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Spring School on Superstring Theory and Related Topics

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Towards holographic duality for condensed matter
Lecture V

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How kinematics protects LFL theory (standard view)

[Abrikosov, Fundsn Thy of Metals]

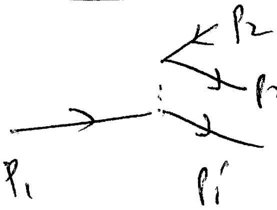
$$\Psi_p^{\text{quasiparticle}}(t) \sim \exp\left(-i\,\xi(p)\,t - t/\tau_p\right)$$

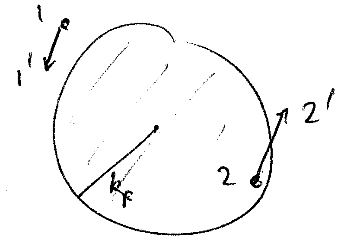
$$\xi_p \equiv \epsilon_p - \epsilon_F = p^2/2m - k_F^2/2m$$

qp is meaningful if $\frac{1}{\tau} \ll |\xi|$ (many cycles before decay)

(i.e $\Gamma \ll \omega_p$)

whence τ? in clean FL w/o phonons





$p_1 > k_F$, $p_2 < k_F$

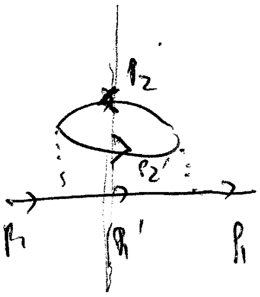
$p_1' > k_F$, $p_2' > k_F$

$$\vec{p}_1 = -\vec{p}_2 + \vec{p}_1' + \vec{p}_2'$$

(momentum of resulting hole)

e w/ $p_1 > k_F$
interacts w/ e in F.Sea, $p_2 < k_F$
kicks it out, creating e w/ p_2'

$\frac{1}{\tau_p} \sim$ scattering prob $=$ Im



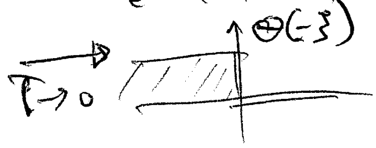
(i.e. Fermi's golden rule)

$$\propto \int d^3p_2\, d^3p_1'\, f(-\xi_{p_2})\, f(\xi_{p_1'})\, f(\xi_{p_2'})\, \delta\big(\epsilon_1 - (-\epsilon_2 + \epsilon_1' + \epsilon_2')\big)$$

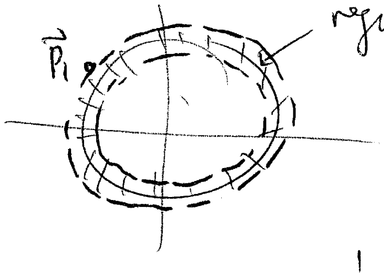
must be filled — must be empty — ϵ of final state

fermi f'n
$$f(\xi) = \frac{1}{e^{\beta\xi}+1}$$

$T \to 0$



claim: if $|\vec{p}_1| \sim k_F$ then so are p_1', p_2, p_2'.



region of integration for $|p_2|, |p_1'|$

fraction of total ~~sf.~~ area: $\left(\frac{\xi_p}{E_F}\right)^2$

$$\longrightarrow \frac{1}{\tau_p}\left(\text{from ee scatt}\right) \sim \xi_p^2 = \omega_*^2(p)$$

4-fermi interactions preserve

$\longrightarrow$ stable q.p.

Loophole in argument for incompatibility of F.L. theory; boson version.

$$S_{eff}[\psi, a] = \int_{k,\omega}\Big[\psi^\dagger_{k\alpha}(-i\omega-\mu+\epsilon_k)\psi_{k\alpha} + \frac{k^2}{e^2}|a(k,\omega)|^2 + a\cdot\mathcal{O}(\psi)\Big]$$

$\alpha = 1..N$

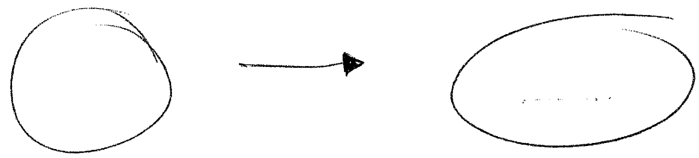
bilinear in ψ's.

(only no kinetic term momentarily)

(Note: no m^2a^2 term by assumption)

whence a? many possibilities

A) Pomeranchuk instability. "nematic order param" shape of FS near critical pt



$$\longrightarrow \mathcal{O} = \frac{1}{\sqrt{N}}\sum_k (\cos k_x l - \cos k_y l)\,\psi^\dagger_{k\alpha}\psi_{k\alpha}$$

(like vector model) lattice spacing.

B) spin-charge separation (ever)

where:

$$\mathcal{L}_{FL} = \psi^\dagger(\partial_t - \xi(p))\psi + V(1..4)\,\psi^\dagger_1\psi_2\psi^\dagger_3\psi_4$$

$V(1..4) = V(p_1,p_2;p_1,p_2)$ is marginal

$$= V\Big(\cos\theta \equiv \frac{\vec{p}_1\cdot\vec{p}_2}{p_1p_2}\Big)$$

$$= \sum_\ell P_\ell(\cos\theta)\,F_\ell$$

Landau params

$$s\frac{d}{ds}F_\ell = -NF_\ell^2 + \mathcal{O}(F^3)$$

Pomeranchuk: if some $F_\ell < 0$

$\langle\psi^\dagger\psi(\ell)\rangle = \phi_\ell \neq 0$

$\rightarrow \mathcal{L} \sim \psi^\dagger(\ldots - \xi(p))\psi + \psi^\dagger\psi\phi_\ell + \ldots$

changes shape of FS

FS + gauge field, relevance to cuprates. (Polchinski cond-mat/9303037)

strong on-site repulsion (Hubbard U)
⇒ no double occupancy ⟹ "spin-charge separation"
"fractionalization"
"the electron shatters"

replace inequality w/ equality: ("slave particle ansatz")
each site is occupied by spin ↑ or ↓ e^- or <u>hole</u>

$$1 = \sum_{\alpha=1}^{2} f_{i\alpha}^{+} f_{i\alpha} + b_i^{+} b_i \qquad \underline{\forall i}.$$

e^- field $\psi_{i\alpha} = \underset{\text{spin dof}}{f_{i\alpha}} \underset{\text{charge dof.}}{b_i^{+}}$ destroys a spin & creates a hole

<u>Redundancy</u>: local $U(1)$ $\begin{cases} f_{i\alpha}(t) \to e^{i\lambda_i(t)} f_{i\alpha}(t) \\ b_i(t) \to e^{+i\lambda_i(t)} b_i(t) \end{cases}$ preserves ψ.

the gauge field arises in implementing this.

More precisely, promote $SU(2) \to SU(n)$, integrate out f
to find $S_{eff}[f_i^{+}f_j \equiv \Delta_{ij}]$

<u>Sometimes</u> $S_{eff} \Rightarrow \langle \Delta_{ij} \rangle = |\Delta_{ij}| e^{i a_{ij}} \neq 0$

<u>Really</u>: ~~mean field~~ ansatz for groundstate.
~~not~~, not a derivation.

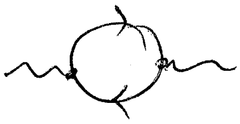
Spin liq. other ansatze possible, eg: $\psi = \underset{\text{fermionic holons}}{f_i^{+}} b_{i\alpha}$

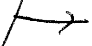
Mott <u>Ins</u>: b's massive. → a gapless

<u>Open Q</u>: why don't b condense in metallic phase? we ignore them.

25.4

(1) "landau damping"

$\delta\Sigma_{boson}(\omega, q) =$  $\underset{\substack{\omega\to 0\\ q\gg\omega}}{\sim} \gamma\left|\frac{\omega}{q}\right|.$ $(\gamma = 1/4\pi)$

 $\equiv \frac{1}{-i\omega-\mu+\epsilon_k}$

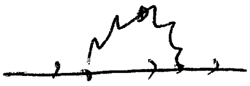
$D_{eff}(\omega, q) \equiv$  $= \left(\frac{q^2}{e^2} + \gamma\left|\frac{\omega}{q}\right|\right)^{-1}$

change scaling : $\omega \sim q^3$ if they compete $(z_b = 3)$

useful generalization $\frac{q^2}{e^2} \longrightarrow \frac{|q|^{z_b-1}}{e^2}$

[HLR : $\frac{1}{2}$-filled landau level , Nayak-Wilczek .]

(2) what happens to ψ when it couples to a ?

$\delta\Sigma_{ferm}(\omega, k) =$  $\sim -i\frac{1}{\lambda N}\,\mathrm{sgn}(\omega)|\omega|^{2/z_b}$

$\lambda = 4\pi \sin\frac{2\pi}{z_b}\,\gamma^{\frac{z_b-2}{z_b}}$

- $\underbrace{\delta\Sigma \sim \omega^{2/z_b}}_{\text{wins}} \gg \omega$ for $\omega < e^{\#}$. $(z_b > 1)$

~~new scaling for ψ~~

- now $\frac{\Gamma(k_\perp)}{\omega_*(k_\perp)} \xrightarrow{k_\perp\to 0} \mathrm{const}$, $Z \to 0$ kills qp.

- new scaling for $\psi \Longrightarrow a\psi^\dagger\psi$ is marginal for $z_b \in [2, 3]$!

- origin of boson irrelevant here. (not phonons! extra qs!)
(so far)

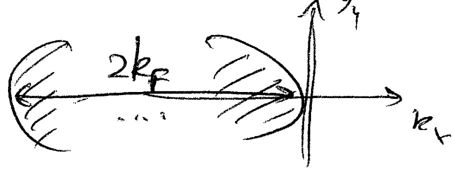
③ universal bit: zoom in on patch (+ antipode) 　　　　 LS.5

$$S_{\text{2-patch}} = \int d^2x\, d\tau \sum_{s,\alpha} \psi^\dagger_{s,\alpha}\left(\eta\partial_\tau - is\partial_x - \partial_y^2\right)\psi_{s\alpha}$$

$\eta = 0^+$. 　　　　 $+ \frac{g_s}{\sqrt{N}}\, a\, \psi^\dagger_{s\alpha}\psi_{s\alpha} \quad + \int_{k,\omega} |k_y|^{z_b - 1} |a|^2$

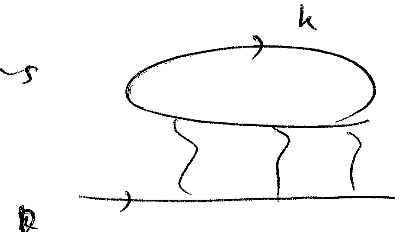
$s = \pm$ for ψ_R

$$g_s = \begin{cases} + & \text{nematic} \\ (\pm)^s & \text{gauge som.} \end{cases}$$



④ Q: what justifies 1-loop bubble sum ("RPA")?
(note: self-consistent) 　　　　 large N?

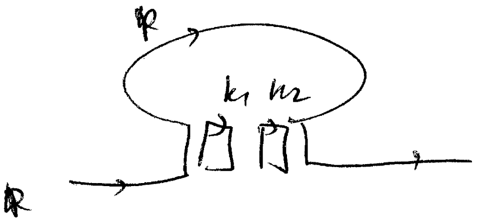
⑤ [SSL] 0905.4532: violations
keep just $s=+1$.



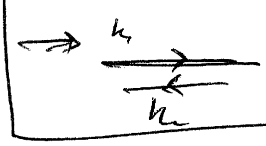
keep k on FS $\Rightarrow \epsilon_k = 0$

$\rightsquigarrow g = \left(\frac{\omega^{2/3}}{N} + k_y\right)^{-1} \propto N$

condition for staying on FS: ~~diagram~~ diagram may be drawn as double line



connects two pts on FS k_1, k_2

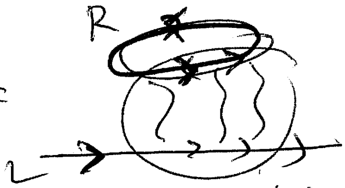


$\rightarrow$ planar diagrams from vector model!

~~[MS]: two patches $\Rightarrow$ further violations.~~

LS.6

⑥ [MS] two patches gets worse.. ($z_b = 3$)
1001.1153

A) $\delta \Sigma^{(3)}_{\substack{\text{fermi} \\ \text{2-patch}}}(p, w=0) =$  $\propto g_s^3 \, \cancel{N} \, \epsilon_p \ln \frac{\Lambda}{\epsilon_p^{3/2}}$

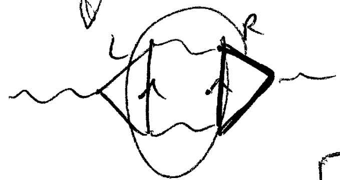
$N^{-2} \cdot N^{+2}$

ϕ

enhancement

note: Cooper vertex ~~Ampere~~.

changes fermion dim by $O(1)$

B) $\delta \Sigma_{\text{boson}}(w=0, q) \sim$  $\propto - \underbrace{g_s^3}_{\sim} \cdot \underbrace{\frac{\cancel{\epsilon} N}{N}}_{O(1)} \cdot \underbrace{|q|^{z_b - 1}}_{\text{corrects kinetic term.}}$

( < 0 for gauge boson)

negative for nematic! $\Rightarrow$ instability!

⑦ [Mross et al] vary z_b (+ interesting results.)

$\delta \Sigma^{\cancel{3}}_{z_b \sim 2} \propto \cancel{g^3} \frac{1}{N^2} \cdot \frac{1}{z_b - 2} |w|^{2/z_b}$

$= f(N(z_b - 2))$

(note: → $w \ln w$ as $z_b \to 2$)

take $N \to \infty$, $z_b \to 2$, fix $\lambda \equiv (z_b - 2) N$.

A), B) become small interesting correction.

A) → suppression of tunneling d.o.s. at nematic transition

Can compute $2k_F$ response. suppressed for ~~gauge field~~ nematic

Cooper is funny. ($z_b - 2$'s cancel!)

interesting for gauge field (competition betw spreading of FS + amperean attraction.

proposed ~~phase~~ phase diagram in N, z_b.