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Superstring multiloop amplitudes and non-renormalization theorems Lecture IV - Derivation of Results -

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IV. Derivation of multiloop results

A. Explicit computations

One-loop: $\mathcal{A}_{g=1} = \int d\tau \left\langle V_1 \int dz_2 U_2 \ldots \int dz_N U_N \left(\int \mu b\right) \mathcal{N}_0 \right\rangle$

Need $\theta^{16} (p_{\bar{z}})^{16}\ r^{11} (s_{\bar{z}})^{11}$ for non-vanishing amplitude

$\mathcal{N}_0 \to (ds)^{11} (r\theta)^p$, $U \to d_\alpha W^\alpha(x,\theta)$, $b \to rdd$

$\Rightarrow$ Need at least 3 U's

$$\mathcal{A}^{open}_{g=1} \sim \int d\tau \left(\frac{\partial}{\partial\theta}\right)^6 (A\,W\,W\,W) \sim F^4$$

$$\mathcal{A}^{closed}_{g=1} \propto \int \left| d\tau \left(\frac{\partial}{\partial\theta}\right)^6 (AWWW)\right|^2 \sim R^4$$

(Mafra)

Answer agrees with RNS including normalization (Gomez)

5-pt also agrees with RNS (Mafra + Stahn)

Two-loop: $\mathcal{A}_{g=2} = \int d^3\tau \left\langle \int dz_1 U_1 \ldots \int dz_N U_N \left(\int \mu b\right)^3 \mathcal{N}_0 \right\rangle$

Need $\theta^{16} (p)^{32}\ r^{11} (s)^{22}$ for non-vanishing amp.

$\mathcal{N}_0 \to (ds)^{22} (r\theta)^p$, $U \to d_\alpha W^\alpha$, $b \to rdd$

$\Rightarrow$ Need at least 4 U's

$$\mathcal{A}^{open}_{g=2} \propto \int d\tau \left(\frac{\partial}{\partial\theta}\right)^8 (WWWW) \sim D^2 F^4$$

$$\mathcal{A}^{closed}_{g=2} \propto \int \left| d\tau \left(\frac{\partial}{\partial\theta}\right)^8 (WWWW)\right|^2 \sim D^4 R^4$$

Answer agrees with duality conjecture including normalization (Gomez + Mafra)

RNS computation (w/o normalization) is very difficult.

Three-loop: $\mathcal{A}_{g=3} = \int d^6\tau \left\langle \left(\int dz\, U\right)^4 \left(\int \mu b\right)^6 \mathcal{N}_0 \right\rangle$

Need $\theta^{16} (p)^{48}\, r^{11} (s)^{33}$, $\mathcal{N}_0 \to (ds)^{33} (r\theta)^p$

For 4-pt. amp's, b ghosts cannot provide $r^{12} d^{11}$ $\Rightarrow$ do not need $\mathcal{N}_R$

$$\left(\int \mu b\right)^6 \longrightarrow \text{(A)}\ (rdd)^5 (\Pi d) \quad \text{OR} \quad \text{(B)}\ (rdd)^6$$

(A) $\int d^6\tau \left(\frac{\partial}{\partial\theta}\right)^{10} (WWWW) \sim \partial^3 F^4 \Rightarrow \partial^4 F^4$

$\int \left| d^6\tau \left(\frac{\partial}{\partial\theta}\right)^{10} (WWWW)\right|^2 \sim \partial^6 R^4$

(B) ~~$\int d\tau$~~ Extra d non-zero mode allows <u>inverse derivative factor</u> coming from colliding vertex operators

$$\int dz_1\, V_1 \int dz_2\, V_2 \sim \int dz_1 \int dz_2\, |z_1 - z_2|^{-1+k_1\cdot k_2} f(z_1) \sim \int dz_1 \frac{f(z_1)}{k_1\cdot k_2}$$

if $V_1(z_1)\, V_2(z_2)$ ~~has~~ $\to |z_1 - z_2|^{-1+k_1\cdot k_2} f(z_1)$.

$$V_1 = W^\alpha_{(1)} d_\alpha \ , \quad V_2 = W^\beta_{(2)} d_\beta \Rightarrow V_1 V_2 \to |z_1 - z_2|^{-1+k_1\cdot k_2} f(z_1)$$

where $f(z_1) = \left(W^\alpha_{(1)} D_\alpha W^\beta_{(2)} - W^\alpha_{(2)} D_\alpha W^\beta_{(1)}\right) d_\beta(z_1)$

$$\Rightarrow \int d^6\tau \left(\frac{\partial}{\partial\theta}\right)^{11} \frac{(W(DW)\, WW)}{k_1\cdot k_2} \sim \partial^2 F^4.$$

Symmetry properties $\Rightarrow$ only possible for $Tr(F^4)$

For closed strings, need $V_1 V_2 \to |z_1 - z_2|^{-2+k_1\cdot k_2} f(z_1)$

$\Rightarrow$ colliding vertex op's do not ~~affec~~ affect result

Result: $\mathcal{A}_{g=4} \to \partial^4 (Tr F^2)^2 \ , \ \partial^2 (Tr F^4) \ , \ \partial^6 R^4$

Four-loop: $\mathcal{A}_{g=4} = \int d^9\tau \left\langle \left(\int dz\, U\right)^4 \left(\int \mu b\right)^9 \mathcal{N}_0 \right\rangle$

Need $\theta^{16} (p)^{64}\, r^{11}\, (s)^{44}$. $\mathcal{N}_0 \to (ds)^{44} (r\theta)^p$

For 4-pt amp's, b ghosts cannot provide $r^{12} d^{16} \Rightarrow$ do not need $\mathcal{N}_R$

$\left(\int \mu b\right)^9 \to$ (A) $(rdd)^7 (\Pi d)^2$ (B) $(rdd)^9$

(A) $\left(\frac{\partial}{\partial\theta}\right)^{12} (WWWW) \sim \partial^4 F^4$

$\left|\left(\frac{\partial}{\partial\theta}\right)^{12} (WWWW)\right|^2 \sim \partial^8 R^4$

(B) Extra 2 d non-zero mode allows <u>two</u> inverse deriv. factors from colliding $V_1 \to V_2$ and $V_3 \to V_4$

$\Rightarrow \int d^6\tau \left(\frac{\partial}{\partial\theta}\right)^{14} \frac{(W\,DW\;W\,DW)}{(k_1 \cdot k_2)(k_3 \cdot k_4)} \sim \partial^2 F^4$

Result: $\mathcal{A}_{g=4} \to \partial^4 (\mathrm{Tr} F^2)^2,\ \partial^8 R^4,\ \partial^2 (\mathrm{Tr} F^4)$

Five-loop: $\mathcal{A}_{g=5} = \int d^{12}\tau \left\langle \left(\int dz\, U\right)^4 \left(\int \mu b\right)^{12} \mathcal{N}_0 \mathcal{N}_R \right\rangle$

Need $\theta^{16} (p)^{80}\, r^{11}\, (s)^{55}$. $\mathcal{N}_0 \to (ds)^{55} (r\theta)^p$

b ghosts can provide $r^{12} d^{24} \Rightarrow$ <u>do</u> need $\mathcal{N}_R$

$\left(\int \mu b\right)^{12} \to (rdd)^{12}$, $\mathcal{N}_R \to (ds)^{[illegible]}$

$\mathcal{N}_0 \to (ds)^{55}$

Have 4 d non-zero modes. Can get inverse derivative factors for open and closed string amp's.

$$d_\alpha W_1^\alpha(y_1)\, d_\beta W_2^\beta(y_2) \to \gamma^m_{\alpha\beta}\, \Pi_m\, W_1^\alpha W_2^\beta\, (y_1-y_2)^{-1+k_1\cdot k_2}$$

$$\cancel{\left(\frac{\partial}{\partial\theta}\right)^{16}}\; d_\alpha W_3^\alpha(y_3)\, d_\beta W_4^\beta(y_4) \to \gamma^m_{\alpha\beta}\, \Pi_m\, W_3^\alpha W_4^\beta\, (y_3-y_4)^{-1+k_3\cdot k_4}$$

$$\int d^{12}\tau \left(\frac{\partial}{\partial\theta}\right)^{16} \frac{(W_1 W_2 W_3 W_4)}{k_1\cdot k_2\; k_3\cdot k_4} \to \partial^2 F^4$$

$$\left|\int d^{12}\tau \left(\frac{\partial}{\partial\theta}\right)^{16} \frac{(W_1 W_2 W_3 W_4)}{k_1\cdot k_2\; k_3\cdot k_4}\right|^2 \to \frac{\partial^{12} R^4}{k_1\cdot k_2\; k_3\cdot k_4} = \partial^8 R^4$$

$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$
$Tr(F^4)$	$\partial^2 Tr(F^4)$	$\partial^2 Tr(F^4)$	$\partial^2 Tr(F^4)$	$\ldots$	
$(Tr(F^2))^2$	$\partial^2 (Tr(F^2))^2$	$\partial^4 (Tr(F^2))^2$	$\partial^4 (Tr(F^2))^2$	$\ldots$	
R^4	$\partial^4 R^4$	$\partial^6 R^4$	$\partial^8 R^4$	$\partial^8 R^4$	$\ldots$

$$\left.\begin{matrix} \partial^2 Tr(F^4) \\ \partial^4 (Tr(F^2))^2 \\ \partial^8 R^4 \end{matrix}\right\} \text{D-terms}$$

$\int d^4x \int d^{16}\theta\, K_{tr(\phi\phi)}$

$\Rightarrow$ $N=4$ $D=8$ sugra has first divergence at 7-loops (agrees with analysis of Bossard, Howe, Stelle and Green, Russo, Vanhove)

$\int d^4x \int d^{16}\theta\, K^2$

$\int d^4x \int d^{32}\theta\, \Phi^4$

Vanishing of 0, 1, 2, 3-pt functions is easily proven at arbitrary loop by counting fermionic zero modes