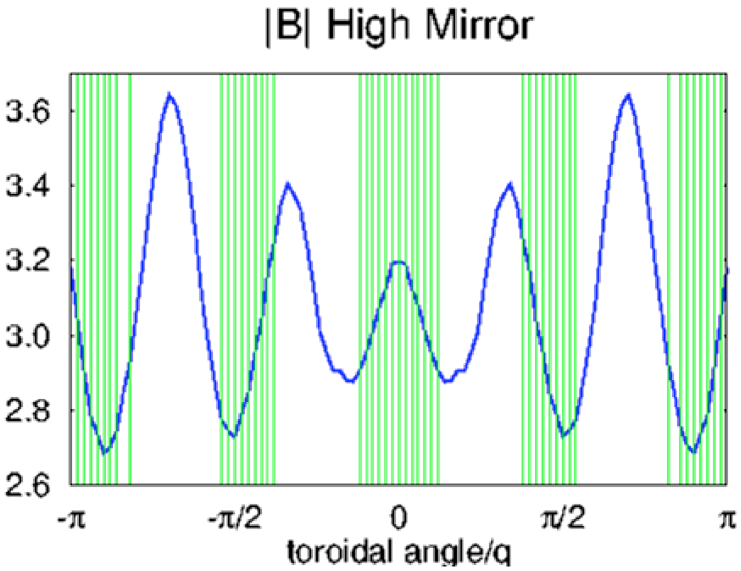
Nonlinear runs exhibit weak ZF activity, no Dimits shift.

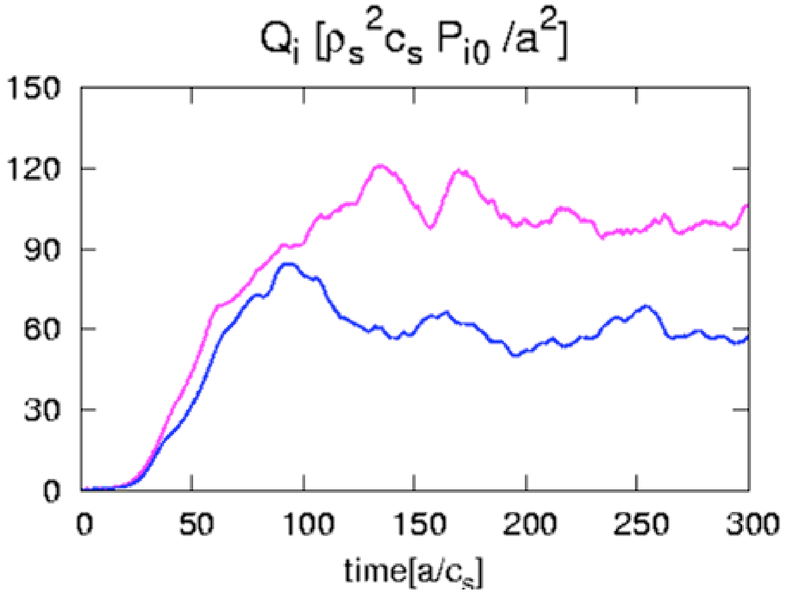


ITG turbulence in W7-X (kinetic electrons)



Bad curvature regions



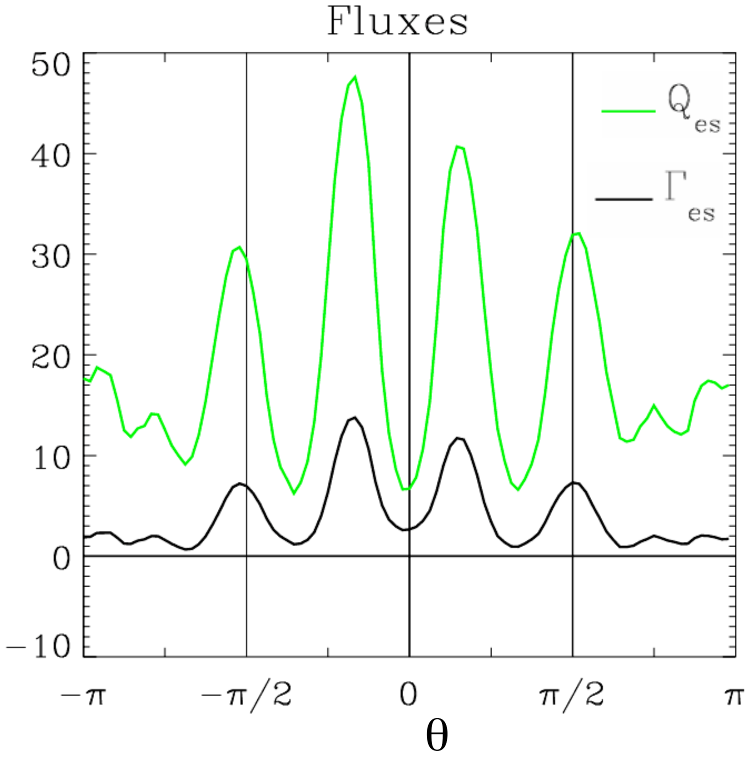


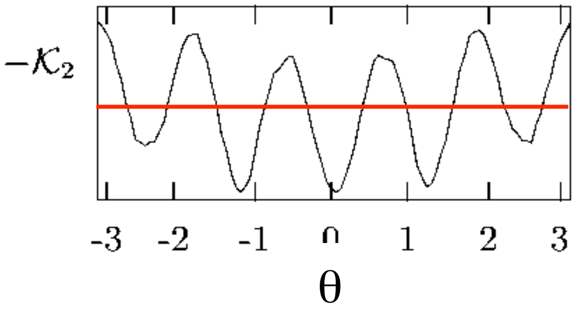
Two configurations which are geometrically virtually identical yield clearly differing results – although the <u>linear physics is almost the same</u>; there must be <u>geometric control of the NL saturation mechanism</u>

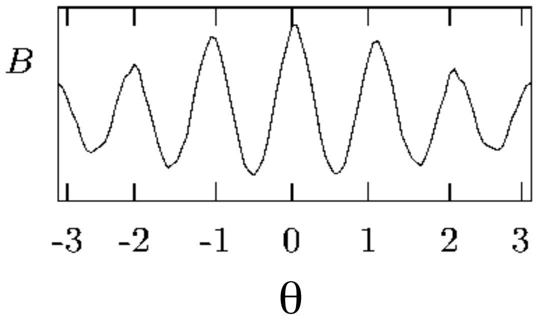
Properties of TEM turbulence in W7-X

- Parallel stucture: Transport reflects the structure of the magnetic wells

- Regions where bad curvature and magnetic wells overlap dominate transport





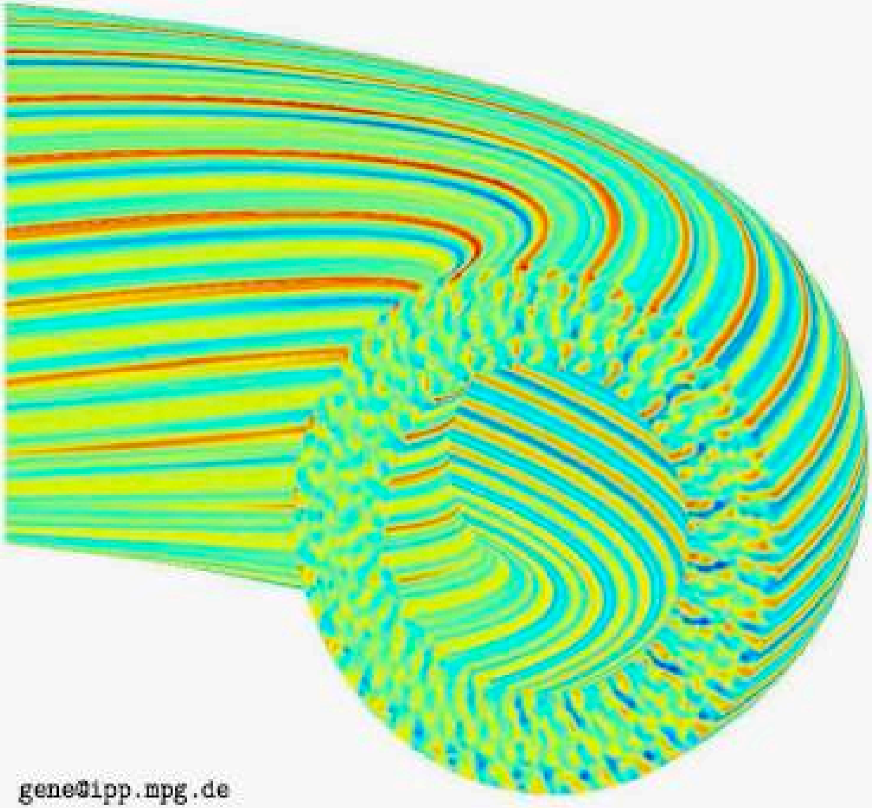


- Side remark: Nonlinear and linear mode structure are quite similar; zonal flows are weak

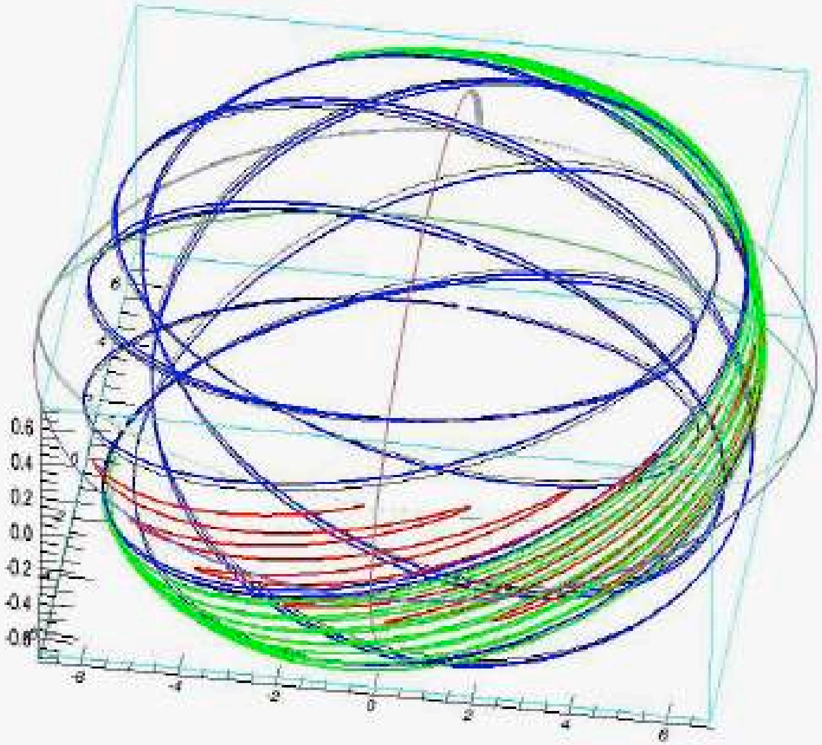
- Potential for turbulence control

Plasma microturbulence: Transport of fast particles?

Turbulent structures and energetic particle trajectories

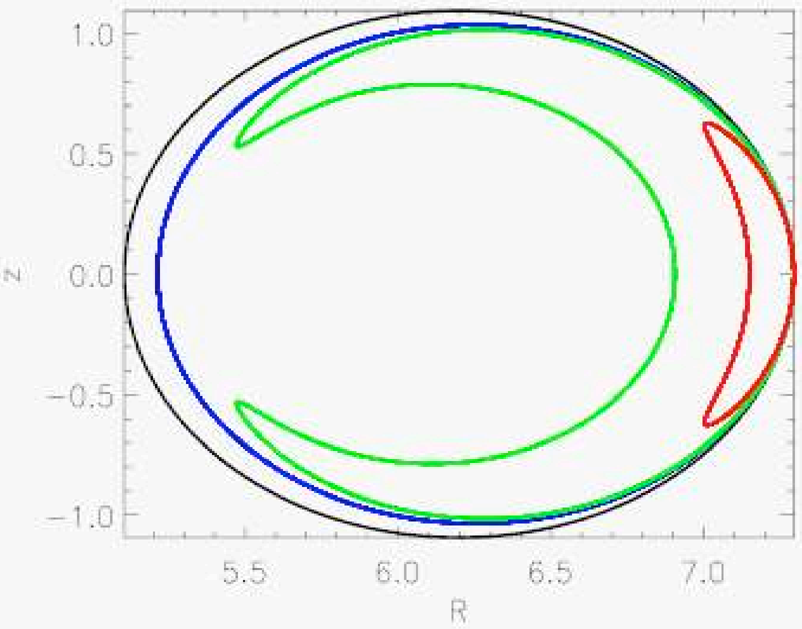


Turbulence in a torus (GENE)

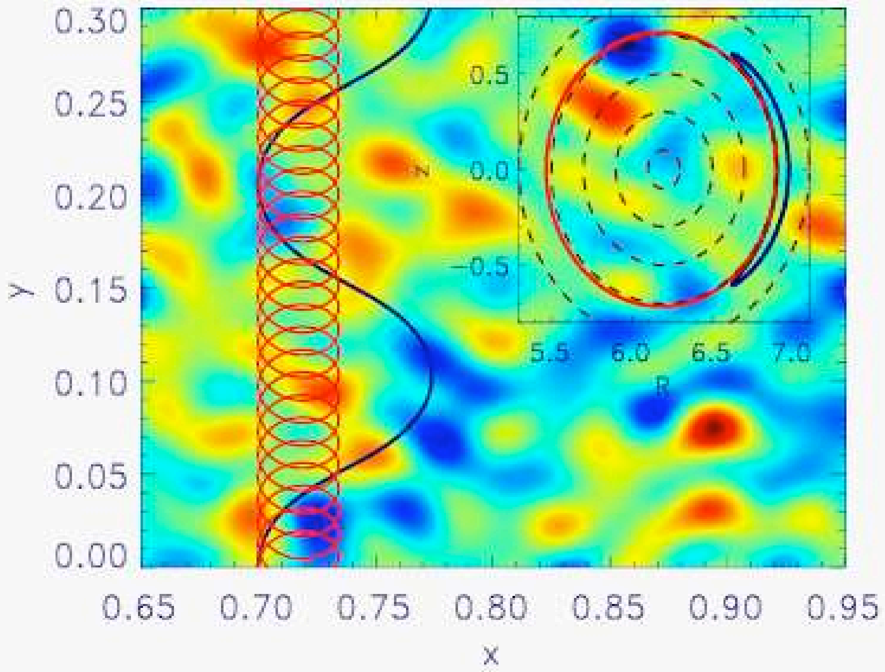


Drift orbits in the torus

Energetic particle trajectories (cont'd)



Drift orbits in the R-z plane



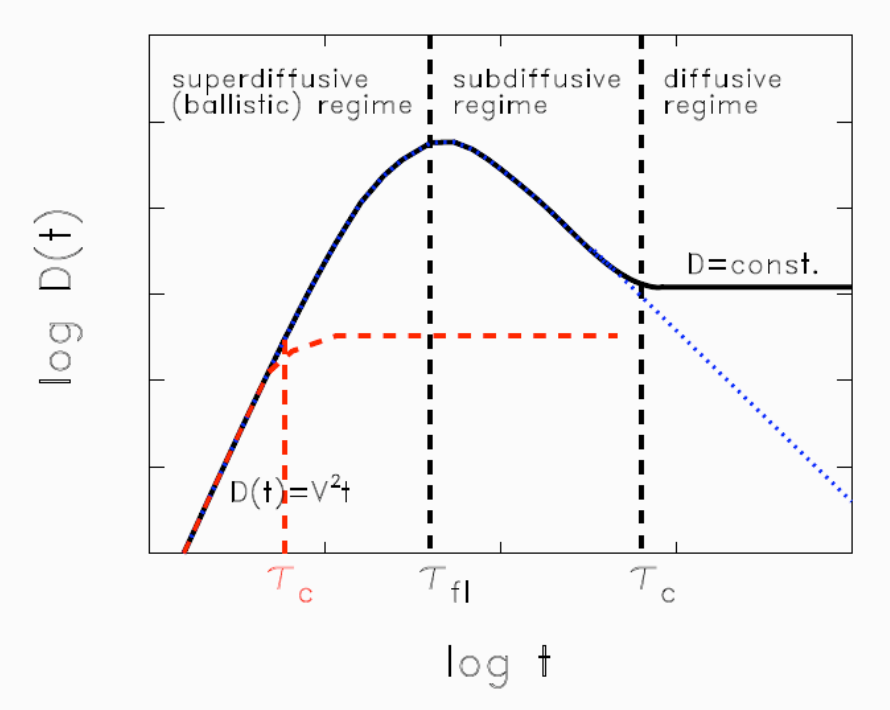
Drift orbits in field aligned coordinates

Standard view since late 1970s: Fast particles orbit-average out the fluctuations and therefore don't feel the turbulence.
Is this correct?

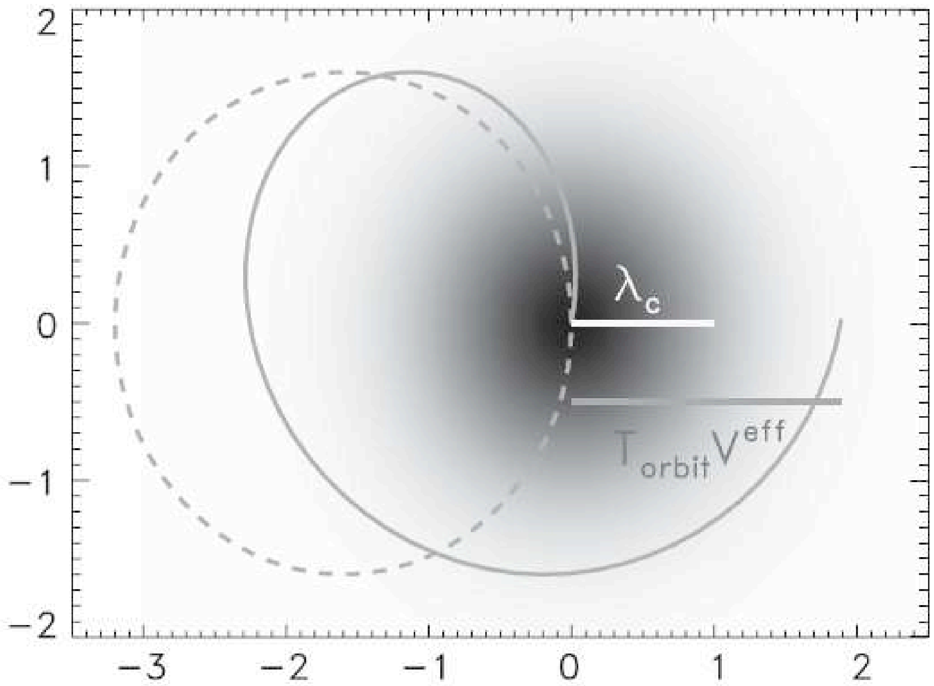
Diffusivities and correlations

Test particle approach

- Diffusion equation: $\partial_t n(\mathbf{x},t) = D(t)\nabla^2 n(\mathbf{x},t)$
- Einstein (1905): $D_x(t) = \frac{1}{2}\frac{d}{dt}\langle \delta x^2(t)\rangle$
- Taylor (1920): $D_x(t) = \frac{1}{2}\frac{d}{dt}\langle \delta x^2(t)\rangle = \int_0^t d\xi\, \langle v_x(0) v_x(\xi)\rangle \equiv \int_0^t d\xi\, L_{v_x}(\xi)$



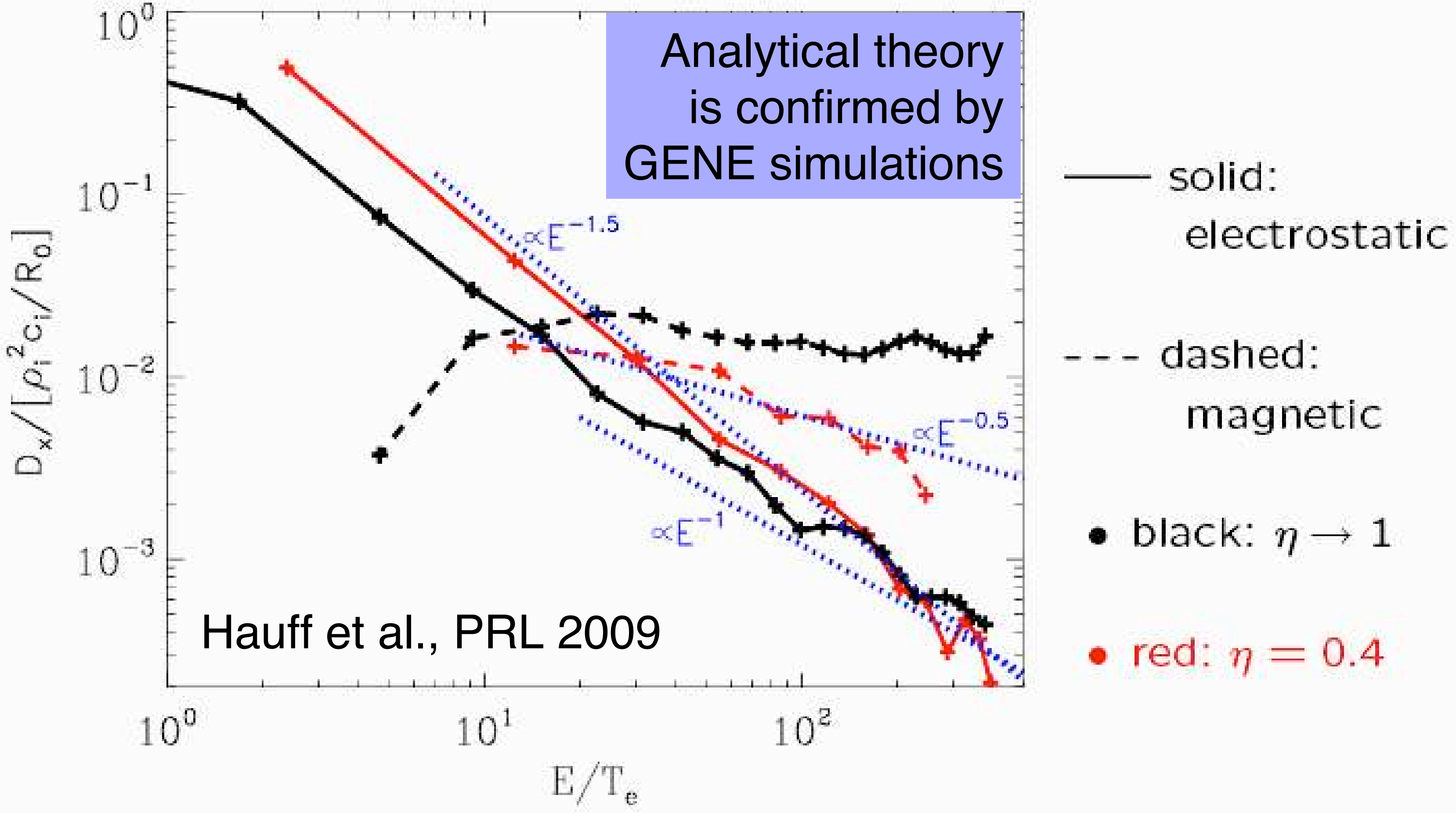
Validity of gyro/orbit averaging



$$\Xi_{\text{o.a.}} \equiv \max\{V_E, |v_{\text{dr}} - v_y|\}\frac{T_{\text{orbit}}}{\lambda_c} < 1, \qquad T_{\text{orbit}} \ll \tau_c$$

Magnetic transport along fluctuating field lines: $v_B \equiv v_{\parallel}(\tilde{B}_r/B_0)$

Scaling laws: Diffusivities vs energy



Analytical theory is confirmed by GENE simulations

Hauff et al., PRL 2009

Beam ions: Electrostatic transport falls off (only) as 1/E, and magnetic transport of beam ions is independent of energy!

Signs of turbulent fast-ion transport (for E/T~10) in DIII-D (Heidbrink et al.)

Conclusions

Some surprising findings

- Importance (or not) of zonal flows
- Role of high-k turbulence
- Turbulence in optimized stellarators
- Interaction of turbulence with fast ions

Gyrokinetic simulation has been crucial to discover these features.

Some outstanding open issues

- Prediction of steady state density and temperature profiles
- Nonlocal, nondiffusive & finite-size effects
- Core and edge transport barriers

To tackle these issues, we are required and prepared to use the next generation(s) of HPC platforms.

The ultimate goal? A virtual fusion plasma!