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Nonlinear vortical structures and light-plasmon turbulence in quantum plasmas

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OUTLINE

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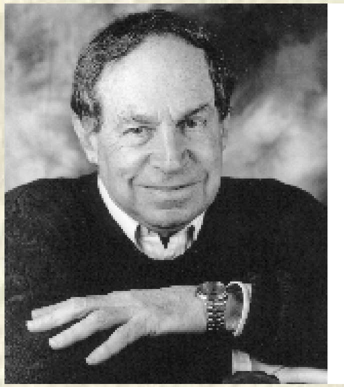
Eugene Paul Wigner (1902–1995). Nobel Prize in Physics in 1963 ”for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles” (Quantum plasma physics: Wigner-Moyal transform, Wigner function)

David Joseph Bohm (1917–1992) (Bohm-diffusion, the Bohm sheath criterion, the quantum Bohm potential. Also the Aharonov-Bohm effect where a charged particle is affected by electromagnetic fields in regions from which the particle is excluded)



David Pines (1924–) Awarded two Guggenheim Fellowships, the Feenberg Medal, Friemann, Dirac, and Drucker Prizes. (Theory of many-body systems and theoretical astrophysics, early works on collective effects in quantum plasma.) Prof. Em. at University of Illinois at Urbana–Champaign.

Introduction

❑ Quantum plasmas are ubiquitous in:

- ☛ ultrasmall electronic devices and micromechanical systems
- ☛ intense laser–solid density plasma interaction experiments
- ☛ microplasmas
- ☛ superdense astrophysical objects (neutron stars and white dwarfs)

❑ Quantum mechanical effects can be important when the de Broglie wavelength of the charge carriers (electrons, positrons) is comparable to:

- ☛ the dimension of the system $\rightarrow$ tunneling effects
- ☛ the mean distance between particles $\rightarrow$ overlapping of wave functions, quantum statistics

Introduction (Continued)

❑ Classical vs. Quantum plasmas

- ☛ Classical plasmas have low density and high temperature.
- ☛ Quantum plasmas have high density and low temperature

❑ Quantum forces due to the

- ☛ strong electron/positron (hole) density correlations (the Bohm potential),
- ☛ the quantum statistical description for a Fermi plasma yields a new pressure law owing to the Fermi-Dirac statistics.

❑ Nonlinear waves and structures

- ☛ dark quantum solitons and vortices
- ☛ nonlinear interaction with electromagnetic waves

Classical Plasmas vs. Quantum Plasmas

- Quantum effects can be measured by the thermal de Broglie wavelength of the particles composing the plasma

$$\lambda_B = \frac{\hbar}{mV_T}, \qquad V_T = \sqrt{\frac{k_B T}{m}}$$

which roughly represents the spatial extension of a particle's wave function due to quantum uncertainty.

- For classical regimes, the de Broglie wavelength is so small that particles can be considered as point-like, and therefore there is no overlapping of the wave functions and no quantum interference.

Classical Plasmas vs. Quantum Plasmas (Continued)

Quantum effects start playing a significant role when

- the de Broglie wavelength is similar to or larger than the average interparticle distance $n^{-1/3}$, i.e. when

$$n\lambda_B^3 \gtrsim 1,$$

or, the temperature is comparable or lower than the Fermi temperature $T_F = E_F/k_B$, where

$$E_F = \frac{\hbar^2}{2m}(3\pi^2)^{2/3}n^{2/3}$$

is the Fermi energy for electrons, so that

$$\chi = \frac{T_F}{T} = \frac{1}{2}(3\pi^2)^{2/3}(n\lambda_B^3)^{2/3} \gtrsim 1$$

Properties of Dense Quantum Plasmas

- The quantum coupling parameter

$$\Gamma_Q = \frac{E_{int}}{E_F} = \frac{2}{(3\pi^2)^{2/3}} \frac{m_e e^2}{\hbar^2 n_e^{1/3}} \sim \left(\frac{1}{n\lambda_F^3}\right)^{2/3} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2,$$

where $E_{int} = e^2 n_e^{1/3}$ is the interaction energy, is analogous to the classical one $\Gamma_C = E_{int}/k_B T_e$ when $\lambda_F \to \lambda_D$, where the Fermi screening scalelength

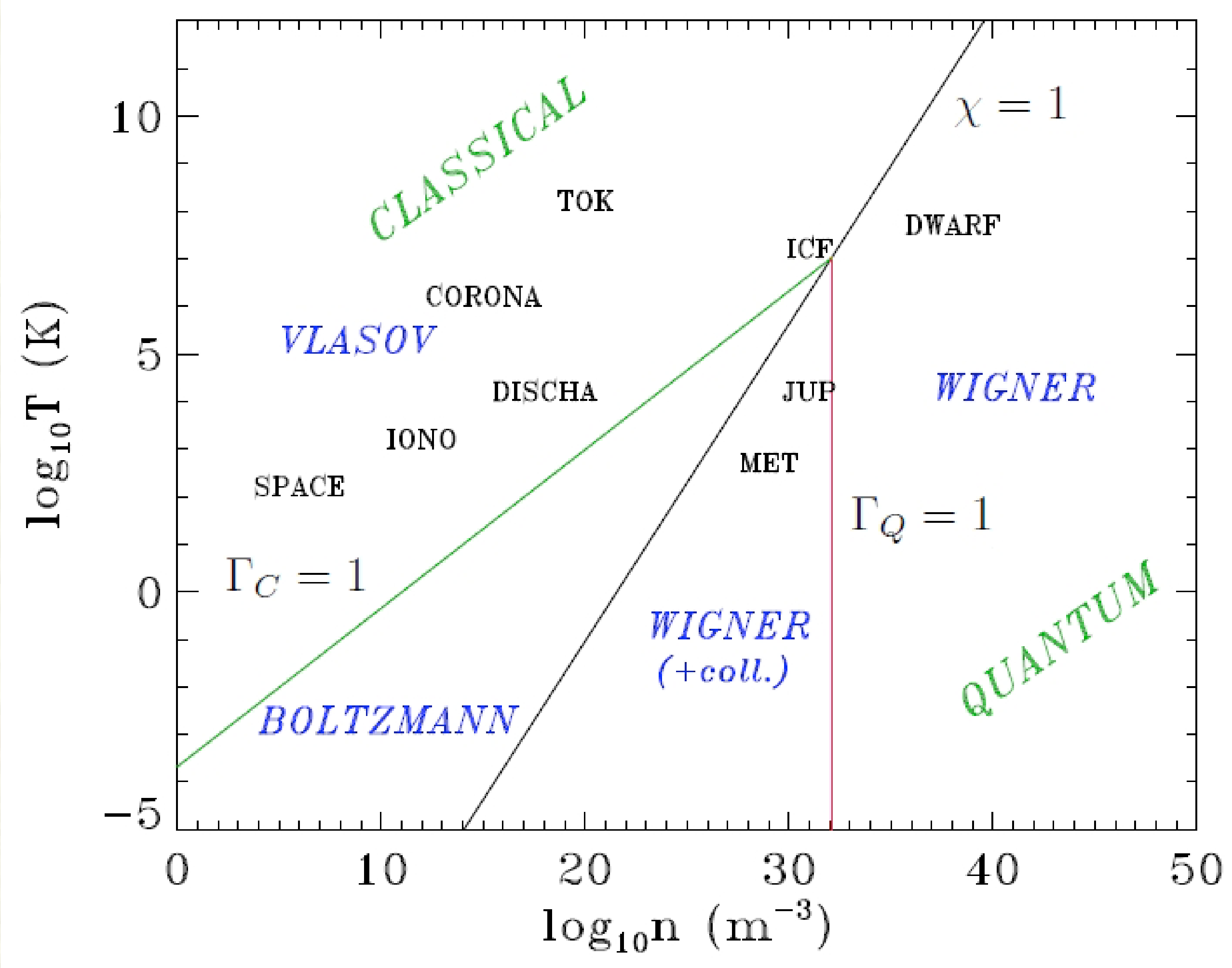
$$\lambda_F = V_F/\omega_p$$

is the quantum analogue of the Debye radius and

$$V_F = (2E_F/m)^{1/2} = \frac{\hbar}{m}(3\pi^2 n)^{1/3}.$$

is the speed of an electron at the Fermi surface. We have Γ_C, $\Gamma_Q < 1$ for weakly collisional and Γ_C, $\Gamma_Q > 1$ for collisional plasmas.

Plasma diagram



Bonitz *et al.*, J. Phys. A: Math. Gen. **36**, 5921 (2003)

Manfredi, Fields Inst. Commun. **46**, 263 (2005); arxiv:quant-ph/0505004.

Equation of state in dense stars

When the mean distance between electrons $d = n^{-1/3}$ is comparable to the Compton length $\lambda_C = h/m_e c \approx 2.4 \times 10^{-12}\,\mathrm{m}$ (or $n_e \gtrsim 10^{35}\,\mathrm{m}^{-3}$), then the Fermi energy $\mathcal{E}_F$ of the electron is comparable to the electron rest energy $m_e c^2$, and the momentum of an electron at the Fermi surface becomes relativistic. Then the pressure P is given by the Fermi pressure of the degenerate electron gas,

$$P = \frac{\pi m_e^4 c^5}{3h^3}[\xi(2\xi^2 - 3)(\xi^2 + 1)^{1/2} + 3\mathrm{arcsinh}(\xi)], \quad (1)$$

where $\xi = p_0/m_e c$ is related to the electron number density by

$$\xi = (3/8\pi)^{1/3} n_e^{1/3} \lambda_C \quad (2)$$

Equation of state in dense stars

In the non-relativistic limit $n_e \ll \lambda_C^{-3}$ (or $\xi \ll 1$), we have

$$P = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{m_e} n_e^{5/3} = \frac{2}{5}\mathcal{E}_F n_0 \left(\frac{n_e}{n_0}\right)^{5/3}, \tag{3}$$

where $\mathcal{E}_F = (3\pi^2 n_0)^{2/3}\hbar^2/2m_e$ is the Fermi energy and $\hbar = h/2\pi$, and in the ultra-relativistic limit $n_e \gg \lambda_C^{-3}$ (or $\xi \gg 1$) we have

$$P = \left(\frac{3}{\pi}\right)^{1/3} \frac{ch}{8} n_e^{4/3}. \tag{4}$$

The decrease of exponent from $5/3$ to $4/3$ leads to the collapse of white dwarfs with masses larger than ~ 1.4 solar masses.

S. Chandrasekhar, The Highly Collapsed Configurations of a Stellar Mass (second paper), MNRAS **95**, pp. 207–225 (1935).

Anti-symmetric wave function — Pauli exclusion principle

- Model for the quantum N body problem: The Schrödinger equation for the N-particle wave function $\psi(q_1, q_2, \ldots, q_N, t)$ where $q_j = (\mathbf{r}_j, s_j)$ (space, spin) of particle j.
  Identical Fermions: The Slater determinant

$$\psi(q_1,\, q_2,\, \ldots,\, q_N, t) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(q_1,t) & \psi_2(q_1,t) & \cdots & \psi_N(q_1,t) \\ \psi_1(q_2,t) & \psi_2(q_2,t) & \cdots & \psi_N(q_2,t) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_1(q_N,t) & \psi_2(q_N,t) & \cdots & \psi_N(q_N,t) \end{vmatrix}$$

  Anti-symmetric under odd numbers of permutations. The Pauli exclusion principle: ψ vanishes if two rows are identical.
  Example $(N = 2)$:
  $\psi(q_1, q_2, t) = \frac{1}{\sqrt{2}}[\psi_1(q_1, t)\psi_2(q_2, t) - \psi_1(q_2, t)\psi_2(q_1, t)]$ so that $\psi(q_2, q_1, t) = -\psi(q_1, q_2, t)$ and $\psi(q_1, q_1, t) = 0$.

Quantum kinetic model

- The quantum analogue to the Vlasov-Poisson system is the Wigner-Poisson model

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = -\frac{iem_e^3}{(2\pi)^3 \hbar^4}$$

$$\times \iint d^3\lambda \, d^3v' e^{im_e(\mathbf{v}-\mathbf{v}')\cdot \boldsymbol{\lambda}/\hbar} \left[\phi\left(\mathbf{x} + \frac{\boldsymbol{\lambda}}{2}, t\right) - \phi\left(\mathbf{x} - \frac{\boldsymbol{\lambda}}{2}, t\right) \right] f(\mathbf{x}, \mathbf{v}', t)$$

and

$$\nabla^2 \phi = 4\pi e \left(\int f d^3v - n_0 \right).$$

- Note that the Wigner equation converges to the Vlasov equation when $\hbar \to 0$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = -\frac{e}{m_e} \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}}$$

Quantum Hydrodynamical (QHD) Model

- We take the moments of the Wigner equation and obtain for the quantum-electron fluid

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$m\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = e\nabla\phi - \frac{1}{n}\nabla P + \mathbf{F}_Q,$$

where ϕ is determined from $\nabla^2\phi = 4\pi e(n - n_0)$, and for the FD plasma we have

$$P = \frac{mV_F^2}{3n_0^2}n^3 \quad \text{and} \quad \mathbf{F}_Q = \frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) \equiv -\nabla\phi_B.$$

Manfredi & Haas, Phys. Rev. B **64**, 075316 (2001),

Manfredi, Fields Inst. Commun. **46**, 263 (2005); arxiv:quant-ph/0505004.

Electrostatic electron waves

Linearization of the NLS-Poisson Eqs. yields the frequency of EPOs

$$\omega_k = \left(\omega_{pe}^2 + k^2 V_{TF}^2 + \frac{\hbar^2 k^4}{4m_e^2}\right)^{1/2}, \qquad V_{TF} = \sqrt{\frac{k_B T_{Fe}}{m_e}}$$

Two distinct dispersive effects:

- ☛ Long wavelength regime: $V_{TF} \gg \hbar k/2m_e$
- ☛ Short wavelength regime: $V_{TF} \lesssim \hbar k/2m_e$
- ☛ Critical wavenumber:

$$k_{crit} = \frac{2\pi}{\lambda_{crit}} = \frac{\pi\hbar}{m_e V_{TF}} \sim n^{-1/3}$$

Similar results obtained by Bohm & Pines, Phys. Rev. **92**, 609 (1953) and Pines, J. Nucl. Energy, Part C: Plasma Physics **2**, 5 (1961).

Effective nonlinear Schrödinger equation

Introduce the effective wave function

$$\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\exp(iS(\mathbf{r},t)/\hbar)$$

where S is defined according to $m\mathbf{u} = \nabla S$ and $n = |\psi|^2$. For this *particular case of potential flow*, the QHD equations are equivalent to

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi + e\phi\psi - \frac{mV_F^2}{2n_0^2}|\psi|^{4/D}\psi = 0$$

and

$$\nabla^2\phi = 4\pi e(|\psi^2| - n_0)$$

Manfredi & Haas, Phys. Rev. B **64**, 075316 (2001),

Manfredi, Fields Inst. Commun. **46**, 263 (2005); arxiv:quant-ph/0505004.

Nonlinear Structures at Quantum Scale

Normalized system of equations for plasmons

$$i\frac{\partial\Psi}{\partial t} + A\nabla^2\Psi + \varphi\Psi - |\Psi|^{4/D}\Psi = 0\ ,$$

$$\nabla^2\varphi = |\Psi|^2 - 1,$$

where A represents the quantum coupling strength. Conserved quantities:

$$N = \int |\Psi|^2\, d^3x$$

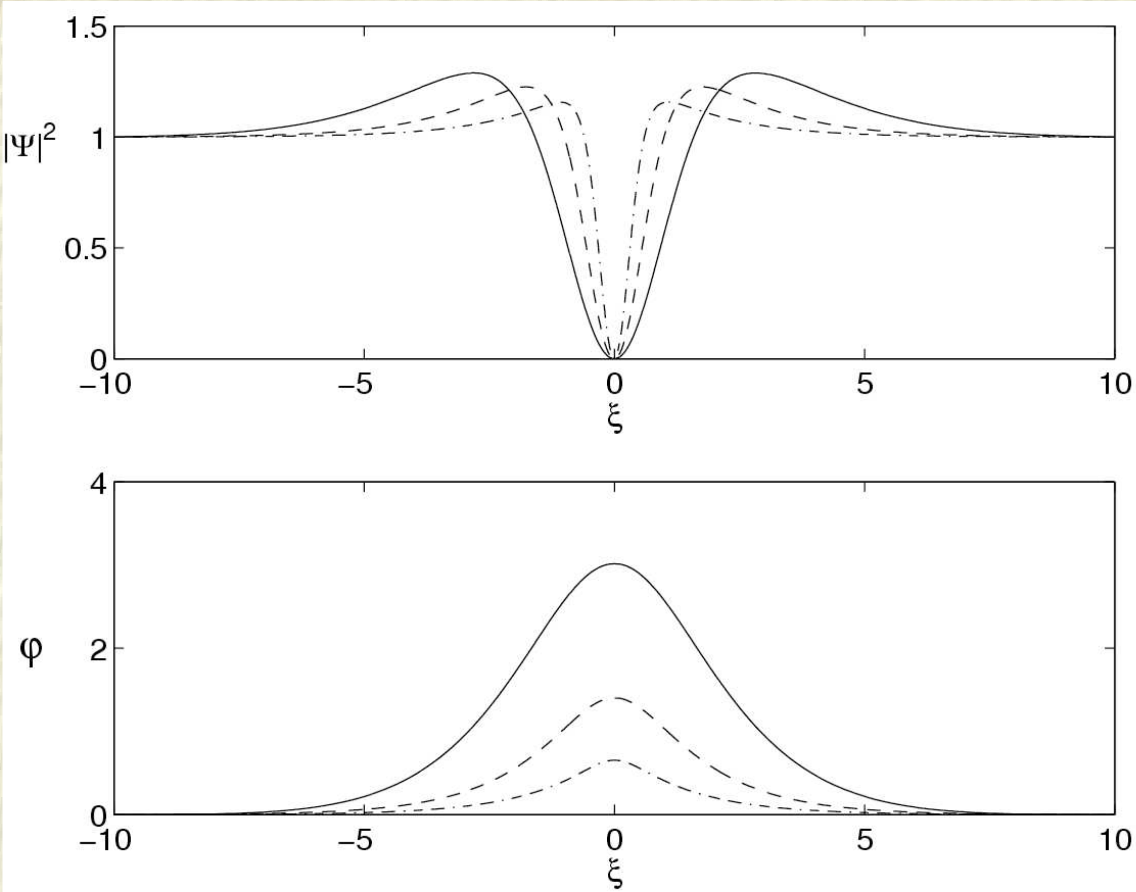
$$\mathbf{P} = -i\int \Psi^*\nabla\Psi\, d^3x$$

$$\mathbf{L} = -i\int \Psi^*\mathbf{r}\times\nabla\Psi\, d^3x$$

$$\mathcal{E} = \int [-\Psi^* A\nabla^2\Psi + |\nabla\varphi|^2/2 + |\Psi|^{2+4/D} D/(2+D)]\, d^3x$$

Shukla & Eliasson, PRL **96**, 245001 (2006)

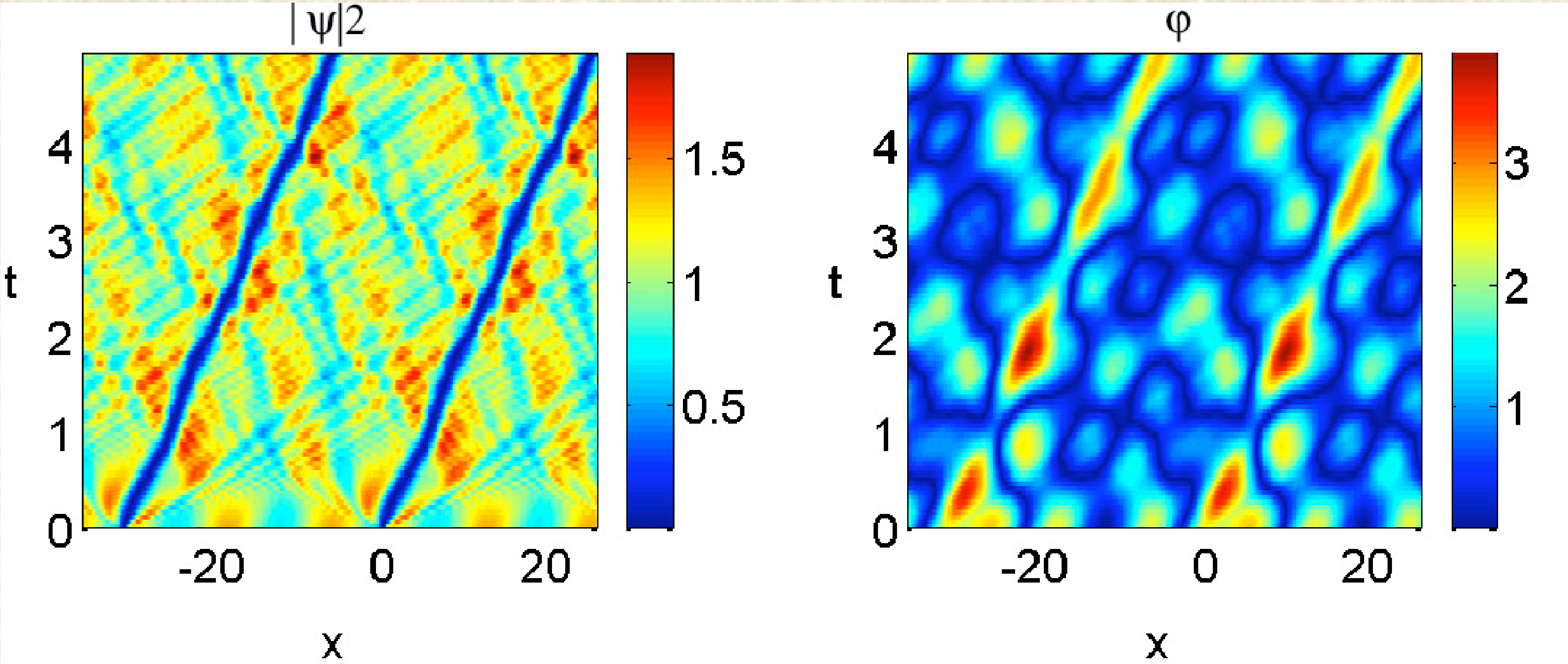
1D Quantum Electron Hole (Dark Soliton)



Shukla & Eliasson, PRL **96**, 245001 (2006)

Dynamics of Quantum Electron Holes

Electron density (left) & electrostatic potential (right)



Shukla & Eliasson, PRL **96**, 245001 (2006)

2D Quantum Electron Vortices

Introduce $\Psi = \psi(r)\exp(in\theta - i\Omega t)$

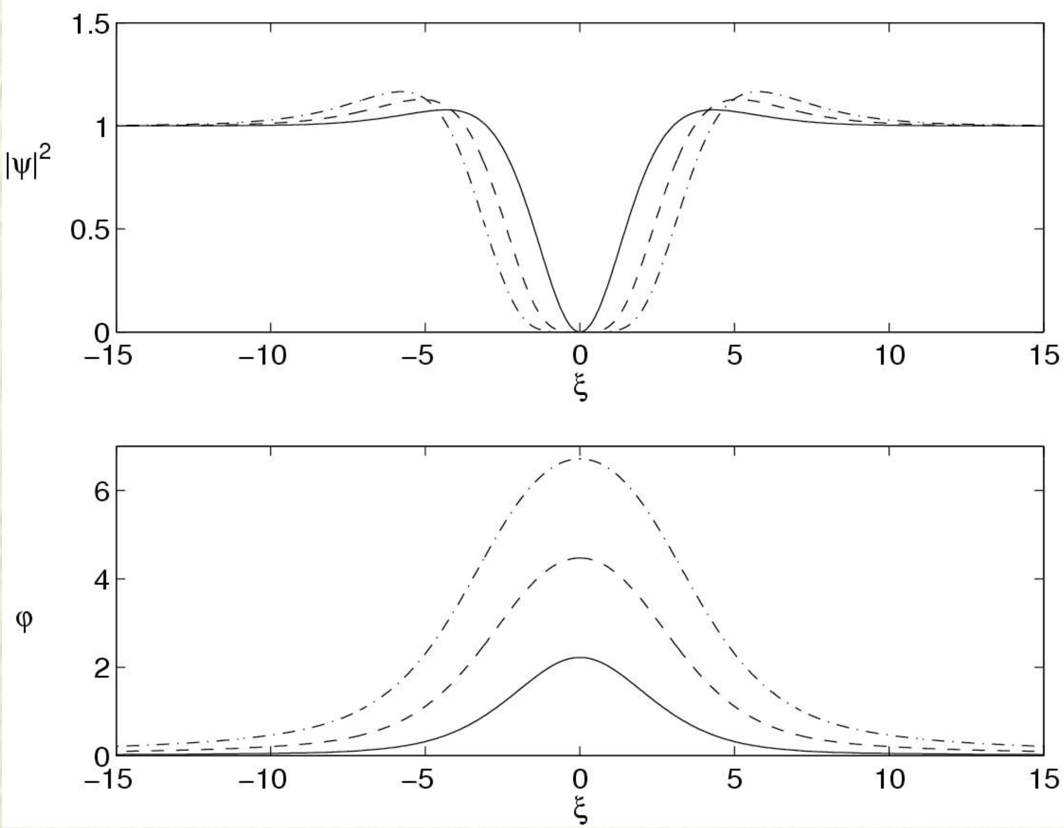
$$\Omega\psi + A\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n}{r^2}\right)\psi + \varphi\psi - |\psi|^2\psi = 0,$$

and

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\varphi = |\psi|^2 - 1,$$

with $\Omega = 1$ (due to $\psi = 1$ and $\varphi = d\psi/dr = 0$ at $r = \infty$) and vortex charge states $n = 0, \pm 1, \pm 2, \ldots$.

2D Quantum Electron Vortices



——— $n = 1$

- - - - - $n = 2$

-·-·-·-· $n = 3$

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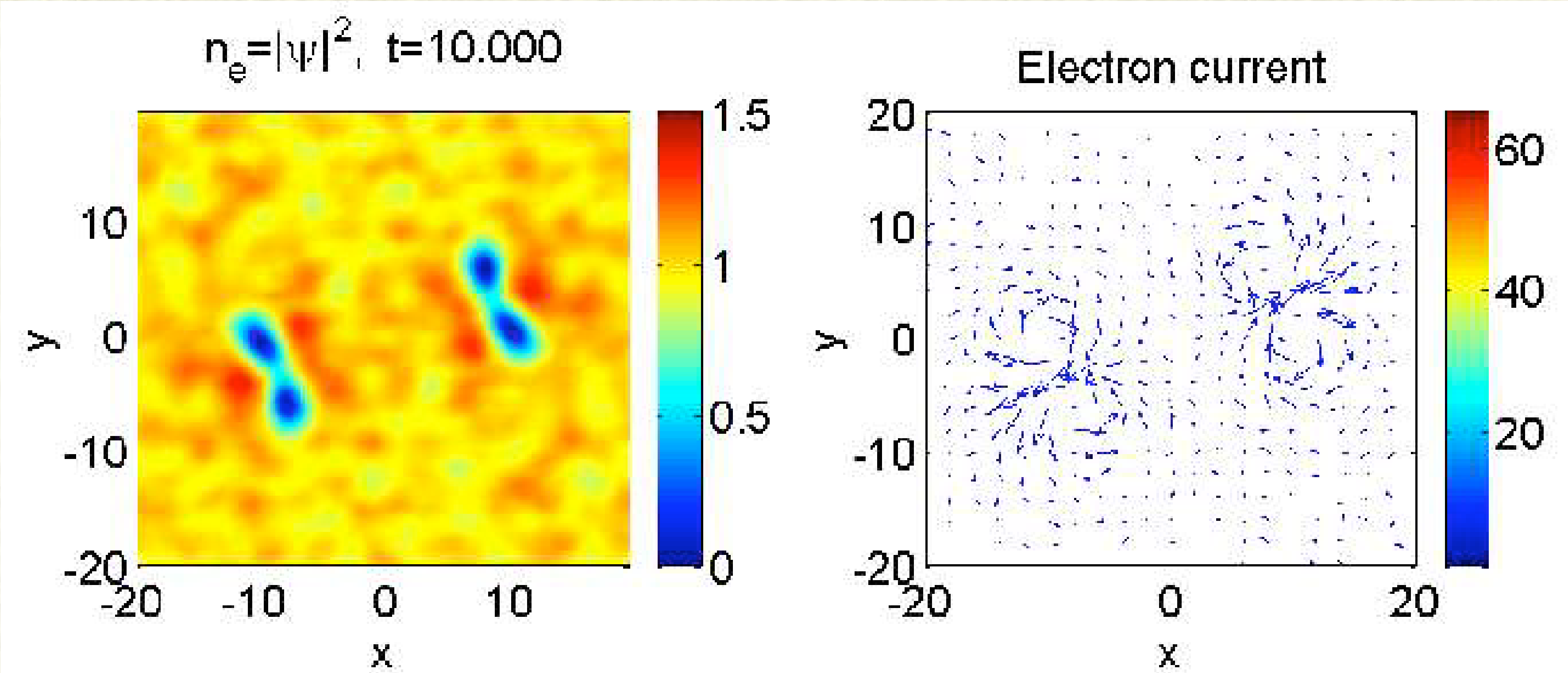
Interacting 2D Quantum Vortices

Single charge states ($n = 1$)



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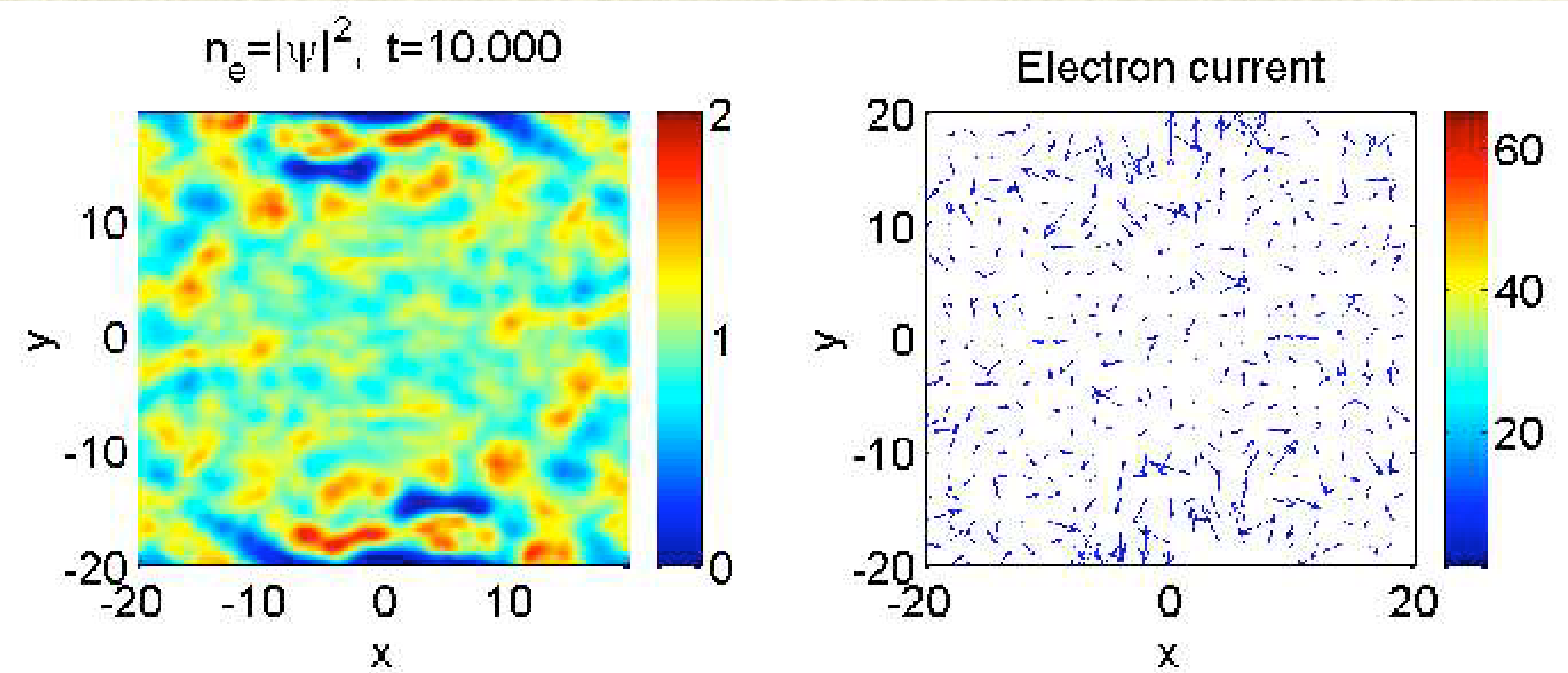
Interacting 2D Quantum Vortices

Double charge states ($n = 2$)



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Nonlinear Photon–Plasmon Interactions

- Photons get trapped into quantum electron holes. The governing dynamical equations are the Schrödinger equations for the photons and plasmons, which are respectively

$$2i\Omega_0\left(\frac{\partial}{\partial t}+V_g\frac{\partial}{\partial x}\right)A_\perp+\frac{\partial^2 A_\perp}{\partial x^2}-\left(\frac{|\psi|^2}{\sqrt{1+|A_\perp|^2}}-1\right)A_\perp=0,$$

and

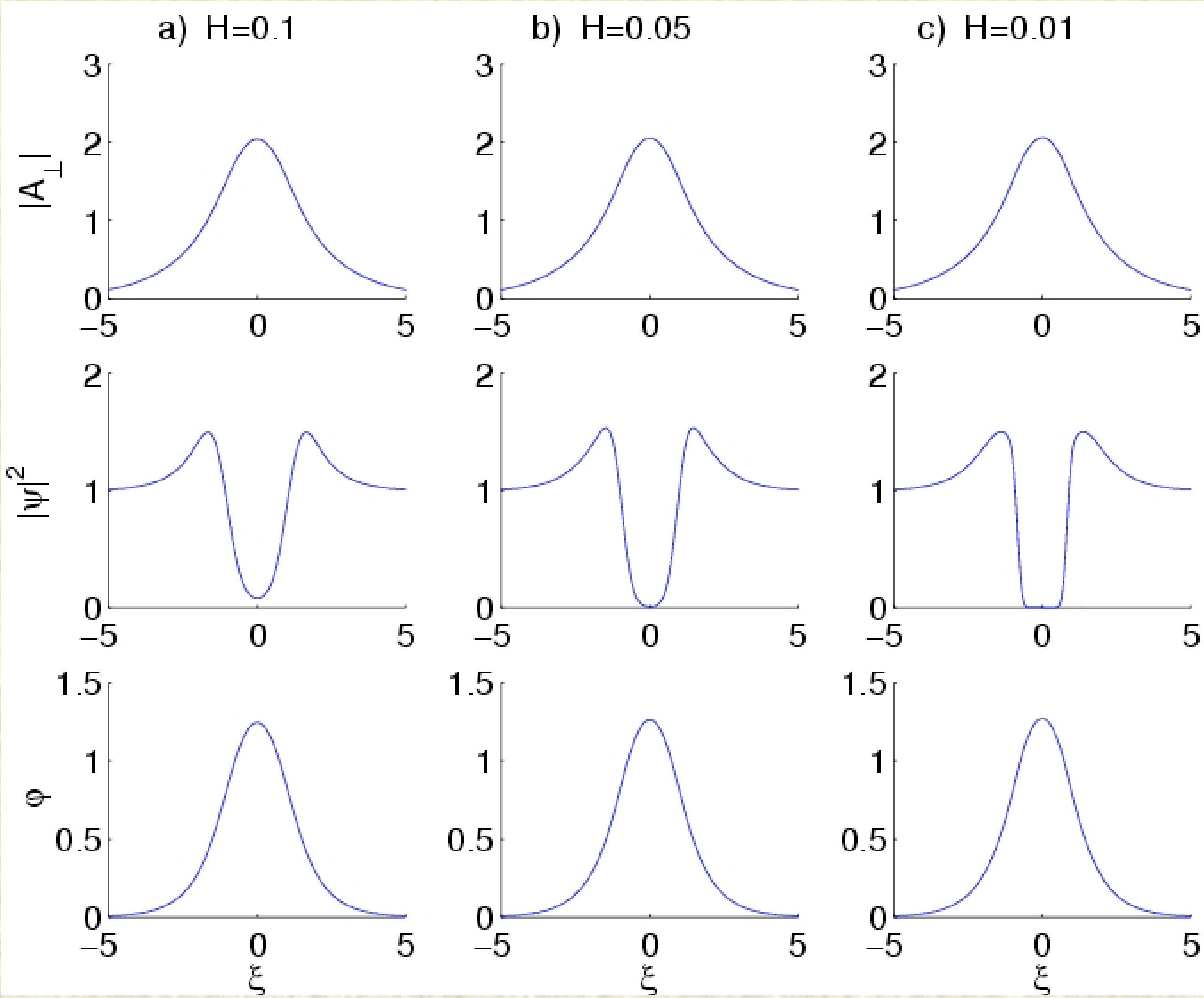
$$iH_e\frac{\partial\psi}{\partial t}+\frac{H_e^2}{2}\frac{\partial^2\psi}{\partial x^2}+(\varphi-\sqrt{1+|A_\perp|^2}+1)\psi=0,$$

where $H_e=\hbar\omega_{pe}/m_ec^2$, and φ follows from the Poisson equation

$$\frac{\partial^2\varphi}{\partial x^2}=|\psi|^2-1$$

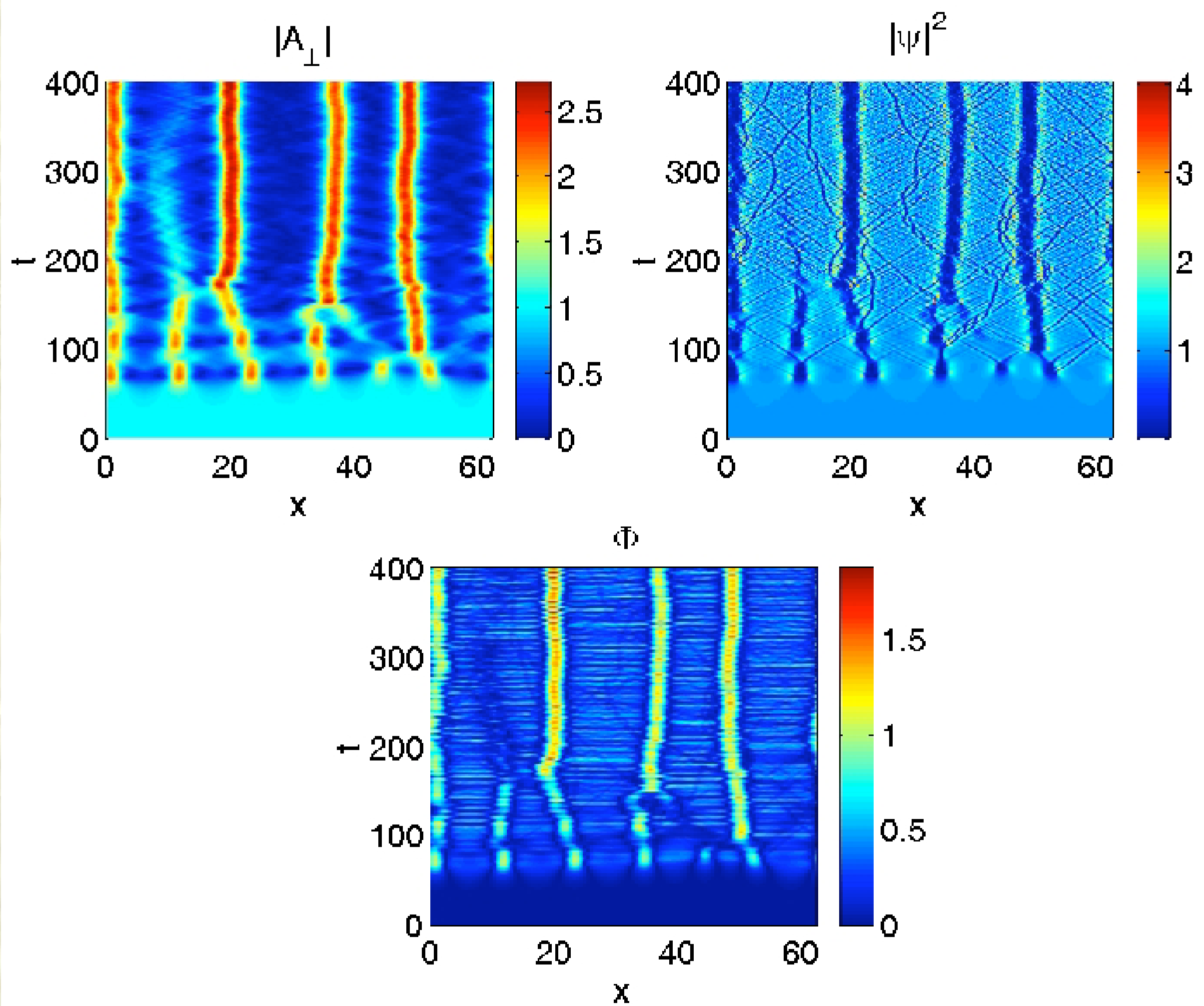
Shukla & Eliasson, PRL **99**, 096401 (2007)

Localized Excitations for Different H_e



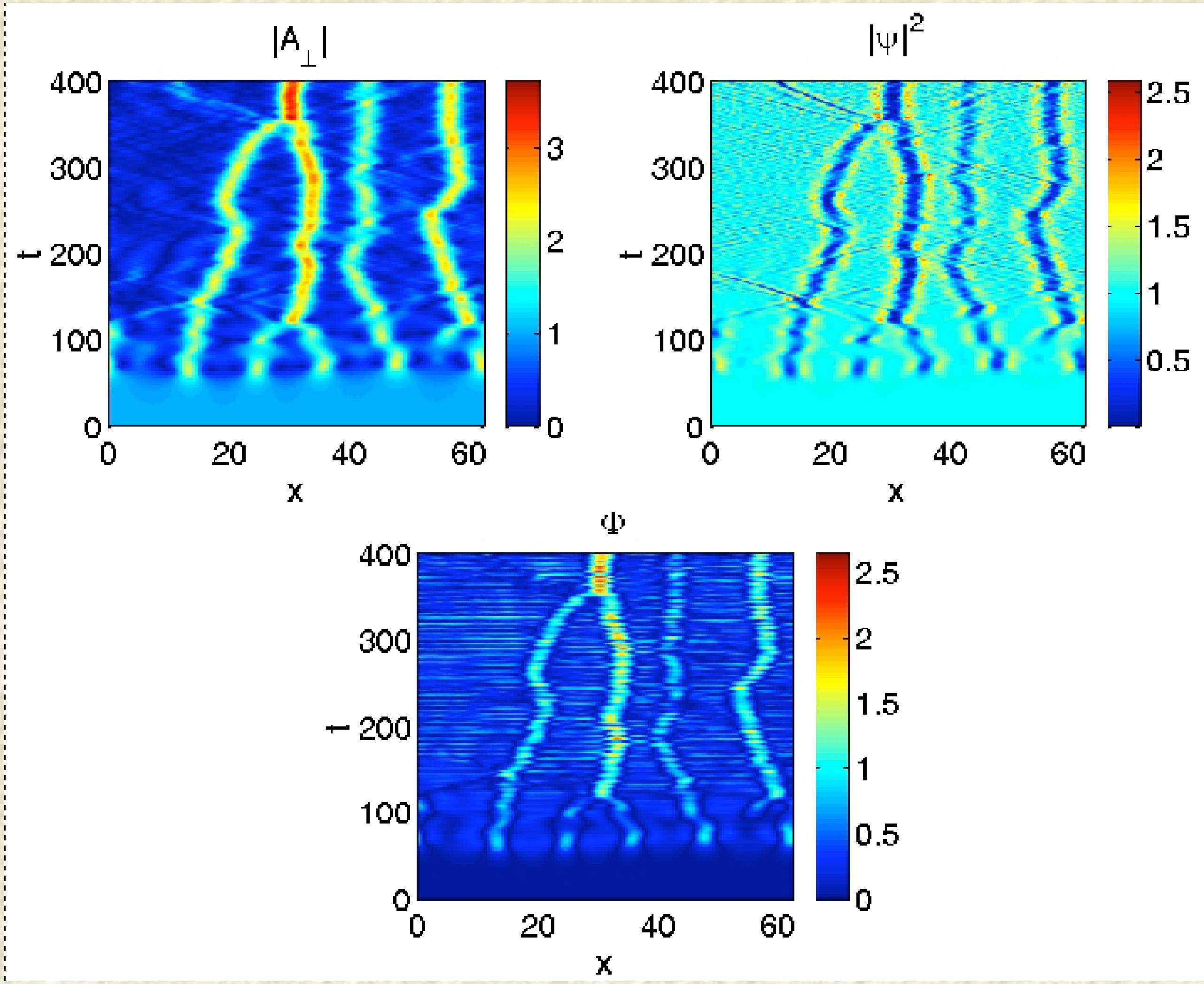
Shukla & Eliasson, PRL **99**, 096401 (2007)

Collapse of Photons into Solitary Structures, $H_e = 0.1$.



Shukla & Eliasson, PRL **99**, 096401 (2007)

Collapse of Photons into Solitary Structures, $H_e = 0.5$.



Shukla & Eliasson, PRL **99**, 096401 (2007)

Summary & Discussions

- We have summarized some properties of quantum plasmas.
  - Transition between classical and quantum plasmas $T \approx T_{Fe}$. Maxwell-Boltzmann $\rightarrow$ Fermi-Dirac statistics.
  - Collisional and collisionless quantum plasmas.
  - Relativistic density in white dwarf stars.
  - Quantum fluid models: Fermi statistical pressure and Bohm potential.
- Quantum models for collective motion of quantum plasmas.
  - Localized electrostatic structures in the form of quantum electron holes and 2D quantum electron vortices
  - Relativistic localization of EM waves in a quantum plasma.