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Drift Wave Turbulence and Zonal Flows Part II

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CSDX –

Drift Turbulence-Zonal Flow Studies in a Laboratory Plasma

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Outline

- Demonstrate Development of "Weak" Turbulence from Drift Waves & Onset of Zonal Flows
- Study Nonlinear dynamics of coupled DWT/ZF system in simple system
- Show evidence for a non-diffusive turbulent stress at plasma boundary acting to drive rotation
- Demonstrate Critical Gradient dynamics of the system

➔ SIMPLE PLASMA SYSTEM EXHIBITS THREE KEY NONLINEAR DYNAMICAL PROCESSES THOUGHT TO OCCUR IN FUSION SYSTEMS

Controlled Shear De-Correlation Experiment



Dimensionless Scales

Length Scales $\rho_i / L_n \sim 0.1$ $\rho_S / L_n \sim 0.5$ $\lambda_e^{mfp} / L_{\parallel} \sim 0.05 - 0.1$

Parallel Dissipation Rate



Collision Rates

$$v_e / \Omega_{C_e} \sim 0.005$$
$$v_{ii} / \Omega_{Ci} \sim 1$$
$$v_{i0} / \Omega_{Ci} \sim 0.01$$

Non-ambipolar G.C. Drifts

Polarization Drift

$$\omega/\Omega_{C_i} \sim 0.1 - 0.3$$

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Ion-Ion Collisional Drift

$$\frac{\mu_{ii}}{L_{\perp}^2 \Omega_{ci}} \approx 0.01 - 0.1$$



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DEVELOPMENT OF DRIFT WAVES





Equilibrium Profiles Evolve as B Field Increases in <u>CSDX Helicon Plasma</u> <u>CSDX</u> –



Burin et al, May 2005 PoP





Typical Result for B>Bcrit





Wave at Onset Consistent w/ Collisional Drift Wave

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Linear Eigenmodes of DW w/ Flow Shear Match Observations Near Onset CSDX –







Close to Onset Measured Dispersion Agrees with Linear Theory



Burin Phys. Plasmas 2005







A Few Useful Tools from Digital Signal Processing





Linear Signal Processing Gives Average Spatio-temporal Scales of TIME STATIONARY Turbulence

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Fourier Transform:

$$f(t) = \sum_{\omega} f(\omega) \exp(i\omega t) \Delta \omega$$
 $f(\omega) = \frac{1}{2\omega} \sum_{\tau} f(t) \exp(-i\omega t) \Delta t$

Correlation Functions & Power Spectra:

$$C_{auto}(\tau) = \frac{1}{N_{ens}} \sum_{n=1}^{N_{ens}} \frac{1}{T} \int_{-T/2}^{T/2} \tilde{f}_i(t) \tilde{f}_i(t+\tau) \qquad P_{auto}(\omega) = \frac{1}{N_{ens}} \sum_{n=1}^{N_{ens}} \left| \tilde{f}_i(\omega) \right|^2$$

$$C_{crs}(\tau) = \frac{1}{N_{ens}} \sum_{n=1}^{N_{ens}} \frac{1}{T} \int_{-T/2}^{T/2} \tilde{f}_i(t) \tilde{g}_i(t+\tau) \qquad P_{crs}(\omega) = \frac{1}{N_{ens}} \sum_{n=1}^{N_{ens}} \tilde{f}_i(\omega) \tilde{g}_i^*(\omega)$$

Inter-relation Between Two Signals: coherence and crossphase

$$\gamma_{12}(\omega) = \frac{|P_{crs}(\omega)|}{\sqrt{P_1P_2}}, \tan \phi(\omega) = \frac{\operatorname{Im}(P_{crs}(\omega))}{\operatorname{Re}(P_{crs}(\omega))}$$





Nonlinear Energy Transfer Comes from 3rd Order Spectrum: Example: Collisional Drift Turbulence Model (Hasegawa-Wakatani 1983)

$$\frac{\partial W^{n}}{\partial t} = \frac{\Gamma_{r}}{L_{n}} + T^{n} + C_{1} \left(W^{n} - \left\langle n^{*} \phi \right\rangle \right)$$
$$\frac{\partial W^{\phi}}{\partial t} = T^{\phi} - C_{2} \left\langle \left| \nabla_{\perp}^{2} \phi \right|^{2} \right\rangle - C_{3} W^{\phi}$$

Internal and Kinetic Energies Defined As

$$W^{n} = \sum_{m} \left| n_{m}(r,t) \right|^{2}$$
$$W^{\phi} = \sum_{m} \left| \nabla_{\perp} \phi_{m}(r,t) \right|^{2}$$

Nonlinear Energy Transfer From Cross-Bispectrum:

$$T_{\theta}^{n}(r,f,f') = -\operatorname{Re}\left\langle \tilde{n}^{*}(r,f)\tilde{v}_{\theta}(r,f-f')\frac{1}{r}\frac{\partial\tilde{n}}{\partial\theta}(r,f')\right\rangle$$
$$T_{ZF}^{\phi}(r,f,f') = -\operatorname{Re}\left\langle V_{ZF}^{*}(r,f)\tilde{v}_{\theta}(r,f-f')\frac{1}{r}\frac{\partial\tilde{v}_{r}}{\partial\theta}(r,f')\right\rangle$$

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Rate and Direction of Energy Transfer Determined by BiSpectrum CSDX –

Bi-spectrum Defined As

$$B(\omega_1,\omega_2) = \langle X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2) \rangle$$

Degree of Phase Coherence Determined by BiCoherence

$$\hat{b}^{2}(\omega_{1},\omega_{2}) = \frac{B(\omega_{1}+\omega_{2})}{\left\langle \left|X(\omega_{1})X(\omega_{2})\right|^{2}\right\rangle \left\langle \left|X^{*}(\omega_{1}+\omega_{2})\right|^{2}\right\rangle} \quad ; \quad 0 < \hat{b} < 1$$

BiPhase Determines Phase Delay Between Interacting Waves:

$$\Theta(\omega_1, \omega_2) \equiv Tan^{-1} \left\{ \frac{\operatorname{Im} \left[B(\omega_1, \omega_2) \right]}{\operatorname{Re} \left[B(\omega_1, \omega_2) \right]} \right\} \quad ; \quad -\pi < \Theta < \pi$$

Energy Transfer Direction and Rate:

$$\left[\operatorname{Re}\left[B(\omega_{1},\omega_{2})\right]=\left|B(\omega_{1},\omega_{2})\right|\cos\left[\Theta(\omega_{1},\omega_{2})\right]\right]$$

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DEVELOPMENT OF DRIFT TURBULENCE FROM LINEAR DRIFT WAVES





Origins of Nonlinear DW Dynamics

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Keep Convective Derivative in Eqn's

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = 0$$
$$\frac{\partial n \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla n \mathbf{u} = RHS$$

In Fourier Domain (I.e. waves) Introduces Convolution in (X,k), e.g.

$$\frac{1}{2} \frac{\partial \langle u^2(\mathbf{k}) \rangle}{\partial t} + \sum_{\substack{\mathbf{k}_1 \mathbf{k}_2 \\ \mathbf{k} = \mathbf{k}_1 \pm \mathbf{k}_2}} \operatorname{Re} \langle u^*(\mathbf{k}) (\tilde{\mathbf{u}}(\mathbf{k}_1) \cdot \nabla) \tilde{u}_{\theta}(\mathbf{k}_2) \rangle = RHS$$





Chu et al Phys. Fluids 1973

3-Wave Resonance Criteria Was Satisfied:

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 \pm \boldsymbol{\omega}_2$$
$$\mathbf{k} = \mathbf{k}_1 \pm \mathbf{k}_2$$

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DEVELOPMENT OF DRIFT TURBULENCE FROM LINEAR DRIFT WAVES

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- Increase Free Energy Source and/or
- Decrease Linear Damping
- Two Experiments:
 - Klinger, 1997: Increase Free Energy (J_{\parallel})
 - Burin, 2005: Increase L_n / ρ_s , Decrease $\mu_{\rm ii}$ viscous damping



Local k-spectra Are Constructed from 2-Point Measurements

Ref: J.M. Beall et al, J. App. Phys. 1982

Measure Fluctuations at 2 Points:



Find Phase Delay due to Propagation $Im(X(f)X^*(f))$

$$\delta \vartheta = Tan^{-1} \frac{\operatorname{IIII}(\langle X_1(f) X_2(f) \rangle)}{\operatorname{Re}(\langle X_1(f) X_2^*(f) \rangle)}$$

Local Wavenumber
$$k_{\theta_{local}} = \frac{\delta \vartheta}{\delta x}$$

Build-up Local k-spectra from Multiple Realizations:



Burin et al, May 2005 PoP

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Evolution of $(\phi_f/kT_e)^2$ Power Spectrum with Increasing Magnetic Field

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- Coherent Drift Waves Appear at ~400G
- Harmonics Develop As B Increases
- Coherent Modes at Intermediate B
- Broadband Spectra at 1kG

Burin et al, May 2005 PoP

3-wave Interaction Increases as Turbulence Develops

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Tynan, PoP 2004

Burin, PoP 2005

Burin et al, May 2005 PoP

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Real and Imaginary Frequencies of Linear Eigenmodes

Free Energy Sources Are Known → Can Find Linearly Unstable Region CSDX –

- Include grad-P, Vshear Free Energy Sources
- Include Neutral Flow Drag (Effective at high k), FLR Damping
- Find Stable & Unstable Regions

Implies Energy MUST Be Transferred Into Low-k Region Via Nonlinear Processes

Tynan et al Nov 2004 PoP

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Radially Sheared Azimuthal Fluid & Fluctuation Propagation Occurs at High B-field

J.H.Yu, et al., Journal of Nuclear Materials, V363-365, 728, 2007

Fast Imaging of DWT Fluctuation Propagation

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Radially Sheared Azimuthal Flow & Finite m DWT Fluctuations

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Evidence that Flow is Driven by the Turbulence

Turbulent Azimuthal Momentum Balance

Tynan et al April 2006 PPCF Holland et al, in press, PRL

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Estimate Dissipation from Measurements

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Measured Profile Consistent with Turbulent Momentum Balance

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Tynan et al, April 2006 PPCF, , Holland et al, PRL 2006

No Turbulent Particle Transport Across Zonal Flow

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Tynan et al, April 2006 PPCF, Holland et al, In press, PRL

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Shear Layer Formation in Collisional Drift Turbulence Simulations

Nonlinear Drift Turbulence Simulation

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- (2D) Hasegawa-Wakatani model in cylindrical geometry.
 - Includes ion-neutral flow damping effect *n*, **neglects** nonlocal (finite r_s / L_n) terms, **fixed parallel wavenumber**.

$$\left(\frac{\partial}{\partial t} + \vec{V}_{E \times B} \cdot \vec{\nabla} \right) n + \frac{V^*}{r} \frac{\partial n}{\partial \theta} + \frac{k_{\parallel}^2 v_{_{th_e}}^2}{\omega^* v_e} (n - \phi) = D_n \nabla_{\perp}^2 n$$

$$\left(\frac{\partial}{\partial t} + \vec{V}_{E \times B} \cdot \vec{\nabla} \right) \nabla_{\perp}^2 \phi + \frac{k_{\parallel}^2 v_{_{th_e}}^2}{\omega^* v_e} (n - \phi) = v \nabla_{\perp}^2 \phi + \mu \nabla_{\perp}^4 \phi$$

- Parameters used reflect best estimates for average CSDX values:
 - $r_s = 1 \text{ cm}, L_n = 2 \text{ cm}, W_{\parallel} = 1, n = 0.03 \text{C}_{\text{s}}/\text{L}_{\text{n}},$
 - $D_n = 0.01 r_s^2 C_s / L_n$, $m = 0.4 r_s^2 C_s / L_n$
- Advances eqns by combination of 2nd order RK and implicit treatment of diffusive terms (conserves energy to within 1%).

Iso-Potential Shows Zonal Flow Formation from Drift Vortices

Simulations Show Zonal Flow Formation Vortex Merging

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Iso-Potential Contours:

Time

- Simulation uses 64 x 64 pts, results insensitive to changes in D_n , n, m
- Changing L_n to 10 cm does not qualitatively affect results

Numerical Model Reproduces Key Features of Experiment and Analysis

Tynan et al April 2006 PPCF

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$$\frac{1}{2} \frac{\partial \left\langle u_{\theta_{z}}^{2} \left(\mathbf{k}_{z} \right) \right\rangle}{\partial t} - P_{k_{z}}^{turb} = -\mu \left\langle u_{\theta_{z}}^{2} \left(\mathbf{k}_{z} \right) \right\rangle$$
where $P_{k_{z}}^{turb} = \sum_{\substack{\mathbf{k}_{1}\mathbf{k}_{2}\\\mathbf{k}_{z}=\mathbf{k}_{1}+\mathbf{k}_{2}}} \left[\operatorname{Re} \left\langle u_{\theta_{z}}^{*} \left(\mathbf{k}_{z} \right) \left(\tilde{\mathbf{u}} \left(\mathbf{k}_{1} \right) \cdot \nabla \right) \tilde{u}_{\theta} \left(\mathbf{k}_{z} \right) \right) \right]$

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- Free Energy Source Releases Energy On One Scale
- Nonlinear Energy Transfer Moves Energy to Dissipation Region
- Shear Flows Develop Via Transfer of Energy to LARGE SCALES (small k)

Nonlinear Energy Transfer

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Internal energy

$$T_{\text{int ernal}} = -\operatorname{Re} \sum_{\substack{\omega_1, \omega_2\\ \omega = \omega_1 + \omega_2}} n_{\omega}^* (\hat{z} \times \nabla_{\perp} \phi_{\omega_1} \cdot \nabla_{\perp}) n_{\omega_2}$$

Kinetic energy

$$T_{Kinetic} = -\operatorname{Re} \sum_{\substack{\omega_1, \omega_2\\ \omega = \omega_1 + \omega_2}} u_{\omega}^* (u_{\omega_1} \cdot \nabla_{\perp}) u_{\omega_2}$$

where $u_{\omega} = \frac{1}{B} \hat{z} \times \nabla_{\perp} \phi_{\omega}$

Nonlinear Kinetic Energy Transfer Confirms Turbulence Drive

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Direct Visualization of Drift Vortex-ZF Interaction

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- Vortex Born at Gradn Max
- Moves in r, θ plane
- Stretched Out by ZF

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Merges w ZF

Outline

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- Show evidence for a non-diffusive turbulent stress at plasma boundary acting to drive rotation
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➔ SIMPLE PLASMA SYSTEM EXHIBITS THREE KEY NONLINEAR DYNAMICAL PROCESSES THOUGHT TO OCCUR IN FUSION SYSTEMS

Flow Evolves at Slow Time Scale (250-300Hz)

Reynolds Stress Modulated In Phase Coherent Manner with ZF

•Kinetic energy transferred between higher frequency turbulence and low frequency flow.

Direct Visualization of Turbulent Decorrelation w/ High ZF Shearing Rate

• Shear flow decorrelating turbulence structure is observed at strong shear case, but not at weak shear case.

Anti-correlation between Flow Shear and Turbulence Radial Correlation Length

Shear flow de-correlates turbulent structures

Correlation between

$$\omega_s = r \frac{d}{dr} \left(\frac{v_{\theta}}{r} \right)$$
 and

radial correlation length

Outline

CSDX –

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- Demonstrate Development of "Weak" Turbulence from Drift Waves & Onset of Zonal Flows
- Study Nonlinear dynamics of coupled DWT/ZF system in simple system
- Show evidence for a non-diffusive turbulent stress at plasma boundary acting to drive "intrinsic" rotation
- Demonstrate Critical Gradient dynamics of the system

➔ SIMPLE PLASMA SYSTEM EXHIBITS KEY NONLINEAR DYNAMICAL PROCESSES THOUGHT TO OCCUR IN FUSION SYSTEMS

Finite Nondiffusive Stress at Boundary is Required to Drive NET Instrinsic Plasma Rotation:

Azimuthal momentum balance w/o dissipation:

Synthesize Diffusive Flux from Measurements:

Non-diffusive Stress Localized to Boundary Region

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Nonlinear Kinetic Energy Transfer into m=0 Flow LOCALIZED TO EDGE

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Schematic of Sheared Flow Dynamics

Z. Yan et al, Submitted for Publication

UCDDJacoos

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Results Similar to Momentum Transport in Tokamaks

Rice et al, Nuc Fusion 2004, Lee et al, PRL 2003

Toroidal Momentum Transport Source Appears Localized to Edge & Diffused/ Convected Inwards

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- Nonlinear dynamics of coupled DWT/ZF system in simple system
- Evidence for a non-diffusive turbulent stress at plasma boundary acting to drive rotation
- Demonstrate Critical Gradient dynamics of the system

Magnetic field scaling of shear flow

• Equilibrium density

• Density fluctuation amplitude and radial particle flux

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Magnetic field scaling of shear flow (2)

 Divergence of turbulent Reynolds stress at shear layer increase with magnetic fields

CSDX —

Development of Shear Flow w/ B-Field

Shear flow increases with magnetic fields (~ 700G or above)

Z. Yan PhD Dissertation, UCSD 2009

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Re-casting in terms of Critical Gradient

Linear Dispersion Relation, including Cylindrical Geometry, Ion Viscosity, Ion-Neutral Damping:

$$-ik_{mn}^2\omega_{mn}^2 + \left\{\omega_{\parallel}\left(1+k_{mn}^2\right)+v_k\right\}\omega_{mn} - \omega_{\parallel}\left(m\frac{\rho_s}{L_n}-iv_k\right) = 0$$

Where damping and wavenumbers are given by:

$$v_k = (v_{i-n} + \mu_{ii}k_{mn}^2)k_{mn}^2$$

$$k_{mn} = X_{mn}/a$$
, where X_{mn} is the n^{th} zero of $J_m(x)$.

And Eigenmodes have the form:

$$\tilde{\phi}(r \,\theta, t) = \operatorname{Re}\sum_{m,n} \tilde{\phi}_{mn} J_m(k_{mn}r) \exp\{i(m\theta - \omega_{mn}t)\}$$

Critical Gradient Development vs. |B|

B (Tesla)

Evolution of DWT-ZF System vs. Critical Gradient

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Result is Similar to Tokamak Thermal Transport

Conclusions

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- Nonlinear dynamics of coupled DWT/ZF system in simple system qualitatively consistent with theory
- Show evidence for a non-diffusive turbulent stress at plasma boundary & diffusive transport in center acting to drive rotation
- Demonstrate Critical Gradient dynamics of the system

SIMPLE PLASMA SYSTEM EXHIBITS THREE KEY NONLINEAR DYNAMICAL PROCESSES THOUGHT TO OCCUR IN FUSION SYSTEMS

