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Nuclear reactor dynamics - IV & V

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A unifying approach to reactor kinetics by variational methods

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Outline

- Introduction
- Variational formulation of kinetic models
  - Multipoint
  - Modal Methods
- Results and performance comparison
- Conclusions

Introduction

- Space-energy models are needed for the time-dependent analysis of nuclear reactors
- Approximate kinetic models can be of use to provide accurate results with reduced computational effort
- The variational approach is useful in the study of approximate methods ...

Introduction

- All kinetic models can be given a unifying variational foundation
  - Point kinetics
  - Quasi-statics
  - Multipoint kinetics
  - Synthesis methods

Favorite J. A., Stacey W. M. Jr, A new variational functional for space-time neutronics, *Nuclear Science and Engineering*, **125**, 101–106 (1997).

Favorite J. A., Stacey W. M. Jr, Variational estimates for use with the improved quasistatic method for reactor dynamics, *Nuclear Science and Engineering*, **126**, 282–292 (1997).

Dulla S., Picca P., Consistent multipoint kinetics for source-driven systems, *Progress in Nuclear Energy*, **48**, 617–628 (2006).

- The variational approach is very useful to discuss potentialities and shortcomings of various methods

Variational formulation of kinetic models

- The variational approach is based on the following steps:
  - Definition of a general functional with physical significance
  - Introduction of the balance equations + BC and IC with *proper* Lagrange multipliers
  - Determination of equations for the Lagrange multipliers
  - Introduction of suitable trial functions

Variational formulation of kinetic models

$$H[\varphi,\mathscr{E}_i,\varphi^\dagger,\mathscr{E}_i^\dagger,\vartheta,\Gamma,\Lambda]=G[\varphi]+\int_0^t dt\int d\mathbf{x}\varphi^\dagger\left[-\frac{1}{v}\frac{\partial\varphi}{\partial t}+\left(-\hat{\mathbf{L}}\varphi+\hat{\mathbf{M}}_p\varphi\right)\right.$$
$$\left.+\sum_{i=1}^{R}\mathscr{E}_i+S\right]+\sum_{i=1}^{R}\int_0^t dt\int d\mathbf{x}\mathscr{E}_i^\dagger\left[\frac{1}{\lambda_i}\frac{\partial\mathscr{E}_i}{\partial t}+\mathscr{E}_i-\hat{\mathbf{M}}_i\varphi\right]$$
$$+\int d\mathbf{x}\Gamma(\mathbf{x})\left[\varphi(\mathbf{x},t=0)-\varphi_0(\mathbf{x})\right]+\sum_{i=1}^{R}\int d\mathbf{x}\Lambda_i(\mathbf{x})\left[\mathscr{E}_i(\mathbf{x},t=0)-\mathscr{E}_{i,0}(\mathbf{x})\right]$$
$$+\int_0^t dt\int_{\partial V}d\mathscr{A}\int dE\int_{\Omega_{in}}d\Omega\vartheta\left\{\varphi(\mathbf{r}_s,E,\mathbf{\Omega}_{in},t)\right.$$
$$\left.-f(\mathbf{r}_s,E,\mathbf{\Omega}_{in},t)-\left[\hat{\mathbf{R}}\varphi\right](\mathbf{r}_s,E,\mathbf{\Omega}_{in},t)\right\}.$$

Variational formulation of kinetic models

- Definition of operator G is physically crucial (*detector*);
- e.g., power at time τ :

$$G[\varphi] = P_{th}(\tau) = \int_V d\mathbf{r} \int dE \oint d\Omega \gamma(\mathbf{r}, E) \Sigma_f(\mathbf{r}, E, \tau) \varphi(\mathbf{r}, E, \mathbf{\Omega}, \tau).$$

- The choice of G can be tailored to the problem considered (more general view of the dynamic problem)

Variational formulation of kinetic models

- Variations with respect to flux and precursor emissivities yield equations for Lagrange multipliers (adjoint model)

$$\begin{cases} -\dfrac{1}{v}\dfrac{\partial\varphi^{\dagger}(\mathbf{r},E,\Omega,t)}{\partial t} = \left[-\hat{\mathbf{L}}^{\dagger}(t)\,\varphi^{\dagger}+\hat{\mathbf{M}}_{p}^{\dagger}(t)\,\varphi^{\dagger}\right](\mathbf{r},E,\Omega,t) \\ \qquad\qquad +\displaystyle\sum_{i=1}^{R}\left[\hat{\mathbf{M}}_{i}^{\dagger}\mathscr{E}_{i}^{\dagger}\right](\mathbf{r},E,t)+Q^{\dagger}(\mathbf{r},E,\Omega,t) \\ -\dfrac{1}{\lambda_{i}}\dfrac{\partial\mathscr{E}_{i}^{\dagger}(\mathbf{r},E,t)}{\partial t} = -\mathscr{E}_{i}^{\dagger}(\mathbf{r},E,t)+\dfrac{1}{4\pi}\oint d\Omega\varphi^{\dagger}(\mathbf{r},E,\Omega,t)\,, \qquad i=1,...,R \end{cases}$$

Note: adjoint for delayed emissivities also appear → *importance*

Variational formulation of kinetic models

- Introduction of trial functions:

$$\varphi(\mathbf{x},t) = \sum_{j=1}^{J} \left(N_j(t)u_j(\mathbf{x})\right) \phi(\mathbf{x};t) = \sum_{j=1}^{J} N_j(t)\phi_j(\mathbf{x};t),$$

$$\mathscr{E}_i(\mathbf{x},t) = \sum_{j=1}^{J} \left(E_{i,j}(t)u_j(\mathbf{x})\right) \epsilon_i(\mathbf{x};t) = \sum_{j=1}^{J} E_{i,j}(t)\epsilon_{i,j}(\mathbf{x};t),$$

$$\varphi^\dagger(\mathbf{x},t) = \sum_{j=1}^{J} \left(N_j^\dagger(t)u_j(\mathbf{x})\right) \phi^\dagger(\mathbf{x};t) = \sum_{j=1}^{J} N_j^\dagger(t)\phi_j^\dagger(\mathbf{x};t),$$

$$\mathscr{E}_i^\dagger(\mathbf{x},t) = \sum_{j=1}^{J} \left(E_{i,j}^\dagger(t)u_j(\mathbf{x})\right) \epsilon_i^\dagger(\mathbf{x};t) = \sum_{j=1}^{J} E_{i,j}^\dagger(t)\epsilon_{i,j}^\dagger(\mathbf{x};t).$$

Variational formulation of kinetic models

- With proper choices, standard models can be retrieved:

$$\varphi(\mathbf{x},t) = \sum_{j=1}^{J} \left(N_j(t)u_j(\mathbf{x})\right)\phi(\mathbf{x};t) = \sum_{j=1}^{J} N_j(t)\phi_j(\mathbf{x};t),$$
$$\mathscr{E}_i(\mathbf{x},t) = \sum_{j=1}^{J} \left(E_{i,j}(t)u_j(\mathbf{x})\right)\epsilon_i(\mathbf{x};t) = \sum_{j=1}^{J} E_{i,j}(t)\epsilon_{i,j}(\mathbf{x};t),$$
$$\varphi^\dagger(\mathbf{x},t) = \sum_{j=1}^{J} \left(N_j^\dagger(t)u_j(\mathbf{x})\right)\phi^\dagger(\mathbf{x};t) = \sum_{j=1}^{J} N_j^\dagger(t)\phi_j^\dagger(\mathbf{x};t),$$
$$\mathscr{E}_i^\dagger(\mathbf{x},t) = \sum_{j=1}^{J} \left(E_{i,j}^\dagger(t)u_j(\mathbf{x})\right)\epsilon_i^\dagger(\mathbf{x};t) = \sum_{j=1}^{J} E_{i,j}^\dagger(t)\epsilon_{i,j}^\dagger(\mathbf{x};t).$$

  - Full time-space factorization → PK
  - Use of different time scales → QS

- How can Multipoint (MPK) and Modal approaches be retrieved ?

Multipoint kinetics

- The expression chosen for $u_j(\mathbf{x})$ in the formula

$$\varphi(\mathbf{x},t) = \sum_{j=1}^{J} \left(N_j(t) u_j(\mathbf{x})\right) \phi(\mathbf{x};t) = \sum_{j=1}^{J} N_j(t) \phi_j(\mathbf{x};t),$$

  can lead to different models

- In MPK, macrodomains Γ_j are introduced:

$$u_j(\mathbf{x}) = \begin{cases} 1 & \text{on } \Gamma_j \\ 0 & \text{otherwise.} \end{cases}$$

Modal approach

- In the modal approach, a set of modes on the full phase-space are introduced;
- In both cases, the kinetics of the system is described by the superposition of separate evolutions, obeying a coupled systems of equations (obtained setting to zero the variations $\delta N_j^\dagger$):

$$\frac{d}{dt}|X\rangle = \hat{M}|X\rangle + |S\rangle$$

The coefficients here appearing are adjoint-weighted integrals

$$|X\rangle^t = |N_1, N_2, ..., N_K, E_{1,1}, ..., E_{1,K}, E_{2,1}, ..., E_{R,K}\rangle^t$$

$$|S\rangle^t = \left|\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_K, 0, ..., 0\right\rangle^t,$$

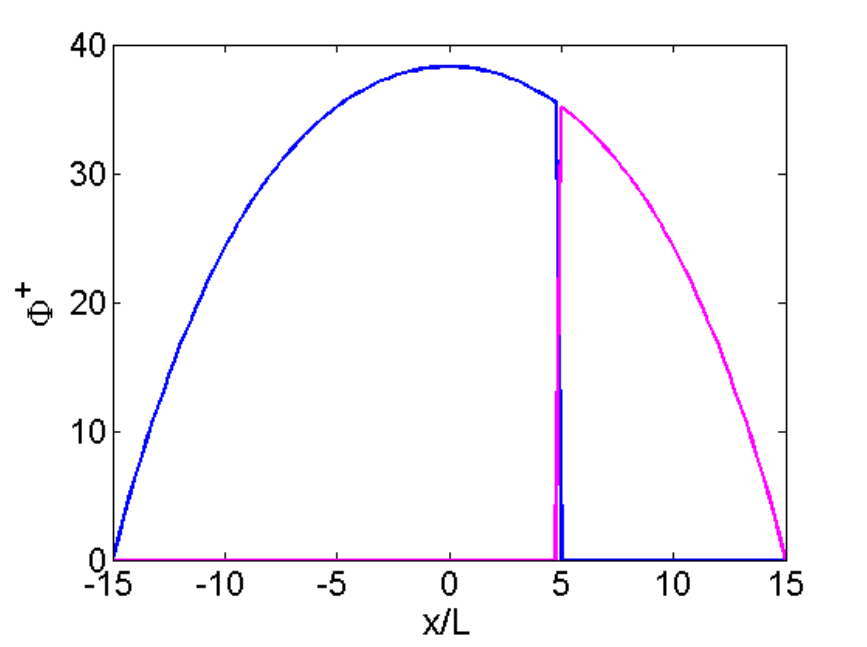
Results and performance comparison

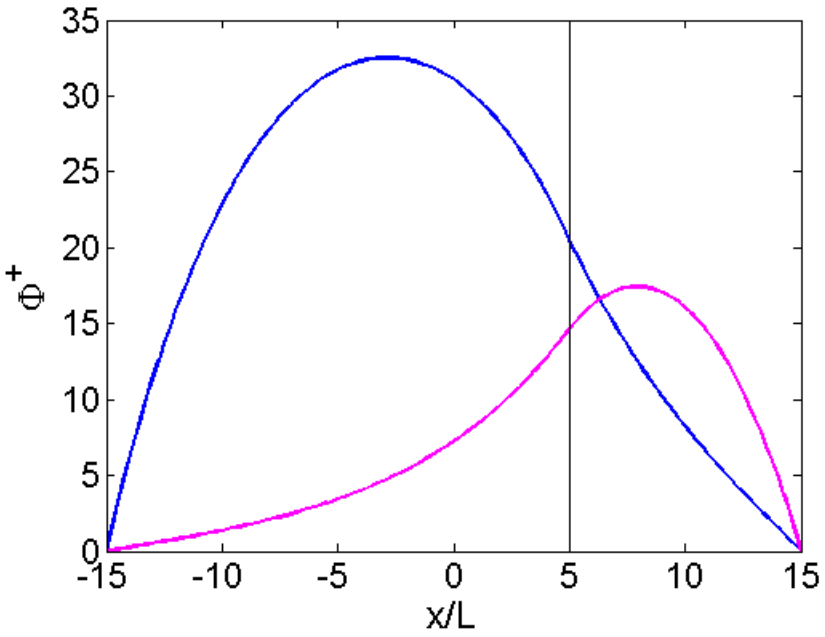
- *Objective*: to show different physical features, advantages and shortcomings of models
- The modal method is constructed using the steady-state harmonics (solutions of the homogeneous problem)
- Two alternative MPK are considered (based on a different choice of trial functions for the adjoint)

The MPK versions

- For initially critical systems, the adjoint is used, restricted to each single subdomain
- For subcritical systems, two possibilities arise:
  - MPK-1: definition of a *global* adjoint source and solution obtained restricted to each single subdomain
  - MPK-2: definition of *local* adjoint source for each of K subdomains → K adjoints obtained

The MPK versions



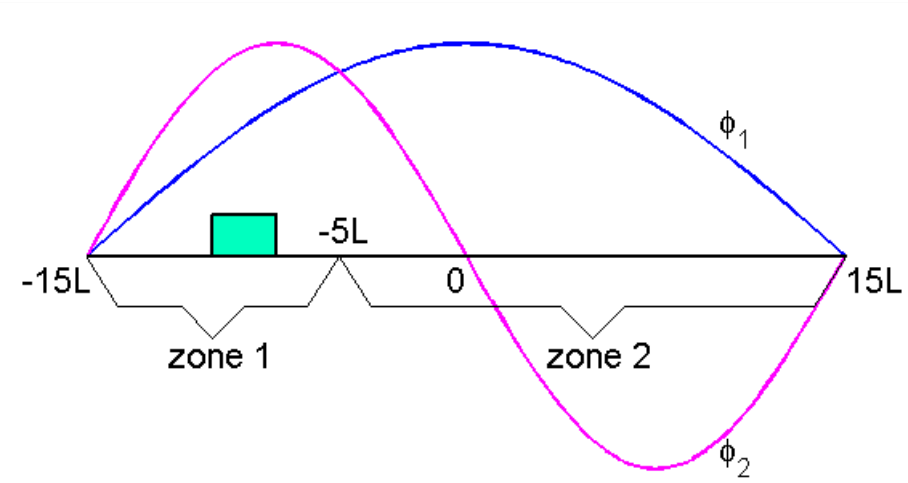


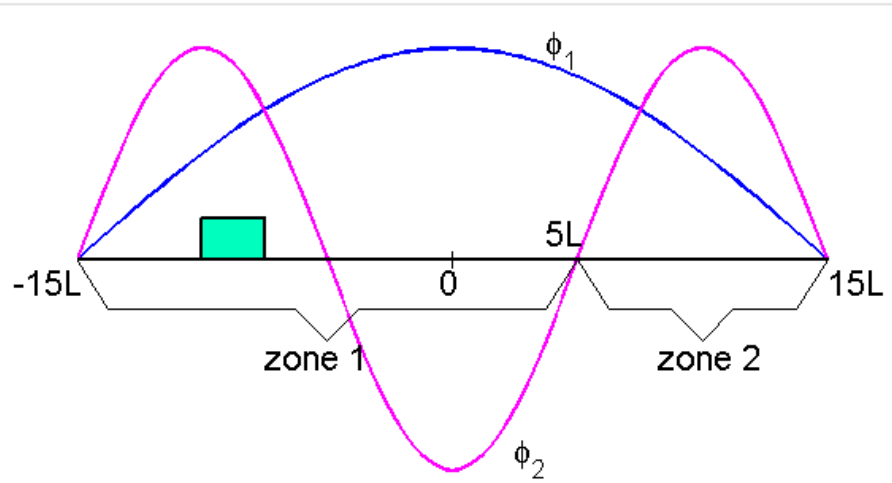
System configuration

- Test-case in 1D diffusion theory
  - Simple calculations
  - Good physical insight
  - Reference results are obtained by direct highly accurate numerical inversion
- Transients induced by localized perturbations:
  - MPK: role of subdomain definition
  - Modal: choice of spatial modes

Critical case

- Localized asymmetrical perturbation of absorption → +1257 pcm
- Comparison of MPK with Modal, using different subdomain division and modes





MPK (case a) Modal (case c)

MPK (case b) Modal (case d)

Critical case

- Percentage error on power

time [ms]	modal (case c)	modal (case d)	MPK-1 (case a)	MPK-1 (case b)	PK
0.5	−0.14	−0.14	−0.13	−0.15	−0.14
1	−0.24	−0.24	−0.23	−0.25	−0.25
10	−0.60	−0.64	−0.92	−0.96	−0.95
50	−1.58	−1.80	−3.42	−3.53	−3.53
100	−2.65	−3.07	−6.28	−6.44	−6.46
500	−11.00	−12.91	−26.53	−27.13	−27.17
1000	−21.35	−24.89	−47.51	−48.41	−48.47

Modal:

Affected by the choice of the modes

Works better than MPK

MPK:

No large influence of the subdomain definition → the system is physically coupled

Error increases with time → necessity to couple with a QS algorithm

Subcritical case

- The same configuration is considered in subcritical status (k_{eff}=0.97)
- An external source is placed in the center of the system, spanning over an interval of 3L
- The introduction of the localized perturbation amounts to +1311 pcm

Subcritical case

- The use of the two MPK approaches leads to different coupling, as seen in matrix M:

$$\frac{d}{dt}|X\rangle = \hat{M}|X\rangle + |S\rangle$$

MPK-1				MPK-2			
-0.98	1.0	$4 \cdot 10^{-5}$	0	-0.66	1.0	$1 \cdot 10^{-3}$	$-2 \cdot 10^{-4}$
0.17	-0.20	0	$6 \cdot 10^{-6}$	0.09	-0.62	$-1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
$8 \cdot 10^{-4}$	0	$-6 \cdot 10^{-6}$	0	$7 \cdot 10^{-3}$	0.01	$-1 \cdot 10^{-4}$	0
0	$5 \cdot 10^{-3}$	0	$-6 \cdot 10^{-6}$	$6 \cdot 10^{-3}$	0.06	0	$-1 \cdot 10^{-4}$

Relative values

Subcritical case

- Percentage error on power

time [ms]	modal (case c)	modal (case d)	MPK-1 (case a)	MPK-1 (case b)	MPK-2 (case a)	MPK-2 (case b)	PK
0.1	0.03	−0.26	−0.11	−0.19	−0.02	−0.24	−0.18
0.5	−0.004	−1.40	−0.97	−1.08	−0.28	−1.27	−1.07
1	−0.10	−2.70	−2.10	−2.23	−0.64	−2.44	−2.22
5	−1.12	−8.86	−8.09	−8.33	−2.79	−8.18	−8.33
100	−2.69	−12.11	−11.45	−11.72	−4.51	−11.48	−11.73

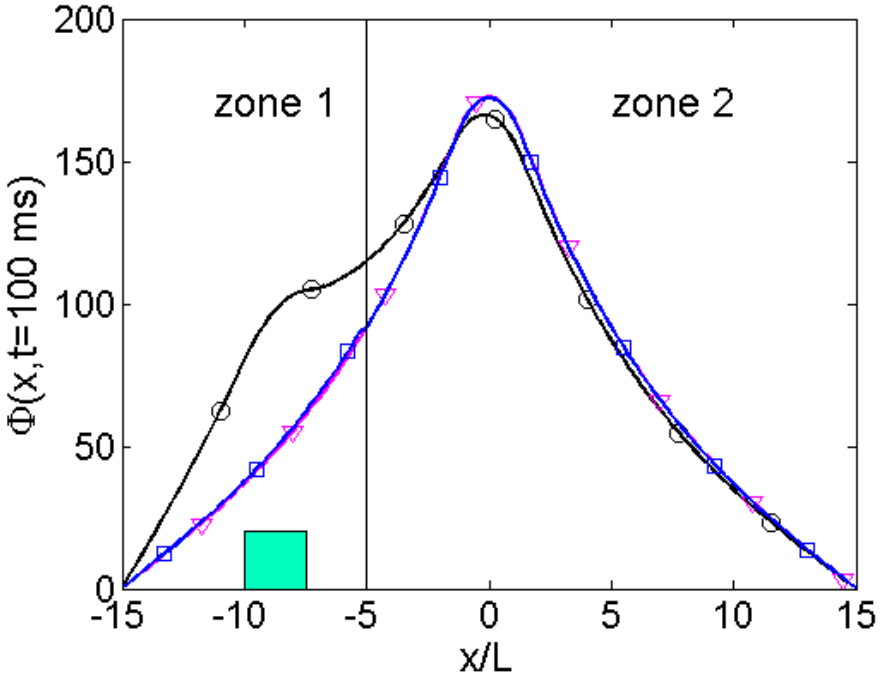
Subcritical case

time [ms]	modal (case c)	modal (case d)	MPK-1 (case a)	MPK-1 (case b)	MPK-2 (case a)	MPK-2 (case b)	PK
0.1	0.03	−0.26	−0.11	−0.19	−0.02	−0.24	−0.18
0.5	−0.004	−1.40	−0.97	−1.08	−0.28	−1.27	−1.07
1	−0.10	−2.70	−2.10	−2.23	−0.64	−2.44	−2.22
5	−1.12	−8.86	−8.09	−8.33	−2.79	−8.18	−8.33
100	−2.69	−12.11	−11.45	−11.72	−4.51	−11.48	−11.73

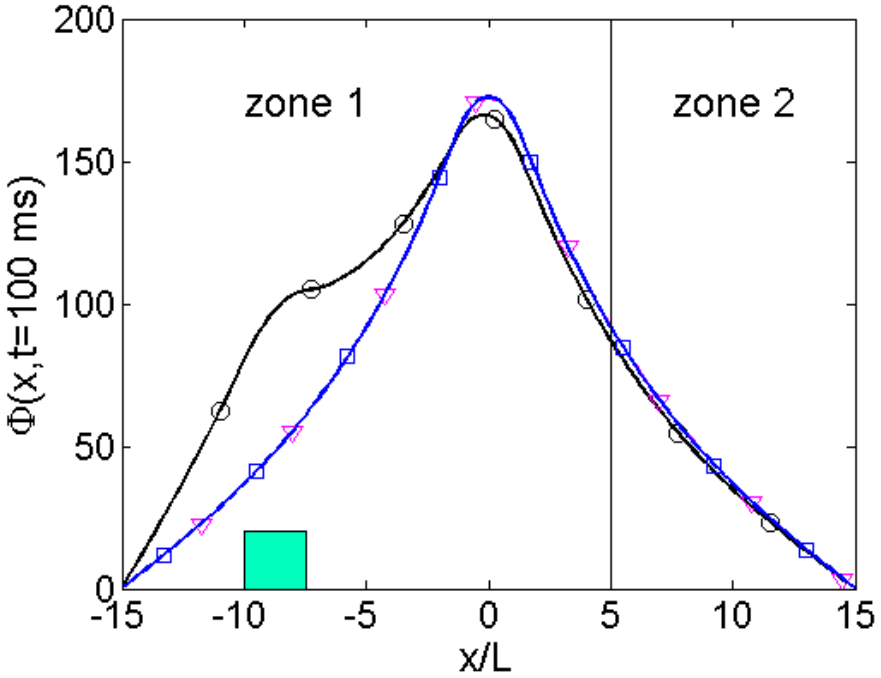
- Multipoint kinetics
  - MPK-2 largely improves with respect to MPK-1
  - Choice of subdomains plays a major role
- Modal approach
  - Choice of modes is important as well
- With inappropriate choices, nothing better than PK can be obtained

Subcritical case

- MPK-1



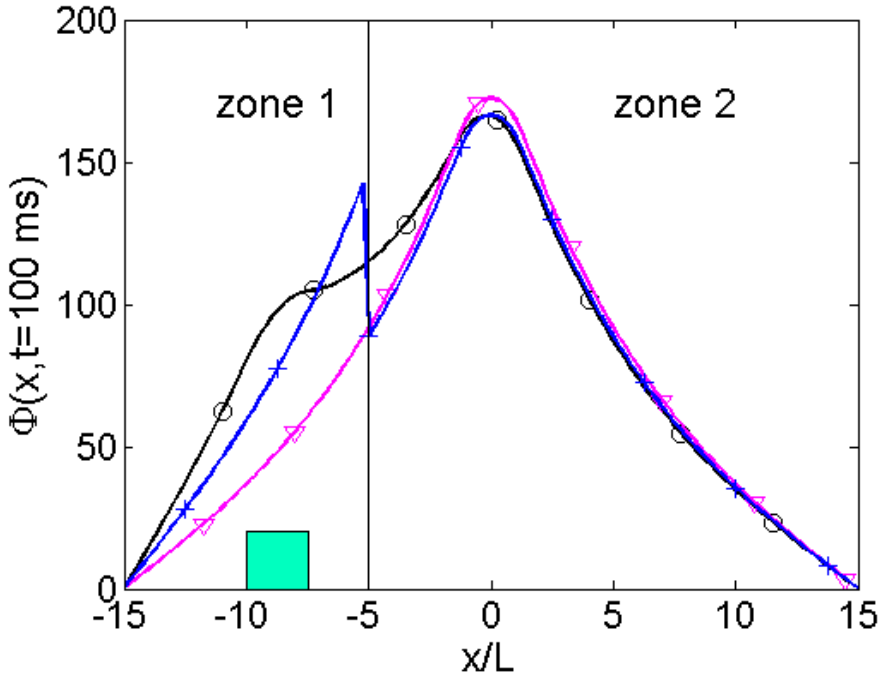
MPK-1 case a



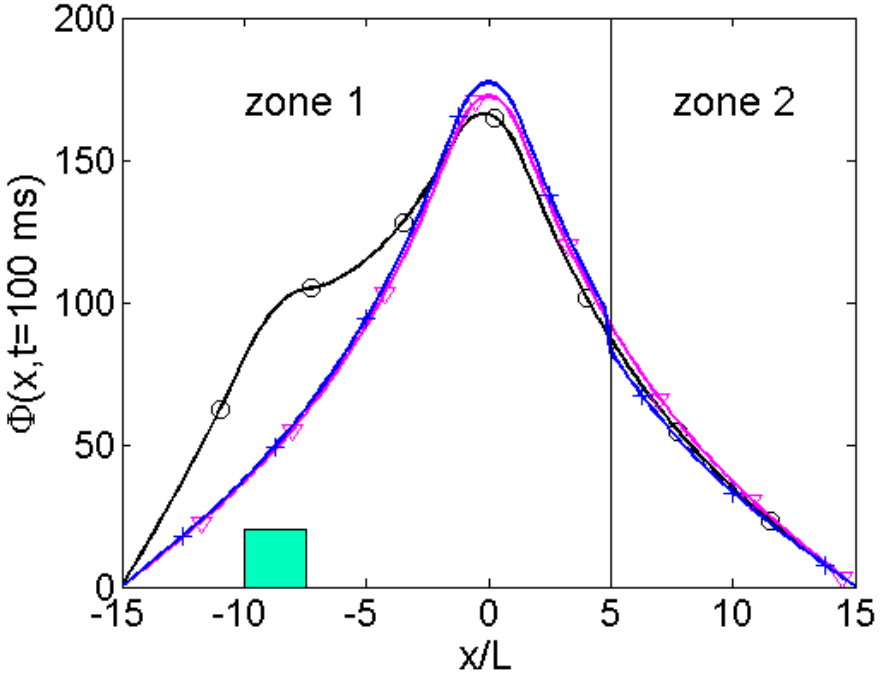
MPK-1 case b

Subcritical case

- MPK-2



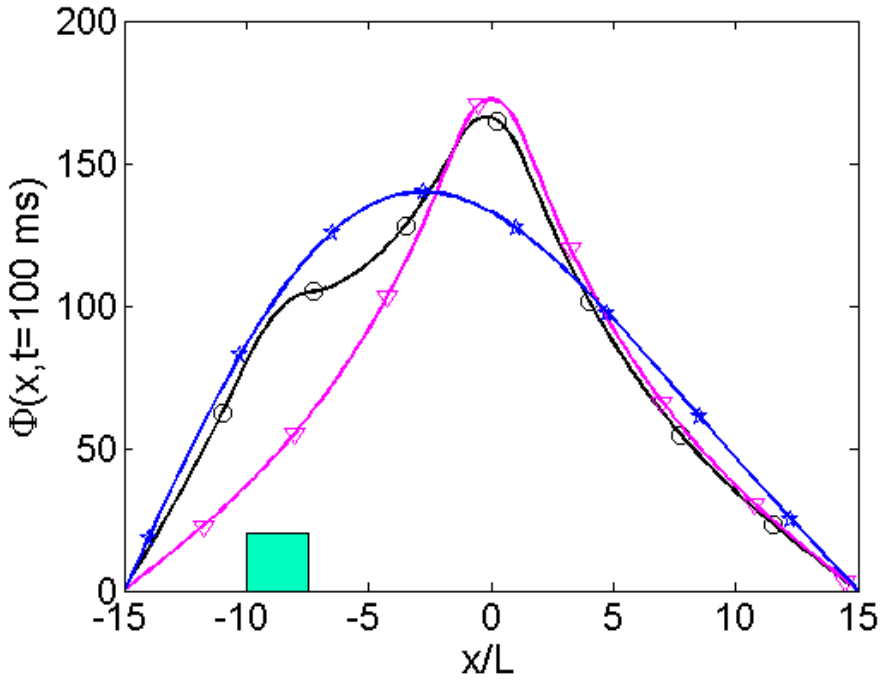
MPK-2 case a



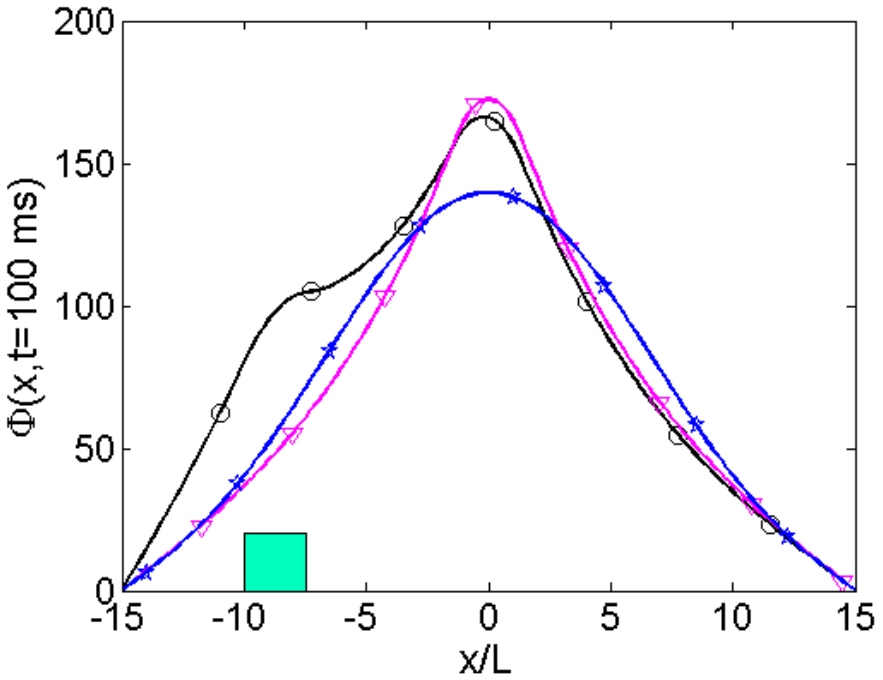
MPK-2 case b

Subcritical case

- Modal



Modal case c



Modal case d

Conclusions

- The variational technique allows consistent and unambiguous definition of integral parameters, for any type of systems
- Results show that, to obtain good performance, choice of modes and subdomain subdivision is crucial
- Need of adaptive algorithms
- Need of automatic scale choice