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International Centre for Theoretical Physics

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2134-17

Spring School on Superstring Theory and Related Topics

22 - 30 March 2010

Superstring multiloop amplitudes and non-renormalization theorems
Lecture I
- Motivation and Summary of Results -

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Lecture Plan: Multiloop superstring amp's and non-renorm. thms

I. Motivation and Summary of results
II. Basics of Pure Spinor Formalism
III. Loop amplitude prescription
IV. Derivation of results

I. Motivation for studying superstring amp's

A. Duality conjectures

Type IIB superstrings have $SL(2,Z)$ duality

$$\Omega = \underset{\Omega_1}{a} + i\,\underset{\Omega_2}{e^{-\varphi}} \rightarrow \frac{A\Omega + B}{C\Omega + D} \qquad AD - BC = 1$$

$$(B = C = -1 \Rightarrow \varphi \to -\varphi)$$

Type IIB effective action: $\int d^{10}x \sqrt{-g}\, \Omega_2^{1/2} Z_{3/2}(\Omega, \bar\Omega) R^4 + \ldots$

$\Omega_2^{1/2}$: from string gauge

$$Z_s = \sum_{(m,n)\neq(0,0)} \frac{\Omega_2^s}{|m+n\Omega|^{2s}} = c_{1,s}\Omega_2^s + c_{2,s}\Omega_2^{1-s} + \sum_{k\neq 0} d_{k,s}\, e^{-2\pi(|k|\Omega_2 - ik\Omega_1)}$$

$$\ldots = \int d^{10}x \sqrt{-g}\, \Omega_2^{-1/2} Z_{5/2}(\Omega, \bar\Omega) D^4 R^4 + \text{more complicated } SL(2,Z)\text{-inv. fncns}$$

(Einstein gauge $\int d^{10}x \sqrt{-g}\, R \rightarrow \int d^{10}x \sqrt{-g}\, \Omega_2^2 R$ String gauge)

$$g \to g e^{-\varphi/2} = g\, e^{-\frac{4\varphi}{d-2}}$$

Green, Russo, Vanhove 0610299

R	R^4	D^4R^4	$\ldots$
only tree	tree, one-loop, non-pert	tree, two-loop, non-pert	?

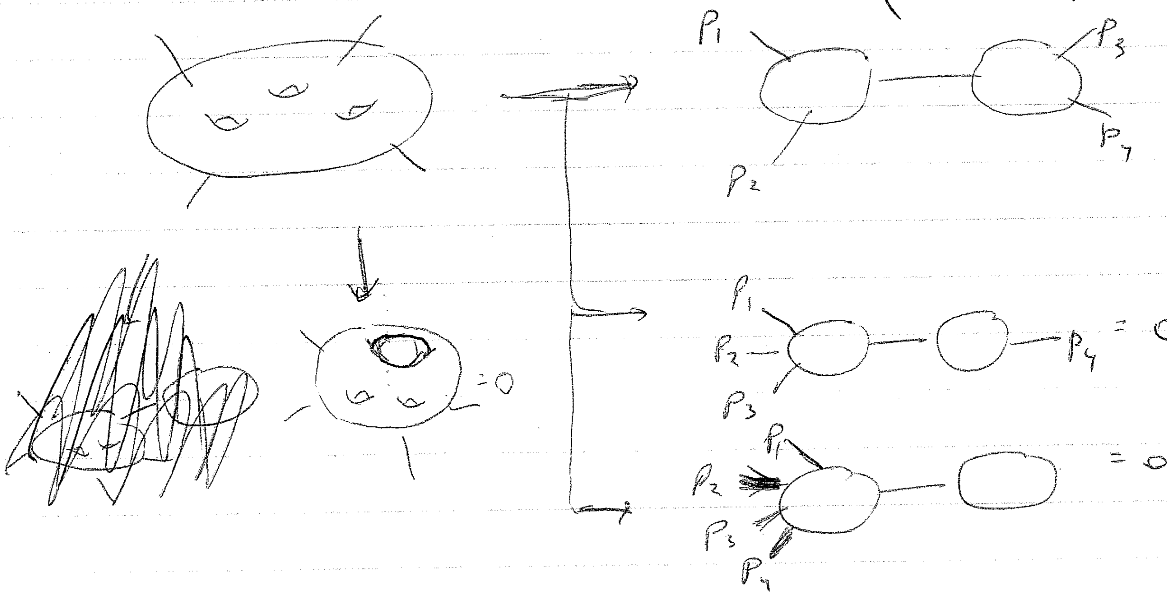
(Also have Heterotic - Type I duality ⇒ structure of $\mathrm{Tr}(F^N)$ from Born-Infeld in Type I)

B. Finiteness conjectures

1. No unphysical poles in superstring amplitudes

⇒ 0-pt, 1-pt amp's get no ~~too~~ contributions (no cosm. constant and no tadpole)

2-pt massless amp. gets no loop contributions (no mass shift ∃ of massless particles)



Can analytically continue to remove pole

2. 3-pt massless amp. gets no loop contributions

⇒ coupling constant not renormalized

3. Finiteness of $N=4$ $d=8$ sugra

Dimensional analysis implies that $\int d^dx\sqrt{g}\, e^{(2\overset{\text{loop}}{h}-2)\phi} D^k R^4$

diverges as $\Lambda^{(d-2)(h-1)-k-8+d} = \Lambda^{(d-2)h-6-k}$

If $k \geq 2h$, no divergence when $\Lambda \to \infty$ if $d=4$.

Exp. evidence: $h=2 \Rightarrow k\geq 4$, $h=3 \Rightarrow k \geq 6$, $h=4 \Rightarrow k\geq 8$

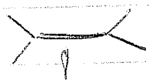
4. Finiteness of ~~d=~~ $N=4$ super-YM above $d=4$

$$\int d^dx\; e^{-4}\,\mathrm{Tr}(F^2) + \int d^dx\; e^{(h-1)4}\, D^k\, \mathrm{Tr}(F^4) + \int d^dx\; e^{(h-1)4}\, D^k \left(\mathrm{Tr}(F^2)\right)^2$$

~~diverges~~

diverges like $\Lambda^{(h-1)(d-4)-8-k+d} = \Lambda^{(d-4)h-4-k}$

Corrections proportional to $(t_8 F^4)$ and $(t_8 t_8 R^4)$ because of maximal susy



massive

$C^{ab}_A\, C^{cd}_A \Rightarrow$ polarization dependence is fixed by A

massive has $2^4 \times 2^4 = 2^8$ comp's (or $2^8 \times 2^8 = 2^{16}$ comp's)

(Alday + Maldacena)

C. Superstring methods

1. RNS

Multiloop amplitudes are complicated since need to sum over spin structures $\Rightarrow$ spacetime susy is not manifest.

Explicit computations up to 4-point NS-NS scattering at 2-loops (d'Hoker, Phong; Zhu, et al.) $\Rightarrow$ finiteness up to 2-loops

Difficult to prove non-renorm. theorems beyond 2-loops.

2. GS

Amplitudes can be computed in light-cone gauge. No sum over spin structures, but need "interaction pt. operators" and contact terms. Computations only up to 4-pt. one-loop (Green+Schwarz),

3. Pure Spinor formalism

~~In flat backgro~~ Can compute loop amplitudes with manifest
$d=10$ Poincare inv. and spacetime susy

Explicit computations up to ~~5pt~~ 4-pt two-loop with NS or R external states.
~~Normalization of two-loop agrees with duality conjecture~~ (Matone, Gomez)
~~(cannot be checked directly in RNS)~~

Can prove various non-renorm. theorem to all loops

D. Results

Can prove (assuming factorization) that no ~~loop~~ contribution at any loop
~~any loops~~ to 0-pt, 1-pt, ~~or~~ 2-pt, 3-pt massless amplitudes

Can prove that first term which can contribute at h-loops is

$h=1$	2	3	4	5	6	...
R^4	D^4R^4	D^6R^4	D^8R^4	D^8R^4	D^8R^4	...
$Tr(F^4)$	$D^2Tr(F^4)$	$D^2Tr(F^4)$	$D^2Tr(F^4)$	...		
$(Tr(F^2))^2$	$D^2(Tr(F^2)^2)$	$D^4(Tr(F^2)^2)$	$D^4(Tr(F^2)^2)$	$D^4(Tr(F^2)^2)$	...	

$\Rightarrow$ Divergence at h-loops when $d \geq d_c$ where $d_c =$

	$h=1$	2	3	4	5	6	7
Sugra	8	7	6	$11/2$	$24/5$	$13/3$	4
$Tr(F^4)$	8	7	6	$11/2$	$26/5$	5	$34/7$
$(Tr(F^2))^2$	8	7	$20/3$	6	$28/5$	$16/3$	$36/7$

Suggests $N=4$ $d=8$ Sugra diverges at 7 loops

D-terms vs. F-terms