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ICTP/FANAS Conference on trends in Nanotribology

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Contact mechanics with applications

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Heat transfer: contact resistance

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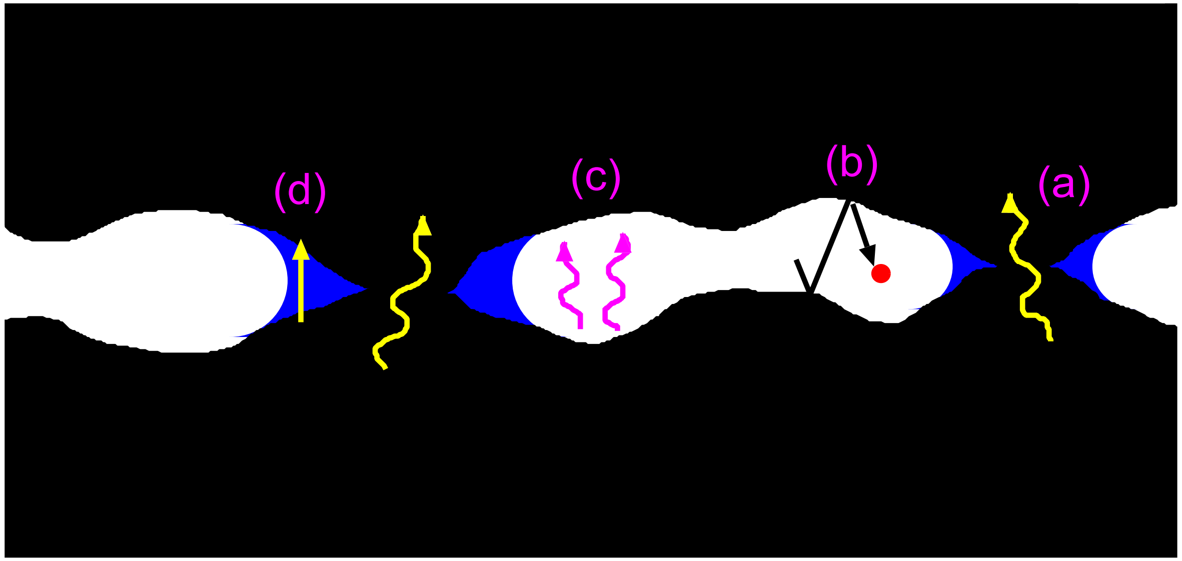
Research Center Jülich

www.multiscaleconsulting.com

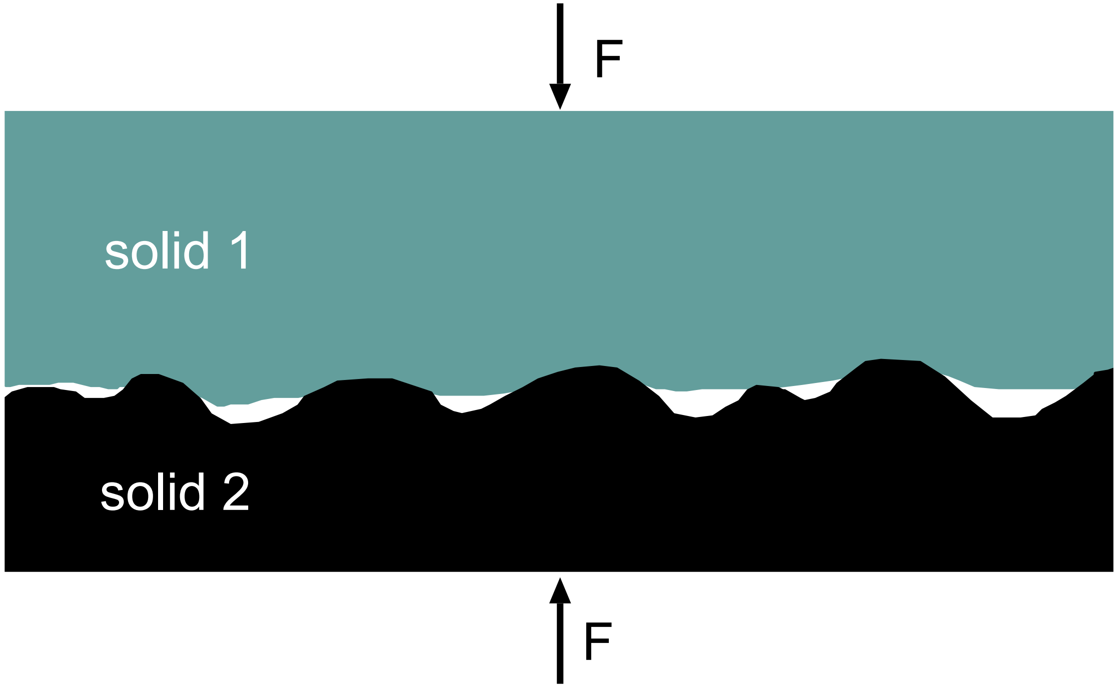
- Contact mechanics
- Heat transfer

Heat transfer processes



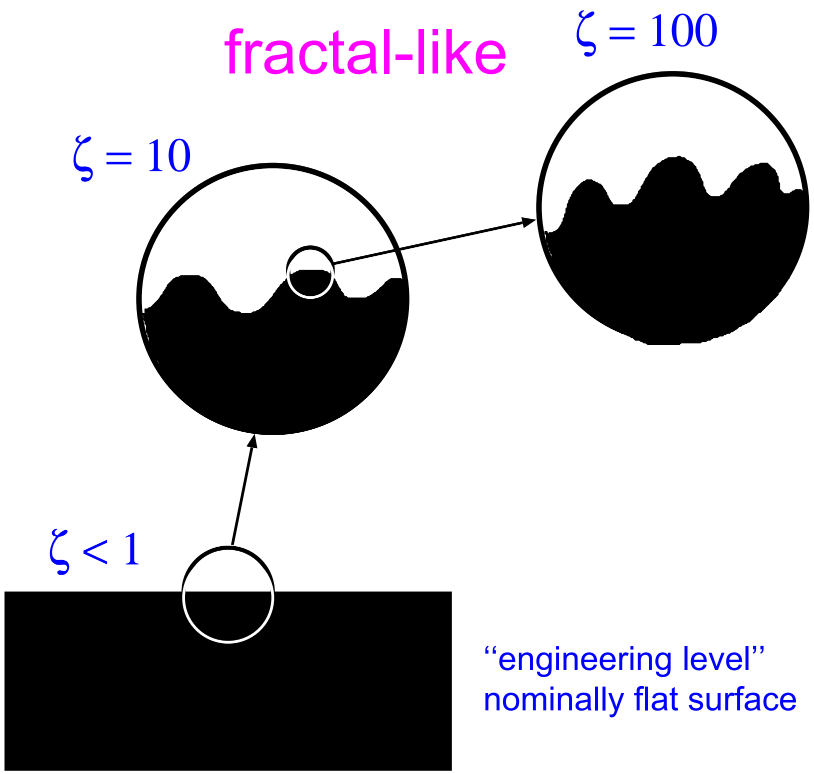
(a) Heat transfer via the area of real contact
(b) Heat transfer via surrounding gas (or liquid)
(c) Radiative (electromagnetic wave) heat transfer
(d) Heat diffusion in fluid capillary bridges

Contact mechanics for rough surfaces



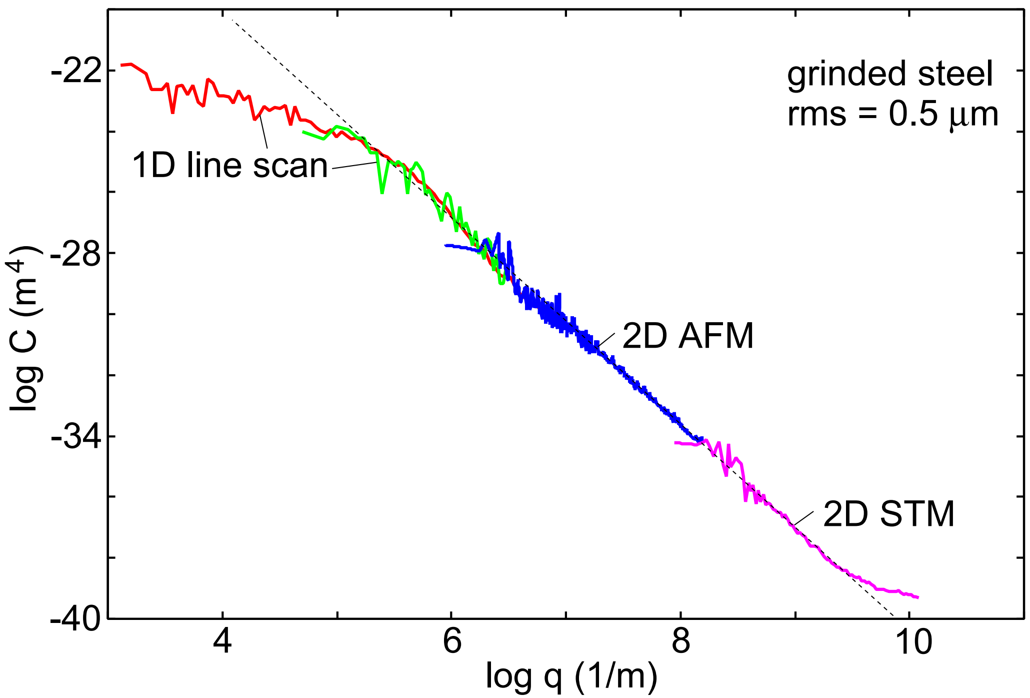
- Contact area?
- Stress distribution?
- Interfacial separation?

Roughness on many length scales



When a nominally flat surface is studied at increasing magnification ζ, new surface roughness is detected.

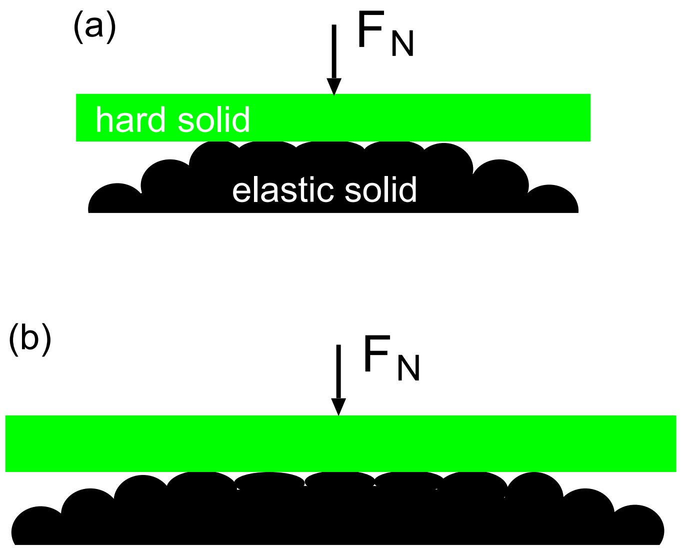
Surface roughness power spectrum $C(q)$



Experiment: A. Wohlers, IFAS, RWTH Aachen

$$h(\mathbf{q}) = \int d^2x \; h(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}$$
$$C(q) = |h(\mathbf{q})|^2$$

Role of long-ranged elastic deformation



Asperity contact mechanics models neglect the long-range elastic coupling

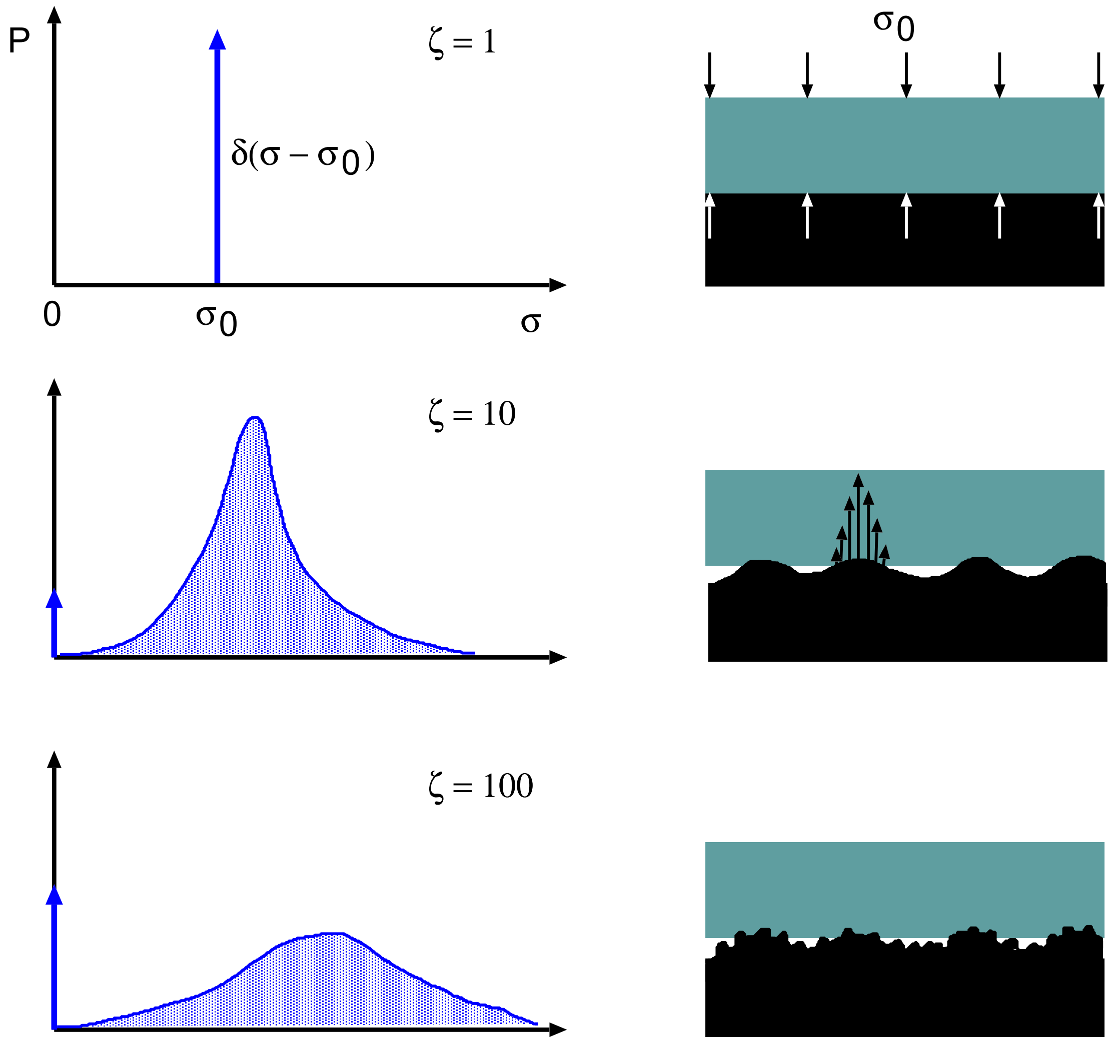
Greenwood–Williamson theory; theory of Thomas et al; theory of Bhushan et al, theory of Popov et al, ...

$$p = a \exp\left(-b\bar{u}^2\right)$$

Correct result:

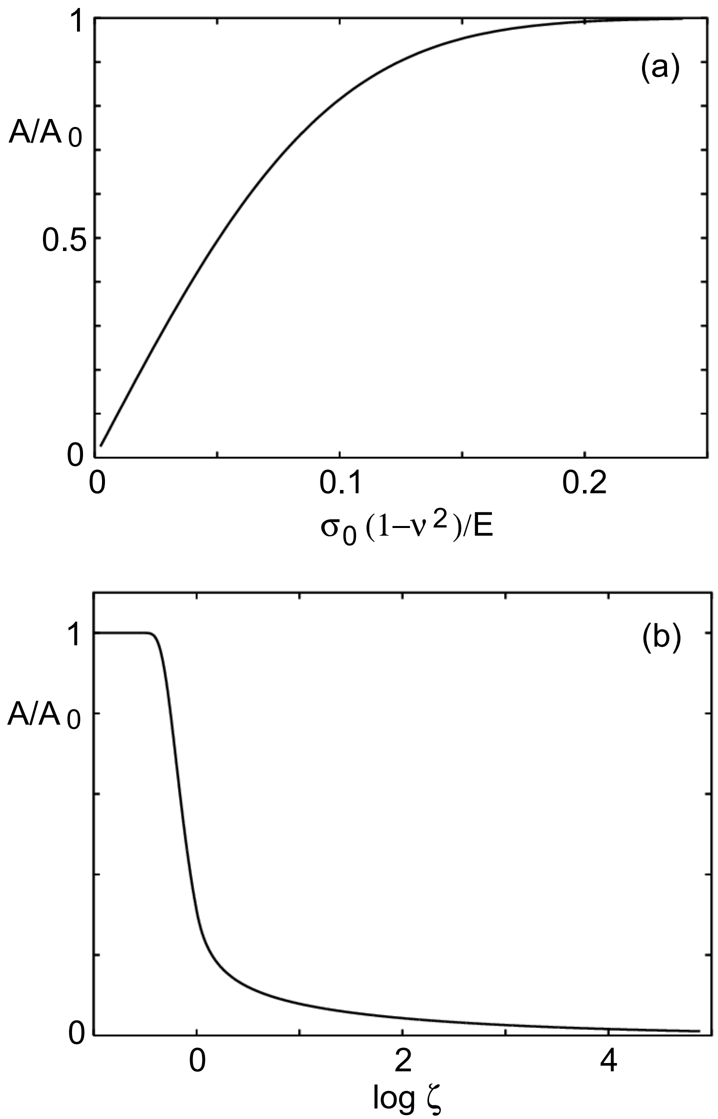
$$p = a \exp(-b\bar{u})$$

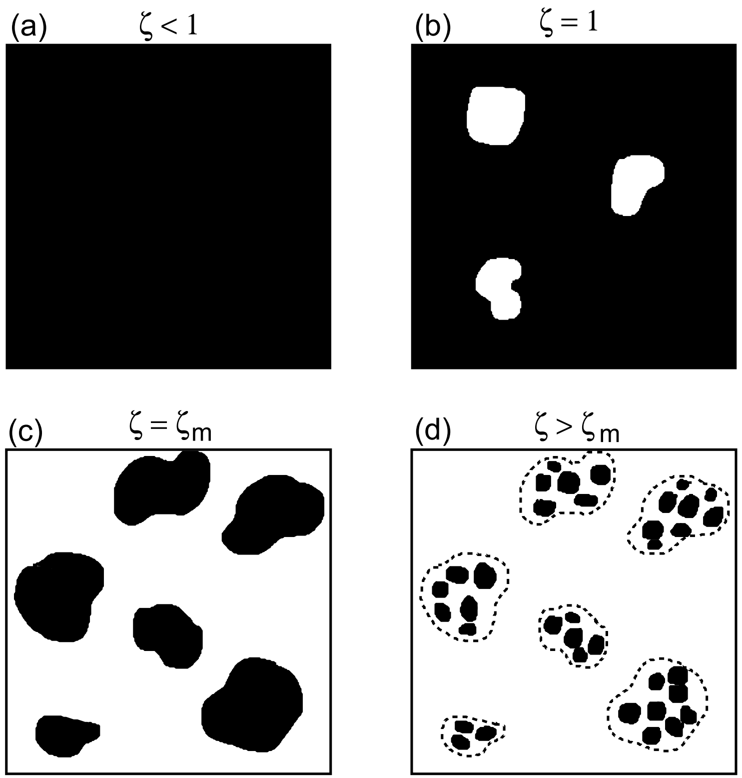
Interfacial stress distribution.



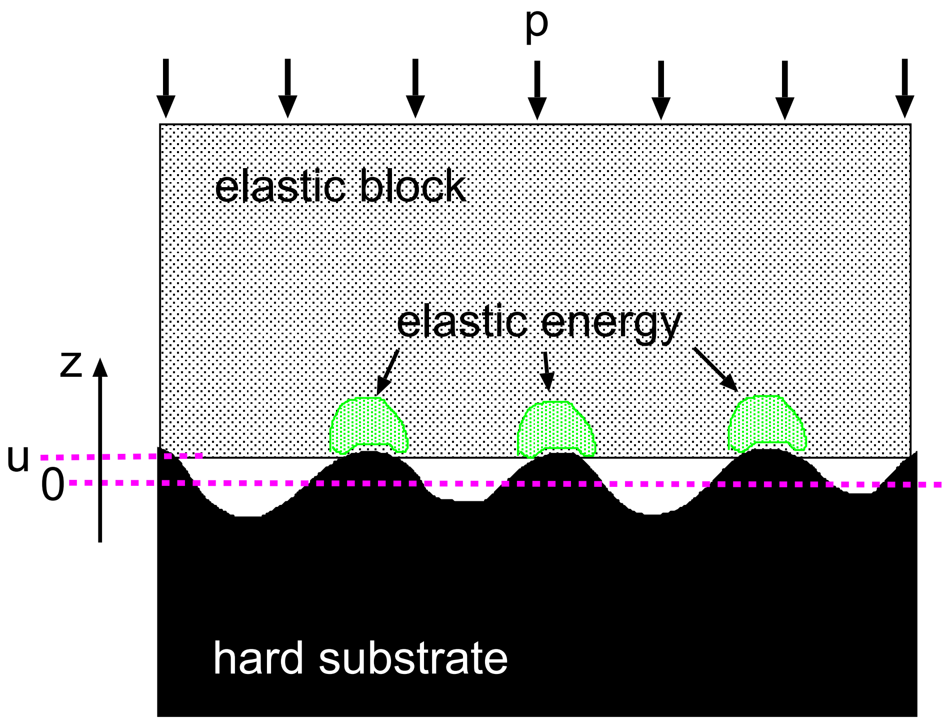
The stress distribution $P(\sigma, \zeta)$ for different magnifications ζ.

Contact area as a function of load and magnific.



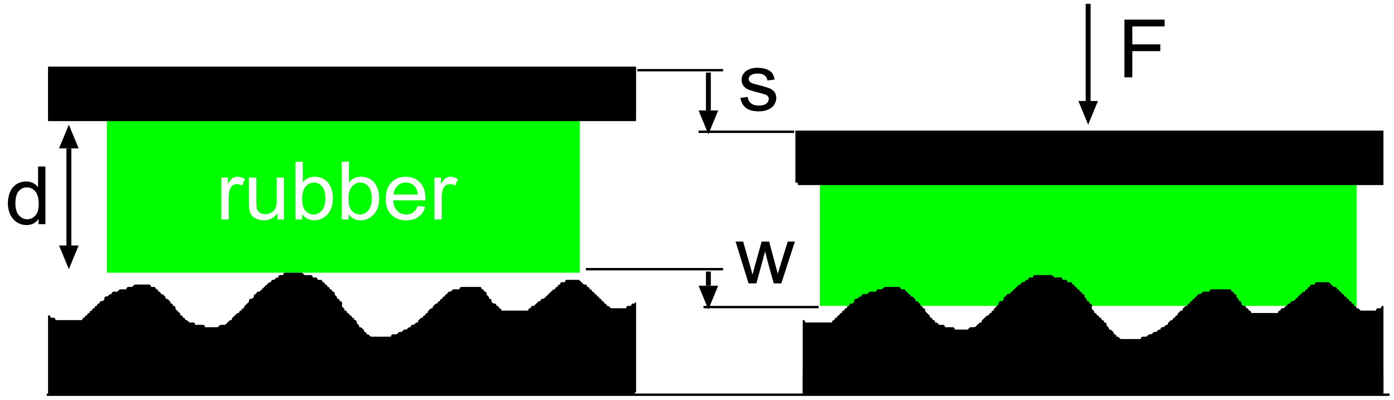


Interfacial separation: theory



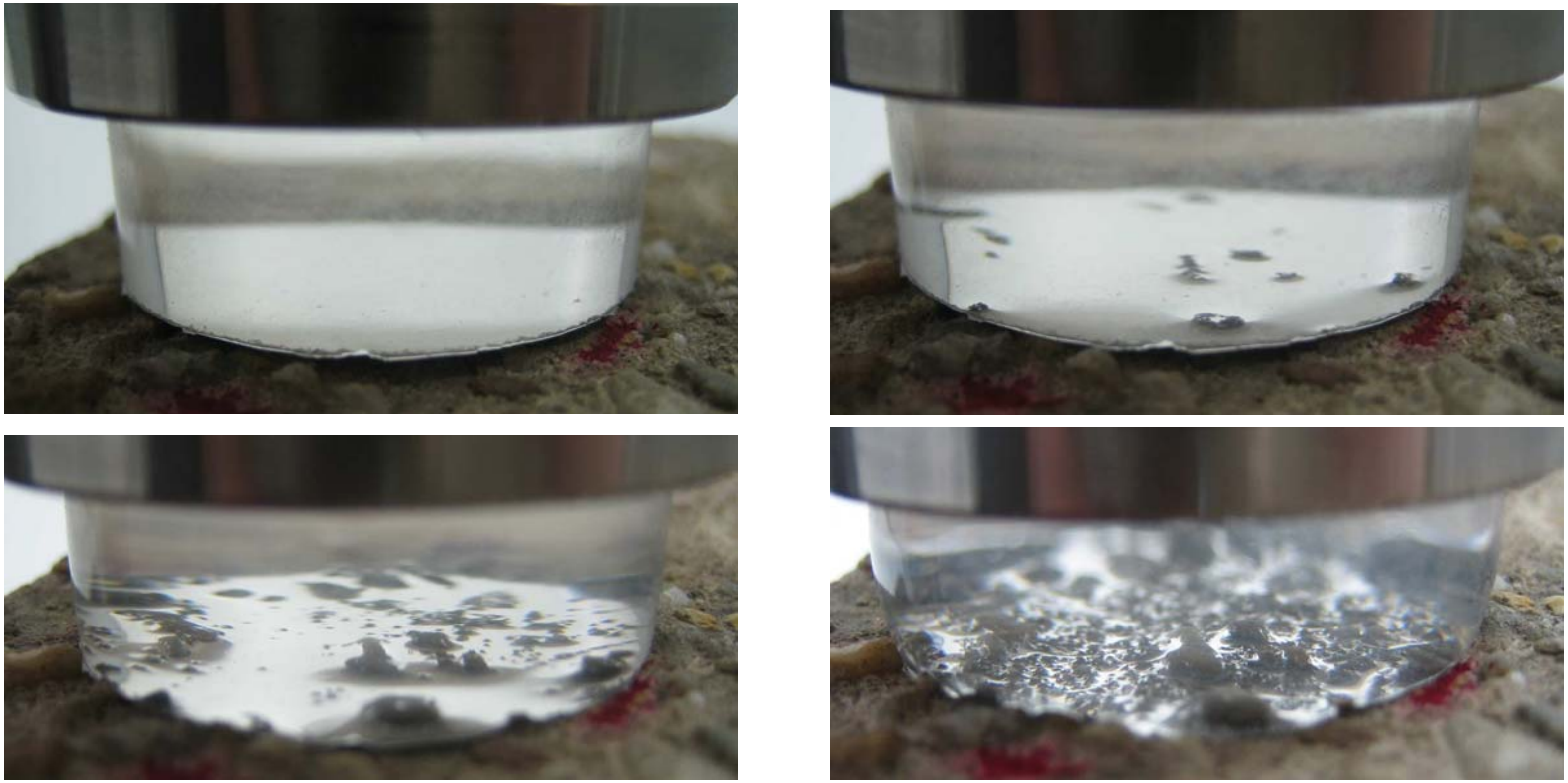
$$p(u) = -\frac{dU_{\rm el}}{du}(u), \qquad U_{\rm el} \sim p(u), \qquad p(u) \sim e^{-u/u_0}$$

Interfacial separation



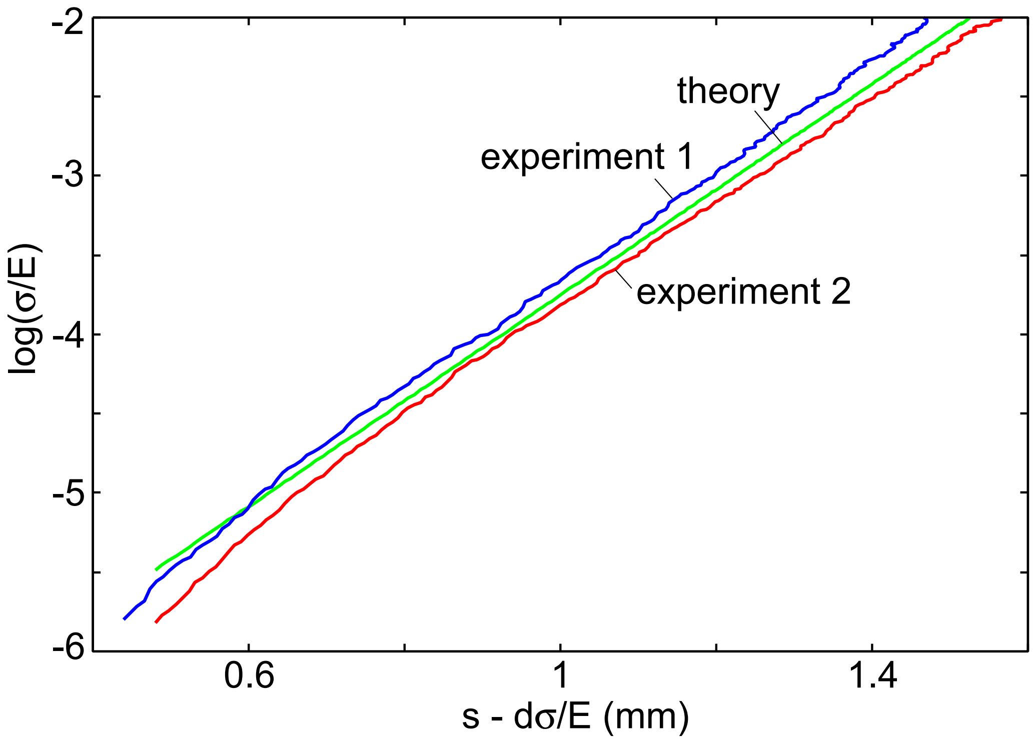
$$\log\left(\frac{\sigma}{E}\right) = B + \frac{1}{u_0}\left(s - d\frac{\sigma}{E}\right)$$

Interfacial separation: experiment



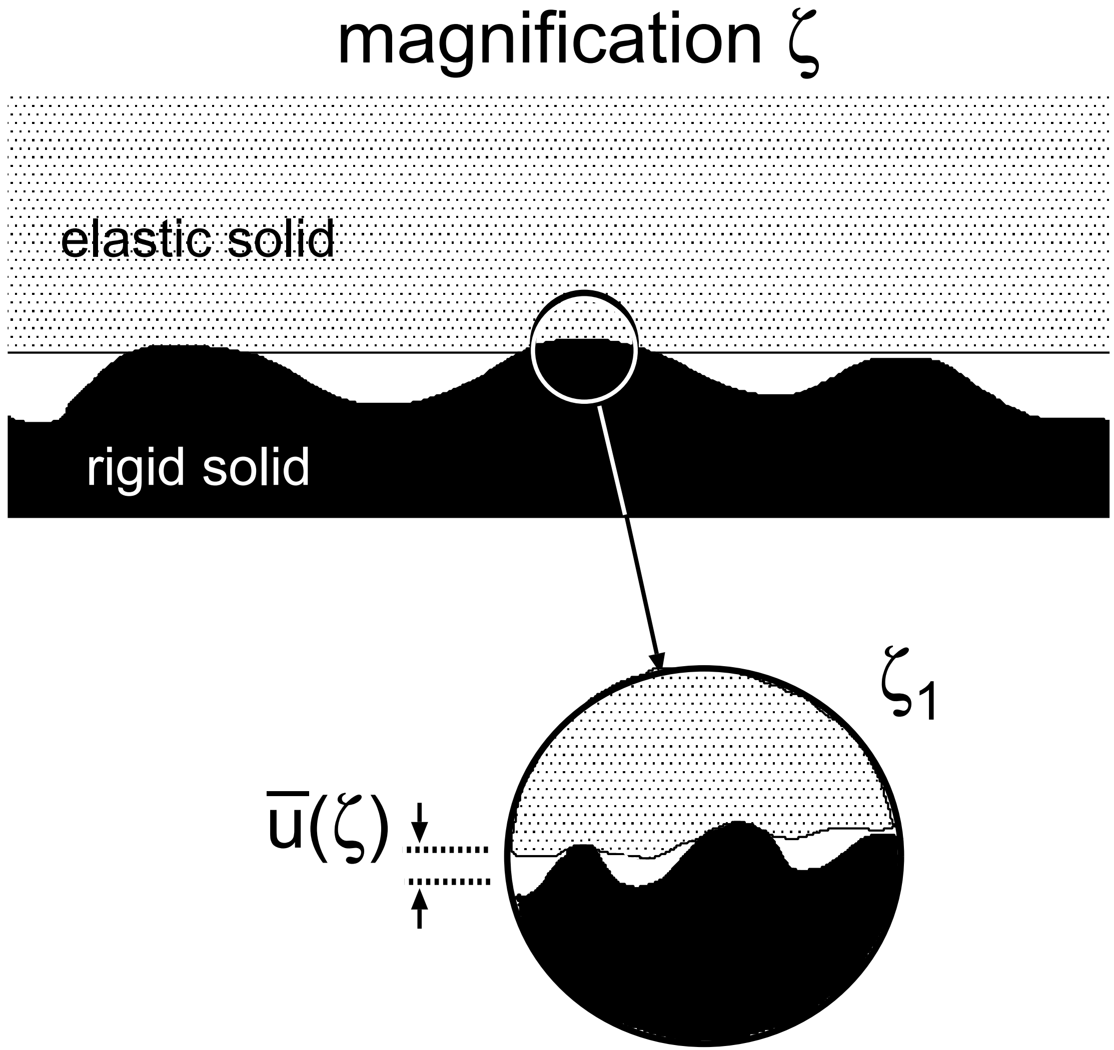
Experiment: B. Lorenz, IFF, FZ-Jülich

Interfacial separation: experiment

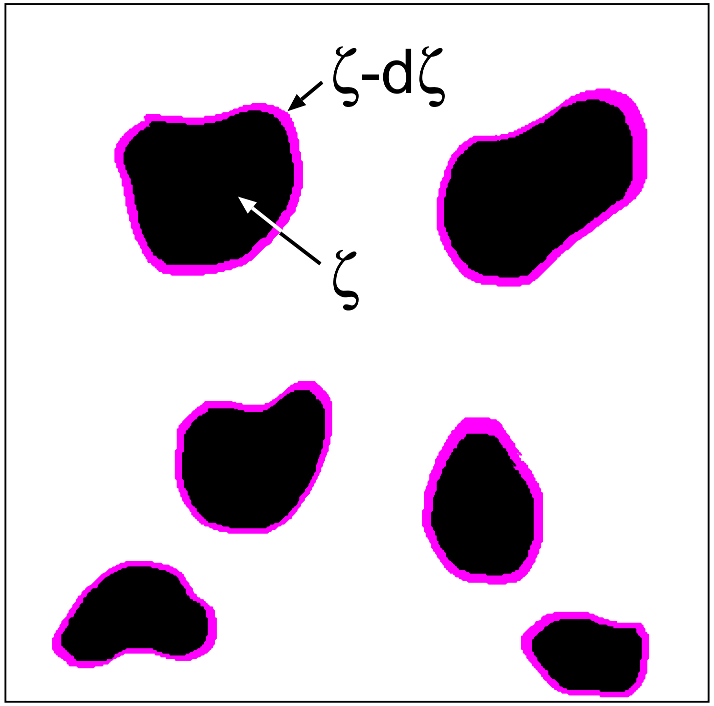


B. Lorenz and B.N.J. Persson, J. Phys. Condens. Matter **21**, 15003 (2009)

Interfacial separation $\bar{u}(\zeta)$



Interfacial separation $u_1(\zeta)$



$\bar{u}(\zeta)$ = average surface separation in black area

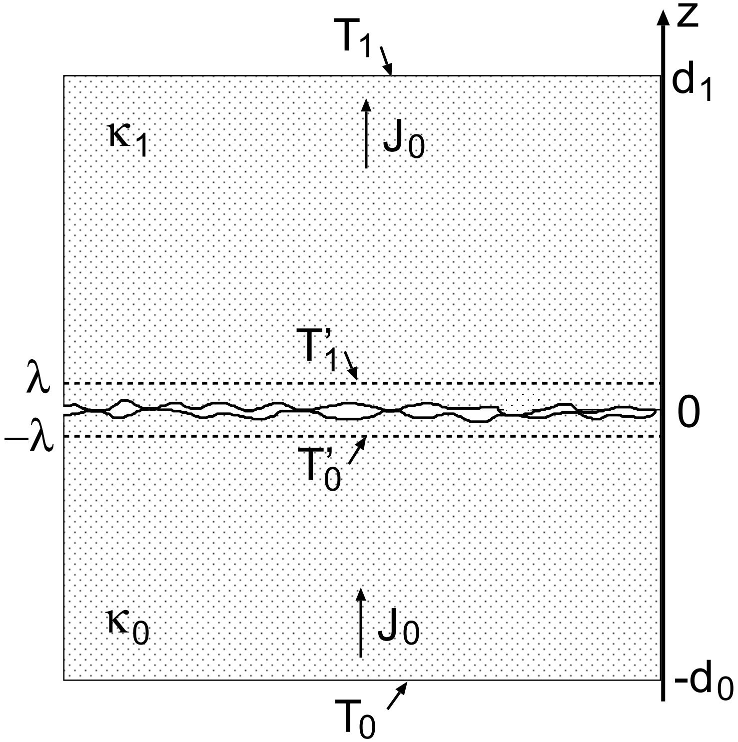
$u_1(\zeta)$ = average surface separation in the pink area

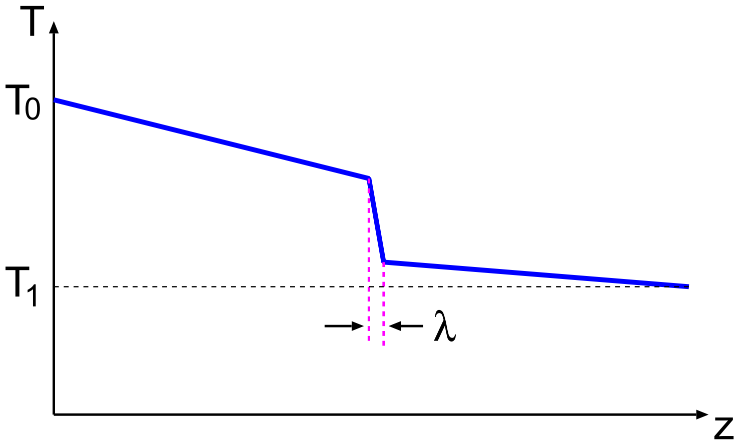
Definition: $u_1(\zeta)$ is the (average) height separating the surfaces which appear to come into contact when the magnification decreases from ζ to $\zeta - d\zeta$

The new contact mechanics theory is very flexible and can be applied to many important topics including

- Elastic and Viscoelastic Contact Mechanics
- Rubber Friction
- Elastoplastic Contact Mechanics
- Adhesion
- Capillary Adhesion
- Leak-rate of Seals
- Elastohydrodynamics at low sliding velocity
- Thermal and Electric Contact Resistance

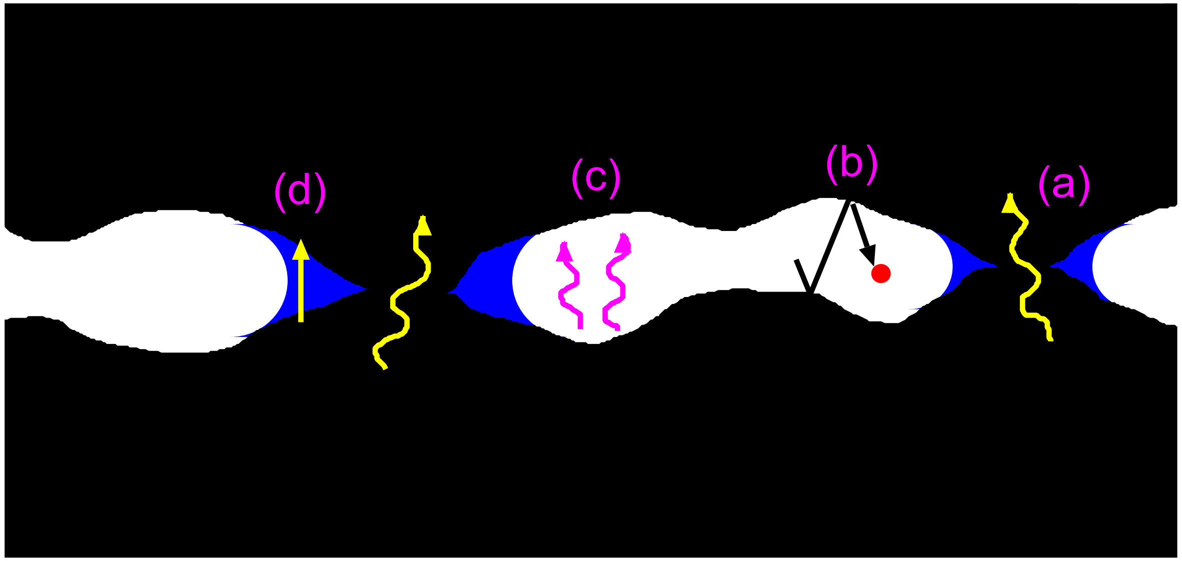
Thermal contact resistance





$$J_0 = \alpha(T'_0 - T'_1)$$

Heat transfer processes



(a) Heat transfer via the area of real contact $\alpha_{\rm cont}$

(b) Heat transfer via surrounding gas (or liquid) $\alpha_{\rm gas}$

(c) Radiative (electromagnetic wave) heat transfer $\alpha_{\rm rad}$

(d) Heat diffusion in fluid capillary bridges $\alpha_{\rm cap}$

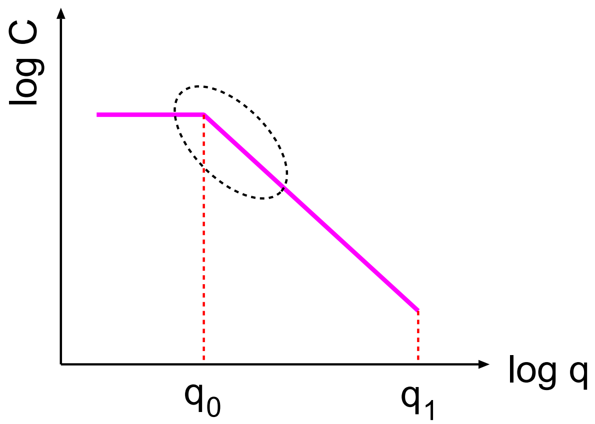
$\alpha \approx \alpha_{\rm cont} + \alpha_{\rm cap} + \alpha_{\rm rad} + \alpha_{\rm gas}$

(a) Heat transfer via the area of real contact

$$\alpha_{\rm cont} = \frac{\kappa p_0}{E^* u_0}$$
$$\frac{1}{\kappa} = \frac{1}{\kappa_0} + \frac{1}{\kappa_1}$$
$$\frac{1}{E^*} = \frac{1}{E_0} + \frac{1}{E_1}$$

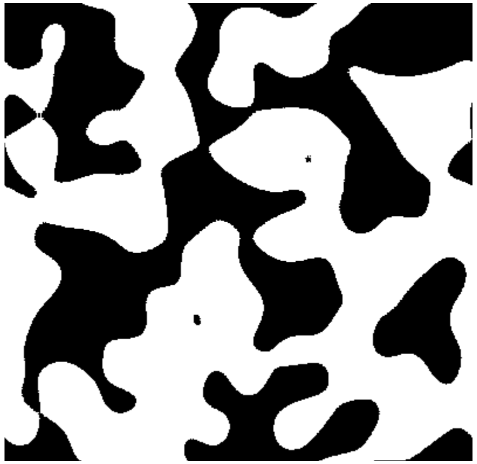
For self-affine fractal surface ($D_{\rm f} = 3 - H$):

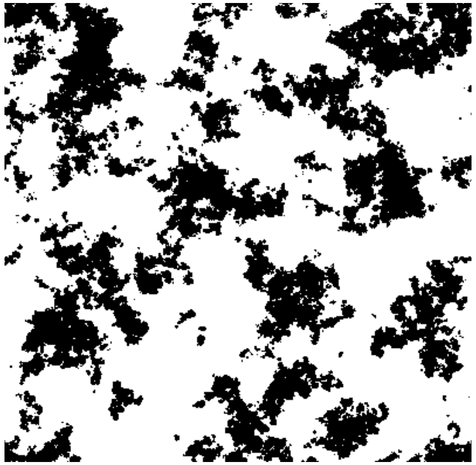
$$u_0 \sim h_{\rm rms} \left[r(H) - \left(\frac{q_0}{q_1}\right)^H \right]$$



Paradox: Contact heat transfer without contact!!!

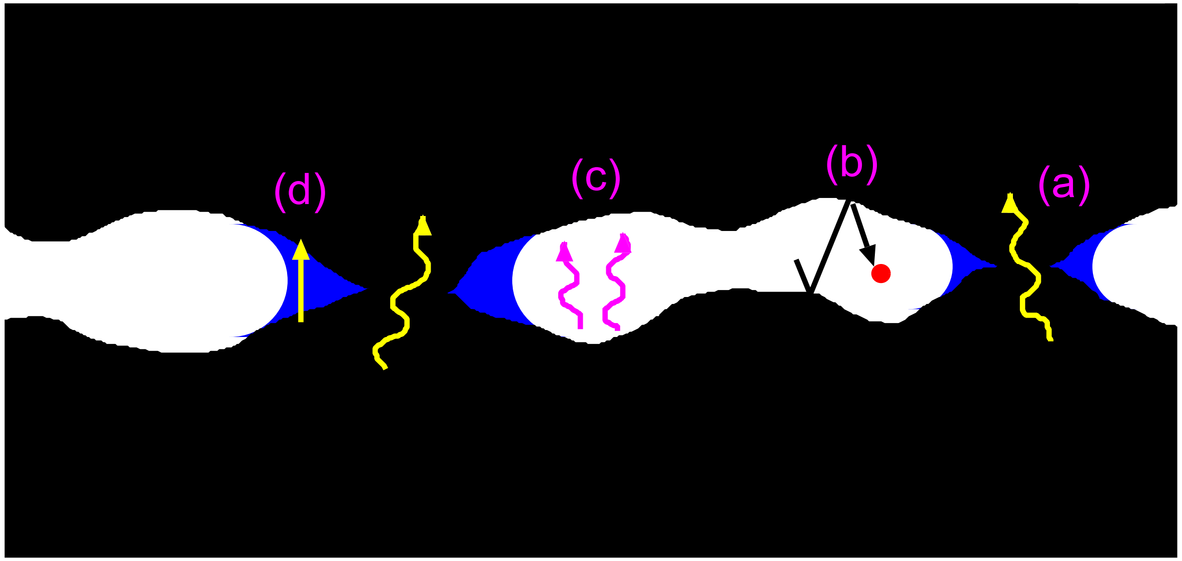
The apparent contact at two magnifications





The heat transfer is determined accurately by the nature of the contact at relative low magnification (left).

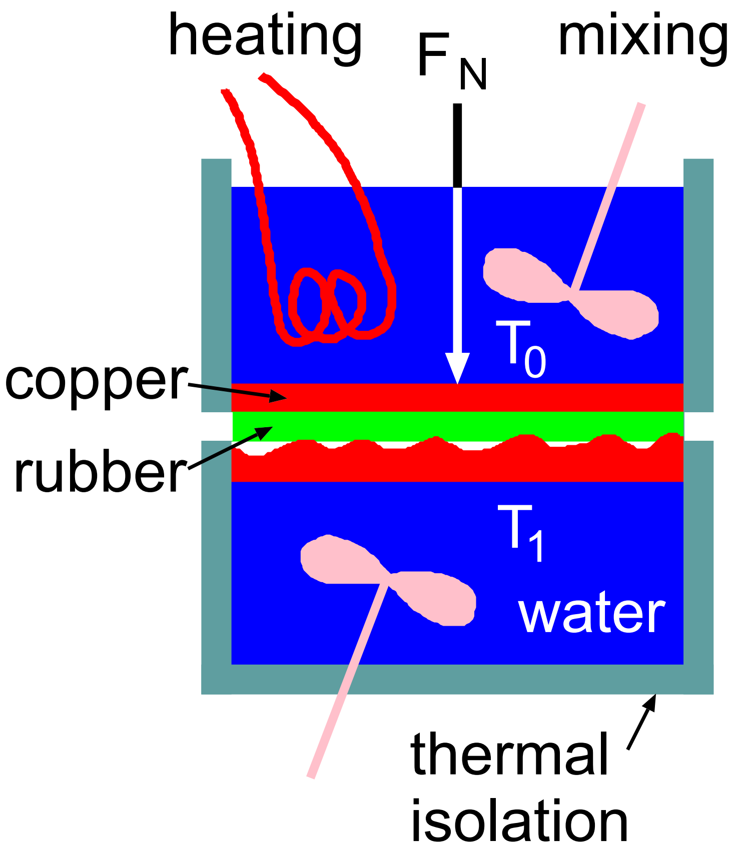
(b) Heat transfer via surrounding gas (or liquid)



$$J_0 = \frac{\kappa_{\rm gas}\Delta T}{d+a\Lambda}$$

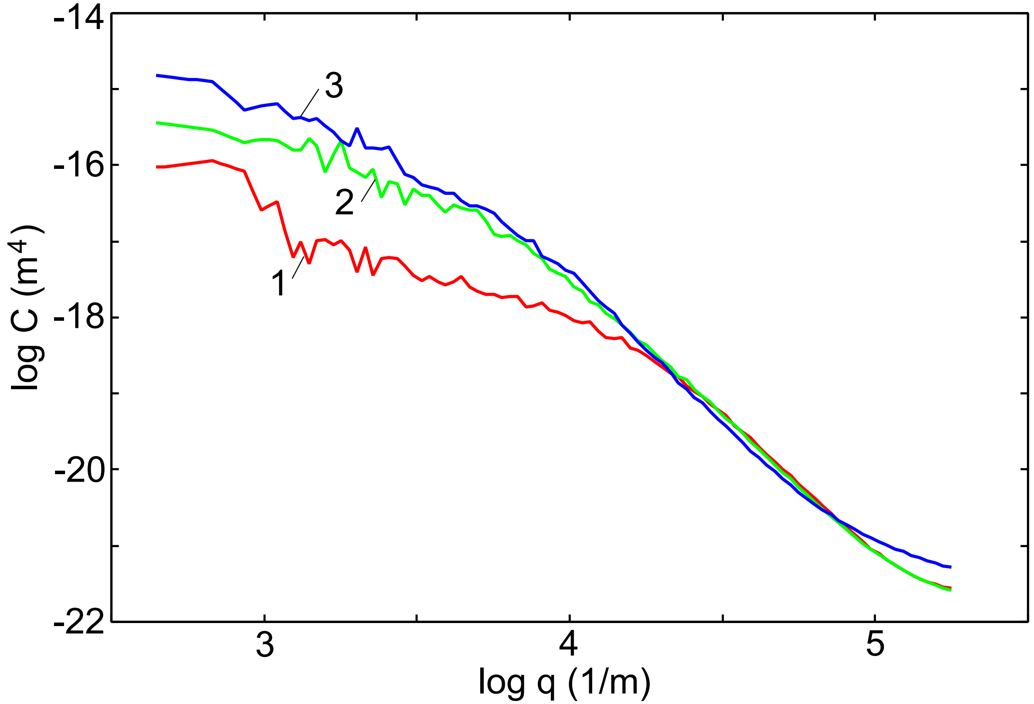
$$\alpha \approx \kappa_{\rm gas}\langle (d+\Lambda)^{-1}\rangle = \kappa_{\rm gas}\int_{u_c}^{\infty} du\, P(u)(u+\Lambda)^{-1}$$

Heat transfer experiment



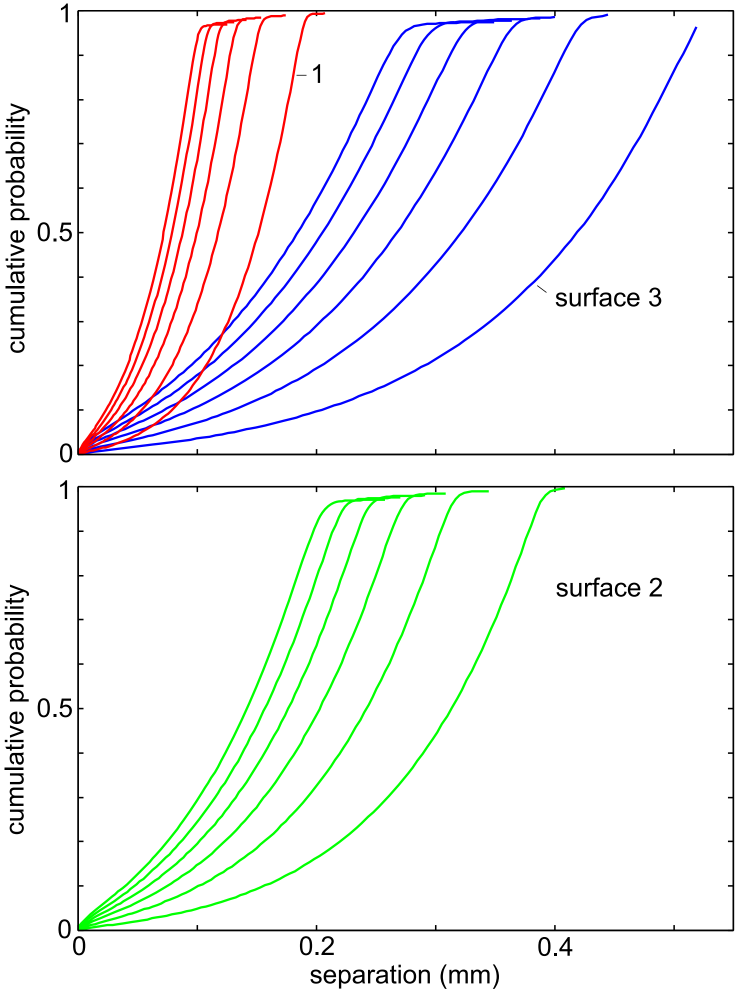
The heat transfer is determined by the temperature increase $T_1(t)$ in the lower container.

Surface roughness power spectrum



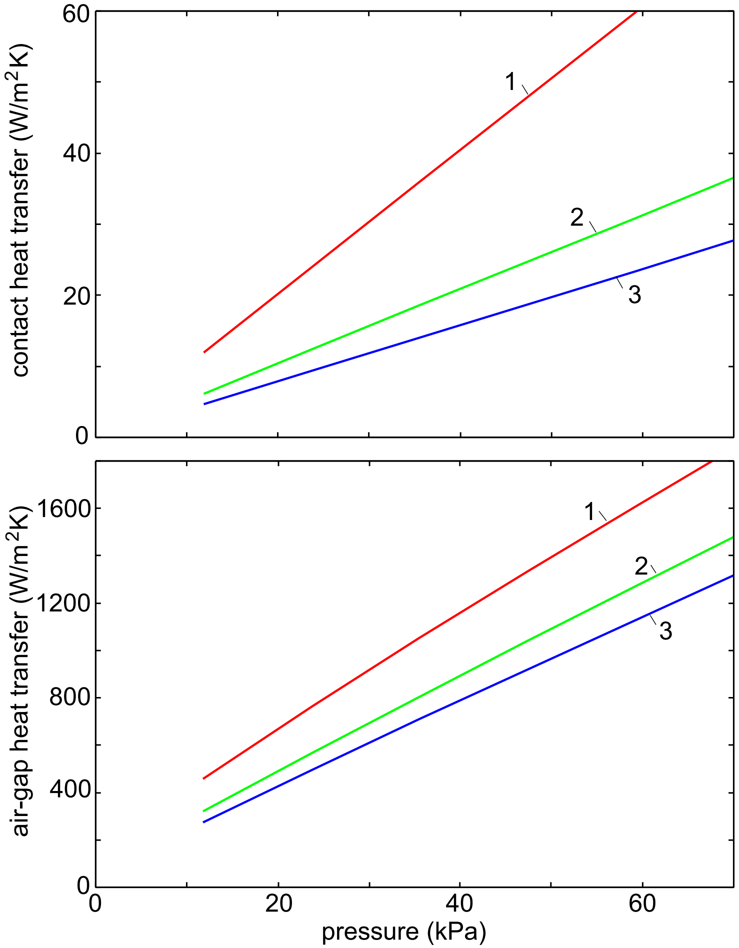
The power spectrum $C(q)$ of the copper surfaces used in the experiment.

Interfacial separation



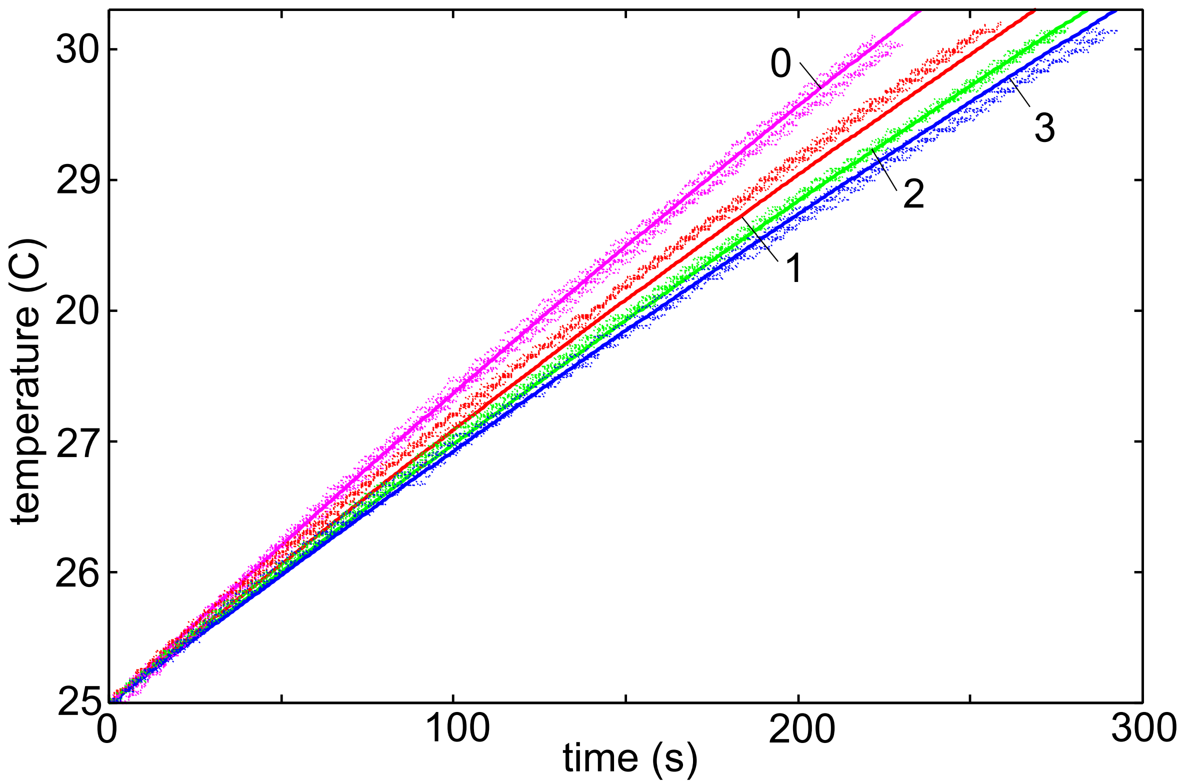
The cumulative height distribution as a function of the interfacial separation for several squeezing pressures.

Calculated heat transfer coefficients



The heat transfer coefficient as a function of the squeezing pressure.

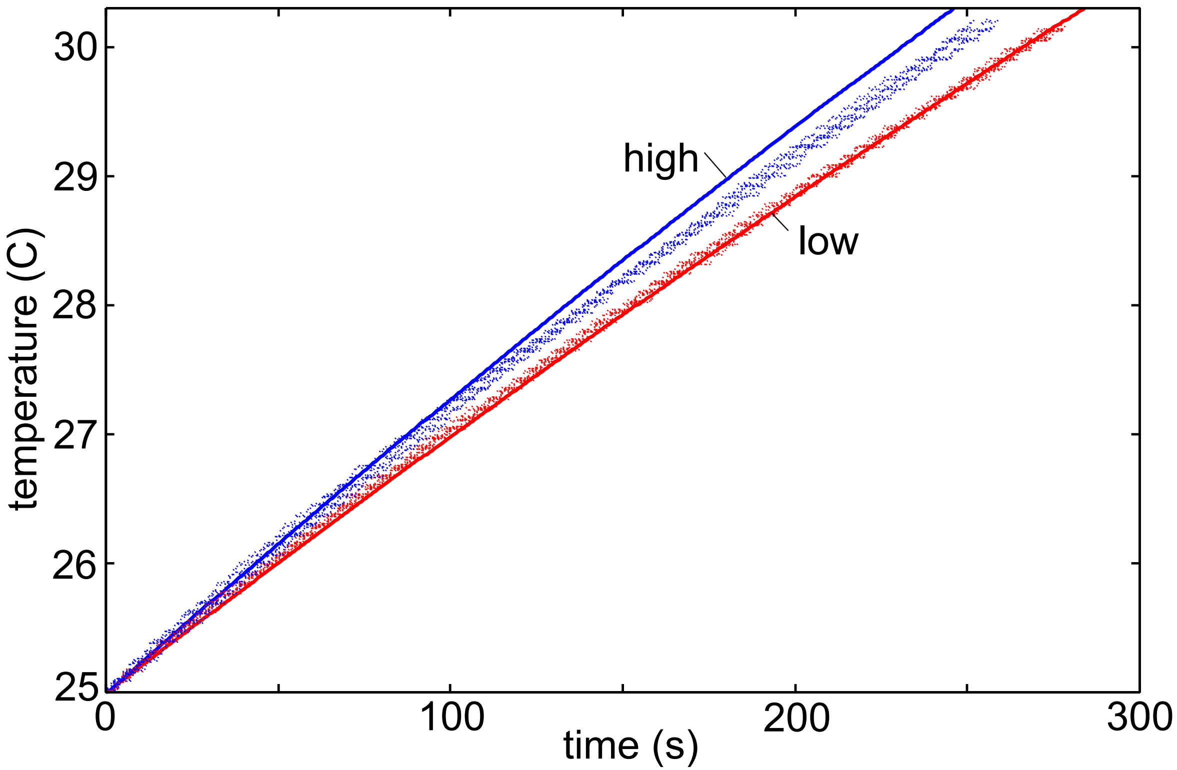
Experiment: temperature increase



The increase with time of the temperature at fixed squeezing pressure for all 4 surfaces.

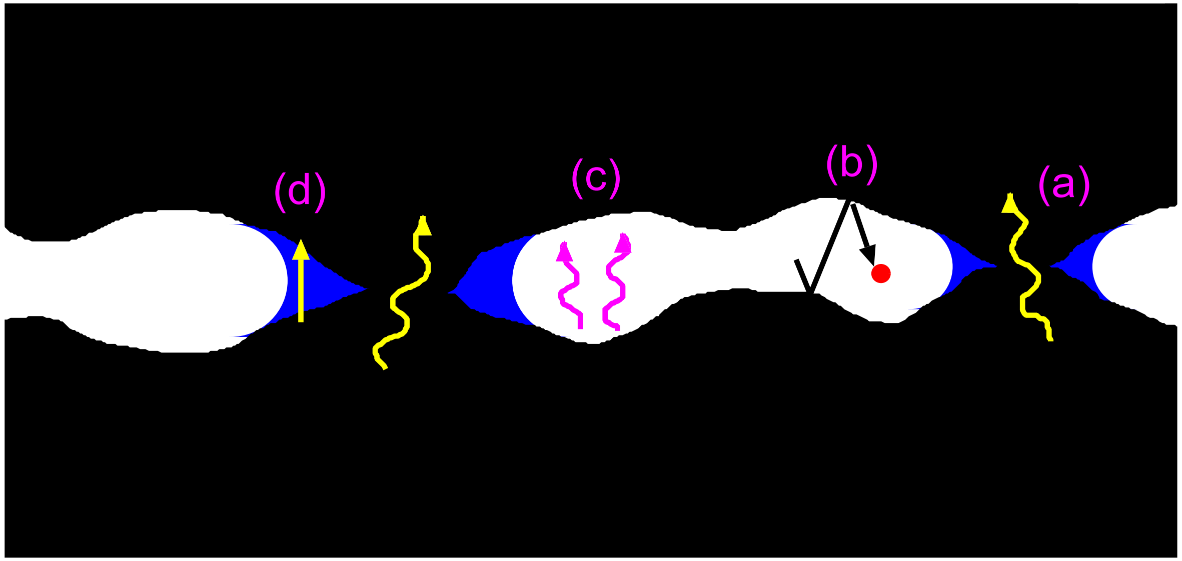
Experiment: B. Lorenz

Experiment: temperature increase

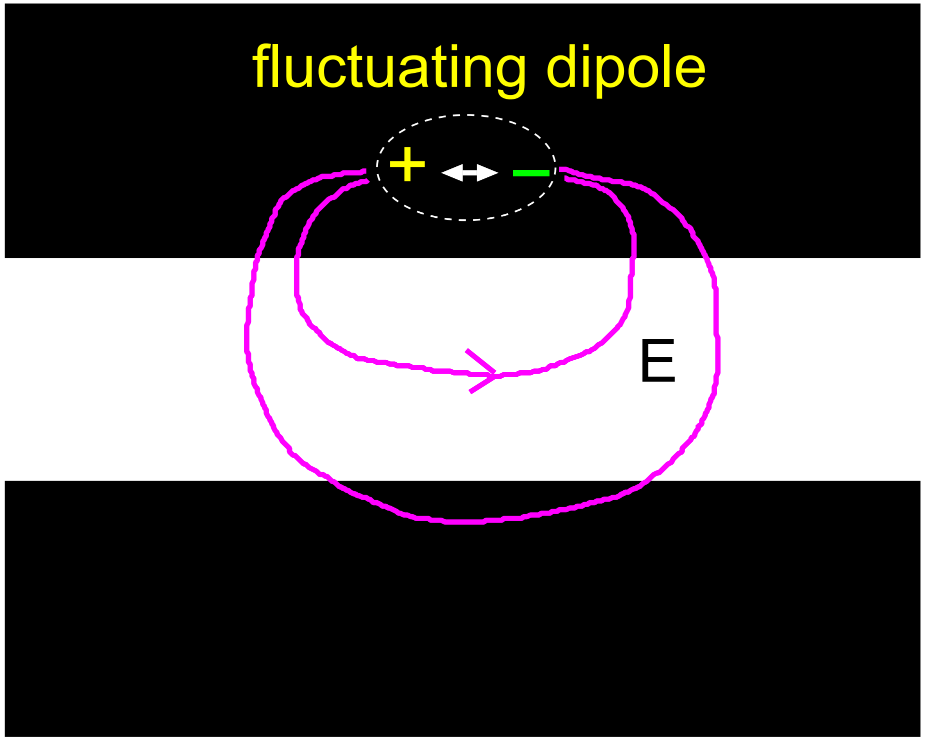


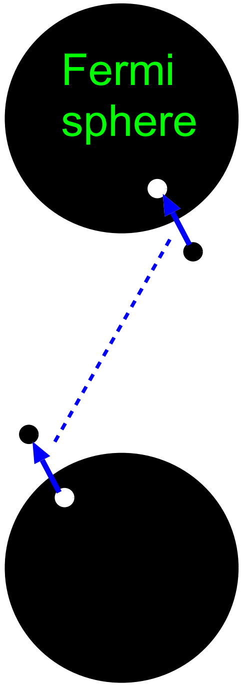
The increase with time of the temperature for surface 2 for two squeezing pressures.

(c) Radiative heat transfer



Radiative heat transfer: theory





Radiative heat transfer: theory

For $d >> d_T = c\hbar/k_B T$ Stefan-Boltzmann law:

$$J_0 = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}\left(T_0^4 - T_1^4\right)$$

$$\alpha_{\rm rad} \approx \frac{4\pi^2 k_B^4}{60\hbar^3 c^2} T_0^3$$

For $T_0 = 300\ \rm K$:

$\alpha_{\rm rad} \approx 6\ \rm W/m^2K$

For $d < d_T$:

$$J_0 = F(\epsilon_1, \epsilon_2, T_0) d^{-2} \Delta T$$

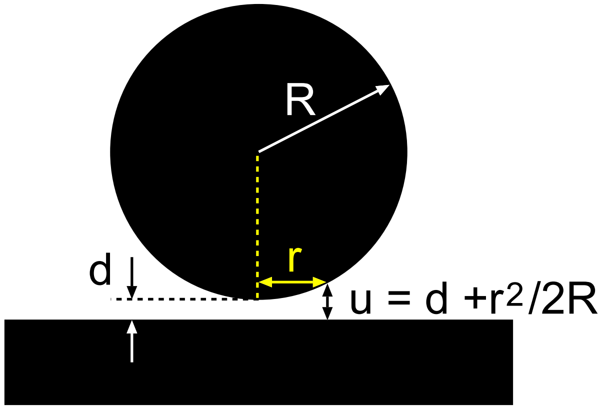
For (amorphous) $\rm SiO_2$ for $d = 1\ \rm nm$ and $T_0 = 300\ \rm K$:

$\alpha_{\rm rad} \approx 2 \times 10^6\ \rm W/m^2K$

For non-uniform surface separation:

$$\alpha_{\rm rad} = F(\epsilon_1, \epsilon_2, T_0) \int_{u_c}^{\infty} du\ P(u) u^{-2}$$

Heat transfer ball-plane

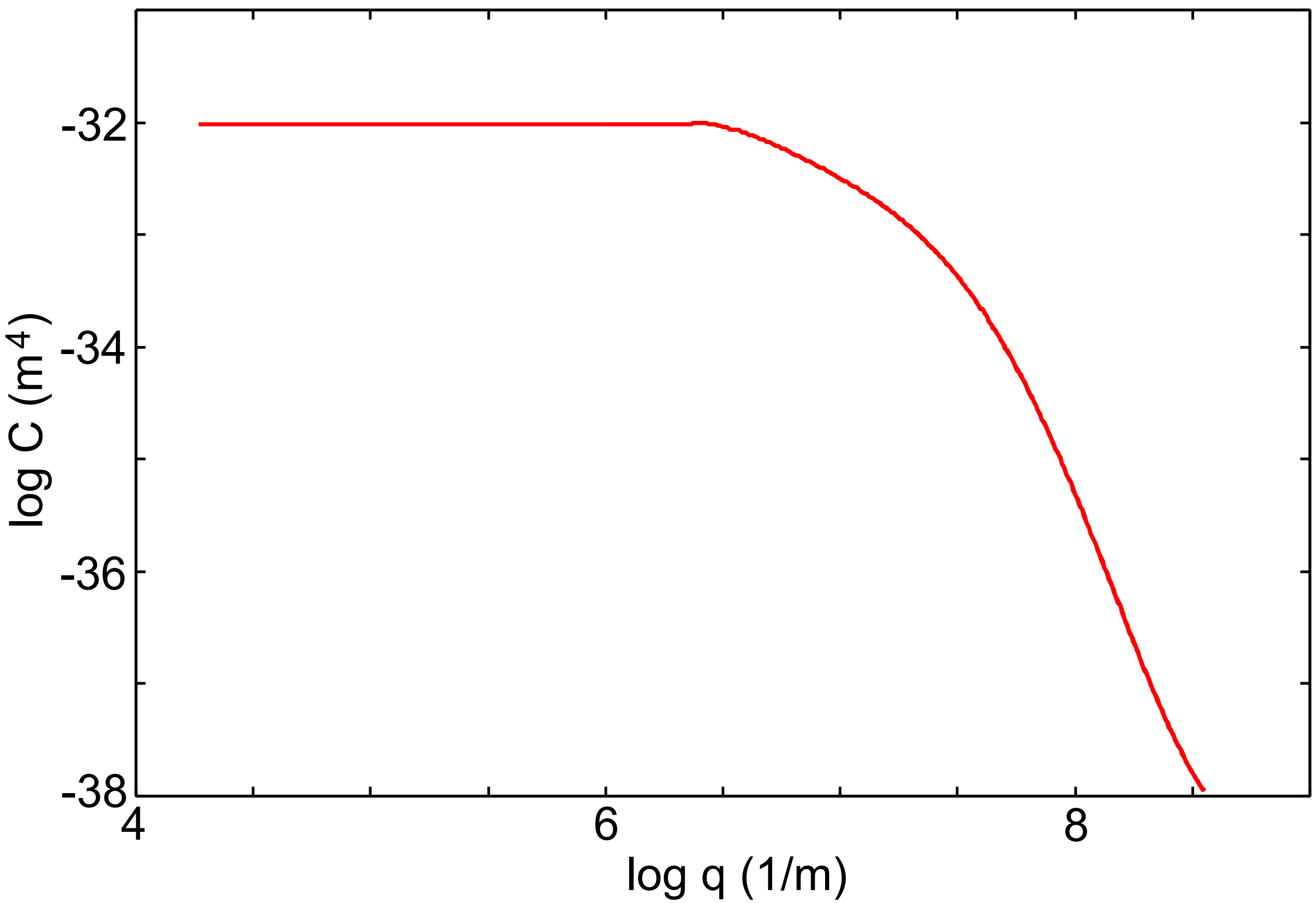


$$\dot{q} = \int d^2x \; J_0 \sim \int_0^\infty dr r \; u^{-2}(r) \sim d^{-1}$$

S. Shen et al., Nano Letters **9**, 2909 (2009)

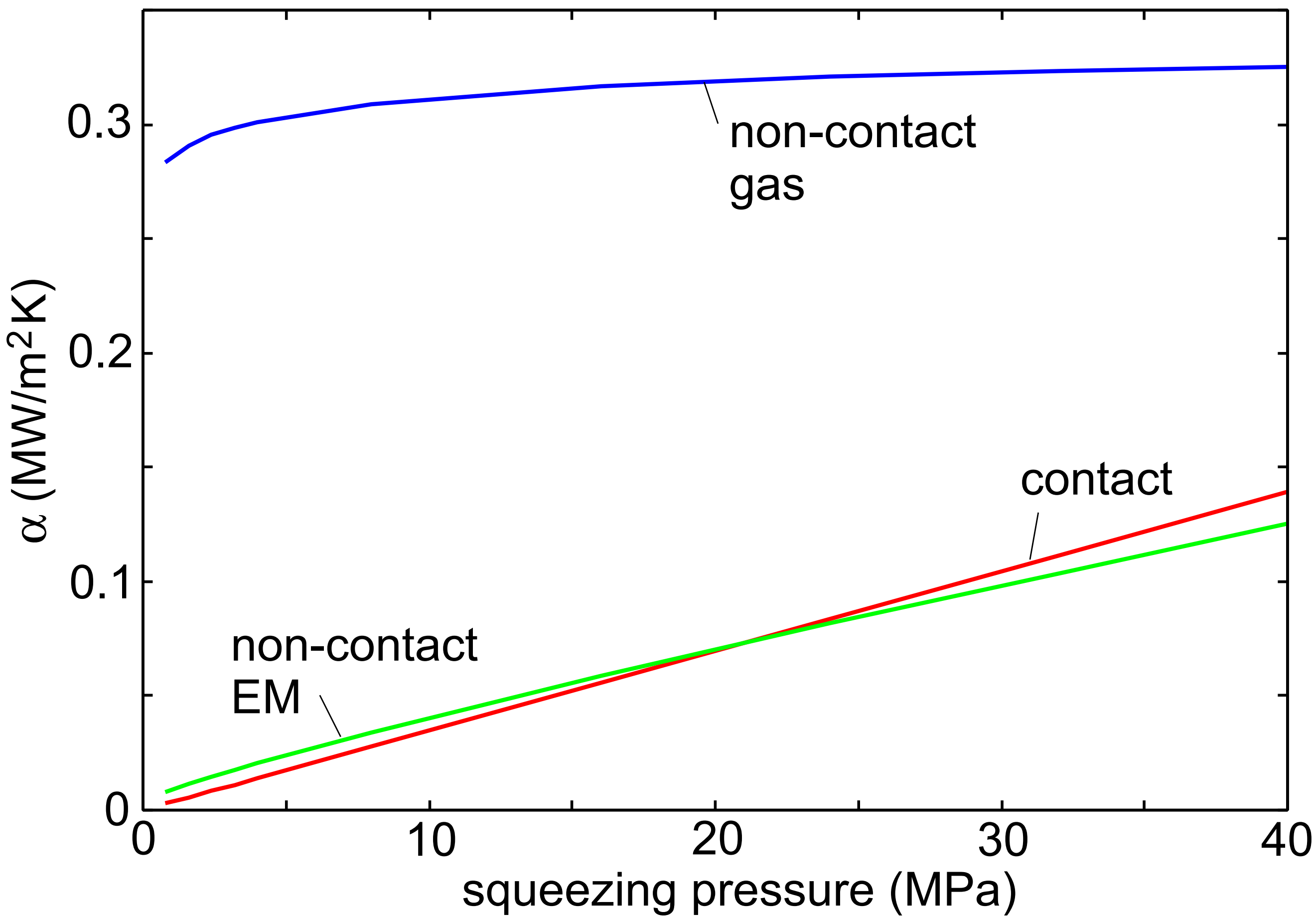
E. Rousseau et al., Nature photonics **3**, 514 (2009)

Power spectrum $C(q)$ of a MEMS surface

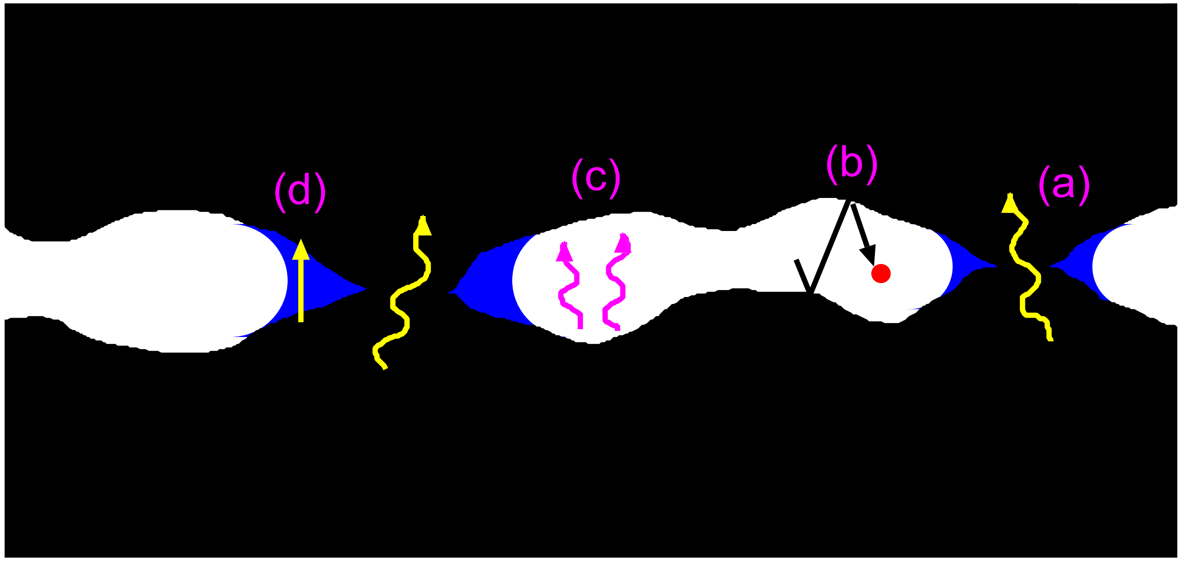


Root-mean-square roughness amplitude: 2.5 nm

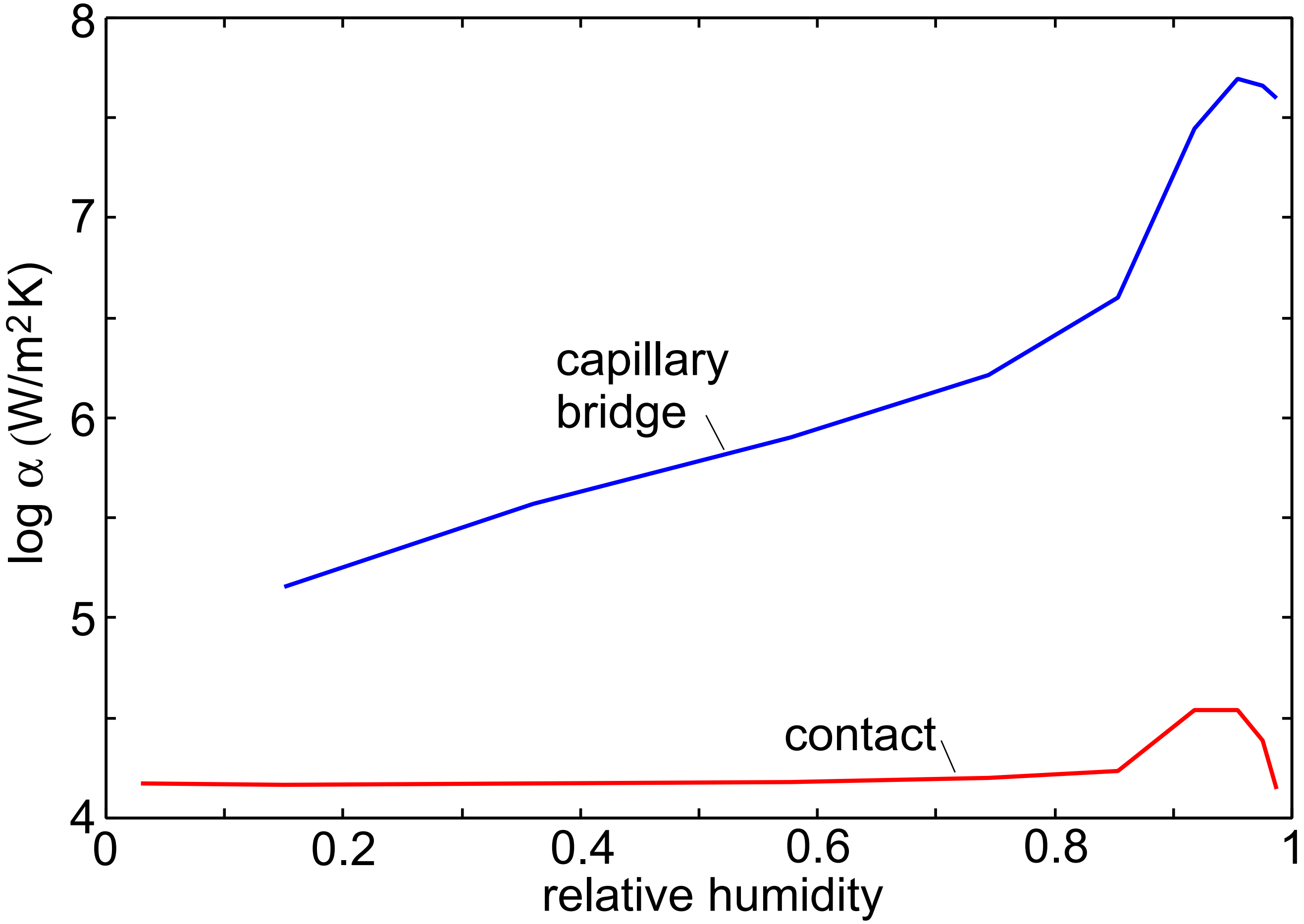
Numerical results: MEMS surfaces



(d) Heat diffusion in fluid capillary bridges



Numerical results: MEMS surfaces



Thank you for your attention!

Multiscale

Stress distribution $P(\sigma, \zeta)$

$$\frac{\partial P}{\partial \zeta} = f(\zeta) \frac{\partial^2 P}{\partial \sigma^2}$$

where

$$f(\zeta) = \frac{\pi}{4} \left(\frac{E}{1-\nu^2} \right)^2 q_L q^3 C(q)$$

where $q = \zeta q_{\mathrm{L}}$. Boundary conditions:

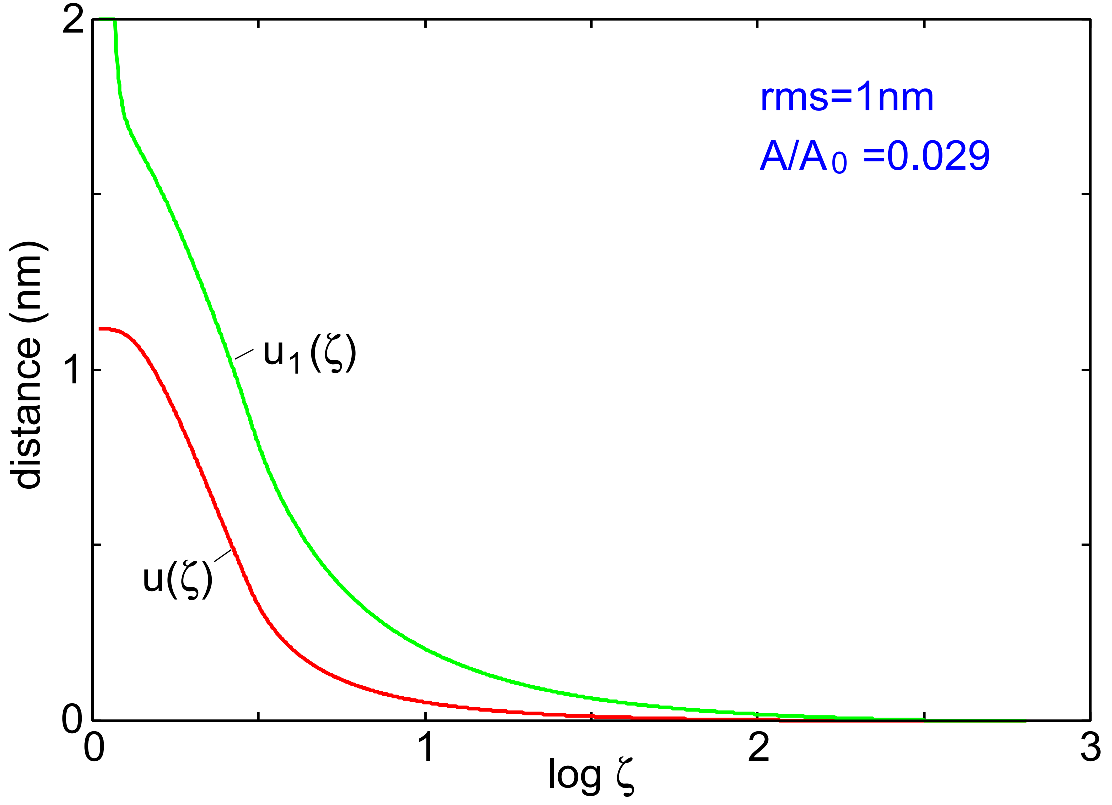
$$P(0, \zeta) = 0, \qquad P(\infty, \zeta) = 0$$

Initial condition:

$$P(\sigma, 0) = \delta(\sigma - \sigma_0)$$

Bo Persson, J. Chem. Phys. 115, 3840 (2001)

Numerical results for $\bar{u}(\zeta)$ and $u_1(\zeta)$



$$u_1(\zeta) = \bar{u}(\zeta) + \bar{u}'(\zeta)A(\zeta)/A'(\zeta)$$

Comment on asperity contact mechanics models

The Greenwood–Williamson theory and the theory of Bush et al predict:

- $p(u) \sim u^{-a} e^{-bu^2}$
- $\langle \sigma(\mathbf{q}) \sigma(-\mathbf{q}) \rangle \sim q^{-2(1+H)}$
- For multi-scale roughness, contact area depends non-linearly on the load already for very small A/A_0

I predict:

- $p(u) \sim e^{-u/u_0}$
- $\langle \sigma(\mathbf{q}) \sigma(-\mathbf{q}) \rangle \sim q^{-(1+H)}$
- Contact area depends linearly on the load as long as the relative contact area $A/A_0 \lesssim 0.1$

Contact mechanics: outlook

$$\frac{\partial P}{\partial \zeta} = f(\zeta) \frac{\partial^2 P}{\partial \sigma^2}$$

where

$$f(\zeta) = \frac{\pi}{4} \left(\frac{E}{1-\nu^2} \right)^2 q_L q^3 C(q) S(q)$$

Original theory assumes $S(q) = 1$. If one writes

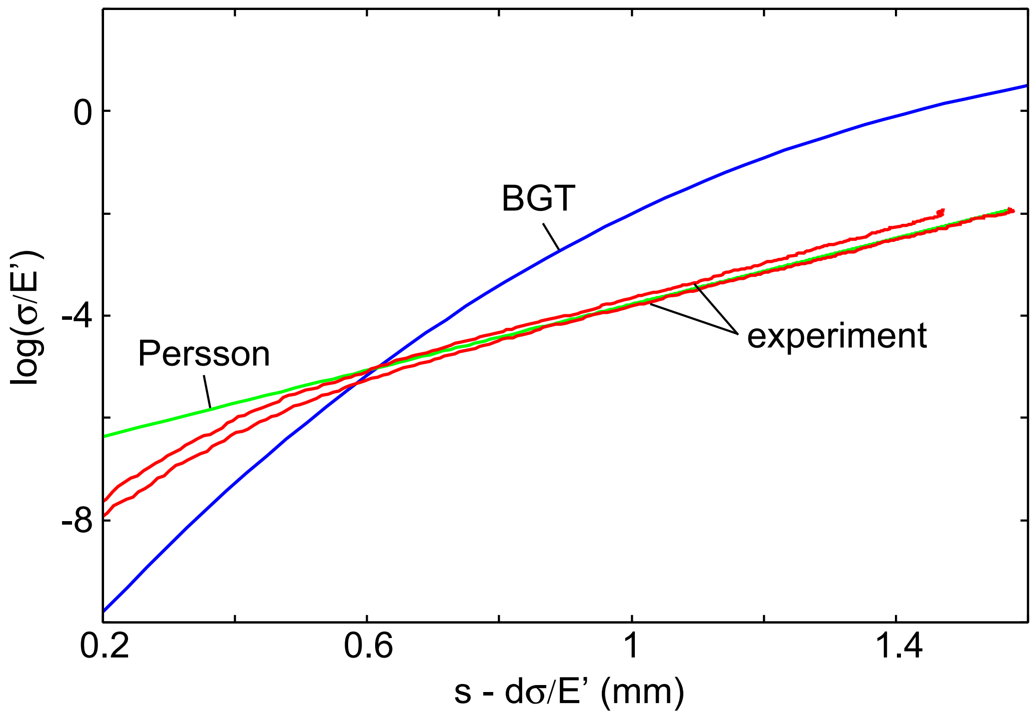
$$U_{\mathrm{el}} = \frac{E^* A_0}{4} \int d^2 q \; q C(q) W(q)$$

then we have the exact result $S(q) = W(q)/P(q)$ where

$$P(q) = \int_0^\infty d\sigma \; P(\sigma, \zeta)$$

Improved theory with $S(q) = \beta + (1-\beta) P^2(q)$ with $\beta \approx 0.5$.

Interfacial separation: experiment



Role of plastic deformation: MEMS surfaces