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Pure adhesion in friction

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Pure Adhesion in Friction

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Outline

Friction

Friction Coefficient

- Macroscopic friction coefficient μ
- Amontons law 1699
- Defined as ratio between the frictional force and the normal force

Idea

Can friction be (partly) explained by a simple adhesive model?

Friction

Adhesion

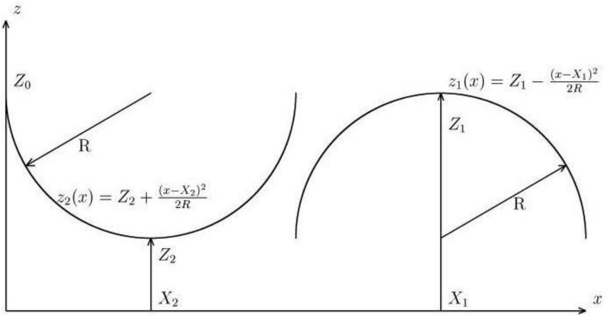
- Attractive force between two bodies
- Van der Waals force
- Lennard-Jones Potential
- Surface energy density γ_{12}

JKR Theory

- Hertz contact theory for contact between rigid surface and elastic sphere 1882
- Johnson, Kendall and Roberts 1971
- Improvement by inclusion of surface effects
- Surface energy $\gamma_{12} \to 0$ gives back Hertz theory

The Geometric Model

Geometrical Set up



Round Asperities

Parabolas

- Round spheres approximated to first order as parabolas

$$
\begin{aligned}
z_1(x) &= Z_1 - \frac{(x - X_1)^2}{2R} \\
z_2(x) &= Z_2 + \frac{(x - X_2)^2}{2R}
\end{aligned}
$$

Indentation Depth

Indentation

- Difference in z coordinates
- Indentation

$$\hat{d} \equiv z_1\left(\frac{X_1+X_2}{2}\right) - z_2\left(\frac{X_1+X_2}{2}\right) = Z_1 - Z_2 - \frac{(X_1-X_2)^2}{4R}$$

- Actual indentation: Multiply by cosine of angle
- Approximated by unity

Uniform Distribution of Asperities

- Uniform distribution of asperities along x axis
- Macroscopic length l

$$\Psi_1(X_1) = \frac{1}{2l}, \quad \Psi_2(X_2) = \frac{1}{2l}$$

- Stochastic averaging

$$\langle g \rangle_x = \int \mathrm{d}X_0 \, \frac{1}{2l} \, g(X_0)$$

with

$$X_0 \equiv X_1 - X_2$$

Normal Distribution of Heights

- Gaussian distribution of heights
- Standard deviation L, macroscopic distance Z_0

$$\Phi_1(Z_1) = \frac{1}{\sqrt{2\pi}L}\mathrm{e}^{-\frac{Z_1^2}{2L^2}}, \quad \Phi_2(Z_2) = \frac{1}{\sqrt{2\pi}L}\mathrm{e}^{-\frac{(Z_2-Z_0)^2}{2L^2}}$$

- Inspired by Greenwood and Williamson 1966
- Stochastic averaging

$$\langle g \rangle_z = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\gamma}{\sqrt{4\pi}} \frac{d_c}{L} e^{-\frac{d_c^2}{4L^2}(\gamma+z_0)^2} g(\gamma)$$

$$\gamma \equiv \frac{Z_1 - Z_2}{d_c}, \quad z_0 \equiv \frac{Z_0}{d_c}$$

JKR Theory

Adhesion Theories

- Models for adhesion
- Johnson, Kendall and Roberts 1971
- Bradley 1932
- Derjaguin, Müller and Toporov 1975
- Tabor 1976
- Potential elastic energy
- Surface energy

Formulation

Content

- Force F due to elastic deformation and adhesion
- Indentation depth d
- Contact radius a

$$
\begin{aligned}
F &= \frac{4E^{*}a^{3}}{3R} - \sqrt{8\pi\gamma_{12}a^{3}E^{*}} \\
d &= \frac{a^{2}}{R} - \sqrt{\frac{2\pi\gamma_{12}a}{E^{*}}}
\end{aligned}
$$

JKR Corrections

- Effective Young's modulus E^* if two different materials interact

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

ν_i - Poisson number

- Radius R: harmonic mean if two round surfaces adhere

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Critical Values

Critical Values for Detachment

- Maximal negative force F_c
- Associated contact radius a_c
- "Indentation" depth d_c - length of neck

$$F_c = -\frac{3}{2}\pi\gamma_{12}R$$

$$a_c = \sqrt[3]{\frac{9\pi\gamma_{12}R^2}{8E^*}}$$

$$d_c = -\sqrt[3]{\frac{3\pi^2\gamma_{12}^2 R}{64E^{*2}}}$$

Dimensionless Quantities

- Dimensionless relations

$$\tilde{a} \equiv \frac{a}{|a_c|}$$

$$\tilde{F} \equiv \frac{F}{|F_c|} = \tilde{a}^3 - 2\,\tilde{a}^{\frac{3}{2}}$$

$$\tilde{d} \equiv \frac{d}{|d_c|} = 3\,\tilde{a}^2 - 4\,\tilde{a}^{\frac{1}{2}}$$

- Implicit F - d - relation: Solvable by Cardano's formula
- Instead: Approximation by power law

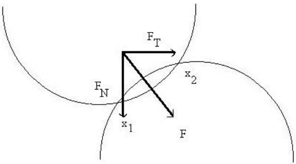
$$\tilde{F} \approx \alpha(\tilde{d}+1)^{\beta} - 1, \quad -1 \leq \tilde{d} \leq 10$$

$$\alpha \approx \frac{1}{9}, \quad \beta \approx \frac{5}{3}$$

Decomposition of Forces

Decomposition

- Adhesive force F acts along the line connecting the sphere centers
- Decomposition necessary
- Small angles - first order approximation
- Normal force $F_N = F$
- Friction force $F_T = F \frac{X_0}{2R}$



Contact Region

First Contact

- First Contact
- Intersection of parabolas

$$z_1 = z_2 \Rightarrow Z_1 - \frac{(x - X_1)^2}{2R} = Z_2 + \frac{(x - X_2)^2}{2R}$$

$$x_{1,2} = \frac{X_1 + X_2}{2} \pm \sqrt{R(Z_1 - Z_2) - \frac{(X_1 - X_2)^2}{4}}$$

- Condition for just one intersection point

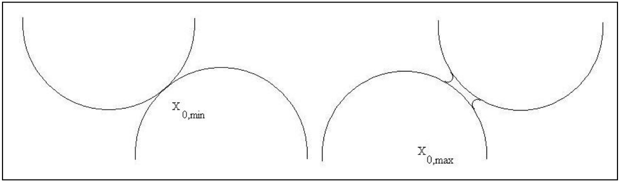
$$x = \frac{X_1 + X_2}{2}, \qquad X_1 - X_2 = \pm 2\sqrt{R(Z_1 - Z_2)}$$

Boundaries

Contact

- First contact at intersection of parabolas
- Breaking of contact delayed by adhesion
- Scope of interaction extended by critical indentation d_c
- Boundary

$$
\begin{aligned}
X_{0,min} &= -2\sqrt{R\,(Z_1 - Z_2)} \\
X_{0,max} &= 2\sqrt{R\,(Z_1 - Z_2) + Rd_c}
\end{aligned}
$$



Horizontal Mean

- Normal and tangential forces are horizontally averaged quantities

$$\langle \tilde{F}_N \rangle_x = \frac{1}{2l} \int_{X_{0,min}}^{X_{0,max}} \mathrm{d}X_0 \left[\alpha \left(1 + \frac{1}{d_c} \left(Z_1 - Z_2 - \frac{X_0^2}{4R} \right) \right)^{\beta} - 1 \right]$$

$$\langle \tilde{F}_T \rangle_x = \frac{1}{2l} \int_{X_{0,min}}^{X_{0,max}} \mathrm{d}X_0 \left[\alpha \left(1 + \frac{1}{d_c} \left(Z_1 - Z_2 - \frac{X_0^2}{4R} \right) \right)^{\beta} - 1 \right] \frac{X_0}{2R}$$

Frictional force

Horizontally averaged frictional force

- Integration of the frictional force F_T easily performed

$$\langle \tilde{F}_T \rangle_x \quad = \quad \frac{1}{2} \frac{d_c}{l} \left(\frac{\alpha}{1+\beta} - 1 \right)$$

- Frictional force proportional to d_c but independent of R

Normal Force

Horizontally averaged normal force

- Normal force

$$\langle \tilde{F}_N \rangle_x = \sqrt{\frac{R}{d_c}\frac{d_c}{l}} \left[\frac{\alpha}{2}(1+\gamma)^{\beta+\frac{1}{2}} \left[B\left(\frac{1}{2},\beta+1\right) + B\left(\frac{\gamma}{\gamma+1},\frac{1}{2},\beta+1\right) \right] - \sqrt{\gamma} - \sqrt{\gamma+1} \right]$$

- (Incomplete) Beta function

$$B(z,a,b) \equiv \int_0^z t^{a-1}(1-t)^{b-1}\,\mathrm{d}t$$

$$B(a,b) \equiv B(1,a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Height Mean

Averaging of frictional force

- Stochastic mean over the height coordinates Z_i
- Frictional force

$$\langle\langle \tilde{F}_T \rangle\rangle = \int_0^\infty \frac{\mathrm{d}\gamma}{\sqrt{4\pi}} \frac{d_c}{L} \mathrm{e}^{-\frac{d_c^2}{4L^2}(\gamma+z_0)^2} \frac{d_c}{2l} \left(\frac{\alpha}{1+\beta} - 1 \right)$$
$$= \frac{F_c}{4} \frac{d_c}{l} \left(\frac{\alpha}{1+\beta} - 1 \right) \operatorname{erfc}\left[\frac{d_c z_0}{2L} \right]$$

- Complementary error function *erfc*

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \mathrm{d}t \, \mathrm{e}^{-t^2}$$

- Frictional force depends linearly on indentation depth d_c

Height Mean

Averaging of normal force

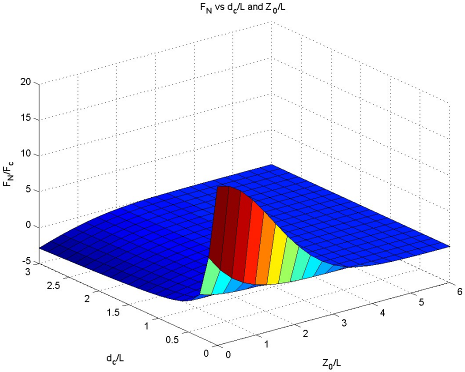
- Normal force

$$\langle\langle \tilde{F}_N \rangle\rangle = -\frac{\sqrt{d_c R}}{l} \int_0^\infty \frac{\mathrm{d}\gamma}{\sqrt{4\pi}} \frac{d_c}{L} \, \mathrm{e}^{-\frac{d_c^2}{4L^2}(\gamma+z_0)^2} \left[\sqrt{\gamma+1} + \sqrt{\gamma} - \frac{\alpha}{2}(1+\gamma)^{\beta+\frac{1}{2}} \left[B\left(\frac{1}{2}, \beta+1\right) + B\left(\frac{\gamma}{\gamma+1}, \frac{1}{2}, \beta+1\right) \right] \right]$$

- No closed analytic expression
- Numerically evaluated

Normal Force

Normal Force $\tilde{F}_N$ as function of the normalized macroscopic distance Z_0/L and the normalized indentation depth d_c/L



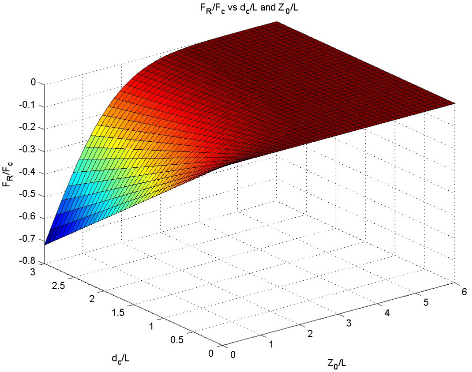
Normal Force

Normal force

- Normal force turns from negative to positive
- Negative: Adhesive effects prevail
- Positive: Compression takes over

Frictional force

Normalized friction force $\tilde{F}_T$ as function of the normalized macroscopic distance Z_0/L and the normalized indentation depth d_c/L



Friction Coefficient

- Macroscopic friction coefficient as statistical mean

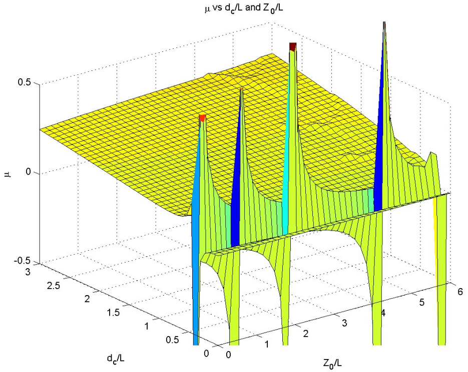
$$\mu = \frac{\langle\langle F_T \rangle\rangle}{\langle\langle F_N \rangle\rangle}$$

$$\mu = \frac{\frac{\sqrt{\pi}}{2}\sqrt{\frac{L^2}{Rd_c}}\left(1 - \frac{\alpha}{1+\beta}\right)\mathrm{erfc}\left[\frac{Z_0}{2L}\right]}{\int\limits_0^\infty \mathrm{d}\gamma\, \mathrm{e}^{-\left(\frac{d_c\gamma+Z_0}{2L}\right)^2}\left[\sqrt{1+\gamma}+\sqrt{\gamma}-\frac{\alpha}{2}(1+\gamma)^{\beta+\frac{1}{2}}\left[B\left(\frac{1}{2},\beta+1\right)+B\left(\frac{\gamma}{\gamma+1},\frac{1}{2},\beta+1\right)\right]\right]}$$

- Numerically evaluated integral

Diagram of Friction Coefficient

Friction coefficient μ as function of the normalized macroscopic distance Z_0/L and the normalized critical indentation depth d_c/L

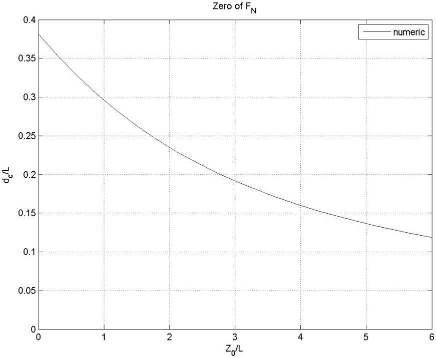


Discontinuity

- Discontinuity in friction coefficient
- Reason: Zero of normal force
- Not observed in experiments

Zero of Normal Force

Parametric plot of the values of the normalized macroscopic distance Z_0/L and the critical indentation depth d_c/L for vanishing normal force



Length Scales

Hierarchy

1. Macroscopic length l
2. Macroscopic separation Z_0
3. Curvature radius of asperities R
4. Stochastic length scale L
5. Indentation depth d_c

Conclusions and Outlook

- Too simplistic model
- Discontinuity in friction coefficient
- Vanishing of normal force at finite frictional force
- Inclusion of different physical effects necessary

Thanks

Thank you for your attention!