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Spring School on Superstring Theory and Related Topics

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Towards holographic duality for condensed matter
Lecture III

J.A. McGreevy

MIT (Massachusetts Institute of Technology)
Cambridge
USA

Towards
Physical Applications of Holographic Duality
Part 3

John McGreevy, MIT

Outline

1. Holographic duality with a view toward condensed matter
   [review: JM, 0909.0518]
   (almost done)
2. Gravity duals of non-relativistic QFTs

   [Son, 0804.3972
   Koushik Balasubramanian, JM, 0804.4053
   Herzog, Rangamani, Ross, 0807.1099
   Maldacena, Martelli, Tachikawa, 0807.1100
   Allan Adams, KB, JM, 0807.1111
   KB, JM, to appear]
3. Effective field theory and the Fermi surface
   [Polchinski, hep-th/9210046]
4. Non-Fermi liquids from non-holography
5. Non-Fermi liquids from holography

An example of a theory with a known gravity dual

$\mathcal{N} = 4$ SYM is a CFT, (a supersymmetric, relativistic gauge theory)
each of these red words is bad from our point of view.
The $\mathcal{N} = 4$ SYM action is schematically

$$\mathcal{L}_{\mathsf{SYM}} \sim \mathrm{tr} \left(F^2 + (D\Phi)^2 + i\bar{\Psi}\Gamma \cdot D\Psi + g^2[\Phi, \Phi]^2 + ig\bar{\Psi}[\Phi, \Psi] \right)$$

this gauge theory comes with 2 parameters:
a coupling constant $\lambda = g^2 N$ (with $\beta_\lambda \equiv 0$)
an integer, the number of colors N.

$$\boxed{\mathcal{N} = 4 \ \mathrm{SYM}_{N,\lambda}} = \boxed{\mathrm{IIB\ strings\ in\ } AdS_5 \times S^5 \mathrm{\ of\ size\ } \lambda, \hbar = 1/N}$$

[Maldacena 1997]

- large N makes gravity classical (suppresses splitting and joining of strings)
- strong coupling (large λ) makes the geometry big.

'IIB strings in ...' specifies a list of bulk fields and interactions.
$\exists$ *infinitely many* other examples of dual pairs [*e.g.* Hanany, Vegh et al...]

Remarks on the role of supersymmetry (susy)

- susy constrains the form of interactions.
  fewer candidates for dual.
- in susy theories, $\exists$ more coupling-independent quantities, hence $\exists$ more checks.
- susy allows *lines* of fixed points (*e.g.* $\mathcal{N} = 4$ SYM)
  coupling $=$ dimensionless parameter
- for these applications, susy is broken by finite T, μ, anyway. it's not clear what influence it has on the resulting states.

Remarks on the role of string theory

1. What are consistent ways to UV complete our gravity model?
   - So far, no known constraints that aren't visible from EFT.
   - Suggests interesting resummations of higher-derivative terms, protected by stringy symmetries.
     e.g. the DBI action $L_{DBI} \sim \sqrt{1 - F^2}$ is 'natural' in string theory because its form is protected by the T-dual Lorentz invariance.
2. What is a microscopic description of the dual QFT?
   - Such a description is crucial for the detailed checks that make us believe the duality.
   - A weak coupling limit needn't exist (isolated fixed points are generic).
   - A Lagrangian description needn't exist
     (*e.g.* minimal models) gravity plus matter in AdS provides a much more direct construction of CFT.
   - Honesty: Any L_{micro} that we would get from string theory is so far from $L_{Hubbard}$ anyway that it isn't clear how it helps.

Lessons for how to use AdS/CFT to do physics

- critical exponents depend on ‘landscape issues’
  (parameters in bulk action)
- thermodynamics is not so different between weak and strong coupling
  (in examples: $\mathcal{N} = 4$ SYM, lattice QCD)
- transport is very different
  transport by weakly-interacting quasiparticles is less effective

$$\left(\frac{\eta}{s}\right)_{\rm weak} \sim \frac{1}{g^4 \ln g} \quad \ll \quad \left(\frac{\eta}{s}\right)_{\rm strong} \sim \frac{1}{4\pi}.$$

Galilean CFT Liquid from Holography

(towards cold atoms at unitarity)

Motivation

Some relativistic CFTs have an effective description at strong coupling in terms of gravity (more generally strings) in AdS space.

It would be great if we had a gravity dual for a system which can be created in a laboratory.

(solutions of strong-coupling problems, quantum gravity experiments)

Some laboratory systems have critical points described by relativistic CFTs.

– QCD a little above T_c acts like a CFT

– some quantum-critical condensed matter systems have emergent lightcones

Alternative approach (later): ask questions which don't care about the short-distance symmetries.

More precisely

- It would be great if we had a gravity dual for a real system

  which lives longer than a fermi/c,

  and which can be created in a laboratory more convenient than RHIC.

- Piles of atoms have a rest frame.

(even if present, lightcone need not be shared by different degrees of freedom.)

So, in searching for experiments with which string theory has some interface, it's worth noting that:

non-relativistic CFTs exist.

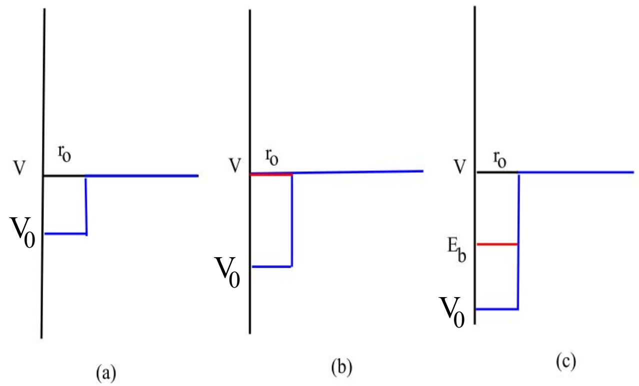
Cold atoms at unitarity

Most of the work on AdS/CFT involves relativistic CFTs.

Strongly-coupled Galilean-invariant CFTs exist, even experimentally.

[Zwierlein et al, Hulet et al, Thomas et al]

Consider nonrelativistic fermionic particles ('atoms') interacting via a short-range attractive two-body potential $V(r)$, *e.g.*:



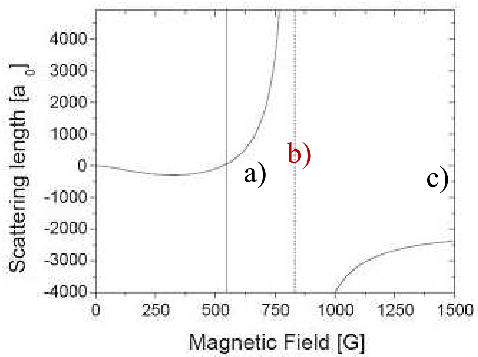
Case (b): σ saturates bound on scattering cross section from unitarity

Range of interactions $\to 0$, scattering length $\to \infty \implies$ no scale.

Lithium atoms

have a boundstate with a different magnetic moment.

Zeeman effect $\implies$ scattering length can be controlled using an external magnetic field:



Strongly-coupled NRCFT

The fixed-point theory ("fermions at unitarity") is a strongly-coupled nonrelativistic CFT ('Schrödinger symmetry') [Nishida-Son].

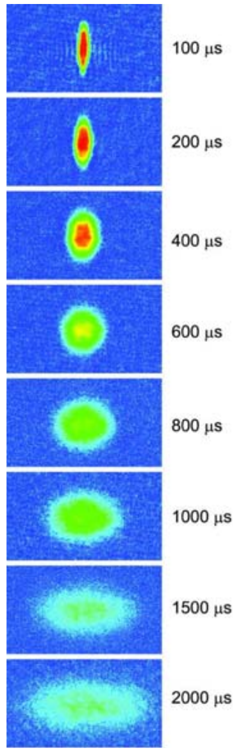
universality: it also describes neutron-neutron scattering [Mehen-Stewart-Wise]

Two-body physics is completely solved.

Many body physics is mysterious.

Experiments: very low viscosity, $\frac{\eta}{s} \sim \frac{5}{4\pi}$ [Thomas, Schafer]

$\longrightarrow$ strongly coupled.



AdS/CFT?

Clearly we can't approximate it as a *relativistic* CFT.

Different hydro: conserved particle number.

A holographic description?

Method of the missing box

AdS : relativistic CFT

$\boxed{?}$: galilean-invariant CFT

Note restriction to Gal.-invariance $\partial_t - \vec{\nabla}^2$

distinct from: Lifshitz-like fixed points $\partial_t^2 - (\vec{\nabla}^2)^2$

are not relativistic, but have antiparticles.

gravity duals of those: S. Kachru, X. Liu, M. Mulligan, 0808.1725

before guessing what's in the box, more about this symmetry and its realizations

Galilean scale invariance

$i, j = 1...d$ spatial dims (sorry for the notation change)

Symmetries of free schödinger equation $i\partial_t\psi = \partial_x^2\psi$

Galilean symmetry:

translations P_i, rotations M_{ij}, time translations H,
Galilean boosts K_i, number or mass operator N:

$[K_i, P_j] = \delta_{ij} iN$ (in 'non-relativistic natural units': $\hbar = M = 1$)

---

dilatations D: $[D, P] = -iP$ (D measures length dimensions)

$[D, H] = -izH$ ($z \equiv$ dynamical exponent: $x \to \lambda x$, $t \to \lambda^z t$)

closure of algebra $\longrightarrow$ $[D, K] = i(z-1)K$, $[D, N] = i(z-2)N$.

Schrödinger symmetry:

In the special case $z = 2$, there is an additional conformal generator, $C = ITI$

$$[M_{ij}, C] = 0, \quad [K_i, C] = 0, \quad [D, C] = -2iC, \quad [H, C] = -iD.$$

comments

- there's only *one* special conformal symmetry, not $d+1$ like in relativistic case.
- we're using 'non-relativistic natural units' where $\hbar = M = 1$, so $\hat{N}$ measures particle number or mass.
- this 'schrödinger' algebra $\subset SO(d+1,2)$
  (the relativistic conformal group)
- [Nishida-Son] irreps of Schrod ($z=2$) labelled by $\Delta_0, N_0 \equiv \ell$.
- [Tachikawa] unitarity bound: $\Delta \geq \frac{d}{2}$ (independent of spin.)

QFT realization

free fermions (or free bosons): $S_0 = \int dt d^d x \left(\psi^\dagger i\partial_t \psi + \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi \right)$

$$n(\vec{x}) \equiv \psi^\dagger \psi, \quad \vec{j}(\vec{x}) \equiv -\frac{i}{2}\left(\psi^\dagger \vec{\nabla}\psi - \vec{\nabla}\psi^\dagger \psi \right)$$

$$N = \int d^d x \; n(\vec{x}), \; P_i = \int j_i(\vec{x}), \; M_{ij} = \int (x_i j_j(\vec{x}) - x_j j_i(\vec{x}))$$

$$K_i = \int x_i n(\vec{x}), \; D = \int x_i j_i(\vec{x}), \; C = \int \frac{x^2 n(\vec{x})}{2}$$

satisfy all the commutation relations not involving the Hamiltonian.
With $H_0 = \int d\vec{x} \frac{1}{2} \nabla_i \psi^\dagger \nabla_i \psi$, ψ saturates unitarity bound.
towards interacting NRCFT:

$$\Delta S = \frac{1}{2} \int dt \int d\vec{x} d\vec{y} \psi^\dagger(\vec{x}) \psi^\dagger(\vec{y}) \underbrace{V(|\vec{x} - \vec{y}|)}_{\equiv V(r)} \psi(\vec{y}) \psi(\vec{x})$$

geometric realization

A metric whose isometry group is the schrödinger group:

$$L^{-2} ds^2_{\mathrm{Schr}^z_d} = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - 2\beta^2 \frac{dt^2}{r^{2z}}$$

$\boxed{?}$ = 'schrödinger space' [Son; Balasubramanian, JM]

Compare to AdS in light-cone coordinates:

$$ds^2_{AdS_{d+3}} = \frac{-d\tau^2 + dy^2 + d\vec{x}^2 + dr^2}{r^2}$$

$$= \frac{2d\xi dt + \vec{dx}^2 + dr^2}{r^2}$$

without the β^2 term, ∂_t is lightlike.

comments

1. only $z = 2$ has conformal symmetry.
2. if $\xi \in \mathbb{R}$, we can scale away $2\beta^2$ by (remnant of boost) $\begin{cases} t \mapsto \frac{t}{\sqrt{2}\beta} \\ \xi \mapsto \sqrt{2}\beta\xi \end{cases}$

   but discrete spectrum requires compact $\xi \simeq \xi + L_\xi$

   $\frac{\beta}{L_\xi}$ is an invariant parameter $\sim M$.
3. dual to *vacuum* of a gal. inv't field theory (no antiparticles!).

   the ξ-circle is *null*. (light winding modes?)

   (this is the phase of the wavefunction of a state with no particles!)

   at finite temperature or density, not so.
4. all curvature scalars are constant.
5. this spacetime is conformal to a pp-wave.

   conformal boundary is one-dimensional. [Hubeny-Rangamani]

   Nevertheless, we will compute correlators of a CFT with d spatial dims.

What holds it up?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} - \delta^t_\mu \delta^t_\nu g_{tt} \mathcal{E}$$

$\Lambda = -\frac{(d+1)(d+2)}{2L^2}$: CC $\mathcal{E}$: a constant energy density ('dust')

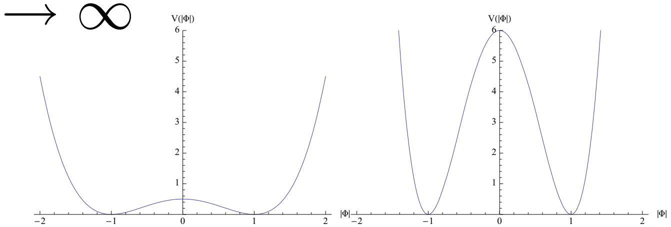
A realization of the dust: metric is sourced by *e.g.* the ground state of an Abelian Higgs model in its broken phase.

$$S = \int d^{d+3}x \sqrt{g} \left(-\frac{1}{4} F^2 + \frac{1}{2} |D\Phi|^2 - V\left(|\Phi|^2\right) \right)$$

with $D_a\Phi \equiv (\partial_a + ieA_a)\Phi$, with a Mexican-hat potential

$$V\left(|\Phi|^2\right) = g \left(|\Phi|^2 - \frac{z(z+d)}{e^2} \right)^2 + \Lambda$$

extreme type II limit : $g \to \infty \Longrightarrow m_h^2 \to \infty$



$$L_{bulk} = -\frac{1}{4} F^2 - \frac{m^2}{2} A^2 - \Lambda, \qquad m^2 = z(z+d)$$

Holographic dictionary

Basic entry: bulk fields $\leftrightarrow$ operators in dual QFT
Irreps of schrod labelled by $\Delta, \hat{N} = \ell$, so we work at fixed ξ-momentum, ℓ: $\phi(r,t,\vec{x},\xi) = f_{\omega,k,\ell}(r)e^{i(\ell\xi - \omega t + \vec{k}\cdot\vec{x})} \leftrightarrow \mathcal{O}_{\ell,\Delta}(\omega,\vec{k})$ scalar operator.
Consider a probe scalar field:
$S[\phi] = -\int d^{d+1}x\sqrt{g}\left((\partial\phi)^2 + m^2\phi^2\right).$
or: δg^x_y also satisfies this equation
Scalar wave equation in this background:

$$\left(-r^{d+3}\partial_r\left(\frac{1}{r^{d+1}}\partial_r\right) + r^2(2l\omega + \vec{k}^2) + r^{4-2z}l^2 + m^2\right) f_{\omega,\vec{k},l}(r) = 0.$$

For $z \leq 2$, the behavior of the solution near the boundary ($r \sim 0$) is:

$$f \propto r^{\Delta}, \qquad \Delta_\pm = 1 + \frac{d}{2} \pm \sqrt{\left(1+\frac{d}{2}\right)^2 + m^2 + \delta_{z,2}l^2}.$$

For $z > 2$, not power law. (??!)

some basic checks (focus on $z = 2$)

1) $\Delta_+ + \Delta_- = d + 2$ matches dimensional analysis on

$$S_{bdy} \ni \int dt d^d x \, \phi_0 \mathcal{O}$$

(ϕ_0 is the source for $\mathcal{O}$)

$$[x] = -1, [t] = -2, [\phi_0] = \Delta_-, [\mathcal{O}] = \Delta_+.$$

2) unitarity bound $\Delta \geq \frac{d}{2}$ matches requirement on m to prevent bulk tachyon instability (analog of BF-bound).

3) the correlators are of the expected form

$$\rightarrow \langle \mathcal{O}_1(x,t) \mathcal{O}_2(0,0) \rangle \propto \delta_{\Delta_1, \Delta_2} \theta(t) \frac{1}{|\epsilon^2 t|^\Delta} e^{-iMx^2/2|t|}$$

consistent with (determined by [Nishida-Son]) NR conformal Ward ids.

But: the vacuum of a galilean-invariant field theory is extremely boring: no antiparticles! no stuff!

How to add stuff?

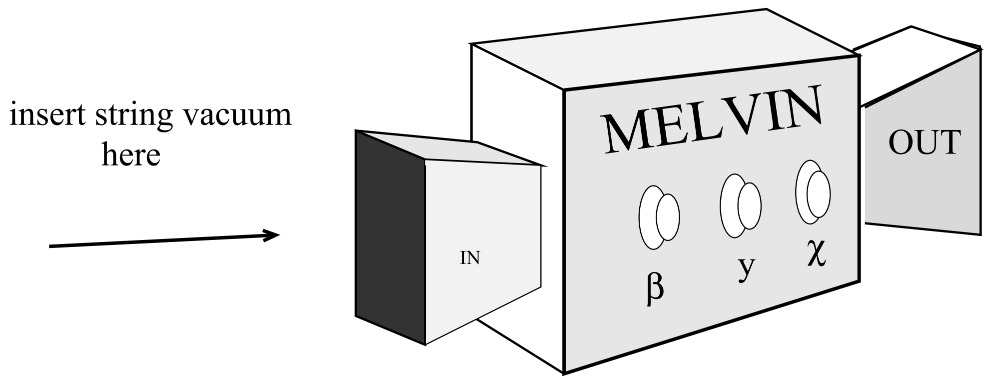
A holographic description of more than zero atoms?

A black hole (BH) in schrödinger spacetime.

[A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was extremely useful:

A solution-generating machine named Melvin [Ganor et al]



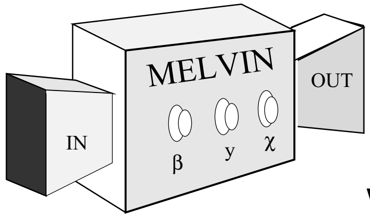
IN: $AdS_5 \times S^5$ OUT: schrodinger $\times S^5$

→ A hint about *which* NRCFTs we are describing:

we can also feed $\mathcal{N} = 4$ SYM to Melvin.

IN: AdS_5 BH $\times S^5$ OUT: schrodinger BH $\times$ squashed S^5

"Null Melvin Twist"

is a machine 

which generates new type II SUGRA solutions from old Ganor et al, Gimon et al. (with different asymptotics)

Previous work: dials set to 'highly non-commutative'.

Choose two killing vectors $(\partial_y, \partial_\chi)$ and:

1. Boost along y with boost parameter γ
2. T-dualize along y.
3. Twist: replace $\chi \to \chi + \alpha y$, α constant
4. T-dualize back along y
5. Boost back by $-\gamma$ along y
6. Scaling limit: $\gamma \to \infty$, $\alpha \to 0$ keeping $\beta = \frac{1}{2}\alpha e^{\gamma}$ fixed.

Schrödinger spacetime in string theory

Input solution of type IIB supergravity: $AdS_5 \times S_5$

$$ds^2 = \frac{-d\tau^2 + dy^2 + d\vec{x}^2 + dr^2}{r^2} + ds^2_{S_5} \qquad \vec{x} \equiv (x^1, x^2).$$

$ds^2_{S_5} = ds^2_{\mathbb{P}^2} + \eta^2$. $\eta \equiv d\chi + \mathcal{A} =$ vertical one-form of Hopf fibration

Feeding this to the melvinizer gives:

$$ds^2 = \frac{1}{r^2}\left(-\left(1 + \frac{\beta^2}{r^2}\right) d\tau^2 + \left(1 - \frac{\beta^2}{r^2}\right) dy^2 + 2\frac{\beta^2}{r^2} d\tau dy + d\vec{x}^2 + dr^2 \right) + ds^2_{S_5}$$

Defining $\xi \equiv \frac{1}{2\beta}(y - \tau)$, $t \equiv \beta(\tau + y)$, and reducing on the 5-sphere:

$$\longrightarrow ds^2 = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - \frac{dt^2}{r^4} \qquad (\mathrm{Schr}^{z=2}_{d=2})$$

The ten-dimensional metric is sourced by

$$B = \beta r^{-2} \eta \wedge (d\tau + dy), \;\; F_5 = (1 + \star)\mathrm{Vol}(S^5) \;\xrightarrow{5d}\; A = r^{-2} dt, \;\; m^2 = 8, \;\; \Lambda.$$

– No higgs field, alas.

– This can be done for $S^5 \to$ any Sasaki-Einstein 5-manifold. – It works similarly for the AdS BH, but then the sphere gets squashed.

Thermodynamics

BH is saddle point of $Z = \mathrm{tr}\ e^{-\frac{1}{T}(H-\mu N)} = \mathrm{tr}\ e^{-\frac{1}{T}\left(i\partial_\tau - \mu i \partial_\xi\right)}$

Temperature & Chemical Potential: euclidean regularity requires

$$it \simeq it + \tfrac{n}{T}, \xi \simeq \xi + L_\xi \mu n \quad \Longrightarrow \quad T = \frac{\kappa}{2\pi} = \frac{1}{\pi \beta r_H}, \mu = -\frac{1}{2\beta^2}$$

note: $\mu < 0$!

We got finite density for free. Which is good because $S_{BH} \neq 0$, but no antiparticles.

Entropy: $S_{BH} = \dfrac{1}{4G_N}\dfrac{L_y}{r_H^3} = VL_\xi \dfrac{\pi^2 N^2 T^3}{16\mu^2}$

Free energy : $F = S_{\mathrm{onshell}} T = VL_\xi \dfrac{\pi^2 N^2 T^4}{32\mu^2}$

S_{onshell} is renormalized by adding local boundary counterterms

fancy reason: makes the variational problem well defined

Mystery: we are forced to add *extrinsic* boundary terms for the massive gauge field: $S_{\mathrm{bdy}} \ni \int n^\mu A_\nu F^{\mu\nu}$

The required coefficient is exactly the one that changes the boundary conditions on A_μ from Dirichlet to Neumann.

Boundary stress tensor

$$S_{\rm bdy} = \int \sqrt{\gamma}\left(\Theta + c_0 + c_1\Phi + c_2\Phi^2 + n^\mu A^\mu F_{\mu\nu}\left(c_5 + c_6\Phi\right)\right)$$

Vary metric at boundary:

$$T^\mu_\nu = -\frac{2}{\sqrt{\gamma}}\frac{\delta S_{\rm onshell}}{\delta\gamma^\nu_\mu} = \Theta^\mu_\nu - \delta^\mu_\nu\Theta - {\rm c.t.}|_{bdy} \qquad \Theta = \text{extrinsic curvature}$$

Fix counterterm coeffs w/

–Ward identity: $2E = dP$ = residual bulk gauge symmetries

–first law of thermodynamics: $(E + P = TS + \mu N)$

$$\longrightarrow \quad \mathcal{E} = \frac{E}{V} = -\int \sqrt{\gamma}\, T^t_t = \frac{1}{16\pi G r_H^4} = \frac{\pi^2 N^2}{64} L_\xi \frac{T^4}{\mu^2}$$

Who is T^ξ_t? Just as T^χ_μ is the R-charge current,

Density: $$\rho = \int \sqrt{\gamma}\, T^\xi_t = \frac{\beta^2}{16\pi G r_H^4} = \frac{\pi^2 N^2 T^4}{32\mu^3} L_\xi$$

Note: $T^\xi_\xi, T^t_\xi = \infty$ with naive falloffs on $\delta_{\mu\nu}$. We don't care about these anyway.

Results so far

This black hole gives the thermo and hydro of some NRCFT

('dipole theory' [Ganor et al]).

$$\text{Einstein gravity} \overset{[Iqbal-Liu]}{\Longrightarrow} \quad \frac{\eta}{s} = \frac{1}{4\pi}.$$

Satisfies laws of thermodynamics, correct scaling laws, correct kubo relations.

[Rangamani-Ross-Son, McEntee-JM-Nickel]

But it's a very different class from unitary fermions:

$$F \sim \frac{T^4}{\mu^2}, \quad \mu < 0$$

(note: scaling symmetry $\implies F \sim T^{\frac{D+2}{2}} g(T/\mu)$)

Q: why is $g(x) = x^2$?

A [MMTv5]: a) if solution arises from DLCQ, an extra (boost) symmetry: $t \to \alpha t, \xi \to \alpha^{-1}\xi \quad \implies T \to \frac{T}{\alpha}, \mu \to \frac{\mu}{\alpha^2}, F \to F \implies F(T,\mu) = g\left(\frac{\mu}{T^2}\right)$

b) melvin twist doesn't change planar amplitudes

(bulk explanation: symmetry of tree-level string theory

boundary explanation: 'non-commutative phases' cancel)

New gravity realizations of Schrod

This is *not* a necessary consequence of $\exists$ gravity dual.

[K Balasubramanian, JM, to appear]

Unnecessary assumption: All of Schrod must be realized geometrically.

We now know how to remove this assumption, can find more realistic models.

Gravity solutions with a ξ dimension are like DLCQ of rel CFT: periodically identify $x^+ \equiv x + t$. Clear from *e.g.* [Barbon-Fuertes]

(Schrod$_D$ is the subgroup of $SO(d+1,2)$ which is preserved.)

We thought the ξ direction was required since $[K, P] = iN$

LHS must be realized geometrically.

This action has solutions with $Schrod_d$ asymptotics:

$$S_{d+2} = \int d^d x dt dr \sqrt{g} \left[R - 2\Lambda e^{-\sigma} - \frac{e^{3\sigma}}{4} (dB)^2 - \frac{3}{2} (\partial\sigma)^2 \right]$$

$$ds^2 = e^{\sigma} \left(-Q \frac{dt^2}{r^6} + \frac{d\vec{x}^2}{r^2} + \frac{dr^2}{r^2} \right), \quad B = Q \frac{f dt}{r^4}, \quad e^{2\sigma} = \frac{r^2}{Q}$$

Realization of symmetries

Symmetry generators of the lower dimensional theory realize Schröd:

- Particle Number:

$$B \to B + d\lambda, \quad \Phi \to e^{i\ell\lambda}\Phi$$

(we take Φ to vanish in the solution shown above.) ℓ is the mass of the associated particle.

- Translations and rotations are realized as-usual by isometries.
- Galilean Boosts act by:

$$K^i = -t\partial_i + \text{Gauge shift}$$

where the gauge transformation parameter is $\lambda = \frac{1}{2}v^2 t + \vec{v}\cdot\vec{x}$.

$$t \to t, \quad \vec{x} \to \vec{x} - \vec{v}t, \quad \varphi \to \varphi + \ell\left(\frac{1}{2}v^2 t + \vec{v}\cdot\vec{x}\right),$$

where $\Phi \equiv e^{i\varphi}|\Phi|$ Role of ξ played by φ.

- Scale symmetry acts by

$$D = -2t\partial_t - x^i\partial_i - r\partial_r + \text{shift in } \sigma;$$

wave equation:

$$\left(-\omega^2 r^6 + m^2 + r^2(2\ell\omega + k^2)\right)\Phi - r^{d+3}\partial_r\left(r^{-d-1}\partial_r\Phi\right) = 0 \qquad (\star)$$

$(\star)$ is the eom from:

$$S_{\rm probe}[\Phi] = \int \sqrt{g}\left[|(\partial - i\ell B)\Phi|^2 - \left(\ell^2 e^{-3\sigma} + m^2 e^{-\sigma}\right)|\Phi|^2\right] \quad .$$

coupling to scalar σ req'd to realize Schröd.

First term in $(\star)$ is unimportant for the boundary behavior ($r \to 0$), but does spoil the Schrödinger invariance of the equation.

Note: we can't find a solution which preserves all of schrod

(recall our surprise at finding a vacuum solution earlier)

But, black hole solution:

$$ds^2 = e^{\sigma}\left(-Qf\frac{dt^2}{r^6} + \frac{d\vec{x}^2}{r^2} + \frac{dr^2}{r^2 f}\right), \quad B = Q\frac{fdt}{r^4}, \quad e^{2\sigma} = \frac{r^2}{Q}$$

where $f = 1 - r^4/r_H^4$

Same thermo as before (obtained by dim'l reduction and scaling).

Another system which realizes schrod

$$S_4^E = \int d^4x(-g_4)^{1/2}\Big[R_4 - 2\Lambda e^{-\sigma} - \frac{e^{3\sigma}}{4}(dB)^2 - \frac{3}{2}(\partial\sigma)^2 - \frac{1}{2}(\partial\Psi)^2\Big]$$

A solution with asymptotic *Sch* symmetry:

$$ds_E^2\left(\widehat{Sch}\right) = e^{\sigma}\left(-QK_x^2\frac{dt^2}{r^6} + K_x\frac{d\vec{x}^2}{r^2} + \frac{dr^2}{r^2}\right)$$

$$B = Q\frac{(1 - r^4/r_0^4)dt}{r^4}, \quad e^{2\sigma} = \frac{r^2}{Q}, \quad e^{2\Psi/\sqrt{5}} = \frac{1 - r^4/r_0^4}{1 + r^4/r_0^4}$$

$$K_x^2 = 1 - r^8/r_0^8.$$

Geometry ends at $r = r_0$ with a curvature singularity.

related solutions: [Gubser-Rocha, Goldstein-Kachru-Prakash-Trivedi]

This curvature singularity at $r = r_0$ can be resolved by oxidation!

Lift to ten and eleven dimensions

The action S_4^E is a consistent truncation of

$$S_5^E = \int d^5x(-g_5)^{1/2}\Big[R_5 - 2\Lambda - \frac{1}{2}(\partial\Psi)^2\Big]$$

which is a consistent truncation of type IIB supergravity [MMT].
Lift to 10d:

$$ds^2_{10} = ds^2_E\left(\widehat{Sch}\right) + e^{2\sigma}\left(d\xi + B\right)^2 + ds^2\left(S^5\right),$$
$$F_5 = Q\left(\Omega_5 + \star\Omega_5\right), \text{ and} \tag{1}$$
$$e^{2\Phi} = e^{2\Psi}$$

is still singular at $r = r_0$.
T-dualize on the Hopf dir [Duff-Pope] and lift to 11d SUGRA:

$$ds^2_{11} = e^{-\Psi/2}\Big[ds^2_E\left(\widehat{Sch}\right) + e^{2\sigma}\left(d\xi + B\right)^2 + ds^2\left(\mathbb{CP}^2\right) + d\chi_1^2\Big] + e^{4\Psi/3}d\chi_2^2$$

and $G_4 = ...$

Consequences of lift

The important point: the 11d geometry ends smoothly at $r = r_0$.

(like the geometries describing confining gauge theories.) [KS, MN]

This determines the boundary conditions on fields

(like origin of polar coords.)

The solution has non-zero energy, pressure, density and free energy, but has zero entropy (no horizon).

Real boundary conditions.

for example on B_μ, which computes current correlators.

This is a "Mott" "insulator":

$\rho \neq 0$ but there is a gap to charge excitations

("Mott": there are strong interactions,

and it's not a band insulator or an Anderson insulator)

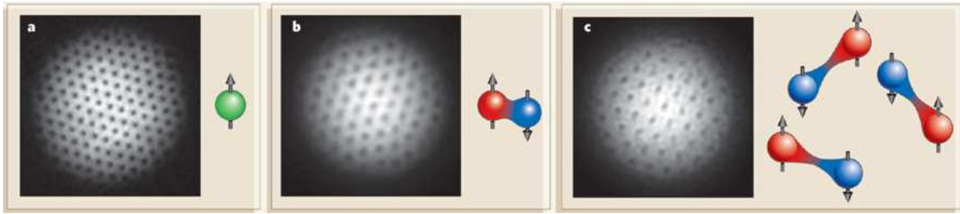
But: translation invariance $\Longrightarrow \sigma(\Omega) \propto \delta(\Omega)$.

As specified, it's a perfect conductor.

Conjecture: if we pinned down the center-of-mass mode, it would be an insulator.

Some future questions

- How close can we get to unitary fermions with a gravity dual?
- Can we realize the superfluid phase?



Should break ξ-isometry, cut off IR geometry.