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Spring School on Superstring Theory and Related Topics

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Models of electroweak symmetry breaking and the TeV scale
Lecture II

C. Csaki
Cornell University,
Ithaca
USA

EXTRA DIMENSIONS

- Idea of extra dimensions around for a very long time (1920's Kaluza & Klein)
- Did not seem to be very interesting for low-energy particle physics till ~ last decad

Kaluza-Klein decomposition

Take a scalar field in 5D. Reason why we have not seen 5^{th} dim yet → compactified. Assume 5^{th} dimension compactified on a circle. Free scalar:

$$S = \int d^5x \, \frac{1}{2} \partial_M \phi \, \partial^M \phi \qquad M = 0,1,2,3,5$$

$$x_5 = y$$

Since y compact → can do Fourier decomposition

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^n(x) \, e^{i\frac{n}{R} y}$$

Since ϕ real $\quad (\phi^{(n)})^+ = \phi^{(-n)}$

What will action look like?

$$\partial_M \phi \partial^M \phi = \partial_\mu \phi \partial^\mu - (\partial_y \phi)^2$$

By orthogonality of Fourier terms

$$S = \int d^4x \sum_{mn} \left(\int dy \frac{1}{2\pi R} e^{i\frac{(m+n)}{R} y} \right) \cdot$$

$$\cdot \frac{1}{2} \left[\partial_\mu \phi^{(m)}(x) \, \partial^\mu \phi^{(n)}(x) + \frac{mn}{R^2} \phi^{(m)}(x) \, \phi^{(n)}(x) \right.$$

$$= \frac{1}{2} \int d^4x \sum_n \left[\partial_\mu \phi^{(-n)} \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \phi^{(-n)} \phi^{(n)} \right.$$

From 4D point of view → 1 5D scalar = tower of 4D particles, with masses $= \frac{n}{R}$ ≡ "KK- tower" of fields.

If more than one extra dimension, i.e. n- dimensional torus $R_5, R_6, \ldots$

$$m^2_{n_5, n_6, \ldots} = m_0^2 + \frac{n_5^2}{R_5^2} + \frac{n_6^2}{R_6^2} + \ldots$$

the higher dimensional mass of the field.

If I have a more complicated field, say gauge field A_M.

Do same KK decomposition

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_n A_M^{(n)}(x^\mu)\, e^{i\frac{n}{R}y}$$

$$S = \int d^4x\, dy \left(-\tfrac{1}{4} F_{MN} F^{MN}\right)$$

$$= \int d^4x\, dy \; -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2}\left(\partial_\mu A_5 - \partial_5 A_\mu\right)\left(\partial^\mu A_5 - \partial_5 A^\mu\right)$$

$$= \int d^4x \sum_n -\tfrac{1}{4} F_{\mu\nu}^{(-n)} F^{(n)\mu\nu} \ldots$$

$$+ \tfrac{1}{2}\left(\partial_\mu A_5^{(-n)} + i\frac{n}{R} A_\mu^{(-n)}\right)\left(\partial^\mu A_5^{(n)} - i\frac{n}{R} A^{(n)\mu}\right)$$

mixing between A_5, A_μ!

Do gauge transformation

$$A_\mu^{(n)} \to A_\mu^{(n)} - \frac{i}{n/R}\, \partial_\mu A_5^{(n)}$$

$$A_5^{(n)} \to 0 \quad \text{for } n \neq 0 \quad (n=0 \text{ does not mix anyway})$$

$$\to S = -\tfrac{1}{4}\left(F_{\mu\nu}^{0}\right)^2 + \tfrac{1}{2}\left(\partial_\mu A_5^{(0)}\right)^2$$

$$+ \sum_{n\geq 1} 2\left(-\tfrac{1}{4} F_{\mu\nu}^{(-n)} F^{\mu\nu(n)} + \tfrac{1}{2}\frac{n^2}{R^2} A_\mu^{(-n)} A^{\mu(n)}\right)$$

→ tower of massive gauge bosons

+ zero mode GB, zero mode scalar

$$A_M = \left(\frac{A_\mu}{A_5}\right)$$

4D gauge boson: 2 physical DOF (massless) 3 DOF (massive)

5D gauge boson (massless) : 3 DOF

at massive level just a massive 4D gauge boson

massless level : scalar + GB

(A_5 scalar eaten at the massive level).

In more then 5D: ~~4~~ n extra dim.

$$A_M \to \quad A_\mu \quad \text{massive KK}$$
$$(n-1) \quad \varphi \quad \text{massive scalars}$$
$\Big\}$ massive

&

massless: gauge boson + n scalars...

Similarly for graviton

$$g_{MN} = \begin{array}{cc} & \overbrace{\hphantom{A_\mu A_\mu}}^{n} \end{array} \\ \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline & \end{array}\right)$$

massive level:

graviton + (n−1) gauge fields

$+ \frac{n(n+1)}{2} - n$ scalars

⋮

massless graviton: 2 DOF

massive graviton: 5 DOF = graviton + eaten GB + eaten scalar

Matching of couplings:

gauge field: $D_\mu = \partial_\mu - i g_5 A_\mu =$

$$= \partial_\mu - i g_5 \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)} + \dots$$

$$\boxed{g_4 = \frac{g_5}{\sqrt{2\pi R}}}$$

for general n dim's

$$g_4^2 = \frac{g_{(n)}^2}{V_n}$$

Matching of gravitational coupling:

$$S_{4+n} = - M_{(4+n)}^{2+n} \int d^{4+n} \sqrt{g}\, R_{(4+n)}$$

$\uparrow$ 4+n dim'l Planck scale $\equiv M_*$

$$= - M_*^{2+n} V_n \int d^4x \sqrt{g^{(4)}}\, R_{(4)} + \dots$$

$$= - M_{Pl}^2 \int d^4x \sqrt{g}\, R$$

$$\boxed{M_{Pl}^2 = M_*^{2+n} V_n}$$

matching of gravity.

If there is a 4+n dim'l theory with just a single scale M_*, compactification radius r

$$g_* \sim \frac{1}{M_*^{n/2}}$$

matching : $$1/g_4^2 = V_n M_*^n \sim r^n M_*^n$$

$$M_{Pl}^2 = V_n M_*^{n+2} \sim r^n M_*^{n+2}$$

$$\rightarrow \quad r \sim \frac{1}{M_{Pl}} \, g_4^{\frac{n+2}{n}}$$

extremely tiny size, not interesting.

New ingredient in mid '90's (Polchinsky, Horava-Witten, ...)

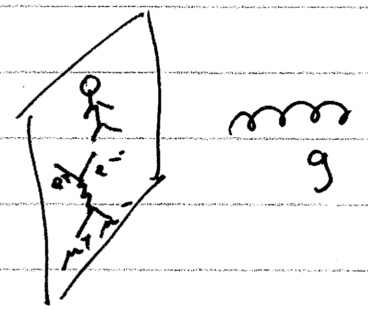
Branes could trap fields. Not all fields have to propagate in all dimensions!

This can decouple particle physics & gravity, & size of extra dimension could possibly be much larger?

LARGE EXTRA DIMENSIONS

(Arkani-Hamed, Dimopoulos, Dvali)
ADD

Assume all of particle physics stuck to a 3-brane, but gravity propagates in extra dimensions



How large could extra dimension be now?
Just need matching of 4+n dim gravity to 4D gravity:

$$M_{Pl}^2 = M_*^{2+n} \, r^n$$

$$r = \frac{1}{M_*} \left(\frac{M_{Pl}}{M_*} \right)^{2/n}$$

or $$1/r = M_* \left(\frac{M_*}{M_P} \right)^{\frac{2}{n}}$$

Could it be that M_*, the "fundamental scale" of gravity is $M_* \sim \text{TeV}$?

if $M_* = 1\,\text{TeV}$ $\quad \frac{1}{r} = (\text{TeV}) \; 10^{-\frac{32}{n}}$

using $1\,(\mathrm{GeV})^{-1} = 2\cdot 10^{-14}\,\mathrm{cm}$

$$\rightarrow \boxed{r \sim 2\cdot 10^{-17}\; 10^{\frac{32}{n}}\;\mathrm{cm}}$$ **for ADD**

for $n=1$ $\quad r = 10^{15}\,\mathrm{cm}$ size of solar system
not possible

$n=2$ $\quad r \approx 0.1\,\mathrm{cm}$ barely not allowed
by gravitational Cavendish expt.

$n=3$ $\quad r < 10^{-6}\,\mathrm{cm}$ ...

What do we know about short distance gravity? Surprisingly little.
Gravitational Cavendish experiments test $\frac{1}{r^2}$ law down to $\sim 10^{-4}\,\mathrm{m}$
(Eöt-wash experiment).
Direct bound on 2 extra dim $r < 37\,\mu\mathrm{m}$.

$n = 3, \dots 6$ allowed to have $M_* = 1\,\mathrm{TeV}$.

For $n=2$ the bound implies $M_* > 1.4\,\mathrm{TeV}$

If this was the case:
- gravity & particle physics fundamentally not different
- $M_* \sim 1\,\mathrm{TeV}$ is the fundamental scale, no need to worry about hierarchy problem.

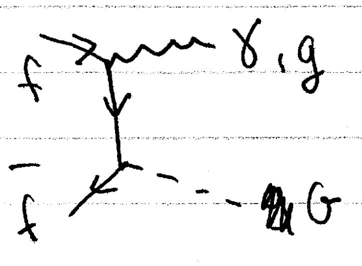
- gravity *appears* weaker at short distances because its flux is diluted in more dimensions. Once we go below scale r notice gravity $4+n$ dim'l.

- BUT : in a completely natural theory expect $r \sim 1/M_* \sim 1(\mathrm{TeV})^{-1}$, while here $r \sim \frac{1}{M_*}\left(\frac{M_P}{M_*}\right)^{\frac{2}{n}}$.

Hierarchy in mass scales translated to hierarchy in $r/(1/M_*)$. Just reformulated hierarchy problem to a problem of radius stabilization. Pretty hard to solve...

- Other interesting topics related to ADD

• graviton production in colliders

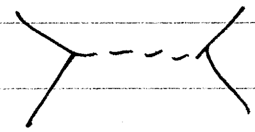


$f\bar{f} \to \gamma + G$

photon + missing energy

- virtual graviton exchange

$e^+e^- \to f\bar{f}$



contributes
gives a bound on M_*

- Supernova cooling: Supernovae can cool due to graviton emission (similar to axion bounds from supernova cooling). Strongest bound on $n=2$ theories $M_* \gtrsim 100\,\text{TeV}$.

- Black hole production in colliders, cosmic rays

For $E_{CM} > M_*$ black holes can be formed

$$R_S \sim \frac{1}{M_*}\left(\frac{M_{BH}}{M_*}\right)^{\frac{1}{n+1}}$$

$$\sigma_{BH} \sim \pi R_S^2 \,, \quad \text{can be big} \sim 400\,\text{pb}$$

Decays via Hawking radiation

$$T_H \sim \frac{1}{R_S} \sim M_* \left(\frac{M_*}{M_{BH}}\right)^{\frac{1}{n+1}}$$

decays equally to all degrees of freedom, 10% leptons, 2% photons...

WARPED EXTRA DIMENSIONS

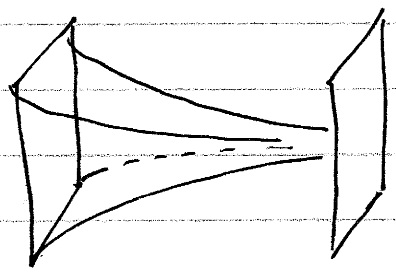
(Randall-Sundrum '99 / Maldacena '97)

Large extra dimensions interesting phenomenology, but does not REALLY solve hierarchy, since large radius unexplained... More interesting possibility: warped extra dim's.

AdS$_5$ metric:

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

Take a slice of AdS, cut off space at $z = R$ & $z = R'$



Then natural physical scales will be warped down by appropriate factors of

$\frac{z}{R}$.

(original RS proposal)

Example: take a Higgs scalar at $z = R'$

$$\int d^4x \sqrt{g^{ind}} \left[\partial_\mu \varphi \, \partial_\nu \varphi \, g^{\mu\nu} - \left(|\varphi|^2 - \frac{v^2}{2}\right)^2 \right]$$

Say $v = 1/R$ of the order of the large scale

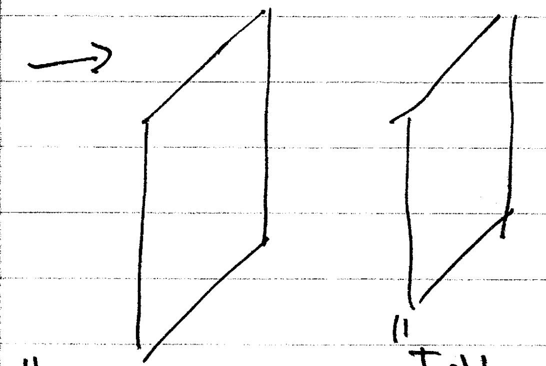
$$S = \int d^4x \left(\frac{R}{z}\right)^4 \left[(\partial_\mu \varphi \partial^\mu \varphi)\left(\frac{z}{R}\right)^2 - \lambda \left(|\varphi|^2 - \frac{v^2}{2}\right)^2 \right|_{z=R'}$$

Kinetic term $\left(\frac{R}{R'}\right)^2 (\partial_\mu \varphi)^2$,

canonically normalized higgs $\tilde{\varphi} = \left(\frac{R}{R'}\varphi\right)$

$$S_{eff} = \int d^4x \; (\partial_\mu \tilde{\varphi})^2 - \lambda \left[|\tilde{\varphi}|^2 - \frac{1}{2}\left(v \frac{R}{R'}\right)^2 \right]^2$$

warped down mass scale will be natural scale of Higgs VEV.



"Planck brane"

UV brane

"TeV brane"

IR brane

If SM Higgs peaked on or towards IR brane, while gravity peaked on UV brane $\rightarrow$ hierarchy problem solved!

Best explanation in terms of AdS/CFT

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

$z \to \alpha z$, $x \to \alpha x$ scale invariance of metric. This means: moving along z direction equivalent to rescaling 4D energy scales ("holographic running")
z small corresponds to high energies as expected from before, z large to small energies

bulk of AdS $\longrightarrow$ CFT_4 as in Maldacena

difference here: have only a slice of AdS_5.
<u>UV brane:</u> an explicit UV cutoff at $\Lambda = 1/R$ of theory.

<u>IR brane:</u> more tricky. Right interpretation: CFT spontaneously broken by appearance of IR brane, which provides a mass gap for KK modes. Conformality spontaneously broken, theory becomes strongly interacting, confines & produces a mass gap.

Important checks: Spectrum of a bulk field (for example bulk gauge fields, bulk scalar):

In appropriate gauge, eom will be:

$$A_\mu(x,z) = \epsilon_\mu(p)\, f(z)\, e^{ipx}$$

$$-m^2 f - z\,\partial_z \frac{1}{z}\,\partial_z f = 0$$

$$f'' - \frac{1}{z} f' + m^2 f = 0$$ Bessel eq. after $f = zg$ replacement

$$f(z) = z\left[A J_1(mz) + B Y_1(mz)\right]$$

~~Get~~ / ~~this~~ With BC $f'(R) = f'(R') = 0$

get 0 mode ← flat

+ discrete spectrum with spacing $\frac{1}{R'}$

$\sim$ TeV

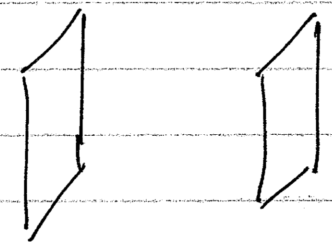
all peaked on IR brane.

Interpretation: KK modes = composite spin 1 mesons generated via confinement!

How about gauge vs. global symmetries?

Maldacena: $\mathcal{N}=4$ has $SO(6) = SU(4)_R$ global symmetry, shows up as a gauge field in AdS_5, which will couple to global current J_μ on boundary. Due to infinite AdS no normalizable 0 mode.

Here:



finite slice, gauge 0 mode normalizable!

If $(+,+)$ BC $\rightarrow$ global symmetry of CFT weakly gauged.

If $(-,+)$ BC $\rightarrow$ just a global symmetry (no 0 mode)

If $(+,-)$ BC $\rightarrow$ global symmetry weakly gauged, & broken by IR dynamics $\longrightarrow$ TECHNICOLOR

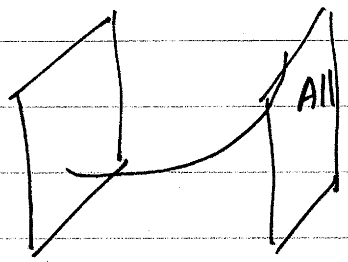
$\downarrow$

will be basis of higgsless models

If $(-,-)$ BC $\rightarrow$ global symmetry broken $\rightarrow$ get Goldstone boson (A_5 massless)

The various warped space models

RS1



Interpretation: All of SM is composite, including gauge symmetries, SM fermions, Higgs

Possible, but very hard to imagine, why higher dimensional op's on IR brane suppressed.

Cut-off scale on IR brane

$\sim \frac{4\pi}{R'} \sim 10\ \text{TeV}$. Then why is the Higgs mass not at least $\frac{1}{16\pi^2}\Lambda^2$?

Little hierarchy problem.

Higgs composite $\rightarrow$ solves big hierarchy problem (this is essence of the RS model), but still no understanding why $\frac{m_h^2}{\Lambda^2} \sim 10^{-2}$.

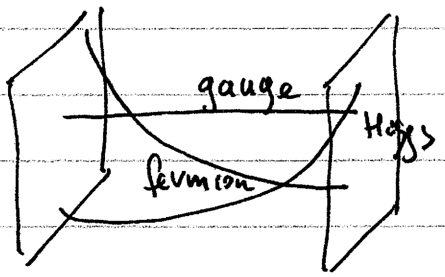
Also, why is $\frac{1}{\Lambda^2}(\bar{\psi}\psi)(\bar{\psi}\psi)$ on IR brane suppressed by

$\Lambda_{Flavor} \sim 10^4 - 10^5$ TeV?

EWP?

S-parameter? ρ-parameter

A possible step: move fermions & gauge fields into bulk.



Only Higgs needs to peak on IR brane. Can put gauge & fermion into bulk.

If fermion peaked on UV brane: fermions mostly elementary, flavor violation could be suppressed.

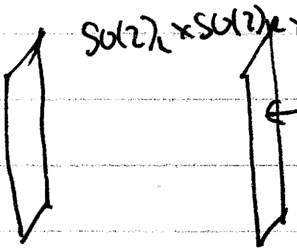
Still need to protect ρ-parameter! Can do it using custodial symmetry

Remember: custodial symmetry $SU(2)_L \times SU(2)_R$
$\uparrow$ gauge $\quad \uparrow$ global

Agashe, Delgado, May, Sundrum

Realistic RS

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$



Higgs on IR brane breaks gauged $SU(2)_L \times U(1)_Y$ just like SM

$\underbrace{SU(2)_R \times U(1)_{B-L}}$

$U(1)_Y$ by BC's on Planck $\rightarrow$ ensures global symmetry

Realistic RS: - solves (sort of) flavor issue
- solves ~~T~~ ρ-parameter
- bound on KK mass from remaining EWP bound ("S-parameter")
$m_{KK} \gtrsim 3$ TeV
- little hierarchy remains
- main signal search for KK gluon, W', Z' mostly decaying to 3^{rd} gen. quarks.

To solve little hierarchy 2 options

- Higgsless breaking
- pseudo-Goldstone boson composite Higgs.

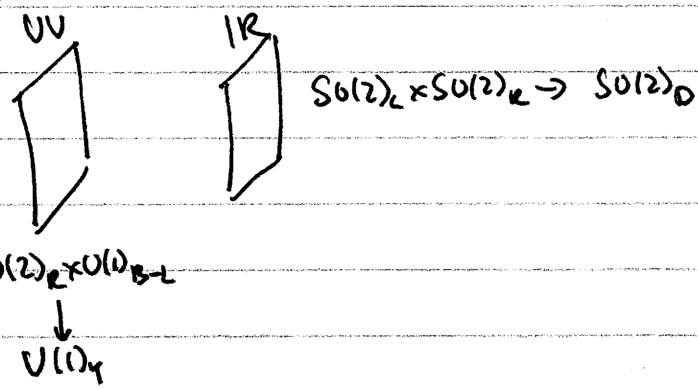
Idea of higgsless breaking:

Little hierarchy on IR brane: why is higgs VEV (& mass) of order 100 GeV on IR brane. What happens when we raise the Higgs VEV?

BC on IR brane for gauge field will slowly be —
Gauge field repelled from IR, but NOT

infinitely heavy!

Higgsless BC:



together like SM breaking pattern, but Higgs decoupled.

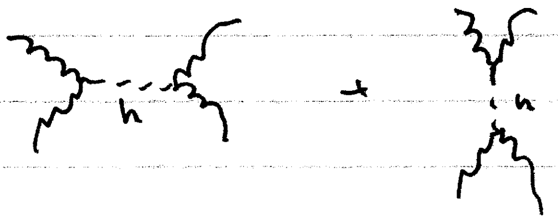
$$M_W^2 = \frac{1}{R'^2 \log R'/R}$$

W, Z masses no longer set by higgs VEV, but by $1/R'$. Higher KK modes

$$M_{W_n} \sim \frac{\pi}{2} (n + 1/2)\, 1/R'$$

GAP between W, W' set by $\log R'/R$.

~~the~~ Unitarization of W-W scattering via exchange of W', Z' KK modes



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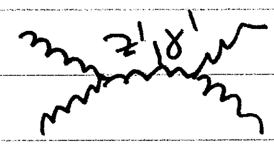
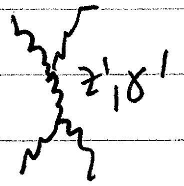




main problem: S-parameter (an EWP measure) large, need to tune a parameter to cancel it → like little hierarchy!

- Pseudo-goldstone composite Higgs (minimal composite Higgs)

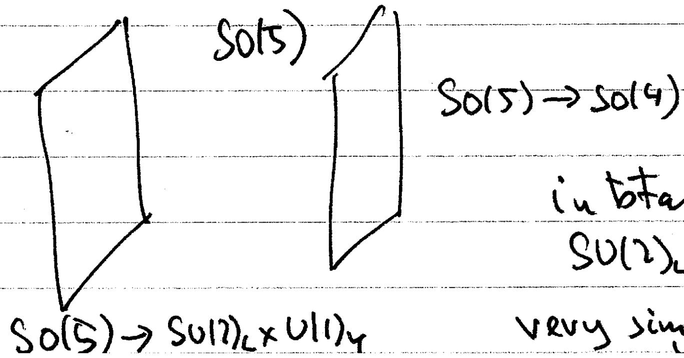
Enlarge global symmetry, & ensure that a higgs remains a pseudo-Goldstone boson. In this case ~~higgs VEV $\sim \frac{1}{4\pi}$~~

f = global symmetry breaking scale $\sim 1\,\text{TeV}$ $(\sim 1/R')$

Λ = cutoff $= 4\pi f$

higgs VEV can be naturally small $\sim \frac{f}{4\pi}$

The concrete model in warped space (minimal composite Higgs)



in total 1 doublet of $SU(2)_L \times U(1)_Y$ Goldstone..
very similar to little Higgs, will discuss that!

LITTLE HIGGS

Explicit 4D realization of the Higgs as pGB idea.

f global symmetry breaking scale $\sim 1\,TeV$

$v \sim \frac{f}{4\pi} \sim 100\,GeV$

$\Lambda \sim 4\pi f \sim 10\,TeV$ cutoff.

On its own can solve little hierarchy problem (but not the full hierarchy problem). For full hierarchy either need to make it a composite Higgs (see before), or embed into SUSY, make it strongly interacting.

How to make Higgs a Goldstone?

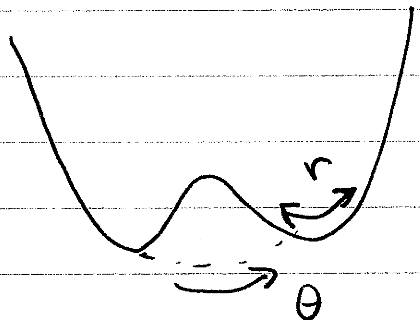
Remember Goldstone thm: for spontaneously broken global symmetry → massless scalar, that is only derivatively coupled. Lagrangian below scale f: non-linear σ-model

Example: $U(1)$ $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$

$V(\phi)$ minimum at $\phi^* \phi = f^2$

Paremetrize $\phi = \frac{1}{\sqrt{2}} \left(f + r(x)\right) e^{i\theta(x)/f}$

radial mode   phase = GB



$$\partial_\mu \phi = \frac{\partial_\mu r}{\sqrt{2}} e^{i\theta/f} = \frac{1}{\sqrt{2}} (f+r)\, i \frac{\partial_\mu \theta}{f} e^{i\theta}$$

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu r + \left(1 + \frac{r}{f}\right) i \partial_\mu \theta \right|^2 - V(r)$$

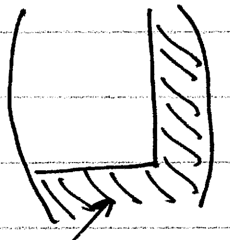
$$= \frac{1}{2}(\partial_\mu r)^2 + \frac{1}{2}(\partial_\mu \theta)^2 + \frac{1}{2}\frac{r^2}{f^2}(\partial_\mu \theta)^2 + \frac{r}{f}(\partial_\mu \theta)^2 - V(r)$$

- Only derivative interaction for θ
- $\theta \to \theta + \alpha$ shift symmetry forbids mass

But we need a whole doublet of Goldstones.
How to get more complicated reps?

$SU(N) \to SU(N-1)$ via VEV of fundamental of $SU(N)$

broken gen's: $(N^2-1) - (N-1)^2 - 1$
$= 2N-1$ real fields



broken generators of $SU(N)$

$$\phi_0 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ f \end{pmatrix}$$

$2N-1$ of them.

<u>Parametrization:</u> non-linear σ-model

$$\Sigma = e^{i\Pi^a T^a / f}\,\phi_0$$

Coleman-Wess-Zumino thm: this always works, and they always give equivalent Lagrangian irrespective of what representation we choose.

In this case

$$\Pi^a T^a = \begin{pmatrix} \Pi_0 & & & & \Pi_1 \\ & \Pi_0 & & & \Pi_2 \\ & & \ddots & & \vdots \\ & & & \ddots & \Pi_{N-1} \\ \Pi_1^* & \Pi_2^* & \cdots & \Pi_{N-1}^* & -\Pi_0(N-1) \end{pmatrix}$$

← up to some normalization...

effective Lagrangian:

$$\mathrm{Tr}\,(\partial_\mu \Sigma)^\dagger(\partial^\mu \Sigma) + \frac{1}{f^2}(\partial_\mu \partial^\mu \partial_\nu \Sigma)^\dagger(\partial^\nu \Sigma)$$

$$+ \cdots$$

Systematic expansion in momenta, valid below $E \ll f$

For example

$SU(3) \to SU(2)$

$$\Pi = \left(\begin{array}{cc|c} -\frac{\eta}{2}\sqrt{\frac{2}{3}} & & h \\ & -\frac{\eta}{2}\sqrt{\frac{2}{3}} & \\ \hline & h^+ & \eta\sqrt{\frac{2}{3}} \end{array}\right) \begin{array}{l} \leftarrow \text{doublet} \\ \\ \leftarrow \text{singlet} \end{array}$$

$$\partial_\mu \phi^\dagger \partial^\mu \phi = \left| \partial_\mu e^{i\Pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \right|^2$$

$$e^{i\Pi/f} \begin{pmatrix} 0 \\ f \end{pmatrix} \approx \begin{pmatrix} ih \\ f - \frac{h^\dagger h}{2f} \end{pmatrix} + \cdots$$

$$|\partial_\mu \phi|^2 = |\partial_\mu h|^2 + \frac{|\partial_\mu h|^2 h^\dagger h}{f^2} + \cdots$$

Cutoff scale $\Lambda = 4\pi f$

This produces $SU(2)$ Goldstones, but no gauge symmetry yet.

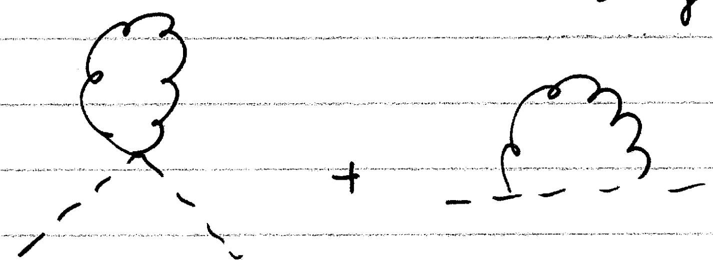
Try to gauge $SU(2) \subset SU(3)$

$$\partial_\mu h \to \left(\partial_\mu - i g W_\mu^a \frac{\tau^a}{2}\right) h$$

But this explicitly & completely breaks original $SU(3)$ global symmetry!

i.e. contains interaction term

$|g W_\mu h|^2$ $\rightarrow$ usual quadratic divergence just like SM!



$\propto \frac{g^2}{16\pi^2} \Lambda^2 h^+ h$ no gain, get SM divergences

($\equiv$ introduced $SU(3)$ global, but explicitly broke it in a way that divergences reintroduced).

How to restore $SU(3)$?

Gauge entire $SU(3)$! Then it is true that gauging does not break symmetry, no quadratic divergences generated.

<u>BUT</u> now entire h Goldstone eaten.

$SU(3) \to SU(2)$ 5 Generators broken $\to$ 5 Goldstones eaten $\to h + \eta$ non-physical ...

<u>Final fix:</u> Two copies of fields ϕ_1, ϕ_2

$$\phi_1 = e^{i\Pi_1/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

$$\phi_2 = e^{i\Pi_2/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

for simplicity assumed VEVs aligned & $f_1 = f_2 = f$

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2$$

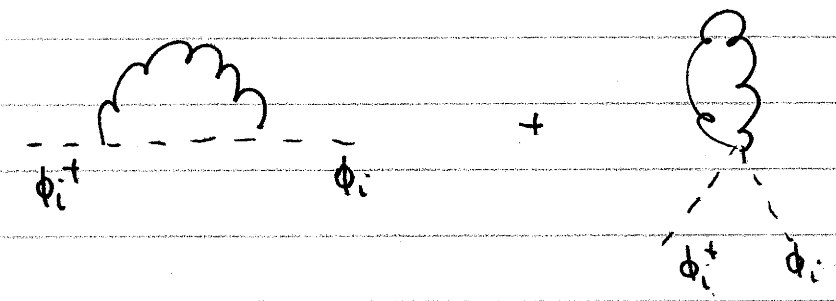
quadratic divergences:

$$\frac{\Lambda^2}{16\pi^2} g^2 \underbrace{(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2)}_{2f^2}$$

$\to$ no potential for any GB!

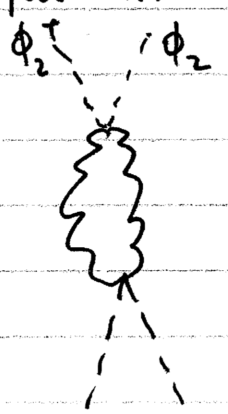
$\Pi_1 + \Pi_2$ eaten, but $\Pi_1 - \Pi_2$ remains physical GB

Why no quadratic divergence?



As long as just one ϕ field appears in diagram: symmetry unbroken $\rightarrow$ can not generate SU(3) breaking term.

If you have more than one field:



$$\propto \frac{g^4}{16\pi^2} \left|\phi_1^+ \phi_2\right|^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

Will give you log divergent mass (more insertion $\rightarrow$ less divergence)

$$\Delta m_H^2 \propto \frac{g^4}{16\pi^2} f^2 \log\left(\frac{\Lambda^2}{\mu^2}\right) \sim (100\,\text{GeV})^2 \text{ OK.}$$

Principle underlying

Collective breaking

2 types of interactions

$$|g_1 A_\mu \phi_1|^2 + |g_2 A_\mu \phi_2|^2 \qquad g_1 = g_2 = g$$

w/o gauging

$$SU(3) \xrightarrow{\phi_1} SU(2) \qquad SU(3) \xrightarrow{\phi_2} SU(2)$$

each produces own Goldstone

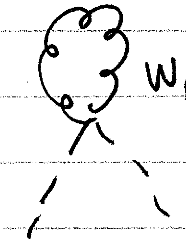
gauging diagonal breaks $SU(3) \times SU(3) \to SU(3)_D$
eats one set of Goldstones.

Imagine $g_1 \to 0$ $SU(3)$ symmetry on A, ϕ_2 unbroken $\to$ no quad. div. in this limit

$g_2 \to 0$ $SU(3)$ on A_{μ}, ϕ_1 unbroken

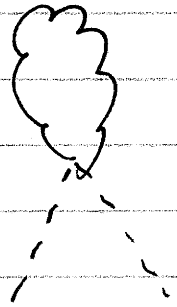
If one of couplings turned off, symmetry larger, GB protected. Need to have $g_1 g_2$ inserted, not quad. div.

So what cancels the divergences of SM gauge bosons to higgs.?



W, Z

$W_\mu^{1,2,3} \to +3$ Coupling of heavy gauge bosons



$$\left.\begin{array}{l} X_\mu^+ X^{\mu -} \to -1 \\ Y_{1,2} \to -1 \\ A_\mu^8 \to -1 \end{array}\right\} -3$$

Where $A_\mu^a T^a = \begin{pmatrix} 0 & W^+ & X^+ \\ W^- & 0 & Y_1^0 + iY_2^0 \\ X^- & \bar{Y} & 0 \end{pmatrix}$

$A_3 + A_8$

In this language cancellation may seem miracolous. But we understand reason for cancellation is collective breaking.

<u>upshot:</u> quadratic divergences cancelled by <u>same spin</u> TeV-scale partners of SM particles. Related by global symmetry to SM particles!

This is basic of SIMPLEST little Higgs of Schmaltz & Kaplan.

To also get fermions (protection against 1-loop Yukawas) : need SU(3) symmetry in fermions

$SU(2)_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \rightarrow \begin{pmatrix} t \\ b \\ T \end{pmatrix} \equiv \Psi$

LH top partner

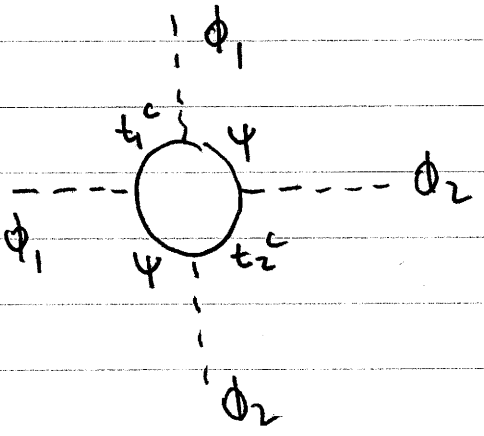
$t_R \rightarrow t^c, T^c$ two RH tps

$b_R \rightarrow b^c$

$$\mathcal{L}_{Yuk} = \lambda_1 \, \phi_1^+ \Psi \, t_1^c + \lambda_2 \, \phi_2^+ \Psi \, t_2^c$$

Again SU(3) symmetry collectively broken. If $\lambda_2 = 0 \rightarrow$ first term SU(3) invariant. If $\lambda_1 = 0 \rightarrow$ second term SU(3) inv.

Need BOTH λ_1 & λ_2 to generate PGB mass $\rightarrow$ will not be quad. divergent!



$$\propto \boxed{\frac{\lambda_1^2 \lambda_2^2}{16\pi^2} |\phi_1^+ \phi_2|^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)}$$

again log div.

$\sim (100 \text{ GeV})^2$ OK

Hardest part to add Higgs quartic self-coupling!

need to write $V(\Phi_1, \Phi_2)$

- no mass term for Higgs
- contains quartic
- quartic is collective

Impossible in pure SU(3),

- can enlarge gauge group
- add op's with small coeff's
- go to SUSY & use D-term quartic...