



**The Abdus Salam  
International Centre for Theoretical Physics**



**2065-7**

**Advanced Training Course on FPGA Design and VHDL for Hardware  
Simulation and Synthesis**

***26 October - 20 November, 2009***

**Digital Arithmetic (contd.)**

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# Outline

- Digital CMOS design

- Arithmetic operators

  - Adders

  - Comparators

  - Shifters

  - Multipliers



# Comparators

Comparing a natural number to a constant : =

Let consider a natural number  $a$  coded on 8 bits using Natural Binary Code

$a_7 \ a_6 \ a_5 \ a_4 \ a_3 \ a_2 \ a_1 \ a_0$

= ?

0 0 0 0 0 0 0 0



0 / 1

# Comparators

Comparing a natural number to zero :=

Boolean function

Null = 1 if

$$\bar{a}_7 \cdot \bar{a}_6 \cdot \bar{a}_5 \cdot \bar{a}_4 \cdot \bar{a}_3 \cdot \bar{a}_2 \cdot \bar{a}_1 \cdot \bar{a}_0 = 1$$

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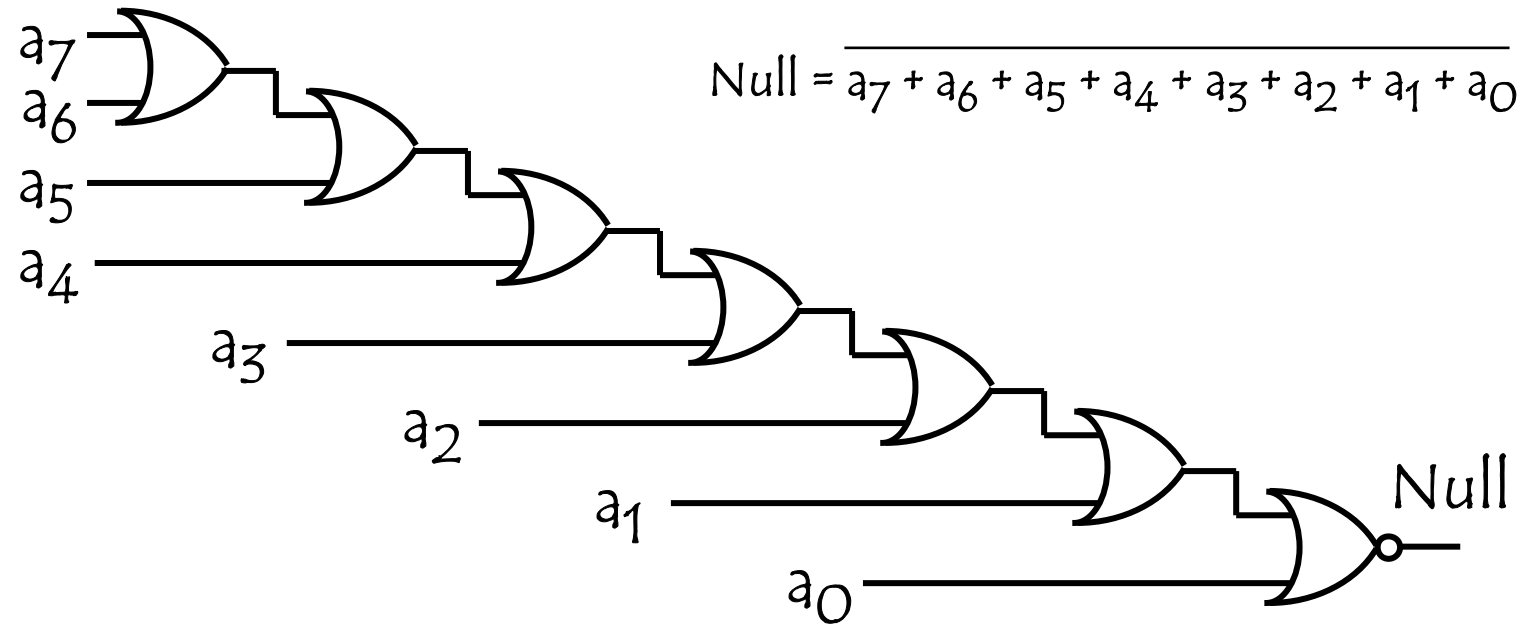
$$\text{Null} = a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0$$



# Comparators

Comparing a natural number to zero :=

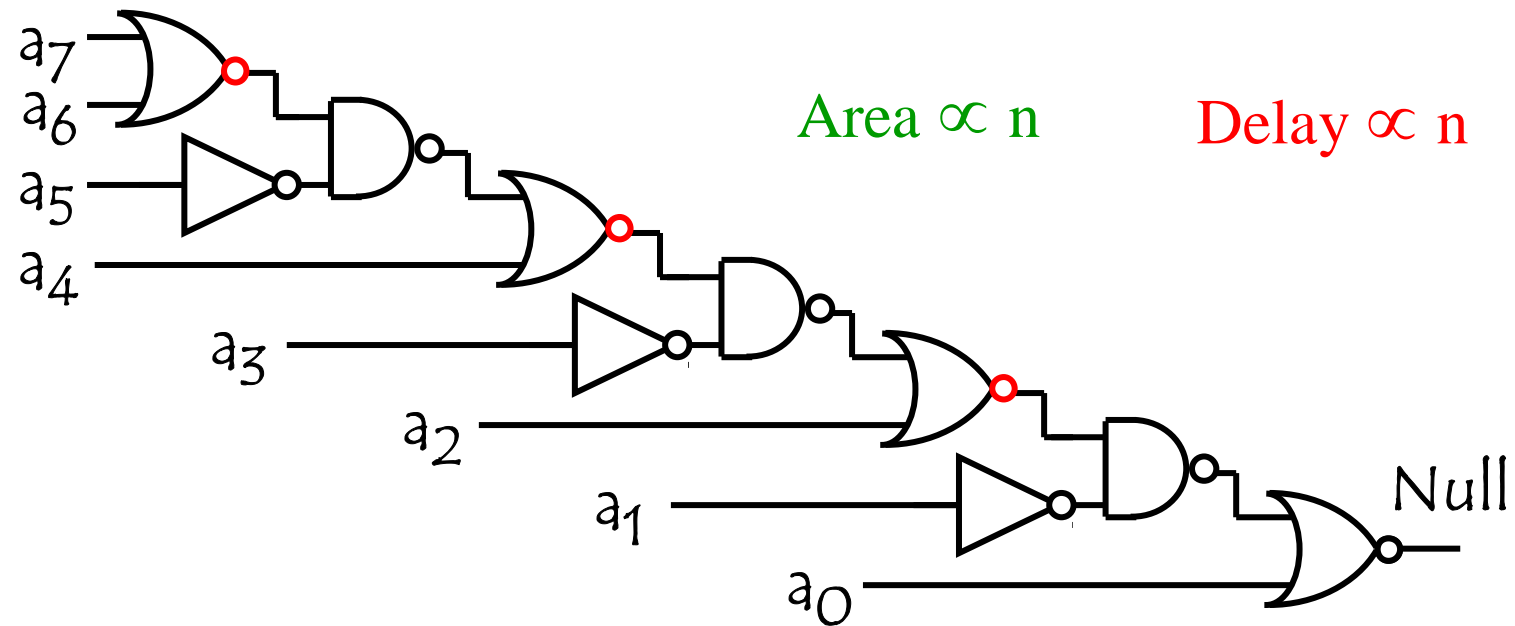
## Implementation



# Comparators

Comparing a natural number to zero : =

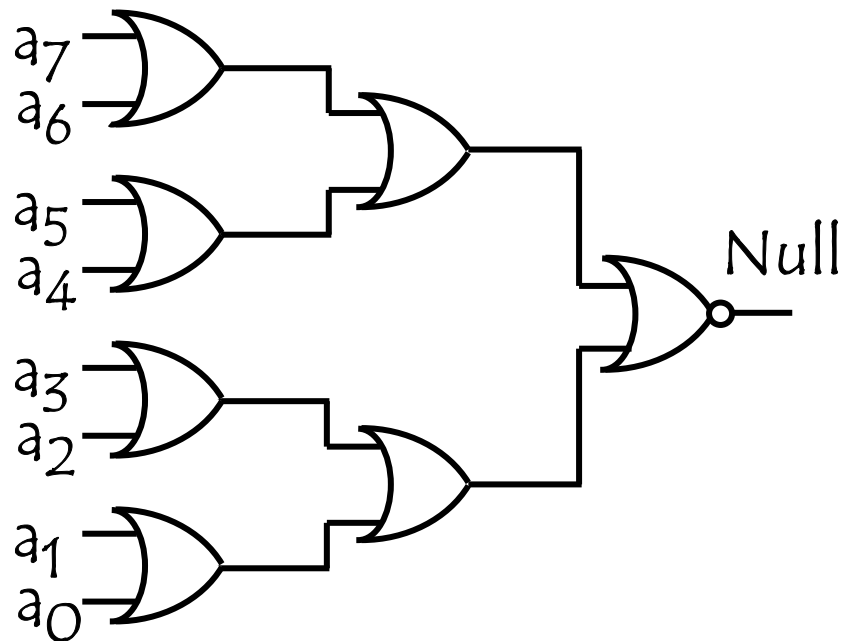
Implementation



# Comparators

Comparing a natural number to zero : =

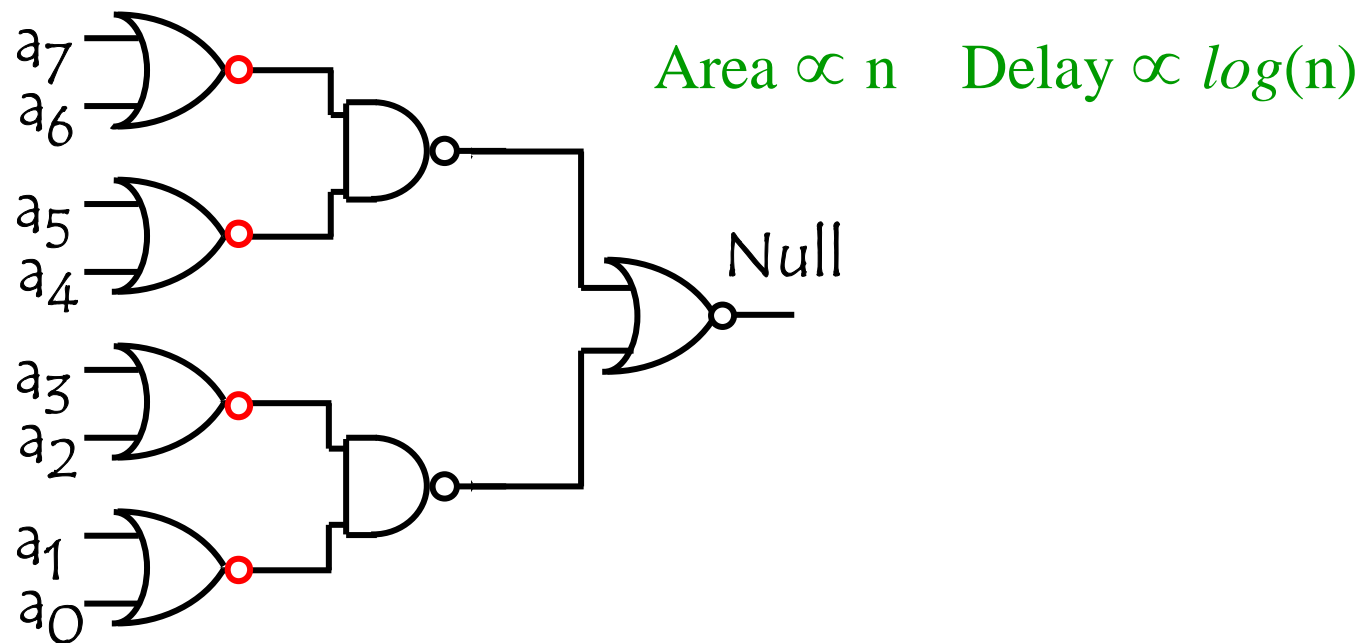
Implementation improvement



# Comparators

Comparing a natural number to zero :=

Implementation improvement





# Comparators

Comparing two natural numbers : =

Let consider two natural numbers  $a$  and  $b$   
coded on 8 bits using Natural Binary Code

$$\begin{array}{cccccccc} a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ & & & = ? & & & & \\ b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\ & & & \downarrow & & & & \\ & & & 0 / 1 & & & & \end{array}$$

# Comparators

Comparing two natural numbers :=

Boolean function

a Equal b if :  $a_7=b_7$  and  $a_6=b_6$  and ... and  $a_0=b_0$

a Equal b if :  $\overline{(a_7 \oplus b_7)} \cdot \dots \cdot \overline{(a_0 \oplus b_0)} = 1$

$$\text{Equal} = \overline{(a_7 \oplus b_7)} + \dots + (a_0 \oplus b_0)$$

$$\text{Equal} = (e_7) + \dots + (e_0)$$

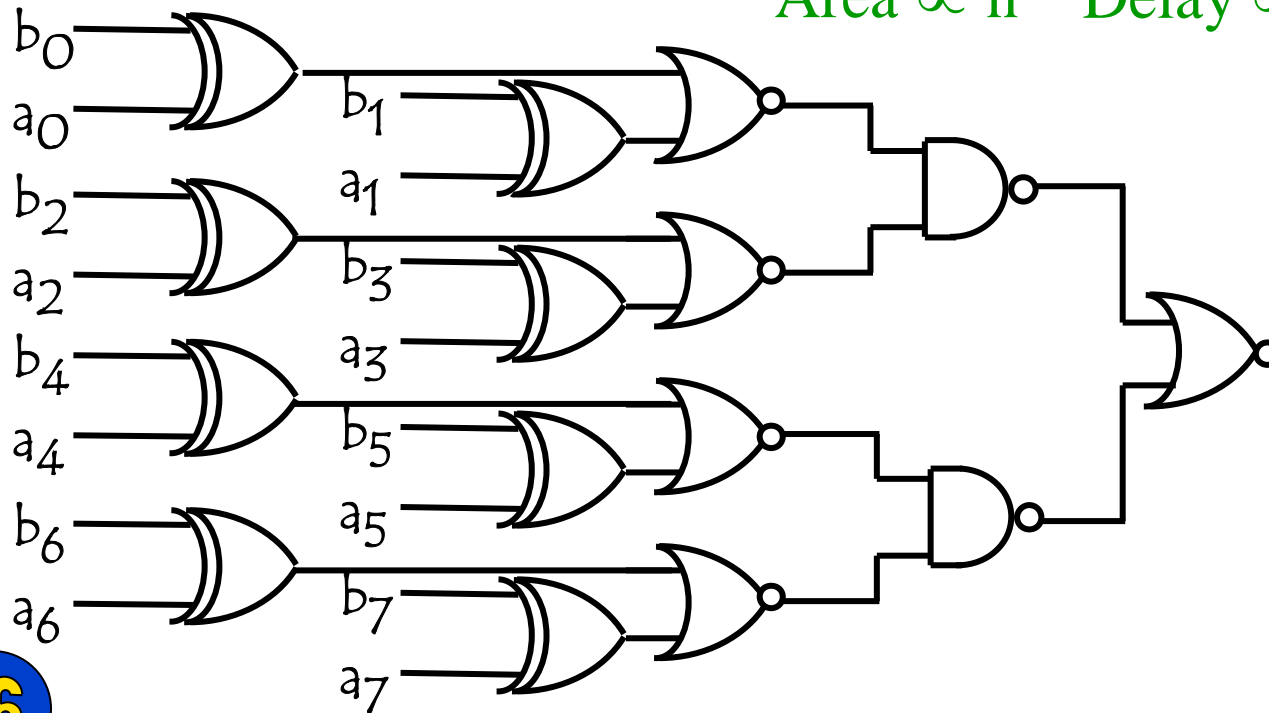


# Comparators

Comparing two natural numbers :=

Implementation

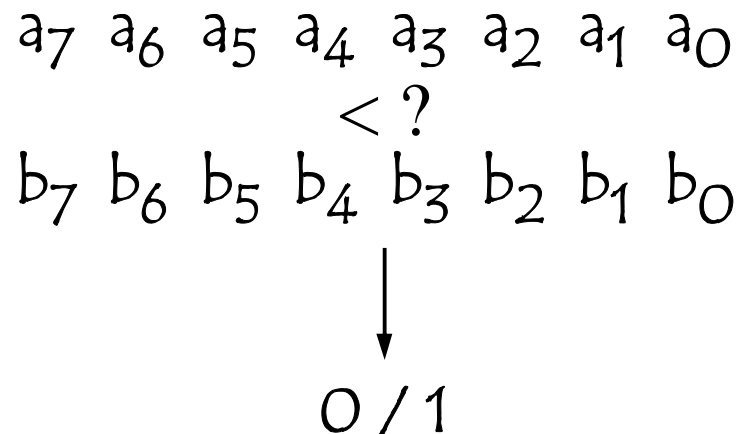
Area  $\propto n$  Delay  $\propto \log(n)$



# Comparators

Comparing two natural numbers : <

Let consider two natural numbers  $a$  and  $b$   
coded on 8 bits using Natural Binary Code



# Comparators

Comparing two natural numbers : <

Boolean function

$a < b$  if :  $a_7 < b_7$  or  $(a_7 = b_7$  and  $(a_6 < b_6$  or  $(a_6 = b_6$  and ... )))

$a_7 \ a_6 \ a_5 \ a_4 \ a_3 \ a_2 \ a_1 \ a_0$

< ?

$b_7 \ b_6 \ b_5 \ b_4 \ b_3 \ b_2 \ b_1 \ b_0$



0 / 1



# Comparators

Comparing two natural numbers : <

Boolean function

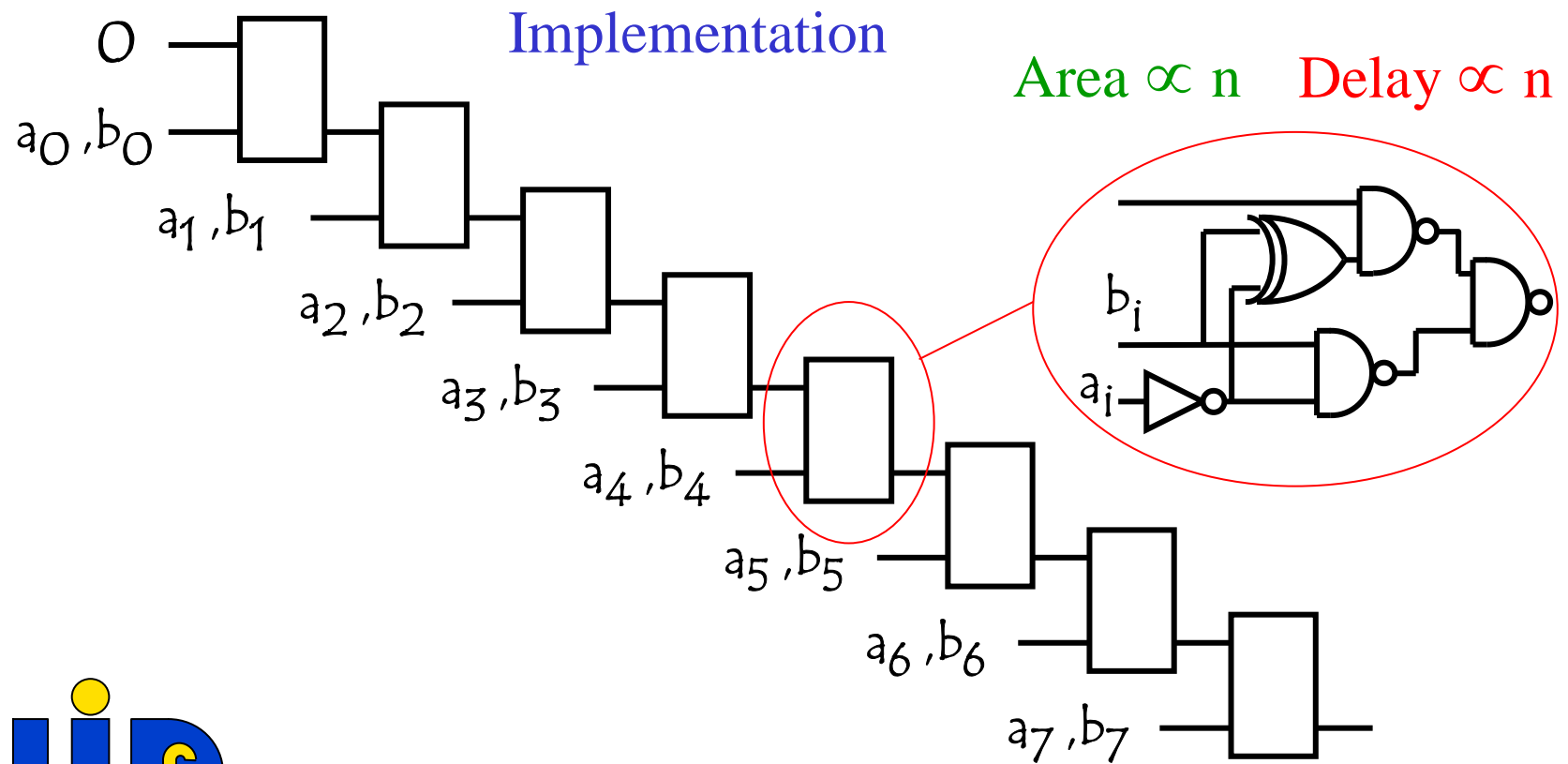
$a < b$  if :  $a_7 < b_7$  or  $(a_7 = b_7$  and  $(a_6 < b_6$  or  $(a_6 = b_6$  and ... )))

$a < b$  if :  $\overline{a_7}b_7 + ((\overline{a_7 \oplus b_7}) \cdot (\overline{a_6}b_6 + ((\overline{a_6 \oplus b_6}) \cdot \dots )))$



# Comparators

Comparing two natural numbers : <



# Comparators

Comparing two natural numbers : <

## Implementation Improvement

$a < b$  if :  $a_7 < b_7$  or ( $a_7 = b_7$  and ( $a_6 < b_6$  or ( $a_6 = b_6$  and ... )))

$a < b$  if :  $\overline{a_7}b_7 + ((\overline{a_7} \oplus b_7) \cdot (\overline{a_6}b_6 + ((\overline{a_6} \oplus b_6) \cdot \dots )))$

$\overline{a_i}b_i + (\overline{a_i} \oplus b_i) \cdot \text{previous}$

Propagation  
Generation

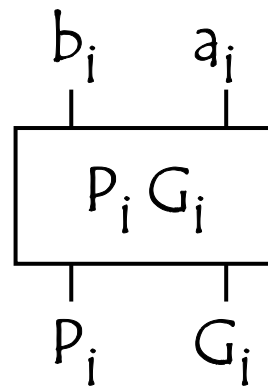




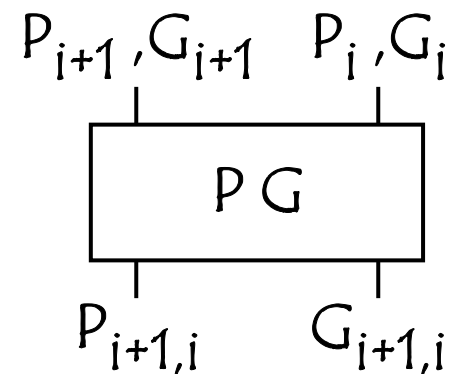
# Comparators

Comparing two natural numbers : <

Implementation Improvement

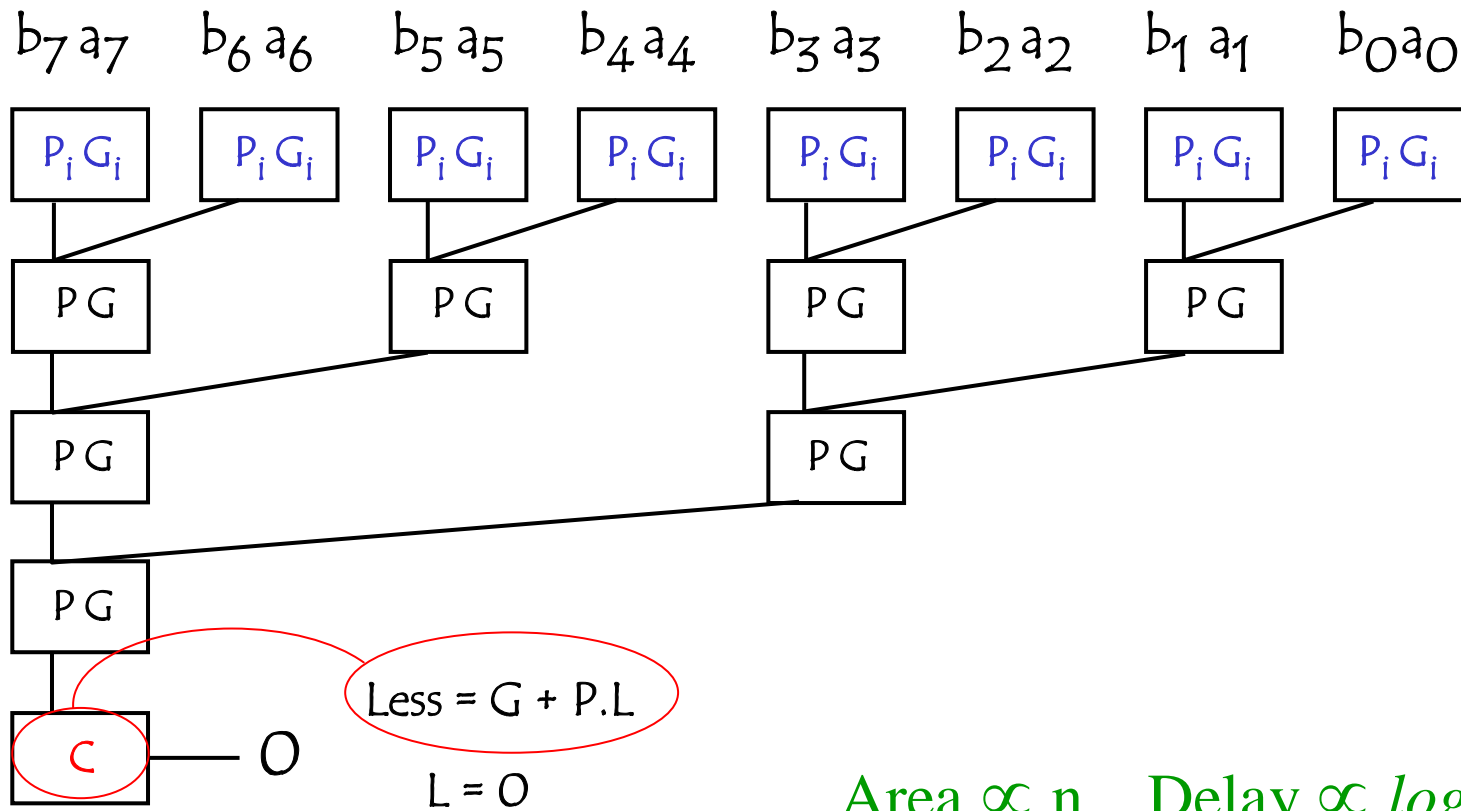


$$G_i = \bar{a}_i b_i$$
$$P_i = \bar{a}_i \oplus b_i$$



$$G_{i+1,i} = G_{i+1} + G_i \cdot P_{i+1}$$
$$P_{i+1,i} = P_i \cdot P_{i+1}$$

# Comparators



Area  $\propto n$  Delay  $\propto \log(n)$



# Comparators

