The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2052-42

Summer College on Plasma Physics

10 - 28 August 2009

Periodic nonlinear surface waves in plasmas: nonlinear electrostatic oscillations in a sharp plasma interface

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Periodic nonlinear surface waves in plasmas:

nonlinear electrostatic oscillations in a sharp plasma interface

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Plan of the talk

- Linear plasma surface waves
- Gradov-Stenflo equation
- New periodic solutions to the Gradov-Stenflo equation

Linear electrostatic surface plasma waves

$$z < 0, \quad \varepsilon = 1, \quad \phi = \phi_0 \exp(ikx + kz)$$

vacuum

$$z > 0, \quad \varepsilon = 1 - \frac{\omega_p^2}{\omega^2}, \quad \phi = \phi_0 \exp(ikx - kz)$$

plasma

Properties

1) Propagation paralell to the surface z = 0
2) Restricted basically to that surface (since k > 0)
3) Satisfy $\nabla \bullet (\varepsilon \nabla \phi) = 0$
4) From the later: continuity of

   $\varepsilon \dfrac{\partial \phi}{\partial z}$ at z=0

Linear dispersion relation

Therefore: $$-k = (1 - \frac{\omega_p^2}{\omega^2})\, k$$

or $$\omega = \frac{\omega_p}{\sqrt{2}}$$

Mismatch factor: $$\Delta = 2 - \frac{\omega_p^2}{\omega^2}$$

- Reasonable agreement with experiments
- Physics of low-temperature *bounded* plasmas: not as well developed as that of high-temperature fusion plasmas
- Many industrial applications (*e.g.* TV screens)

Nonlinear plasma surface waves

- A veritable *zoo* of special solutions
- Some sort of expansion in powers of the amplitude of the electrostatic potential

$$\Rightarrow$$

nonlinear corrections to the dielectric function

The Gradov-Stenflo equation

$$\frac{8ik^2}{\omega}\frac{\partial \phi_0}{\partial t} + \frac{\partial^2 \phi_0}{\partial x^2} - \frac{1}{\phi_0}\left(\frac{\partial \phi_0}{\partial x}\right)^2 +$$

$$+ 2\beta\gamma k\,|\phi_0|\,\phi_0 + \beta^2\,|\phi_0|^2\,\phi_0 + 2k^2\Delta\phi_0 = 0$$

Derivation

Sharp plane plasma boundary at z = 0, or: a plasma with a fixed homogeneous ionic background for z > 0 and vacuum for z < 0.

Scalar potential: $\phi(x,z,t)\,\exp[i(kx-\omega t)]$

For z > 0:

$$\phi = \phi_0(x,t)\,\exp[\,-k\,z + \int_0^z dz'\,k_z(x,z')\,]\,,$$

$$k_z << k$$

ϕ_0 : a slowly varying envelope.

Plus the analytic continuation for z < 0.

- From the cold plasma fluid equations and a perturbative treatment:

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} + 2i\frac{\omega_p^2}{\omega^3}\frac{\partial}{\partial t}\ln\phi - \frac{\beta^2}{2k^2}|\phi|^2$$

- <u>Assumptions:</u> $\frac{\partial}{\partial t}\ln\phi << \omega, \quad \frac{\partial}{\partial x}\ln\phi << k$

- Previously known exact solutions: *stationary localized structures* (without the $\sim \gamma$ and $\sim \Delta$ terms).

O. M. Gradov and L. Stenflo, Phys. Fluids **25**, 983 (1982).

Similar to the recently derived *oscillon* solutions (in an external periodic flow oscillating at twice the natural surface wave frequency), but that‘s another history!

A peculiar property

- Remark:

$$\phi_0 \neq 0$$

- Remember Kibble‘s objection in the context of dissipative quantum mechanics
- The scalar potential is small, but nonzero!

A conservation law

It can be shown that

$$\frac{d}{dt}\int |\ln \phi_0|^2 \, dx = 0,$$

$$\int |\ln \phi_0(x,t)|^2 \, dx = \int |\ln \phi_0(x,0)|^2 \, dx$$

Exact periodic solution

- Let $\psi = \ln \phi_0$

- A modified nonlinear Schrödinger equation:

$$\frac{8ik^2}{\omega}\frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + 2\gamma\beta k\,|\exp\psi| + \beta^2\,|\exp(2\psi)| + 2k^2\Delta = 0$$

Madelung transformation

$$\psi = A\exp(iS),$$

$$A = A(X), \quad S = S(X),$$

$$X = x - ut,$$

$$\varphi = \ln(A/A_0), \quad A_0 > 0$$

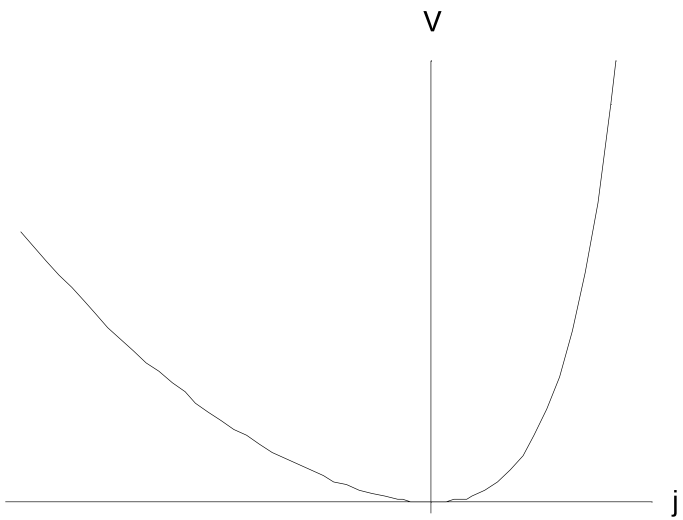
Sagdeev potential

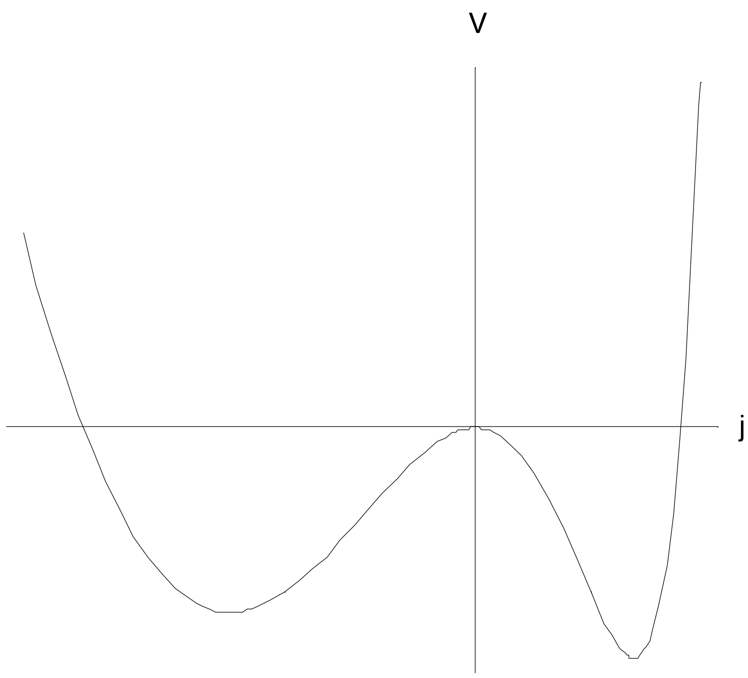
$$\frac{d^2\varphi}{dX^2} = -\frac{dV}{d\varphi},$$

$$V = V(\varphi) =$$

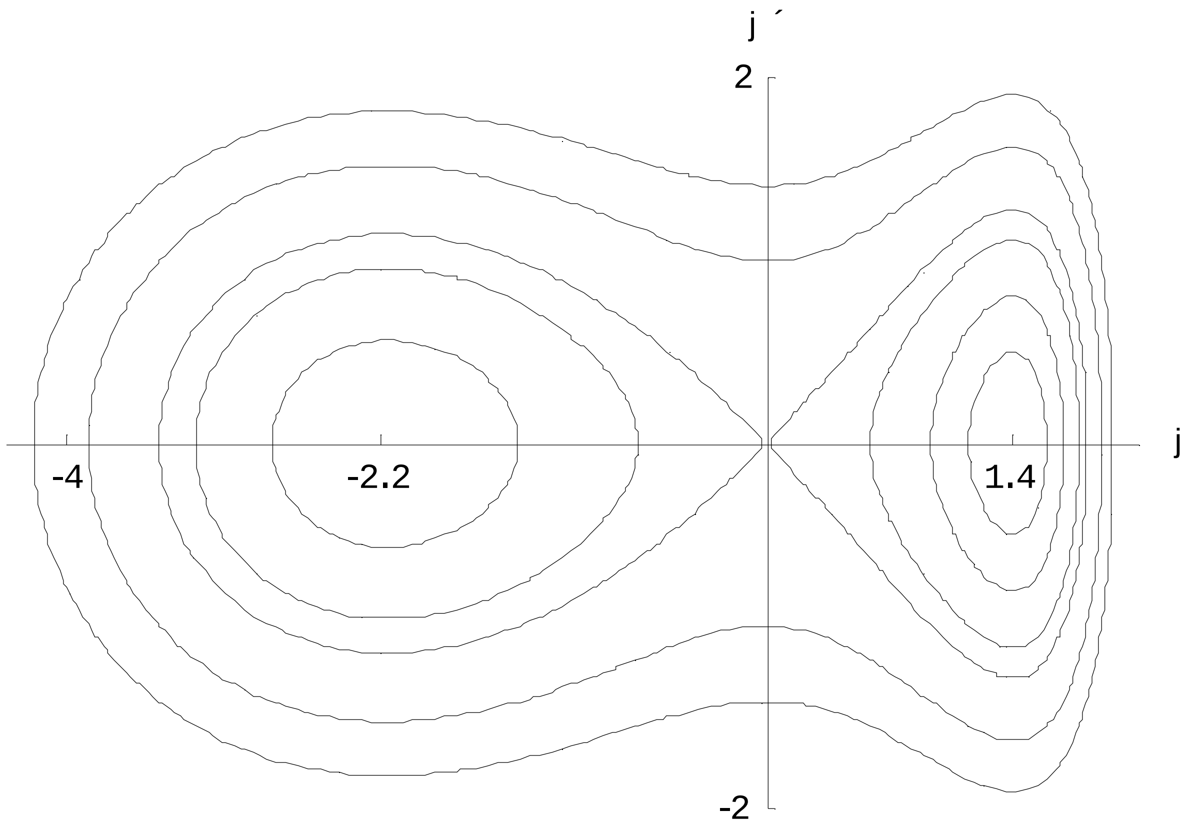
$$= 32\left(\frac{k^2 u}{\omega}\right)^2 \varphi^2 + 2\gamma\beta k A_0 (e^{\varphi} - \varphi - 1) + \frac{\beta^2 A_0^2}{2}(e^{2\varphi} - 2\varphi - 1)$$

Stable or bistable oscillations

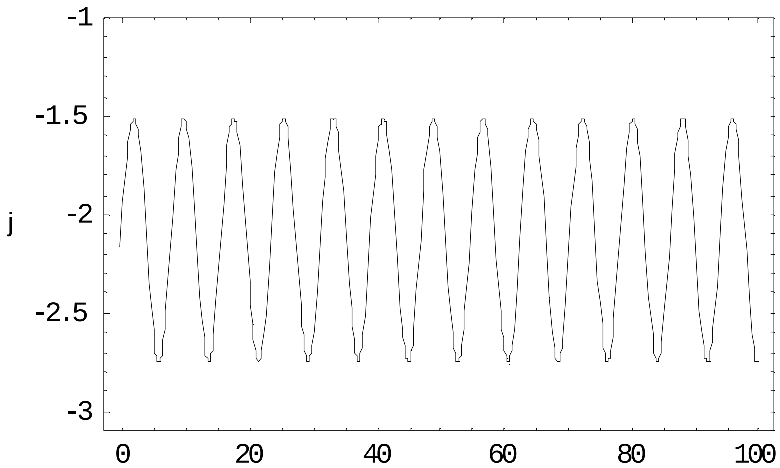


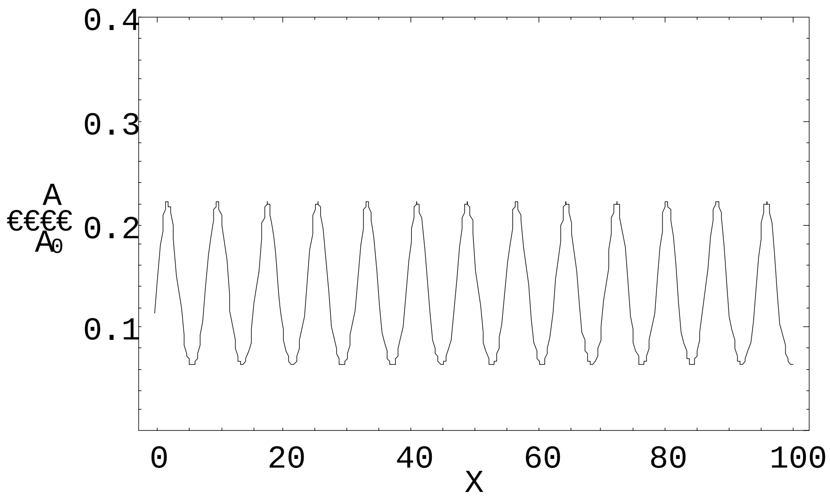


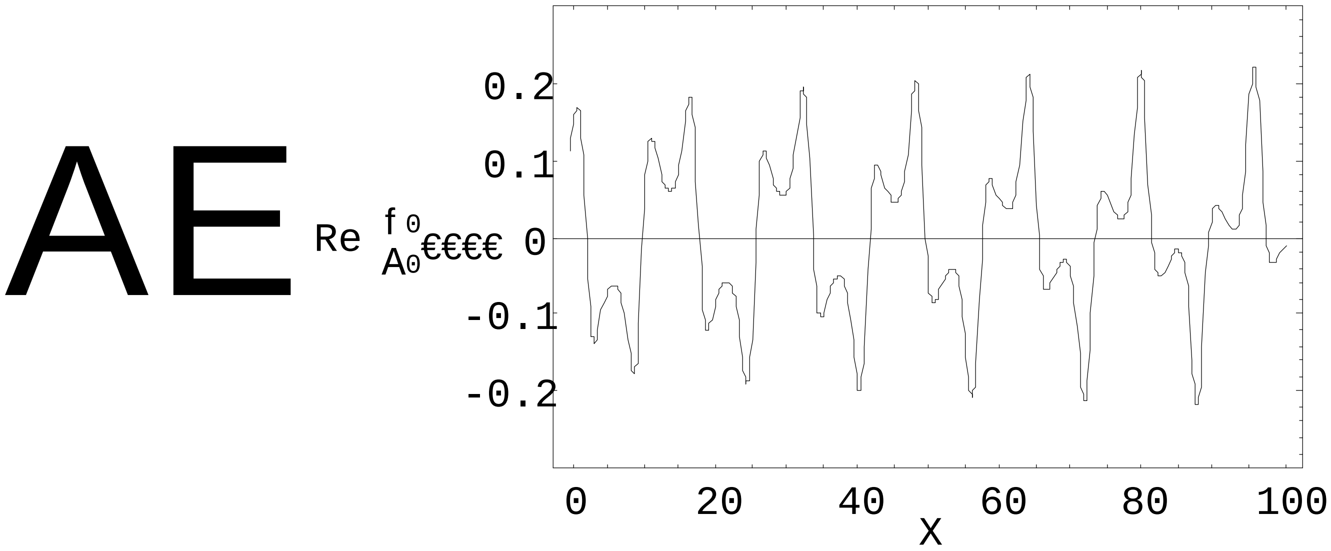
Phase space trajectories in the bistable case



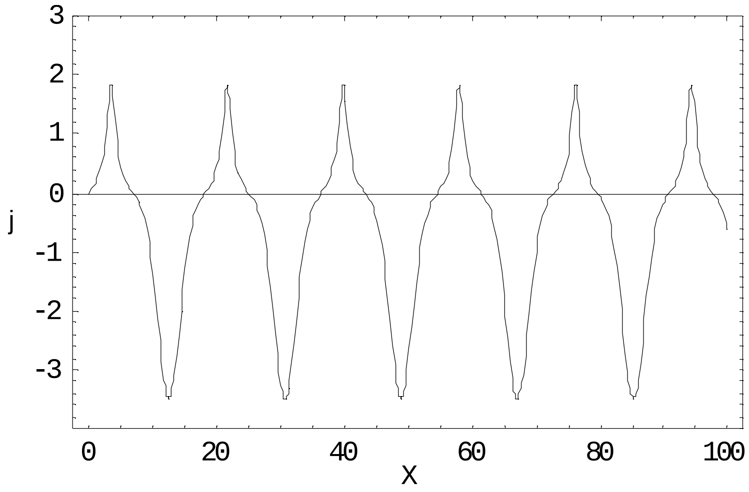
Near the bottom trajectories:

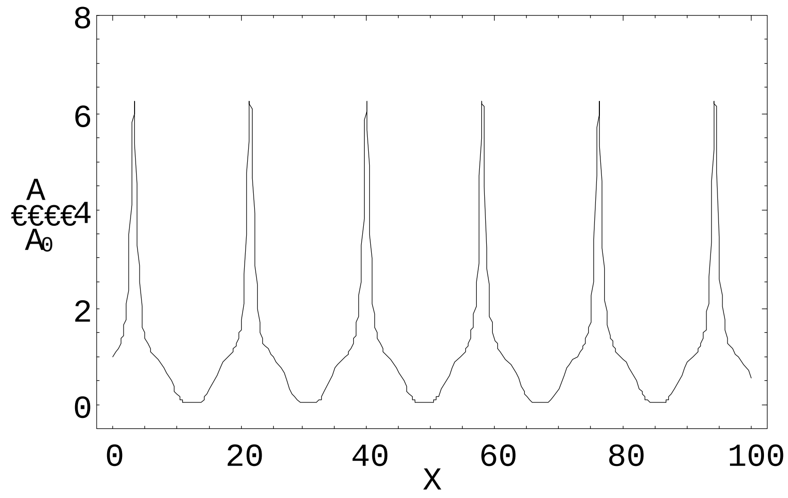


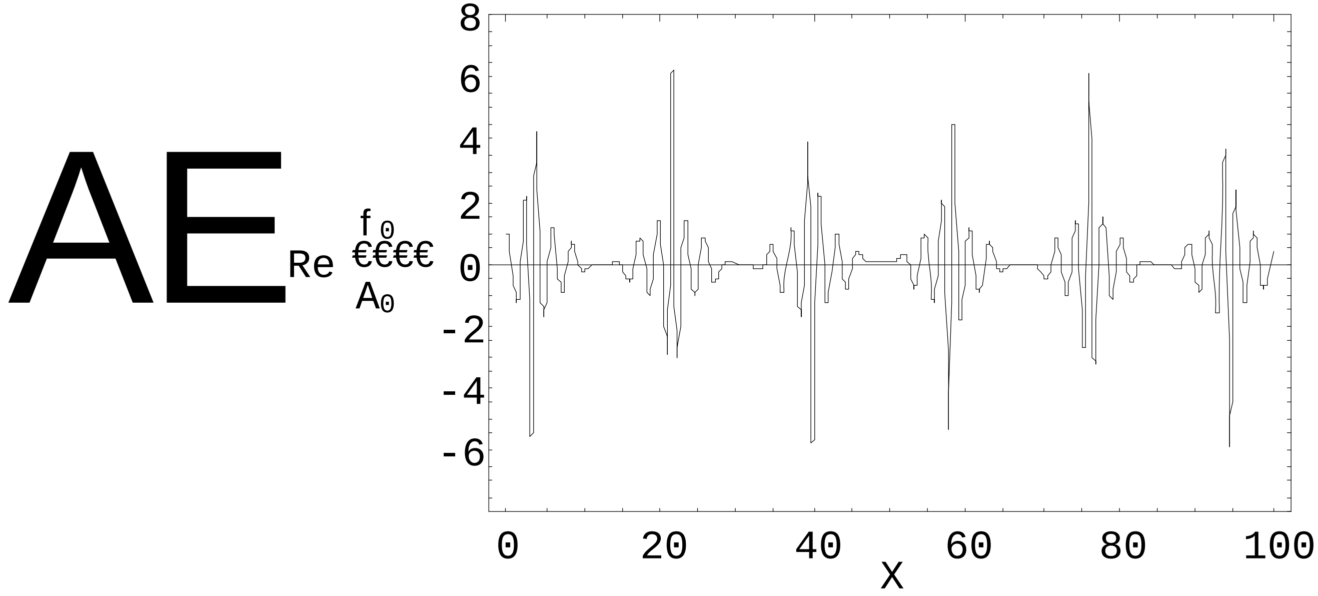




Near separatrix trajectories:







To conclude

Madelung transformation for the Gradov-Stenflo equation

$$\Rightarrow$$

exact translating oscillatory solution

Possible improvements: dissipative or transverse effects; physical modeling of parameters; moving boundaries...