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Spring School on Superstring Theory and Related Topics

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Models of electroweak symmetry breaking and the TeV scale
Lecture I

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TRIESTE 2010 LECTURES

on

BSM PHYSICS, EWSB, TeV SCALE

CSABA CSÁKI (CORNELL)

OUTLINE:

1.) THE SM, HIERARCHY PROBLEM
ELECTROWEAK PRECISION, LITTLE HIERARCHY

2.) THE MSSM, SUPERSYMMETRY
BREAKING. GAUGE MEDIATION. LITTLE
HIERARCHY of MSSM, μ -PROBLEM

3.) EXTRA DIMENSIONS. LARGE EXTRA DIM.
WARPED EXTRA DIM. UED. COMPOSITE
HIGGS

4.) LITTLE HIGGS

THE SM

Description of strong-weak-E&M.
Gives unified framework in terms of non-abelian gauge theories

↙ gauge group

	$SU(3)_c$	$\times\ SU(2)_L$	$\times\ U(1)_Y$
Q_L	3	2	1/6
u_R	3	1	2/3
d_R	3	1	-1/3
L_L	1	2	-1/2
e_R	1	1	-1

fermion matter fields

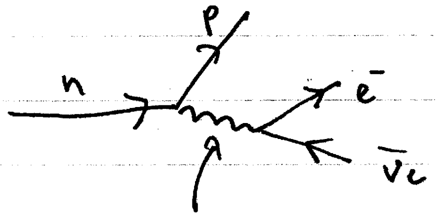
strong interaction QCD

unified theory of weak + E&M

- Left handed fields $SU(2)_L$ doublets
- Right handed $SU(2)_L$ singlets
- quarks charged under $SU(3)_c$
- all anomalies cancel

Important: - weak interactions have finite range

e.g. β decay $\quad n \to p^+ + e^- + \bar{\nu}_e$



the W gauge boson should be massive

Well described by an effective

4-fermi op.

$$\mathcal{L}_{weak} = \frac{G_F}{\sqrt{2}} \bar{\Psi}_p (1-\gamma_5) \Psi_n \bar{\Psi}_\nu (1-\gamma_5) \Psi_e$$

produced if massive gauge boson W

$$\frac{g^2}{M_W^2} \sim G_F \quad \rightarrow \quad M_W \sim 100 M_p \sim 100 \text{GeV}$$

How do we get massive gauge bosons?

Higgs mechanism (spontaneous gauge sym. breaking)

Lagrangian invariant under symmetries, but vacuum not!

Need to do it w/o breaking Lorentz invariance. Vacuum needs to be Lorentz scalar, but $SU(2) \times U(1)$ non-singlet!

→ Need scalar Higgs field

$H \quad 2_{1/2}$ under $SU(2)_L \times U(1)_Y$

complex doublet $H = \begin{pmatrix} h_1 + i h_2 \\ h_3 + i h_4 \end{pmatrix}$

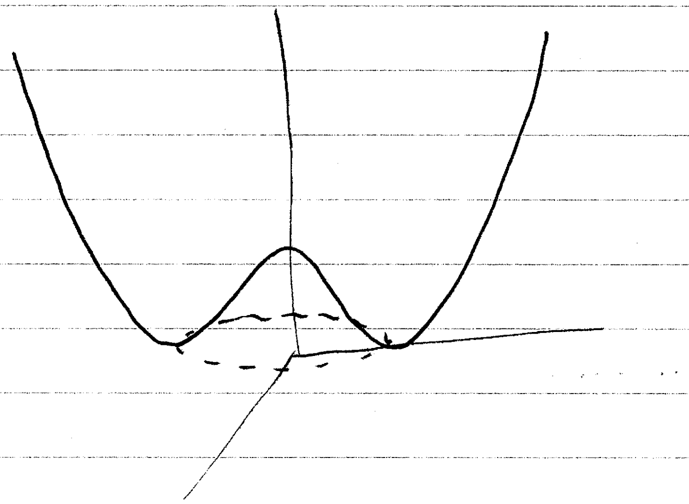
potential could break EWS

$$V(H) = \lambda \left(H^+ H - \frac{v^2}{2}\right)^2$$

(at this point equivalent to

$-\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$

Minimum <u>not</u> at origin, Mexican hat pot'l



$v^2 = \frac{\mu^2}{\lambda}$ minimum.

$H = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ Symmetry broken, but <u>NOT</u> completely, remaining piece provides QED massless gauge bos.

SU(2) gen $\frac{\tau^a}{2}$

$U(1)_Y$ gen $1/2$

$T_3 + Y = \begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & \\ & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$

annihilates vacuu

$Q = T_3 + Y$ unbroken $\to$ QED

Gauge boson masses from $|D_\mu H|^2$

$$D_\mu H = \Big(\partial_\mu - ig \frac{\tau^a}{2} \underset{\substack{\uparrow \\ SU(2) \\ \text{gb.}}}{W_\mu^a} - \frac{ig'}{2} \underset{\substack{\uparrow \\ U(1)_Y \text{ gb.}}}{B_\mu} \Big) H$$

mass term: $$\left| \frac{1}{2} \begin{pmatrix} gW^3 + g'B & g(W^1 - iW^2) \\ g(W^1 + iW^2) & -gW^3 + g'B \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \right|^2$$

$$= \frac{g^2 v^2}{8} \underbrace{|W^1 - iW^2|^2}_{W^\pm} + \frac{v^2}{8} \underbrace{(-gW^3 + g'B)^2}_{Z}$$

$$W^\pm = \frac{W^1 \mp iW^2}{\sqrt{2}} \qquad Z = \frac{gW^3 - g'B}{\sqrt{g^2 + g'^2}}$$

$$\equiv \cos\theta_W W^3 - \sin\theta_W B$$

$$M_W^2 = \frac{g^2 v^2}{4} \qquad \boxed{\frac{M_W^2}{M_Z^2} = \cos^2\theta_W}$$

$$M_Z^2 = \frac{(g^2 + g'^2) v^2}{4}$$

firm prediction of tree-level SM Higgs mechanism

Experimentally:

$$M_W \sim 80\,\text{GeV}$$
$$M_Z \sim 90\,\text{GeV}$$
$$\sin^2\theta_W \sim 0.23$$

Relation important, because fermion couplings fixed:

after rewriting in terms of $W^\pm, Z, A$

W coupling: $\frac{g}{\sqrt{2}} \bar{\Psi}_{u_L} \gamma_\mu \Psi_{d_L} W^{\mu +} + h.c.$

Z coupling $\frac{g}{\cos\theta_W} \bar{\Psi}_i \gamma_\mu (T_3 - s_W^2 Q) \Psi_i Z^\mu$

A coupling $e \bar{\Psi}_i \gamma_\mu \Psi A^\mu$

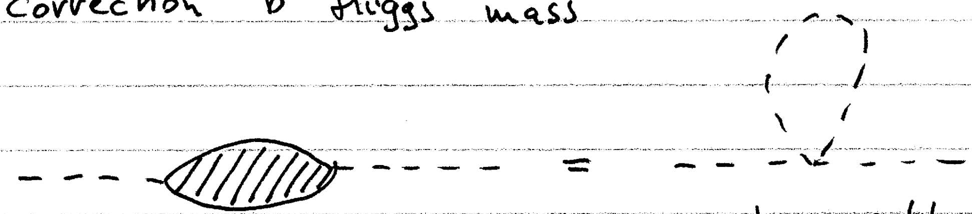
$$e = g \sin\theta_W$$

SM well tested (will talk about it soon), still think we need physics BSM at $\sim$ 1TeV, main reason: <u>hierarchy problem</u>

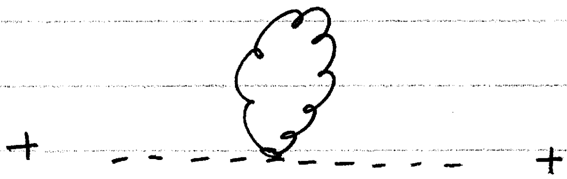
HIERARCHY PROBLEM

Traditional presentation

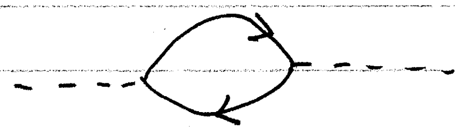
correction to Higgs mass



1 loop higgs self coupling



gauge loops



fermion loops (especially top quark)

quadratically divergent

$$\propto \int \frac{d^4k}{(k^2-m^2)} \frac{1}{(2\pi)^4} \propto \Lambda^2$$

Λ ← cut-off scale

$$\delta m^2 = \frac{\Lambda^2}{32\pi^2}\left(\underset{\substack{\uparrow \\ \text{self-coupling} \\ (800\,\text{GeV})^2}}{6\lambda} + \underbrace{\tfrac{1}{4}\left(9g^2+3g'^2\right)}_{(600\,\text{GeV})^2} - \underset{\substack{\uparrow \\ -(1.5\,\text{TeV})^2}}{6y_t^2}\right)$$

If $\Lambda \gg 10\,\text{TeV}$ (for example $\Lambda \sim M_{P\ell}$)

$\delta m_H^2 \gg m_H^2$ → hierarchy problem.

Higgs mass sensitive to <u>ANY</u> scale of new physics

NOTE: this problem is specific to elementary scalars.

For fermions: $m_e \to 0$, new chiral symmetry shows up

For GB's: $m_W \to 0$ gauge symmetry restored

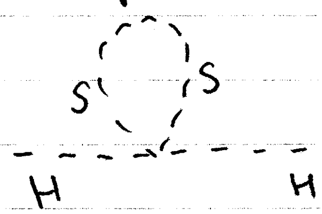
$$\left.\begin{array}{l} \Delta m_e \propto m_e \log\left(\frac{\Lambda}{M}\right) \\ \Delta M_W^2 \propto M_W^2 \log\left(\frac{\Lambda}{M_W}\right) \end{array}\right\} \text{ no quadratic sensitivity!}$$

Isn't this dependent on what kind of regularization I use (e.g. in dim reg no quadratic divergences, $1/\epsilon$ poles $\sim$ log div's).

NO! any new physics coupled to Higgs will introduce it

Example: a new phsics at scale m_S,

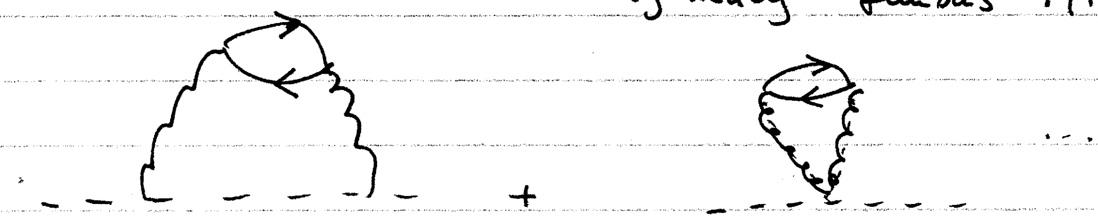
for example $\lambda_S |H|^2 |S|^2$



$$\delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \log\frac{\Lambda_{UV}}{m_S} + \text{finite}\right]$$

even if $\Lambda_{UV} \to 0$, log divergent & finite contributions $\propto m_S^2$ ...

Even if NOT directly coupled to Higgs, but coupled to some SM fields. Say heavy fermions $F, \bar{F}$



2 loop: $\delta m_H^2 \propto \left(\frac{g^2}{16\pi^2}\right)^2 \left[a \Lambda_{UV}^2 + 48\, m_F^2 \log + \ldots\right]$

Real formulation of hierarchy problem:

m_H sensitive to any high scale in theory, if indirectly coupled to SM

Other words: Higgs mass relevant operator, relative importance grows toward IR. Only relevant operator in SM!

Usual solution: there needs to be new physics at TeV scale that eliminates large loop corrections above TeV scale!

Possibilities:

- Relate elementary scalar to fermions via SUSY. Chiral sym. of fermions ensures together with SUSY cancellation of div's.
- Relate to elementary gauge field (gauge-Higgs unification)
- There is no higgs boson, just a condensate dynamically generated (technicolor, higgsless)
- There is a Higgs, but it is not elementary. At ~TeV start feeling large form factors, that suppress corrections (~ "Higgs dissolves...", Composite higgs, warped extra dim's, RS
- The higgs is a pseudo-Goldstone boson, that gives some protection (usually 1-loop) from quadratic divergences. Still need to combine with some of the other mechanisms (little Higgs)
- The fundamental scale of all of new physics is actually 1TeV (large extra dimensions)

Electroweak precision & little hierarchy problem

SM tested very precisely for ~ 2 decades

LEP '89-95 on Z-peak } e^+e^- collision
LEP2 '96-00 90-210 GeV }
Tevatron late 80's - 2011(?) $p\bar{p}$ @ 2 TeV

About 2 dozen quantities measured to ~ 0.1% precision

example @ LEP1

Γ_Z total width

σ_h $e^+e^- \to$ hadrons @ Z-peak

$$R_h = \frac{\Gamma(Z \to had)}{\Gamma(Z \to \mu^+\mu^-)}$$

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to had)}$$

$$A^{\ell}_{FB} = \left(\frac{\sigma(e^+e^- \to \ell\bar{\ell})_F - \sigma(e^+e^- \to \ell\bar{\ell}) ...}{... + ...} \right.$$

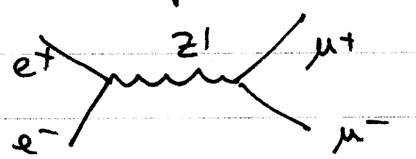
@ LEP2 $\sigma(e^+e^- \to \mu^+\mu^-)$ at $\sqrt{s}$ on many values between 100-21...

Tevatron m_{top}, M_W } precise determination

- All measurements within 2σ of SM predictions!
- Problem: if new particles at TeV scale: would generically contribute to EWP.

How large would it be?

For example:



$$\sim \frac{g^2}{M_{Z'}^2}$$

$$\text{vs.} \quad \frac{g^2}{M_Z^2}, \quad \frac{g^2}{(200\,\text{GeV})^2}.$$

Needs to be $\lesssim$ few $\times$ 0.1%

$$\frac{M_Z^2}{M_{Z'}^2} \lesssim \text{few} \times 10^{-3}$$

$$M_{Z'} \gtrsim M_Z \times 30 \sim \boxed{3\text{–}5\,\text{TeV}}$$

tree-level exchange of TeV scale particles would have likely been already indirectly observed.

Little hierarchy problem:

Need TeV scale particles to solve hierarchy problem.

- However generic new physics constrained to about 5-10 TeV already via EWPO, new physics scale ~ 10 TeV

$\rightarrow$ little hierarchy of order $\left(\frac{1\,\text{TeV}}{10\,\text{TeV}}\right)^2 \sim 1\%$ type fine tuning.

In different models will have different ~~...~~ specific realizations.

Possible way out:

- Certain quantities can be protected via symmetry (ie. custodial symmetry for ρ parameter)

- All tree-level ~~quan~~ exchanges can be protected via discrete symmetry called T-parity

1.) T-parity:

A Z_2 discrete symmetry

$$(\text{SM particle}) \xrightarrow{T} (\text{SM particle})$$

$$(\text{BSM particle}) \xrightarrow{T} -(\text{BSM particle})$$

Then no coupling

SM–SM–BSM allowed.

Every vertex contains at least 2 BSM particles → can only contribute at 1 loop.

correction size: $\left(\frac{M_Z}{M_{Z'}}\right)^2 \longrightarrow \frac{1}{16\pi^2}\left(\frac{M_Z}{M_{Z'}}\right)^2$

→ extra 4π factor allows 1TeV BSM particles.

<u>Example</u>: – R-parity in SUSY
– KK parity in UED
– T-parity of little Higgs

2.) Symmetry for ρ parameter

<u>"Custodial SU(2)"</u>

In SM $\frac{M_W^2}{M_Z^2} = \cos^2\theta_W$

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \quad \text{at tree-level.}$$

This is <u>NOT</u> a coincidence in SM, but a consequence of a symmetry.

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2}\right)^2 =$$
$$= \lambda \left(h_1^2 + h_2^2 + h_3^2 + h_4^2 - \frac{v^2}{2}\right)^2$$

→ $SO(4)$ global symmetry of potential

$SO(4) \sim SU(2)_L \times SU(2)_R$ property of $SO(4)$

$$H_i \to (H^*)^i \xrightarrow{i\sigma^2 = \epsilon} (H^*)_i$$ like H_i from $SU(2)_L$

$SU(2)_R$ ← exchanges H with (ϵH^*)

$$SU(2)_L \longrightarrow \overbrace{\begin{pmatrix} i\sigma^2 H^* & H \end{pmatrix}}^{SU(2)_R}$$

in this notation
$H = (2,2)$ of $SU(2)_L \times SU(2)_R$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & \\ & v \end{pmatrix}$$

$SU(2)_L \times SU(2)_R \to SU(2)_D$

The remaining $SU(2)_D$ NOT broken at all by Higgs VEV (though broken by hypercharge, Yukawa couplings)

$(W_\mu^1, W_\mu^2, W_\mu^3)$ triplets under $SU(2)_L$, also under $SU(2)_D$

Need to have <u>same</u> mass when $g' \to 0$

$\frac{1}{2} M_Z^2 Z_\mu Z^\mu = \frac{1}{2} M_Z^2 \left(\cos\theta_W W_{3\mu} - s_\theta B_\mu\right)^2$

$= \underline{\frac{1}{2} M_Z^2 \cos^2\theta_W (W_3^\mu)^2} + \ldots$

$\to \boxed{M_Z^2 \cos^2\theta_W = M_W^2}$ by $SU(2)_D$ ...

$\rho = 1.$ Special to choice of Higgs rep.

For other reps (T, Y)

$$\rho = \frac{T(T+1) - T_3^2}{2T_3^2} \ldots 1 \ldots$$

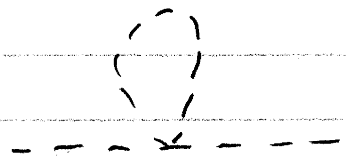
for triplet $T = 1, \; T_3 = 1$ $\boxed{\rho = 1/2}$

Why off ...

SUPERSYMMETRY

- The most promising extension of SM
- Due to SUSY & chiral symmetry higgs quadratic divergences will be cancelled.

For example, top contribution:



$\delta m_H^2 = -2\Lambda_{UV}^2 \lambda_f^2$ $\qquad$ $\delta m_H^2 = +\Lambda_{UV}^2 \lambda_S$

$\uparrow$ Yukawa $\qquad$ $\uparrow$ quartic

if $f\bar{f}$ (L+R fermions) $\to$ 2 complex scalars

& $|\lambda_f|^2 = \lambda_S$ $\to$ would cancel out

In SUSY these conditions automatic.
In fact in SUSY due to non-renormalization theorem all loop order cancelation will be guaranteed!

I assume everyone somewhat familiar with SUSY. Will build $\mathcal{N}=1$ extension of SM called MSSM.

Lagrangian for $\mathcal{N}=1$ gauge theories

every SM fermion $\psi_i \longrightarrow \Phi_i$ chiral SF

$$\Phi = \varphi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

sfermion

$y^\mu \hat{=} x^\mu + i\theta\sigma^\mu\bar{\theta}$

fermion

F-term auxiliary field

SM gauge fields $A_\mu \longrightarrow W_\alpha$ vector SF

$$W_\alpha = -i\lambda_\alpha(y) + \theta_\beta\left[\delta_\alpha^\beta D(y) - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)^\beta_\alpha F_{\mu\nu}\right.$$
$$+\theta^2 \sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}}(y)$$

gaugino

D-term auxiliary field

gauge field

Lagrangian in superspace:

$$\int d^4\theta\, \Phi_i^+ e^{gV}\Phi_i + \frac{1}{4g^2}\int d^2\theta\, W^{\alpha a}W_\alpha^a + h.c.$$
$$+\int d^2\theta\, W(\Phi) + h.c.$$

gauge + gaugino kinetic term

superpotential

gauge invariant kinetic terms for matter fields

The Matter content of the MSSM

same gauge group as SM, but need 2 Higgs chiral superfields (otherwise $SU(2)^2 U(1)_Y$, $SU(2)^3$ (witten) anomaly) Write in terms of LH chiral superfields only to maintain holomorphy of SUSY...

	SU(3)	× SU(2)$_L$	× U(1)$_Y$	B	L
L	1	2	$-1/2$	0	1
$\bar{E}$	1	1	$+1$	0	-1
Q	3	2	$1/6$	$1/3$	0
$\bar{U}$	$\bar{3}$	1	$-2/3$	$-1/3$	0
$\bar{D}$	$\bar{3}$	1	$1/3$	$-1/3$	0
H_u	1	2	$1/2$	0	0
H_d	1	2	$-1/2$	0	0

Possible superpotential terms:

$$W^{(good)} = \underbrace{\lambda_u^{ij} Q^i H_u \bar{U}^j + \lambda_d^{ij} Q^i H_d \bar{D}^j + \lambda_e^{ij} L^i H_d \bar{E}}_{\text{SUSY extensions of SM Yukawa couplings}}$$

$$+ \underbrace{\mu H_u H_d}_{\text{Supersymmetric Higgs mass (μ-term)}}$$

Need these:
- give masses to SM fermions
- give mass to Higgsinos (eliminate axion)

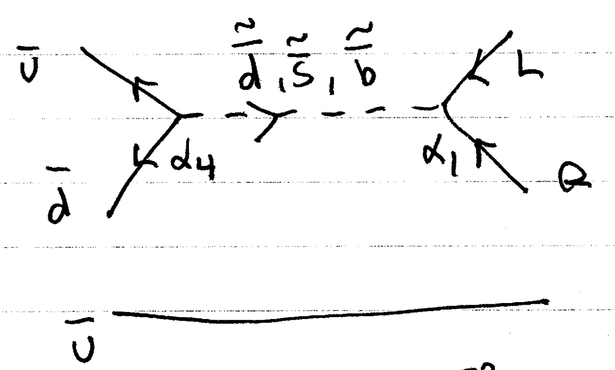
$$W^{(bad)} = \overbrace{\alpha_1^{ijk}\, Q^i L^j \bar{D}^k + \alpha_2^{ijk}\, L^i L^j E^k}^{\Delta L=1} + \underbrace{\alpha_3^{i}\, L^i H_u}_{\Delta L=1} + \underbrace{\alpha_4^{ijk}\, \bar{D}^i \bar{D}^j \bar{U}^k}_{\Delta B=1}$$

would violate baryon & lepton #
renormalizable interaction

- VERY different from SM : in SM all terms allowed by gauge invariance also conserve B, L . B, L accidental global symmetry. In SM B, L violation $\propto \frac{1}{M}$ where M can be a very high scale.
- In MSSM : new fields (superpartners) that also carry B, L , more renormalizable terms.

Need to forbid $W^{(bad)}$!

could give proton decay



couplings completely unsuppressed!

$$p^+ \to e^+\pi^0, \; e^+K^0 \quad \text{or} \quad \bar{\nu}\pi^+, \; \bar{\nu}K^+$$

Forbid $W^{(bad)}$ by <u>matter parity</u>, $\mathbb{Z}_2$ symmetry

quark, lepton χSF $\quad P_M = -1$
Higgs $\quad P_M = +1$
~~gauge~~ vector SF $\quad P_M = +1$

$W^{(good)}$: all have $P_M = +1$
$W^{(bad)}$: all have $P_M = -1$

<u>can check:</u>

$$P_M = (-1)^{3(B-L)}$$

<u>variation:</u>

<u>R-parity:</u> $\quad P_R = (-1)^{3(B-L)+2s}$

spin of field

If matter parity conserved, R-parity also conserved, $(-1)^{2s} \rightarrow$ always need even # of fermions by Lorentz.

R-parity:

$$\left.\begin{matrix} \text{(SM fields)} \rightarrow +1 \\ \text{(superpartners)} \rightarrow -1 \end{matrix}\right\} \text{like a } \mathbb{T}\text{-parity.}$$

Forbids all tree-level EWP corrections, chance SUSY is right...

Important consequences of R-parity

(usually quoted as consequences of SUSY, but it really just follows from R-parity)

- Lightest R-parity odd particle stable
  $\equiv$ LSP lightest superpartner
  if LSP electrically neutral, color singlet: candidate for WIMP-like DM

- Each sparticle other than LSP will decay, at the end will contain odd # (usually one) LSP's

- Collider experiments: initial state $P_R = +1$ $\rightarrow$ only even # of superpartners can be produced, must be pair produced. At the end decay to LSP's $\rightarrow$ missing energy signal in colliders.

Will postulate that MSSM has exact R-parity conservation (somewhat ad-hoc assumption)

SUPERSYMMETRY BREAKING

SUSY unbroken if $Q_\alpha|0\rangle = 0$
$\bar{Q}_{\dot\alpha}|0\rangle = 0$

Then using SUSY algebra $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$

$$\rightarrow P^\nu = 1/4\, (\bar\sigma^\nu)^{\dot\beta\alpha} \{Q_\alpha, \bar{Q}_{\dot\beta}\}$$

$$H = P^0 = 1/4\, \left(Q_1\bar{Q}_{\dot 1} + \bar{Q}_{\dot 1}Q_1 + Q_2\bar{Q}_{\dot 2} + \bar{Q}_{\dot 2}Q_2\right)$$

if SUSY unbroken $\langle 0|H|0\rangle = 0$
if SUSY broken $\langle 0|H|0\rangle > 0$

Scalar potential

$$V(\phi) = \underbrace{\sum_i{}' \left|\frac{\partial W}{\partial \phi_i}\right|^2}_{\sum_i |F_i|^2} + \sum_a{}' \frac{1}{2} g^2 \left|\sum_i \phi_i^+ T^a \phi_i\right|^2$$

$$\underbrace{\sum_i |F_i|^2}_{V_F} + \underbrace{\sum_a \frac{1}{2}\, g\, D^a D^a}_{V_D}$$

SUSY breaking $\langle F_i\rangle \neq 0$ or $\langle D\rangle \neq 0$

F-type breaking D-type breaking

If SUSY breaking $\rightarrow \exists$ massless fermion

Goldstino.

For example if $\langle F \rangle \neq 0$, the SUSY transformation of ψ

$\delta_\xi \psi = 2\xi \langle F \rangle$ → shift symmetry for fermion → fermion in multiplet where $\langle F \rangle = 0$ massless..

If more than one field:

$$\delta \psi_i = 2\xi \langle F_i \rangle$$

$$\psi_{\text{Goldstino}} = \sum_i \frac{F_i}{\sqrt{\sum_i F_i^2}} \psi_i$$ → always just one Goldstino.

How to apply to MSSM?

SUM rule for broken SUSY

Fermion masses:

$$i\sqrt{2}g \, (T^a)^i{}_j \, (\psi_i \bar{\lambda}^a \bar{\psi}^j - \varphi^* \lambda \psi)$$ ← superpartner of D-terms

$$-\frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j + \text{h.c.}$$

from superpotential

$$\boxed{F_i = \frac{\partial W}{\partial \varphi_i}, \quad \bar{F}_i = \frac{\partial \bar{W}}{\partial \bar{\varphi}} \qquad D^a = g \sum_i \varphi_i^* T^a \varphi_i}$$

Fermion mass matrix:

$$(\psi_i \ \lambda_a) \begin{pmatrix} F_{ij} & \sqrt{2} D_{bi} \\ \sqrt{2} D_{aj} & 0 \end{pmatrix} \begin{pmatrix} \psi_j \\ \lambda_b \end{pmatrix}$$

$$F_{ij} \equiv \frac{\partial F_i}{\partial \varphi_j} \quad , \quad D_{ai} = \frac{\partial D_a}{\partial \varphi_i} = g \varphi_i^* T^a$$

$$m^{j=1/2} = \begin{pmatrix} F_{ij} & \sqrt{2} D_{bj} \\ D_{ai} & 0 \end{pmatrix}$$

<u>Scalar mass</u>

$$m^{2\, j=0}_{ij} = \begin{bmatrix} \frac{\partial^2 V}{\partial \varphi_i \partial \varphi^*_j} & \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j} \\ \frac{\partial^2 V}{\partial \varphi_i^* \partial \varphi_j^*} & \frac{\partial^2 V}{\partial \varphi_i^* \partial \varphi_j} \end{bmatrix}$$

$$= \begin{bmatrix} \bar{F}^{ik} F_{kj} + D_a{}^i D_{aj} + D_a{}^i{}_j D_a & \bar{F}^{ijk} F_k + D_a{}^i D_a{}^j \\ F_{ijk} \bar{F}^k + D_{ai} D_{aj} & F_{ik} \bar{F}^{jk} + D_{ai} D_a{}^j + D_a{}^j{}_i D_a \end{bmatrix}$$

<u>GB mass matrix</u>

$$\sum_i g^2 \left| A_\mu^a T^{a\alpha}{}_\beta \phi_{i\alpha} \right|^2 = \left| A_\mu^a D_a^i \right|^2$$

$$m^{2\,(j=1)}_{ab} = D_a^i D_{bi} + D_{ai} D_b^i$$

<u>Traces:</u> $\mathrm{Tr}\ m^{(j=1/2)} (m^+)^{(j=1/2)} = F_{ij} \bar{F}^{ij} + 4 |D_{ai}|^2$

$$\mathrm{Tr}\ m^{2\,(j=0)} = 2 F^{ij} \bar{F}_{ij} + 2 D_a{}^i D_{ai} + 2 D_a D_a{}^i{}_i$$

$$\mathrm{Tr}\left(m^{2(j=1)}\right) = 2 D_{ai} D_a{}^i$$

$$\mathrm{STr}\, M^2 = \mathrm{Tr}\,(2j+1)\,(-1)^{2j}\, M^2$$

$$= -2F\bar{F} - 8\,(D_{ai})^2 + 2F\bar{F} + 2D_a{}^i D_{ai}$$

$$+ 2 D_a D_a{}^i{}_i + 3\cdot 2\, D_{ai} D_a{}^i$$

$$= 2 D_a (D_a)^i{}_i$$

$\langle D_a \rangle \neq 0$ only for $U(1)$'s

$$= 2 D\, D^i{}_i \qquad D^i{}_i = \sum q_i \quad \text{sum of all } U(1) \text{ charges}$$

$$\mathrm{STr}\, M^2 = 2 D_\alpha \sum_i q_i^\alpha$$

α: $U(1)$ factors

usually $\sum q_i^\alpha = 0$ due to anomaly cancellation!

$\rightarrow$ $\boxed{\mathrm{STr}\, M^2 = 0}$

This is a very bad relation for the MSSM
Tells that <u>SOME</u> superpartners <u>lighter</u> than SM masses

Application to the MSSM (Dimopoulos & Georgi)

Assume sum rule applies. Consequence:
one squark lighter than m_u or m_d (experimentally impossible)

Scalar mass matrix:

$$M^{2\,i}{}_j = \begin{bmatrix} \bar{F}^{ik} F_{kj} + \frac{1}{2} D_a^i D_{aj} + \frac{1}{2} D^i_{ja} D_a & \bar{F}^{ijk} F_k + \frac{1}{2} D_a^i D_a^j \\ \bar{F}^k F_{ijk} + \frac{1}{2} D_{ia} D_{ja} & F_{jk}\bar{F}^{ki} + \frac{1}{2} D_{ai} D^{aj} + \frac{1}{2} D_a{}^j{}_i D_a \end{bmatrix}$$

Specify to squark mass matrix. Squarks should NOT get VEV (color not broken) $D_a^i = 0$

quarks only get mass from superpotential, since squark VEV

$D_{color} = 0$, $D_{1,2} = 0$ only $D_3, D_Y \neq 0$

$$M^2_{2/3} = \begin{bmatrix} m_{2/3} m_{2/3}^+ + \left(\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y\right) \mathbb{1} & \Delta \\ \Delta^+ & m_{2/3}^+ m_{2/3} - \frac{2}{3} g' D_Y \end{bmatrix}$$

$$M^2_{1/3} = \begin{bmatrix} m_{1/3} m_{1/3}^+ + \left(-\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y\right) \mathbb{1} & \Delta' \\ \Delta'^+ & m_{1/3}^+ m_{1/3} + \frac{1}{3} g' D_Y \end{bmatrix}$$

sum of all D-terms $= 0$ at least one ≤ 0

Assume for example $\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y \leq 0$

If β eigenvector of $m_{2/3}$ $(\beta^+, 0)\, M^2_{2/3} \begin{pmatrix} \beta \\ 0 \end{pmatrix} \leq m_0^2$

There must be a squark mass less than m_u or m_d
$\rightarrow$ not possible.

SUM RULES must be broken!

Need to violate assumption leading to sum rule

- renormalizable
- tree-level

Need to assume that no renormalizable interaction between ~~SUSY~~ sector & SM

For example:
- only transmitted through gravity structure of SUGRA Lagrangian (non-renormalizable) allows more terms
- a "messenger sector" mediates between SM fields & ~~SUSY~~ sector

If we don't want to specify, try to parametrize what kind of terms will we get from non-renormalizable interactions that violate SUM rule?

<u>Assume</u> ~~SUSY~~ field S, has <u>only</u>

non-renormalizable couplings to visible sector (either through gravity, quantum loops, ...) What operators could be generated?

$$\langle S \rangle = \ldots + \theta^2 \langle F \rangle$$

<u>Possible terms:</u>

- $$\int \Phi^\dagger \Phi \frac{S^\dagger S}{M^2} d^4\theta \longrightarrow \varphi^* \varphi \left(\frac{F}{M}\right)^2$$

  Scale at which new physics is integrated out
  M_{PL} for gravity
  M_{mess} for GM

  <u>Scalar mass</u>
  $m^2 \sim \left(\frac{F}{M}\right)^2$

(of course could also add terms like
$\int \Phi^\dagger \Phi^\dagger \Phi \frac{S^\dagger S}{M^3} d^4\theta$ get much more suppressed terms...)

- $$\int \Phi^2 S \, d^2\theta \longrightarrow F \left(\varphi^2 + \varphi^{*2}\right)$$

  b-term, natural size $\sim F$ w/o symmetry.

- $$\int \frac{S}{M} \Phi^3 d^2\theta \longrightarrow \frac{F}{M} \left(\varphi^3 + \varphi^{*3}\right) \to A \left(\varphi^3 + \varphi^{*3}\right)$$

  $A \sim m$, same order as scalar m

$$-\int W_\alpha W^\alpha \frac{S}{M} d^2\theta \quad \to \frac{F}{M}\lambda\lambda + h.c.$$

gaugino mass

$$m_\lambda \sim \frac{F}{M} \sim m \sim A$$

<u>Find:</u> — scalar mass
— gaugino mass
— scalar holomorphic cubic (A) & quadratic (b) terms

<u>Note</u>: $STrM^2 = \underbrace{2\sum_i m_i^2 - 2\sum_a m_{\lambda_a}^2}_{\text{no reason to vanish!}}$

This is the rationale for

<u>SOFT</u> breaking terms for the MSSM!

So full MSSM Lagrangian

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{soft}$$

$$\mathcal{L}_{soft} = -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B}\right) + h.c.$$

$$-\left(a_u \tilde{Q} H_u \tilde{\bar{u}} + a_d \tilde{Q} H_d \tilde{\bar{d}} + a_e \tilde{L} H_d \tilde{\bar{e}}\right) + h.c.$$

$$- \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{\bar{u}}^\dagger m_u^2 \tilde{\bar{u}}$$

$$- \tilde{\bar{d}}^\dagger m_d^2 \tilde{\bar{d}} - \tilde{\bar{e}}^\dagger m_e^2 \tilde{\bar{e}} - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d$$

$$-\left(b H_u H_d + h.c.\right)$$

$a_{u,d,e}$: 3×3 matrices in flavor space, 1–1 correspondence to Yukawa matrices

$m^2_{Q,L,u,d,e}$: 3×3 matrices in flavor space

We assume: $M_{1,2,3}$. $a_{u,d,e}$ $\sim m_{soft}$

$m^2_{QudLe,H_uH_d,b}$ $\sim m_{soft}^2$

m_{soft} $\sim$ few $\times$ 100 GeV – TeV

A LOT of new parameters : 105 new masses, phases, mixing angles on top of SM.

BUT: most of it ALREADY excluded from flavor & CP violating processes!

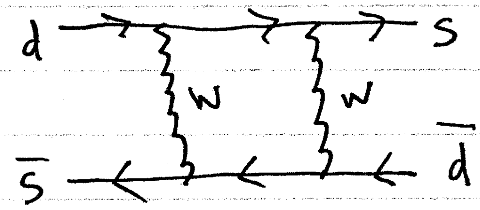
In SM: no tree-level FCNC's, loop level suppressed by GIM mechanism. Similarly lepton flavor # violation strongly suppressed

Example: FCNC in $K-\bar{K}$ mixing (one of the best tested processes).

$K = d\bar{s}$

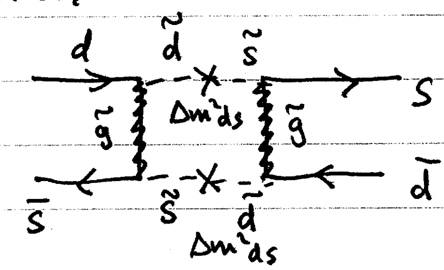
$\bar{K} = \bar{d}s$

in SM



CKM unitarity implies additional suppression

In MSSM :



Additional strongly coupled contribution. No GIM suppression

$$\sim \int \frac{d^4 p}{p^{10}} \sim \frac{1}{m_{SUSY}^6}$$

$$\mathcal{M}_{K\bar{K}}^{MSSM} \propto \alpha_3^2 \left(\frac{\Delta m^2_{ds}}{m_{SUSY}^2}\right)^2 \frac{1}{m_{SUSY}^2}$$

compare to exp'l bound

$$\frac{\Delta m_{ds}^2}{M_{SUSY}^2} \lesssim 4 \cdot 10^{-3} \left(\frac{m_{SUSY}}{500\ \mathrm{GeV}}\right)$$

Off-diagonal terms need to be strongly suppressed...

Similar constraints from $\mu \to e\gamma$, CP violating phases...

Organizing "principle":

<u>Soft-breaking universality</u>

1.) Soft breaking masses are universal ($\propto \mathbb{1}$) for all types of particles

$$m_Q^2 = \mathbb{1}\, m_Q^2$$
$$m_{\bar{u}}^2 = \mathbb{1}\, m_{\bar{u}}^2$$
$$\vdots$$

2.) If A-terms not flavor universal, after Higgs VEV will induce similar mixings

$$A\,(Q\bar{U}H_u + Q\bar{D}H_d + L\bar{E}H_d)$$

assume A itself proportional to Yukawa matrix! Whatever rotation you do on quarks, can also do on squarks → will be diagonal in same basis!

$$A_{ij}\, Q_i \bar{U}_j H_u \rightarrow A_u\, \lambda^u_{ij}\, Q_i \bar{U}_j H_u$$

3.) to avoid CP violation, assume all non-trivial phases beyond SM CKM vanishes

Ultimately want to explain this, for example gauge mediation!

ELECTROWEAK SYMMETRY BREAKING IN MSSM, LITTLE HIERARCHY

Need Higgs potential (assume squarks, sleptons don't get VEVs)

quartic: only from D-terms

$$V_D = \frac{1}{2} g^2 \left|H_u^+ H_d\right|^2 + \frac{1}{8}\left(g^2+g'^2\right)\left(|H_u|^2 - |H_d|^2\right)$$

Important: higgs quartic $\sim g^2, g'^2$
Higgs mass $\sim \sqrt{\lambda}\, v$ $\rightarrow$ Higgs mass related to M_Z!

Full Higgs potential:

$$V_H = \left(\mu^2 + m_{H_u}^2\right)|H_u|^2 + \left(\mu^2 + m_{H_d}^2\right)|H_d|^2$$

$$- B_\mu H_u H_d + h.c. + \frac{1}{2} g^2 \left|H_u^+ H_d\right|^2$$

$$+ \frac{1}{8}\left(g^2+g'^2\right)\left(|H_u|^2 - |H_d|^2\right)$$

~~No quartic along~~

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$$

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$$

only neutral comp's can get VEV!

In terms of H_u^0, H_d^0:

$$V_H = \left(|\mu|^2 + m_{H_u}^2\right)|H_u^0|^2 + \left(|\mu|^2 + m_{H_d}^2\right)|H_d^0|^2$$

$$-B_\mu \left(H_u^0 H_d^0 + h.c.\right) \quad + \frac{1}{8}\left(g^2+g'^2\right)\left(|H_u^0|^2 - |H_d^0|^2\right)^2$$

no quartic along $H_u^0 = H_d^0$

condition for EWSB: one direction along origin destabalized, but direction with no quartic has positive $(\text{mass})^2$!

$$\begin{vmatrix} |\mu|^2 + m_{H_u}^2 & -B_\mu \\ -B_\mu & |\mu|^2 + m_{H_d}^2 \end{vmatrix} < 0$$

$$B_\mu^2 > \left(|\mu|^2 + m_{H_u}^2\right)\left(|\mu|^2 + m_{H_d}^2\right) \qquad \text{negative } m^2$$

$$2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 - 2B_\mu > 0 \qquad \text{stability}$$

NO solution for $m_{H_u}^2 = m_{H_d}^2$.

Typically $\quad m_{H_u}^2 < 0$

$\qquad\qquad m_{H_d}^2 > 0$

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} = \frac{v \sin\beta}{\sqrt{2}}$$

$$\langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}} = \frac{v \cos\beta}{\sqrt{2}} \qquad \tan\beta = \frac{v_u}{v_d}$$

Minimizing potential we find:

$$\sin 2\beta = \frac{2 B_\mu}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}$$

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

really weird equation! Connects

M_Z , m_{H_u, H_d}^2 μ

z-mass soft-breaking SUSY mass.

Origin of Little hierarchy!

Evaluate higgs masses, lightest CP-even higgs ~ SM higgs,

$$m_{h^0}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 \pm \sqrt{(M_Z^2 + m_A^2)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta}\,\right]$$

$$m_A^2 = \frac{B_\mu}{s_\beta c_\beta}$$

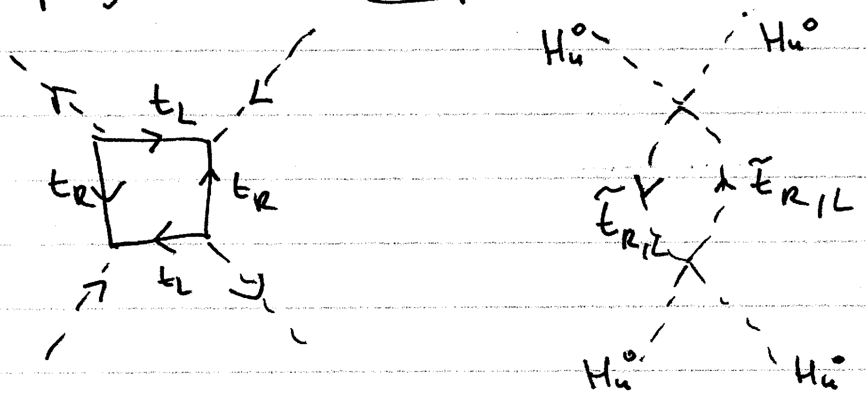
$$m_{h^0} \leq M_Z |\cos 2\beta| \leq M_Z$$

Tree-level upper bound on m_h.

But already know from LEP $m_h \gtrsim 114$ GeV

Tree-level MSSM excluded. Need a <u>large</u> correction to quartic self-coupling. Main effect from top-stop loops!

1.) Want tree-level quartic maximized $\rightarrow$ large $\tan\beta$, VEV mostly in H_u. Light higgs $\sim H_u$. So need mostly H_u^4 coupling. <u>1 loop:</u>



result: $$\lambda(m_t) = \lambda_{SUSY} + \frac{2N_c (y_t)^4}{16\pi^2} \log\left(\frac{m_{\tilde{t}_1} \tilde{m}_{t_2}}{m_t^2}\right)$$

fixed.

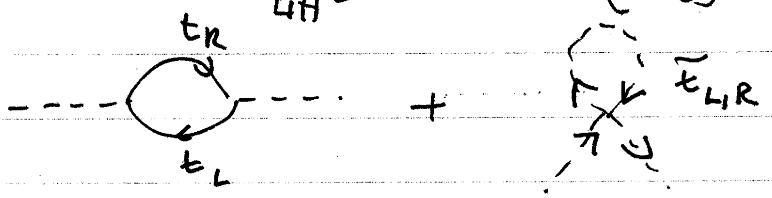
To push up higgs mass $\rightarrow$ need to increase $m_{\tilde{t}}$!

$$\Delta m_{h^0}^2 = \frac{3}{4\pi^2} v^2 y_t^4 \sin^2\beta \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) \lesssim 130\ \mathrm{Ge}$$

Little hierarchy of MSSM

- At tree-level $m_{h^0} \leq M_Z$
- Need a large $m_{\tilde{t}} \sim 1-1.4$ TeV to increase quartic to push $m_{h^0} > 114$ GeV
- But then also get corrections to quadratic in $m_{H_u}^2$

$$\Delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)$$

from 

The bigger $m_{\tilde{t}}$, the larger the shift in $m_{H_u}^2$. But remember weird equation

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

$$\rightarrow m_{H_u}^2 \sim \frac{M_Z^2}{2}$$

but loop correction $\Delta m_{H_u}^2 = \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)$

$$FT \sim \left(\frac{\Delta m_{H_u}^2}{M_Z^2/2}\right) \sim 800 \quad \text{for} \quad m_{\tilde{t}} = 1.2\,\text{TeV},\ \Lambda = 10^{16}$$

$\rightarrow 0.1\%$ tuning.

Little hierarchy of MSSM!

Gauge mediated $\cancel{SUSY}$

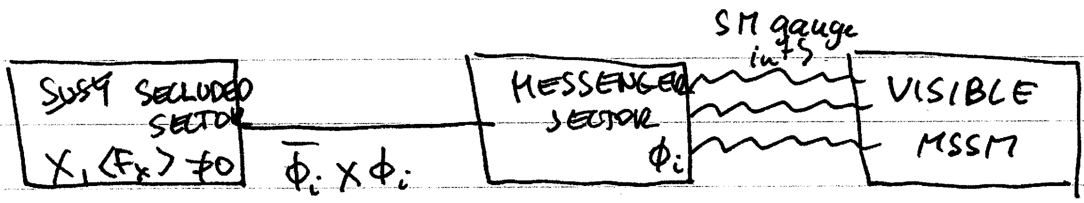
Flavor problem: in SM in limit when Yukawa $\to 0$ $U(3)^5$ flavor symmetry
(3 gen's completely equivalent, 5 types of particles $\underbrace{Q\,u\,d\,L\,e}_{U(3)^5}$)

This flavor symmetry broken at SOME scale Λ_F, below which only imprint is Yukawas.
Λ_F could be very large, so effects of flavor breaking $\propto \frac{1}{\Lambda_F}$ $\to$ could all be very small!

However, if $\cancel{SUSY}$ mediated by gravity, $\cancel{SUSY}$ happens at $M_{Pl} > \Lambda_F$, really NO reason for soft breaking terms to not have $\mathcal{O}(1)$ flavor violation. Even if at tree-level for some reason they are flavor invariant, loop effects of flavor breaking sector will be large.
Would like theory where scale of $\cancel{SUSY}$ mediation $\ll \Lambda_F$. Need to lower relevant mass scale for mediation (& physics of mediation itself should be flavor universal!)

Most important example:

GAUGE MEDIATION



Idea: – generate mass splittings OBEYING sum rules for messengers
– only through messengers in loop will MSSM feel ~~SUSY~~

– Effectively: generate non-renormalizable ops. connecting MSSM & ~~SUSY~~ sector.
– relevant scales M messenger mass
$\langle F_X \rangle$ ~~SUSY~~ VEV
– below M integrate out messenger sector, generate soft ~~SUSY~~ masses
– since interactions ~~of~~ mediating ~~SUSY~~ SM gauge interactions → will be flavor universal (if $\Lambda_F > M$)

Minimal gauge mediation

SM gauge singlet X $\langle X \rangle = M$
$\langle F_X \rangle \neq 0$

~~SUSY~~ sector

<u>Messengers:</u> N_f flavors $\phi_i, \bar{\phi}_i$

$$W = \lambda \bar{\phi} X \phi$$

messenger scalar mass matrix:

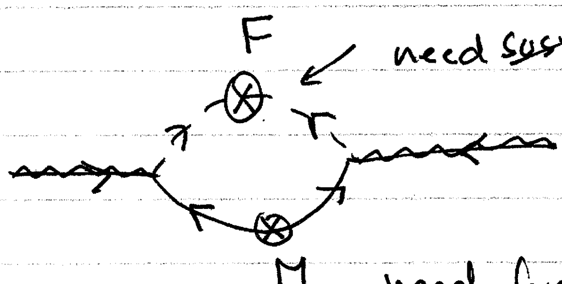
$$\left|\frac{\partial W}{\partial \phi}\right|^2 = |\lambda \langle X \rangle|^2 |\phi|^2 + |\lambda \langle X \rangle|^2 |\bar{\phi}|^2 + \lambda \bar{\phi} \phi F_X$$

masses: $m^2 = \lambda^2 M^2 \pm \lambda F$ scalar
λM fermion

$\left.\begin{matrix}\text{scalar} \\ \text{fermion}\end{matrix}\right\}$ obey sum rule

SM gaugino mass:

need ~~SUSY~~ mass for scalar



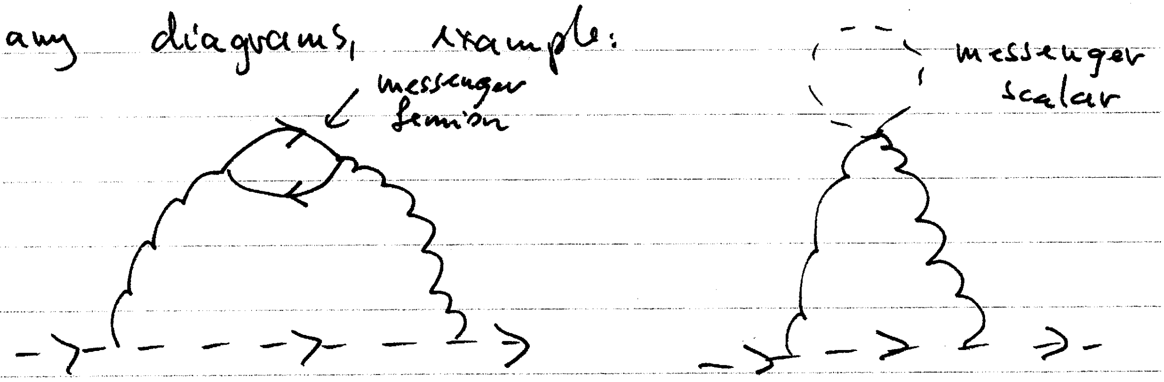
need fermion mass for helicity flip

$$m_\lambda = \frac{F \cdot M}{M^2} \frac{g^2}{16\pi^2} \cdot N_m \qquad \text{exact}$$

$$m_{\lambda_i} = \frac{\alpha_i}{4\pi} N_m \frac{F}{M}$$

Scalar mass: generated @ 2loop only
need both gauge boson & messenger to run in loop

many diagrams, example:



$+ \ldots$

result

$$m_{soft}^2 \propto \frac{g^4}{(16\pi^2)^2} N_m \frac{F^2}{M^2}$$

note $m_{soft}^2 \sim (m_{gaugino})^2$

<u>Important phenomenological consequence:</u>

LSP = gravitino. Why?

~~mass~~

$$m_{3/2} = \frac{F}{M_P}$$ always set by M_P
(like $M_W \sim gv$)

But now F very small

$$\frac{\alpha}{4\pi} N_m \frac{F}{M} \sim m_{EW}$$

$$m_{3/2} = \left(\frac{\sqrt{F}}{100\,\text{TeV}}\right) 2.4\,\text{eV}$$

For relevant F's $m_{3/2} \ll m_{EW}$

Very light, but very weakly coupled!

If $\sqrt{F} \gtrsim 10^6$ GeV : ~~[illegible]~~ NLSP lives so long, for collider physics like ordinary LSP

if $\sqrt{F} \lesssim 10^6$ GeV NLSP decays within detector

→ quite unique signal displaced photons $+ \cancel{E}_T$

The μ-B_μ problem of gauge mediation

μ param: only SUSY preserving mass term. Need to forbid it, then relate to $\cancel{SUSY}$

$$\left.\begin{aligned} H_u &\to e^{i\alpha} H_u \\ H_d &\to e^{i\alpha} H_d \end{aligned}\right\} \text{PQ symmetry forbids it}$$

Assume $\cancel{SUSY}$ breaks PQ symmetry.

In gravity mediation works perfectly

$$\int d^4\theta \, \frac{X^+ H_u H_d}{M_{Pl}} \qquad \langle X \rangle = \theta^2 F \rightarrow$$

get effective μ

$$\mu \sim \frac{F}{M_{Pl}}$$

$$\int d^4\theta \, \frac{X^+ X}{M_{Pl}^2} H_u H_d \qquad B_\mu \sim \frac{F^2}{M_{Pl}^2} \sim \mu^2$$

$B_\mu \sim \mu^2$ good. Only in gravity mediation

In gauge mediation: $F \ll 10^{11}\,\mathrm{GeV}$

μ, B_μ way to small. Need to couple Higgs directly to messengers, but then

$$\left.\begin{aligned} \mu &= \frac{1}{16\pi^2} F/M \\ B_\mu &= \frac{1}{16\pi^2}\left(F/M\right)^2 \end{aligned}\right\} \text{ both at 1-loop,}$$

$B_\mu \gg \mu^2$!

no good EWSB...