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4d N = 2 theory vs. 2d CFT: Plans

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4d $\mathcal{N} = 2$ theory vs. 2d CFT: Plans

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Abstract

This is the summary of what I'll talk about. I'm sorry I haven't been able to provide a nice written-up lecture note in advance. Each bullet is supposed to take 10 minutes. That surely underestimates the time it takes, so I'll need to cut some materials.

1 General Introduction

- Q. Why I'm talking this mathematical stuffs in this year's spring school, which aims for applications of string theory.
- The aim of four lectures: you'll be able to calculate both sides of the equation
$$Z = \langle v|v\rangle \tag{1.1}$$
and check indeed they match. Here, Z is Nekrasov's instanton partition function of $\mathcal{N} = 2$ pure $SU(2)$ gauge theory, and $|v\rangle$ is a certain coherent state in the Verma module of the Virasoro algebra. Don't worry, I'll explain the jargons. You'll also understand why we'd expect such an equality.
- I'll give a very rough idea why there can be such an equality using M5-branes.

2 $\mathcal{N} = 2$ gauge theory basics

- $\mathcal{N} = 2$ multiplets, its Coulomb branch, gauge invariant vevs u and special coordinates/periods/central charges a
- Review of the most classic Seiberg-Witten solution, pure $SU(2)$.

- $SU(2)$ with fundamental flavors
- Pure $SU(N)$.
- How to extract the prepotential from the solution.
- The curve in a 'new' light: M5-branes.
- How you decompose M5-branes into pairs of pants
- How you study pants.

3 Instantons

- very basics about instantons. (non-susy)
- the reason why the instanton integral is hard.
- Nekrasov's way out: Ω-background
- susy QM and the equivariant index
- Explicit formula for Nekrasov's instanton partition function Z: pure case
- With hypermultiplets.

4 CFTs

- Virasoro algebra, Verma module, Kac matrix, etc.
- How to match the curve to the Virasoro algebra
- Coherent state of the Virasoro algebra. $\langle v|v\rangle$
- Conformal blocks: down-to-earth definition
- four-point on the sphere

5 Future Directions

- General N?
- Relation to string dualities