The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2134-15

Spring School on Superstring Theory and Related Topics

22 - 30 March 2010

(Non) Geometric aspects of black holes and their microstates

J. deBoer
University of Amsterdam
Netherlands

(review: see 0811.0263)

- generalities
- AdS BH's
- near horizon physics
- info paradox
- open → closed
  Higgs → Coulomb
- explicit examples
- applications
- no go theorem
- exotic objects
- thermalization

(NON) GEOMETRIC ASPECTS OF BLACK HOLE MICROSTATES

Understanding black holes = important challenge for any quantum theory of gravity

Important progress since '97 or so: entropy computations

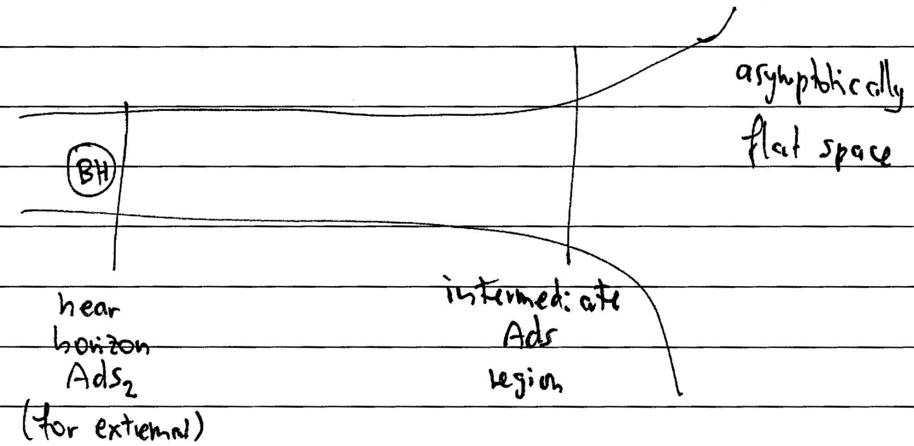
Microphysics $g_s \to 0$

- weakly coupled d.o.f
- D-brane systems & bound states
- Cardy formula in 2d CFT's
  $S \sim 2\pi\sqrt{\frac{c}{6}L_0} + 2\pi\sqrt{\frac{\bar{c}}{6}\bar{L}_0}$
- Modular forms

Gravity $g_s \to \infty$

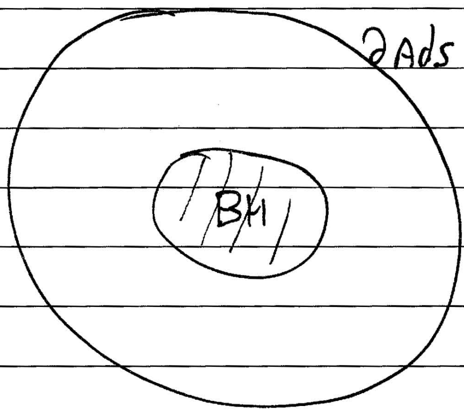
- Bekenstein-Hawking $S = \frac{A}{4G}$
- Wald's formalism
- Higher derivative theories
  $\to$ [OSV] $Z_{BH} \sim Z_{TOP}^2$
- Duality invariance
- Sen's entropy function formalism.

Most of these results are for extremal black holes.
Also, most can be understood in the context of AdS/CFT



Decouple asymptotically flat space à la Maldacena ($\alpha' \to 0$, $\frac{r}{\alpha'}$ fixed)
$\Rightarrow$ Black hole in AdS (drop constants in harmonic functions)

Consider black holes in AdS



To compute BH entropy in dual field theory (which becomes weakly coupled as $g_s \to 0$) need to know what the black hole is dual to.

General recipe : $\begin{pmatrix}\text{stuff in the}\\ \text{boundary}\end{pmatrix} = \sum \begin{pmatrix}\text{bulk geometries with}\\ \text{suitable boundary conditions}\end{pmatrix}$

not clear if this can be inverted to

$$\begin{pmatrix}\text{bulk geometry}\end{pmatrix} = \sum \begin{pmatrix}\text{stuff in the}\\ \text{boundary}\end{pmatrix}$$

Consider eg. Schwarzschild BH in AdS_5

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega_3^2$$

$f = r^2 - c\frac{M}{r^2} + \ldots$ $\quad M =$ mass of black hole.
$\quad = \langle H_{CFT} \rangle$

Suggests that BH is dual to system with $\langle H_{CFT} \rangle = M$.

Go to Euclidean signature: time = periodic,
boundary = $S^1 \times S^3$. Radius of S^1 = fixed by M

~~Assumption~~
Therefore expect black hole to be dual to finite temperature system.

$$Z_{CFT}(\beta) = \sum_{\text{saddle points}} e^{-S_E} (\text{loop corrections} \ldots)$$

Usually, one saddle point dominates. Here, that is
either thermal AdS (S^1 non-contractible) low T
Euclidean BH (S^1 contractible) high T

This is the Hawking-Page transition, dual to the confinement-deconfinement phase transition in $N=4$ SYM.

$\Rightarrow$ For high T, ($R_s \gtrsim \ell_{AdS}$), a black hole is dual to a finite temperature strongly coupled field theory, up to exponential corrections.

Unfortunately, there is in general no known way to compute the partition function of strongly coupled CFT's :
gravity makes a prediction.

$\Rightarrow$ The situation is better in AdS_3, because the finite temperature partition function is strongly constrained by modular invariance of the 2d CFT (is there a higher dimensional analogue?)

For BTZ black holes in AdS_3

$$ds^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}dt^2 + \frac{\ell^2 r^2\, dr^2}{(r^2-r_+^2)(r^2-r_-^2)} + r^2\left(d\phi - \frac{r_+ r_-}{\ell r^2}dt\right)^2$$

Horizon area $\left(L_0 = \frac{c}{24} + \frac{(r_+-r_-)^2}{16 G_3 \ell}, \ \bar{L}_0 = \frac{c}{24} + \frac{(r_++r_-)^2}{16 G_3 \ell} \right), \ c = \frac{3\ell}{2G_3}$

$$\downarrow$$

$$S = \frac{2\pi r_+}{4G_3} = \underbrace{2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} + 2\pi\sqrt{\frac{c}{6}\left(\bar{L}_0 - \frac{c}{24}\right)}}_{\text{Cardy formula}}$$

This always works. Agreement is almost kinematical. As soon as one obtains BTZ, microscopic counting is "trivial"

Why not go all the way to the horizon?

$\Rightarrow$ We would lose the field theory picture.

Consider $ds^2 = -f dt^2 + f^{-1} dr^2 + \ldots$

non extremal: $f \simeq c(r - r_H)$ close to horizon:

take $r = r_H + \varepsilon\rho^2$

$ds^2 \approx \frac{\varepsilon}{c}\left(-c^2\rho^2 dt^2 + 4 d\rho^2\right)$ Rindler space

↗ not dual to a field theory

↘ What does $\varepsilon \to 0$ mean?

extremal: $f \sim c(r - r_H)^2$ close to horizon:

take $r = r_H + \varepsilon\rho$ and $t = \tau/\varepsilon c$

$ds^2 \approx \frac{1}{c}\left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2}\right)$ AdS_2

One might think that one can use AdS_2/CFT_1 in the extremal situation, but AdS_2/CFT_1 is a "non-dynamical" AdS_2/CFT duality. Both AdS_2 and CFT_1 represent some Hilbert space of states, but without further structure, it is difficult to independently count the number of states in CFT_1.

Why non-dynamical? AdS-fragmentation (Maldacena, Strominger)

$$8\pi G_2 S = \int d^2x\sqrt{-g}\left[e^{\psi}\left(R+\frac{2}{\ell^2}\right) - \frac{\ell^2}{4} e^{3\psi} F_{\mu\nu}F^{\mu\nu} + \text{massive fields}\right]$$

$$\text{Solution}\quad \begin{cases} ds^2 = -\frac{\ell^2}{r^2}\left(-dt^2+dr^2\right) \\ F_{tr} = \frac{2Q}{r^2} \\ e^{-\psi} = Q \end{cases}$$

General: assume $ds^2 = e^{2\phi(\sigma^+,\sigma^-)} d\sigma^+ d\sigma^-$, $0 \le \sigma^\pm \le \pi$

Equation of motion: $\nabla_+\nabla_+ e^{\psi} = 8\pi G_2 T_{++}$ (everything)

Integrate against $e^{-2\phi} d\sigma^+$:

$$e^{-2\phi}\partial_+ e^{\psi}\Big|_{\sigma^+=0} - e^{-2\phi}\partial_+ e^{\psi}\Big|_{\sigma^+=\pi} = -8\pi G_2 \int d\sigma^+ e^{-2\phi} T_{++}$$

If $T_{++} \ge 0$ (null energy) $\Longrightarrow$ e^{ψ} must diverge
$\Longrightarrow$ destroys AdS_2 boundary conditions

$\Longrightarrow$ AdS_2 is unstable under generic perturbations

* does not mean we cannot compute AdS_2 partition functions
* fails for $AdS_2 \times$ (noncompact) as in McGreevy's talk, because then $G_2 \to 0$ (but could be unstable for other reasons)
* perhaps $\exists$ AdS_2/CFT_1 duality for multi AdS_2-geometries, but can only see how this might possibly work in multi BH geometries in AdS_3 for example.

⇒ It is instructive to take the near-horizon limit of extremal BTZ (see also arxiv: 0906.3272)

$$ds^2 = -\frac{(r^2-r_+^2)^2}{r^2\ell^2}dt^2 + \frac{\ell^2 r^2 dr^2}{(r^2-r_+^2)^2} + r^2\left(d\phi - \frac{r_+^2}{\ell r^2}dt\right)^2$$

take $r = r_+ + \varepsilon\rho$, $t = \tau/\varepsilon$; $\varepsilon \to 0$:

$$ds^2 = \frac{-4\rho^2}{\ell^2}d\tau^2 + \frac{\ell^2}{4\rho^2}d\rho^2 + r_+^2\left(d\phi - \frac{d\tau}{\ell\varepsilon} + \frac{2\rho}{\ell r_+}d\tau \ldots\right)^2$$

Need to take $\hat{\phi} = \phi - \tau/\ell\varepsilon = \phi - t/\ell$ fixed

~~But at the same time~~ $\cancel{\phi + t/\ell = \hat{\phi} + \frac{2\tau}{\ell\varepsilon}}$

But $\phi + t/\ell = \hat{\phi} + \frac{2\tau}{\varepsilon\ell}$ scales as $\frac{1}{\varepsilon}$

and $\frac{\partial}{\partial(\phi + t/\ell)}$ scales as ε (fixed)

→ left-movers free, right-movers frozen: a chiral CFT

⇒ AdS_2 signals frozen right-movers: ground states

⇒ boundary is a null cylinder ("DLCQ"), it is not clear whether there is any interesting dynamics

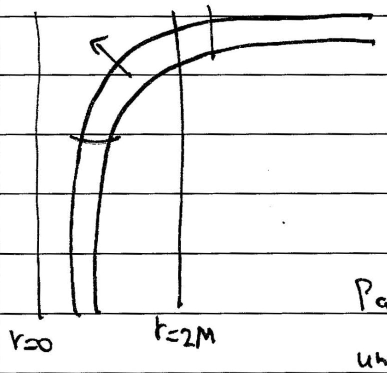
⇒ Somewhat analogous story applies to Kerr/CFT (Strominger et al)

$$ds^2 = A^2(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + B^2(\theta)(d\varphi + r\,dt)^2\right]$$

φ-circle supports $\mathrm{Diff}(S^1)$ → Virasoro → Cardy

Possibilities:
- dual to nothing — Amsel Horowitz Marolf Roberts
- dual to QM — Dias Reall Santos
- dual to chiral CFT — ~~Bredberg, Hartman, Song, Strominger~~
- dual to CFT — Cvetic Larsen

INFORMATION PARADOX (0909.1038)



} nice slices: low curvature, no reason why classical gravity should break down.

Pair creation occurs in the middle region, uncorrelated with other pairs (or infalling matter) (standard Bogoliubov transformation)

state $\sim \left(|0\rangle_{in} |0\rangle_{out} + |1\rangle_{in} |1\rangle_{out} \right)^N$

entanglement entropy = entropy in Hawking radiation $\sim N$.

Process continues until black hole is Planck size: 3 possibilities

1 — "remnants"

2 — ~~QM~~

3 — classical gravity is not reliable on these slices?

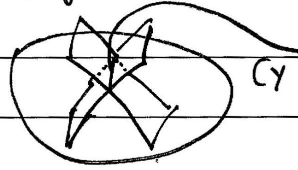
Small corrections do not help!

Will see evidence for scenario 3 later.

Open → Closed, Higgs → Coulomb (cf Denef, hep-th/0206072)

D-brane system $g_s \to 0$ $\Longrightarrow$ gravitational description (black holes) $g_s \to \infty$

prototype example:



light degrees of freedom @ intersection.

$\vec{x}_1$ $\vec{x}_2$

From 4d point of view: quantum mechanics, supersymmetric (4 susys)

$$L = \frac{1}{2}\left(\dot{\vec{x}}^2 + D^2 + 2i\dot{\bar{\lambda}}\lambda\right) - \theta D + |D_t\phi^a|^2 - \left(|\vec{x}|^2 + D\right)|\phi^a|^2 + |F^a|^2$$
$$+ i\bar{\psi}^a D_t \psi^a - \bar{\psi}^a \vec{x}\cdot\vec{\sigma}\,\psi^a - i\sqrt{2}\left(\bar{\phi}^a\psi^a\varepsilon\lambda - \bar{\lambda}\varepsilon\bar{\psi}^a\phi^a\right)$$

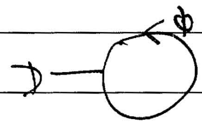
$\vec{X} = \vec{x}_1 - \vec{x}_2$ $\quad$ $\vec{x}, \lambda, A_t, D$: vector multiplet

ϕ^a, ψ^a, F : hyper multiplet

$$V_{classical} = \frac{1}{2}\left(\theta + |\phi^a|^2\right)^2 + |\vec{x}^2||\phi^a|^2$$

$$V_{quantum} = \frac{1}{2}\left(\theta + \frac{N g_s}{|\vec{x}|}\right)^2$$

(integrate out $|\phi^a|$)



If $\theta < 0$, $\exists$ classical Higgs branch. $\left(\sum_a |\phi^a|^2 = -\theta\right)/U(1) = \mathbb{CP}^{N-1}$

But there is also a Coulomb branch $|\vec{x}| = \frac{-N g_s}{\theta}$

Mass of $|\phi|^{a2}$ has to be positive : $|\vec{x}^2| + 2\theta > 0$

$\Rightarrow \frac{N^2 g_s^2}{\theta^2} + 2\theta > 0 \Rightarrow g_s^2 > \frac{2}{N^2}(-\theta^3)$

Higgs good for $g_s \ll \frac{-\theta^{3/2}}{N}$

Coulomb good for $g_s \gg -\theta^{3/2}/N$

What are the states?

⇒ Higgs branch: susy σ-model.

$$A(\phi) + B_a(\phi)\psi^a + C_{ab}(\phi)\psi^a\psi^b + \ldots$$

$$\updownarrow$$

$$A(\phi) + B_a(\phi)\, d\phi^a + \ldots \quad \text{differential form}$$

Susy are like $\partial, \bar\partial$ $\longrightarrow$ $H^*(M_{Higgs}) = H^*(\mathbb{CP}^{N-1}) = \mathbb{Z}N$

⇒ Coulomb branch: $|\vec{X}|$ = fixed: S^2

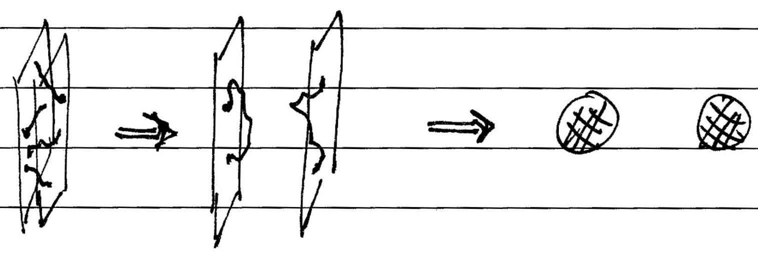
One loop generates a term $\int \vec{A}\,\dot{\vec{X}}\,dt$ as well, with
$\vec\nabla \times \vec{A} = \vec\nabla\left(\frac{N g_s}{|\vec{X}|}\right)$

Symplectic form $\omega \sim \delta x \wedge \delta p \sim \delta x \wedge \delta \vec{A}$

⇒ S^2 is a "phase space", need to quantize it.
pick $\mathcal{L}$ with $c_1(\mathcal{L}) = \omega$, take $H^0(S^2, \mathcal{L})$
$\mathcal{L} = \mathcal{O}(N-1)$, $\dim H^0(S^2, \mathcal{O}(N-1)) = N$

This is similar to Landau levels of a particle on S^2 in a magnetic field.

If one keeps on increasing g_s, we end up with a gravitational description, as is well-known in D-brane physics



Higgs Coulomb supergravity

g_s $\longrightarrow$

* Notice that no states were lost in this process ☺

* Supergravity solutions are known (Lopes Cardoso et al, Denef et al) multi centered black holes with positions $\vec{x}_i$ & charges Γ_i

* susy supergravity solutions $\Longleftrightarrow$ one-loop Coulomb branch equations
supergravity symplectic form $\Longleftrightarrow$ Coulomb branch symplectic form

[in general : $\langle h, \Gamma_i \rangle + \sum_{j \neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|\vec{x}_i - \vec{x}_j|} = 0$

$$\omega = \frac{1}{4} \sum_{i \neq j} \langle \Gamma_i, \Gamma_j \rangle \, \varepsilon_{abc} \frac{\left(\delta(x_i - x_j)^a \wedge \delta(x_i - x_j)^b\right)(x_i - x_j)^c}{|x_i - x_j|^3} \Big]$$

* If we are in type IIA and $\Gamma_i =$ (D6-brane + world volume flux) or $\Gamma = D0$ then geometries are smooth when uplifted to M-theory:

$$\left(ds^2 = -dx_0^2 + \ldots + dx_6^2 + H(dx_7^2 + dx_8^2 + dx_9^2) + H^{-1}(dx^{10} + A_7 dx^7 + A_8 dx^8 + A_9 dx^9)^2 \right), \quad H = 1 + \frac{1}{4\pi|\vec{x}|}, \quad \vec{\nabla} H = \vec{\nabla} \times \vec{A}$$

$\Rightarrow$ Only smooth solutions are completely under control in pure gravity, since they have no sources or singularities

$\Rightarrow$ If $\sum \Gamma_i =$ charge of a black hole, these smooth geometries can be thought of as representing individual degrees of freedom

$\Rightarrow$ Does this make sense? (More later). For now, observe that $|\vec{x}| \sim g_s$, size of smooth geometry grows as g_s (counterintuitive: usually things shrink as $g_s \to \infty$ due to gravitational attraction. Here system has fixed angular momentum, whereas constituents become lighter as $g_s \to \infty$). Size of black hole also grows $\sim g_s$

Before continuing, two other examples of
(Higgs) → Coulomb → Geometry

1) $\frac{1}{2}$ BPS in $N=4$ SYM. "Coulomb branch"

$$S \sim \int DX \, \exp i \left[\int \tfrac{1}{2}\dot{X}^2 - \tfrac{1}{2} m^2 X^2 \right)$$

$X =$ $N \times N$ matrix. Coulomb branch → integrate out
off-diagonal modes of X_{ij}

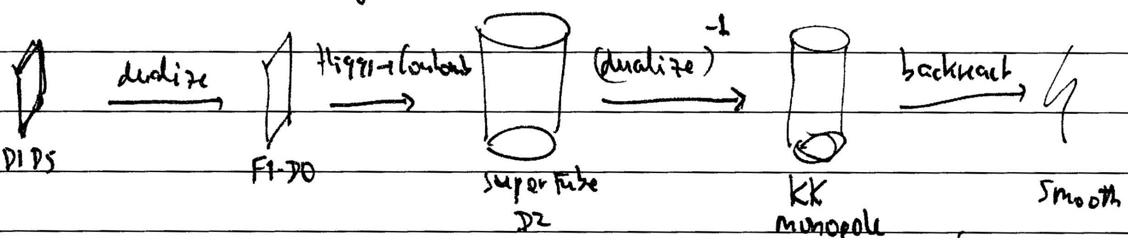
Result $\displaystyle \int \prod d\lambda_i \prod (\lambda_i - \lambda_j)^2 \exp i \left[\tfrac{1}{2} \sum \dot{\lambda}_i^2 - \tfrac{1}{2} \sum m^2 \lambda^2 \right]$

→ fermions in an harmonic oscillator

⇓ backreact

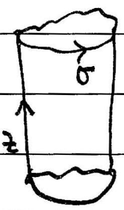
LLM - geometries

2) $\frac{1}{2}$ BPS in D1 - D5 system



(Lunin - Mathur geometries)

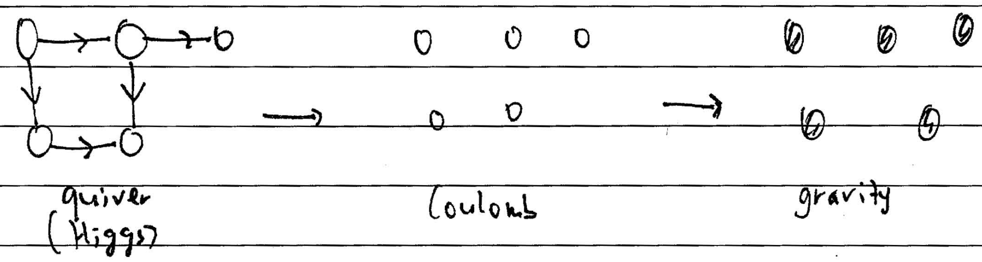
Supertube:



embedding $X^0 = t \quad X^1 = z \quad X^i = X^i(\sigma)$
electric field $F_{tz} = 1$
magn. field $F_{\sigma z} = B(\sigma)$ (Mateos, Townsend)

[always solves the field equations]

back to original case



Can we follow all microscopic degrees of freedom in this way?

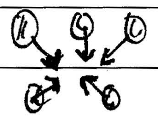
<u>conjecture</u>: for quivers w/o closed loops (so there cannot be a superpotential): yes

[idea: $O \to O \to O \to O \Rightarrow O \to O \to O \Rightarrow O \to O \Rightarrow \ldots$
remove vertices one by one, are always in the local $\mathbb{CP}^N$ setup]

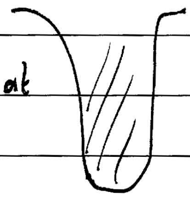
With closed loops, answer is generically <u>no</u>, known in examples (Denef-Moore)

— — — — — —

What do these gravitational solutions look like?
Not like a black hole, unless we can bring all constituents close together (also requires closed loops in quiver)



metric looks like a deep throat



There are only finitely many states if we quantize the Coulomb branch / supergravity
$\Rightarrow$ Very deep throats are quantum, we cannot localize the wavefunction on such a geometry.
Curvatures are very small everywhere:
$\Rightarrow$ <u>Nonlocal breakdown of nice slice argument / effective field theory</u>

Exactly the sort of thing one needs to resolve the information paradox

→ (cf. limits on statistical descriptions of black holes)

Due to lack of phase space, physics becomes nonlocal before the point where one naively expects this

[Consistency check: ~~quantum capping of~~ produces the right gap in the dual CFT]

Can there be enough smooth solutions to explain the black hole entropy?

Here, relevant black holes are 4d black holes. Lift to M-theory, & take a decoupling limit: BTZ black hole in $AdS_3 \times S^2$

[Maldacena, Strominger, Witten]

Quantum Numbers L_0, $\bar{L}_0 = \frac{c}{24}$ susy black hole

Want to find $\sim e^{(cL_0)^{1/2}}$ number of states.

Try to establish an upper bound on the number of smooth supergravity solutions.

Idea: nontrivial metric = coherent state of gravitons

So throw gravitons in AdS, let the system relax, we will get a (possibly) singular geometry.

Cannot control the energy: use BPS gravitons.

We know the complete list of BPS gravitons from the KK reduction of supergravity on $AdS_3 \times S^2 \times CY$

Count number of states in a gas of BPS-gravitons: $S \sim L_0^{2/3}$

This ignores backreaction!

Important: stringy exclusion principle. $S^2 \to SU(2)_k$ current algebra (R-symmetry) in dual CFT. Spin $j \leq \frac{k}{2}$ by unitarity

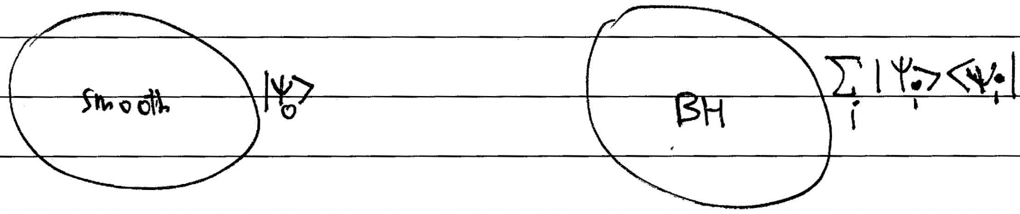
Take this into account: $S_{\text{sugra}} \sim (cL_0)^{1/3} \ll (cL_0)^{1/2} = S_{BH}$.

$\Rightarrow$ supergravity is <u>not</u> enough, need stringy degrees of freedom to "resolve" black holes.

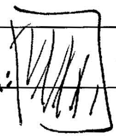
- Quantizing gravity cannot lead to a unitary theory which contains black holes

---

A priori, it is perhaps strange that there exists smooth black hole "microstate geometries" at all.
The claim is that



and that $|\psi_0\rangle$ is one of the states which appears in $\sum_i |\psi_i\rangle\langle\psi_i|$. It coexists with the black hole to which it belongs.

In thermodynamics, this can also happen.
Consider a noninteracting thermal gas in a box:  $\uparrow z$

Most molecule configurations are not very classical.
But some atypical ones are: put all molecules in bottom half and give equal velocity in the z-direction.
Find a classical cloud which travels up and down.

For an interacting gas, don't expect many atypical classical configurations

Here, we have a BPS system, which is similar to a noninteracting gas and that is perhaps why we have such a large number of classical states.
If correct, we don't expect to find many for a Schwarzschild black hole.

One argument why ~~one~~ smooth geometries cannot be part of the black hole ensemble is as follows: recall that [Sen]

$$\underset{\substack{\text{finite temperature} \\ S^3\times S^1}}{Z(\text{CFT})} \simeq \underset{\substack{\downarrow \\ \text{circle not} \\ \text{contractible}}}{\left(\begin{matrix}\text{thermal}\\ \text{AdS}\end{matrix}\right)} + \underset{\substack{\downarrow \\ \text{circle} \\ \text{contractible} \\ \text{at one} \\ \text{point}}}{\left(\begin{matrix}\text{black}\\ \text{hole}\end{matrix}\right)} + \cdots\cdots$$

After Wick rotation, this could be interpreted as

$$Z(\text{CFT}) \simeq \sum\left(\begin{matrix}\text{smooth}\\ \text{solutions}\end{matrix}\right) + \sum\left(\begin{matrix}\text{solutions with}\\ \text{one black hole}\end{matrix}\right) + \sum\left(\begin{matrix}\text{solutions with}\\ \text{two black}\\ \text{holes}\end{matrix}\right) + \cdots$$

<u>But</u> 1) the smooth solutions we are interested in cannot be Wick rotated

2) in more explicit examples (like in the so-called Farey-Tail expansion) not all smooth solutions appear in the first term.

Therefore, this argument is not entirely conclusive.

→ We saw there were insufficiently many smooth geometries
Are there ways out of this?

One possibility I am presently exploring with Masaki Shigemori
is to use "exotic" objects, or non-geometric branes.
Consider the supertube again, in type IIA on T^6

pull up

D0 : 0			D3 0 567
F1 : 0 4	$\xrightarrow{T_{567}}$		F1 0 4
D2 : 0 (12) 4			D5 0 (12) 4 567

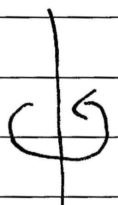
$\downarrow S$

D4 : 0 67 89		D3 0 567
D4 : 0 45 89	$\xleftarrow{T_{589}}$	D1 0 4
5^2_2 : 0 (12) 4 5 67 89		NS_5 0 (12) 4 567

Obers Pioline

5^2_2- branes are "exotic branes" that appear when compactifying down to three dimensions. Tension $\sim \frac{1}{g_s^2} R_{(12)} R_4 R_5 R_6 R_7 (R_8 R_9)^2$

Can write explicit metrics for these. Locally structure is as follows



Monodromy $\in$ U-duality group. $[E_8(\mathbb{Z})$ for type II $/T^7]$

U-duality acts on the scalars one gets from KK reduction of the metric, RR-fields, etc, and generically mixes metric scalars with non-metric scalars.
⇒ These solutions are non-geometric from a higher-d point of view.

Conclusions

- Have shown how to follow individual degrees of freedom
- Found smooth gravitational descriptions of many degrees of freedom, gave an example of (nonlocal) breakdown of nice slice physics.
- In examples, one seems to lose states when passing from Higgs to Coulomb.
- More generally, found a nogo argument that supergravity can never describe large susy black holes
- Briefly described an attempt to go beyond the nogo argument.

Some references:

A general review about black hole microstates is
arXiv: 0811.0263 (Balasubramanian, JdB, El-Showk, Messamah)

Higgs → Coulomb is partially described in
hep-th/0206072 (Denef), see also
hep-th/0606118 (Balasubramanian, Gimon, Levi)

The near-horizon limits of Kerr and BTZ are described in
arXiv: 0906.3272 (Balasubramanian, JdB, Sheikh-Jabbari, Simon)

The nogo theorem is described in
arXiv: 0906.0011 (JdB, El-Showk, Messamah, van den Bleeken)