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A primer on turbulence and gyrokinetics

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A primer on turbulence and gyrokinetics

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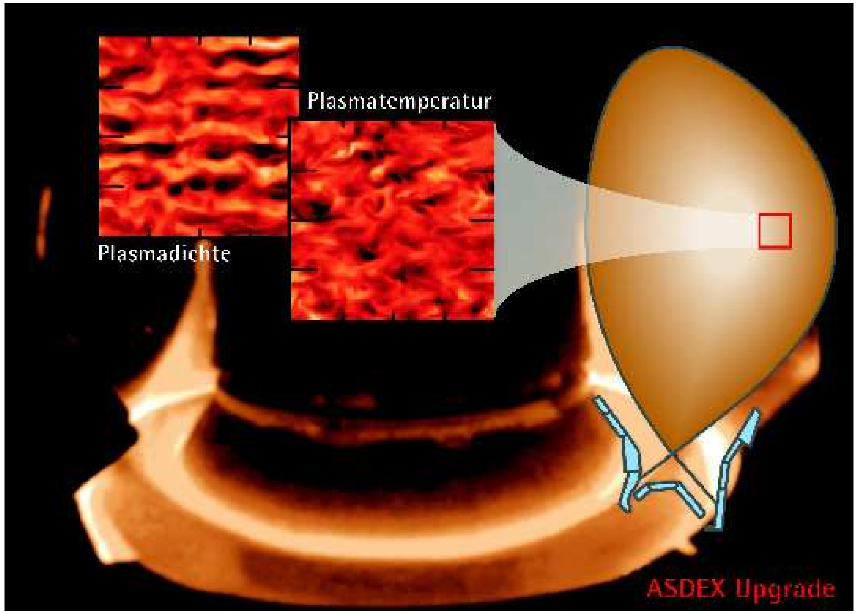
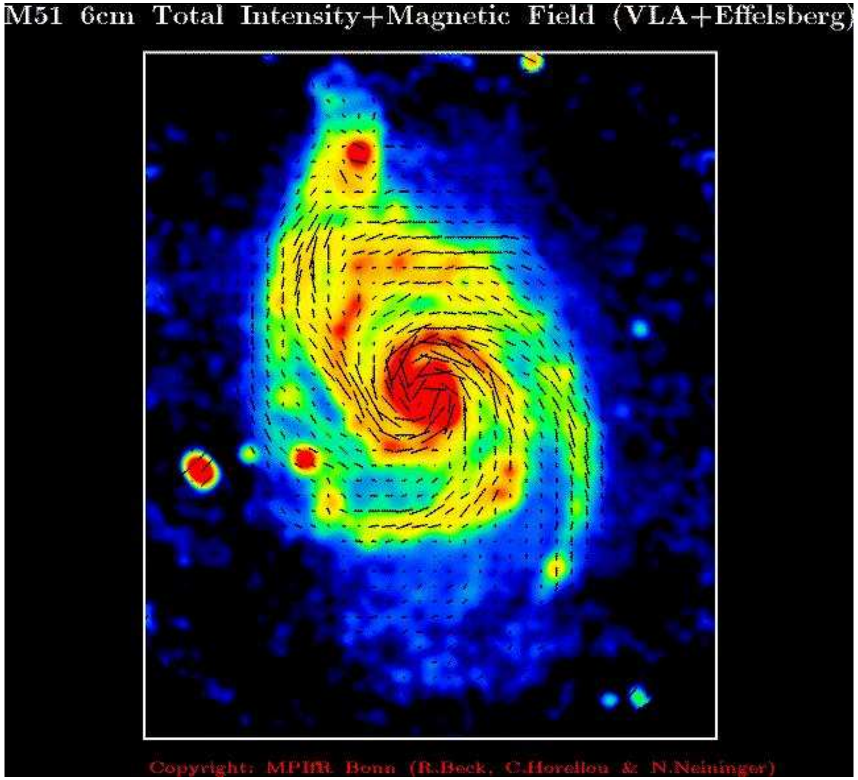
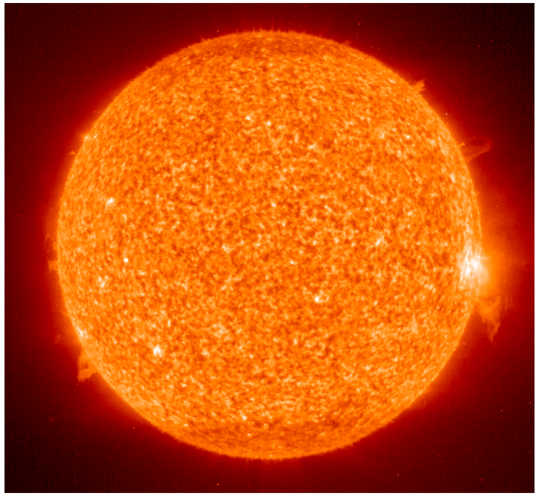
Summer College on Plasma Physics
ICTP, Trieste – Italy, 11 August 2009

Plasma turbulence – an ubiquitous phenomenon

> 99% of the visible universe is in the plasma state, mostly turbulent





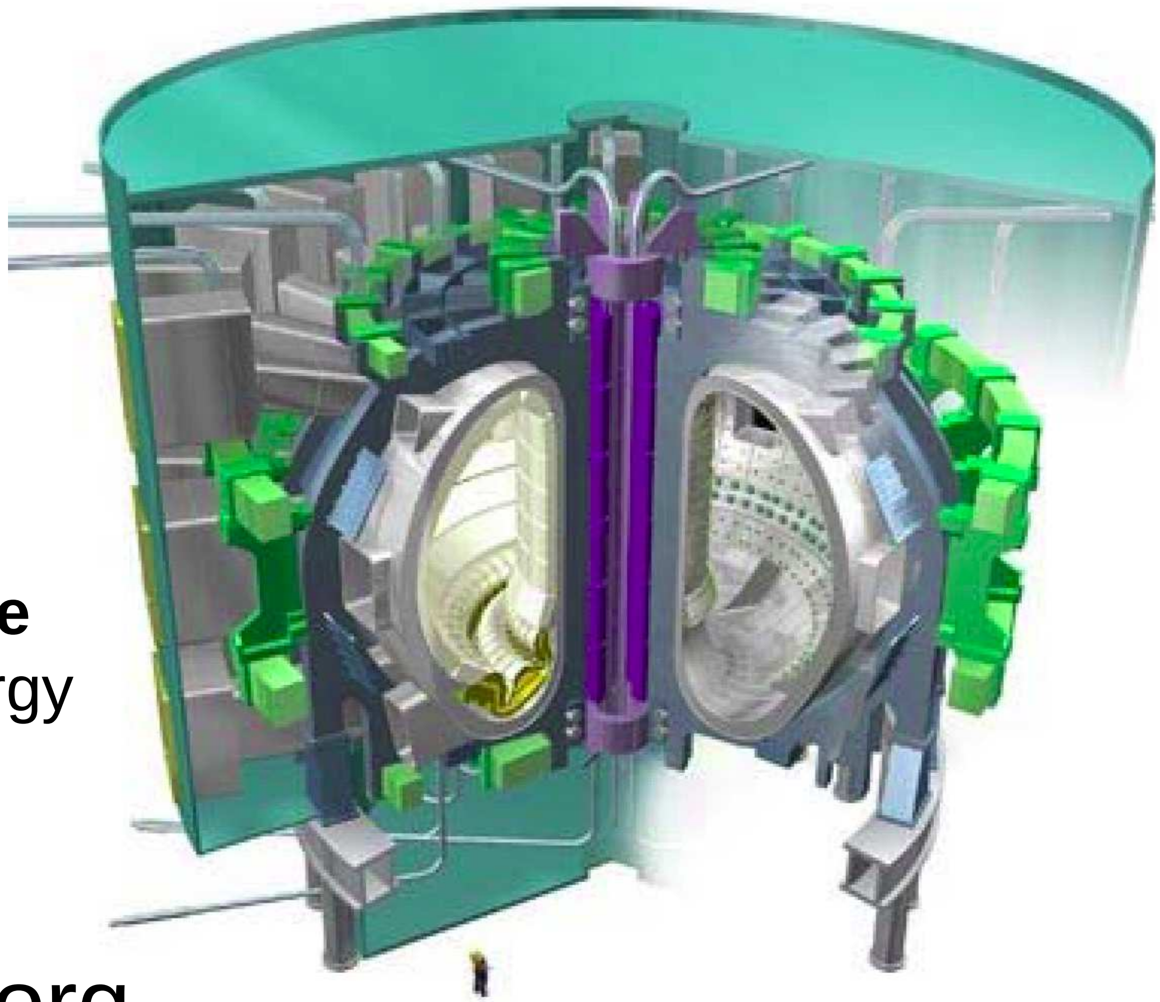


ITER and plasma turbulence

ITER is one of the most challenging scientific projects

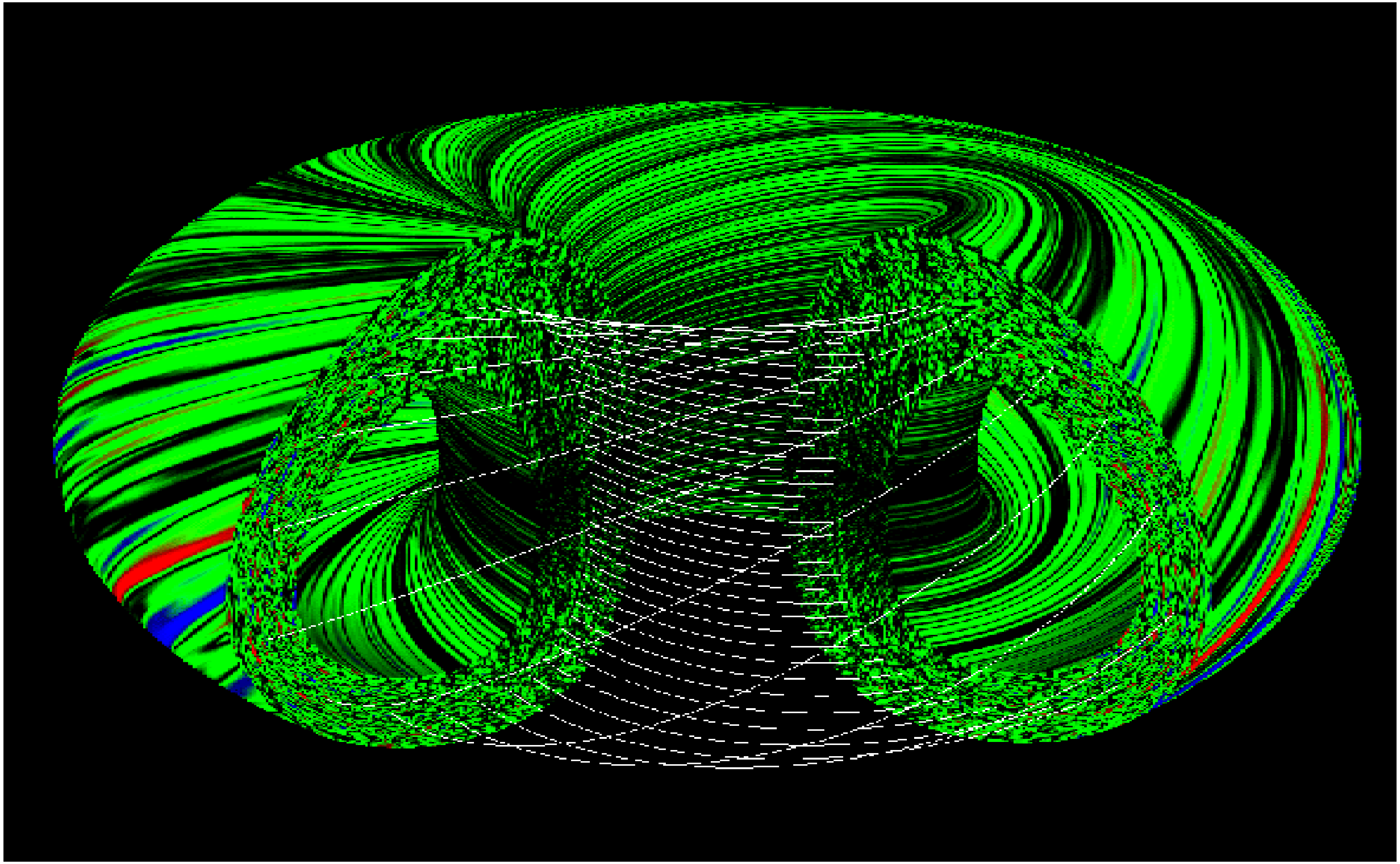
It is currently being built in Cadarache

Plasma turbulence determines its energy confinement time



www.iter.org

Plasma turbulence – a Grand Challenge



Turbulent mixing in a tokamak

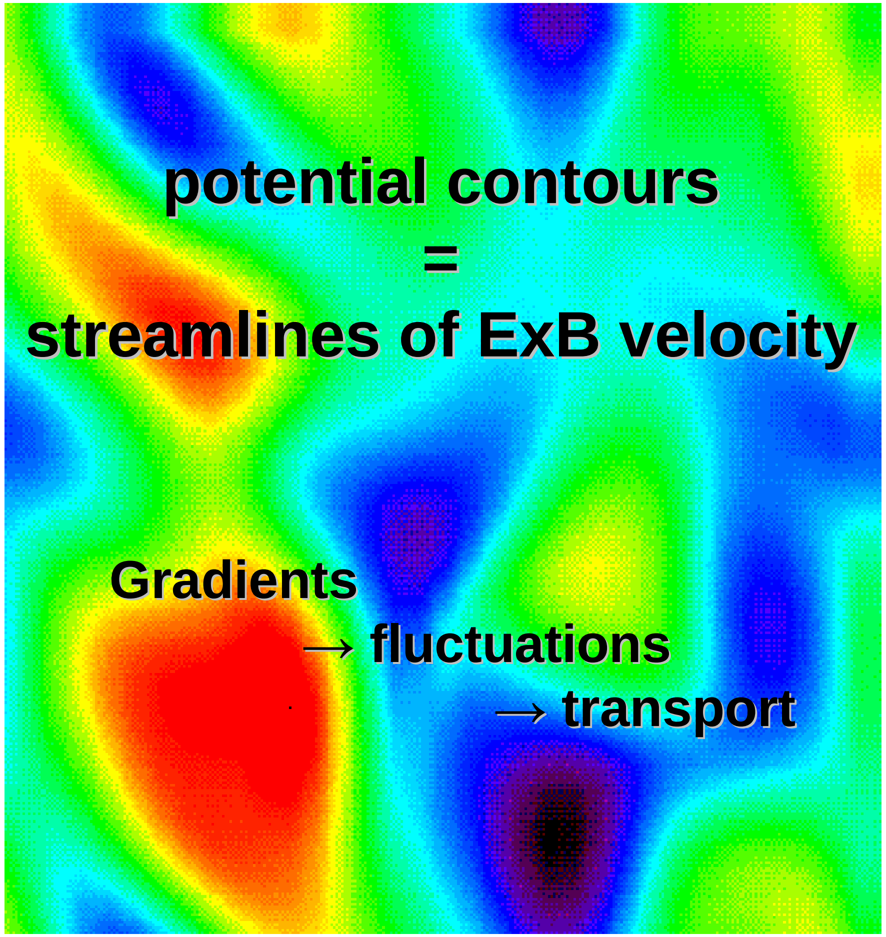
ExB drift velocity

$$\tilde{\mathbf{v}}_E = \frac{c}{B^2}\mathbf{B} \times \nabla\tilde{\phi}$$

$$\mathbf{Q} \equiv \frac{3}{2}\langle \tilde{p}\,\tilde{\mathbf{v}}_E \rangle = -n\chi\nabla T$$

$$\chi \sim \frac{(\delta x)^2}{\delta t} \sim \frac{\rho^2 v_t}{L_T}$$

(random walk/mixing length estimates)



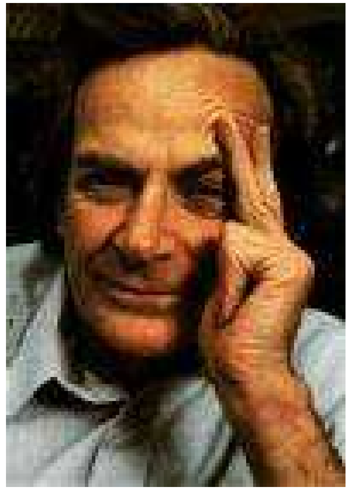
Typical heat and particle diffusivities are of the order of 1 m^2/s.

Hydrodynamic and MHD turbulence

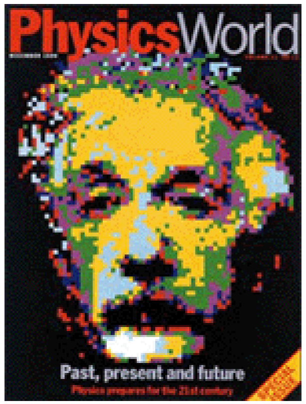
Turbulence – one of the most important unsolved problems in physics

According to a famous statement by Richard Feynman…

…and a survey by the British "Institute of Physics" among many of the leading physicists world-wide…



"Millennium Issue"
(December 1999)



TURBULENCE:
A challenging topic for both basic and applied research

What is turbulence?

Leonardo
da Vinci
(1529)

Turbulence…

- is a nonlinear phenomenon
- occurs (only) in open systems
- involves many degrees of freedom
- is highly irregular (chaotic) in space and time
- often leads to a (statistically) quasi-stationary state far from thermodynamic equilibrium

How to approach turbulence?

Many physicists – including Heisenberg, von Weizsäcker, Onsager, Feynman, and many others – have attempted to tackle turbulence **purely analytically** but with only **very limited success**.

Today, **supercomputers** help to unravel the “mysteries” of turbulence in the spirit of **John von Neumann**:

„There might be some hope to 'break the deadlock' by extensive, but well-planned, computational efforts...“

The Navier-Stokes equation

The NSE in its ‘classical’ form:

$$(\partial_t + \vec{v} \cdot \nabla)\, \vec{v} = -\nabla p + \nu \nabla^2 \vec{v} \qquad \nabla \cdot \vec{v} = 0$$

Expressed in terms of vorticity $\vec{\Omega} = \nabla \times \vec{v}$:

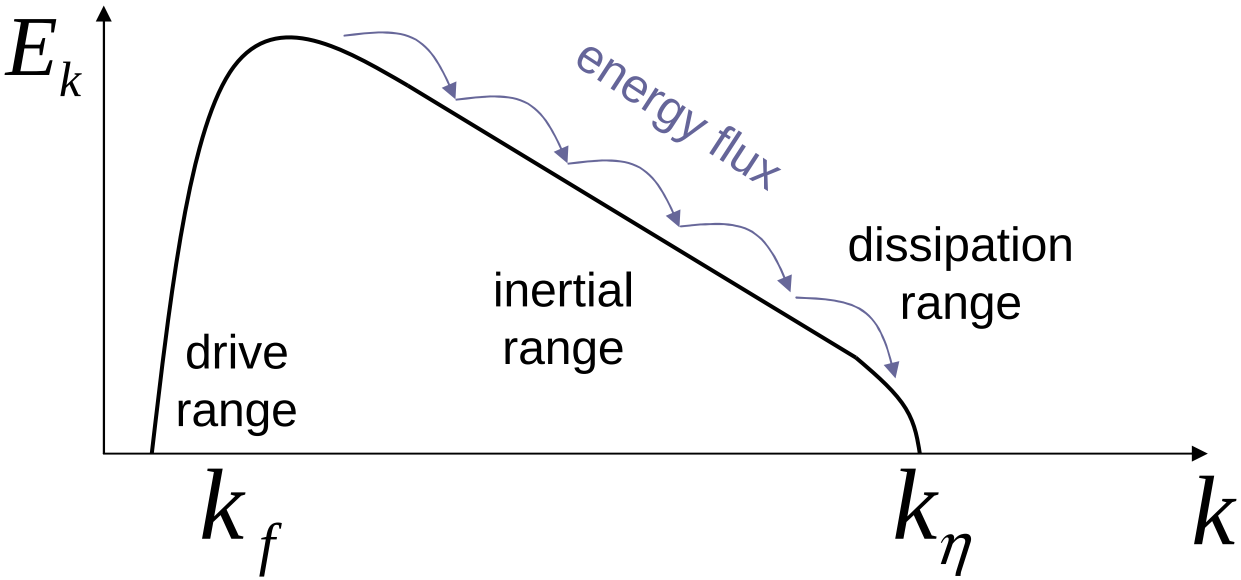
$$(\partial_t + \vec{v} \cdot \nabla)\, \vec{\Omega} = (\vec{\Omega} \cdot \nabla)\, \vec{v} + \mathrm{Re}^{-1} \nabla^2 \vec{\Omega}$$

Reynolds number as single dimensionless parameter:

$$\mathrm{Re} = \frac{LU}{\nu}$$

The Richardson cascade

Turbulence as a **local cascade in wave number space**…



$$\frac{k_\eta}{k_f} \sim \mathrm{Re}^{3/4}$$

Computational effort $\sim \mathrm{Re}^3$

„Big whorls have little whorls, little whorls have smaller whorls that feed on their velocity, and so on to viscosity“

Much turbulence research addresses the **cascade** problem.

Kolmogorov’s theory from 1941

K41 is based merely on intuition and dimensional analysis – it is *not* derived rigorously from the Navier-Stokes equation

Key assumptions:

- Scale invariance – like, e.g., in critical phenomena
- Central quantity: energy flux ε

$$E = \frac{1}{2V}\int v^2 \, d^3x = \int_0^\infty E(k)\, dk$$

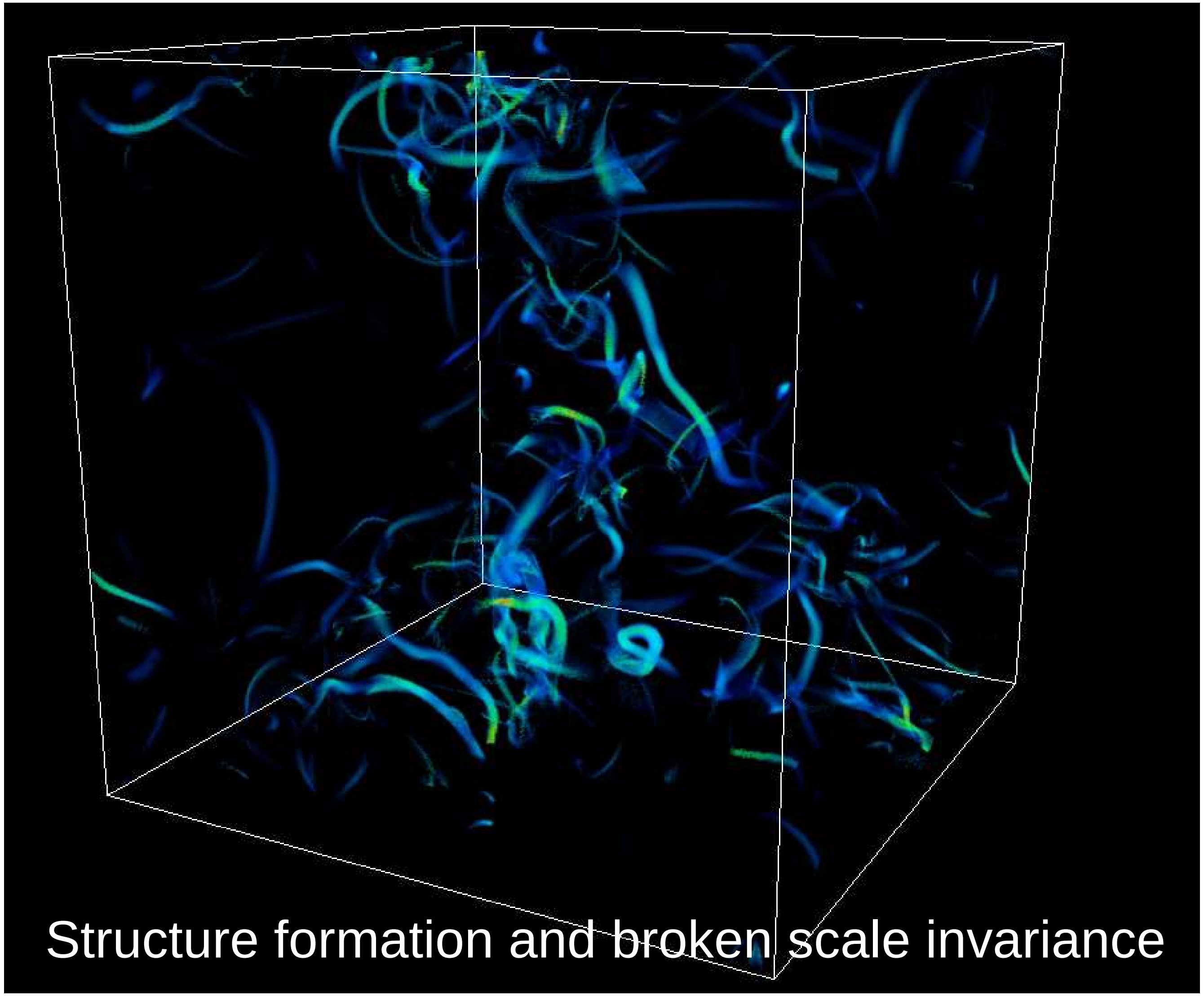
$$\boxed{E(k) = C\, \varepsilon^{2/3} k^{-5/3}}$$

Quantity	Dimension
Wave number	1/length
Energy per unit mass	$\text{length}^2/\text{time}^2$
Energy spectrum $\mathcal{E}(k)$	$\text{length}^3/\text{time}^2$
Energy flux ε	$\text{energy/time} \sim \text{length}^2/\text{time}^3$

This is the most famous turbulence result: the “-5/3” law.

However, K41 is fundamentally wrong: <u>scale invariance is broken</u>!

Direct numerical simulations

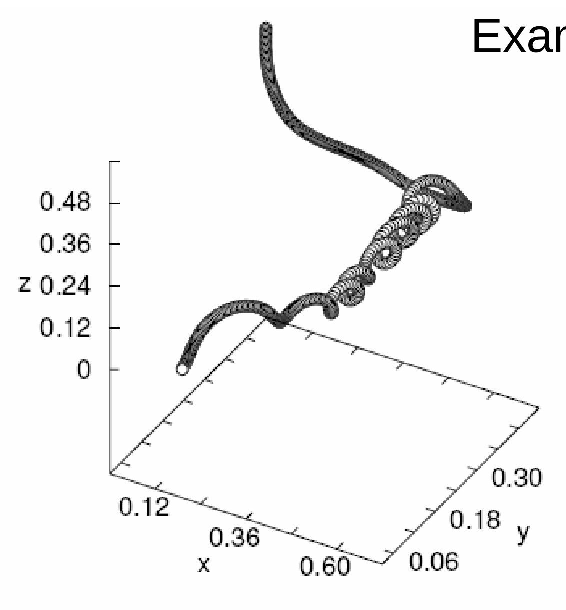


Wilczek *et al.* 2008

Key open issues: Inertial range

- Is the inertial range physics universal (for Re $\rightarrow \infty$)?
- If so, can one derive a rigorous IR theory from the NSE?
- How should one, in general, handle the interplay between randomness and coherence?

Biferale et al. 2005



Example: Trapping of tracers in vortex filaments

Note:

The observed deviations from self-similarity can be reproduced qualitatively by relatively simple vortex models.

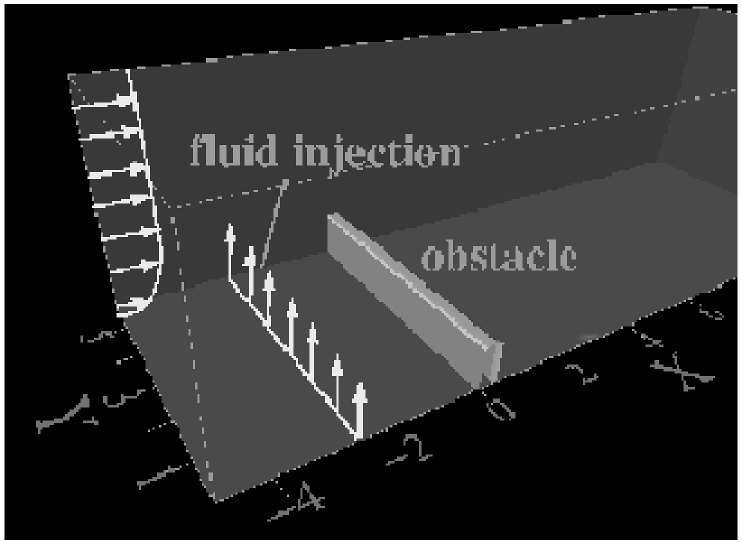
Wilczek, Jenko, and Friedrichs 2008

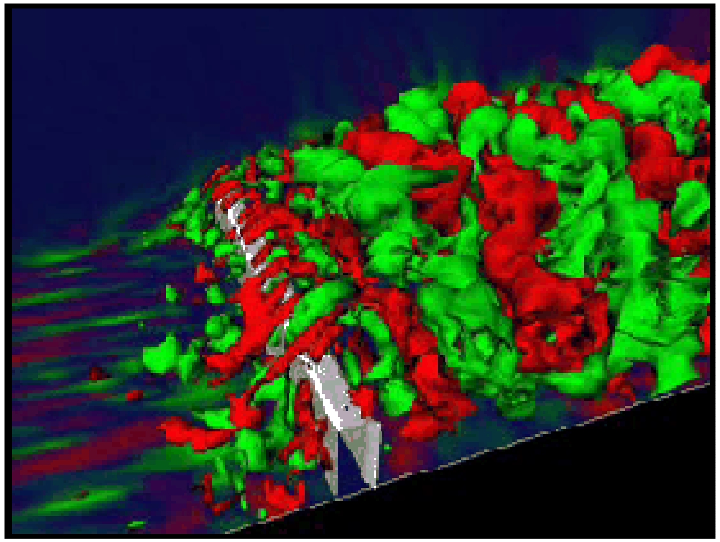
Key open issues: Drive range

- Often, one is interested mainly in the *large* scales. Here, one encounters an interesting interplay between linear (drive) and nonlinear (damping) physics. – Is it possible to remove the small scales?

- Yes: LES, Dynamical Systems Approach etc.

Orellano & Wengle, JT 2001





Magneto-hydrodynamics

A combination of hydrodynamics and non-rel. electrodynamics:

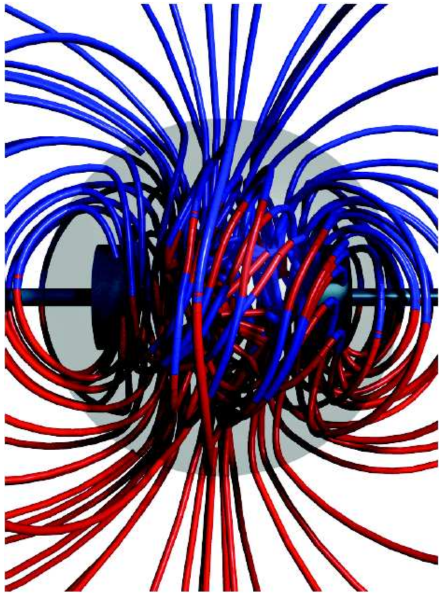
$$\partial_t \vec{\Omega} = \nabla \times (\vec{v} \times \vec{\Omega}) \boxed{+ \nabla \times (\vec{J} \times \vec{B})} + \mathrm{Re}^{-1} \nabla^2 \vec{\Omega}$$

$$\boxed{\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) + \mathrm{Rm}^{-1} \nabla^2 \vec{B}}$$

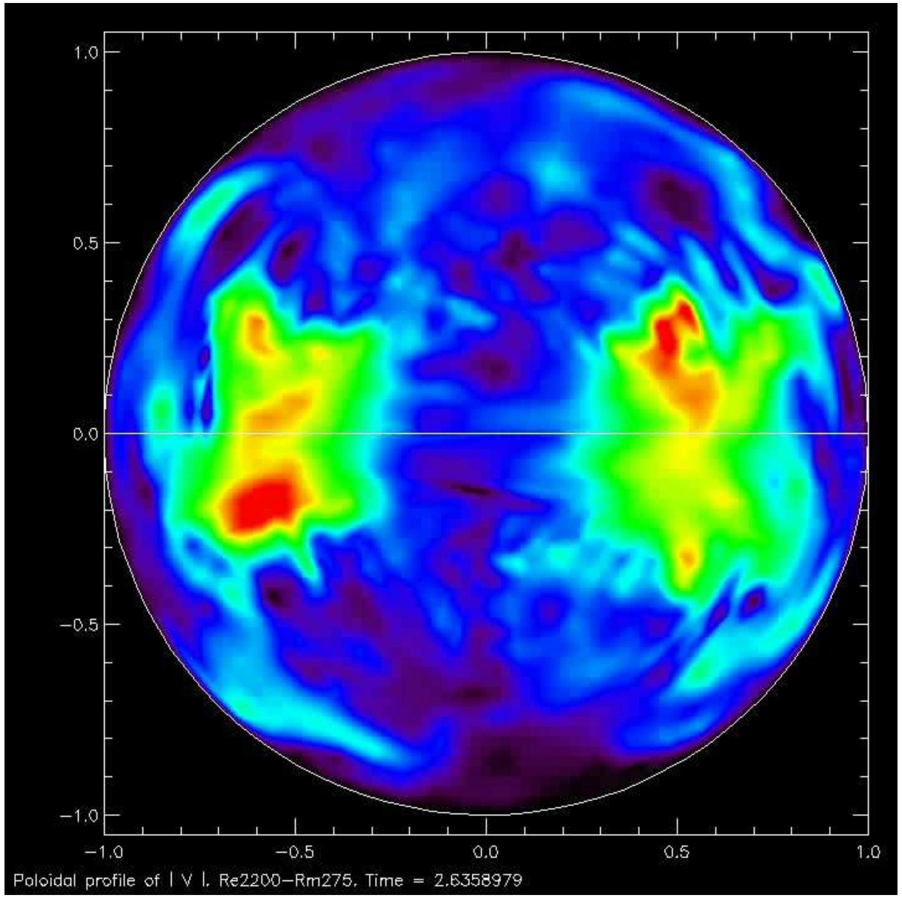
Two dimensionless parameters (Rm = magn. Reynolds number):

$$\mathrm{Re} = \frac{LU}{\nu} \qquad \mathrm{Rm} = \frac{LU}{\eta}$$

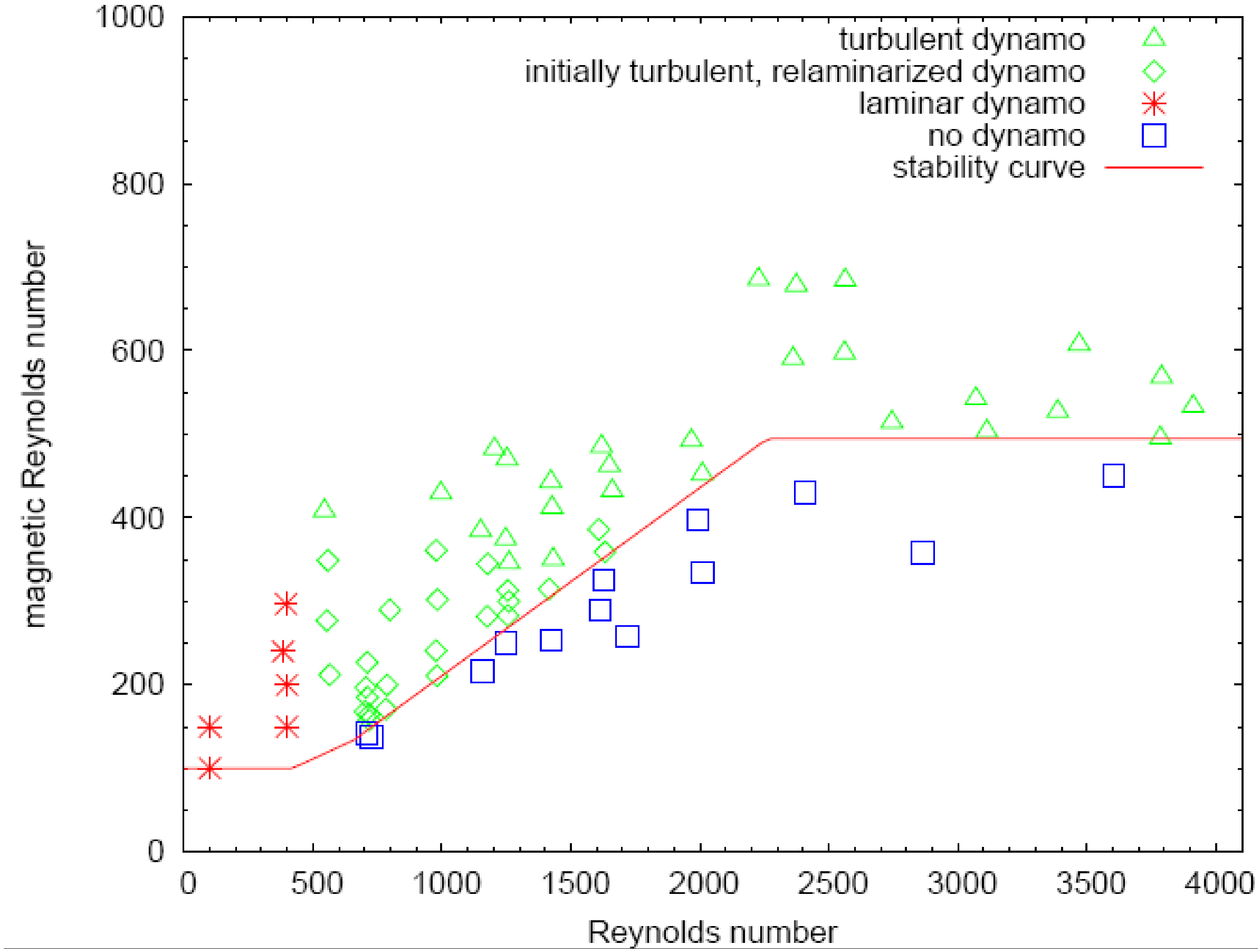
Experiment and simulation



Madison Dynamo Experiment and DYNAMO simulations



Laminar and turbulent dynamos



Reuter & Jenko 2008

An introduction to gyrokinetics

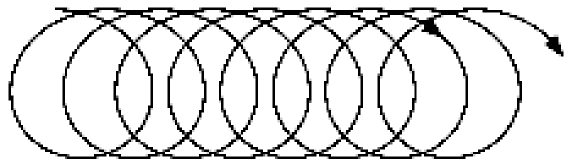
What is gyrokinetic theory?

Dilute and/or hot plasmas are almost collisionless.

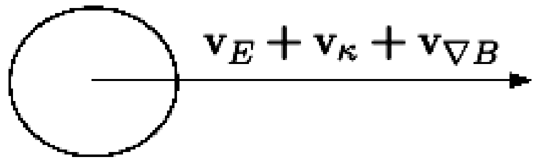
Thus, if kinetic effects (finite Larmor radius, Landau damping, magnetic trapping etc.) play a role, MHD is not applicable, and one has to use a kinetic description!

Vlasov-Maxwell equations $\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{x},\mathbf{v},t) = 0$

Removing the fast gyromotion leads to a dramatic speed-up $\omega \ll \Omega$



Charged rings as quasiparticles; gyrocenter coordinates; keep kinetic effects



Brizard & Hahm, Rev. Mod. Phys. **79**, 421 (2007)

The gyrokinetic ordering

- The gyrokinetic model is a Vlasov-Maxwell on which the GK ordering is imposed:

⇒ Slow time variation as compared to the gyro-motion time scale:

$$\omega/\Omega_i \sim \epsilon_g \ll 1$$

⇒ Spatial equilibrium scale much larger than the Larmor radius:

$$\rho/L_n \sim \rho/L_T \equiv \epsilon_g \ll 1$$

⇒ Strong anisotropy, i.e. only perpendicular gradients of the fluctuating quantities can be large ($k_\perp \rho \sim 1$, $k_\parallel \rho \sim \epsilon_g$):

$$k_\parallel / k_\perp \sim \epsilon_g \ll 1$$

⇒ Small amplitude perturbations, i.e. energy of perturbation much smaller than the thermal energy:

$$e\phi/T_e \sim \epsilon_g \ll 1$$

A. Bottino

A brief historical review

- The word "Gyrokinetic" appeared in the literature in the late sixties.
  Rutherford and Frieman, Taylor and Hastie [1968].

  Goal: Provide a adequate formalism for the linear study of kinetic drift-waves
  in general magnetic configurations, including finite Larmor radius effects.

- First nonlinear set of equations for the perturbed distribution function δF.
  Frieman and Liu Chen [1982].
  $\rightarrow$ Gyrokinetic ordering.

- Littlejohn [1979], Dubin [1983], Hahm[1988], Brizard [1989], ...

  Firm and more transparent theoretical foundation for GK:

  GK equations based on Hamiltonian or Lagrangian variation methods.

Lagrangian
$\Downarrow$
remove gyro-angle dependency in Lagrangian
(change of coordinate system)
$\Downarrow$
equation of motion

A Lagrangian approach

If the Lagrangian of a dynamical system is known...

Example: charged particle motion, in non canonical coordinates $(\vec{x}, \vec{v})$:

$$
\begin{aligned}
L &= \left(\frac{e}{c}\vec{A}(\vec{x},t) + m\vec{v}\right)\cdot\dot{\vec{x}} - H(\vec{x},\vec{v}) \\
H &= \frac{m}{2}v^2 + e\phi(\vec{x},t)
\end{aligned}
$$

with $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla\phi - \partial_t \vec{A}/c$.

...the equation of motion are given by the Lagrange equations:

$$
\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad \text{with } i = 1, \ldots, 6
$$

Lagrange equation of motion for a charged particle:

$$
\begin{aligned}
\vec{v} \Rightarrow &\quad -\tfrac{\partial L}{\partial \vec{v}} = 0 &&\Rightarrow \dot{\vec{x}} = \vec{v} \\
\vec{x} \Rightarrow &\quad \tfrac{\mathrm{d}}{\mathrm{d}t}\tfrac{\partial L}{\partial \vec{v}} - \tfrac{\partial L}{\partial \vec{v}} = 0 &&\Rightarrow \dot{\vec{v}} = \frac{e}{m}\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)
\end{aligned}
$$

Deriving the driftkinetic equations

GK ordering, but $k_\perp \rho \simeq 1 \Rightarrow k_\perp \rho \ll 1$
$\Rightarrow \vec{B}(\vec{x})$, static magnetic field.

- Single particle Lagrangian:

$$L = \left(\frac{e}{c}\vec{A}(\vec{x}) + m\vec{v}\right) \cdot \dot{\vec{x}} - \frac{m}{2}v^2 + e\phi(\vec{x}, t)$$

- Change of coordinates:

particle coordinates $(\vec{x}, \vec{v}) \Rightarrow$ guiding center coordinates $(\vec{R}, v_\parallel, \mu, \varphi)$

$$\begin{aligned}
\vec{x} &= \vec{R} + \vec{\rho} \equiv \vec{R} + \frac{v_\perp}{\Omega}\hat{a}(\vec{R}, \varphi) \\
\mu &= v_\perp^2 / 2B(\vec{R}) \\
v_\parallel &= \vec{v} \cdot \vec{b} \\
\varphi &= \tan^{-1}\left(\frac{\vec{v} \cdot \vec{e}_1}{\vec{v} \cdot \vec{e}_2}\right)
\end{aligned}$$

$\vec{R}$ guiding center position; $\Omega \equiv eB/mc$ gyrofrequency.
$\hat{a} \equiv \cos(\varphi)\ \vec{e}_1 + \sin(\varphi)\ \vec{e}_2$
$\vec{e}_1(\vec{R}, \varphi)$, $\vec{e}_2(\vec{R}, \varphi)$ orthogonal unity vectors in the plane perpendicular to $\vec{b} \equiv \vec{B}/B$.

Driftkinetic equations (cont'd)

- Single particle Lagrangian:

$$L = \left(\frac{e}{c}\vec{A}(\vec{x}) + m\vec{v}\right) \cdot \dot{\vec{x}} - \frac{m}{2}v^2 + e\phi(\vec{x}, t)$$

- $\Rightarrow$ Expand $\vec{A}(\vec{x}) \simeq \vec{A}(\vec{R}) + (\vec{\rho} \cdot \nabla)\vec{A}$
  $\Rightarrow$ Replace $(\vec{x}, \vec{v})$ with the guiding center variables:

$$\begin{aligned} \vec{A}(\vec{x}) \cdot \dot{\vec{x}} &\simeq \vec{A}(\vec{R}) \cdot \dot{\vec{R}} + (\vec{\rho} \cdot \nabla)\vec{A} \cdot \dot{\vec{R}} + \vec{A}(\vec{R}) \cdot \dot{\vec{\rho}} + (\vec{\rho} \cdot \nabla)\vec{A} \cdot \dot{\vec{\rho}} \\ &\ldots \end{aligned}$$

- On each term in the Lagrangian, $\langle\rangle$ average over the gyro-angle φ:

$$\begin{aligned} \langle \vec{A}(\vec{R}) \cdot \dot{\vec{R}} \rangle &= \vec{A}(\vec{R}) \cdot \dot{\vec{R}} \\ \langle (\vec{\rho} \cdot \nabla)\vec{A} \cdot \dot{\vec{R}} \rangle &= 0 \\ &\ldots \end{aligned}$$

- The electrostatic potential?

The GK ordering, ϕ is a small perturbation: $\phi(\vec{x}) \simeq \phi(\vec{R}) + (\vec{\rho} \cdot \nabla)\phi = \phi(\vec{R}) + \mathcal{O}(\epsilon_g^2)$.

Driftkinetic equations (cont'd)

$$L_{DK} = \left(mv_{\|}\vec{b} + \frac{e}{c}\vec{A}(\vec{R})\right) \cdot \dot{\vec{R}} + \frac{\mu B}{\Omega}\dot{\varphi} - H_{DK}$$

$$H_{DK} = \frac{m}{2}v_{\|}^2 + \mu B + q\phi(\vec{R})$$

- Lagrange equations:

$$\dot{\vec{R}} = v_{\|}\vec{b} + \frac{B}{B_{\|}^*}\left(\vec{v}_{E\times B} + \vec{v}_{\nabla B} + \vec{v}_C\right)$$

$$\dot{v}_{\|} = \left(-\mu\nabla B + e\vec{E}\right) \cdot \frac{\dot{\vec{R}}}{mv_{\|}} \quad ; \quad \dot{\mu} = 0 \quad ; \quad \dot{\varphi} = \Omega$$

$\vec{v}_{E\times B} \equiv$	$\frac{c}{B^2}\,\vec{E} \times \vec{B}$	$E \times B$ drift
$\vec{v}_{\nabla B} \equiv$	$\frac{\mu}{m\Omega}\,\vec{b} \times \nabla B$	∇B drift
$\vec{v}_C \equiv$	$\frac{v_{\|}^2}{\Omega}\,\vec{b} \times (\vec{b} \cdot \nabla)\vec{b}$	Curvature drift

with $\vec{B}^* \equiv \vec{B} + (mc/e)v_{\|}\nabla \times \vec{b} = B(1 + \mathcal{O}(\rho_{\|}/L_B))$.

Including fluctuating fields

Particle world, $(\vec{x}, \vec{v})$

$\Downarrow$

static fields

$\Downarrow$

Guiding center, $(\vec{R}, v_{\|}, v_{\perp}, \varphi)$

(remove gyro-angle dependence)

$\Downarrow$

→ Fluctuating fields

$\Downarrow$

Gyrocenter coordinates $(\bar{\vec{R}}, \bar{v}_{\|}, \bar{v}_{\perp}, \bar{\varphi})$

(remove gyro-angle dependence)

$\Downarrow$

Equations of motion

Vlasov equation

Self consistent Poisson equation

(Self consistent Ampere's law)

Gyroaveraged potentials/fields

Drift-kinetic: $\phi(\vec{x}, t) \Rightarrow \phi(\vec{R}, t)$

Gyrokinetic: $\phi(\vec{x}, t) \Rightarrow \langle\phi\rangle(\vec{R}, \mu, t)$

Formally:

$$\langle\phi\rangle(\vec{R}, \mu, t) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi \int \phi(\vec{x}, t)\ \delta(\vec{R} + \vec{\rho} - \vec{x})\ \mathrm{d}\vec{x}$$

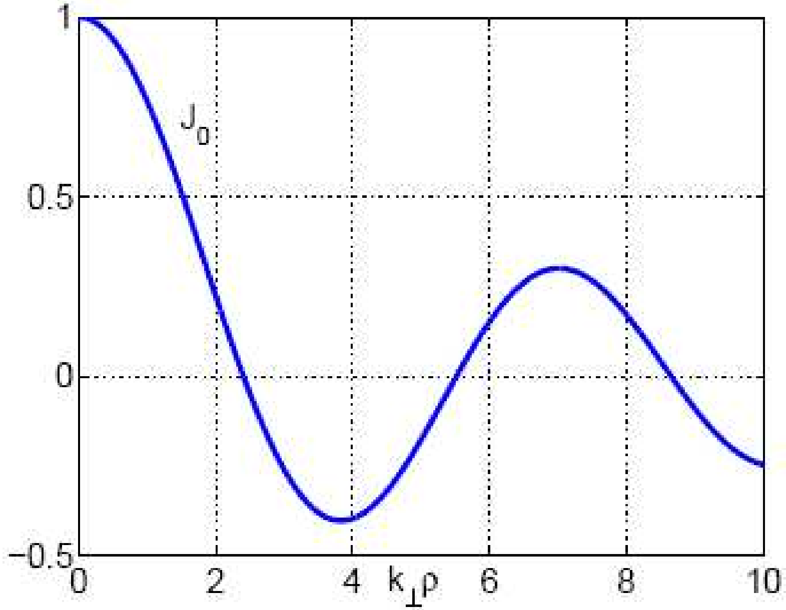
Drift-kinetic : $\vec{E}(\vec{R}, t)$.

Gyro-kinetic : $\langle\vec{E}\rangle$

→ result of an average procedure,
i.e. is "smoother" in space.

→ Fourier representation:

$$\begin{aligned} \langle\phi\rangle(\vec{k}) &= \phi(\vec{k}) J_0(k_\perp \rho) \\ &\simeq \phi(\vec{k})[1 - \frac{1}{4}(k_\perp \rho)^2] \end{aligned}$$



The nonlinear gyrokinetic equations

$$f = f(\mathbf{X}, v_\|, \mu; t)$$

Advection/Conservation equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_\| \frac{\partial f}{\partial v_\|} = 0$$

X = gyrocenter position
$v_\|$ = parallel velocity
μ = magnetic moment

$$\dot{\mathbf{X}} = v_\| \mathbf{b} + \frac{B}{B_\|^*} \left(\frac{v_\|}{B} \bar{\mathbf{B}}_{1\perp} + \mathbf{v}_\perp \right)$$

$$\mathbf{v}_\perp \equiv \frac{c}{B^2} \bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega} \mathbf{b} \times \nabla (B + \bar{B}_{1\|}) + \frac{v_\|^2}{\Omega} (\nabla \times \mathbf{b})_\perp$$

$$\dot{v}_\| = \frac{\dot{\mathbf{X}}}{m v_\|} \cdot \left(e\bar{\mathbf{E}}_1 - \mu \nabla (B + \bar{B}_{1\|}) \right)$$

Appropriate field equations

$$\frac{n_1}{n_0} = \frac{\bar{n}_1}{n_0} - \left(1 - \|I_0^2\|\right) \frac{e\phi_1}{T} + \|x I_0 I_1\| \frac{B_{1\|}}{B}$$

$$\nabla_\perp^2 A_{1\|} = -\frac{4\pi}{c} \sum \bar{J}_{1\|}$$

$$\frac{B_{1\|}}{B} = -\sum \epsilon_\beta \left(\frac{\bar{p}_{1\perp}}{n_0 T} + \|x I_1 I_0\| \frac{e\phi_1}{T} + \|x^2 I_1^2\| \frac{B_{1\|}}{B} \right)$$

Nonlinear integro-differential equations in **5 dimensions**...

Major theoretical speedups

G.Hammett

relative to original Vlasov/pre-Maxwell system on a naïve grid, for ITER $1/\rho_* = a/\rho \sim 1000$

Nonlinear gyrokinetic equations	
eliminate plasma frequency: $\omega_{pe}/\Omega_i \sim m_i/m_e$	x10^3
eliminate Debye length scale: $(\rho_i/\lambda_{De})^3 \sim (m_i/m_e)^{3/2}$	x10^5
average over fast ion gyration: $\Omega_i/\omega \sim 1/\rho_*$	x10^3
Field-aligned coordinates	
adapt to elongated structure of turbulent eddies: $\Delta_{\parallel}/\Delta_{\perp} \sim 1/\rho_*$	x10^3
Reduced simulation volume	
reduce toroidal mode numbers (i.e., 1/15 of toroidal direction)	x15
$L_r \sim a/6 \sim 160\ \rho \sim 10$ correlation lengths	x6
Total speedup	x10^{16}
For comparison: Massively parallel computers (1984-2009)	x10^7

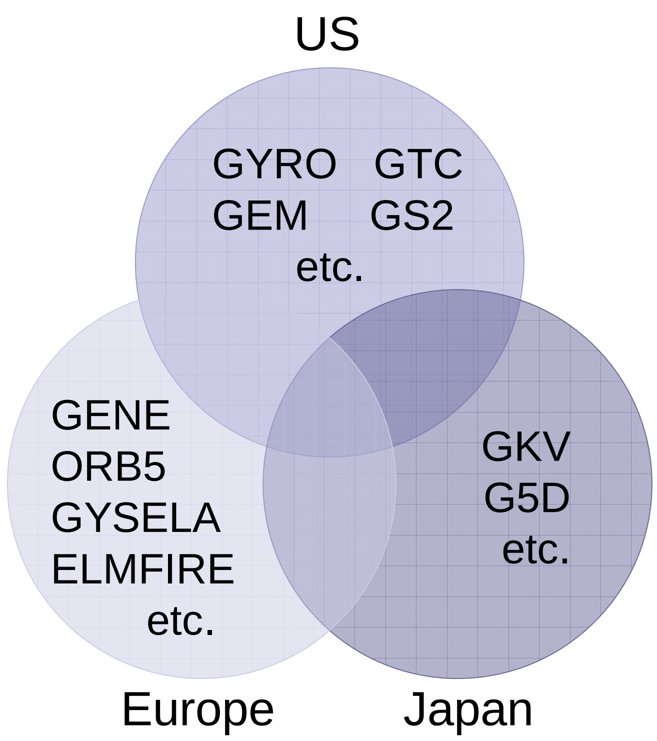
Nonlinear gyrokinetic simulations

Status quo in gyrokinetic simulation

- over the last decade or so, GK has emerged as the standard approach to plasma turbulence

- a variety of nonlinear GK codes is being used and (further) developed

- these codes differ in their numerical schemes (Euler, Lagrange, semi-Lagrange) and physics contents



The simulation code GENE

• GENE is physically comprehensive and computationally efficient CFD-like code with applications to both tokamaks and stellarators

• two main goals: deeper understanding of fundamental physics issues and direct comparisons with experiments (interfaces to MHD codes)

• the differential operators are discretized via a combination of spectral, finite difference, finite element, and finite volume methods; the time stepping is done via a (non-standard) explicit Runge-Kutta method

• GENE is part of the European DEISA benchmark suite and the EU-Japanese IFERC benchmark suite

• GENE is developed cooperatively by an international team, and it is publicly available (www.ipp.mpg.de/~fsj/gene)

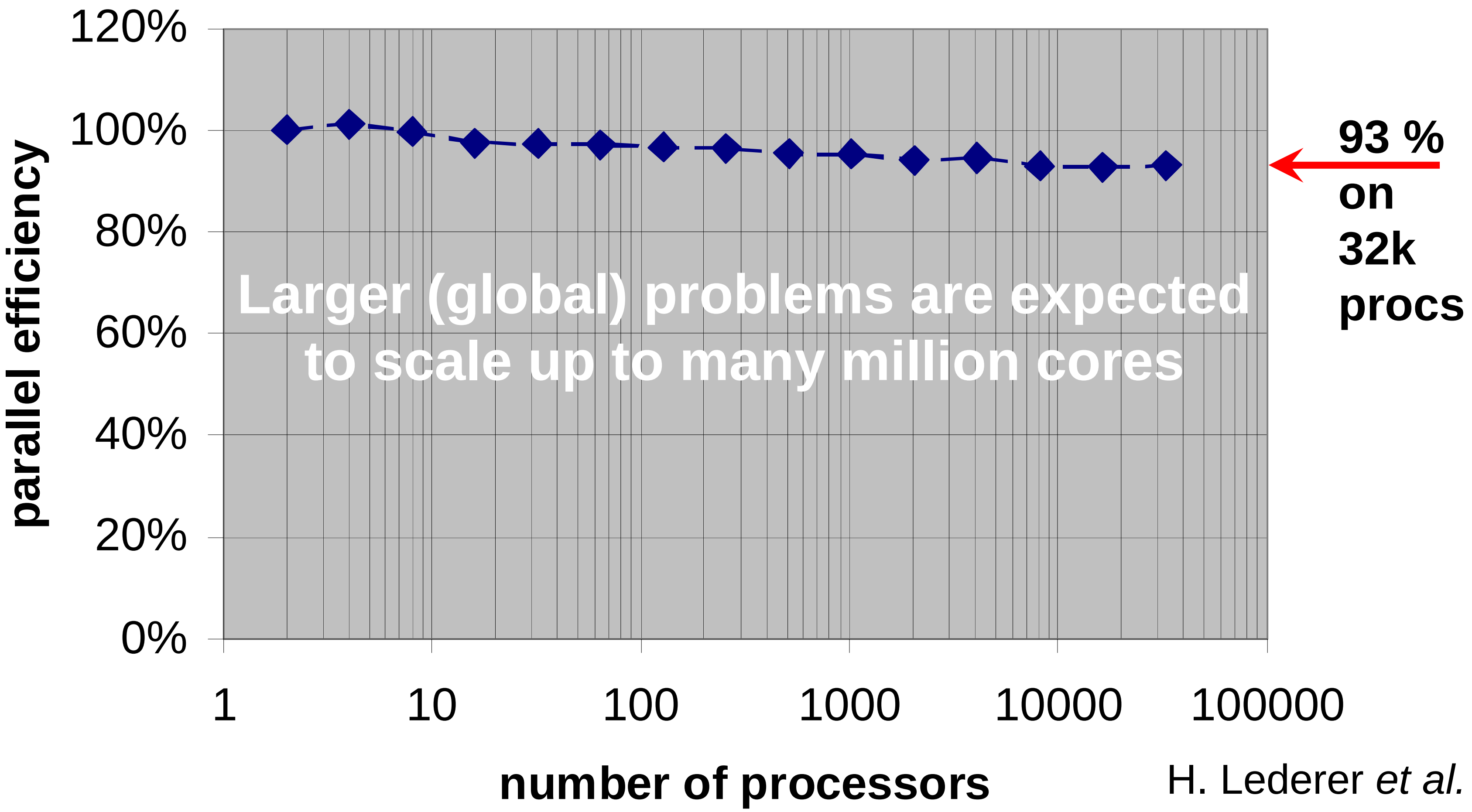
Gyrokinetic Electromagnetic Numerical Experiment

Parallelization of GENE

- Parallelization due to high-dimensional domain decomposition (either pure MPI or mixed MPI/OpenMP paradigm)
- GENE runs very efficiently on a large number of parallel platforms (including IBM BlueGene, IBM Power6, Cray XT4, SGI Altix etc.), on various Linux Clusters, as well as (linearly) on laptops

GENE code on BG/L (weak scaling)

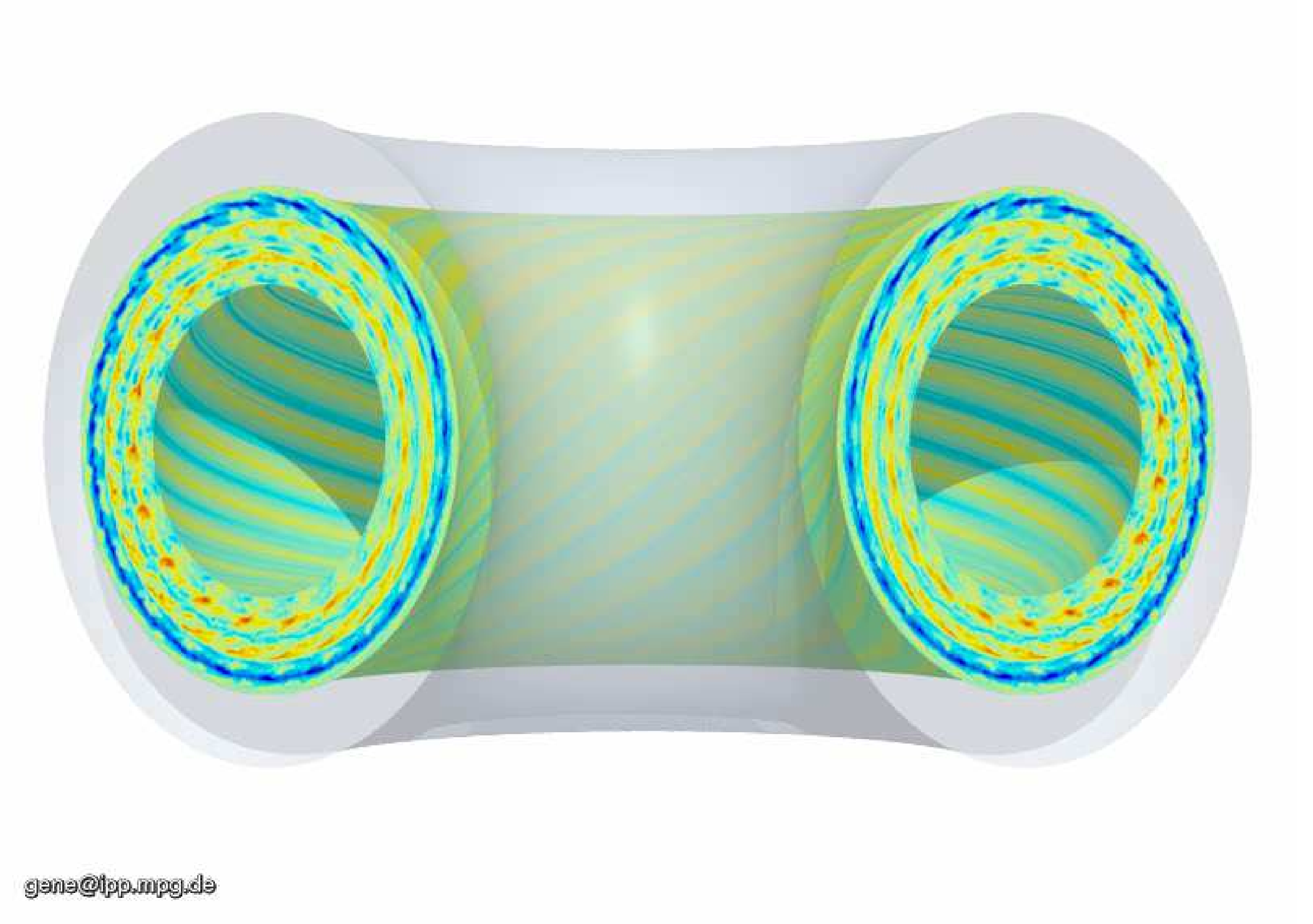
BG/L at Rochester, Minnesota: 2 – 2k procs   BG/L at Watson Research C., NY: 2k – 32k procs



H. Lederer *et al.*

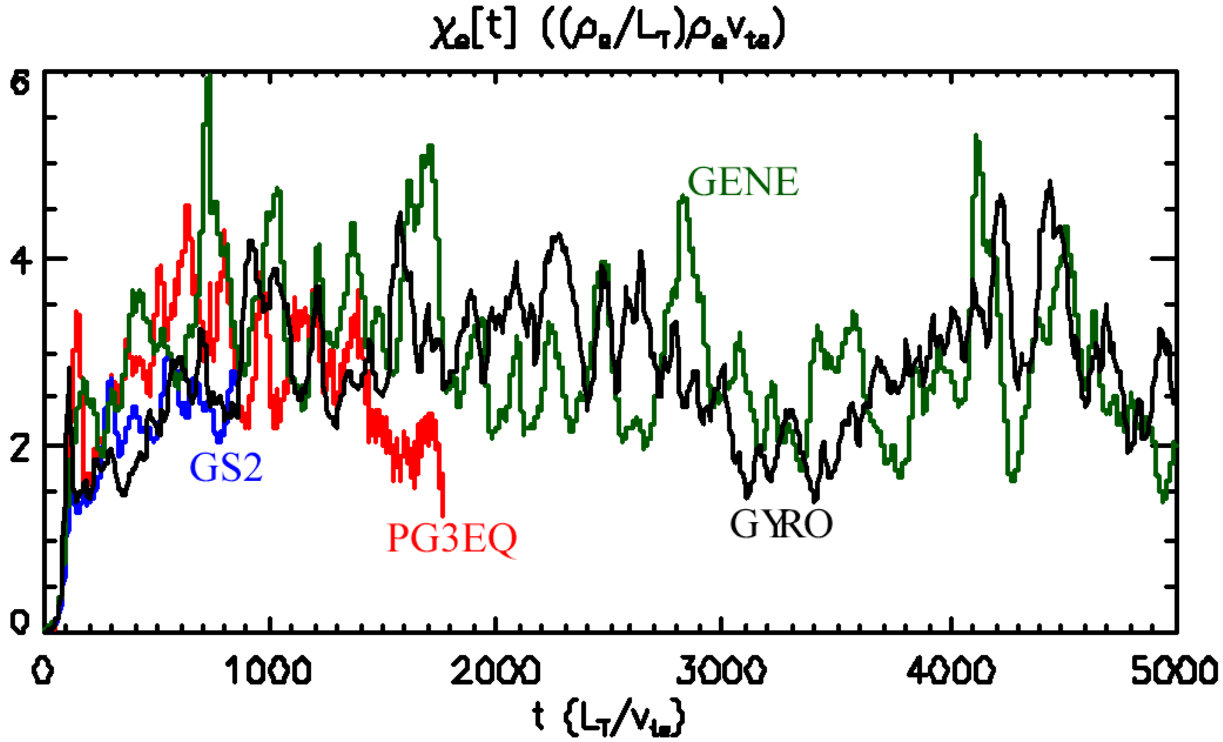
Weak scaling of GENE (15 points covering 4 orders of magnitude)
(problem ~200 MB/proc; measurements in virtual node mode, **normalized to 2 processors**)

Plasma turbulence: GENE simulations



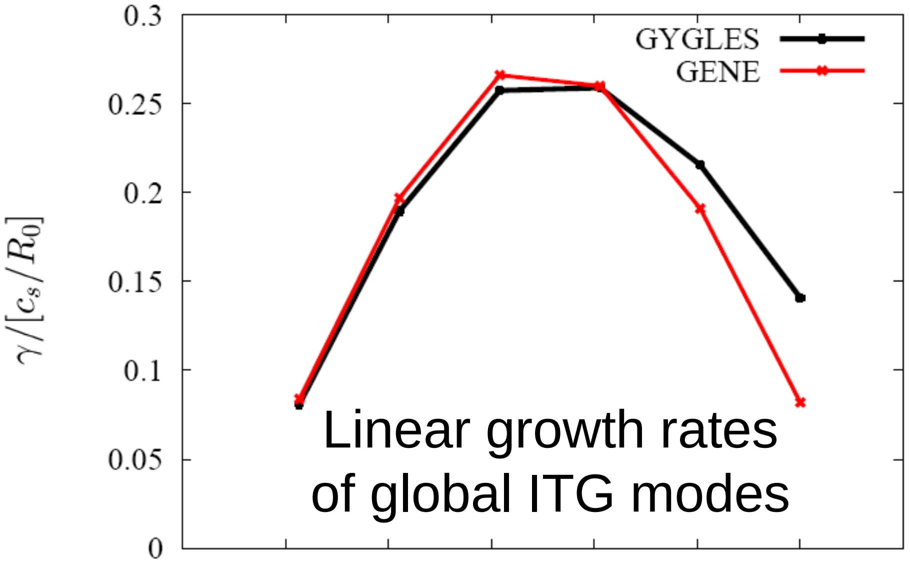
www.ipp.mpg.de/~fsj/gene

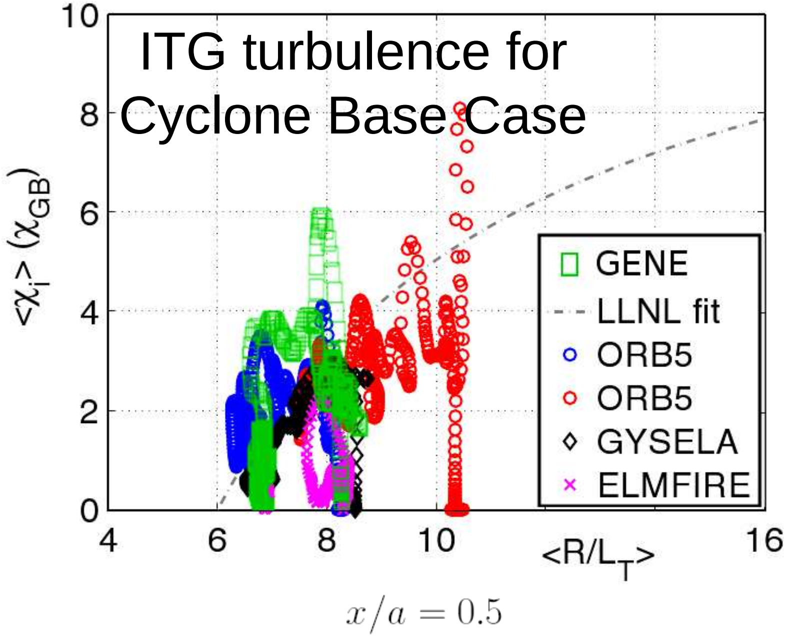
Code benchmarking (local, ETG)

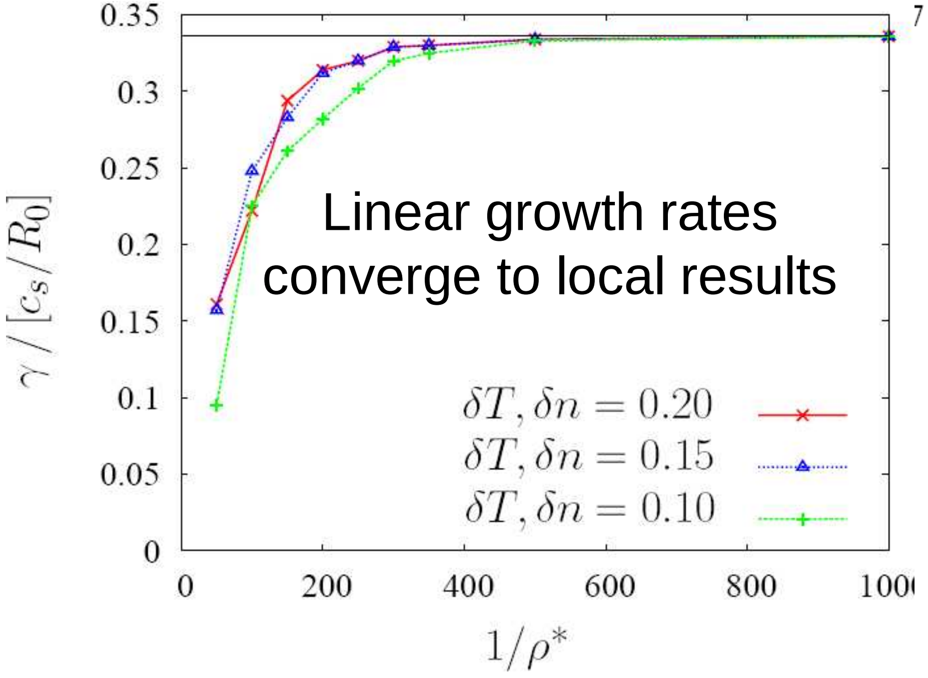


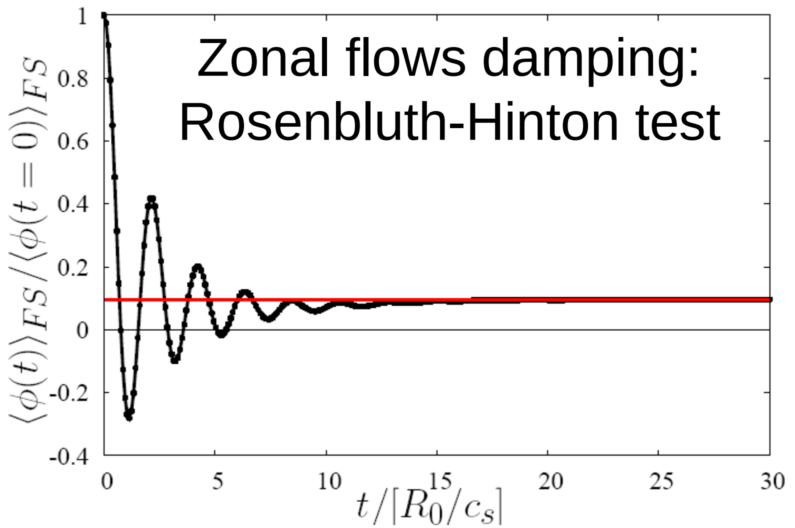
- Are we solving the equations right?
- Yes, according to a recent comparison between 4 codes (GENE, GYRO, GS2, PG3EQ)
- Such efforts are (sometimes) a bit painful but necessary

Code benchmarking (global, ITG)

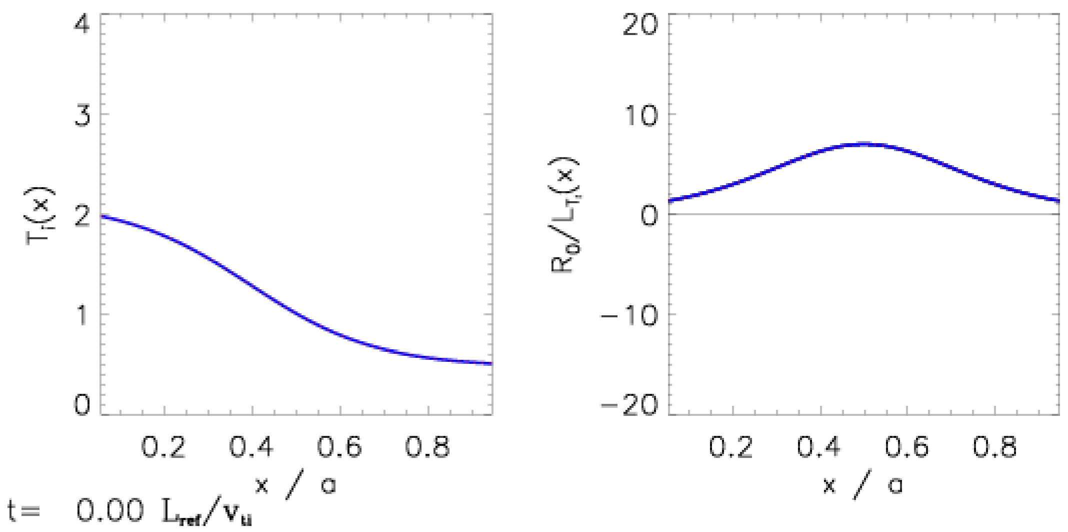








Global GENE results: Avalanches



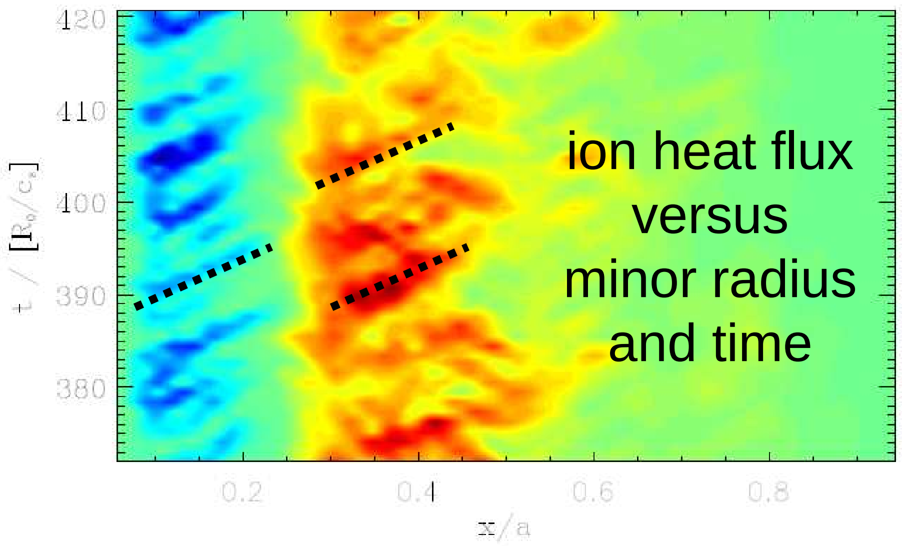
Flux-driven ITG turbulence

temperature profiles & temperature gradient profiles

T. Görler et al.

Heat pulses (avalanches) propagate in the radial direction with constant speed

Detailed investigations are underway



ion heat flux versus minor radius and time

Gyrokinetics in space and astrophysics

Various multiscale challenges in space and astrophysics

Many phenomena involve fluid, ion, electron scales, e.g.:

- magnetic reconnection
- shock waves
- turbulence
- cross-field transport

Simulations are challenged to address these issues.

Fruitful interaction with new generation of experiments like Cross-Scale or Magnetospheric Multiscale to be expected.

Current numerical approaches

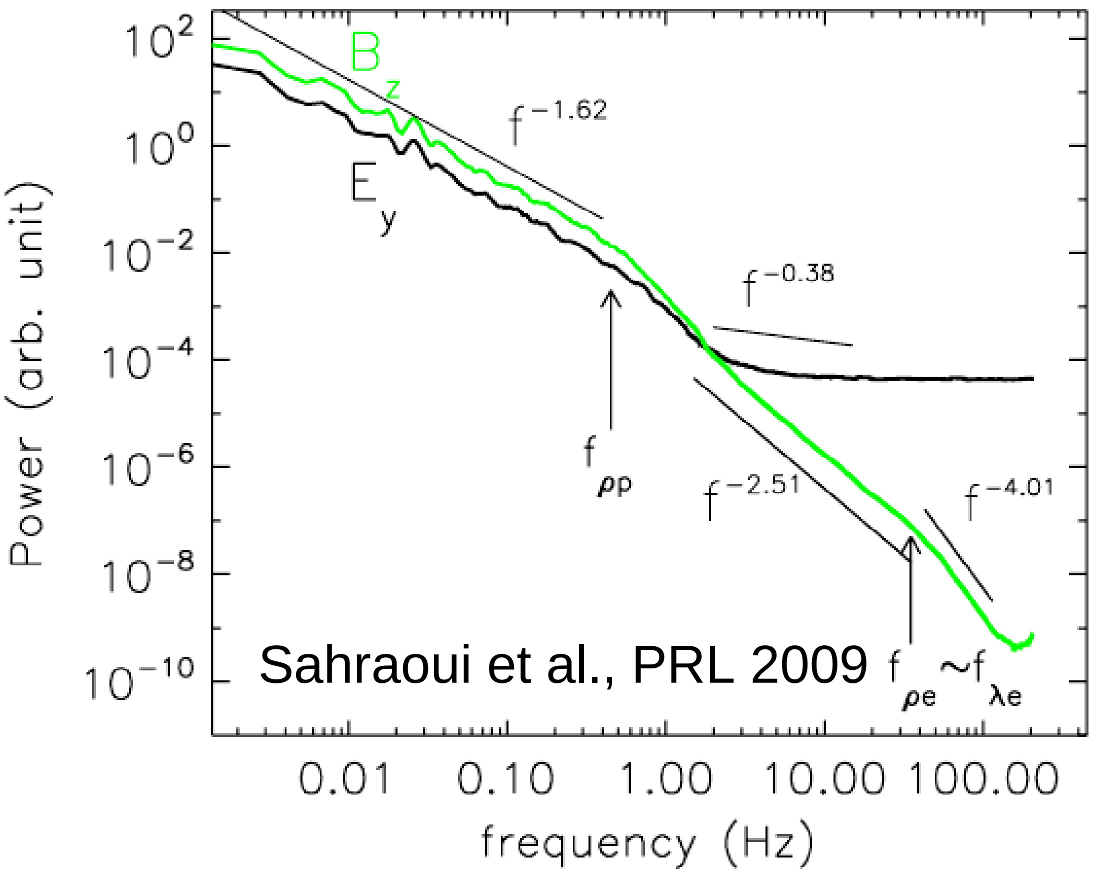
MHD (including multi-fluid extensions)
…the workhorse…

Kinetic (PIC or Vlasov, including δf versions)
…the microphysics laboratory…

Bridging spatio-temporal scales:
- **large-scale kinetics** (test particle approach)
- **hybrid codes** (e.g. kinetic ions & fluid electrons)

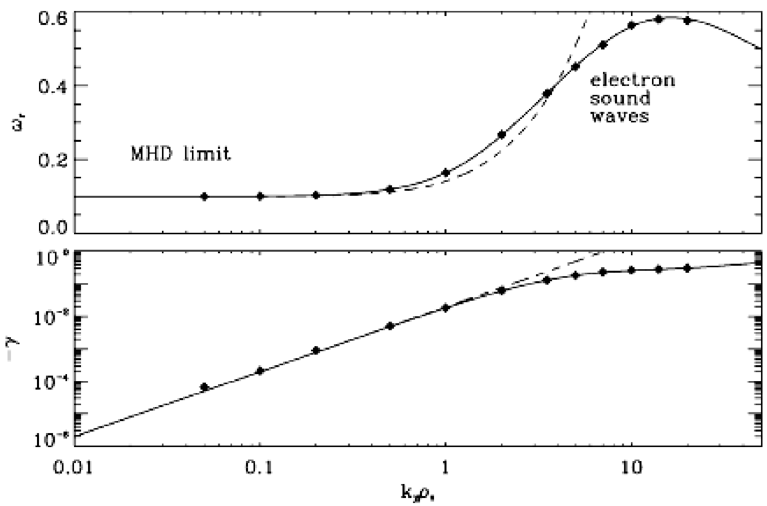
Another option: Gyrokinetics…

Solar wind turbulence: Dissipation?



Collisionless damping of
(shear) Alfvén waves

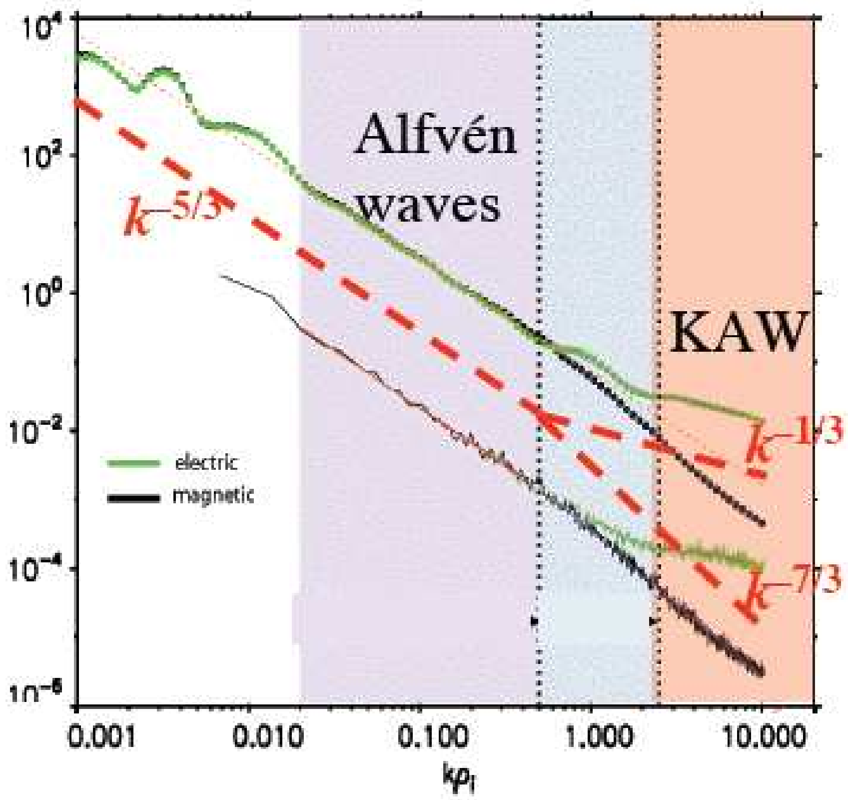
Dannert & Jenko, CPC 2004



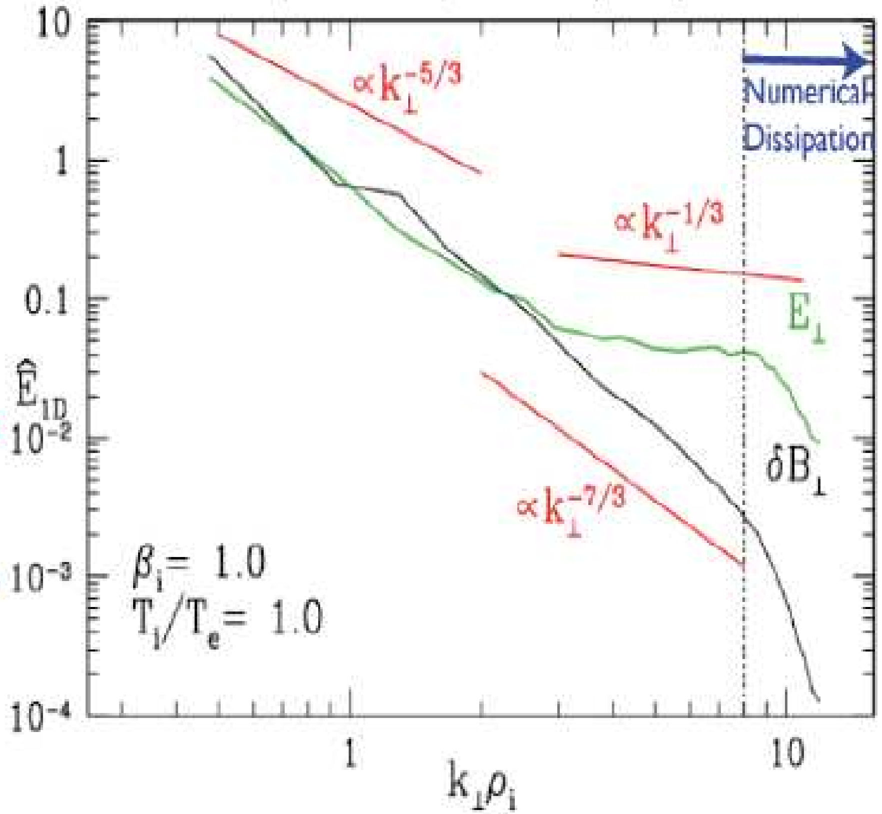
High-k MHD turbulence satisfies the gyrokinetic ordering!

High-k Alfvén wave turbulence: Observations versus simulations

AW turbulence in the solar wind
Bale et al., PRL 2005



GK simulation of AW turbulence
Howes et al., PRL 2008

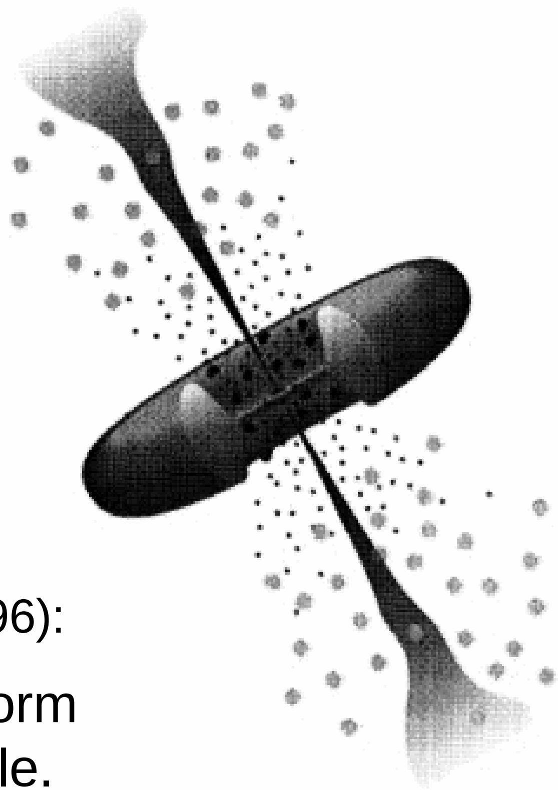


Broad-line regions in AGNs

Measured electromagnetic spectra from AGNs suggest the existence of…

…cold, dense clouds in a hot, dilute, magnetized medium in the central region of AGNs.

How can those cold clouds survive?



Standard model (e.g. Kuncic et al., MNRAS 1996):

Cold clouds are magnetically confined and form filaments; perpendicular transport is negligible.

Gyrokinetic turbulence sets lower limit on cloud size!

Conclusions

Gyrokinetic theory and simulation

- Gyrokinetics is a theory adapted to anisotropic, gyroradius-scale, low-frequency turbulence in magnetized plasmas
- Compared to a (brute-force) fully kinetic approach, it helps to save many orders of magnitude in effort
- Computational gyrokinetics has become a standard tool for theoretical investigations of plasma turbulence
- Some surprising findings from simulations of tokamak and stellarator turbulence will be shown in my second talk