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Pseudochaos and Stable-Chaos in Statistical Mechanics and Quantum Physics

21 - 25 September 2009

Stable chaos: An introduction

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Stable chaos

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Trieste – September 2009
Pseudochaos and Stable-chaos in Statistical Mechanics and Quantum Physics

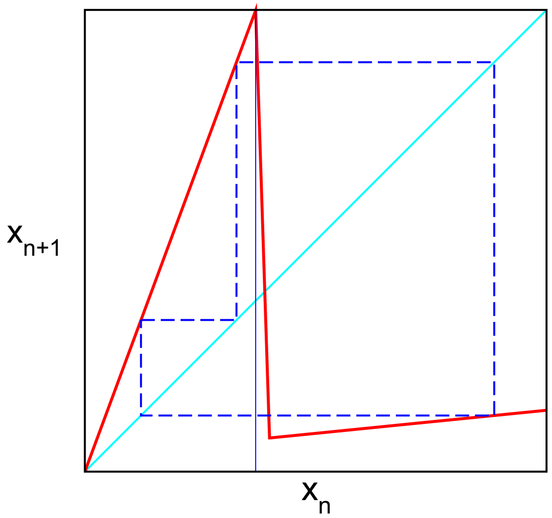
OUTLINE

- COUPLED MAP MODELS AND COUPLED OSCILLATORS
- CHARACTERIZATION
- PROPAGATION OF FINITE-AMPLITUDE PERTURBATIONS
- THE MAXIMUM LYAPUNOV EXPONENT AND ITS FLUCTUATIONS
- ORDER-TO-CHAOS TRANSITIONS?
- CHAINS OF HARD-POINT PARTICLES
- LEAKY INTEGRATE-AND-FIRE NEURONS

LINEARLY STABLE CHAOS IN COUPLED MAPS

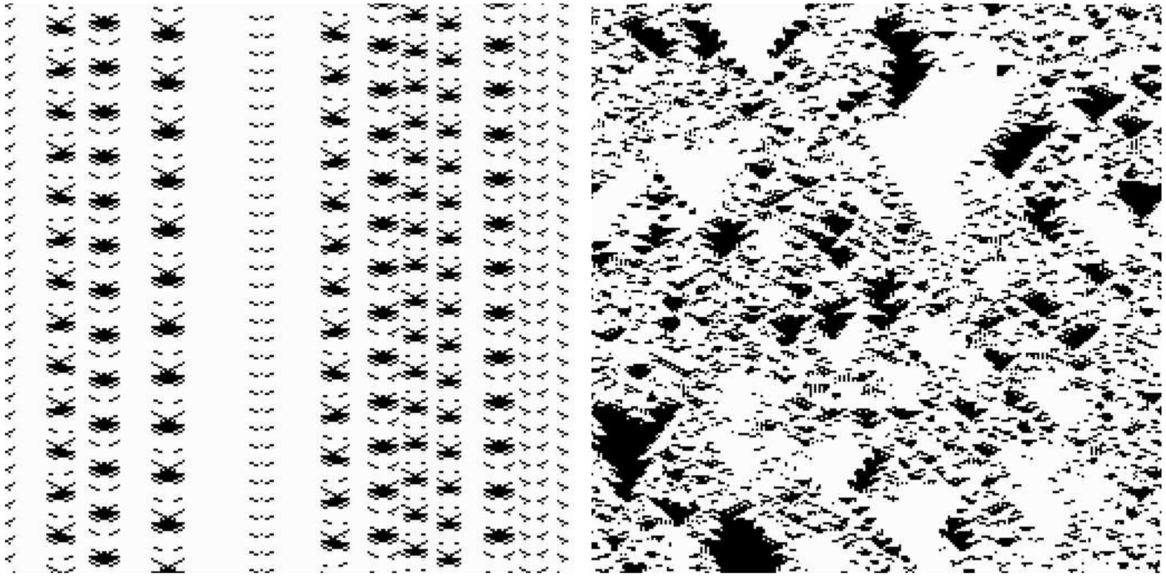
$$x^i_{n+1} = F(\bar{x}^i_n)$$

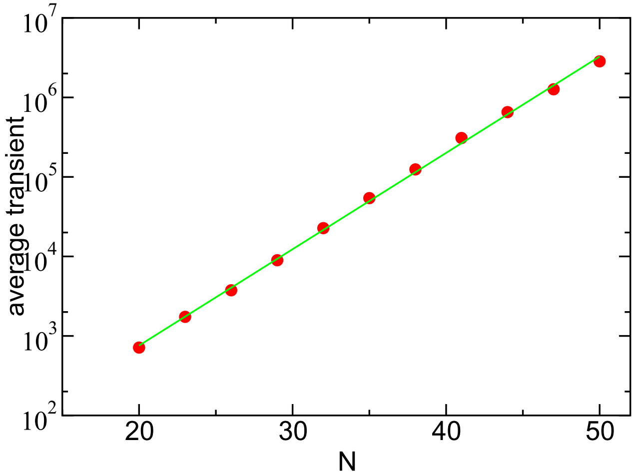
$$\bar{x}^i_n = \frac{\varepsilon}{2} x^{i-1}_n + (1-\varepsilon) x^i_n + \frac{\varepsilon}{2} x^{i+1}_n$$



[J.P.Crutchfield, K.Kaneko, PRL, **60** 2715 (1988)]

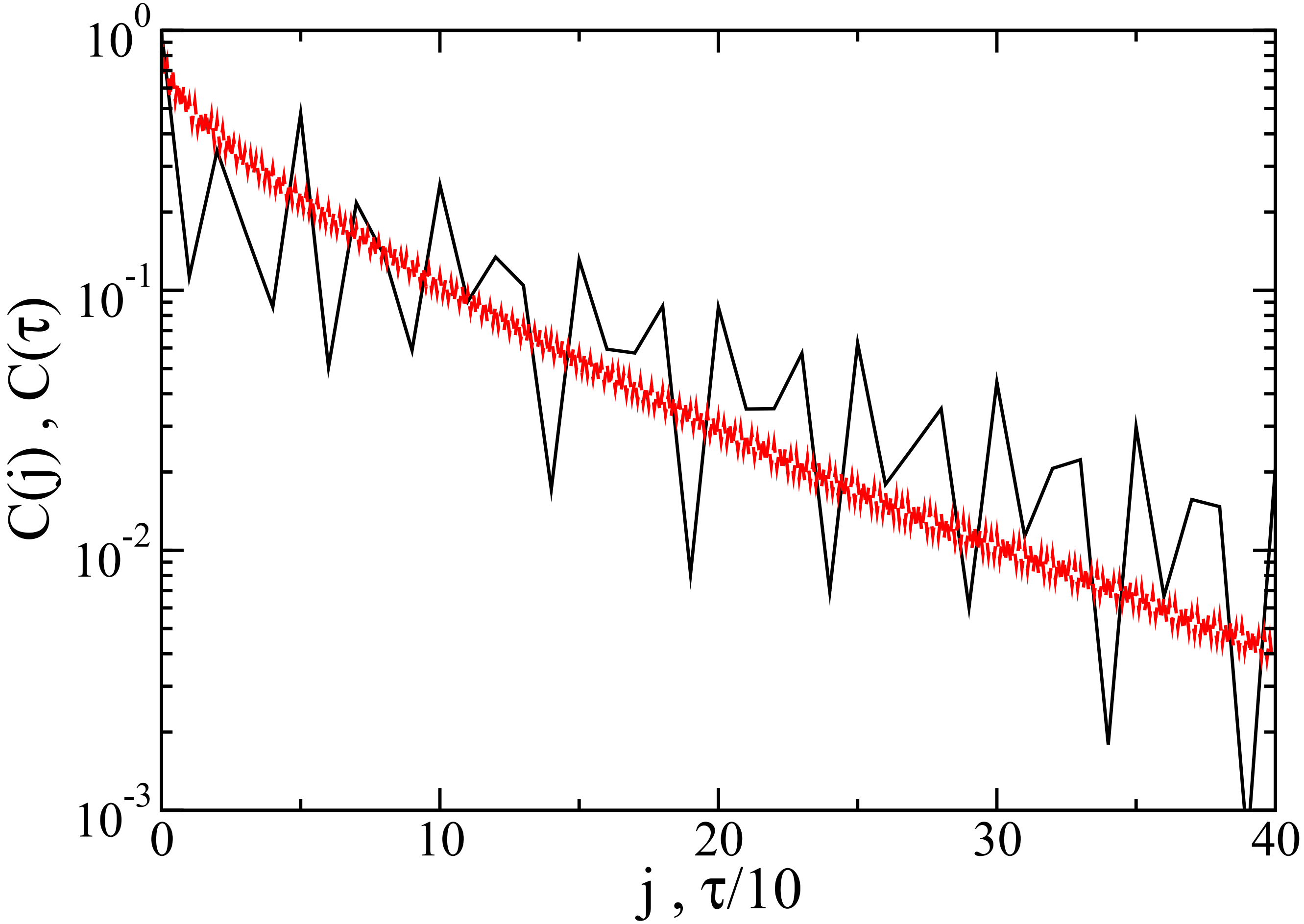
[A.P., R.Livi, G.-L.Oppo, R.Kapral, Europhys. Lett. **22** 571 (1993)]





average transient
N
10^7
10^6
10^5
10^4
10^3
10^2
20
30
40
50

Spatial and temporal correlation functions

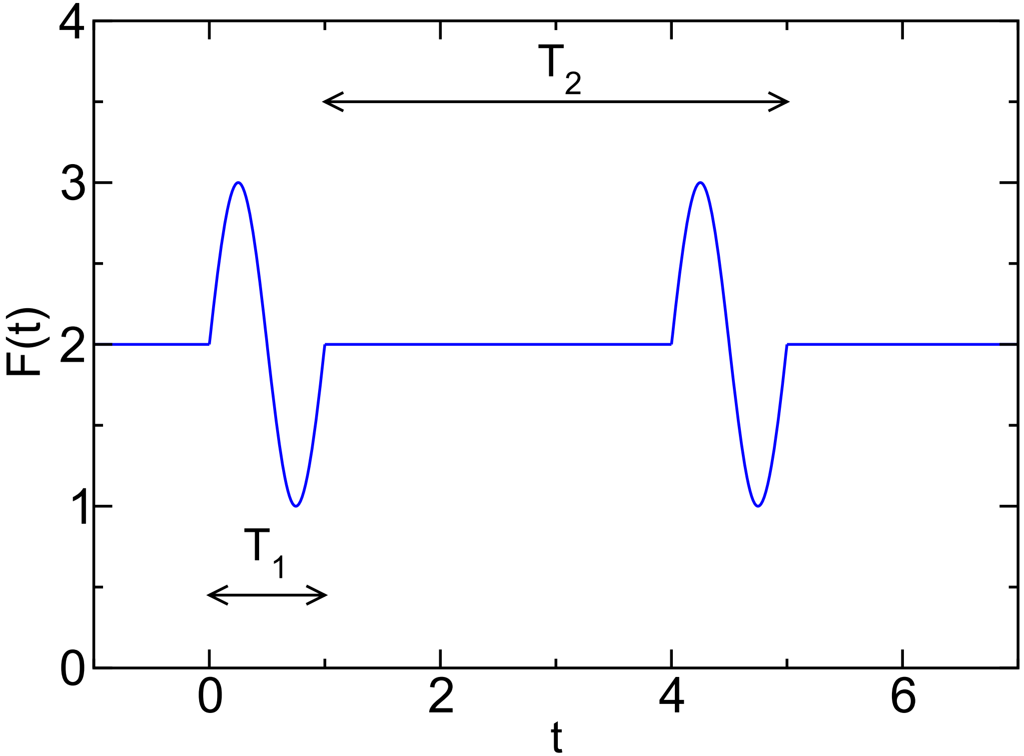


$$C(j) = \frac{\langle x_i(t) x_{i+j}(t) \rangle - \langle x_i(t) \rangle^2}{\langle x_i^2(t) \rangle}$$

$$C(t) = \frac{\langle x_i(t) x_i(t+\tau) \rangle - \langle x_i(t) \rangle^2}{\langle x_i^2(t) \rangle}$$

A CHAIN OF FORCED DUFFING OSCILLATORS

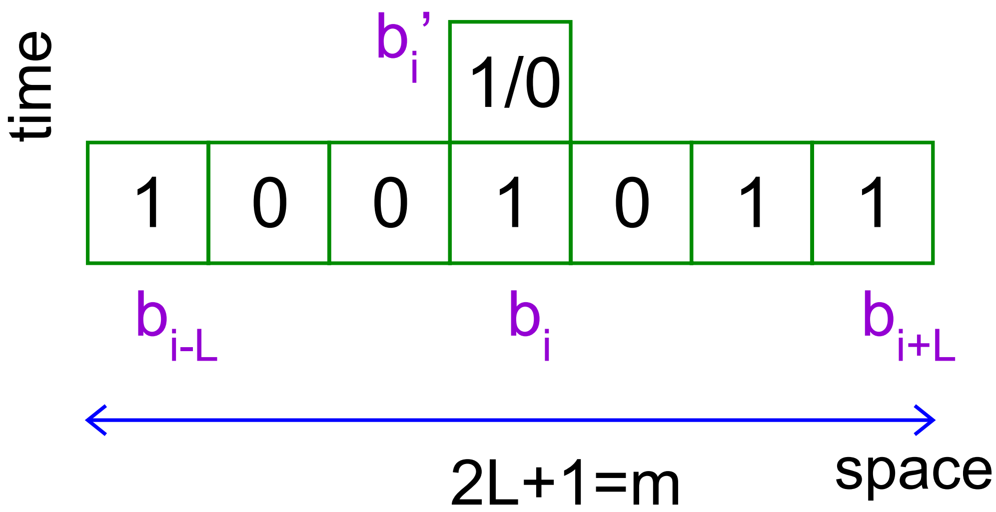
$$\ddot{x}_i = -\gamma \dot{x}_i - x_i^3 + D(x_{i+1} + x_{i-1}) + F(t)x_i$$



CELLULAR AUTOMATA RECONSTRUCTION

$$b_i' = G(S)$$

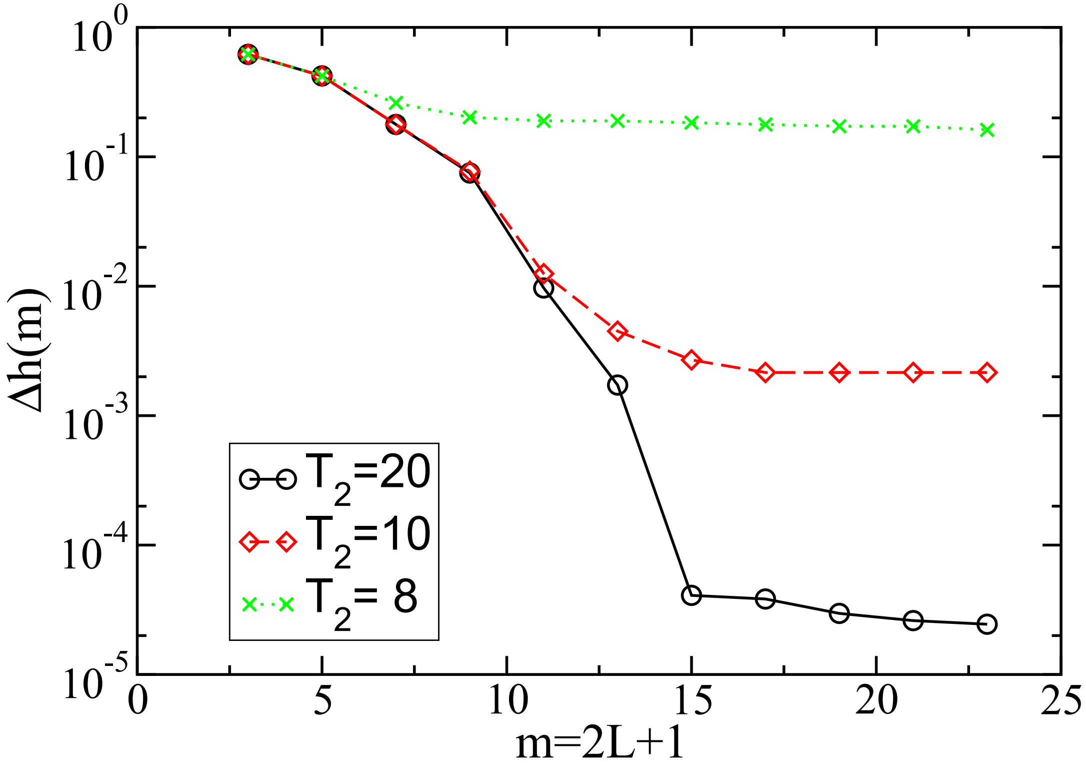
$$S = \{ b_{i-L}, \ldots, b_{i-1}, b_i, b_{i+1}, \ldots, b_{i+L} \}$$



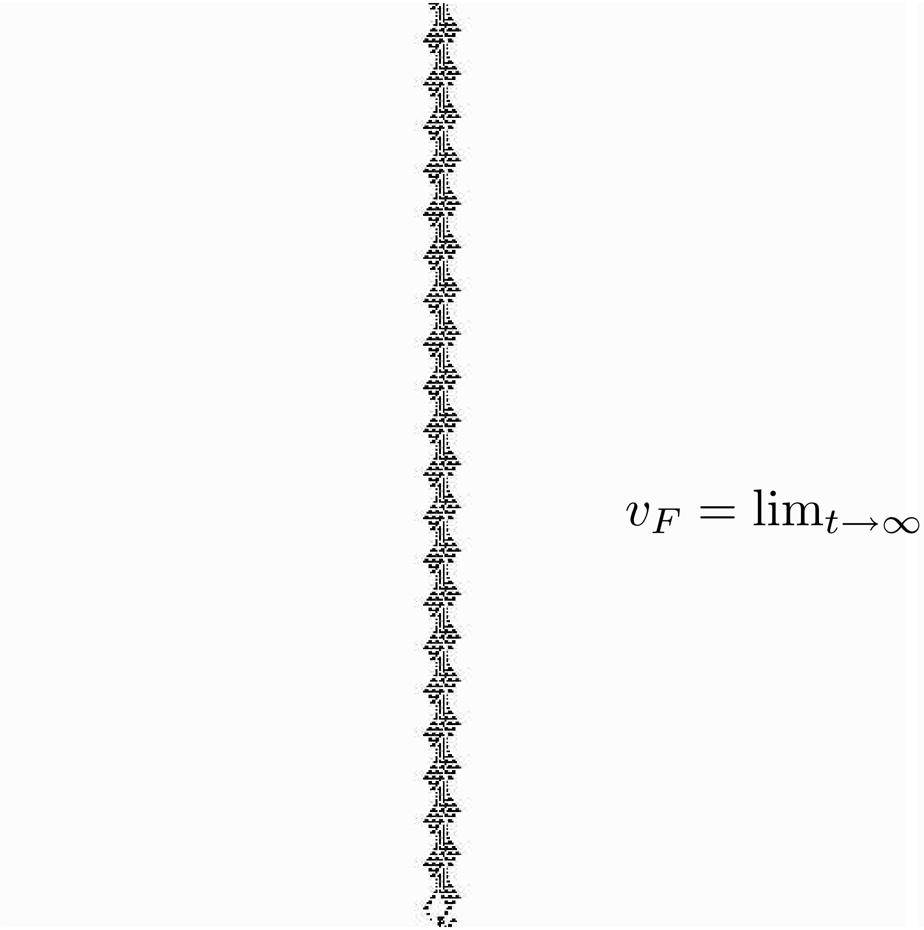
INDETERMINACY OF THE RECONSTRUCTION

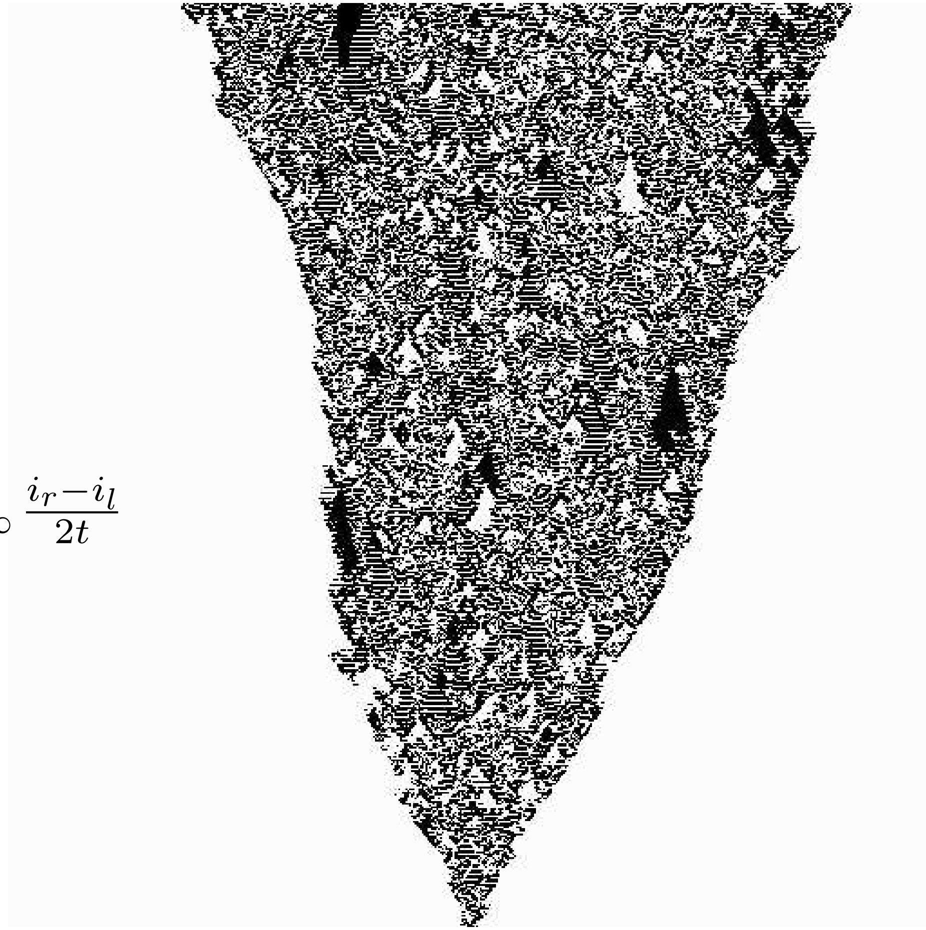
$$\Delta h(m) = -\sum_S \log P(S(m)) \sum_{b'} P(b'|S(m)) \log(P(b'|S(m))$$

$T_1 = T_2/16$



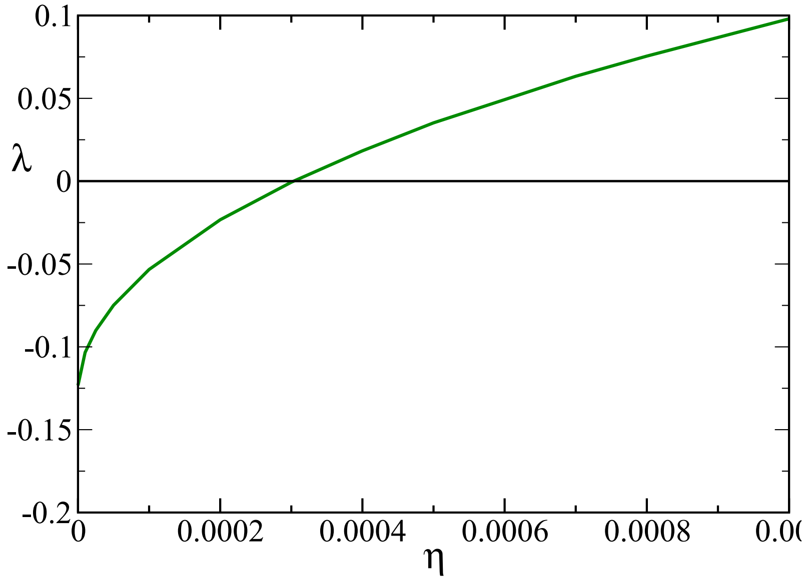
PERTURBATION PROPAGATION





$$v_F = \lim_{t \to \infty} \frac{i_r - i_l}{2t}$$

MAXIMUM LYAPUNOV EXPONENT

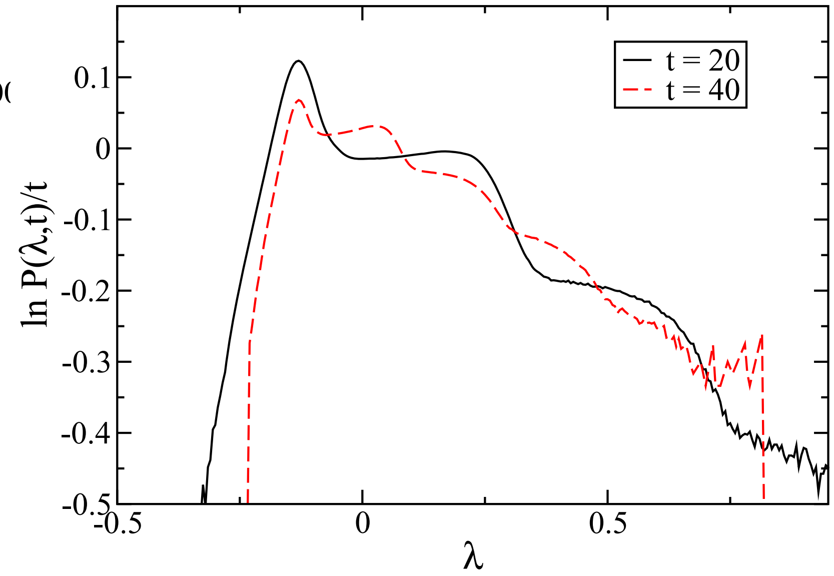


$$\lambda(t) = \left\langle \log \frac{|\delta x(t+t')|}{|\delta x(t')|} \right\rangle$$

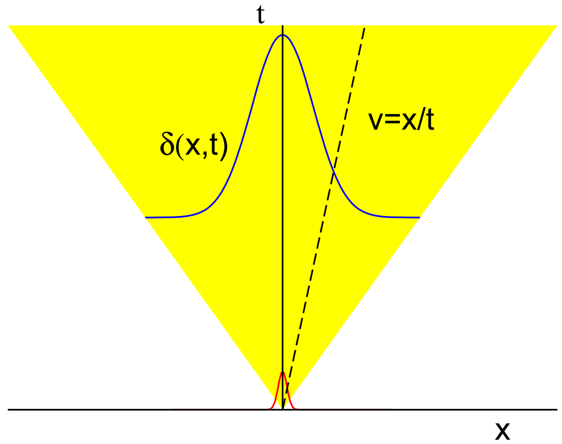
$$P(\lambda, t) = \text{Prob. Lyapunov} \in [\lambda, \lambda + d\lambda]$$

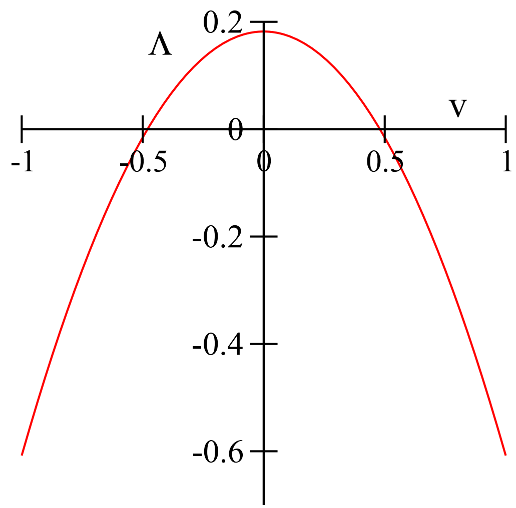
$$P(\lambda, t) \approx \exp\left[tG(\lambda)\right]$$

$$G(\lambda) \leq 0 \qquad G(\lambda_0) = 0$$



CONVECTIVE LYAPUNOV EXPONENTS





$$\delta(x,t) \approx \exp[\Lambda(x/t), t]$$

$$\Lambda(v = x/t) = \lim_{t\to\infty} \frac{1}{t} \ln\left(\frac{|\delta(x,t)|}{|\delta(0,0)|}\right)$$

[R.J.Deissler, K. Kaneko **119** 397 (1987)]

$$\dot{x}_i = F(x_i, x_{i-1}, x_{i+1})$$

$$\dot{\delta x}_i = F'_- \delta x_{i-1} + F'_0 \delta x_i + F'_+ \delta x_{i+1}$$

$$\delta x_i = \mathrm{e}^{\mu i} \delta_i$$

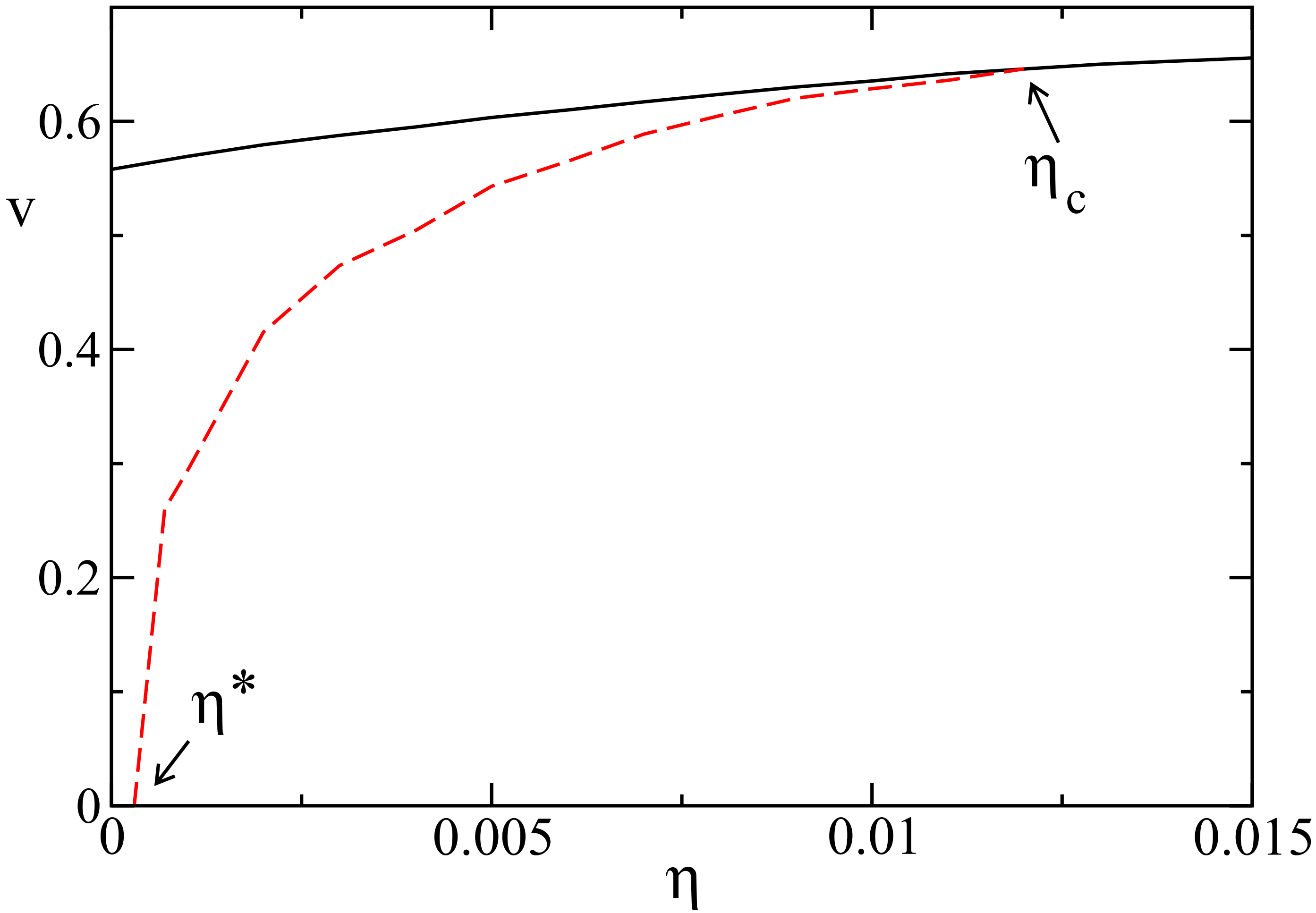
$$\dot{\delta}_i = F'_- \delta_{i-1} \mathrm{e}^{-\mu} + F'_+ \delta_{i+1} \mathrm{e}^{\mu} + F'_0 \delta_i$$

LEGENDRE TRANSFORM

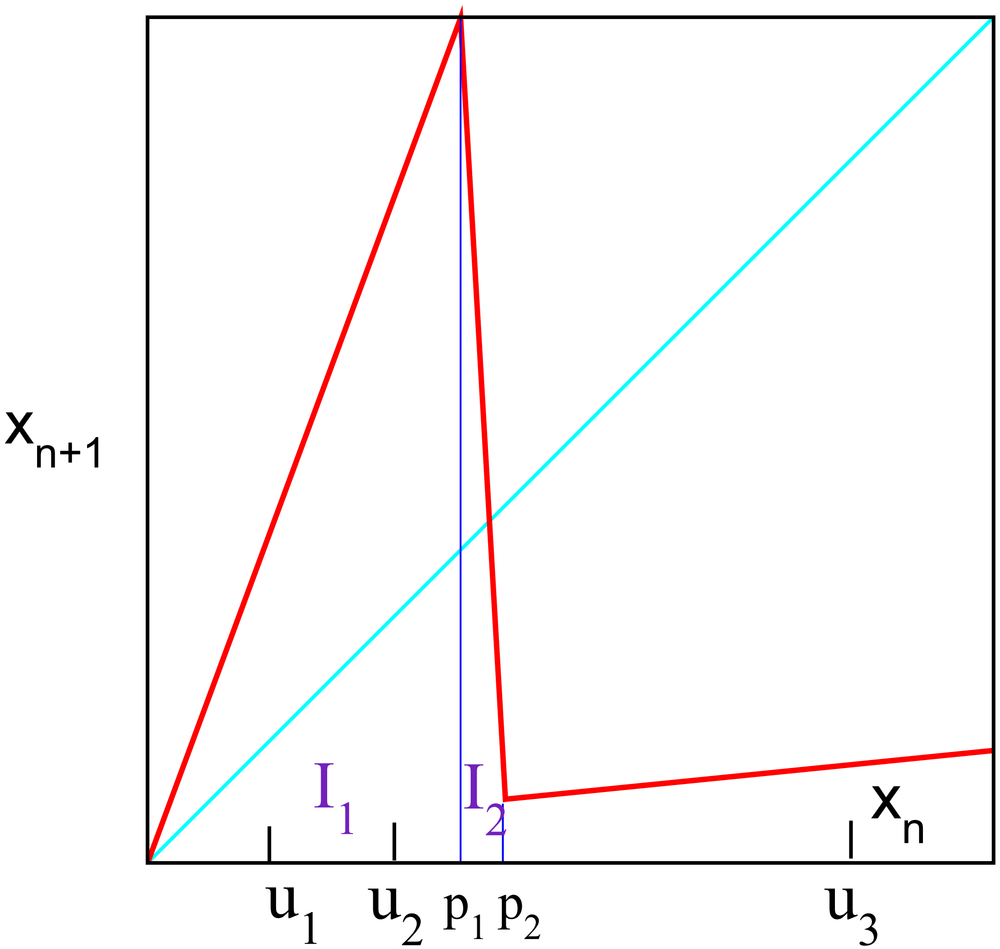
$$\Lambda(V) = \lambda_M(\mu) + \mu \tfrac{d\lambda_M}{d\mu} \qquad \mu = \tfrac{d\Lambda}{dV} \qquad V = \tfrac{d\lambda_M}{d\mu}$$

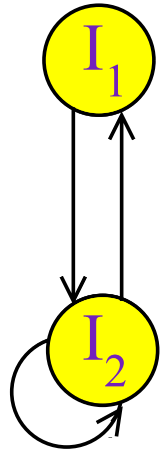
[A.P., A.Torcini, CHAOS **2** 293 (1992)]

PROPAGATION VELOCITIES

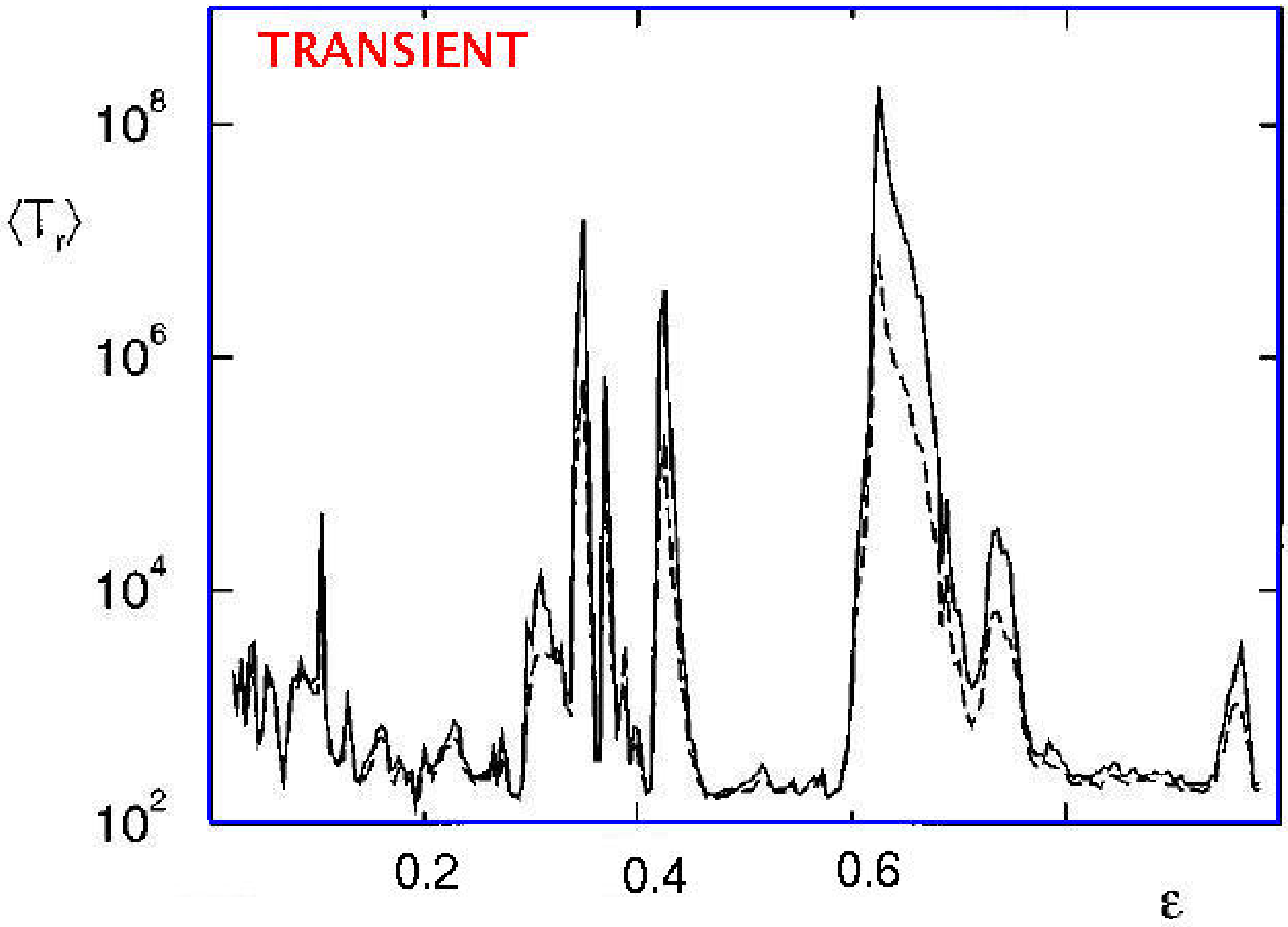


TOPOLOGICAL ENTROPY

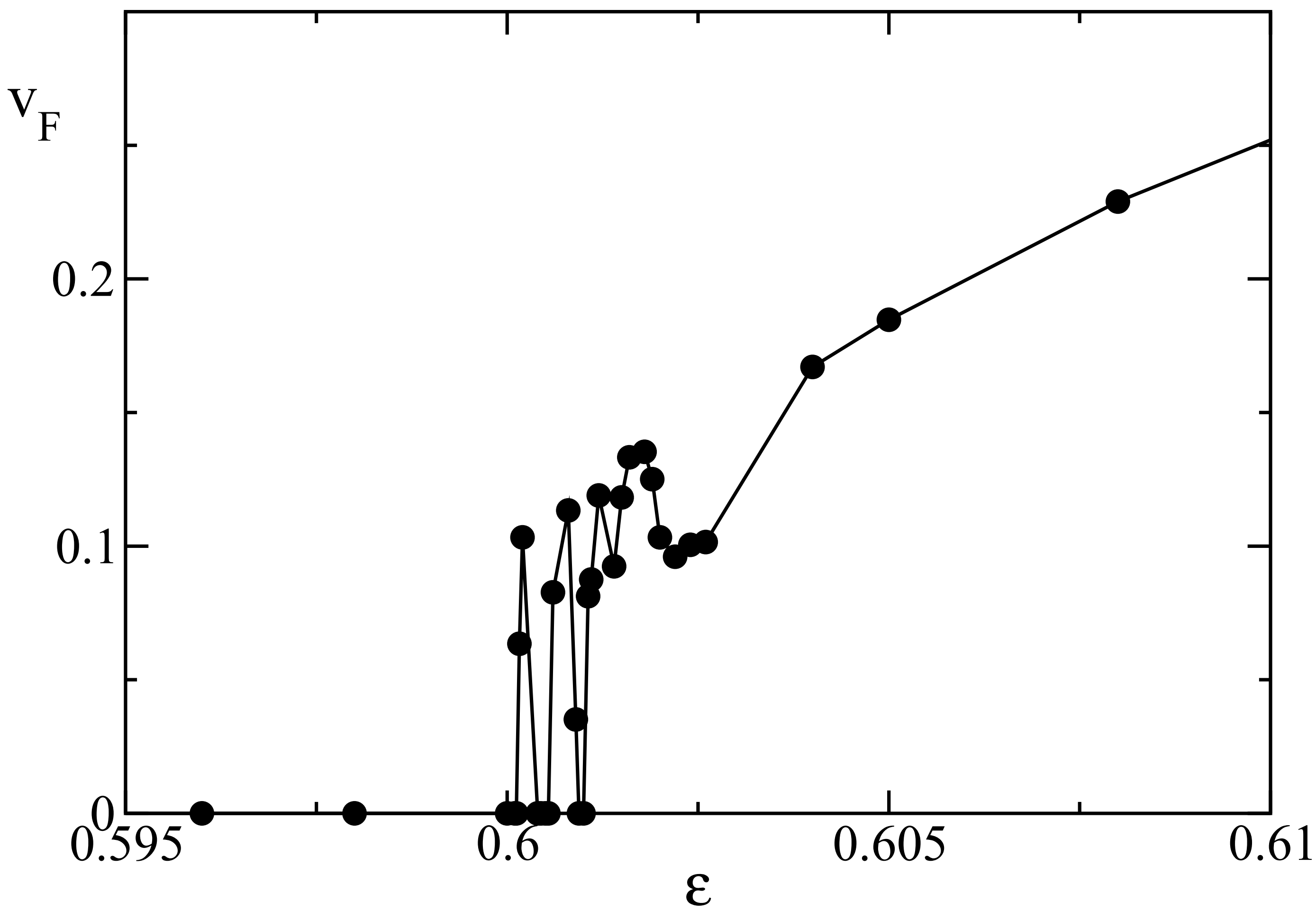




ORDER-TO-CHAOS TRANSITION?



F. Cecconi, R. Livi, AP, Phys. Rev. E **57** 2703 (1998)

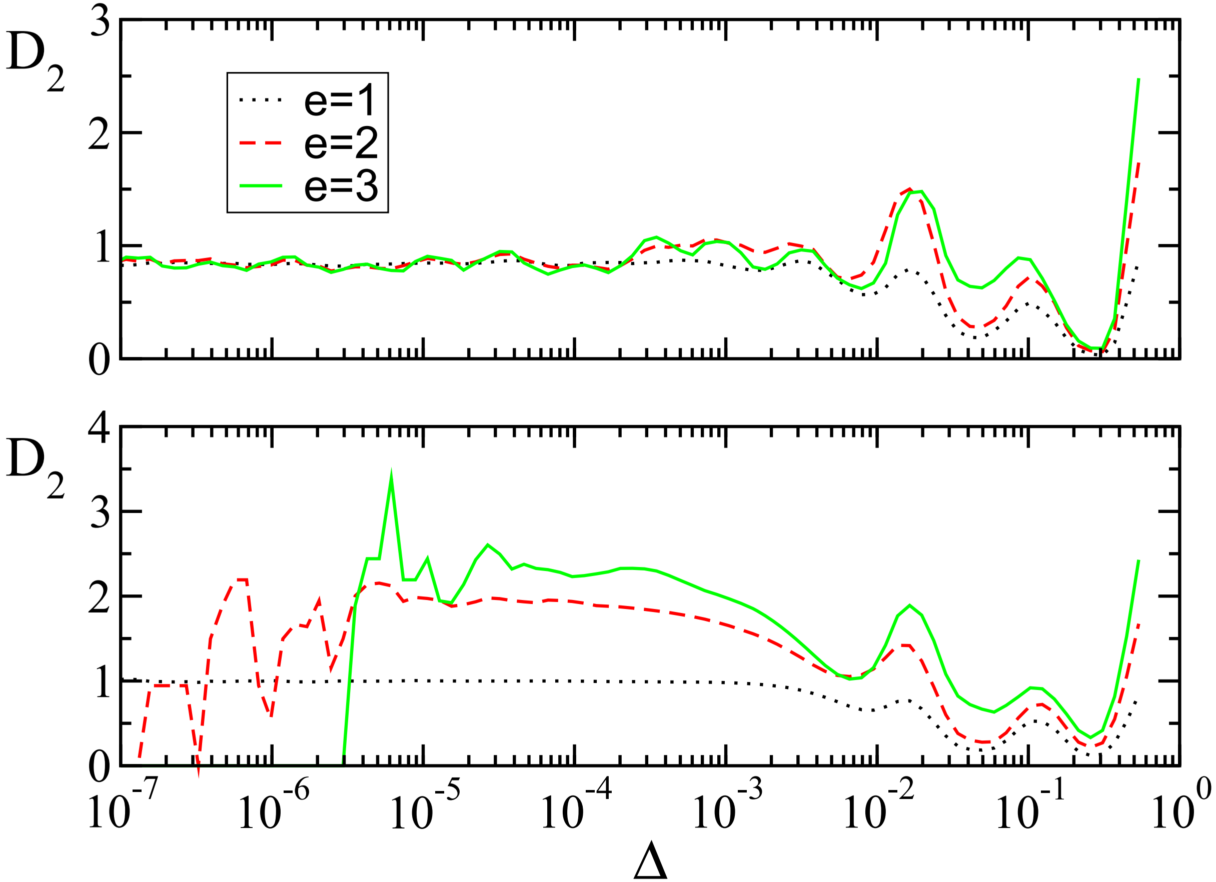


v_F
0.2
0.1
0
0.595
0.6
0.605
0.61
ε

SPATIAL EMBEDDING

$$D_2(e,\Delta) = \frac{\partial \log \mathcal{N}}{\partial \log \Delta}$$

e = embedding dimension



A STOCHASTIC MODEL

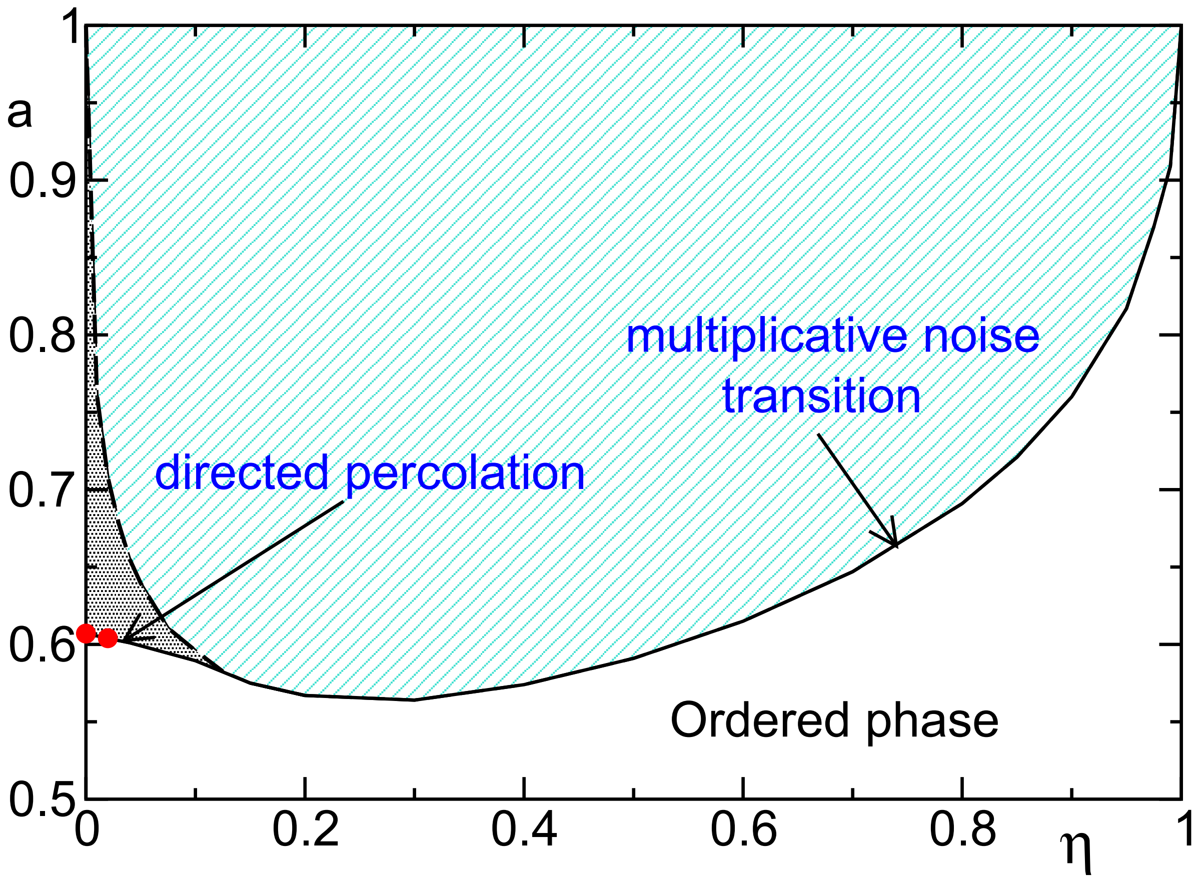
$$v_i(t+1) = (1-\varepsilon)w_i(t+1) + \frac{\varepsilon}{2}\big[w_{i-1}(t+1) + w_{i+1}(t+1))\big]$$

$$w_i(t+1) = \begin{cases} v_i(t)/\eta & \text{w.p.} \quad p = a\eta \\ a v_i(t) & \text{w.p.} \quad 1-p \end{cases} \qquad \text{if} \quad v_i(t) < \eta$$

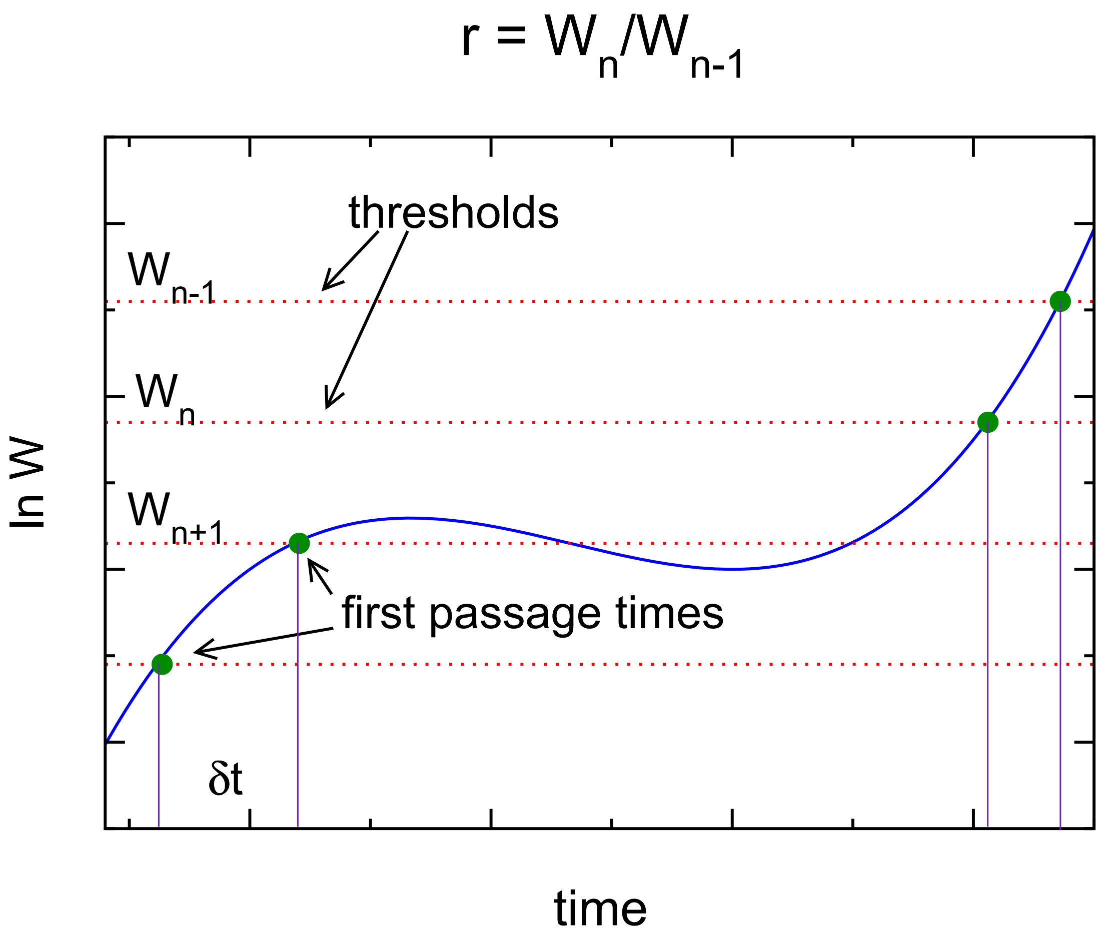
$$w_i(t+1) = \begin{cases} 1 & \text{w.p.} \quad a v_i(t) \\ a v_i(t) & \text{w.p.} \quad 1-p \end{cases} \qquad \text{if} \quad v_i(t) \geq \eta$$

F. Ginelli, R. Livi, A.P. A. Torcini, Phys. Rev. E **67** 046217 (2003)

PHASE DIAGRAM



FINITE-AMPLITUDE LYAPUNOV EXPONENT(s)

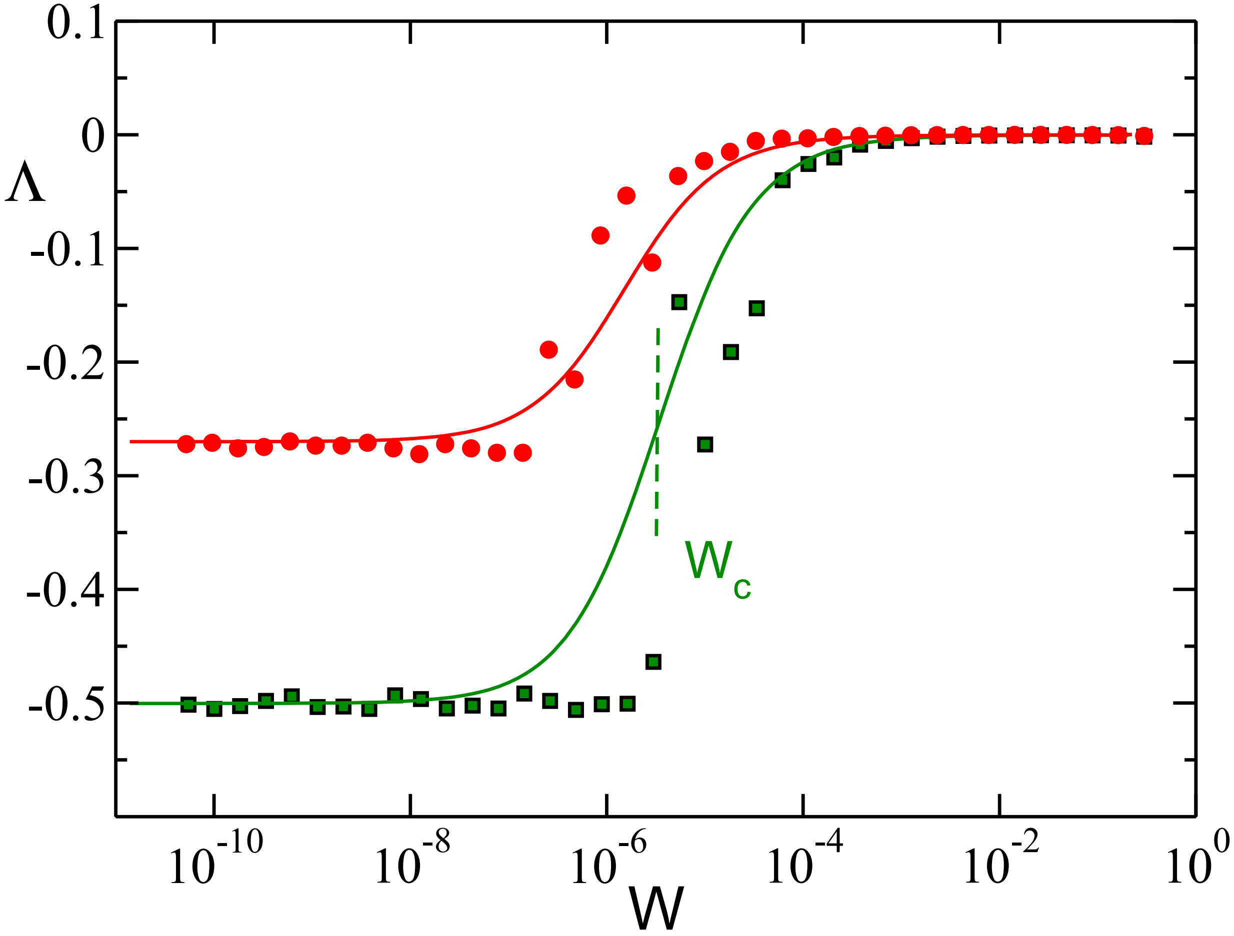


$$\Lambda(W_n) = \frac{\log r}{\tau(W_{n+1}) - \tau(W_n)}$$

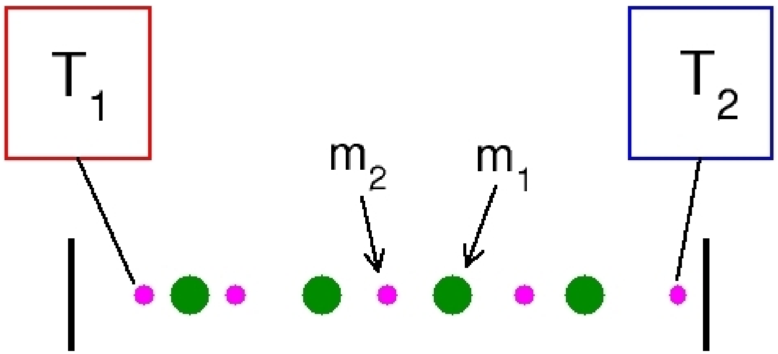
In the limit $r \to 1$

$$\Lambda(W) = \left[\frac{d\tau(W)}{d\log W}\right]^{-1}$$

Vulpiani et al. Phys. Rev. Lett. **77** 1262 (1996)


0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Λ
W
c
10
-10
10
-8
10
-6
10
-4
10
-2
10
0
W

HARD-POINT GAS



$$v_i' = v_j \pm \frac{1-r}{1+r}(v_i - v_j)$$

$r = \frac{m_1}{m_2}$ mass ratio

Evolution in tangent space equal to that in real space

$\Downarrow$

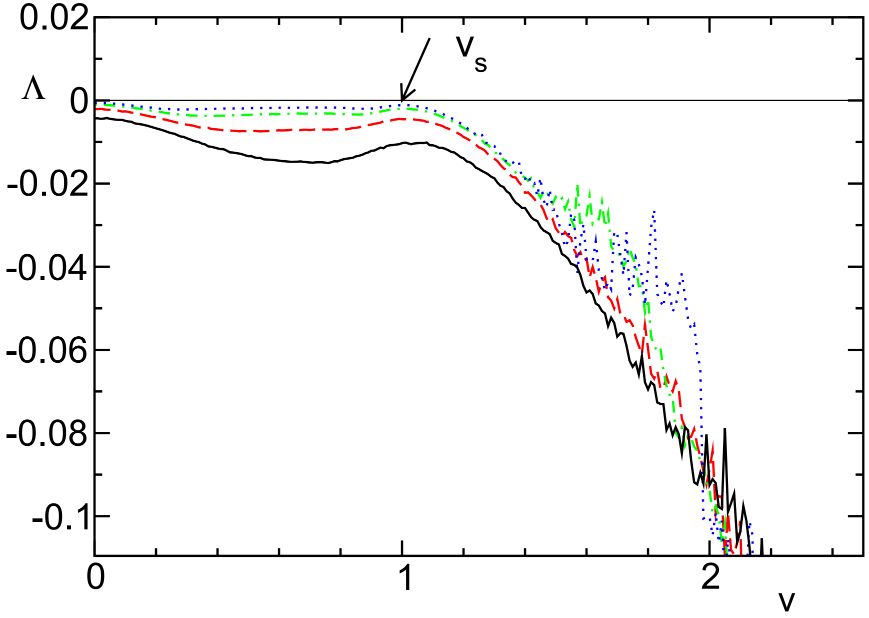
Conservation of the Euclidean norm of infin. perturbations

$\Downarrow$

NO STANDARD CHAOS

SCALING BEHAVIOUR OF HEAT CONDUCTIVITY IN THE 1D HARD POINT GAS

Convective exponents

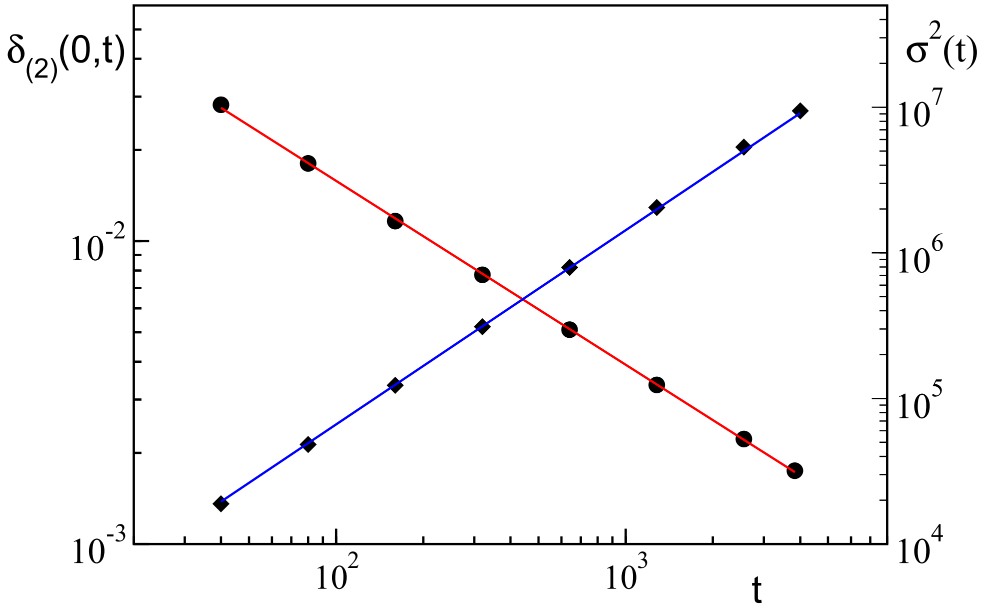


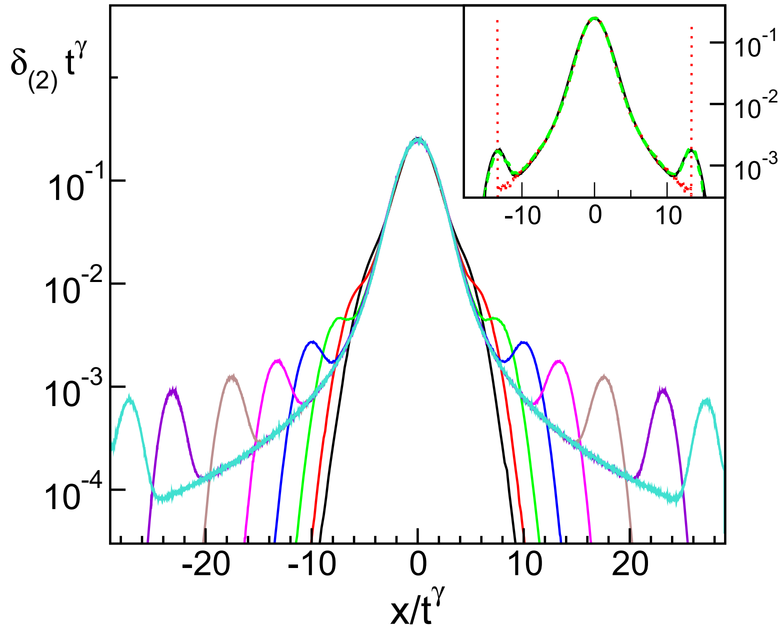
$$\delta_{(2)}(i,t) = m_i \delta v_i^2$$

$$\Lambda(v) = \lim_{t\to\infty} \frac{\log \delta_{(2)}(vt,i)}{2t}$$

$$v_s = \texttt{adiabatic velocity} = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S}$$

Convective exponents revisited





$$\delta_{(2)}(vt, i) \approx t^{-\gamma_1} F\left(i/t^{\gamma_2}\right)$$

CONNECTION WITH LEVY WALKS

A particle moves ballistically with velocity v for a time τ

$\psi(\tau) \propto \tau^{-\mu-1}$ time distribution

$\mathcal{P}(v) = (\delta(v+1) + \delta(v-1))/2$ velocity distribution

$$\mu = 1/\gamma = 5/3$$

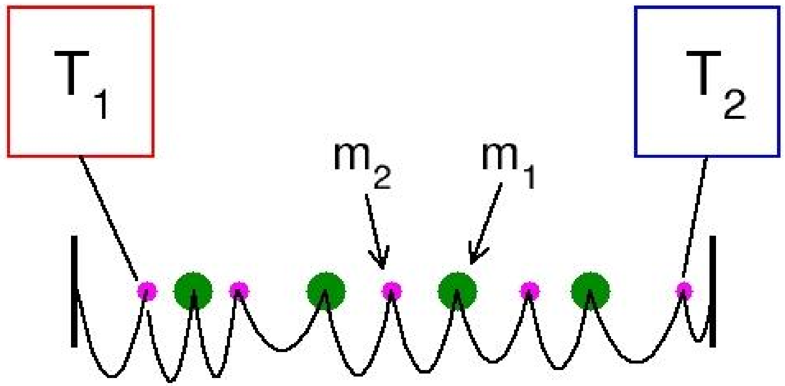
Energy diffusion

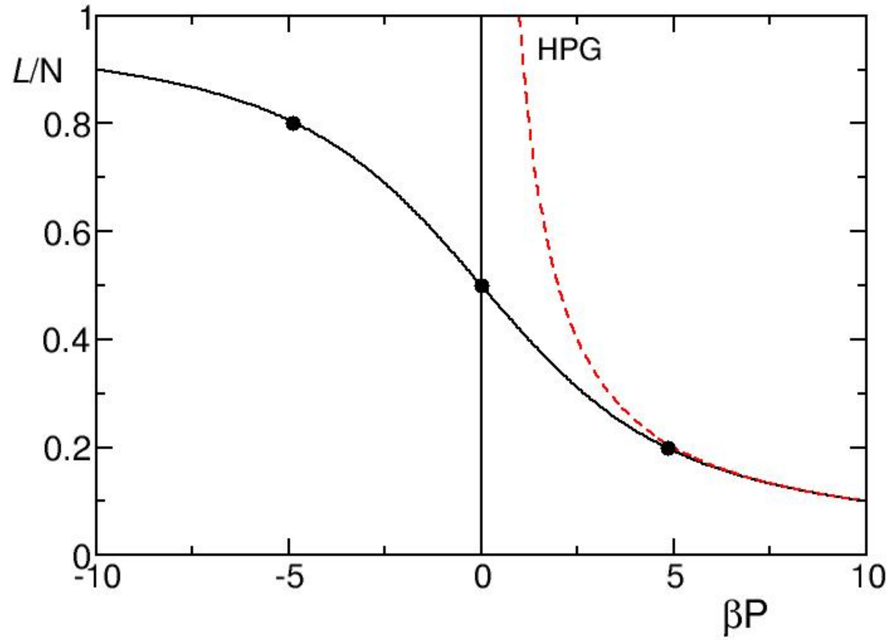
$$\Delta E_i(t) = \frac{m_i}{2}(v_i(t) + \Delta v_i(t))^2 - \frac{m_i}{2}v_i^2(t) = m_i v_i(t)\Delta v_i(t) + \frac{m_i}{2}(\Delta v_i(t))^2$$

$$\alpha = \beta - 1 = 2 - \mu = 1/3$$

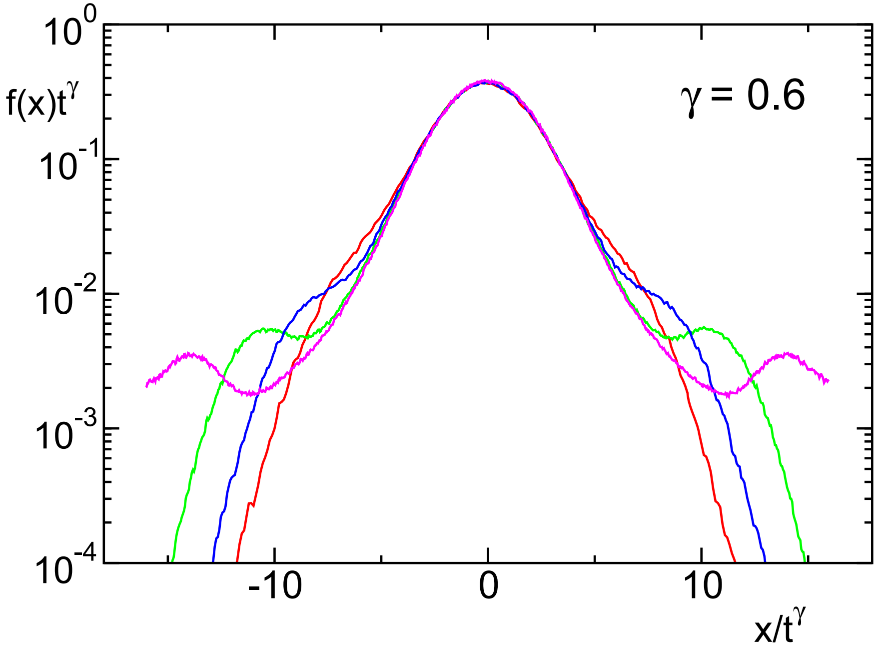
[S.Denisov, J.Klafter, M.Urbakh, PRL **91** 194301 (2003)]

THE HARD-POINT CHAIN

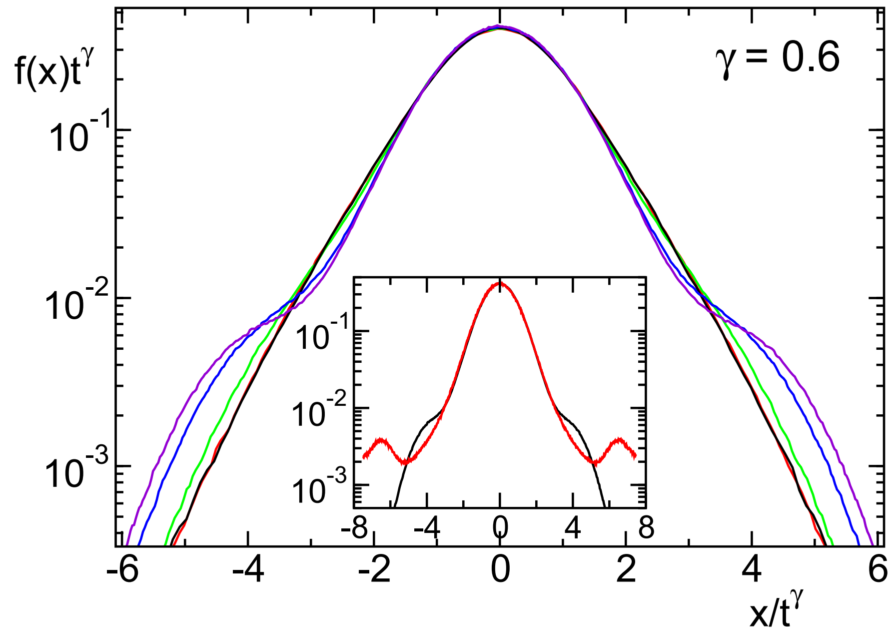




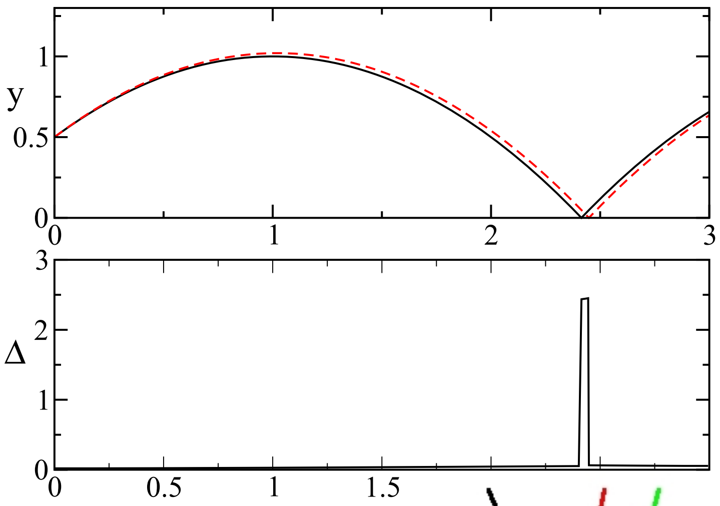
TANGENT SPACE EVOLUTION

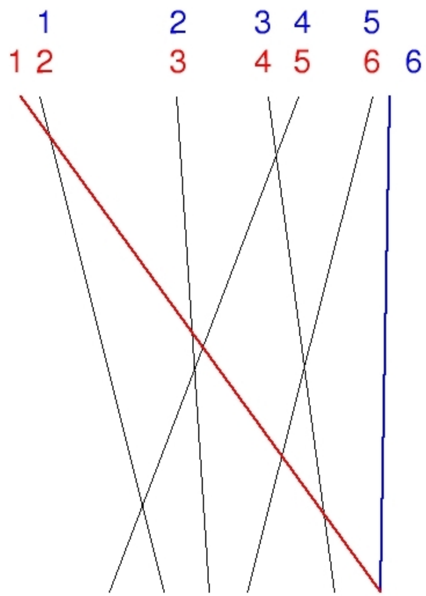


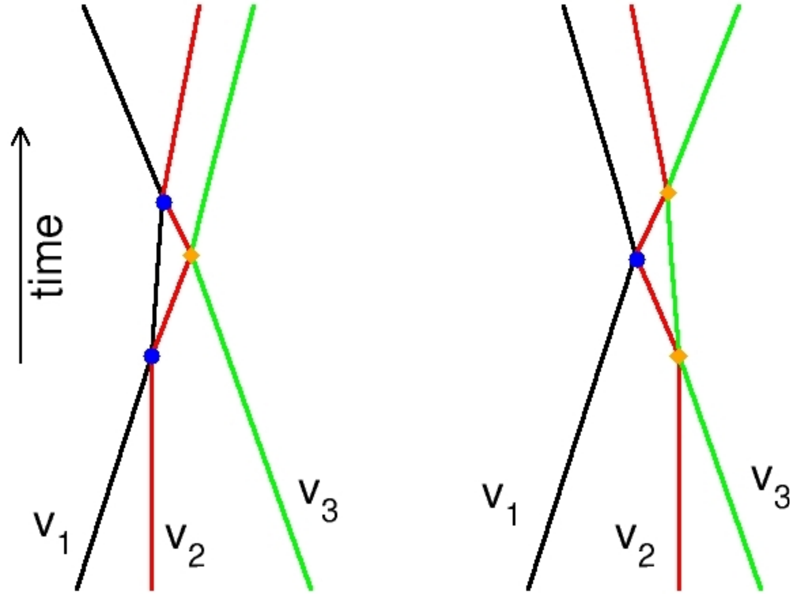
average distance 0.2

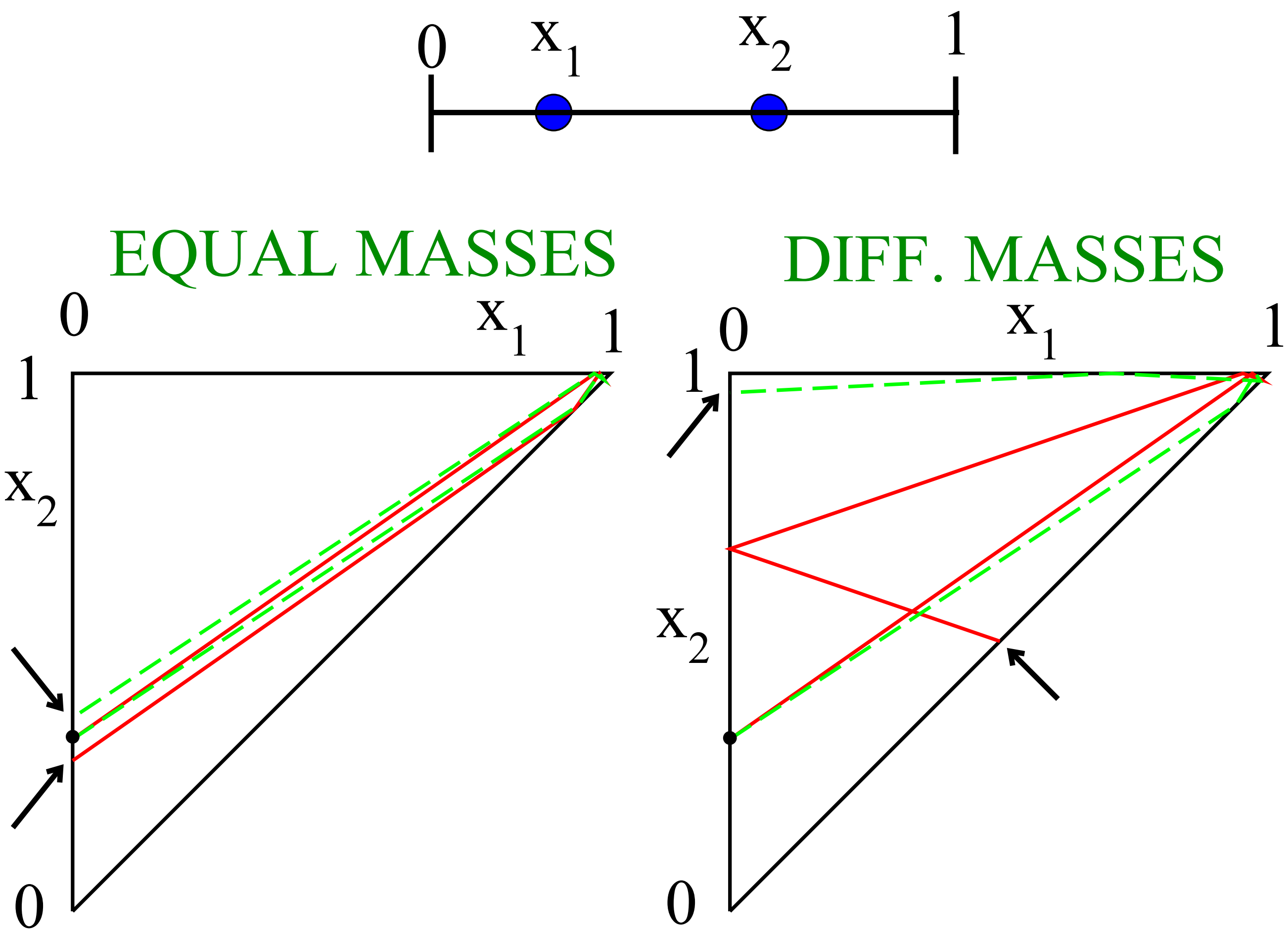


average distance 0.4


y
1
0.5
0
0
1
2
3
Δ
3
2
1
0
0
0.5
1
1.5


1 2 3 4 5
1 2 3 4 5 6 6


time
v_1 v_2 v_3
v_1 v_2 v_3


0
x1
x2
1
EQUAL MASSES
DIFF. MASSES
0
x1
1
1
x2
0
0
x1
1
1
x2
0

NETWORKS OF LEAKY INTEGRATE-AND-FIRE NEURONS

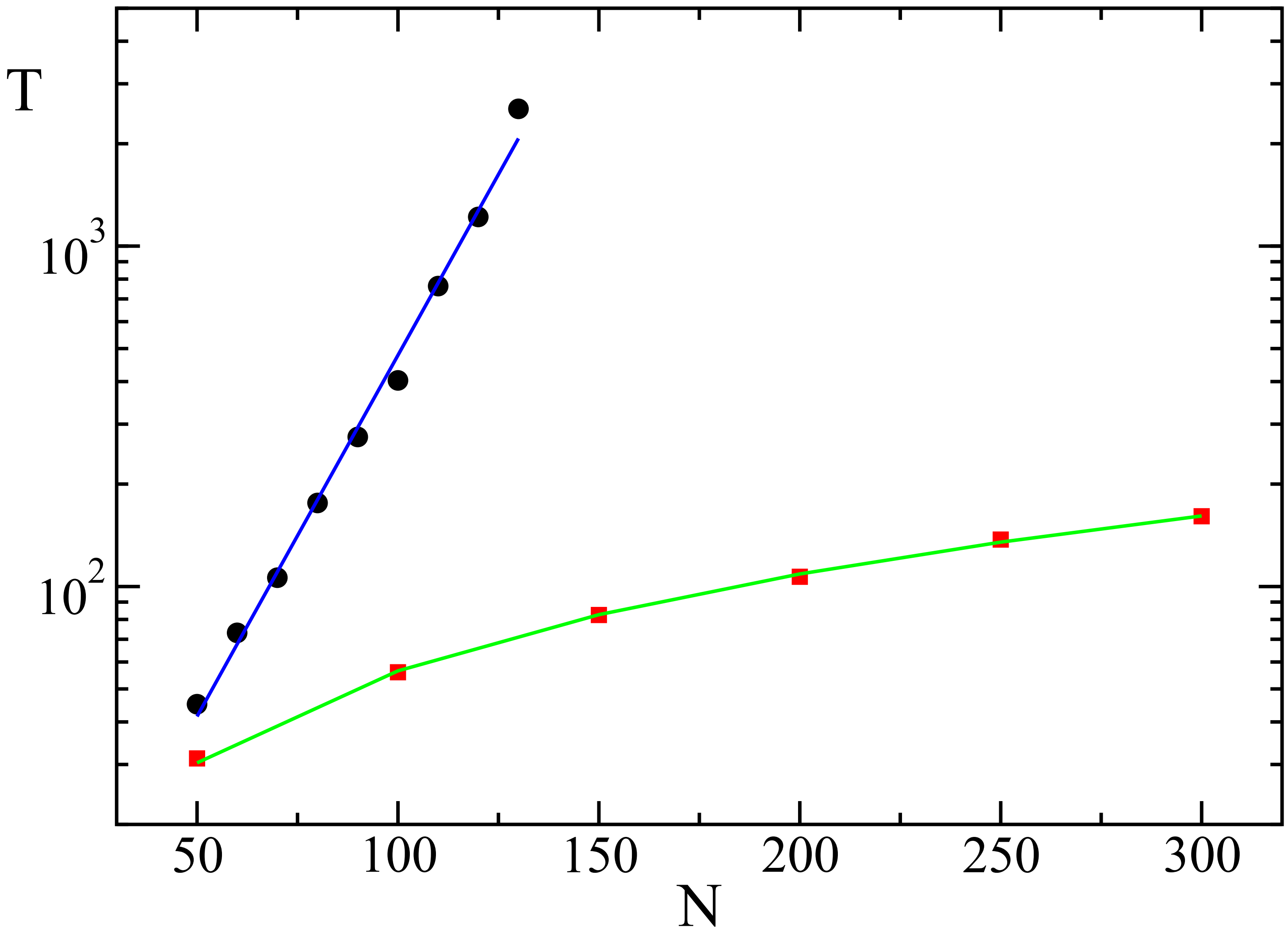
$$\tau \dot{v}_j = L_j - v_j - \frac{\tau}{N} \sum_m \left[G_{j,i(m)}(v_j + E) \right] \delta(t - t_m)$$

[D.Z. Jin, PRL **89**, 208102 (2002)]

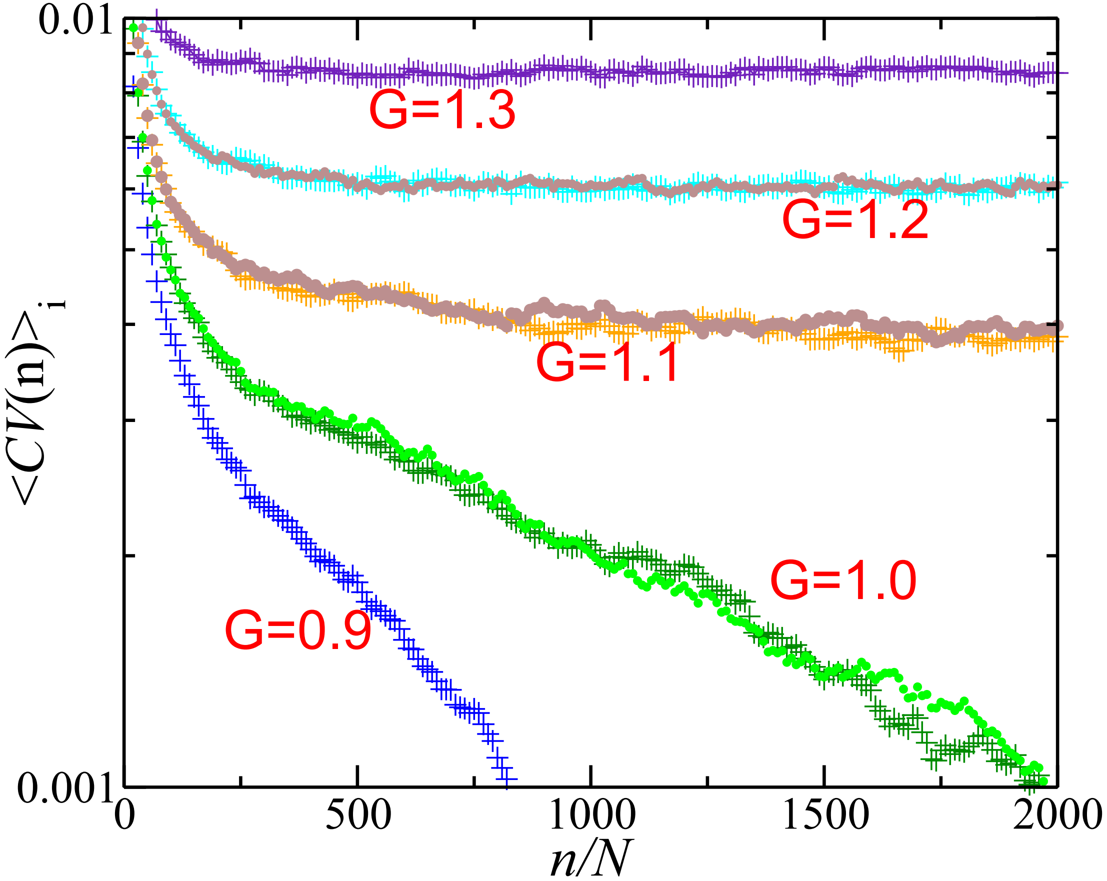
v_j, L_j	membrane, resting potential
$G_{i,j}$	inhibitory synapse conductance
t_m	time of the mth spike emitted by the $i(m)$th neuron
Θ_j	threshold of jth neuron
R_j	resetting potential

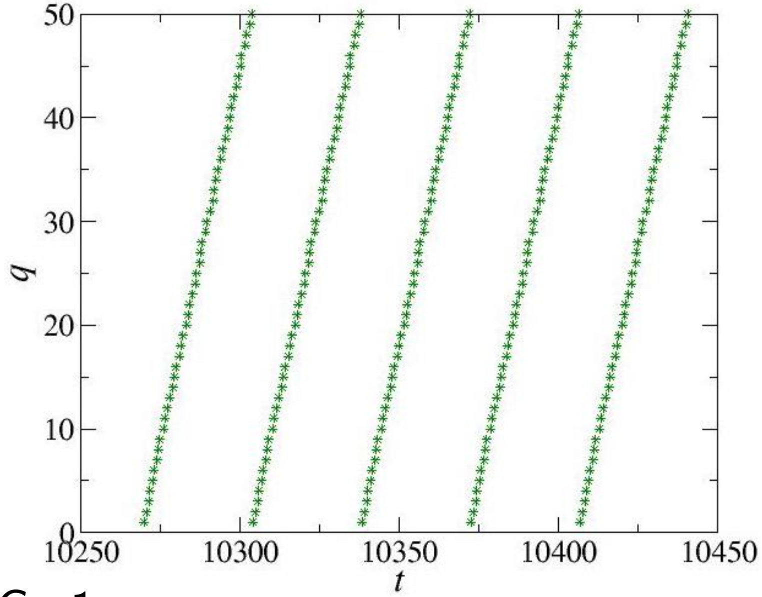
$$\dot{x}_j = a - x_j - G_0 \sum_m F_{j,i(m)}(x_j + r)\delta(t - t'_m) \qquad a = 2,\ r = 4/7$$

Dilution (a given fraction of links is cut)

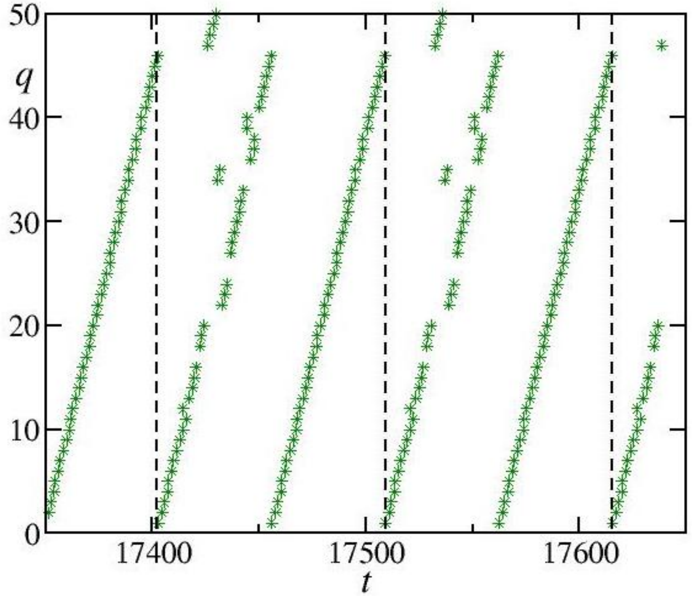


T
10^3
10^2
50
100
150
200
250
300
N

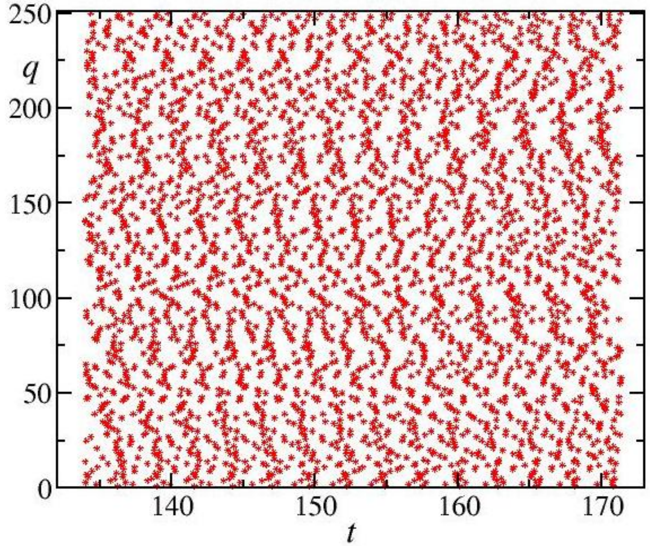

0.01
0.001
0
500
1000
1500
2000
n/N
<CV(n)>i
G=1.3
G=1.2
G=1.1
G=1.0
G=0.9


q
t
0
10
20
30
40
50
10250
10300
10350
10400
10450

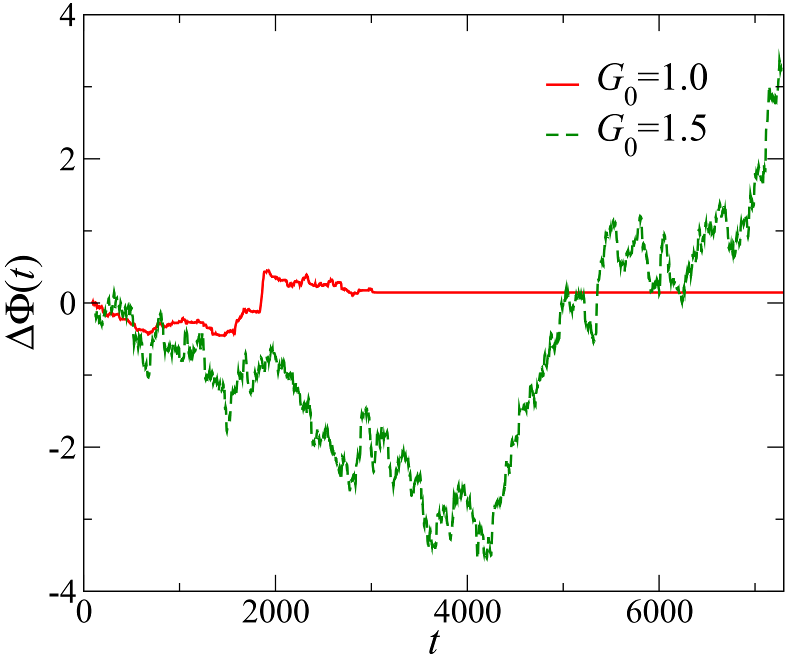
G=1


q
t
0
10
20
30
40
50
17400
17500
17600

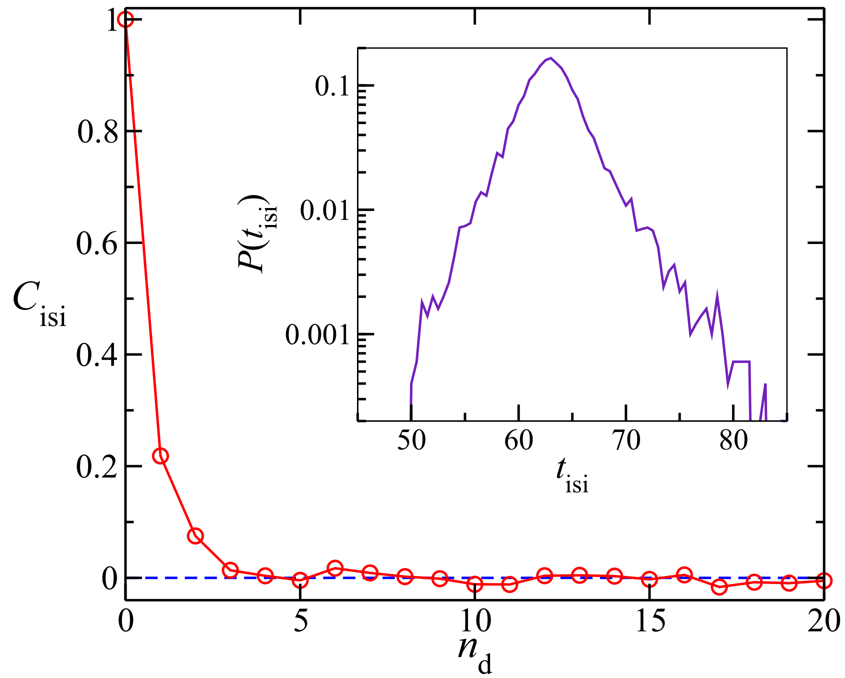
G=2.


q
t
0
50
100
150
200
250
140
150
160
170

G=1.7

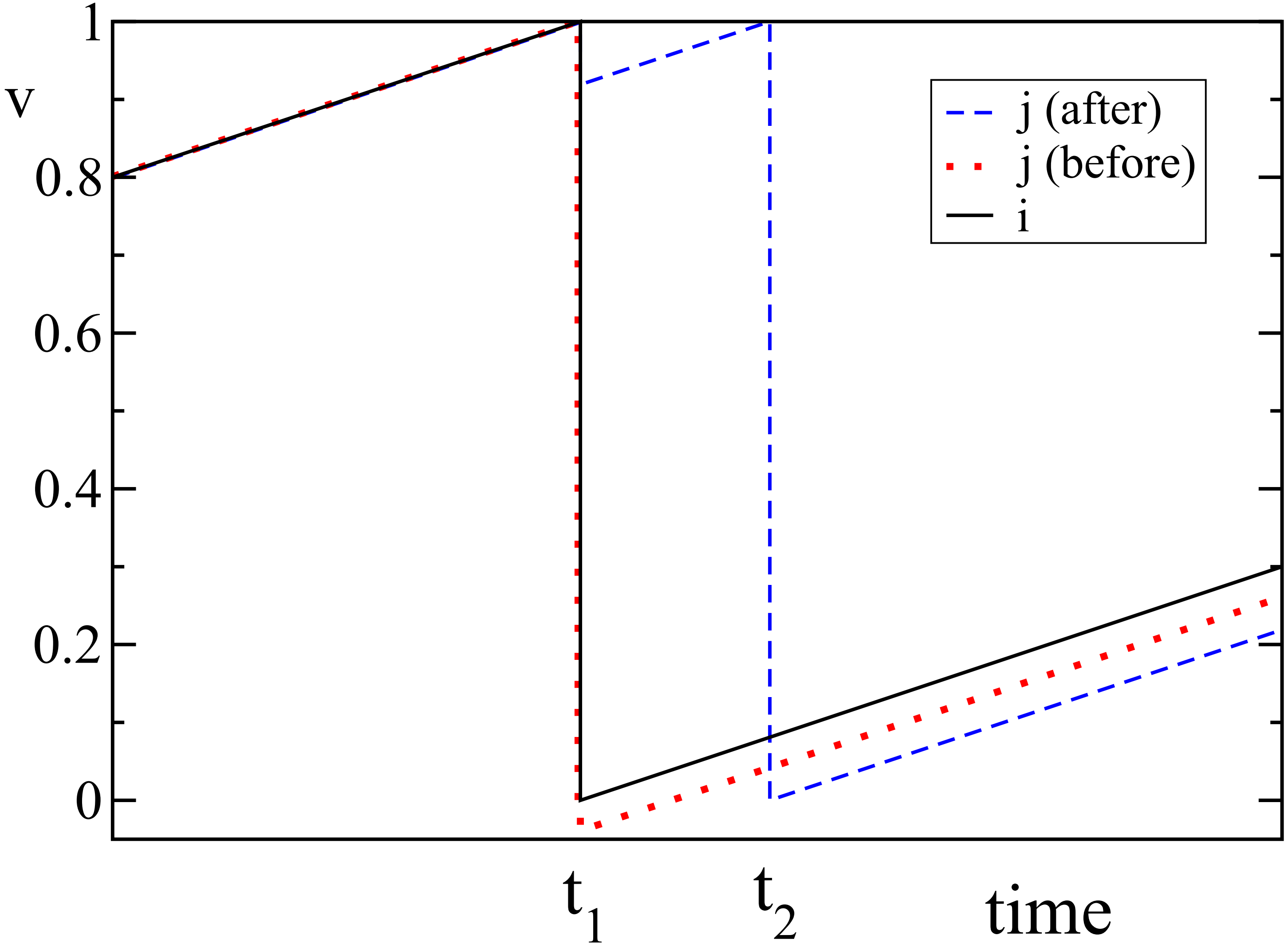


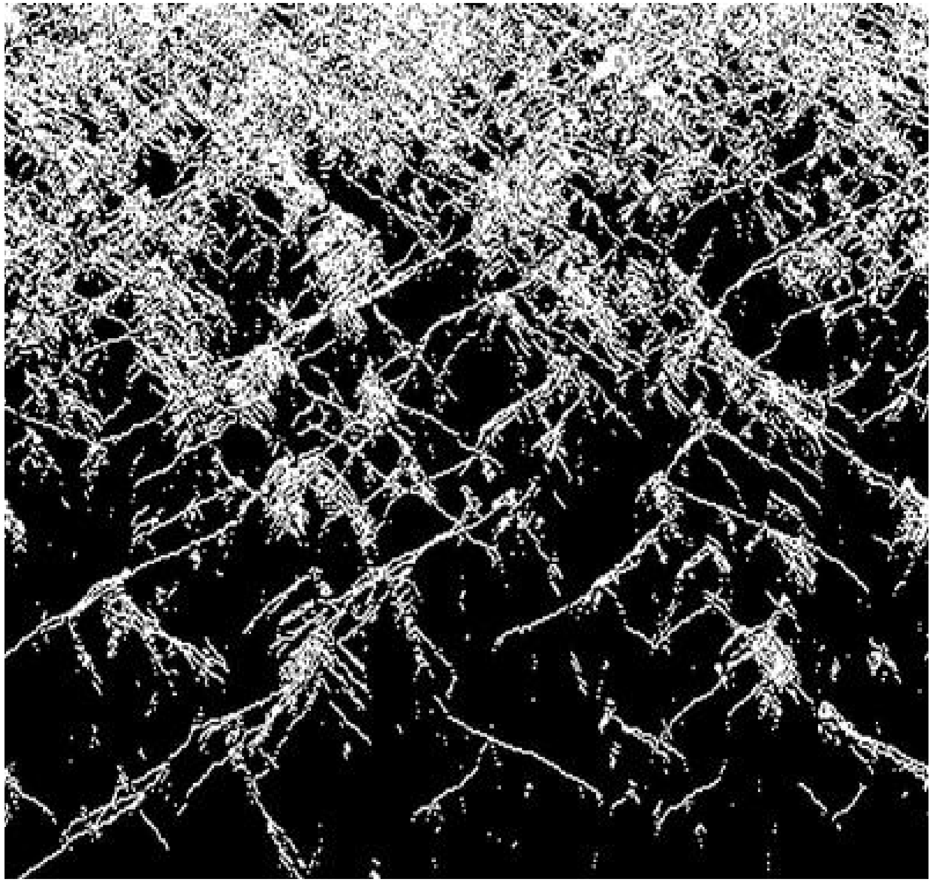
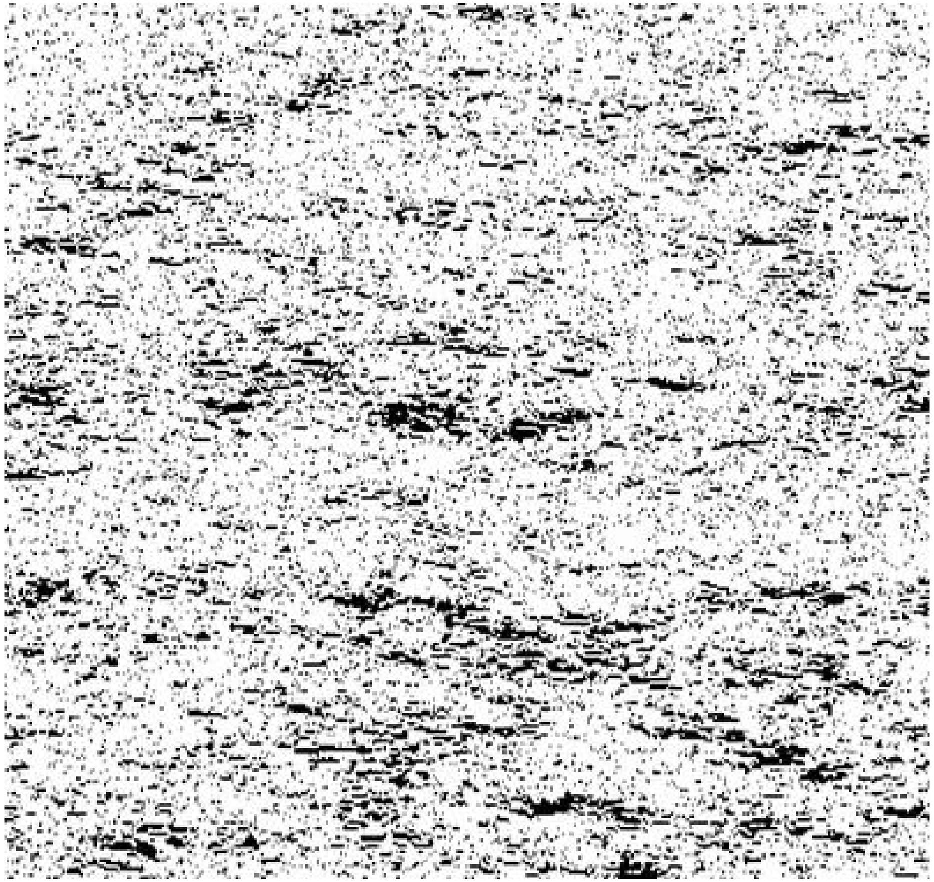
Integrated phase shift



ISI-correlation function

i-th neuron afftects j-th / not vice-versa



Stable chaos

A.P., Alessandro Torcini arXiv:0902.2545v1 [nlin]

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