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Optimal strategy of fishing problem for hermaphrodite population

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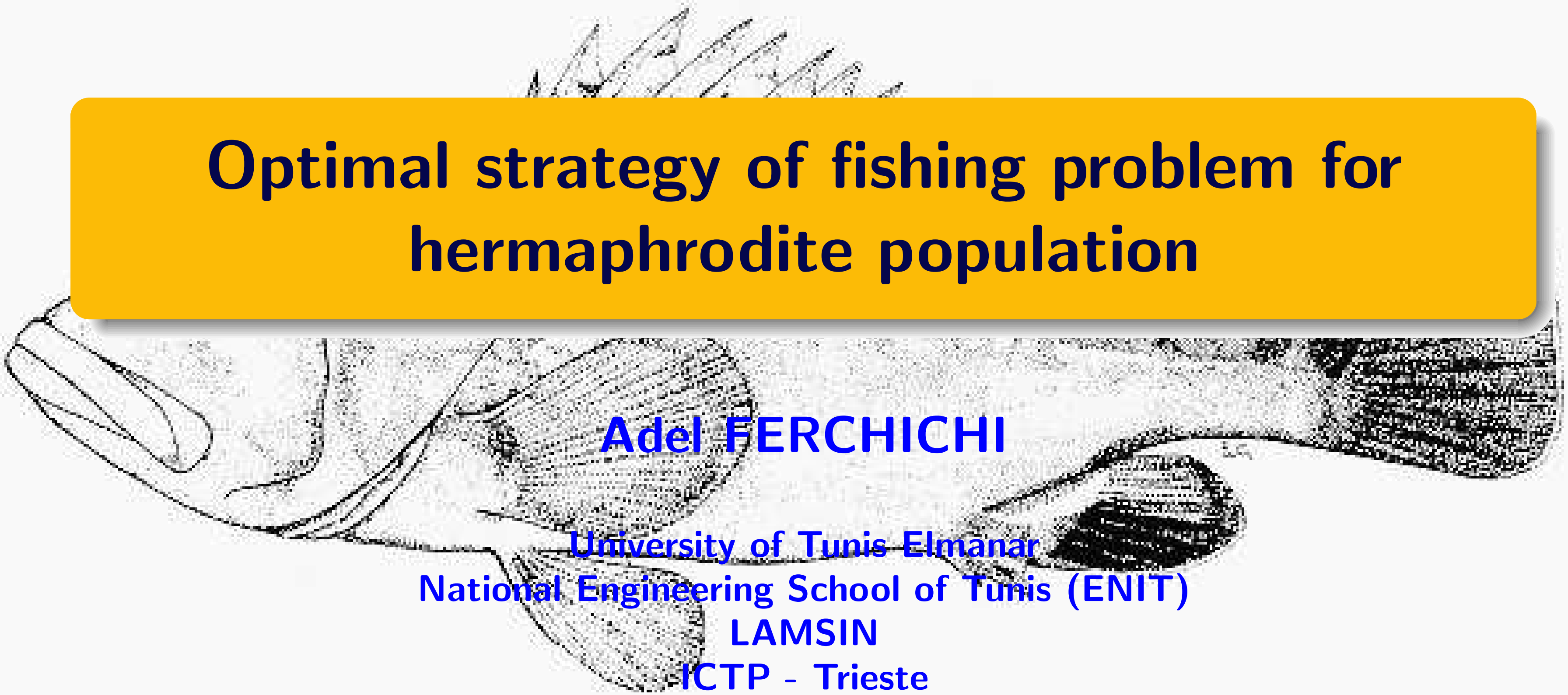

Optimal strategy of fishing problem for hermaphrodite population

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FAO

Outlines

- Introduction
- Model construction
- Demographic model
- Mathematical model
- Optimisation problem
- Optimal control of fishing
- Conclusion and perspectives

Introduction

The effect of fishing activities on marine populations dynamics is currently approximated by the mean of theoretical fisheries models. Models representing the evolution of a stock exploited, are divided in two groups: global models and structured models.

- Global models (Clark et al.).
- Structured models (Armsworth et al, Alonzo et al, R.R Warner et al.).

Our work consists on the study of a structured model representing the grouper population and we try to find an optimal policy of the fishing problem.

Species biology

Some marine fish change from male to female and others change from female to male; these fish are called hermaphrodites.
Protogynous hermaphrodites are those in which, individuals begin life as females and subsequebtly become males.
Grouper are protogynous hermaphrodite because they can produce successively female and then male gametes 9Sadovy et al, R.R Warner).

- After fecundation, the fertilized egg remains in the plankton until it hatches to let the larva out. Then each larva goes down to the bottom of the sea and occupies a small hole, its shelter.
- In spring, when the water warms up, the larva can reach 10 cm in length and thus passes from the larva class to the immature adult class.
- The young grouper is females, immature up to 5 years. The time to move from the class of immature adults to the class of females is 4 years.
- The mechanism of sexual inversion occurs between 9 and 16 years old.

Demographic model

We subdivide the population into 4 classes according to the length of the individual:

- Larva,
- Immature adults,
- Female
- Male.

Let $n_i(t)$ be the number of individuals in the class i $(i = 1, \dots, 4)$ at time t and $N_t = (n_1(t).., n_4(t))^T$ the vector which describes the number of individuals in each class at time t.

Reproduction

The reproduction rate is calculated by taking the ratio of the total number of living immatures at birth by the average number of fertilized females for a given year. Reproduction rate depends on the number of eggs produced per year and on the survival rate from the state eggs to the youthful grouper.

Mortality

Mortality, or mortality rate, is the number of annual deaths reported to the number of individuals in a given territory.
The eggs and larvae suffer from a predation pressure, leading to a high death rate.

Dispersion

There are two types of dispersion:

- A dispersion which affects larvae due to physical and chemical environmental conditions (water currents, wind, nature of water). larvae suffer from a predation pressure, leading to a high death rate, immature adults leave to go and occupy free shelters in nearby territories, but adults have very few predators because of their size, they sedentary.
- A juvenile dispersion due to the competition for shelter.The outcome of the shelter competition between two groups is always in favor of the older group and leads immature adults to completely leave the area.

The diagram of the model

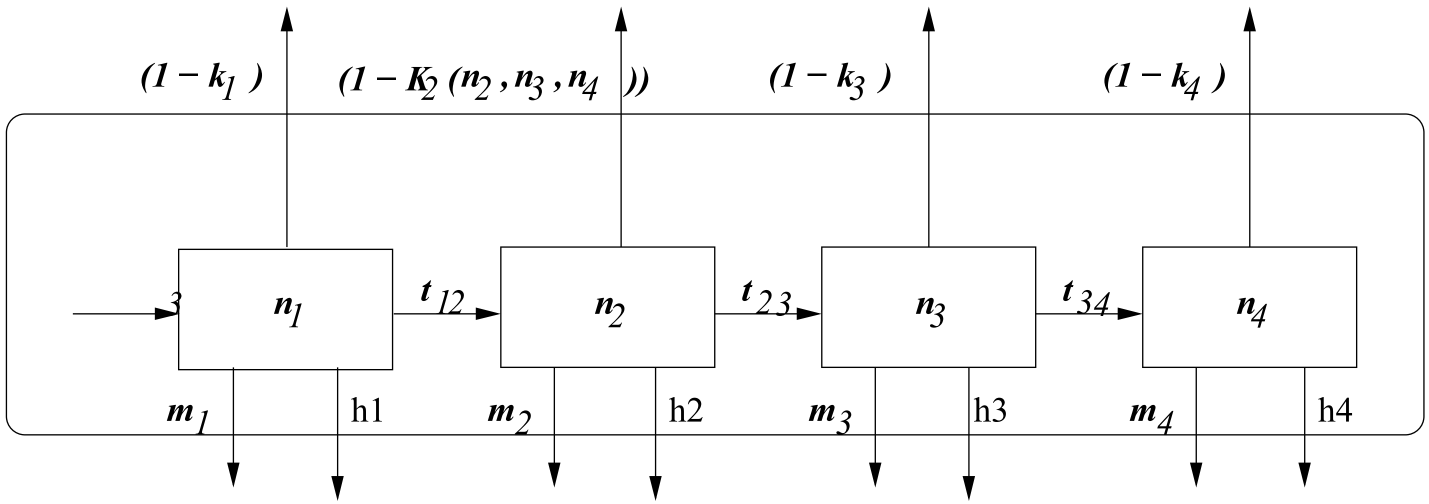


Figure: Conceptual diagram of the Grouper population

The diagram of the model

parameters definition

- s_i: The survival rate of the class i.
- $t_i, i+1$: The transition rate of individuals from the class i to the class $i+1$.
- m_i: The natural mortality of individuals of the class i, we have $(m_i = 1 - s_i - t_i, i+1)$
- $.k_i$: The rate of the remaining individuals in the class i after dispersion.
- K_2: The function giving the rate of remaining immatures after dispersion. We have

$$K(n_2, n_3, n_4) = \frac{T - t_{23}n_2 - (s_3 + t_{34})n_3 - s_4 n_4}{T}$$

This function is density dependent, it gives the rate of remainder immature adults after dispersion.

Mathematical model

When we have a no fishing territory, the model is:

$$N_{t+1} = M(n_2, n_3, n_4) L N_t$$

L: The density dependent matrix associated to the demographic process.
$M(n_2, n_3, n_4)$: The density dependent matrix associated to the dispersal process.

$$L = \begin{pmatrix} s_1 & 0 & f & 0 \\ t_{12} & s_2 & 0 & 0 \\ 0 & t_{23} & s_3 & 0 \\ 0 & 0 & t_{34} & s_4 \end{pmatrix}, \text{ and}$$

$$M(n_2, n_3, n_4) = \begin{pmatrix} k_1 & 0 & 0 & 0 \\ 0 & K_2(n_2, n_3, n_4) & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{pmatrix}.$$

The final model is:

$$\left\{ \begin{array}{lcl} n_1(t+1) & = & k_1 s_1 n_1(t) + k_1 f n_3(t) \\ n_2(t+1) & = & K_2(n_2, n_3, n_4) t_{12} n_1(t) + K_2(n_2, n_3, n_4) s_2 n_2(t) \\ n_3(t+1) & = & k_3 t_{23} n_2(t) + k_3 s_3 n_3(t) \\ n_4(t+1) & = & k_4 t_{34} n_3(t) + k_4 s_4 n_4(t) \end{array} \right. \quad (1)$$

Such that :

$$\forall i = 1...4; n_i(0) = n_{i,0}$$

Mathematical model

Let $H(t) = (h_1(t), ..., h_4(t))'$ the vector which describes the number of individuals captured by fishing. The complete model is :

$$N(t+1) = MLN(t) - H(t) \tag{2}$$

Where, For all i=1...4,

$$h_i(t) = q_i E_i(t) n_i(t)$$

(Beverton Holt)
$E_i(t)$ is the fishing effort(vessels number) between t and $t+1$ and q_i the catchability, it is the capture probability per unit of effort.

Optimisation principle

The system (3) is thus expressed by:

$$\begin{cases} n_1(t+1) &= k_1 s_1 n_1(t) + k_1 f n_3(t) - q_1 E_1(t) n_1(t) \\ n_2(t+1) &= K_2 t_{12} n_1(t) + K_2 s_2 n_2(t) - q_2 E_2(t) n_2(t) \\ n_3(t+1) &= k_3 t_{23} n_2(t) + k_3 s_3 n_3(t) - q_3 E_3(t) n_3(t) \\ n_4(t+1) &= k_4 t_{34} n_3(t) + k_4 s_4 n_4(t) - q_4 E_4(t) n_4(t) \end{cases} \qquad (3)$$

Let $\pi(N_t, H_t)$ the total net revenue in the period t:

$$\pi(N_t, H_t) = pH(t) - cE(t) = (pqN(t) - c)E(t)$$

Thus we have :

$$\begin{aligned} \pi(N_t, H_t) \quad = \quad & (p_1 q_1 n_1(t) - c_1)\, E_1(t) + (p_2 q_2 n_2(t) - c_2)\, E_2(t) + \\ & (p_3 q_3 n_3(t) - c_3)\, E_3(t) + (p_4 q_4 n_4(t) - c_4)\, E_4(t). \end{aligned}$$

The total discounted net revenue on an infinite horizon derived from the exploitation of the resource is given by:

$$J(E_1(t), E_2(t), E_3(t), E_4(t)) = \sum_{t=1}^{\infty} \alpha^{t-1} \pi(N_t, H_t)$$

Where $\alpha > 0$ the constant denoting the rate of discount. Thus, we have :

Our goal is the maximization of the total revenue. Moreover, we assume that we can act on the fishing effort E. Then, we face the following control problem :

$$\max_{E_1,E_2,E_3,E_4} J(E_1(t), E_2(t), E_3(t), E_4(t)) \tag{4}$$

Such that (3) is verified and the controls E_i are constrained:

$$\forall i = 1...4; E_{i,min} \leq E_i(t) \leq E_{i,max}, \forall t \geq 0 \tag{5}$$

Why we use the maximum principle?

General objective

The aim is to maintain the system state fisheries around a point of operation n^* (threshold point or equilibum point) by ensuring a sustainable development of the resource and ensuring at the same time a maximization of fishing revenues by using an optimal control. This control has to minimize or to maximize the efforts of fisheries around this point so as to ensure a conservation of the population and a stable income.

Maximum principle

The maximum principle is formulated in terms of the following expression called the Hamiltonian:

$$H(t) = \alpha^{t-1}\pi(N_t, H_t) + \lambda(t)(MLN(t) - H(t)) \quad (6)$$

Where $\lambda(t) = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t))^T$ is the additional unknown function called the adjoint variable.

Resolution steps

- Define the Hamiltonian
- Use the equation$\forall i = 1...4; \frac{\partial H}{\partial \lambda_i} = 0$ to find a expression of λ_i.
- Use the equation $\lambda_i(t) - \lambda_i(t-1) = -\frac{\partial H(t)}{\partial n_i(t)}$.
- Make the equality between the two equations to find a solution of the problem.

The maximum principle asserts the following equation:

$$\forall i = 1...4; \frac{\partial H}{\partial \lambda_i} = 0$$

Thus we have :

$$\begin{cases} \alpha^{t-1}(p_1 q_1 n_1 - c_1) - \lambda_1(t) q_1 n_1 & = & 0 \\ \alpha^{t-1}(p_2 q_2 n_2 - c_2) - \lambda_2(t) q_2 n_2 & = & 0 \\ \alpha^{t-1}(p_3 q_3 n_3 - c_3) - \lambda_3(t) q_3 n_3 & = & 0 \\ \alpha^{t-1}(p_4 q_4 n_4 - c_4) - \lambda_4(t) q_4 n_4 & = & 0 \end{cases} \quad (7)$$

We deduce from (8) that:

$$\lambda_i(t) = \alpha^{t-1}(p_i - \frac{c_i}{q_i n_i}), \forall i = 1, \ldots, 4 \qquad (8)$$

And consequently :

$$\begin{cases} \lambda_1(t) - \lambda_1(t-1) & = & (\alpha^{t-1} - \alpha^{t-2})(p_1 - \frac{c_1}{q_1 n_1}) \\ \lambda_2(t) - \lambda_2(t-1) & = & (\alpha^{t-1} - \alpha^{t-2})(p_2 - \frac{c_2}{q_2 n_2}) \\ \lambda_3(t) - \lambda_3(t-1) & = & (\alpha^{t-1} - \alpha^{t-2})(p_3 - \frac{c_3}{q_3 n_3}) \\ \lambda_4(t) - \lambda_4(t-1) & = & (\alpha^{t-1} - \alpha^{t-2})(p_4 - \frac{c_4}{q_4 n_4}) \end{cases} \qquad (9)$$

Then, we use the adjoint equation: $\lambda_i(t) - \lambda_i(t-1) = -\frac{\partial H(t)}{\partial n_i(t)}$

We obtain the following equations:

$$
\begin{cases}
\lambda_1(t)-\lambda_1(t-1) = & -\alpha^{t-1}p_1q_1E_1+\alpha^{t-1}(\frac{c_1}{q_1n_1}-p_1)(k_1s_1-q_1E_1)+ \\
& \alpha^{t-1}(\frac{c_2}{q_2n_2}-p_2)K_2(n_2,n_3,n_4)t_{12}-\alpha^{t-1}\frac{c_1}{q_1n_1^2}(k_1s_1n_1+ \\
& k_1fn_3-q_1E_1n_1). \\
\lambda_2(t)-\lambda_2(t-1) = & -\alpha^{t-1}p_2q_2E_2+\alpha^{t-1}(\frac{c_2}{q_2n_2}-p_2)(-t_{23}t_{12}n_1-t_{23}s_2n_2+ \\
& K_2(n_2,n_3,n_4)s_2-q_2E_2)-\alpha^{t-1}\frac{c_2}{q_2n_2^2}[K_2(n_2,n_3,n_4)t_{12}n_1+ \\
& K_2(n_2,n_3,n_4)-q_2E_2n_2]+\alpha^{t-1}(\frac{c_3}{q_3n_3}-p_3)k_3t_{23}. \\
\lambda_3(t)-\lambda_3(t-1) = & -\alpha^{t-1}p_3q_3E_3+\alpha^{t-1}(\frac{c_1}{q_1n_1}-p_1)k_1f+\alpha^{t-1}(\frac{c_2}{q_2n_2}-p_2)(-s_3- \\
& t_{34})(t_{12}+s_2n_2)-\alpha^{t-1}\frac{c_3}{q_3n_3^2}(k_3t_{23}n_2+k_3s_3n_3-q_3E_3n_3)+ \\
& \alpha^{t-1}(\frac{c_3}{q_3n_3}-p_3)(k_3s_3-q_3E_3)+\alpha^{t-1}(\frac{c_4}{q_4n_4}-p_4)k_4t_{34}. \\
\lambda_4(t)-\lambda_4(t-1) = & -\alpha^{t-1}p_4q_4E_4+\alpha^{t-1}(\frac{c_2}{q_2n_2}-p_2)s_4-\alpha^{t-1}\frac{c_4}{q_4n_4^2}(k_4t_{34}n_3+ \\
& k_4s_4n_4-q_4E_4n_4)+\alpha^{t-1}(\frac{c_4}{q_4n_4}-p_4)(k_4s_4-q_4E_4).
\end{cases}
\tag{10}
$$

Making the equality between the two systems (13) and (14), we obtain:

$$\left\{\begin{array}{lcl}(\alpha^{t-1}-\alpha^{t-2})(\frac{c_1}{q_1 n_1}-p_1) & = & \alpha^{t-1}(-p_1 k_1 s_1+(\frac{c_2}{q_2 n_2}-p_2)K_2(n_2,n_3,n_4)t_{12}- \\ & & \frac{c_1}{q_1 n_1^2}k_1 f n_3) \\ (\alpha^{t-1}-\alpha^{t-2})(\frac{c_2}{q_2 n_2}-p_2) & = & \alpha^{t-1}((\frac{c_2}{q_2 n_2}-p_2)(-t_{23}t_{12}n_1-t_{23}s_2 n_2)- \\ & & K_2(n_2,n_3,n_4)[p_2 s_2-\frac{c_2 t_{12} n_1}{q_2 n_2^2}]+(\frac{c_3}{q_3 n_3}-p_3)k_3 t_{23}) \\ (\alpha^{t-1}-\alpha^{t-2})(\frac{c_3}{q_3 n_3}-p_3) & = & \alpha^{t-1}((\frac{c_1}{q_1 n_1}-p_1)+(\frac{c_2}{q_2 n_2}-p_2)(-s_3-t_{34})(t_{12}n_1+ \\ & & s_2 n_2)-\frac{c_3}{q_3 n_3^2}k_3 t_{23} n_2-p_3 k_3 s_3+(\frac{c_4}{q_4 n_4}-p_4)k_4 t_{34}) \\ (\alpha^{t-1}-\alpha^{t-2})(\frac{c_4}{q_4 n_4}-p_4) & = & \alpha^{t-1}((\frac{c_2}{q_2 n_2}-p_2)(t_{12}n_1+s_2 n_2)s_4-\frac{c_4}{q_4 n_4^2}k_4 t_{34} n_3- \\ & & p_4 k_4 s_4)\end{array}\right. \tag{11}$$

we have:

$$n_3^*(n_4) = \frac{(\alpha - 1 - \alpha k_4 s_4) p_4 q_4 n_4^2 + (1-\alpha) c_4 n_4}{\alpha c_4 k_4 t_{34}} \tag{12}$$

$$n_4^*(n_2, n_3) = \frac{\alpha c_4 q_3 n_3^2}{[(1-\alpha) p_3 + p_3 k_3 s_3 + \alpha p_4 k_4 t_{34}] q_3 q_4 n_3^2 + (1-\alpha) c_3 q_4 n_3 + \alpha q_4 c_3 k_3 t_{23} n_2} \tag{13}$$

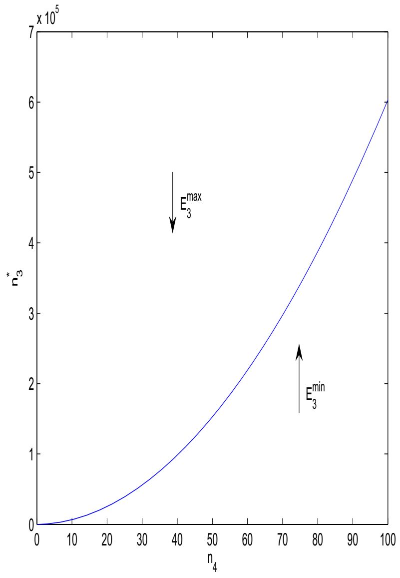
Optimal control of fishing

Theorem

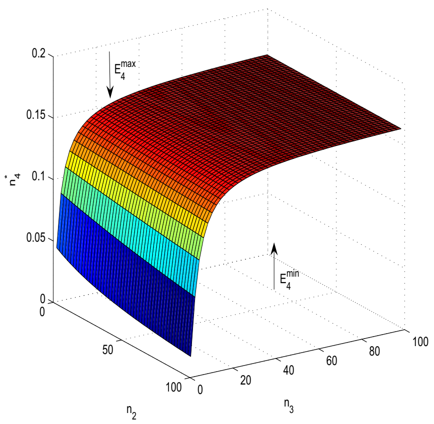
Let the couple $\{n_3, n_4\}$ satisfies (4), then the optimal policy of the fishing problem is given as following:

1. If $n_3 > n_3^*(n_4)$ then the optimal strategy is $E_3^* = E_3^{\max}$.
2. If $n_3 < n_3^*(n_4)$ then the optimal strategy is $E_3^* = E_3^{\min}$.
3. If $n_4 > n_4^*(n_2, n_3)$ then the optimal strategy is $E_4^* = E_4^{\max}$.
4. If $n_4 < n_4^*(n_2, n_3)$ then the optimal strategy is $E_4^* = E_3^{\min}$.

Optimal control of fishing



(a) Curve of the function $n_3^*(n_4)$.



(b) Curve of the function $n_4^*(n_2, n_3)$.

Disscussion and Conclusion

1. Let $n_3 > n_3^*(n_4)$, then we have an important biomass of females. Then, in order to have considerable fishing revenues, we must maximize the fishing effort, then $E_3^* = E_3^{\max}$.
2. Let $n_3 < n_3^*(n_4)$, then we have a minor amount of females and the population is threatened. Thus, in order to preserve it, we must stop fishing this stage. Thus $E_3^* = 0$.
3. Let $n_4 > n_4^*(n_2, n_3)$,. Thus, we must maximize the fishing effort in order to ensure a important income, then $E_4^* = E_4^{\max}$.
4. If $n_4 < n_4^*(n_2, n_3)$, then the population is in danger. So, it is prohibited to capture this stage. Then $E_4^* = 0$.
5. Finally, if $n_3 > n_3^*(n_4)$ and $n_4 > n_4^*(n_2, n_3)$, then the biomass of both males and females is broad and important. The optimal control consists of holding $(E_3^*, E_4^*) = (E_3^{\max}, E_4^{\max})$.

It is easy to verify that

$$n_3^* = A_1 p_4 + B_1$$

and

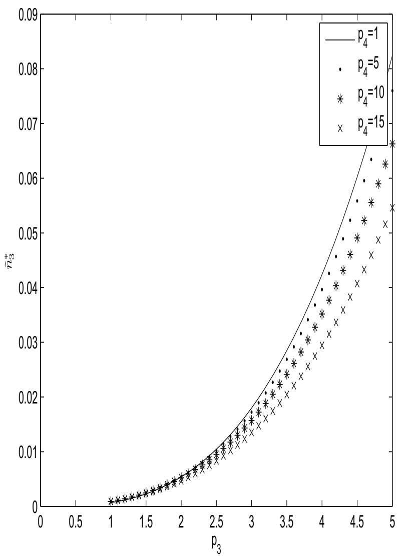
$$n_4^* = \frac{A_2}{B_2 p_3 + C_2 p_4 + D_2}$$

with A_1, A_2, B_1, B_2, C_2 and D_2 are strictly positive constants.
We remark that, at fixed costs, n_3^* depends linearly on p_4, thus n_3^* is proportional to the price of males p_4. But n_4^* is inversely proportional to both p_3 and p_4.
On the other hand, if the price and the cost are are linearly proportional, i.e $p_4 = ap_3$ and $c_4 = bc_3$ where $a > 0$ and $b > 0$ then we have

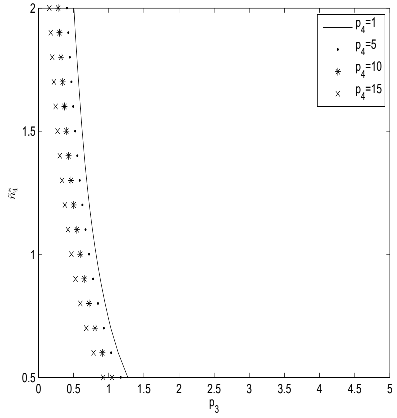
$$n_3^* = E(\frac{p_3}{c_3}) + F$$

and

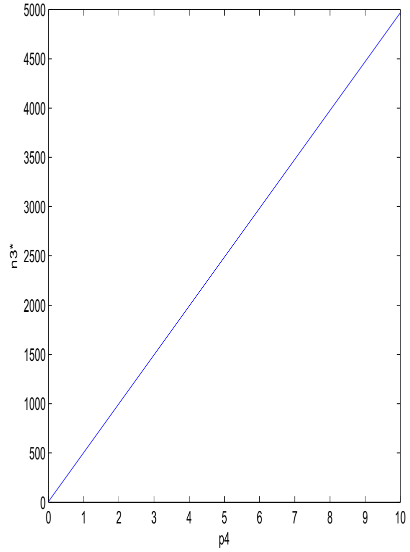
$$n_4^* = (\frac{G}{H}(\frac{p_4}{c_4}) + \frac{I}{H})^{-1}$$



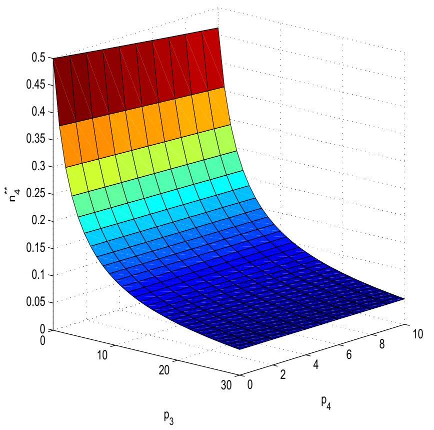
(c) Curve of the function $n_3^*(p_3)$ for different values of p_4.



(d) Curve of the function $n_4^*(p_3)$ for different values of p_4.



(e) Curve of the function $n_3^*(p_4)$.



(f) Curve of the function $n_4^*(p_3, p_4)$.

Conclusion and Perspectives

- Indeed, the grouper fishery is particularly difficult to protect, the slow development of grouper to sexual maturity means that rebuilding a population could take years.
- The effect of fishing on protogynous populations are difficult to measure without very complete information on the reproductive patterns, sex ratios, and other biological aspects of fish stocks.
- It would be interesting to study the impact of the creation of marines reserves on the dynamics of the hermaphrodite population.
- It is also interesting to study the effect of climate change on the dynamics of these hermaphrodite populations.