



*The Abdus Salam*  
**International Centre for Theoretical Physics**



**2065-8**

**Advanced Training Course on FPGA Design and VHDL for Hardware  
Simulation and Synthesis**

***26 October - 20 November, 2009***

**Digital Arithmetic (contd.)**

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# Outline

- Digital CMOS design

- Arithmetic operators

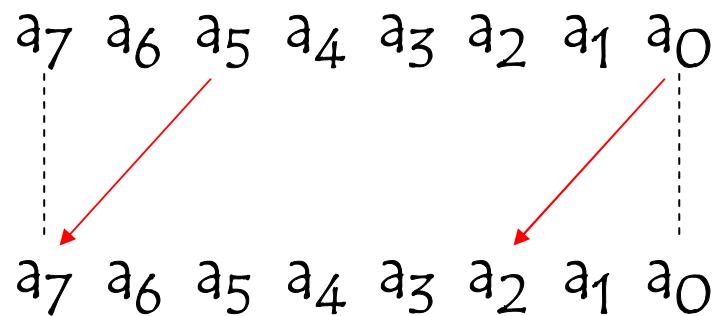
- Adders
- Comparators
- Shifters
- Multipliers



# Shifters

## Shifting a value

Let consider a value  $a$  coded on 8 bits  
 $a$  can be shifted to the left by  $n$  positions ( $0 \leq n < 8$ )



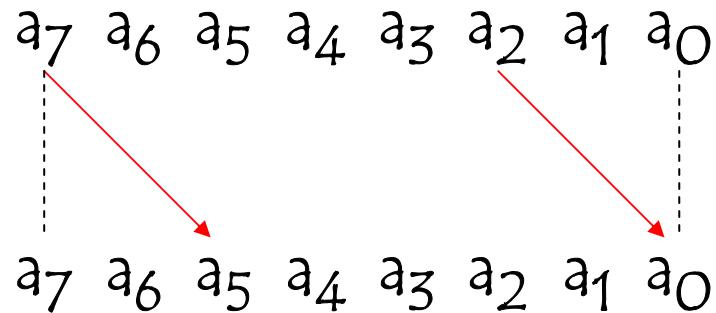
For a natural number, shift left is a multiplication by  $2^n$

# Shifters

## Shifting a value

Let consider a value  $a$  coded on 8 bits  
 $a$  can be shifted to the right by  $n$  positions ( $0 \leq n < 8$ )

logic

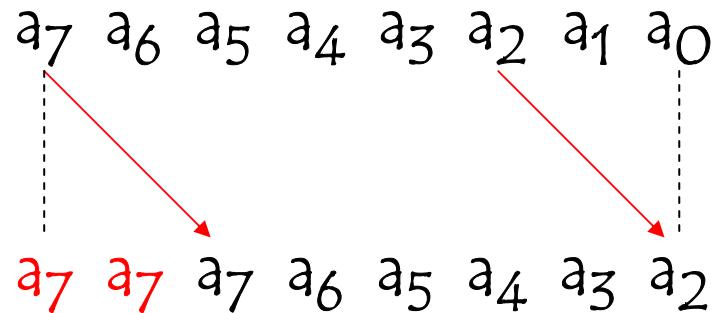


For a natural number, shift right is a division by  $2^n$

# Shifters

## Shifting a value

Let consider a value  $a$  coded on 8 bits  
 $a$  can be shifted to the right by  $n$  positions ( $0 \leq n < 8$ )  
arithmetic

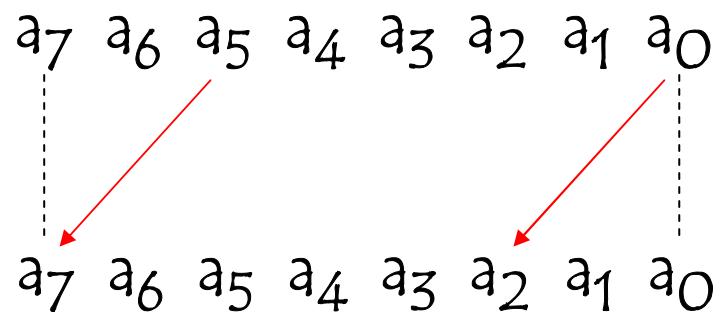


For a relative number, shift right is a division by  $2^n$

# Shifters

## Shifting a value

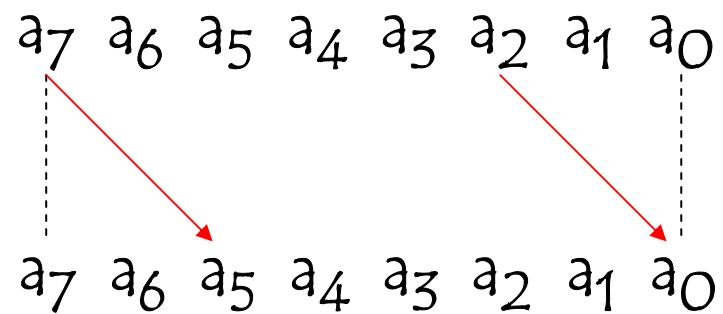
Let consider a value  $a$  coded on 8 bits  
 $a$  can be rotated to the left by  $n$  positions ( $0 \leq n < 8$ )



# Shifters

## Shifting a value

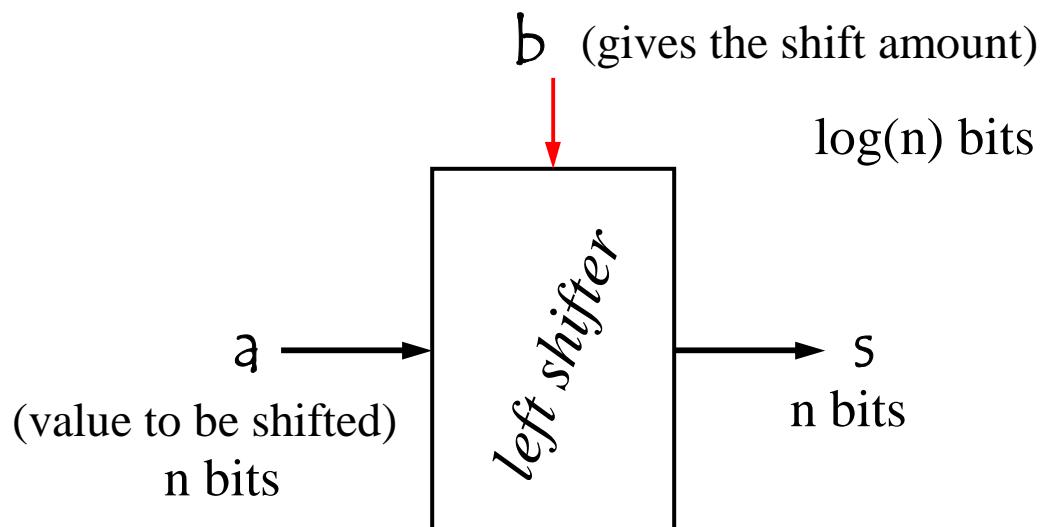
Let consider a value  $a$  coded on 8 bits  
 $a$  can be rotated to the right by  $n$  positions ( $0 \leq n < 8$ )



# Shifters

## Shifting a value

### Implementation (left shifter)



# Shifters

Shifting a value by 1 position to the left

Boolean function

$b$  coded on 1 bit

If  $b = 0$

$$s_i = a_i$$

else

$$s_i = a_{i-1} \text{ assuming } a_{-1} = 0$$

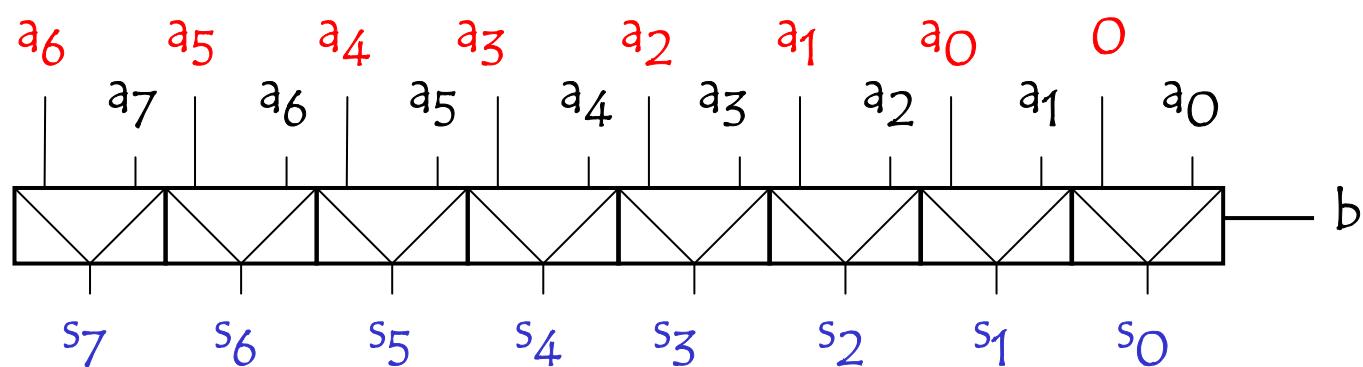
$$s_i = b.a_{i-1} + \bar{b}.a_i \quad \rightarrow \quad \text{2-input Multiplexer}$$



# Shifters

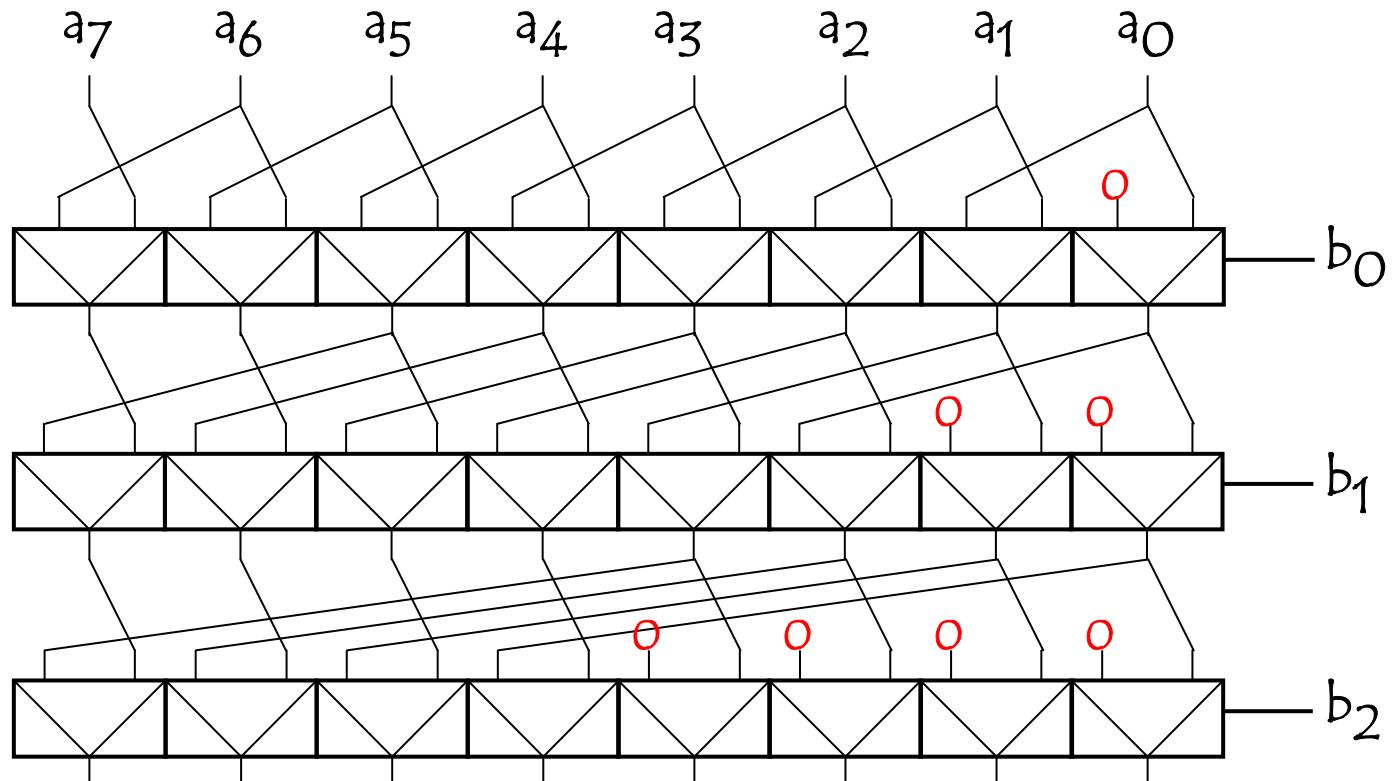
Shifting a value by 1 position to the left

## Implementation



# Shifters

Shifting a value to the left

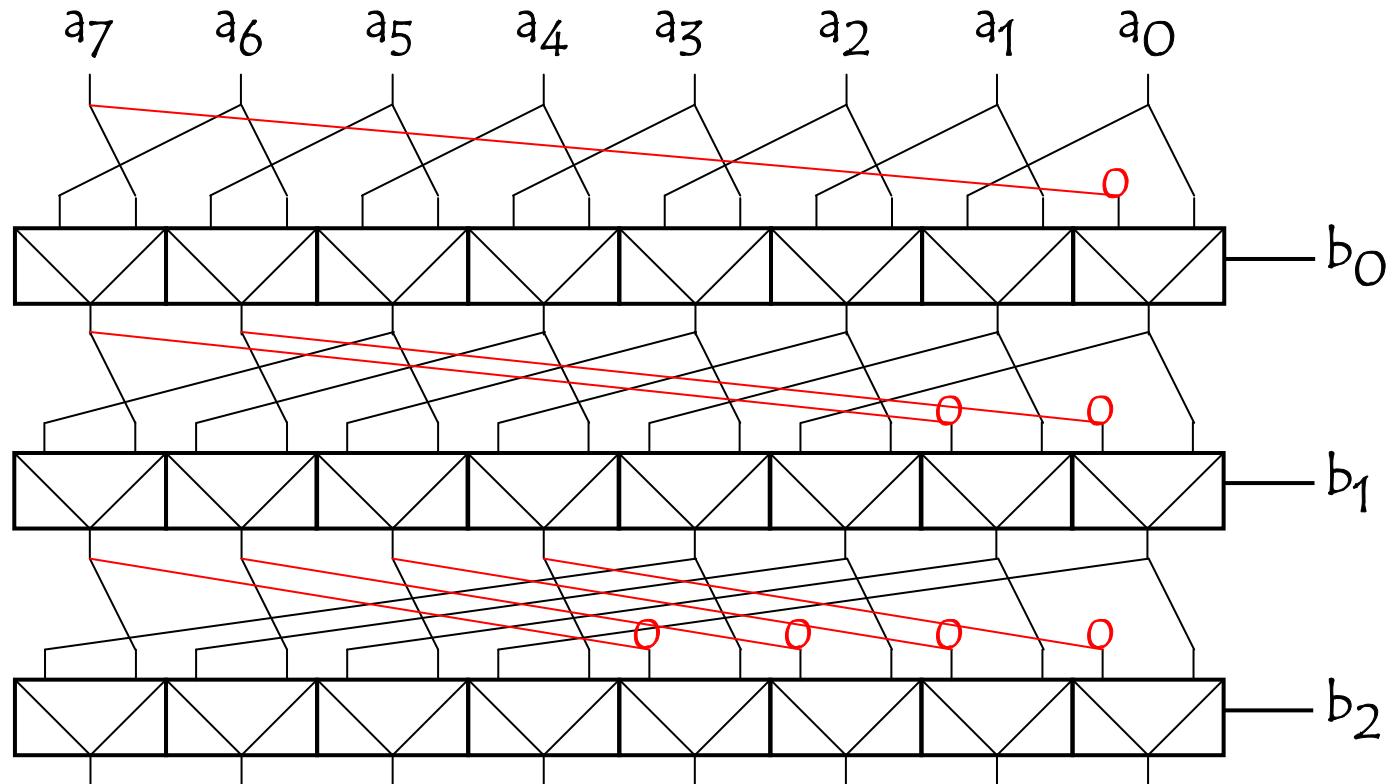


Area  $\propto n \log(n)$  Delay  $\propto \log(n)$



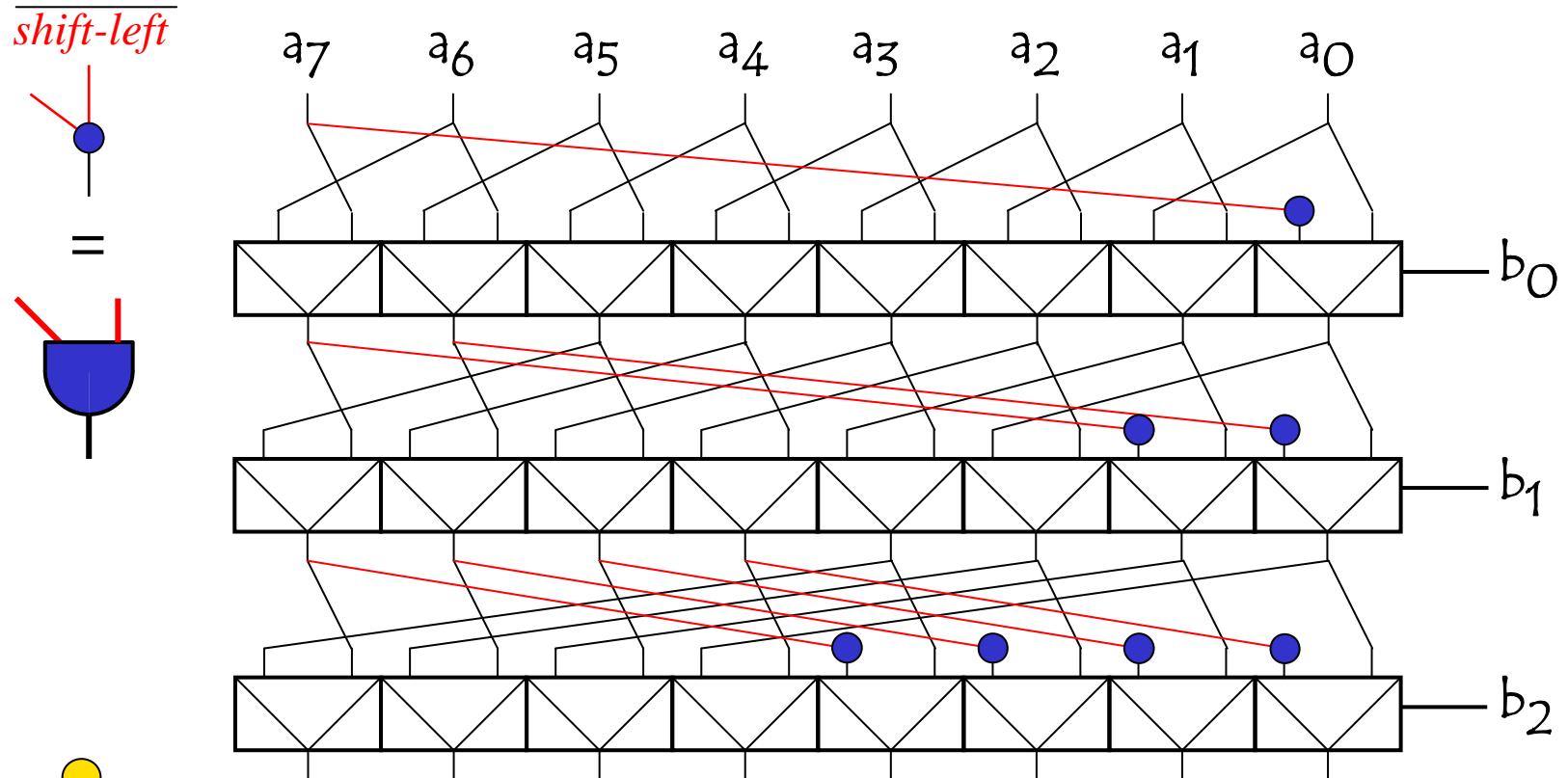
# Shifters

Rotate a value to the left



# Shifters

Shift / Rotate a value to the left



# Shifters

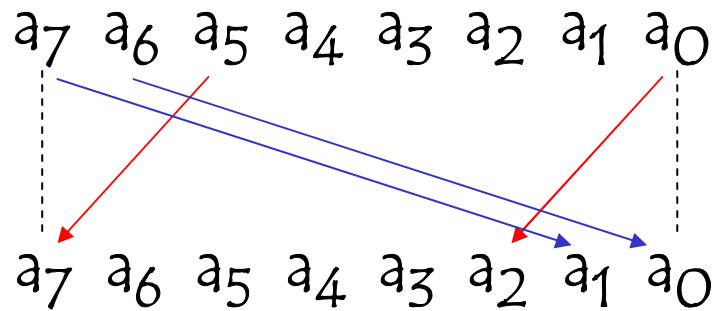
Shifting a value

Rotate left by 2

$6 = -2$

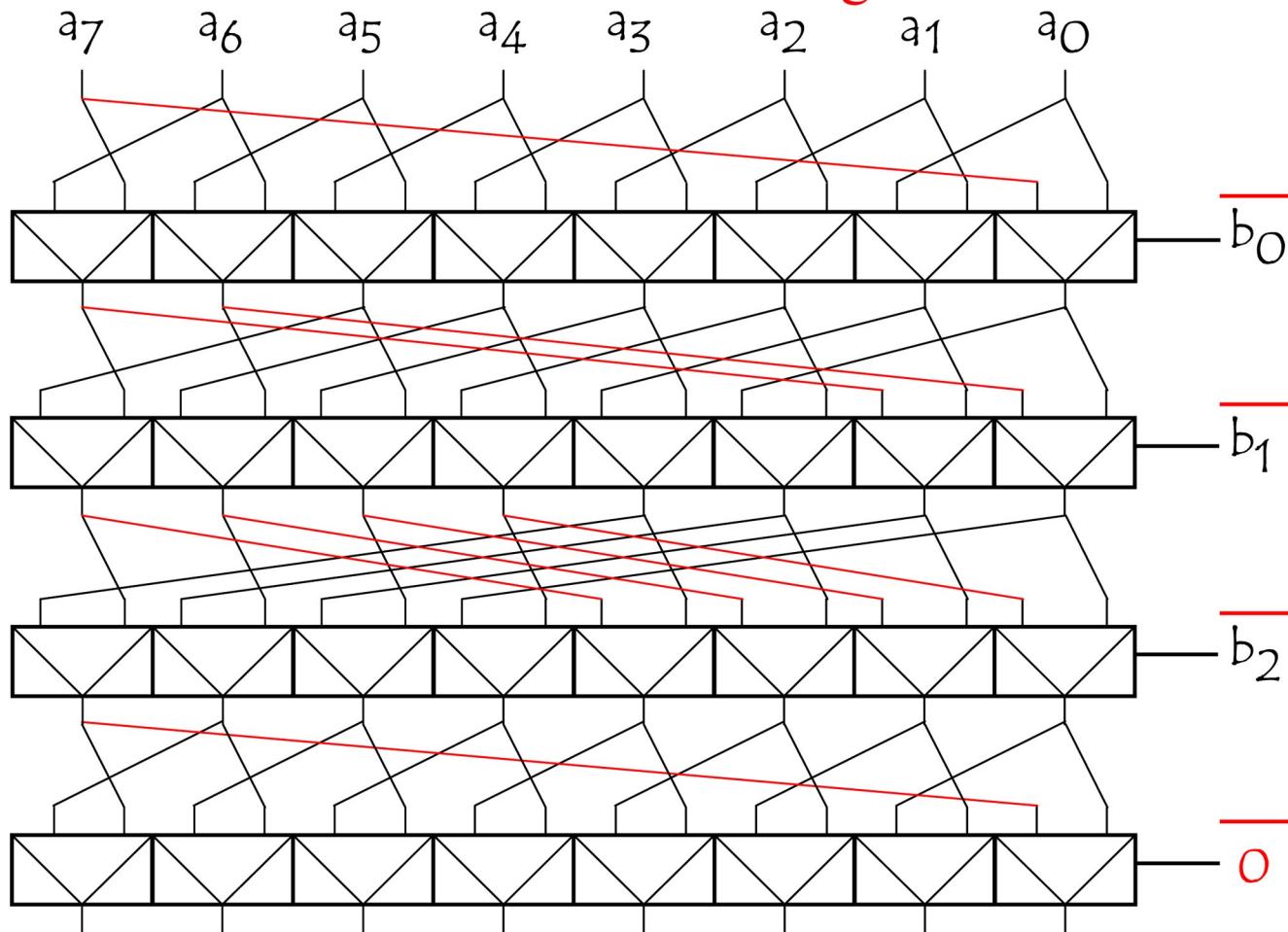
$=$   
Rotate right by 6

$110 = \overline{010} + 1$



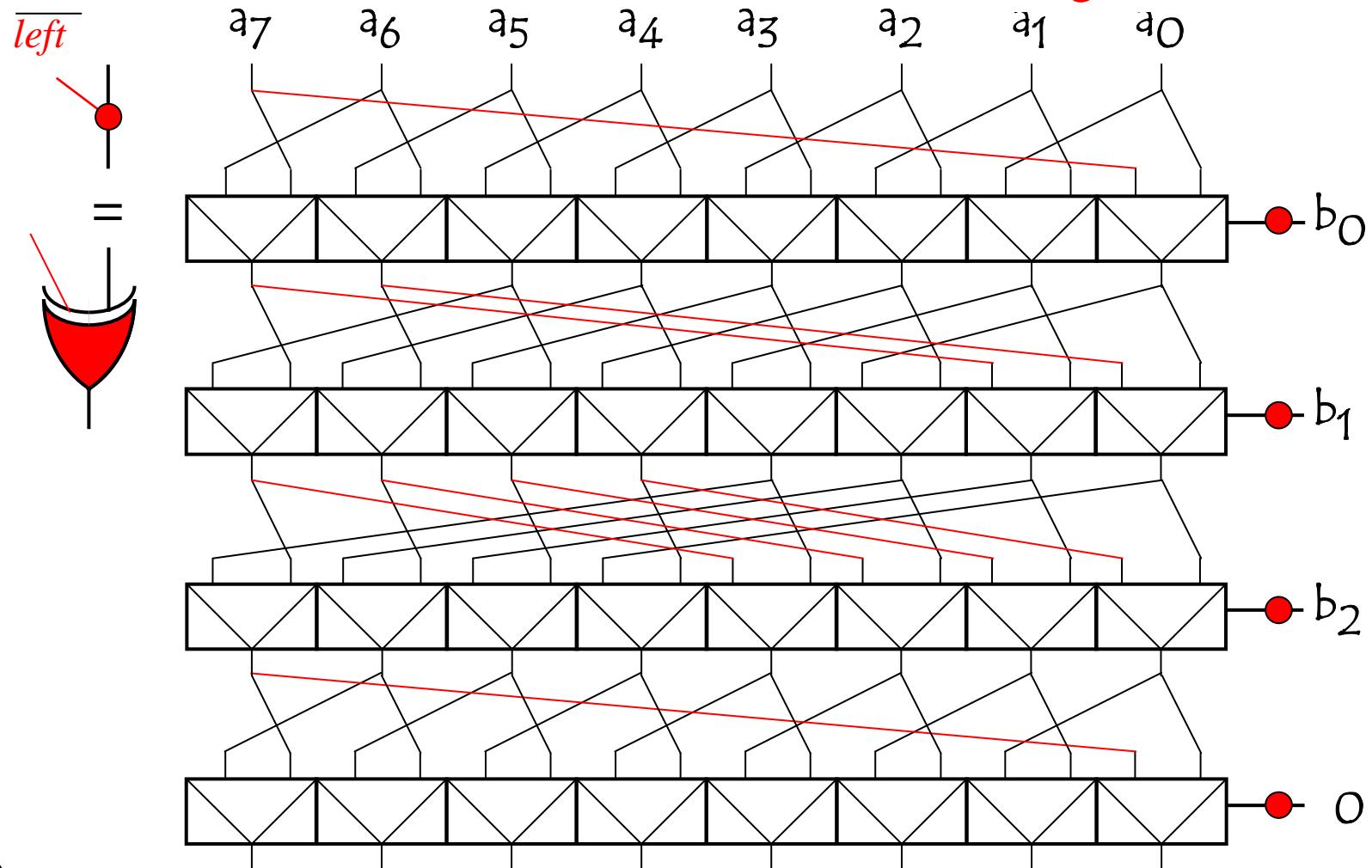
# Shifters

Rotate a value to the **right**



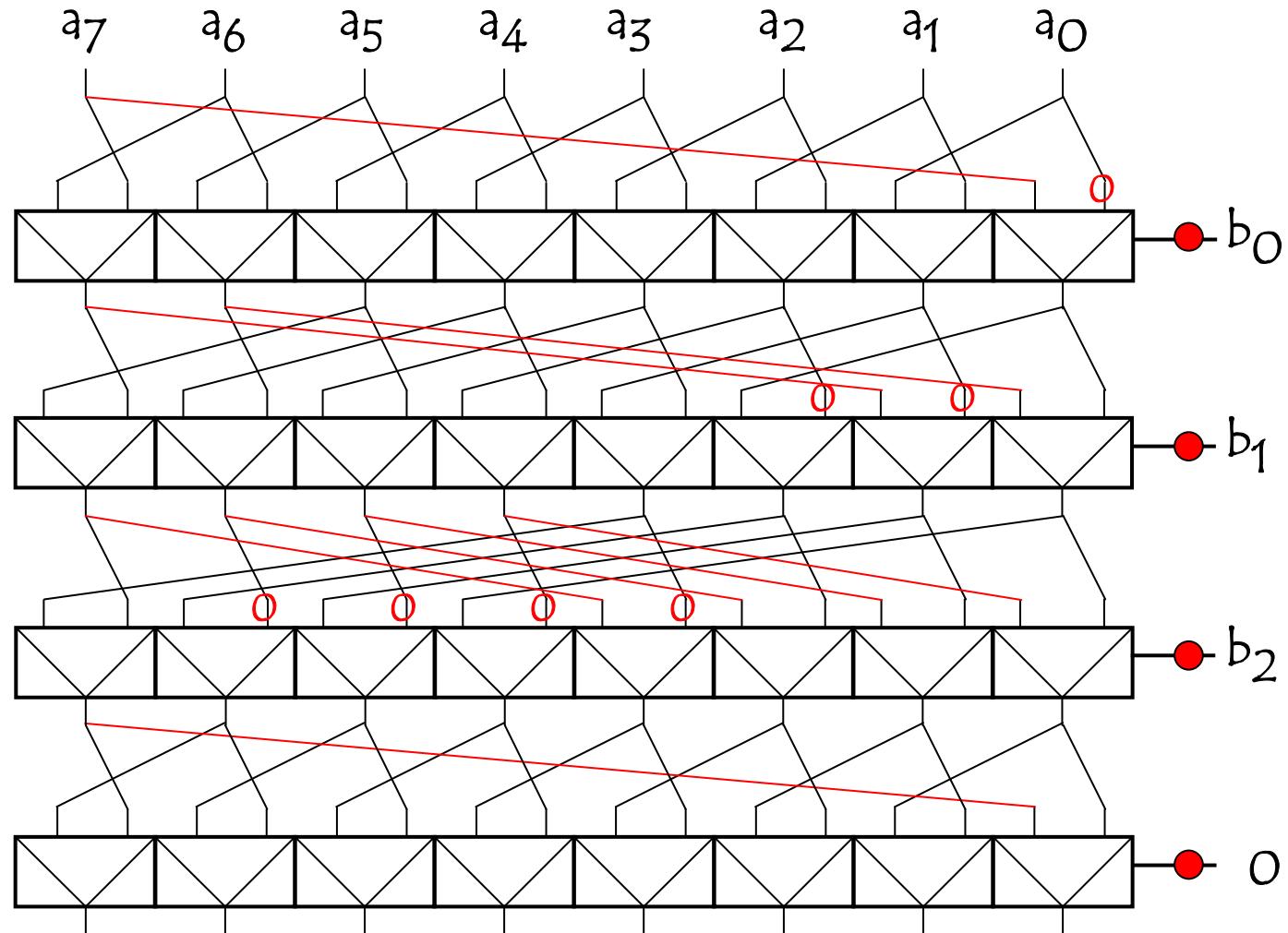
# Shifters

Rotate a value to the **left / right**



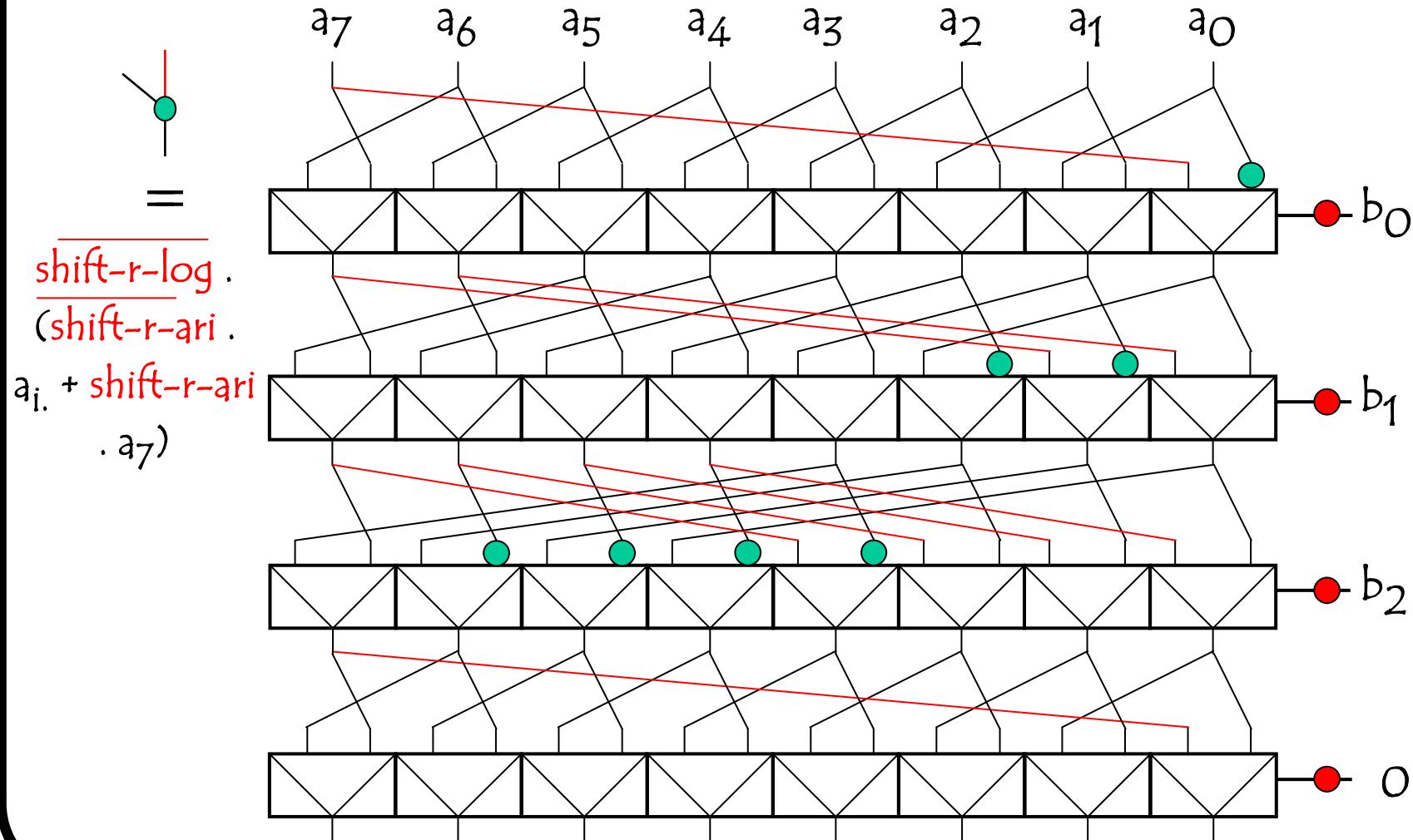
# Shifters

Shift logic a value to the right



# Shifters

Shift / Rotate a value to the right



# Shifters

Shift / Rotate a value to the left / right

