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Spring School on Superstring Theory and Related Topics

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Puzzles and Problems for Gravity and Glue
Lecture IV

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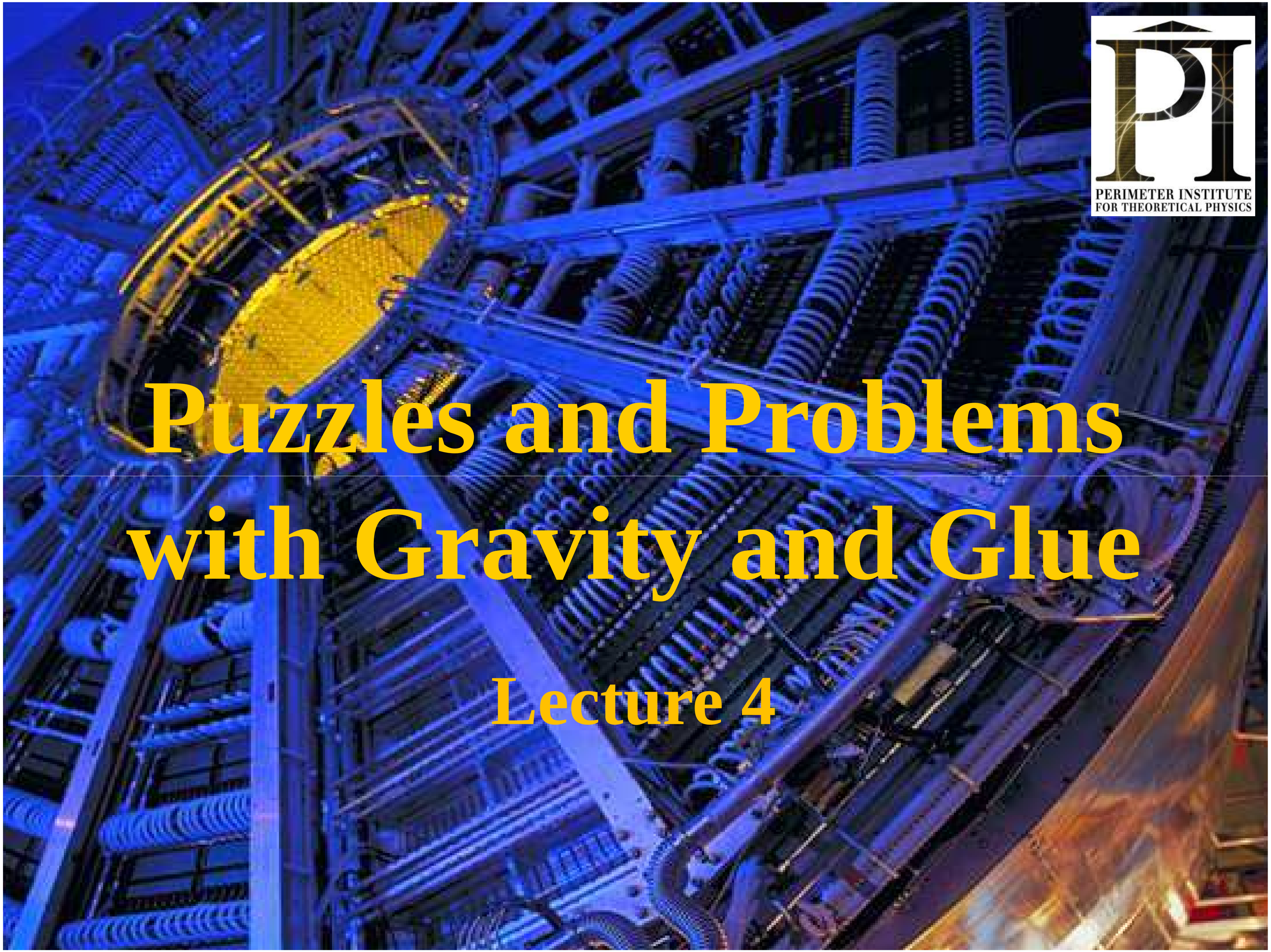

Puzzles and Problems with Gravity and Glue

Lecture 4

Focus:

“role of higher curvature interactions on AdS/CFT calculations”

Overview:

1. Introductory remarks about CFT’s:
   three-point functions, energy fluxes, ….

2. AdS/CFT with Gauss-Bonnet gravity

3. AdS/CFT with quasi-topological gravity

4. Holographic hydrodynamics and viscosity bounds

5. Concluding remarks

Recall for two-dimensional CFT:

$$\langle\, T_{zz}\, T_{ww}\,\rangle = \frac{c/2}{(z-w)^4}$$

$$\langle\, T_{zz}\, T_{ww}\, T_{uu}\,\rangle = \frac{c}{(z-w)^2(w-u)^2(u-z)^2}$$

- conformal symmetry is enough to completely fix form of two- and three-point functions
- additional input (eg, OPE or Ward identities) is needed to relate overall constants
- higher dimensions?? e.g., d=4??

(Osborn & Petkou `93)
(Erdmenger & Osborn `96)

For d-dimensional CFT:

$$\langle T_{ab}(x)\,T_{cd}(0)\,\rangle = \frac{C_T}{x^{2d}}\,\mathcal{I}_{ab,cd}(x)$$

where $$\mathcal{I}_{ab,cd}(x) = \frac{1}{2}\left(I_{ac}(x)I_{bd}(x) + I_{ad}(x)I_{bc}(x)\right) - \frac{1}{d}\delta_{ab}\,\delta_{cd}$$

and $$I_{ab}(x) = \delta_{ab} - 2\frac{x_a\,x_b}{x^2}$$

- fixed by conformal symmetry

- note: $C_T = 4\,c$ for d=2

  $C_T = \dfrac{40}{\pi^4}\,c$ for d=4

(Osborn & Petkou `93)
(Erdmenger & Osborn `96)

For d-dimensional CFT:

$$\langle\, T_{ab}(x)\, T_{cd}(0)\,\rangle = \frac{C_T}{x^{2d}}\, \mathcal{I}_{ab,cd}(x)$$

$$\langle\, T_{ab}(x)\, T_{cd}(y)\, T_{ef}(z)\rangle = \frac{1}{\left((x-y)^2(y-z)^2(z-x)^2\right)^{d/2}}$$

$$\times\ \Big\{ A\, I^{(1)}_{ab,cd,ef}(x,y,z) + B\, I^{(2)}_{ab,cd,ef}(x,y,z) + C\, I^{(3)}_{ab,cd,ef}(x,y,z)$$

$$+ D\, I^{(4)}_{ab,cd,ef}(x,y,z) + E\, I^{(5)}_{ab,cd,ef}(x,y,z) \Big\}$$

where, for example:

$$I^{(1)}_{ab,cd,ef}(x,y,z) = \mathcal{E}^T_{ab,a'b'}\, \mathcal{E}^T_{cd,c'd'}\, \mathcal{E}^T_{ef,e'f'}\, I_{b'c'}(x-y) I_{d'e'}(y-z) I_{f'a'}(z-x)$$

with

$$\mathcal{E}^T_{ab,a'b'} = \frac{1}{2}\left(\delta_{aa'}\delta_{bb'} + \delta_{ab'}\delta_{ba'}\right) - \frac{1}{d}\delta_{ab}\delta_{a'b'}$$

$$I_{ab}(x) = \delta_{ab} - 2\frac{x_a\, x_b}{x^2}$$

(Osborn & Petkou `93)
(Erdmenger & Osborn `96)

For d-dimensional CFT:

$$\langle\, T_{ab}(x)\, T_{cd}(0)\, \rangle = \frac{C_T}{x^{2d}}\, \mathcal{I}_{ab,cd}(x)$$

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$$\times \Big\{ A\, I^{(1)}_{ab,cd,ef}(x,y,z) + B\, I^{(2)}_{ab,cd,ef}(x,y,z) + C\, I^{(3)}_{ab,cd,ef}(x,y,z)$$
$$+ D\, I^{(4)}_{ab,cd,ef}(x,y,z) + E\, I^{(5)}_{ab,cd,ef}(x,y,z) \Big\}$$

- fixed by conformal symmetry and then energy conservation:

$$D = \frac{d^2-4}{2}\, A + \frac{d+2}{2}\, B - 2d\, C$$

$$E = (d^2-4)\, A + \frac{d(d+6)}{2}\, B - \frac{d(d+10)}{2}\, C$$

$\longrightarrow$ three independent parameters: $\boxed{A,\ B,\ C}$

(Osborn & Petkou `93)
(Erdmenger & Osborn `96)

For d-dimensional CFT:

$$\langle\, T_{ab}(x)\, T_{cd}(0)\,\rangle = \frac{C_T}{x^{2d}}\,\mathcal{I}_{ab,cd}(x)$$

$$\langle\, T_{ab}(x)\, T_{cd}(y)\, T_{ef}(z)\rangle = \frac{1}{\left((x-y)^2(y-z)^2(z-x)^2\right)^{d/2}}$$

$$\times\ \Big\{A\, I^{(1)}_{ab,cd,ef}(x,y,z) + B\, I^{(2)}_{ab,cd,ef}(x,y,z) + C\, I^{(3)}_{ab,cd,ef}(x,y,z)$$

$$+D\, I^{(4)}_{ab,cd,ef}(x,y,z) + E\, I^{(5)}_{ab,cd,ef}(x,y,z)\Big\}$$

- fixed by conformal symmetry and then energy conservation:

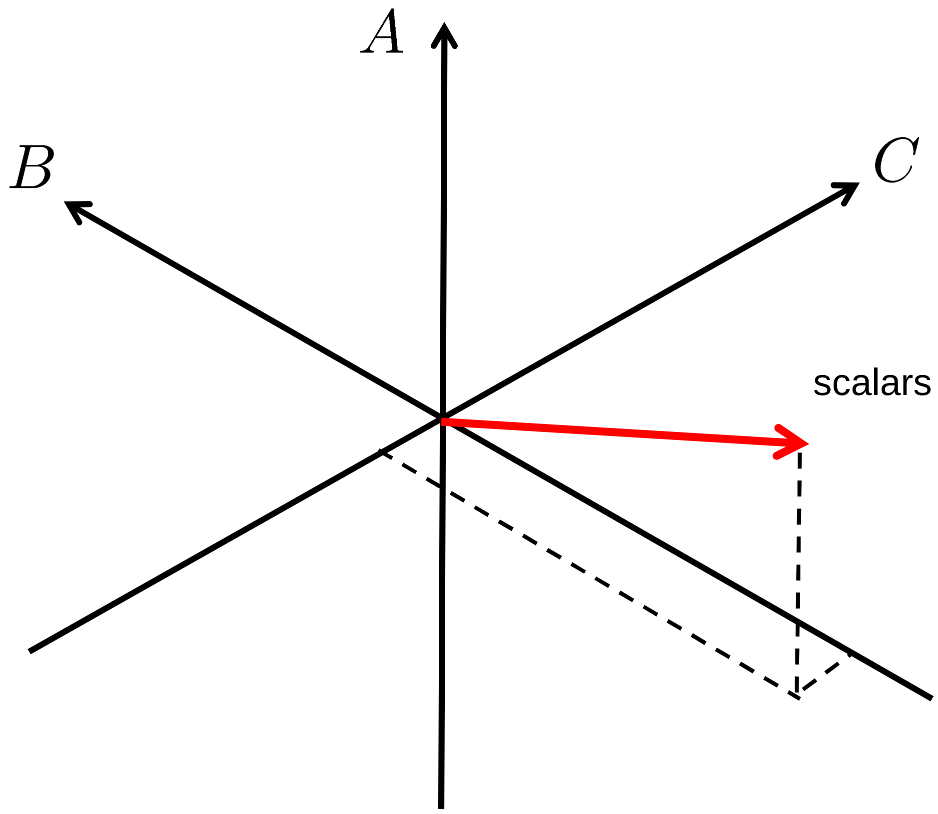
$$D = \frac{d^2-4}{2}\,A + \frac{d+2}{2}\,B - 2d\,C$$

$$E = (d^2-4)\,A + \frac{d(d+6)}{2}\,B - \frac{d(d+10)}{2}\,C$$

- Ward identities: $C_T = \dfrac{\Omega_d}{2}\,\dfrac{(d-1)(d+2)A - 2B - 4(d+1)C}{d(d+2)}$

(Osborn & Petkou `93)

3-pt. function parameters: $A,\ B,\ C$

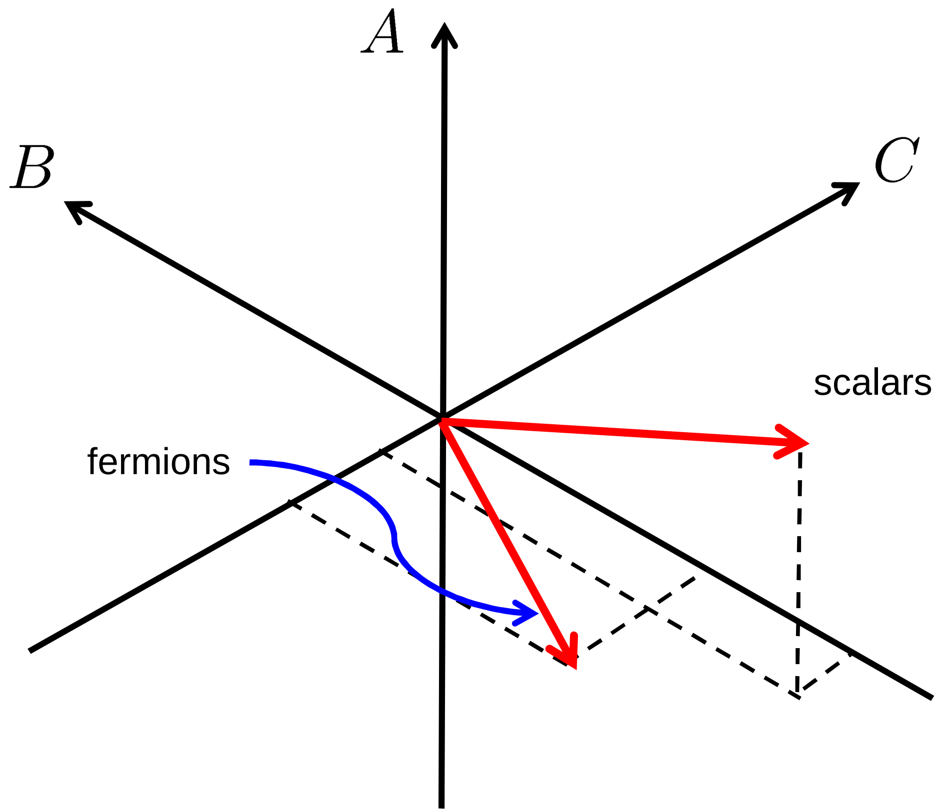


simple d=4 examples: free massless conformal scalars

$$A = \frac{8n_s}{27\pi^6},\ B = -\frac{16n_s}{27\pi^6},\ C = -\frac{2n_s}{27\pi^6}$$

(Osborn & Petkou `93)

3-pt. function parameters: $A,\ B,\ C$

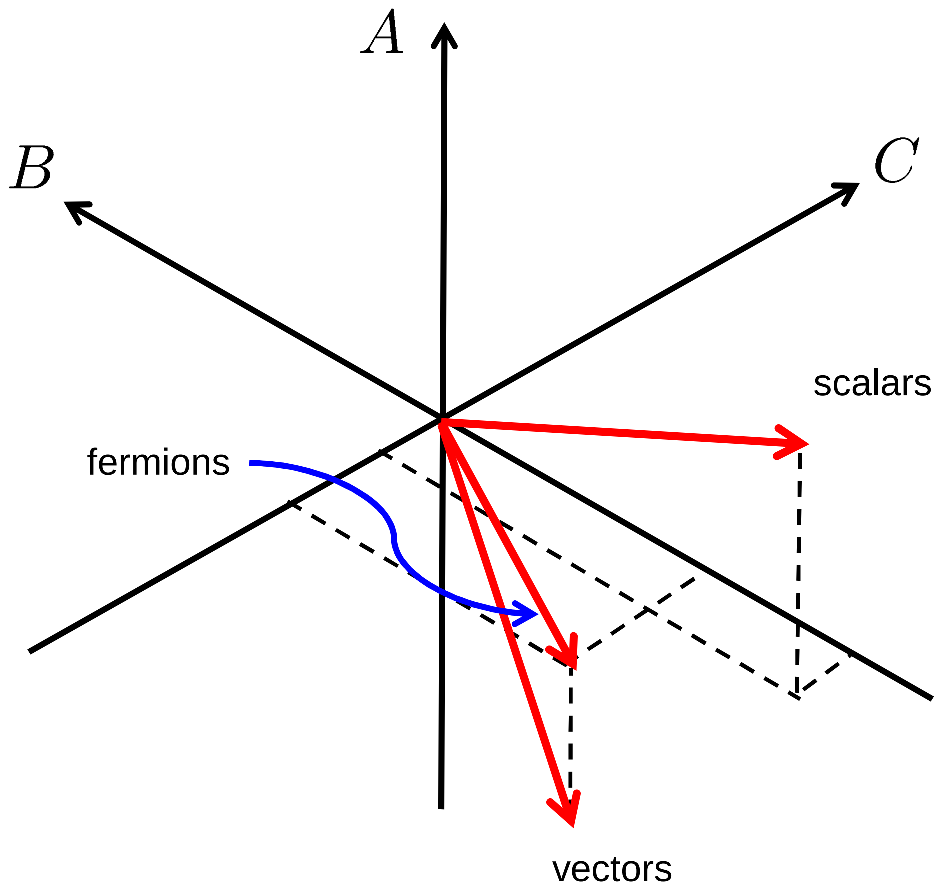


simple d=4 examples: free massless Weyl fermions

$$A = 0,\ B = -\frac{2n_{wf}}{\pi^6},\ C = -\frac{n_{wf}}{\pi^6}$$

(Osborn & Petkou `93)

3-pt. function parameters: $A,\ B,\ C$

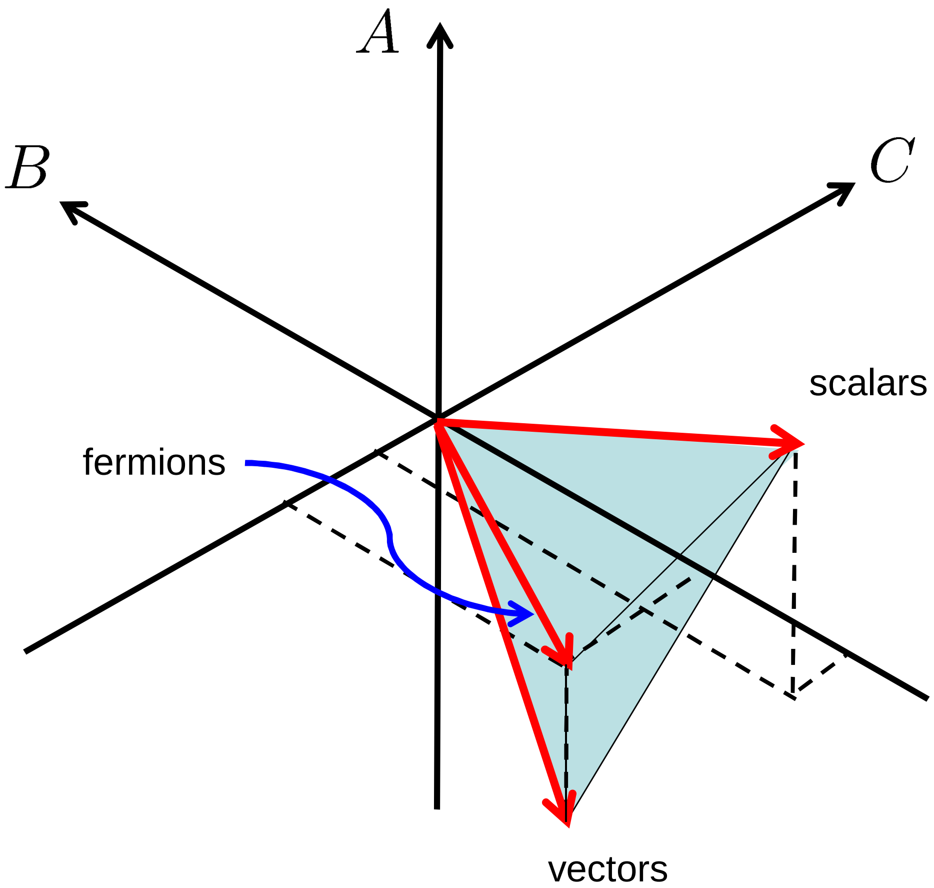


simple d=4 examples: free massless vectors

$$A = -\frac{16 n_v}{\pi^6},\ B = -\frac{32 n_v}{\pi^6},\ C = -\frac{16 n_v}{\pi^6}$$

(Osborn & Petkou `93)

3-pt. function parameters: $A,\ B,\ C$

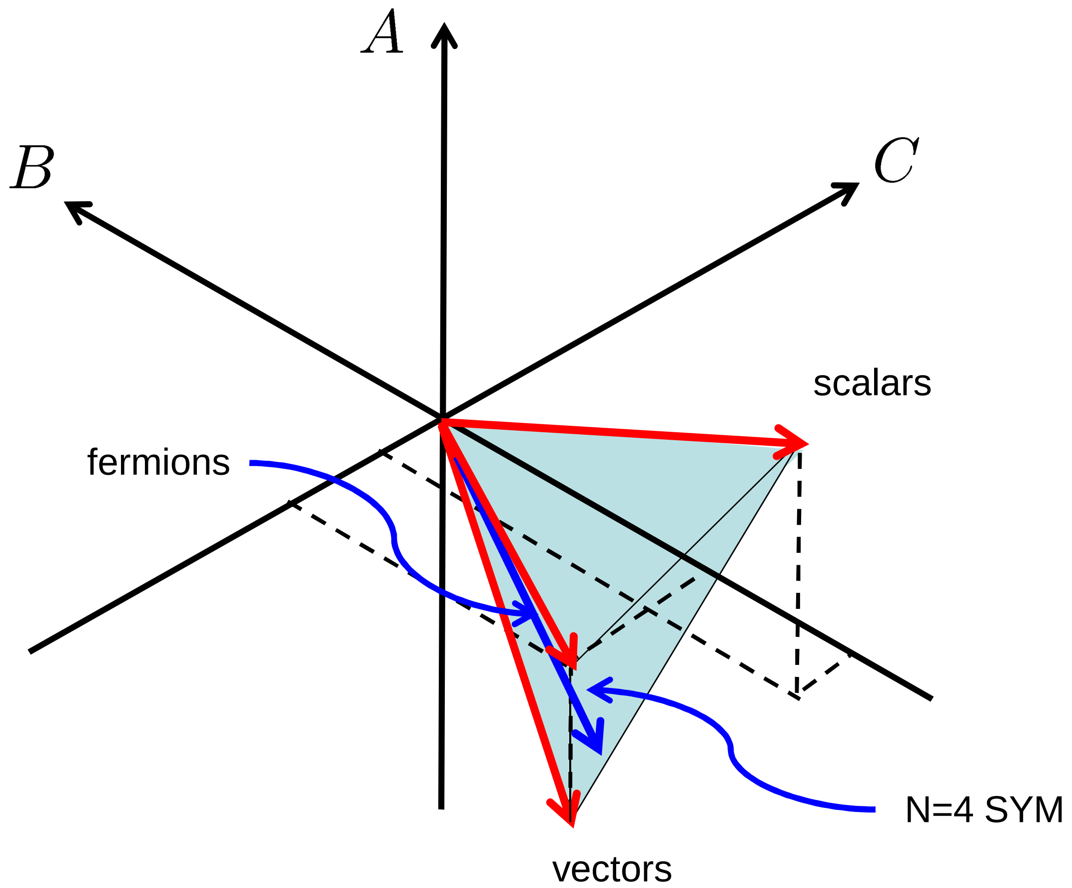


simple d=4 examples: free fields

$$A = \frac{8}{27\pi^6}(n_s - 54 n_v),\ \ B = -\frac{2}{27\pi^6}(8 n_s + 432 n_v + 27 n_{wf}),$$
$$C = -\frac{1}{27\pi^6}(2 n_s + 432 n_v + 27 n_{wf})$$

(Osborn & Petkou `93)

3-pt. function parameters: $A,\ B,\ C$



simple d=4 examples: N=4 U(N_c) super-Yang-Mills

$$A = -\frac{128}{9\pi^6}\,N_c^2,\ B = -\frac{392}{9\pi^6}\,N_c^2,\ C = -\frac{184}{9\pi^6}\,N_c^2$$

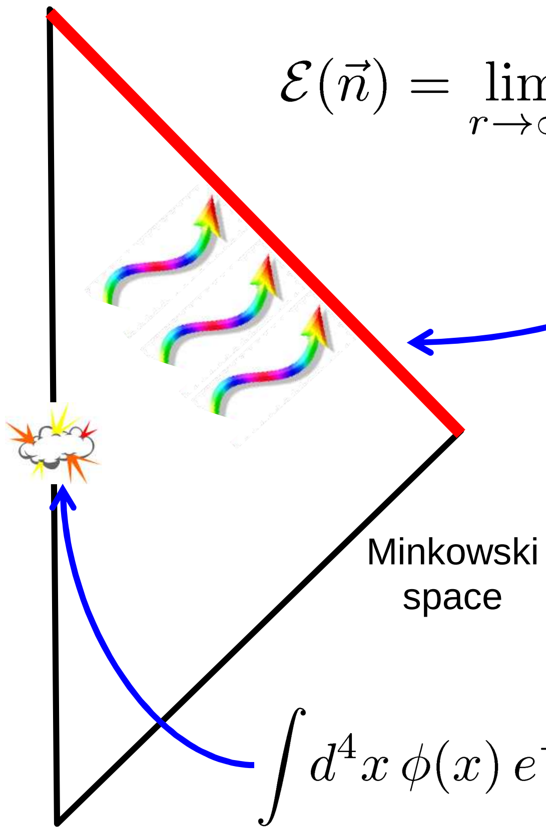
(Arutyunov & Frolov `99)

(Hofman & Maldacena `08)

“Conformal Collider Physics”

- consider scattering “experiments” in d=4 CFT’s
- insert disturbance with stress tensor
- measure energy flux at infinity

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T_i{}^0(t, r n^i)$$



$$\int d^4x\, \phi(x)\, e^{-iEt}\, \epsilon_{ij} T_{ij}$$

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon^*_{ij} T_{ij}\, \mathcal{E}(\vec{n})\, \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon^*_{ij} T_{ij}\, \epsilon_{kl} T_{kl} | 0 \rangle}$$

controlled by 3-pt function!!

(Hofman & Maldacena `08)

"Conformal Collider Physics"

- consider scattering "experiments" in d=4 CFT's

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon^*_{ij} T_{ij} \, \mathcal{E}(\vec{n}) \, \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon^*_{ij} T_{ij} \epsilon_{kl} T_{kl} | 0 \rangle}$$

$$= \frac{E}{4\pi} \left[1 + t_2 \left(\frac{\epsilon^*_{ij} \epsilon_{il} n_i n_j}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{2}{15} \right) \right]$$

- latter fixed by symmetry of "experiment" but can determine:

$$t_2 = \frac{15(5A + 4B - 12C)}{9A - B - 10C} \qquad t_4 = -\frac{15(17A + 32B - 80C)}{4(9A - B - 10C)}$$

(Hofman & Maldacena `08)

"Conformal Collider Physics"

- consider scattering "experiments" in d=4 CFT's

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon^*_{ij} T_{ij}\, \mathcal{E}(\vec{n})\, \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon^*_{ij} T_{ij} \epsilon_{kl} T_{kl} | 0 \rangle}$$

$$= \frac{E}{4\pi} \left[1 + t_2 \left(\frac{\epsilon^*_{ij} \epsilon_{il} n_j n_l}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{2}{15} \right) \right]$$

- demanding $\langle \mathcal{E}(\vec{n}) \rangle \geq 0$ imposes nontrivial constraints

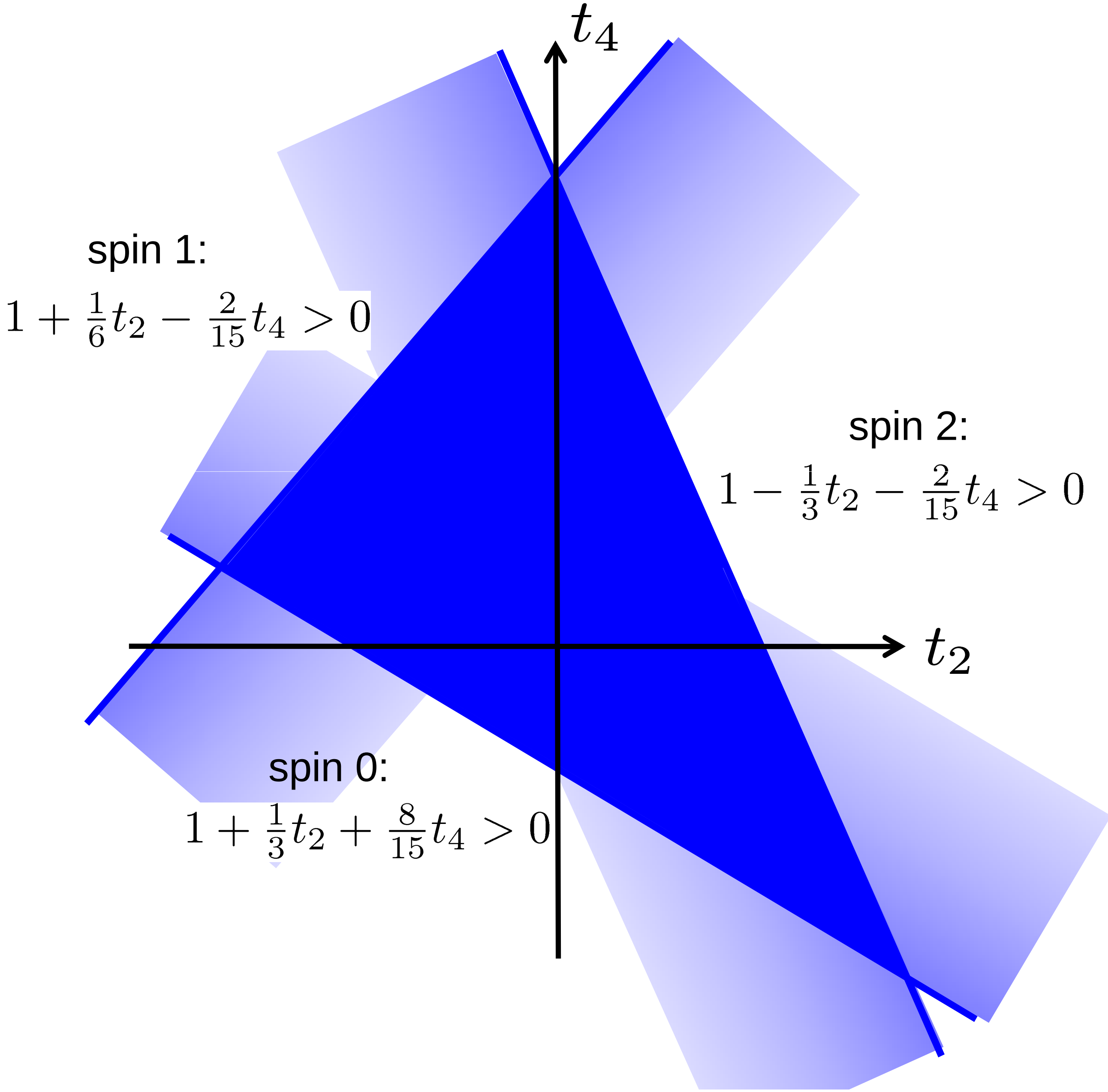
spin 2: $$1 - \frac{1}{3} t_2 - \frac{2}{15} t_4 \geq 0$$

spin 1: $$1 + \frac{1}{6} t_2 - \frac{2}{15} t_4 \geq 0$$

spin 0: $$1 + \frac{1}{3} t_2 + \frac{8}{15} t_4 \geq 0$$

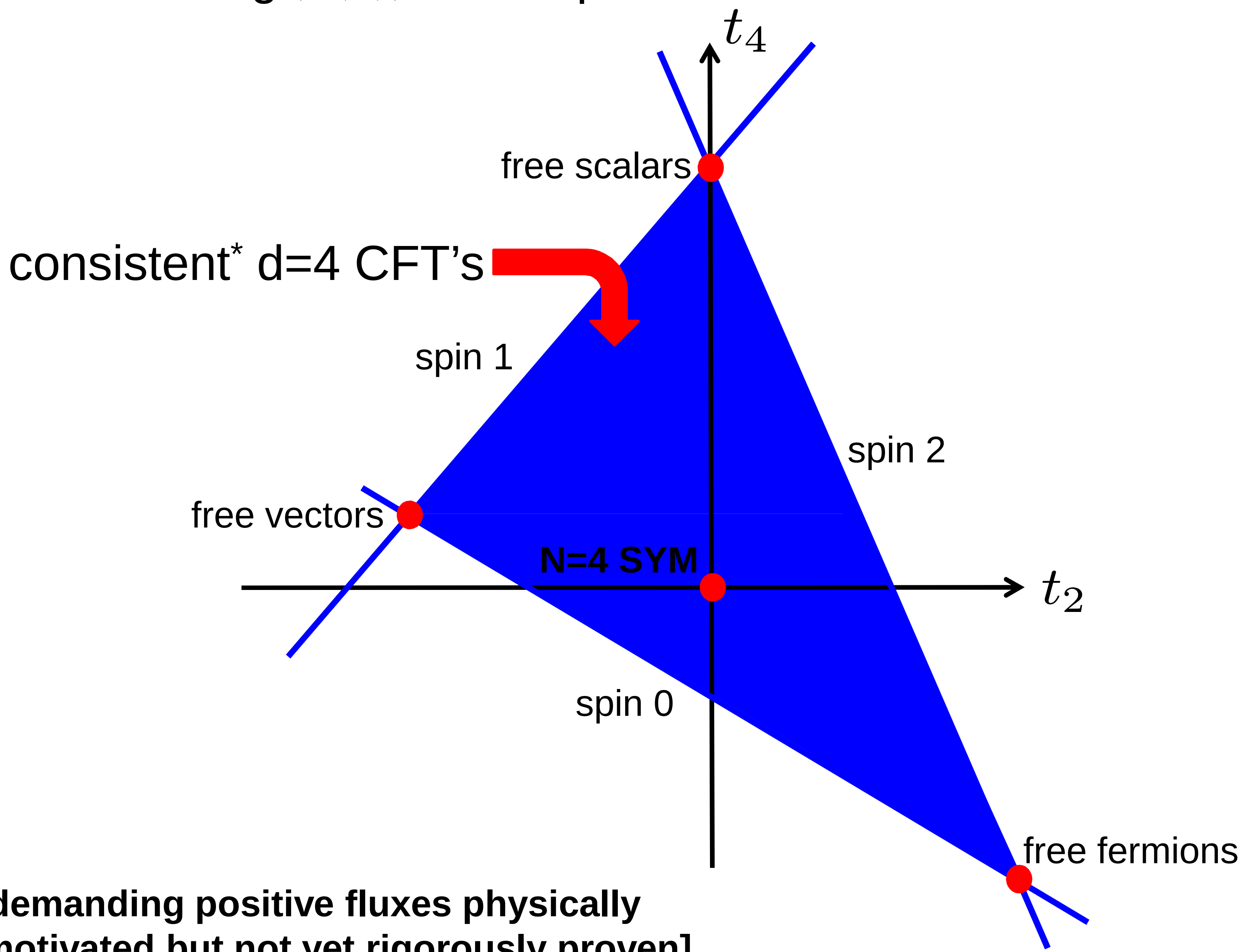
(Hofman & Maldacena `08)

- demanding $\langle \mathcal{E}(\vec{n}) \rangle \geq 0$ imposes nontrivial constraints



(Hofman & Maldacena `08)

- demanding $\langle \mathcal{E}(\vec{n}) \rangle \geq 0$ imposes nontrivial constraints



[*demanding positive fluxes physically motivated but not yet rigorously proven]

(Hofman & Maldacena `08)

“Conformal Collider Physics”

- can we probe positive flux constraints in holographic framework?? (ie, with AdS/CFT correspondence)

- one requires: $t_2 \neq 0 \Longleftrightarrow (R_{abcd})^2$

$$t_4 \neq 0 \Longleftrightarrow (R_{abcd})^3$$

- in strings, sugra action corrected by higher curvature terms

α' corrections: $\alpha'/L^2 \simeq 1/\sqrt{\lambda}$

string loops: $g_s \simeq \lambda/N_c$

- understanding effects of higher curvature terms in sugra is understanding finite $N_c,\ \lambda$ effects in gauge theory
- here need to go beyond perturbative framework to probe constraints (i.e., want to consider finite values of t_2 and t_4)
- if we go to finite parameters where one of above higher curvature terms is important, expect all are important
- ultimately one needs to fully develop string theory for interesting holographic bkgd's
- **instead** consider "toy models" with finite R^n interactions (where we can maintain control of calculations)

$\longrightarrow$ AdS/CFT with Gauss-Bonnet gravity

Brigante, Liu, Myers, Shenker & Yaida

AdS/CFT with Gauss-Bonnet gravity:

$$I = \frac{1}{2\ell_p^3}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2} + R + L^2\frac{\lambda}{2}\,\underbrace{\left(R^{abcd}R_{abcd} - 4R_{ab}R^{ab} + R^2\right)}_{\text{4d Euler density}}\right]$$

- studied in detail for stringy gravity in 1980's

(Zwiebach; Boulware & Deser; Wheeler; Myers & Simon; . . . .)

- higher curvature but eom are still **second order!**
- exact asymptotically flat black hole solutions were found
- exact asymptotically AdS black hole solutions found recently

(Cai; Nojiri & Odinstov; Cho & Neupane)

$\longrightarrow$ perform AdS/CFT calc's but need not treat λ as small!!

original motivation was in holographic hydrodynamics

Brigante, Liu, Myers, Shenker & Yaida

AdS/CFT with Gauss-Bonnet gravity:

$$I = \frac{1}{2\ell_p^3}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2} + R + L^2\,\frac{\lambda}{2}\,\underbrace{\left(R^{abcd}R_{abcd} - 4R_{ab}R^{ab} + R^2\right)}_{\text{4d Euler density}}\right]$$

- shear viscosity: $\frac{\eta}{s} = \frac{1}{4\pi}\,(1 - 4\,\lambda)$, violates KSS bound with $\lambda > 0$
- is theory with $\lambda > 0$ pathological???
- dual CFT is non-gravitational; causality is unambiguous

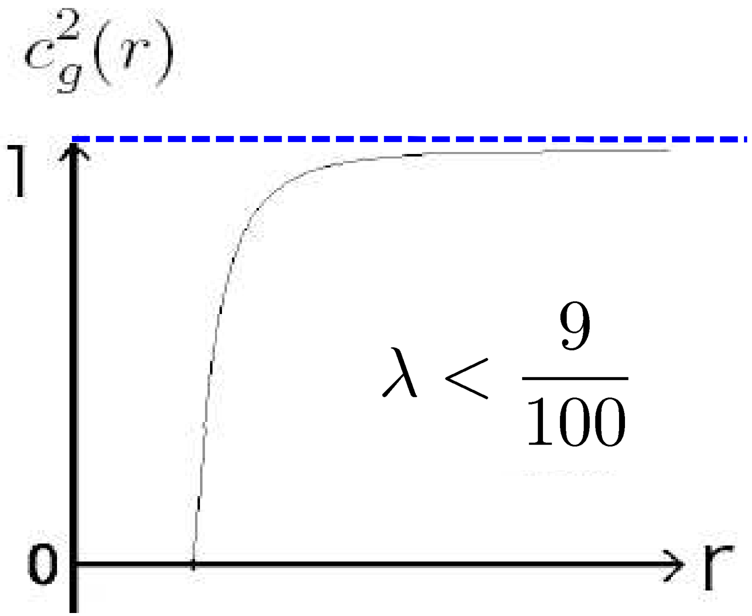
$\longrightarrow$ does bulk theory indicate any problems??

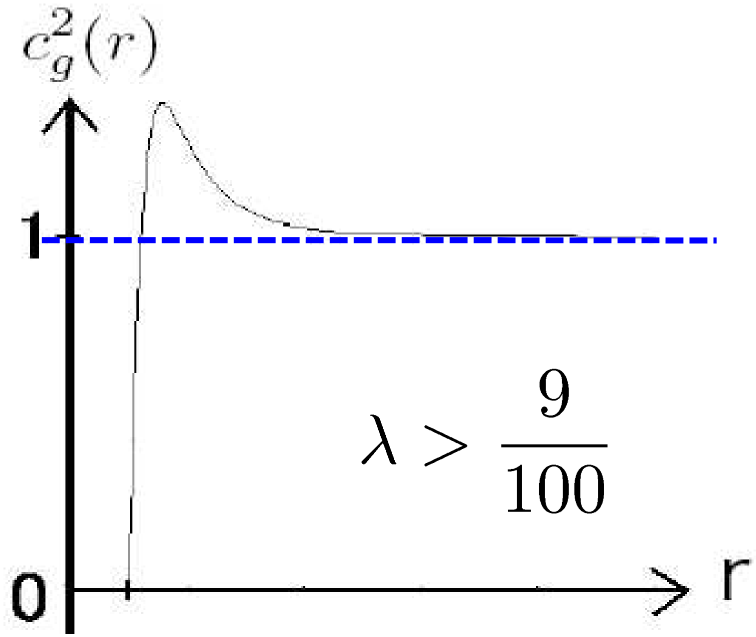
Examine wave equation for tensor h_{xy} in BH background:

→ in WKB/geometric optics limit, can be caste as problem of "null" geodesics in an "effective" metric

$$ds^2_{\mathrm{eff}} = \frac{N_\sharp^2\, f(r)}{c_g^2(r)}\left[-c_g^2(r)\, dt^2 + d\vec{x}^2\right] + \frac{dr^2}{f(r)}$$

$c_g(r)$ = "effective graviton speed" in gauge theory directions



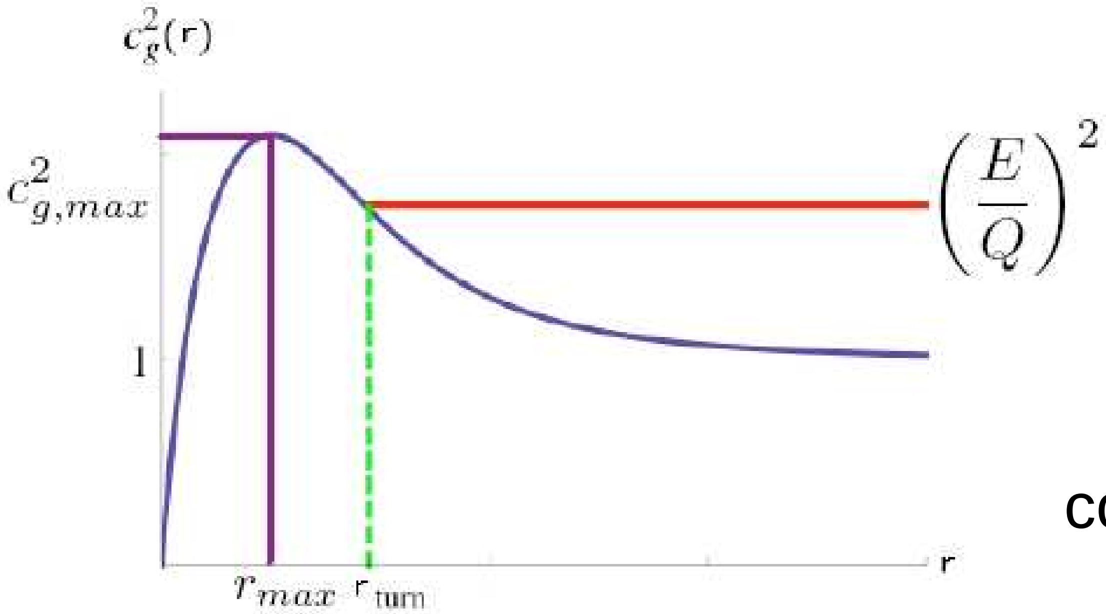


Examine wave equation for $h^x{}_y$ in more detail:

→ in WKB/geometric optics limit, can be caste as problem of "null" geodesics in an "effective" metric

→ for $\lambda > 9/100$, geodesics can bounce back to ∞

geodesic equation: $\left(\frac{dr}{ds}\right)^2 + c_g^2(r) = \left(\frac{E}{Q}\right)^2$



as $E/Q \to c_{g,max}$, geodesic hovers near r_{max}

$$\frac{\Delta x}{\Delta t} \to c_{g,max} > 1$$

couple geodesic to operator 2-point function

(Polchinski, hep-th/9901076)

Brigante, Liu, Myers, Shenker & Yaida

AdS/CFT with Gauss-Bonnet gravity:

$$I=\frac{1}{2\ell_p^3}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2}+R+L^2\frac{\lambda}{2}\underbrace{\left(R^{abcd}R_{abcd}-4R_{ab}R^{ab}+R^2\right)}_{\text{4d Euler density}}\right]$$

- shear viscosity: $\frac{\eta}{s}=\frac{1}{4\pi}\,(1-4\,\lambda)$, violates KSS bound with $\lambda>0$
- is theory with $\lambda>0$ pathological???
- dual CFT is non-gravitational; causality is unambiguous
- studying signals in AdS BH background: in WKB approximation, gravitons follow "null" geodesics in an effective metric

$\longrightarrow$ causality is violated with $\lambda>0$ **but only for** $\lambda>9/100$

How does this translate to the conformal field theory?

$$\lambda=?? \qquad\qquad L/\ell_p=??$$

Brigante, Liu, Myers, Shenker & Yaida

AdS/CFT with Gauss-Bonnet gravity:

$$I = \frac{1}{2\ell_p^3}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2} + R + L^2\frac{\lambda}{2}\,\underbrace{\left(R^{abcd}R_{abcd} - 4R_{ab}R^{ab} + R^2\right)}_{\text{4d Euler density}}\right]$$

- holographic trace anomaly (Henningson & Skenderis; Nojiri & Odintsov; Blau, Narain & Gava)

$$\langle T_\mu{}^\mu\rangle_{CFT} = \frac{c}{16\pi^2}\,I_4 - \frac{a}{16\pi^2}\,E_4$$

$$I_4 = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \quad \text{and} \quad E_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

- GB gravity yields:

$$c = \frac{\pi^2}{2^{3/2}}\,\frac{L^3}{\ell_p^3}\,(1+\sqrt{1-4\lambda})^{3/2}\,\sqrt{1-4\lambda}$$

$$a = \frac{\pi^2}{2^{3/2}}\,\frac{L^3}{\ell_p^3}\,(1+\sqrt{1-4\lambda})^{3/2}\,\left(3\sqrt{1-4\lambda}-2\right)$$

$\longrightarrow$ $\lambda \neq 0$ yields $c \neq a$

AdS/CFT with Gauss-Bonnet gravity:

$\longrightarrow$ turning on t_4 requires R^3 interactions

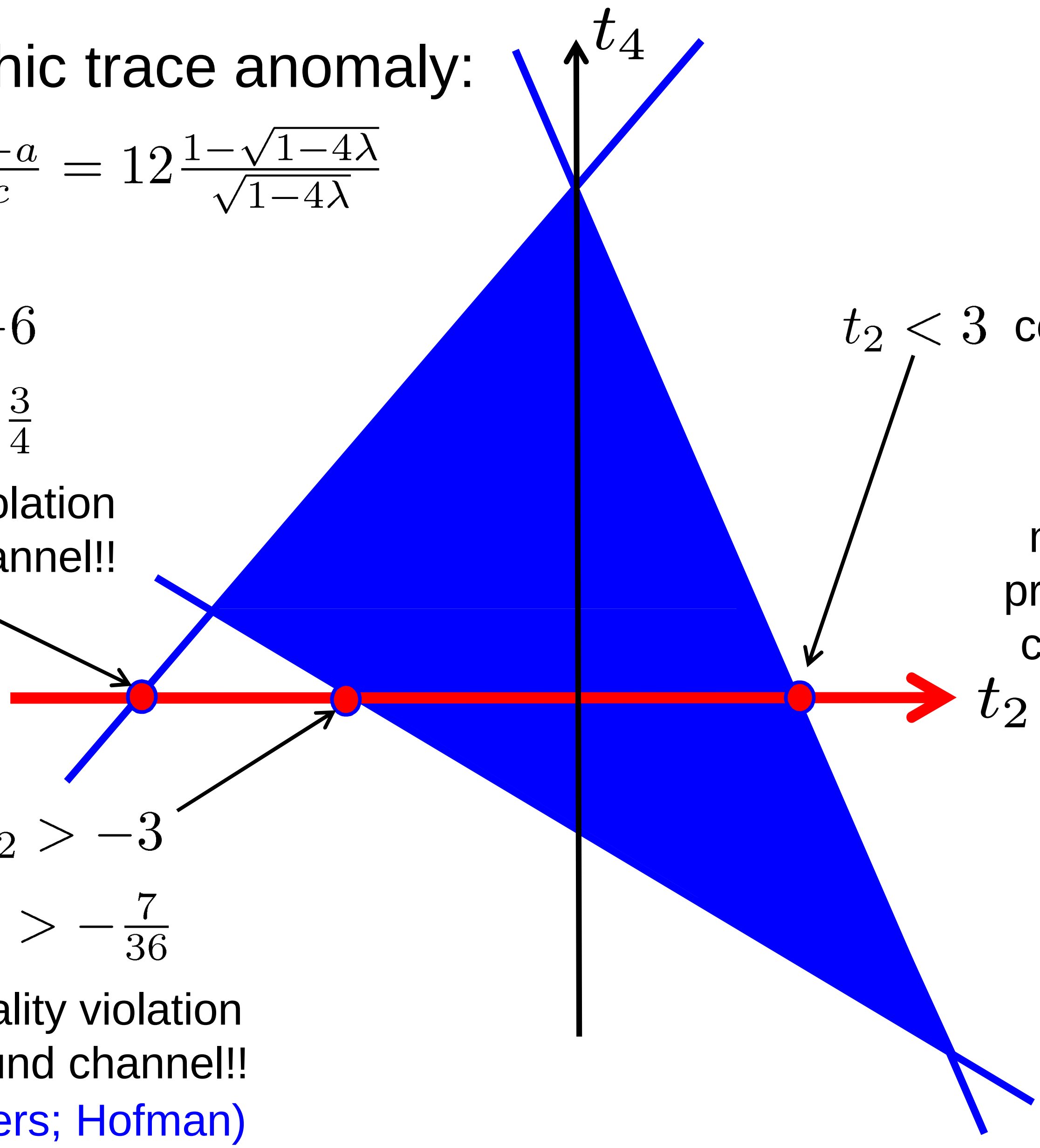
$\longrightarrow$ turning on t_2 requires R^2 interactions

- Gauss-Bonnet gravity corresponds to $(t_4, t_2) = (0, f(\lambda))$
- holographic trace anomaly: $\dfrac{c-a}{c} = 2\left(\dfrac{1}{\sqrt{1-4\lambda}} - 1\right)$
- using t_4=0 and relations of t_4, t_2, c, a in terms of A, B, C

$$t_2 = 6\frac{c-a}{c} = 12\left(\frac{1}{\sqrt{1-4\lambda}} - 1\right)$$

- Gauss-Bonnet gravity corresponds to $(t_4, t_2) = (0, f(\lambda))$
- holographic trace anomaly:

$$t_2 = 6\,\frac{c-a}{c} = 12\,\frac{1-\sqrt{1-4\lambda}}{\sqrt{1-4\lambda}}$$



$t_2 > -6$

$\lambda > -\frac{3}{4}$

causality violation
in shear channel!!

$t_2 > -3$

$\lambda > -\frac{7}{36}$

causality violation
in sound channel!!

(Buchel & Myers; Hofman)

$t_2 < 3$ corresponds to

$$\boxed{\lambda < \frac{9}{100}}$$

matches precisely
previous bound from
causality violation!!

AdS/CFT with Gauss-Bonnet gravity:

$\longrightarrow$ turning on t_4 requires R^3 interactions

$\longrightarrow$ turning on t_2 requires R^2 interactions

- Gauss-Bonnet gravity corresponds $(t_4, t_2) = (0, 12\frac{1-\sqrt{1-4\lambda}}{\sqrt{1-4\lambda}})$

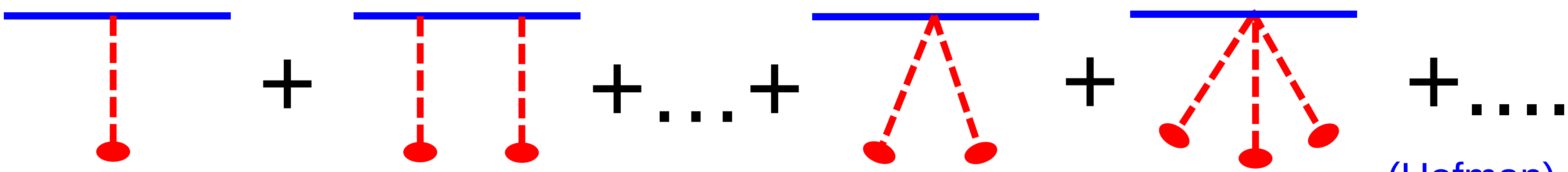
- find direct correlation of negative energy fluxes appearing in CFT calculations and appearance of causality violation in various channels in GB gravity calculations

(Buchel & Myers; Hofman)

NOTE:

- this correlation does **not** extend to any R^2 gravity, eg, $(Cabcd)^2$

- generally, causality depends on more than 3-pt fun. $\langle T_{ab}\, T_{cd}\, T_{ef}\rangle$



(Hofman)

AdS/CFT with Gauss-Bonnet gravity:

→ turning on t_4 requires R^3 interactions

→ turning on t_2 requires R^2 interactions

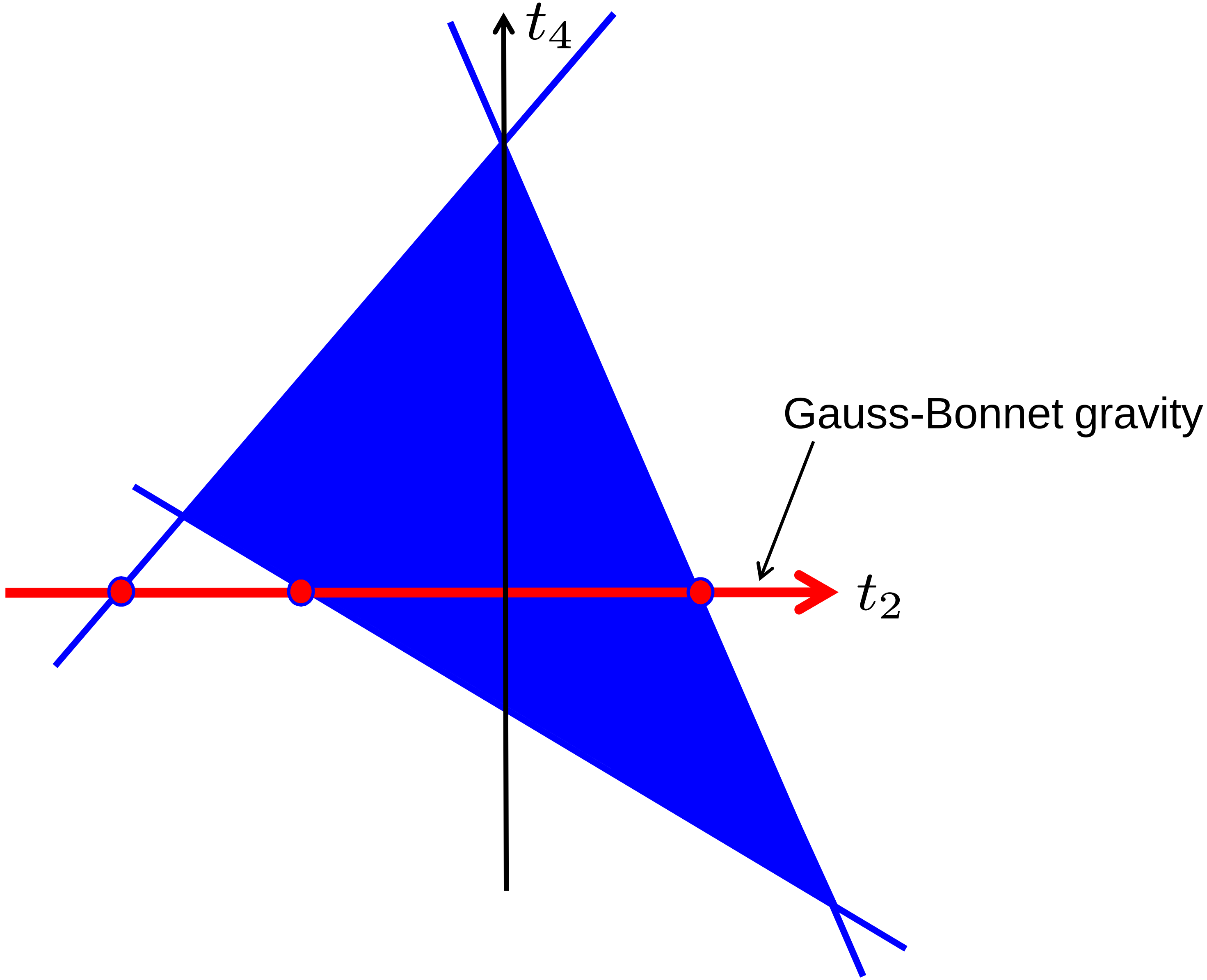
- Gauss-Bonnet gravity corresponds $(t_4, t_2) = (0, 12\frac{1-\sqrt{1-4\lambda}}{\sqrt{1-4\lambda}})$

- find direct correlation of negative energy fluxes appearing in CFT calculations and appearance of causality violation in various channels in GB gravity calculations

(Buchel & Myers; Hofman)

- can we extend AdS/CFT analysis to explore full parameter space of CFT couplings??

- can we extend AdS/CFT analysis to explore t_4 direction??
- need "nice" theory of R^3 gravity ??



- can we extend AdS/CFT analysis to explore t_4 direction??
- need "nice" theory of R^3 gravity ??

- only higher curvature terms yielding 2nd order equations are generalized Euler densities

(Lovelock)

- can we use 6D Euler density??

$$
\begin{aligned}
\chi_6 \;=\;& R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab} - 2R_a{}^c{}_b{}^d R_d{}^e{}_c{}^f R_e{}^a{}_f{}^b - 6R_{abcd}R^{abc}{}_e R^{de} \\
&+\frac{3}{4}R_{abcd}R^{abcd}R + 6R_{abcd}R^{ac}R^{bd} + 4R_a{}^b R_b{}^c R_c{}^a - 3R_a{}^b R_b{}^a R + \frac{1}{4}R^3
\end{aligned}
$$

- all nice properties of GB gravity, but only contributes in $\boxed{D \geq 7}$

(t_4 = 0)

- we also had exact black hole solutions in GB gravity

- there are lots of R^3 terms; so can we tune coefficients to give equally "nice" bh equations in d=5?? **YES**

$$
\begin{aligned}
\mathcal{Z}_5 \;=\;& R_a{}^c{}_b{}^d R_d{}^e{}_c{}^f R_e{}^a{}_f{}^b + \frac{1}{56}\left(-72R_{abcd}R^{abc}{}_e R^{de} + 21R_{abcd}R^{abcd}R\right. \\
&\left.+120R_{abcd}R^{ac}R^{bd} + 144R_a{}^b R_b{}^c R_c{}^a - 132R_a{}^b R_b{}^a R + 15R^3\right)
\end{aligned}
$$

Black holes in Quasi-Topological gravity:

- consider new gravity action:

$$I = \frac{1}{16\pi G}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2} + R + L^2\frac{\lambda}{2}\,\chi_4 + L^4\frac{7\mu}{4}\,\mathcal{Z}_5\right]$$

- metric ansatz for planar asymptotically AdS BH:

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{r^2}\frac{dr^2}{f(r)}$$

- constraint/(t,t)-component equation:

$$\partial_r\left[r^4\left(1 - f + \lambda f^2 + \mu f^3\right)\right] = 0$$

$$\boxed{1 - f + \lambda f^2 + \mu f^3 = \frac{r_+^4}{r^4}}$$

Coefficients tuned
to produce nice result!!

- $$\mathcal{Z}_5 = R_{a\ b}^{\ c\ d} R_{d\ c}^{\ e\ f} R_{e\ f}^{\ a\ b} + \frac{1}{56}\left(\frac{21}{8}R_{abcd}R^{abcd}R - 72R_{abcd}R^{abc}{}_{e}R^{de} + 120R_{abcd}R^{ac}R^{bd} + 144R_a{}^bR_b{}^cR_c{}^a - 132R_a{}^bR_b{}^aR + 15R^3\right)$$

Black holes in Quasi-Topological gravity:

- solutions to cubic equation: $1 - f + \lambda f^2 + \mu f^3 = \frac{r_+^4}{r^4}$
- shift: $f = x - \frac{\lambda}{3\mu}$ $\longrightarrow$ $x^3 + 3px + 2q = 0$

where $p = -\frac{3\mu + \lambda^2}{9\mu^2}$ $\qquad q = \frac{2\lambda^3 + 9\mu\lambda + 27\mu^2(1 - \frac{r_+^4}{r^4})}{54\mu^3}$

- define: $\alpha = \left(-q + \sqrt{q^2 + p^3}\right)^{1/3}$ $\qquad \beta = \left(-q - \sqrt{q^2 + p^3}\right)^{1/3}$

$$\begin{aligned} f_1 &= \alpha + \beta + \frac{\lambda}{3\mu} \\ f_2 &= -\frac{1}{2}(\alpha + \beta) + \imath\frac{\sqrt{3}}{2}(\alpha - \beta) + \frac{\lambda}{3\mu} \\ f_3 &= -\frac{1}{2}(\alpha + \beta) - \imath\frac{\sqrt{3}}{2}(\alpha - \beta) + \frac{\lambda}{3\mu} \end{aligned}$$

- solutions not terribly insightful
- polynomial equation suffices to understand much of physics

Black holes in Quasi-Topological gravity:

- why did this work so well??

eq's of motion are 4^{th} order but complete family of black hole solutions determined by single (nontrivial) integration constant!!

Black holes in Quasi-Topological gravity:

- why did this work so well??

→ "symmetry produces simplicity"

Bonus:

- consider linearized gravitons in AdS vacuum:

$$-\frac{1}{2}\left(1-2\lambda f_\infty-3\mu f_\infty^2\right)\left[\nabla^2 h_{ab}+\nabla_a\nabla_b\, h_c{}^c-\nabla_a\nabla^c h_{cb}-\nabla_b\nabla^c h_{ca}\right.$$
$$\left.-g^{[0]}_{ab}\left(\nabla^2 h_c{}^c-\nabla^c\,\nabla^d h_{cd}\right)+\frac{2}{\tilde{L}^2}\,h_{ab}-\frac{D-3}{\tilde{L}^2}\,g^{[0]}_{ab}\,h_c{}^c\right]=8\pi G_{\rm D}\,\hat{T}_{ab}$$

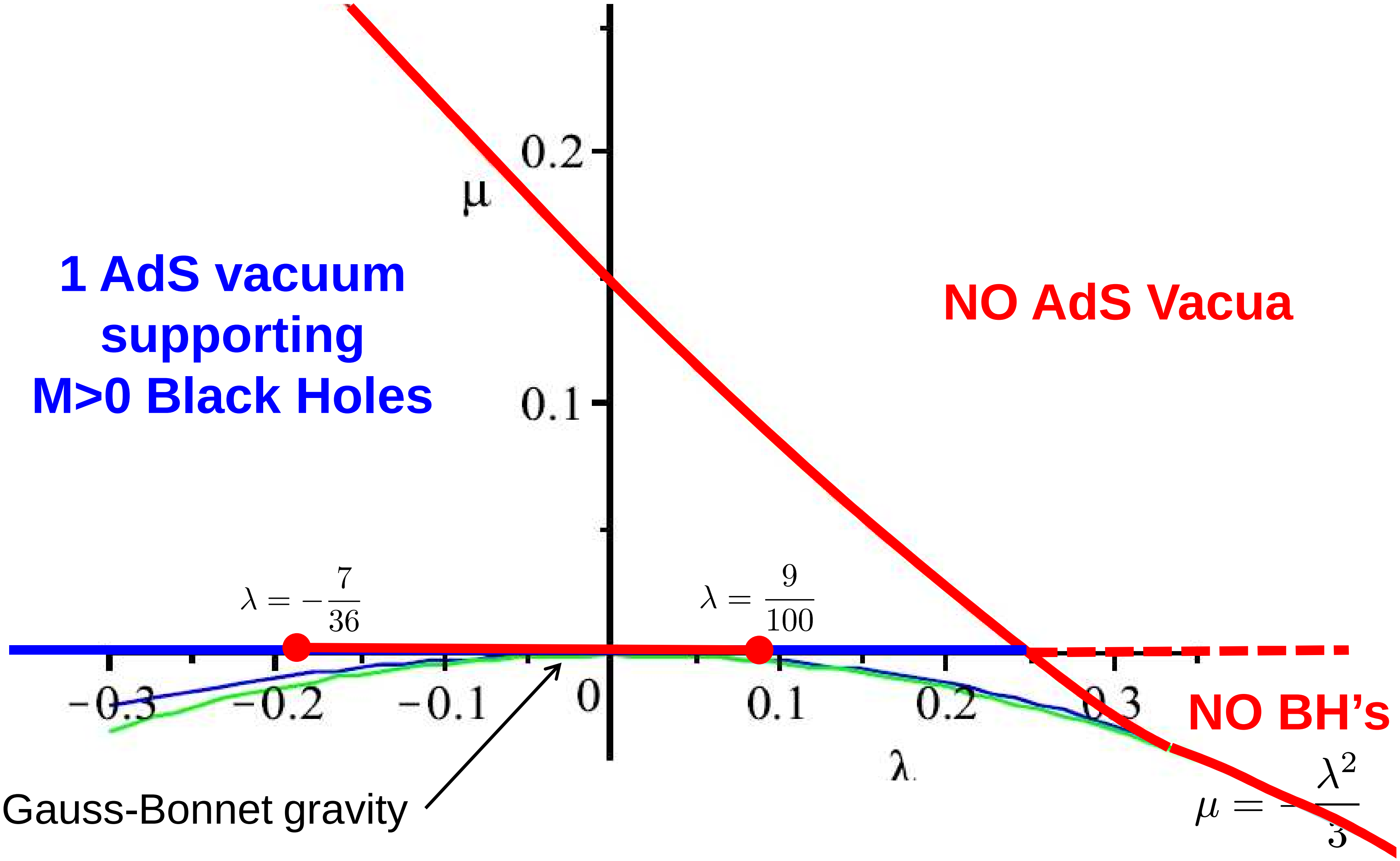
- **second order eom!!**

→ useful for holographic calculations!!

- **not true for general backgrounds!!**

Note: for AdS vacua, set r_+=0 and solve: $1-f_\infty+\lambda f_\infty^2+\mu f_\infty^3=0$

Black holes in Quasi-Topological gravity:



Quasi-Topological gravity:

- need CFT translation for three parameters: $L/\ell_p,\ \lambda,\ \mu$
- holographic trace anomaly gives two central charges: a, c

$$c = \pi^2 \frac{L^3}{\ell_p^3} \frac{1}{f_\infty^{3/2}} \left(1 - 2\lambda f_\infty - 3\mu f_\infty^2\right)$$

$$a = \pi^2 \frac{L^3}{\ell_p^3} \frac{1}{f_\infty^{3/2}} \left(1 - 6\lambda f_\infty + 9\mu f_\infty^2\right)$$

- 3-pt function $\langle T_{ab}\, T_{cd}\, T_{ef} \rangle$ would give three parm's: A, B, C **HARD!**
- consider dual of H&M's scattering experiment: t_2, t_4

$\longrightarrow$ picks out special terms in 3-pt function

Quasi-Topological gravity:

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- 3-pt function $\langle T_{ab}\, T_{cd}\, T_{ef} \rangle$ would give three parm's: A, B, C **HARD!**
- consider dual of H&M's scattering experiment: t_2, t_4

$\longrightarrow$ graviton scattering in shock wave background

(cf, Hofman & Maldacena)

$$t_2 = \frac{24 f_\infty (\lambda - 87 f_\infty \mu)}{1 - 2 f_\infty \lambda - 3 f_\infty^2 \mu}\,, \qquad t_4 = \frac{3780 f_\infty^2 \mu}{1 - 2 f_\infty \lambda - 3 f_\infty^2 \mu}$$

Quasi-Topological gravity:

- need CFT translation for three parameters: $L/\ell_p,\ \lambda,\ \mu$
- constructed AdS/CFT dictionary for 4 parm's: a, c, t_2, t_4
- only three independent underlying parm's: A, B, C

$$\longrightarrow\quad \frac{c-a}{c}=\frac{1}{6}t_2+\frac{4}{45}t_4=\frac{41A-4B-20C}{6(9A-B-10C)}$$

- consistency constraints:

spin 2: $\qquad 1-\frac{1}{3}t_2-\frac{2}{15}t_4\geq 0$

spin 1: $\qquad 1+\frac{1}{6}t_2-\frac{2}{15}t_4\geq 0$

spin 0: $\qquad 1+\frac{1}{3}t_2+\frac{8}{15}t_4\geq 0$

Quasi-Topological gravity:

- need CFT translation for three parameters: $L/\ell_p,\ \lambda,\ \mu$
- constructed AdS/CFT dictionary for 4 parm's: a, c, t_2, t_4
- only three independent underlying parm's: A, B, C

$$\longrightarrow \quad \frac{c-a}{c} = \frac{1}{6}t_2 + \frac{4}{45}t_4 = \frac{41A - 4B - 20C}{6(9A - B - 10C)}$$

- consistency constraints:

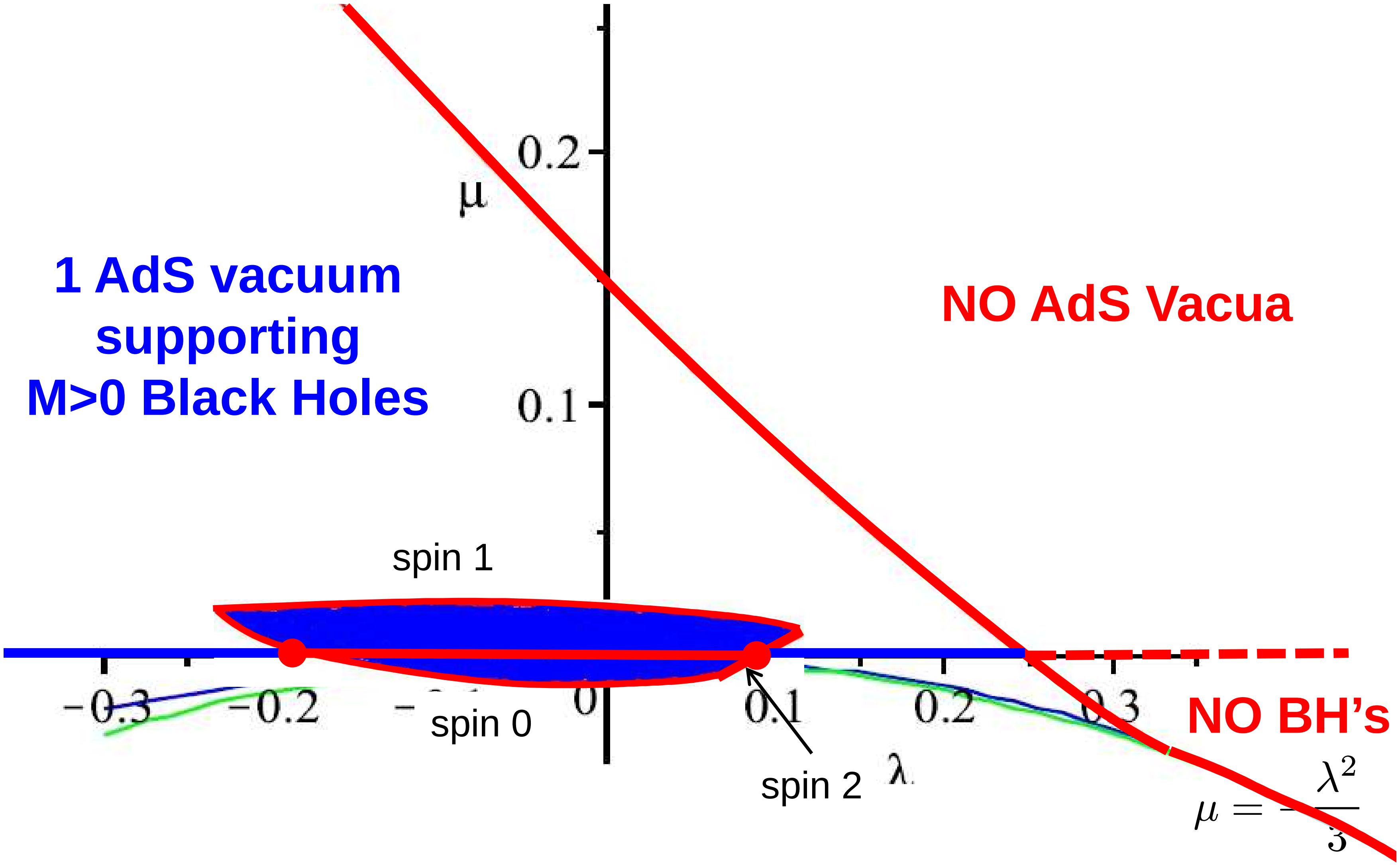
spin 2: $$1 - 10\lambda f_\infty + 189\mu f_\infty^2 \geq 0$$

spin 1: $$1 + 2\lambda f_\infty - 855\mu f_\infty^2 \geq 0$$

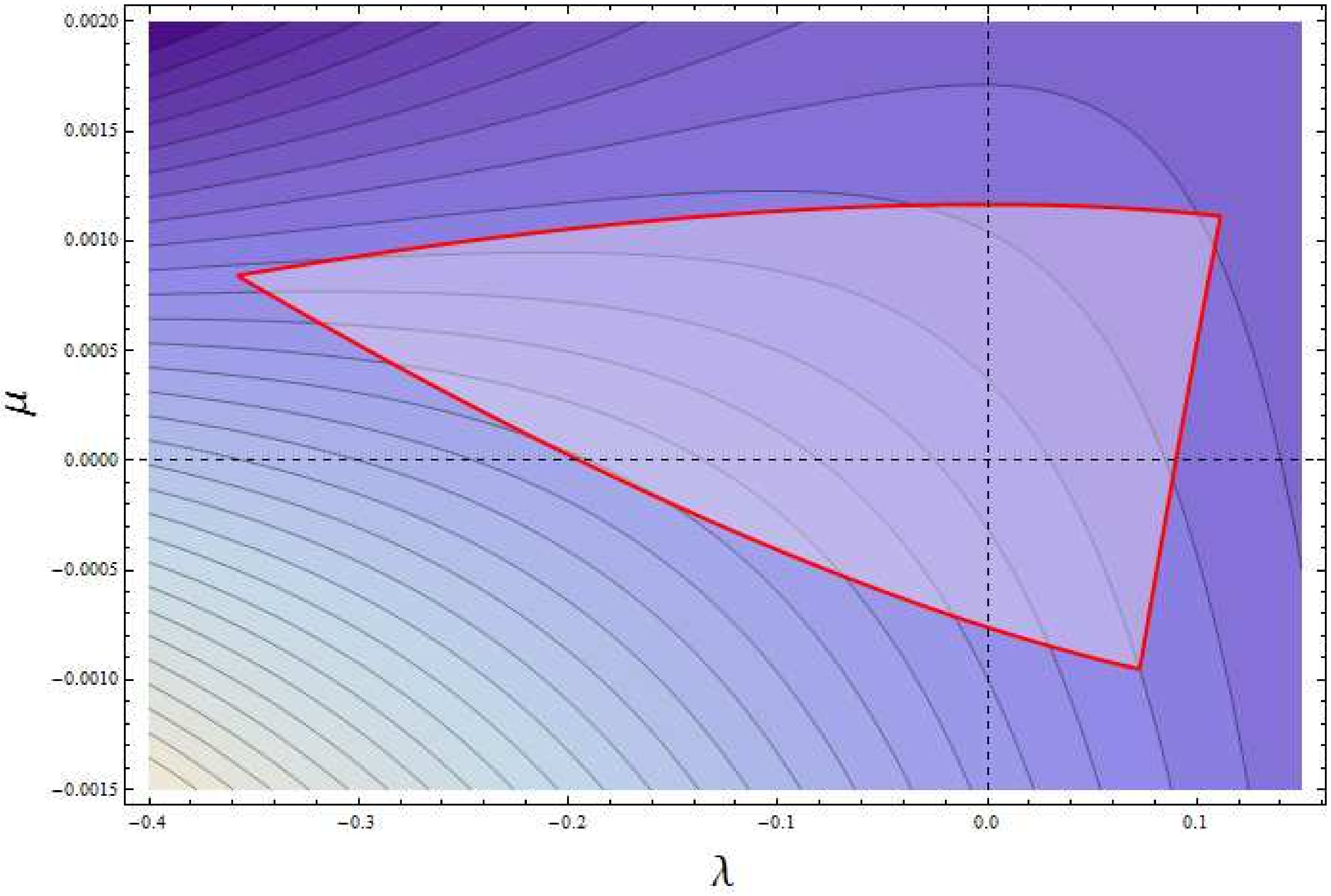
spin 0: $$1 + 6\lambda f_\infty + 1317\mu f_\infty^2 \geq 0$$

Quasi-Topological gravity:

allows us to explore holographic physics over full (A,B,C) space



Quasi-Topological gravity:



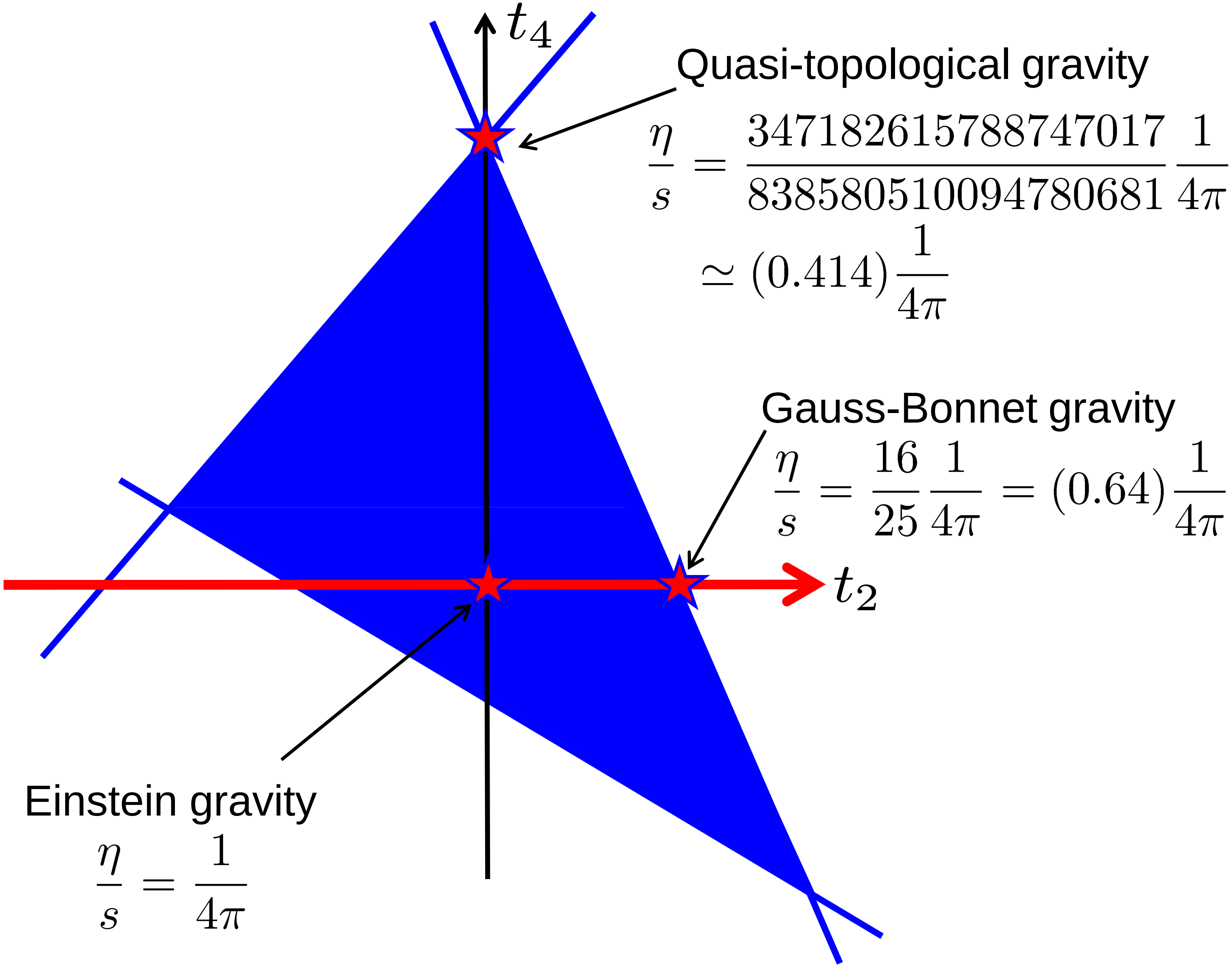
Conjecture? $\boxed{\text{KSS bound: } \eta/s \geq 1/4\pi}$

- string and SUSY gauge theories violate bound perturbatively
- **but does a lower bound exist for η/s???**
- general arguments still suggest some quantum bound

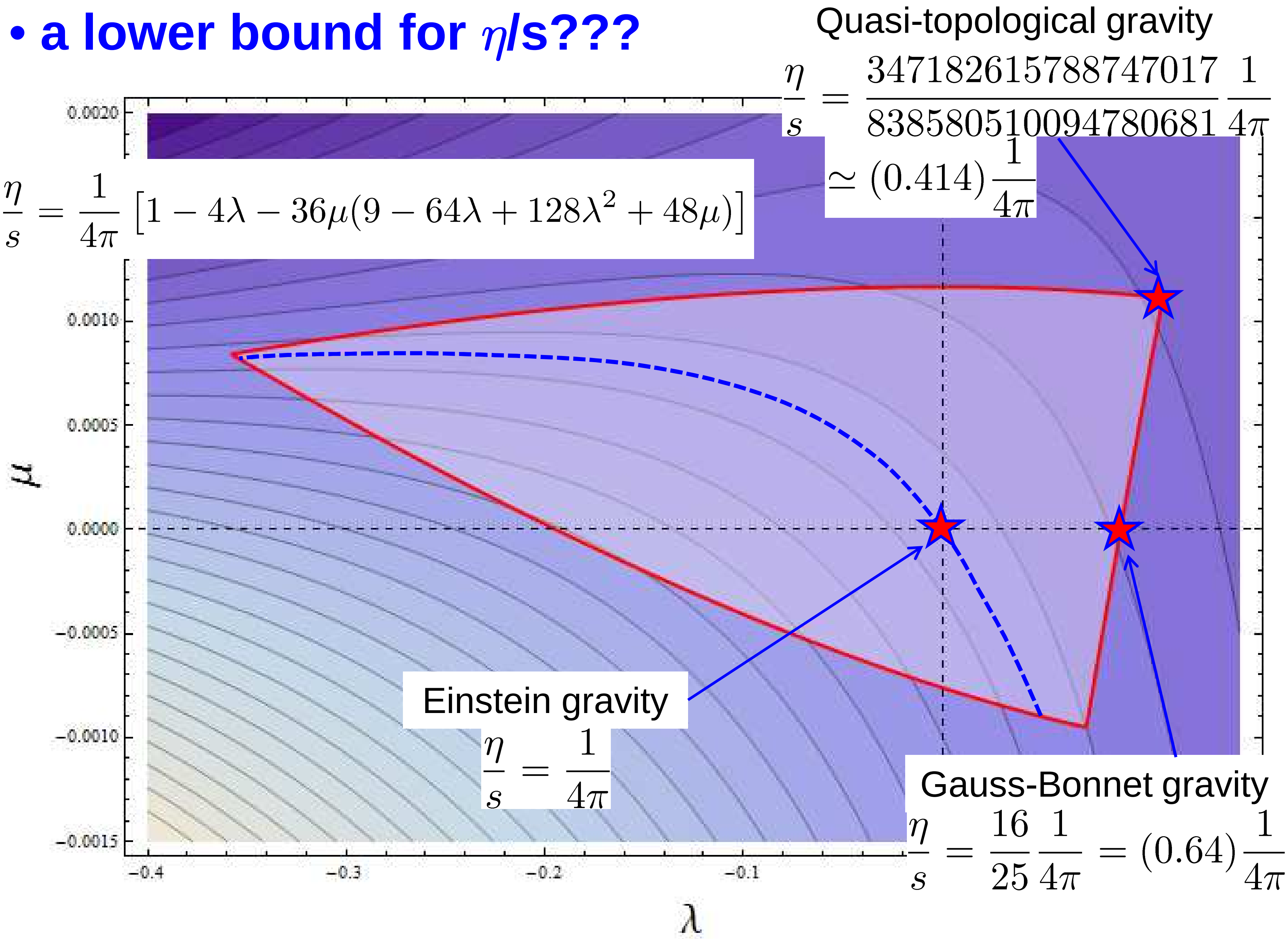
$$\eta/s \ \gtrsim \ \hbar/k_B$$

- holography provides an interesting technique to study fluids in a new "KSS regime" (with unusually low η/s) – so can we use AdS/CFT to address this question??

- **but does a lower bound exist for η/s???**
- explorations with higher curvature theories give us new data



• **a lower bound for η/s???**



- **lower bound for η/s???**
- explorations with higher curvature theories give us new data
- quasi-topological gravity: $\frac{\eta}{s} = \frac{3471826157884747017}{8385805100947806681}\frac{1}{4\pi} \simeq (0.4140)\frac{1}{4\pi}$
- no clear evidence of a solid bound in holographic theories

(Brigante, Liu, RCM, Shenker & Yaida; Ge & Sin; de Boer, Kulaxizi & Parnachev; Camanho & Edelstein; Buchel,Escobedo,RCM,Paulos,Sinha & Smolkin)

- explorations of higher dimensions:
- consider Gauss-Bonnet gravity in D≥5; find minimum:

$$\eta/s \simeq (0.4139)/(4\pi) \quad \text{for} \quad D \simeq 9.207$$

- consider Lovelock R^3 theory in D ≥ 7 (dual to d≥6 CFT's)

$\longrightarrow$ seem to be able to tune η/s to zero (and negative values!)

BH's/plasmas unstable for much of parm. space! (Paulos)

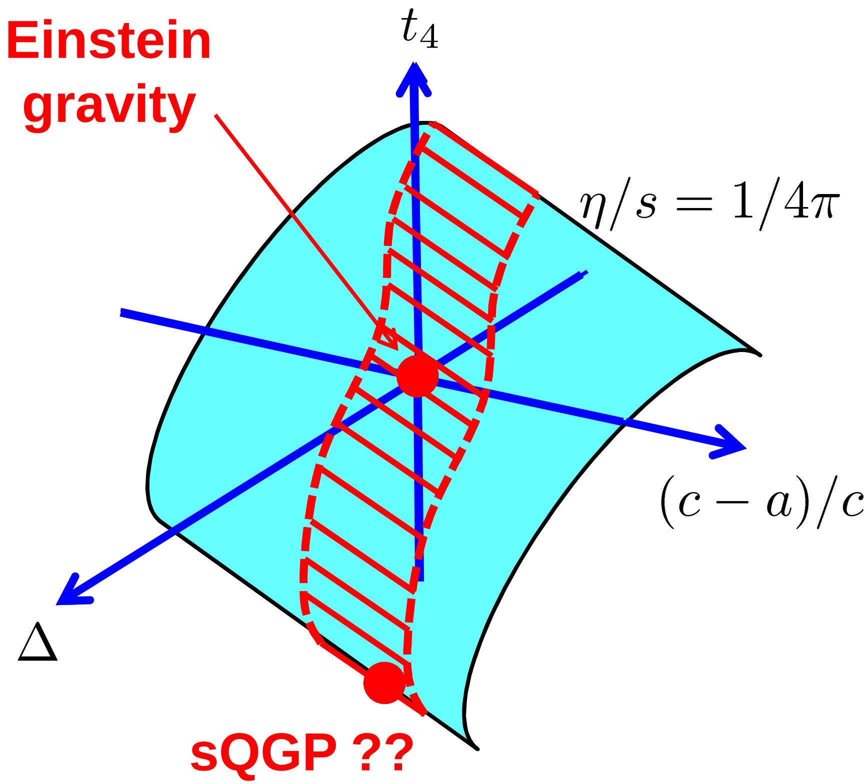
minimum for reliable regime: $\eta/s \sim (0.3938)/(4\pi)$

- it is possible that by adding further embellishments that one can systematically lower η/s closer and closer to zero
- η/s will **not** depend only on parm's in 3-pt function $\langle T_{ab}\, T_{cd}\, T_{ef}\rangle$

What about sQGP??

- holography is great tool to study strongly coupled "KSS" fluids
- CFT parameters translate to couplings in dual gravity theory

$$c,\ a,\ t_4,\ \Delta,\ \cdots \longleftrightarrow L^3/\ell_P^2,\ \lambda,\ \mu,\ \cdots$$



- $\eta/s = 1/4\pi$ only defines surface in parameter space
- where is dual of sQGP?
- consistency constrains our range in parameter space

Lots to explore!

Conclusions:

- AdS/CFT correspondence (gauge/gravity duality) has proven an excellent tool to study strongly coupled gauge theories
- higher-R theories extend duality to broader classes of CFT's

→ maintain calculational control with GB or quasi-top. gravity

- consistency (causality & positive fluxes) constrains couplings
- other limitations/instabilities in higher curvature gravity??
- higher curvature theories certainly violate KSS bound
- open question whether there is a clear bound for η/s??
- further implications for holographic dualities??
- implications for quark-gluon plasma??

Lots to explore!

[Supplementary Material]

- under z $\rightarrow$ u = u(z), quasi-primary operators transform

$$\Phi(z,\bar{z}) = \left(\frac{\partial u}{\partial z}\right)^{h} \left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^{\bar{h}} \Phi(u(z),\bar{u}(\bar{z}))$$

- clearly, these transformations constraint n-point functions

• under z → u = u(z), quasi-primary operators transform

$$\Phi(z,\bar{z}) = \left(\frac{\partial u}{\partial z}\right)^{h} \left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^{\bar{h}} \Phi(u(z),\bar{u}(\bar{z}))$$

• consider the two-point function, as an example:

$$\langle \Phi(z,\bar{z})\Phi(w,\bar{w})\rangle = f(z,\bar{z},w,\bar{w})$$

• focus on global/SL(2,$\mathbb{C}$) transformations:

• translations: $z \to z + a,\ \bar{z} \to \bar{z} + \bar{a}$

$$\begin{aligned} \langle \Phi(z,\bar{z})\Phi(w,\bar{w})\rangle &= \langle \Phi(z+a,\bar{z}+\bar{a})\Phi(w+a,\bar{w}+\bar{a})\rangle \\ &= f(z+a,\bar{z}+\bar{a},w+a,\bar{w}+\bar{a}) \end{aligned}$$

hence $f(z,\bar{z},w,\bar{w}) = f(z-w,\bar{z}-\bar{w})$

- under z → u = u(z), quasi-primary operators transform

$$\Phi(z,\bar z) = \left(\frac{\partial u}{\partial z}\right)^h \left(\frac{\partial \bar u}{\partial \bar z}\right)^{\bar h} \Phi(u(z),\bar u(\bar z))$$

- consider the two-point function, as an example:

$$\langle \Phi(z,\bar z)\Phi(w,\bar w)\rangle = f(z,\bar z,w,\bar w)$$

- focus on global/SL(2,$\mathbb{C}$) transformations:
    - (complex) scalings: $z \to \lambda z,\ \bar z \to \bar\lambda \bar z$

$$\begin{aligned} \langle \Phi(z,\bar z)\Phi(w,\bar w)\rangle &= \lambda^{h_1+h_2}\bar\lambda^{\bar h_1+\bar h_2} f(\lambda(z-w),\bar\lambda(\bar z-\bar w)) \\ &= f(z-w,\bar z-\bar w) \end{aligned}$$

hence $f(z-w,\bar z-\bar w) = \dfrac{C_{12}}{(z-w)^{h_1+h_2}(\bar z-\bar w)^{\bar h_1+\bar h_2}}$

- under z → u = u(z), quasi-primary operators transform

$$\Phi(z,\bar z) = \left(\frac{\partial u}{\partial z}\right)^{h} \left(\frac{\partial \bar u}{\partial \bar z}\right)^{\bar h} \Phi(u(z),\bar u(\bar z))$$

- consider the two-point function, as an example:

$$\langle \Phi(z,\bar z)\Phi(w,\bar w)\rangle = f(z,\bar z,w,\bar w)$$

- focus on global/SL(2,$\mathbb{C}$) transformations:
  - inversion: $z \to -1/z,\ \bar z \to -1/\bar z$ $\quad \left(\frac{\partial u}{\partial z} = \frac{1}{z^2}\right)$

$$\begin{aligned}\langle \Phi(z,\bar z)\Phi(w,\bar w)\rangle &= z^{-2h_1}\bar z^{-2\bar h_1}\, w^{-2h_2}\bar w^{-2\bar h_2} \\ &\times \frac{C_{12}\,(zw)^{h_1+h_2}(\bar z\bar w)^{\bar h_1+\bar h_2}}{(z-w)^{h_1+h_2}(\bar z-\bar w)^{\bar h_1+\bar h_2}}\end{aligned}$$

hence $\langle \Phi(z,\bar z)\Phi(w,\bar w)\rangle = \dfrac{C_{12}}{(z-w)^{2h_1}(\bar z-\bar w)^{2\bar h_1}}$ iff $\begin{cases} h_1 = h_2 \\ \bar h_1 = \bar h_2 \end{cases}$

$(C_{12} = 0\ if\ h_1 \neq h_2\ or\ \bar h_1 \neq \bar h_2)$