Fundamentals of Satellite Navigation

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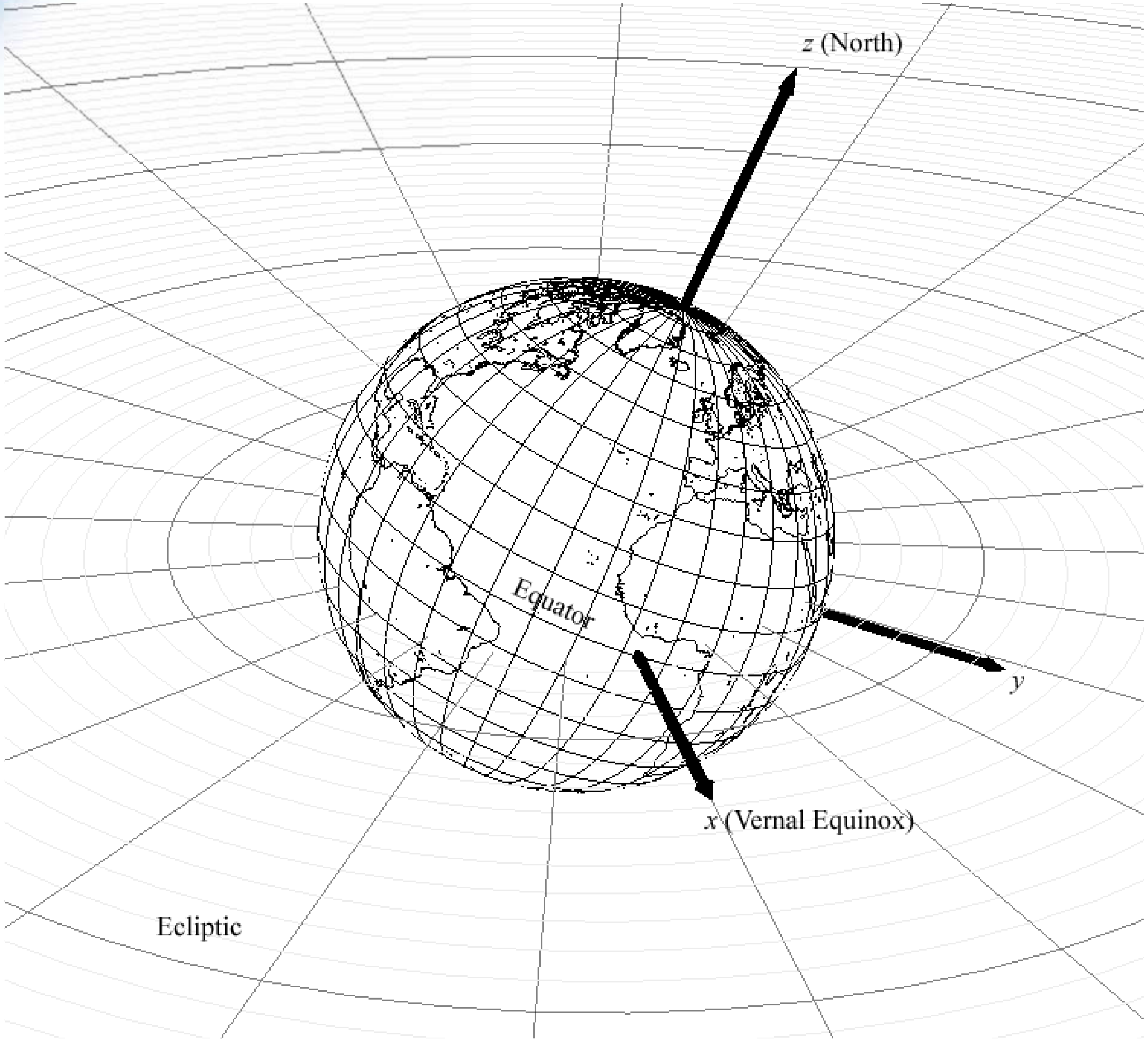
Fundamentals of Satellite Navigation

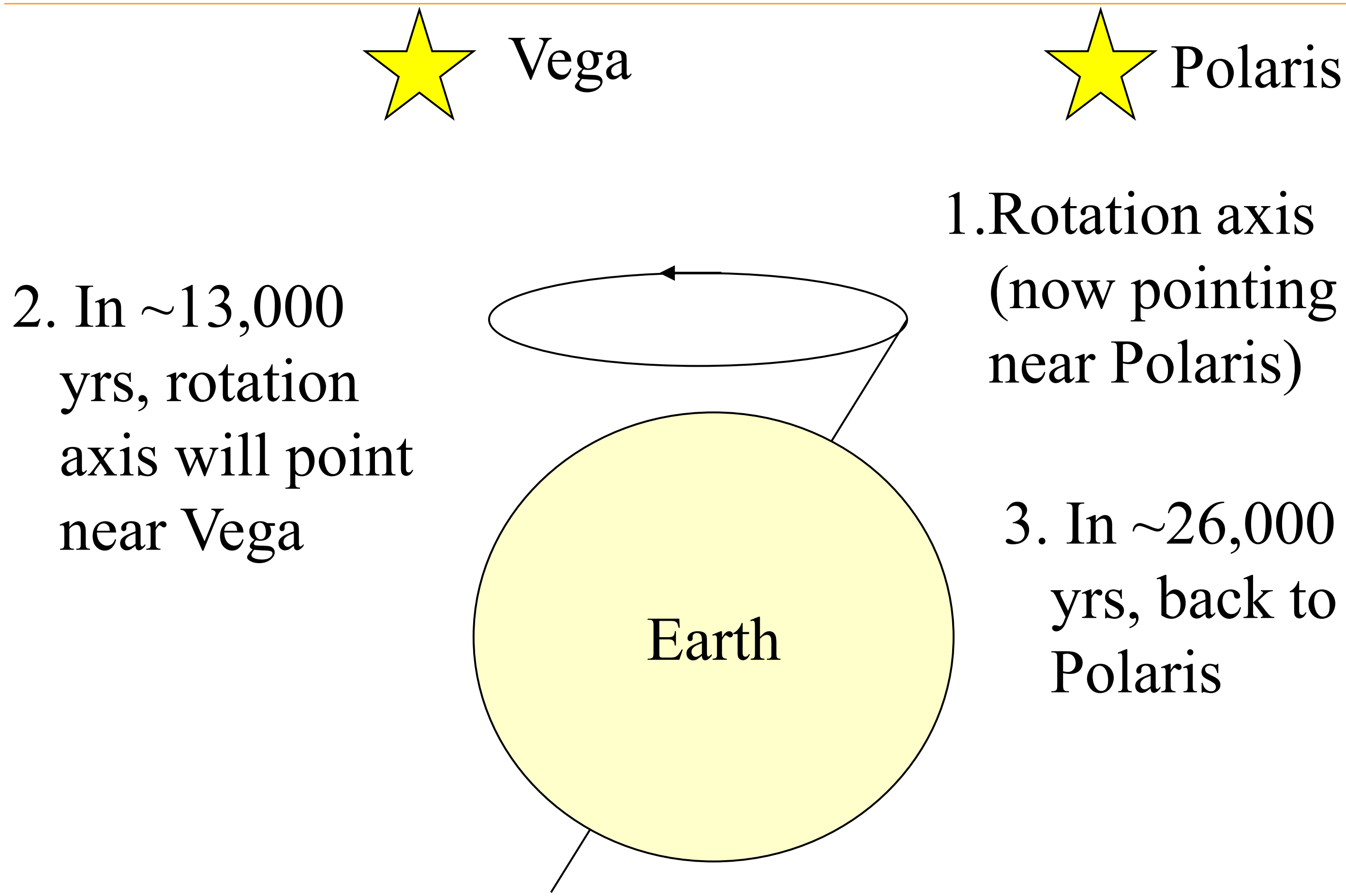
- Geodesy
- Time and clocks
- Satellite orbits
- Positioning

Earth Centered Inertial (ECI) Coordinate System



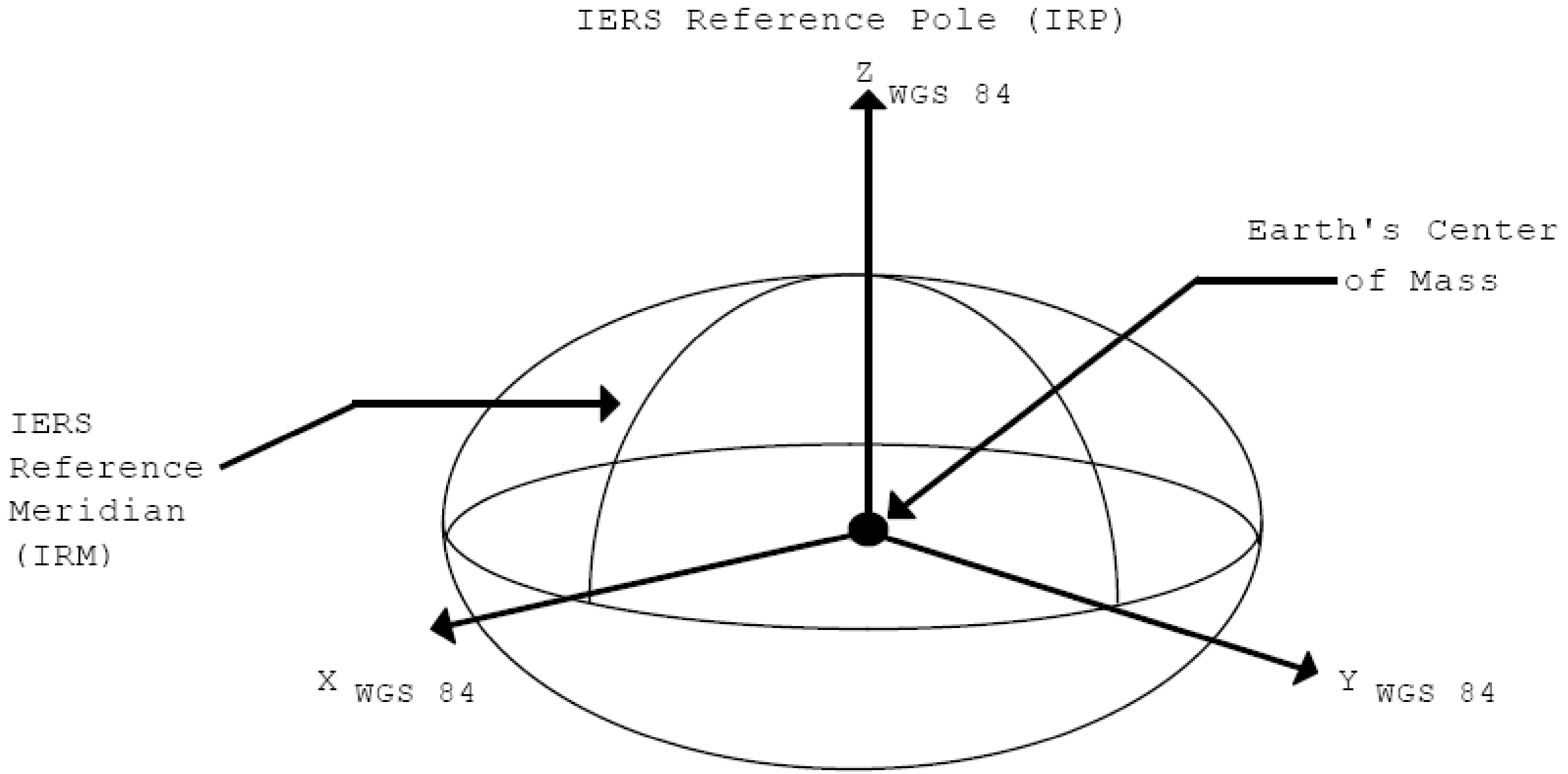
- **Oblateness of the Earth causes direction of axes to move over time**
- **So that coordinate system is truly "inertial" (fixed with respect to stars), it is necessary to fix coordinates**
- **J2000 system fixes coordinates at 1200 hours UTC on January 1, 2000**

Precession and Nutation



- **Precession is the large (23.5 deg half-angle) periodic motion**
- **Nutation is a superimposed oscillation (~9 arcsec max)**

Earth Centered Earth Fixed (ECEF) Coordinate System



Notes: (1) By convention, the z-axis is the mean location of the north pole (spin axis of Earth) for 1900 – 1905, (2) the x-axis passes through 0º longitude, (3) the Earth's crust moves slowly with respect to this coordinate system!

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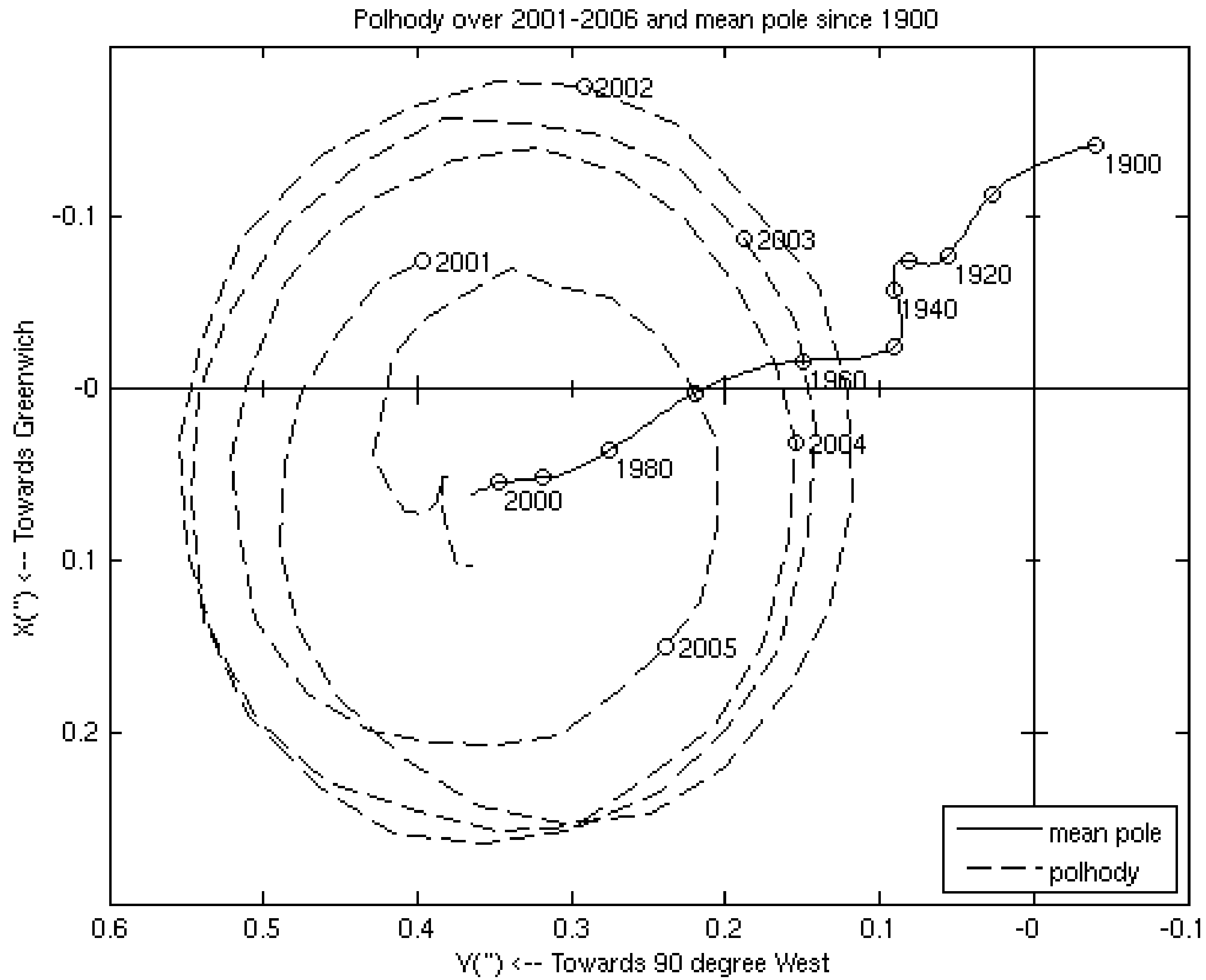
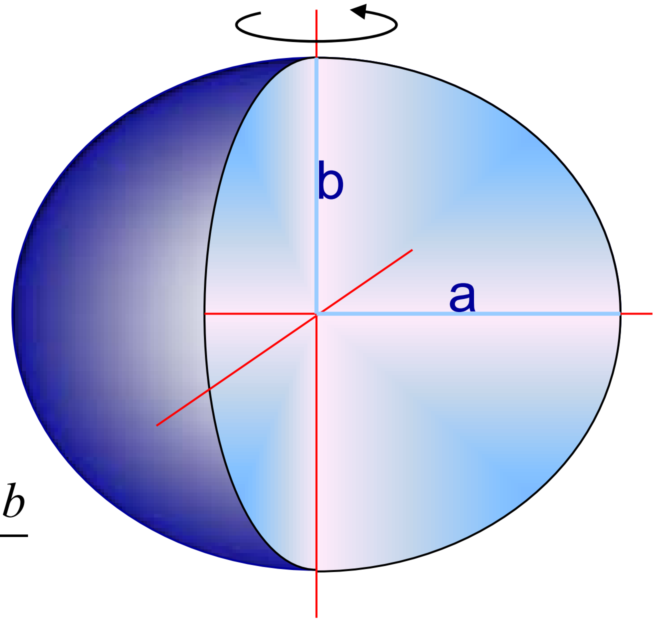
Polar Motion



Source: www.iers.org

Ellipsoid

- **Approximation to Earth's shape**
  - **Flattened at poles**
- **World Geodetic System (WGS)-84 values:**
  - **Semimajor axis, a = 6378137.0 m**
  - **1/f = 298.257223563**

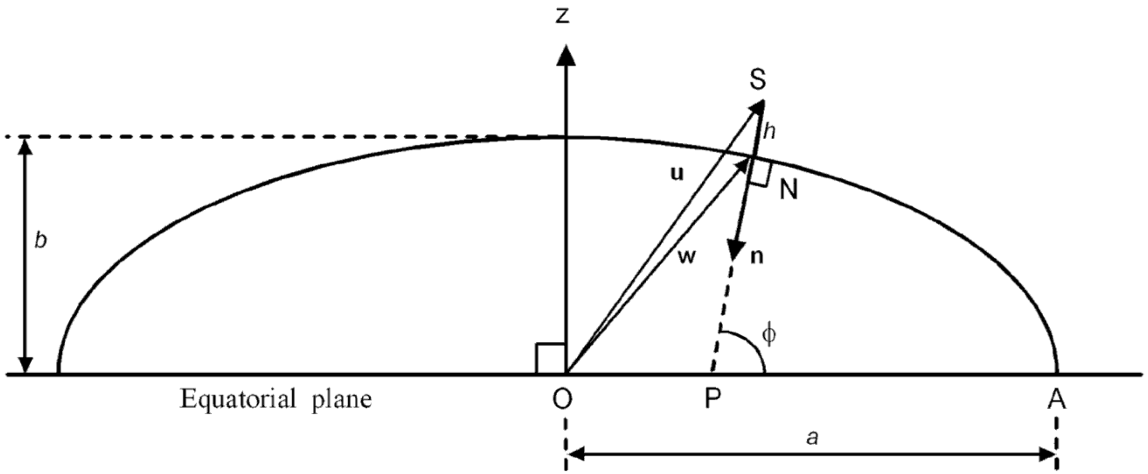


$$f = \frac{a-b}{a}$$

Latitude, Longitude, and Height

Longitude computation

$$
\lambda = \begin{cases} \arctan\left(\dfrac{y_u}{x_u}\right), & x_u \geq 0 \\ 180° + \arctan\left(\dfrac{y_u}{x_u}\right), & x_u < 0 \text{ and } y_u \geq 0 \\ -180° + \arctan\left(\dfrac{y_u}{x_u}\right), & x_u < 0 \text{ and } y_u < 0 \end{cases}
$$



Latitude and height computation

$$p = \sqrt{x^2 + y^2}$$

$$\tan u = \left(\frac{z}{p}\right)\left(\frac{a}{b}\right)$$

Iteration Loop

$$\cos^2 u = \frac{1}{1 + \tan^2 u}$$

$$\sin^2 u = 1 - \cos^2 u$$

$$\tan \varphi = \frac{z + e'^2 b \sin^3 u}{p - e^2 a \cos^3 u}$$

$$\tan u = \left(\frac{b}{a}\right)\tan \varphi$$

until $\tan u$ converges, then

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

$$h = \frac{p}{\cos \phi} - N \qquad \varphi \neq \pm 90°$$

otherwise

$$h = \frac{z}{\sin \phi} - N + e^2 N \qquad \varphi \neq 0$$

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Latitude, Longitude, and Height to ECEF

$$\begin{bmatrix} x_{ecef} \\ y_{ecef} \\ z_{ecef} \end{bmatrix} = \begin{bmatrix} \dfrac{a\cos\lambda}{\sqrt{1+(1-e^2)\tan^2\phi}} + h\cos\lambda\cos\phi \\ \dfrac{a\sin\lambda}{\sqrt{1+(1-e^2)\tan^2\phi}} + h\sin\lambda\cos\phi \\ \dfrac{a(1-e^2)\sin\phi}{\sqrt{1-e^2\sin^2\phi}} + h\sin\phi \end{bmatrix}$$

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Geoid

- Equipotential surface of Earth's gravity field that fits mean sea level on average globally
- Common representations:
  - Spherical harmonic coefficients
  - Grid of values over Earth
- Heights measured relative to geoid are referred to as *orthometric heights*

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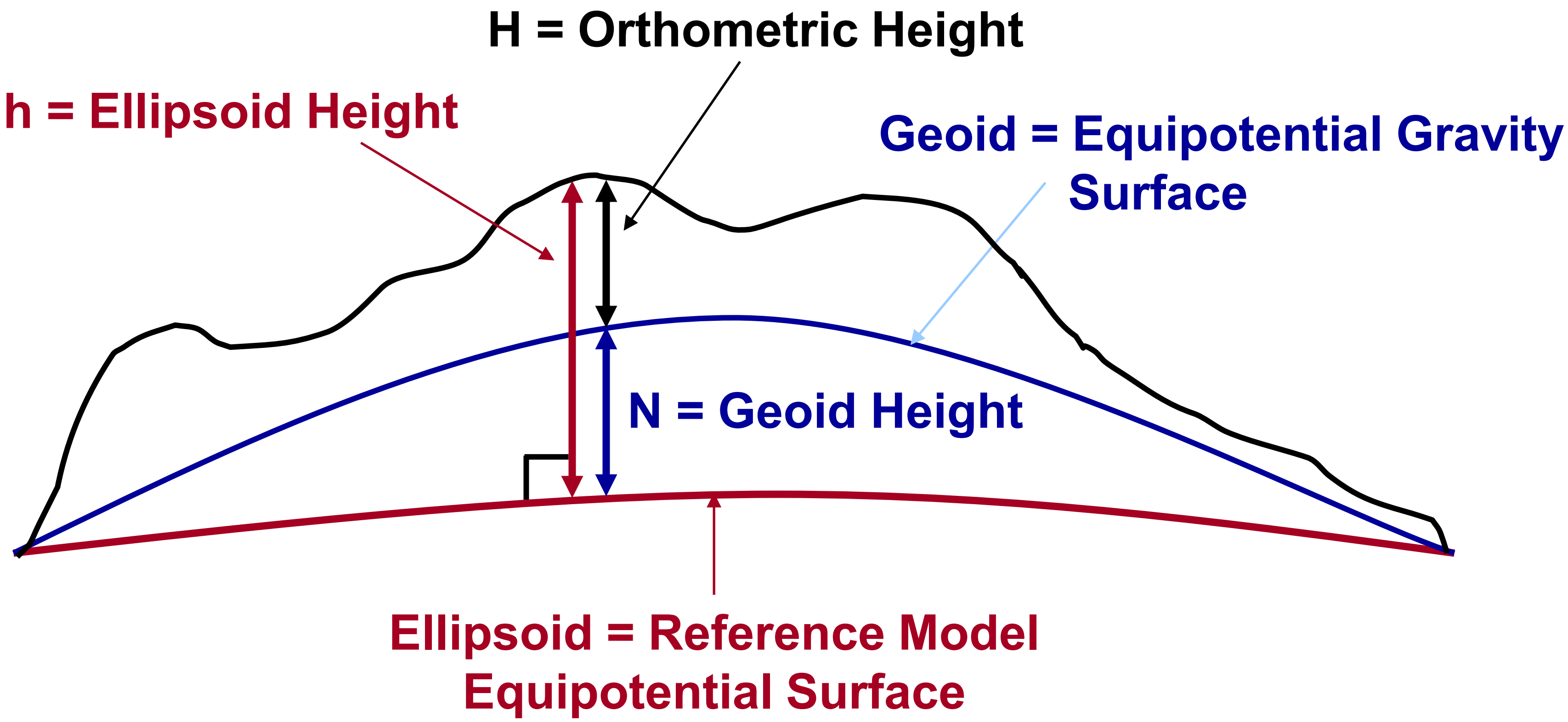
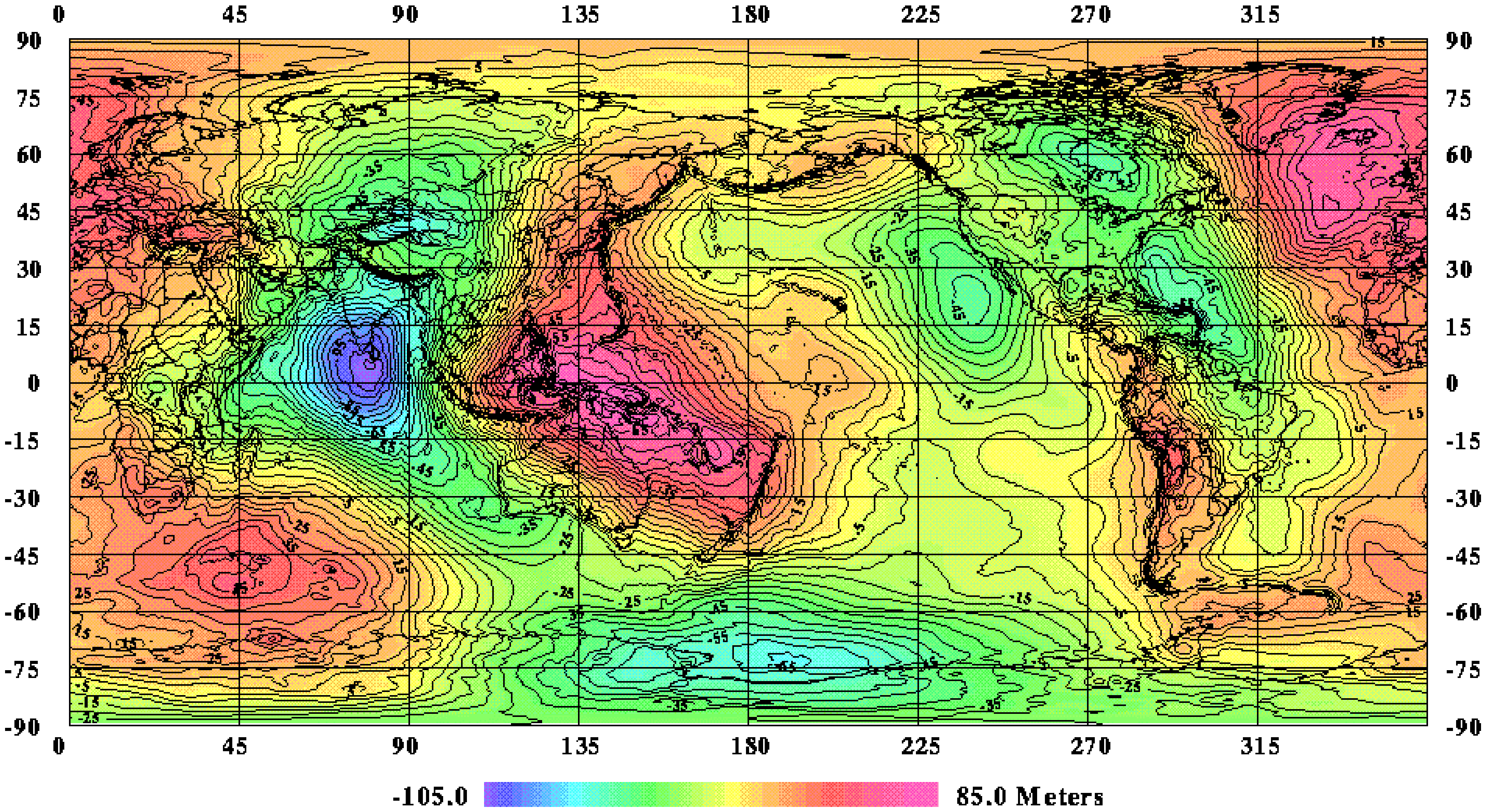
Geodetic Height Definitions



h(Ellipsoid Height) = H(Orthometric Height) + N(Geoid Height)

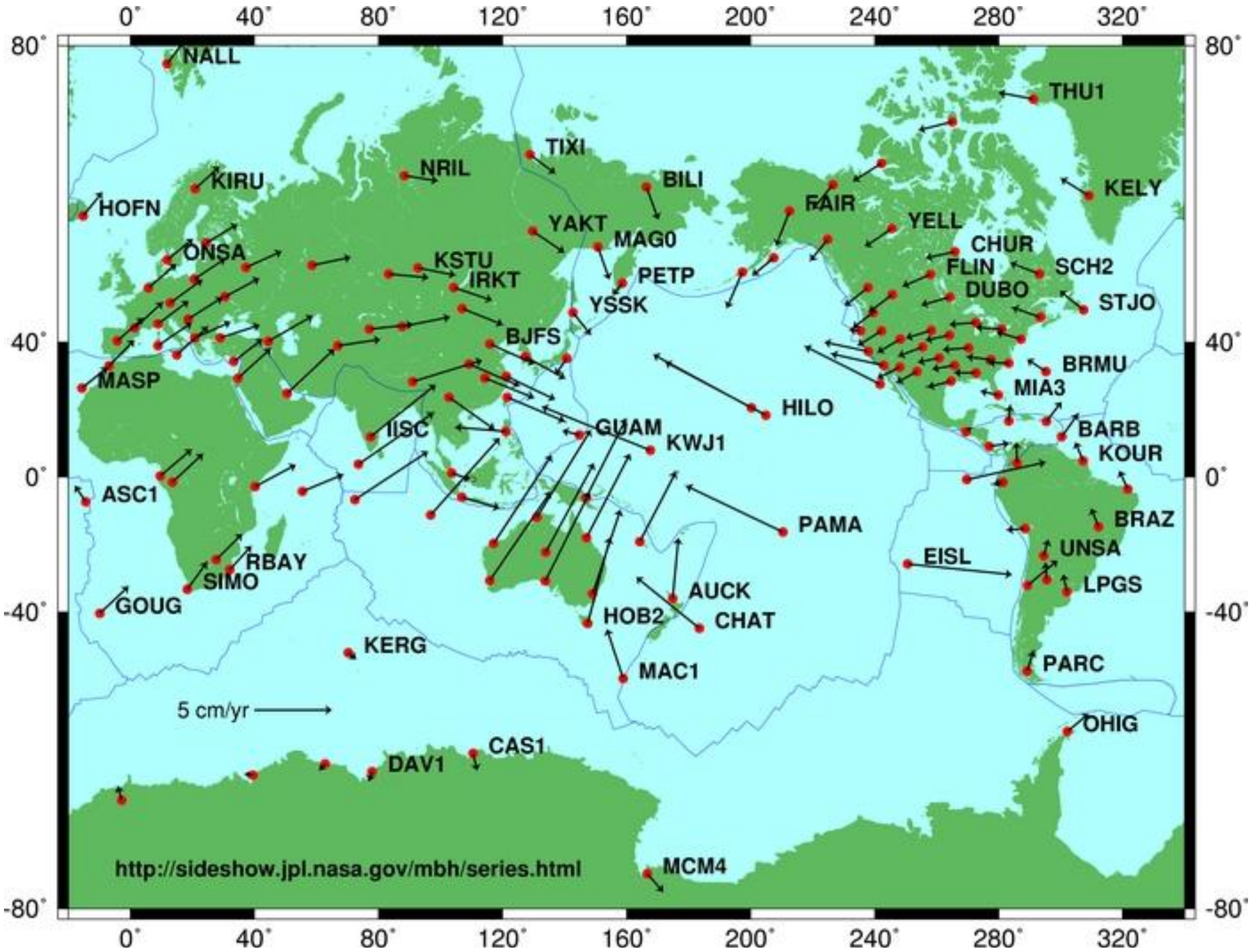
Source: National Geospatial-Intelligence Agency.

The Geoid



Source: National Geospatial-Intelligence Agency.

Tectonic Plates



Source: Jet Propulsion Laboratory.

Solid Earth Tides and Ocean Loading

- **Earth's surface is not rigid, but rather somewhat pliable**
- **Lunar and solar gravity, etc., deform the Earth with strong diurnal and semidiurnal variations over time**
  - **Displacements may be as large as ~30 cm**
- **Ocean tides can cause additional few cm station displacements in coastal locations**
- **Station coordinates often defined to exclude effects of solid Earth tides and ocean loading**

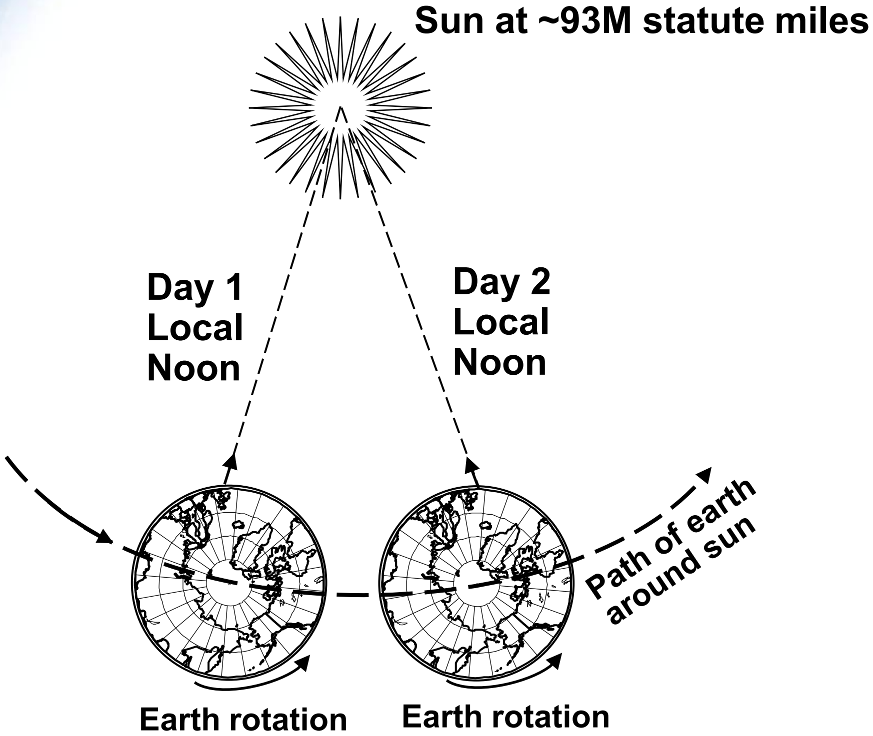
Time

- **Difficult to define time!**
- **For our purposes, time is the quantity that is read off a clock**
- **Any clock includes two main components:**
  - **Periodic event**
  - **Counter of that event**

Timescales

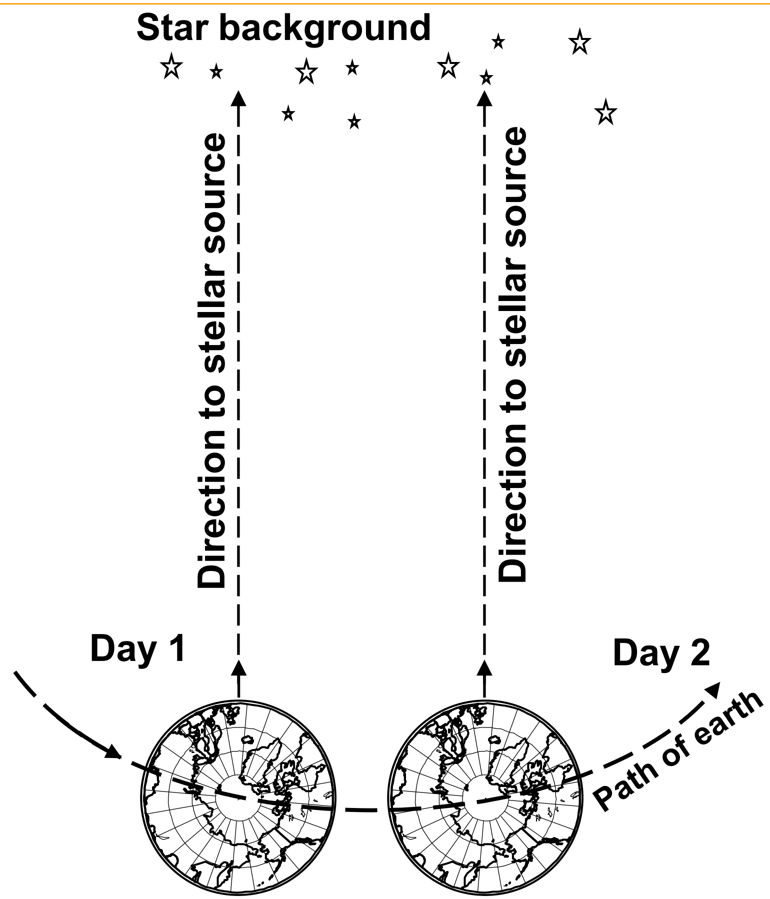
- **Universal Time (UT) - one of several time scales based upon rotation rate of Earth**
  - **UT0 – mean solar time based upon astronomical observations from prime meridian**
  - **UT1 – corrects UT0 for polar motion**
  - **UT2 – corrects UT1 for seasonal variations in Earth rotation rate**
  - **The second used to be defined as 1/86400 of a solar day**
- **Atomic time**
  - **Since 1967, the second has been defined by the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium-133 atom**
  - **International atomic time (TAI) – timescale maintained by international collection of atomic clocks**

Sidereal vs Solar Day



Solar Day

24 h



Sidereal Day
23 h 56 min 3.5 s

Coordinated Universal Time (UTC)

- TAI is a continuous timescale
  - No connection to mean solar time
- UTC differs from TAI by an integer number of "leap" seconds
  - Introduced when needed to keep UTC within 0.9 s of UT1
  - As of April 2010, TAI – UTC = 34 s
- UTC is maintained by the Bureau International des Poids et Mesures (BIPM) in Sevres, France
  - Produced using times maintained by over 250 clocks in 65 nations
  - Not in real-time
- Official U.S. time is the realization of UTC maintained by the U.S. Naval Observatory – UTC(USNO)

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GPS Time

- GPS Time is maintained by the GPS control segment based upon an average of all system clocks
  - Satellites
  - Monitor stations
- It is a continuous time scale
  - TAI – GPS Time = 19 s
  - GPS Time – UTC = 15 s (as of April 2010)
- GPS Time is specified with be within 1 microsecond of UTC(USNO) modulo 1 s

Stability of Frequency Sources - Definitions

- **Consider the oscillator output:**

$$V(t) = A(t)\sin(2\pi f_0 t + \phi(t))$$

- **Instantaneous phase:**

$$\varphi(t) = 2\pi f_0 t + \phi(t)$$

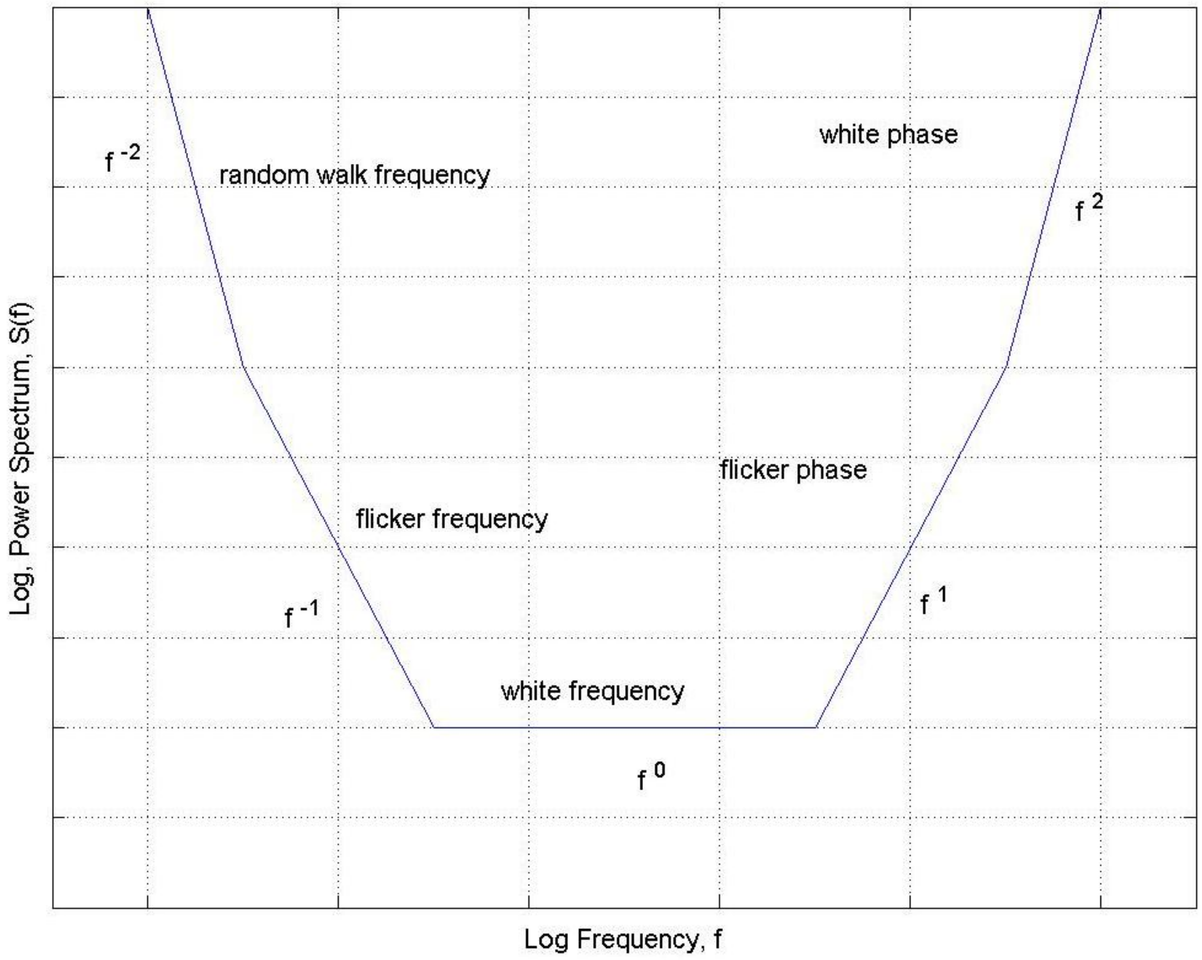
- **Instantaneous frequency:**

$$f(t) = f_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

- **Fractional frequency deviation:**

$$y(t) = \frac{1}{2\pi f_0}\frac{d\phi(t)}{dt}$$

Frequency Noise



Typical characteristics of power spectrum of *y(t)* are illustrated above.

Allan Deviation

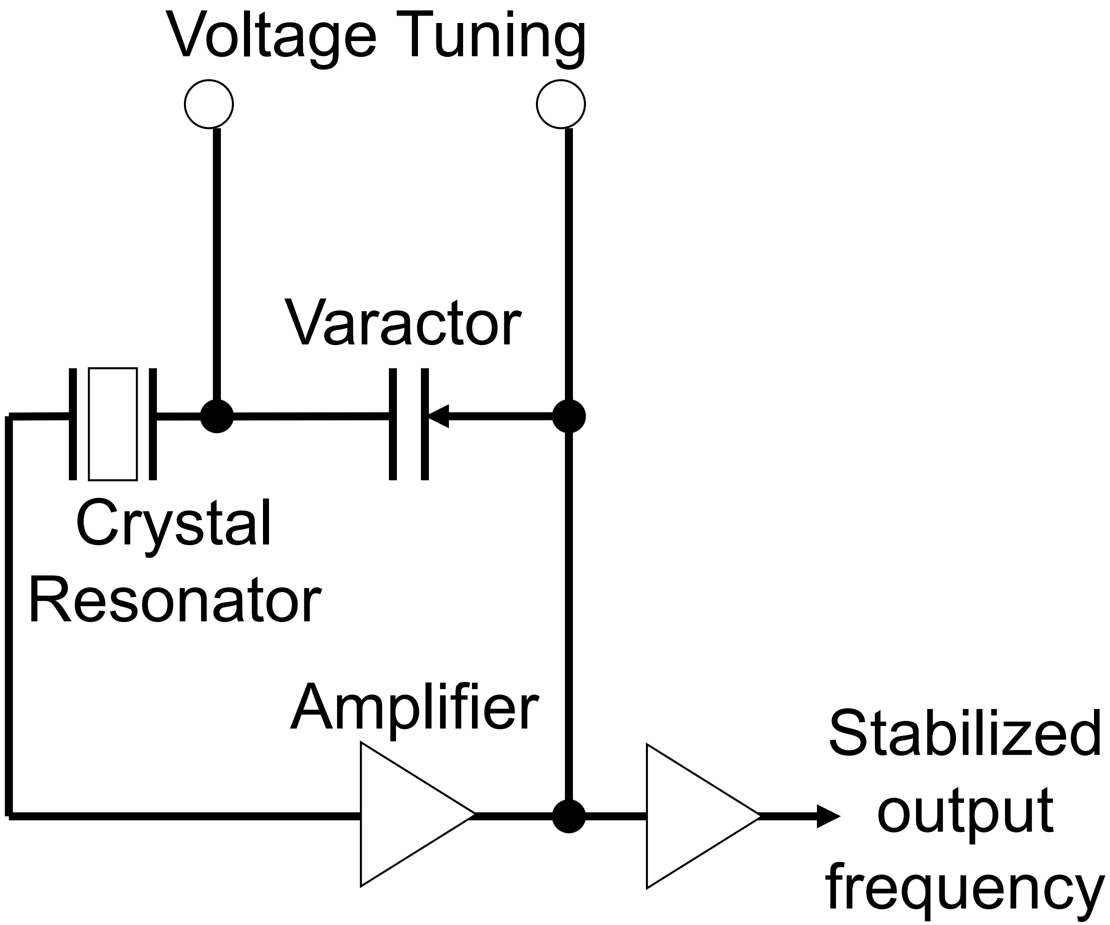
- **Allan deviation is defined as:**

$$\sigma_y(\tau) = \sqrt{\frac{1}{2} E\left[(\bar{y}_{k+1} - \bar{y}_k)^2 \right]}$$

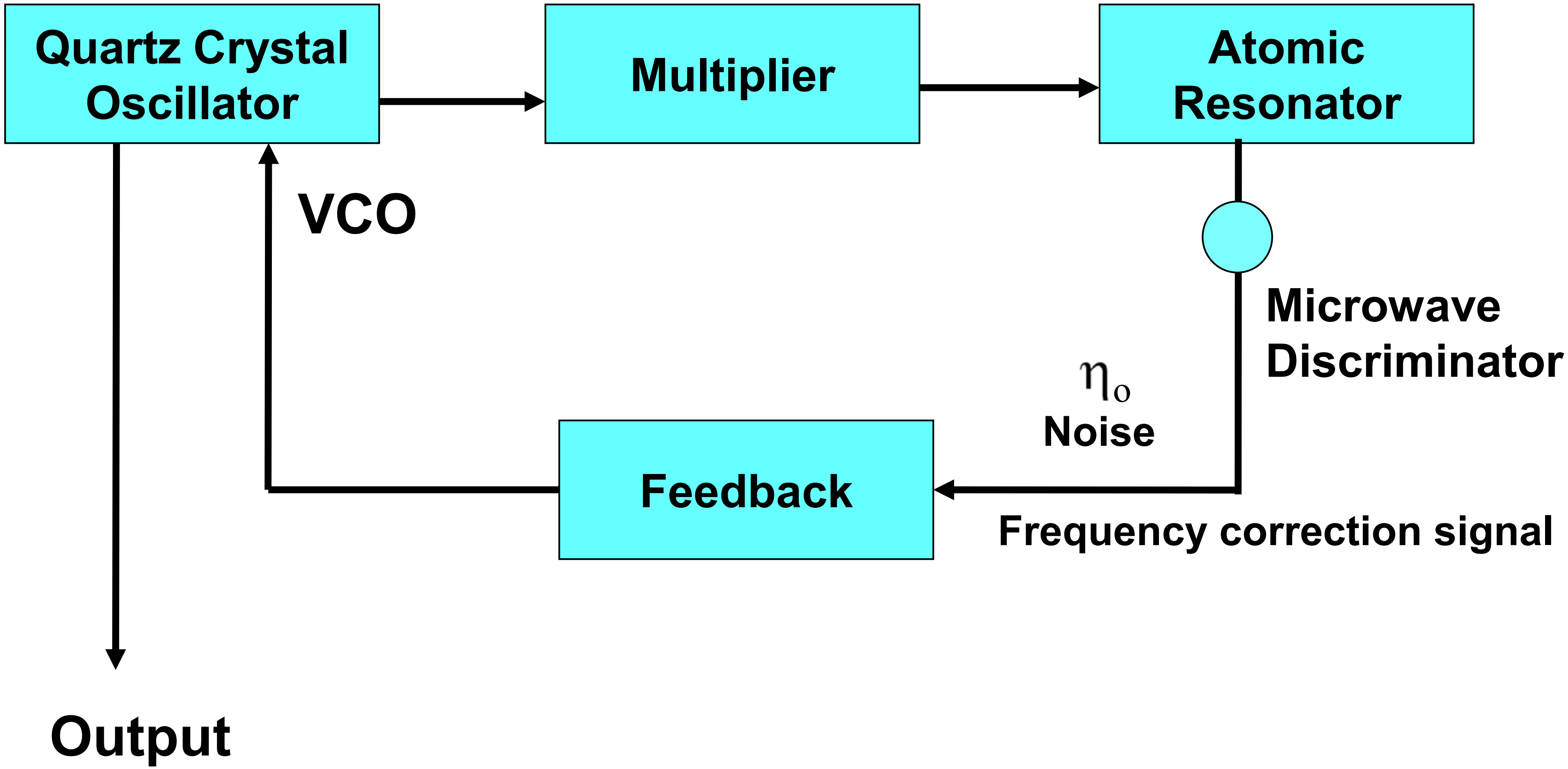
- **where**

$$\bar{y}_k = \frac{\phi([k+1]\tau) - \phi(k\tau)}{2\pi f_0 \tau}$$

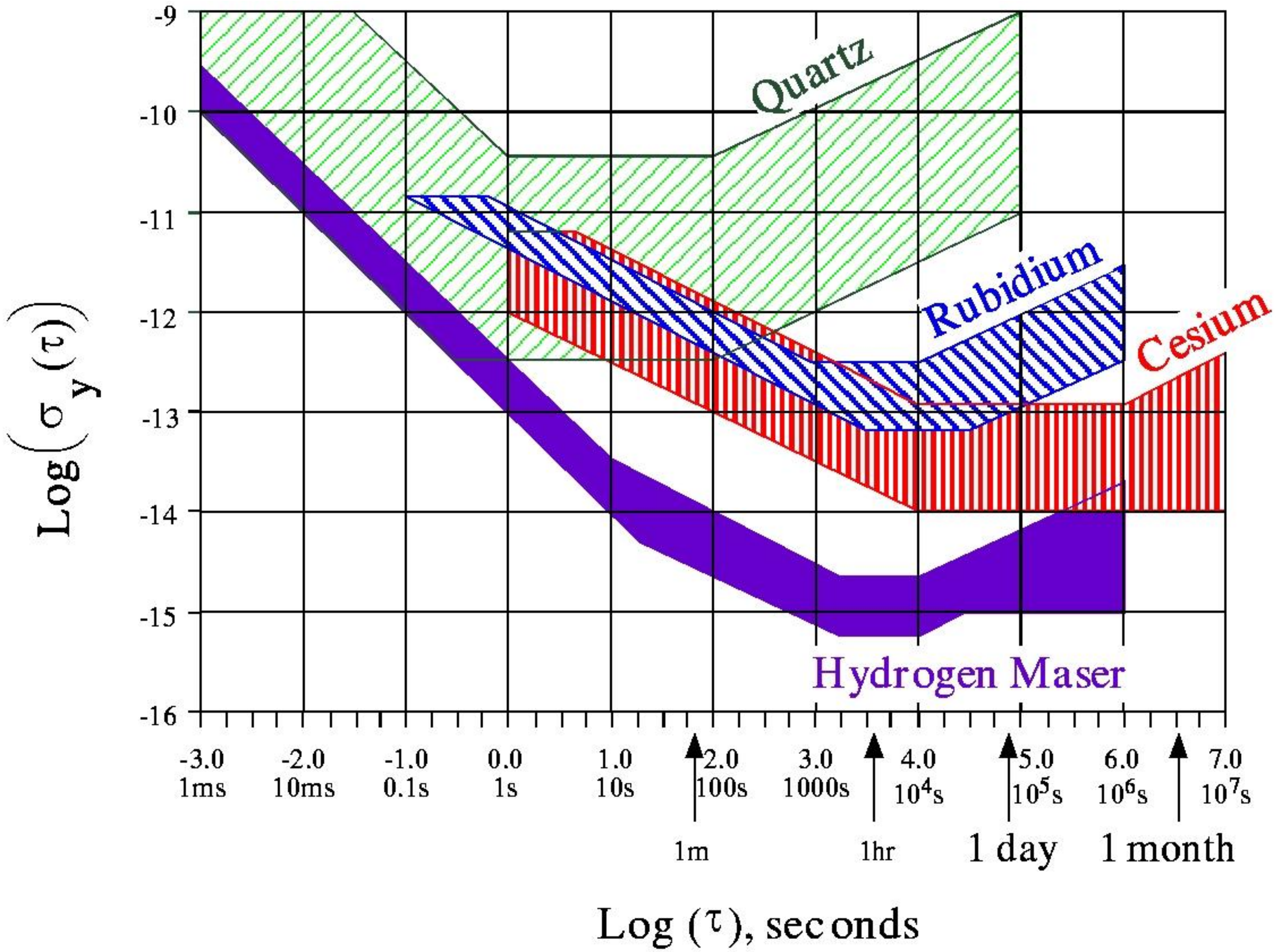
Crystal Oscillator



Atomic Frequency Standard



Stability of Various Clock Types

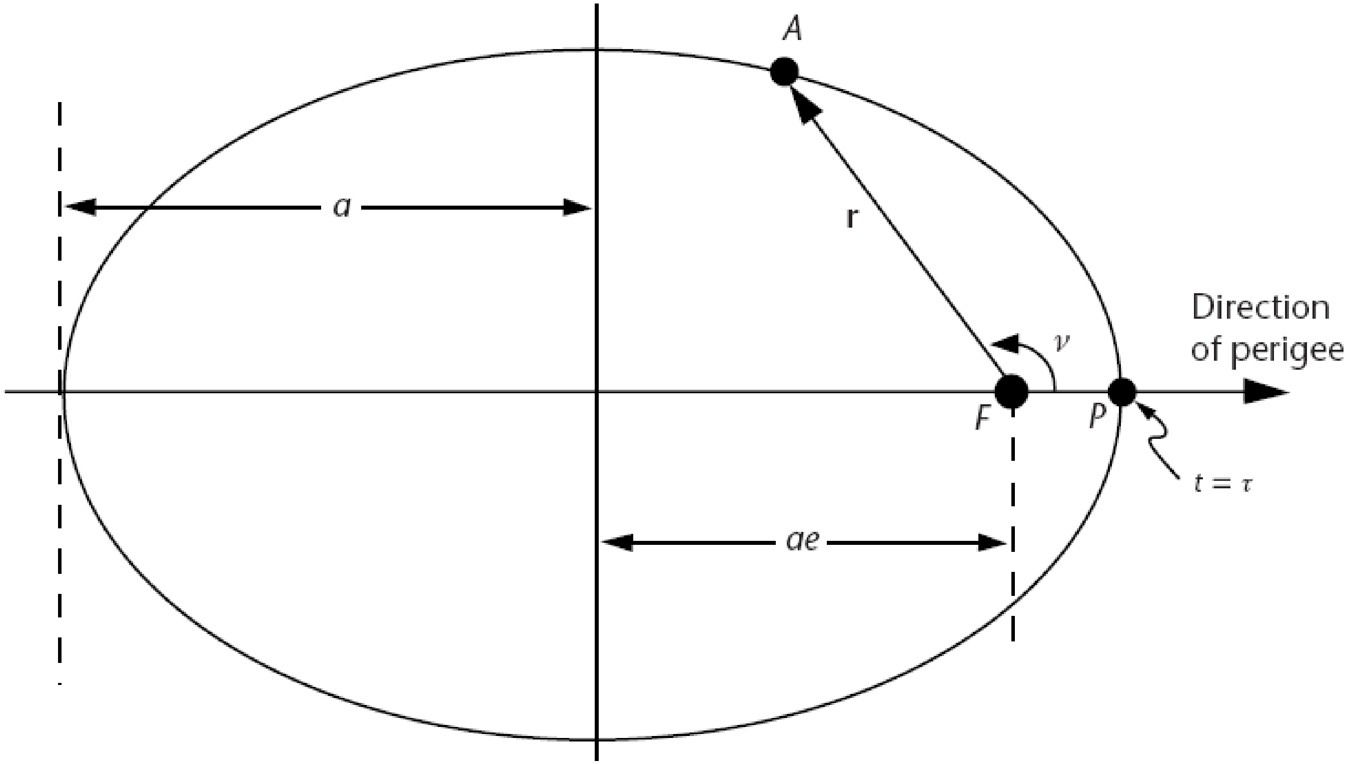


Source: John Vig IEEE tutorial.

Kepler's Laws

- A satellite's orbit is in the shape of an ellipse with the Earth at one focus
- The radius drawn from the center of the Earth to the satellite sweeps out equal area in equal times
- The square of the orbital period of a satellite is proportional to the cube of the semi-major axis

Keplerian Elements – Shape of Orbit



Keplerian elements:

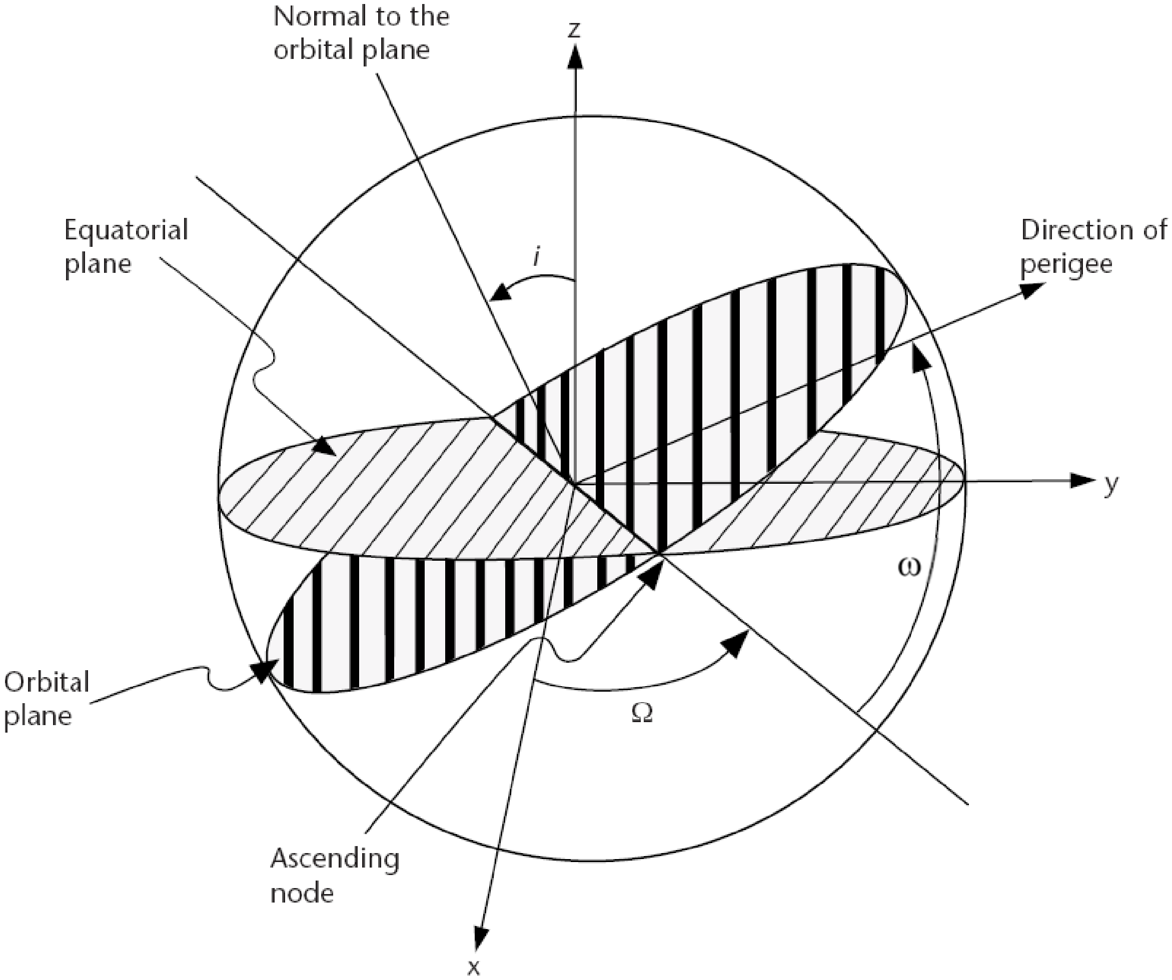
a = **semimajor axis**

e = **eccentricity**

τ = **time of perigee passing**

A = location of satellite, F = focus (Earth center), P = perigee (lowest point on orbit)

Keplerian Elements – Orientation of Orbit

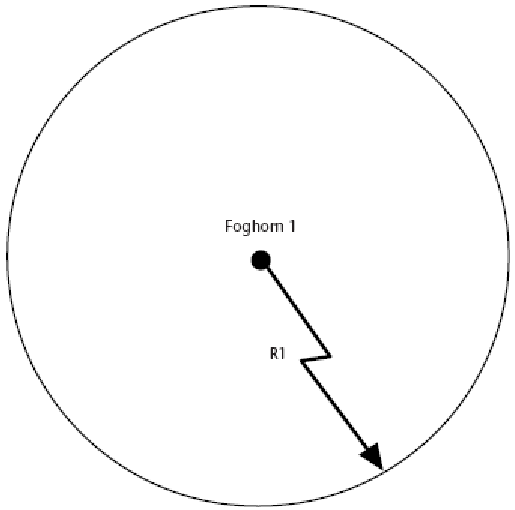


Ω = right ascension of ascending node, i = inclination, ω = argument of perigee

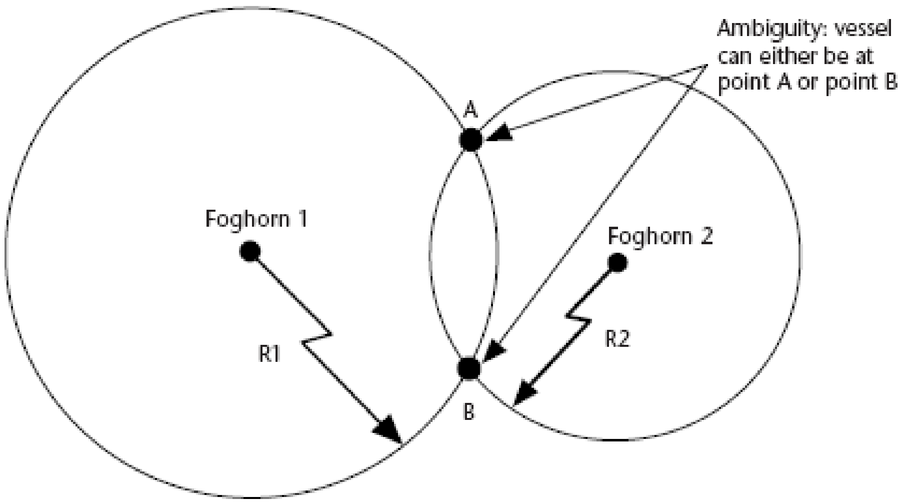
Perturbations to Orbits

- Satellite orbits are not perfect ellipses due to various perturbations:
  - Earth's oblateness
  - Third-body effects – sun, moon, etc.
  - Solar radiation pressure
- As a result, more orbital elements are needed to accurately convey satellite locations
  - Example: Broadcast GPS orbits use basic Keplerian elements plus some rate terms plus sine/cosine corrections to some elements

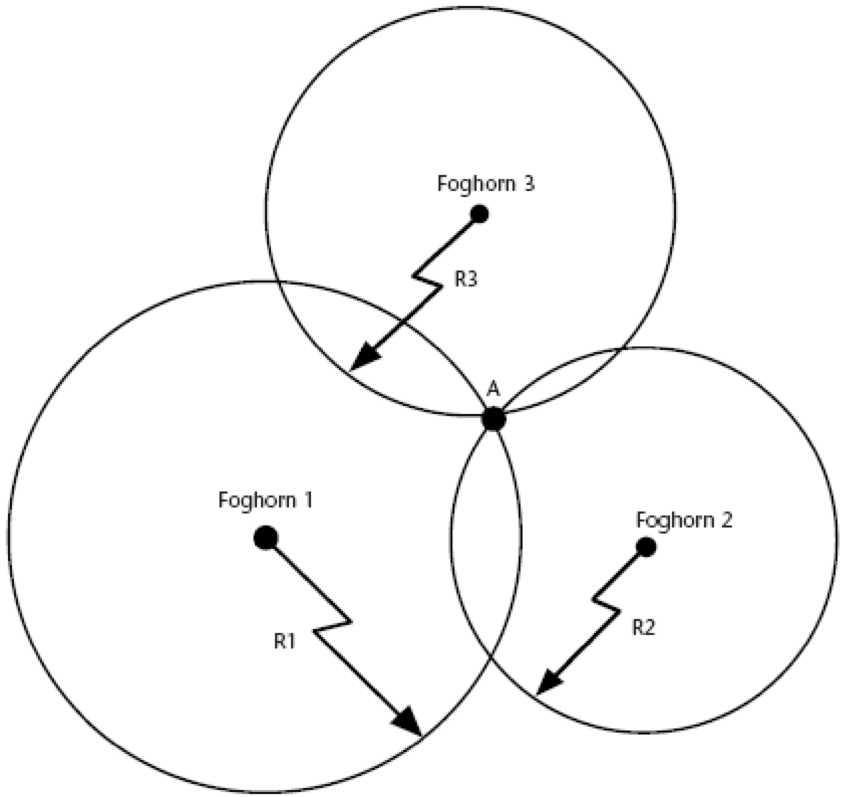
Positioning in Two-Dimensions



One measurement

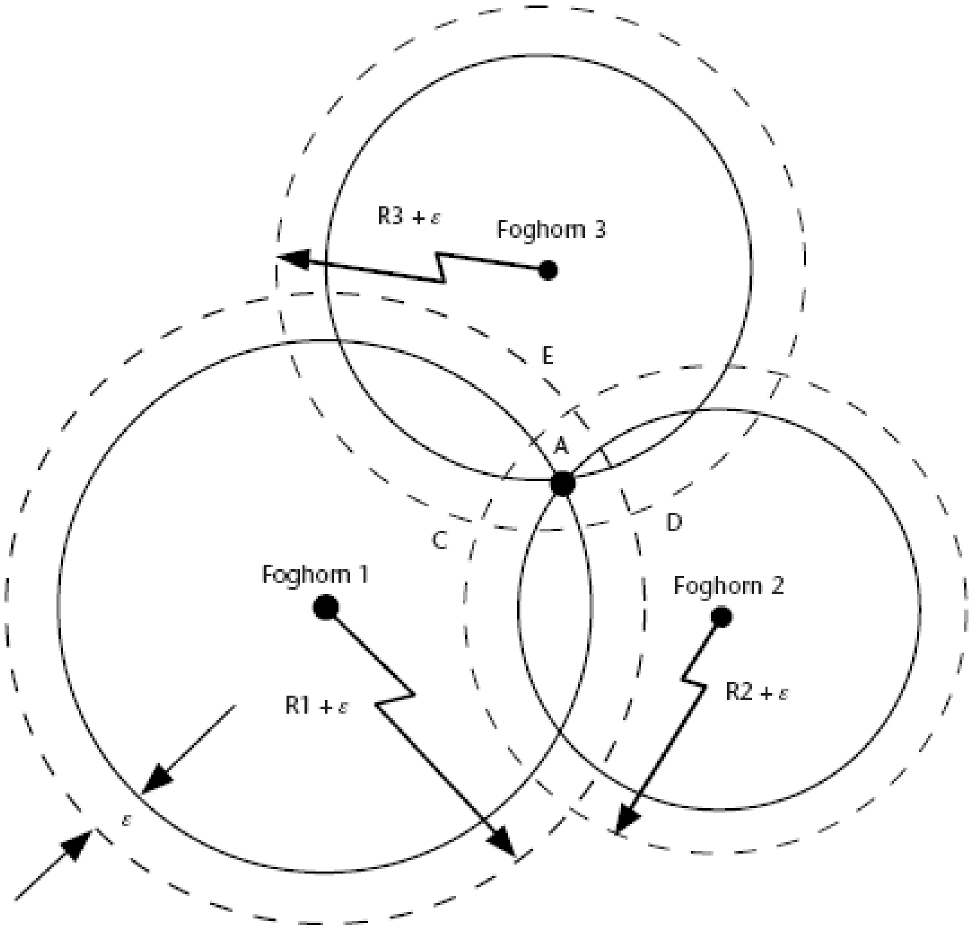


Two measurements



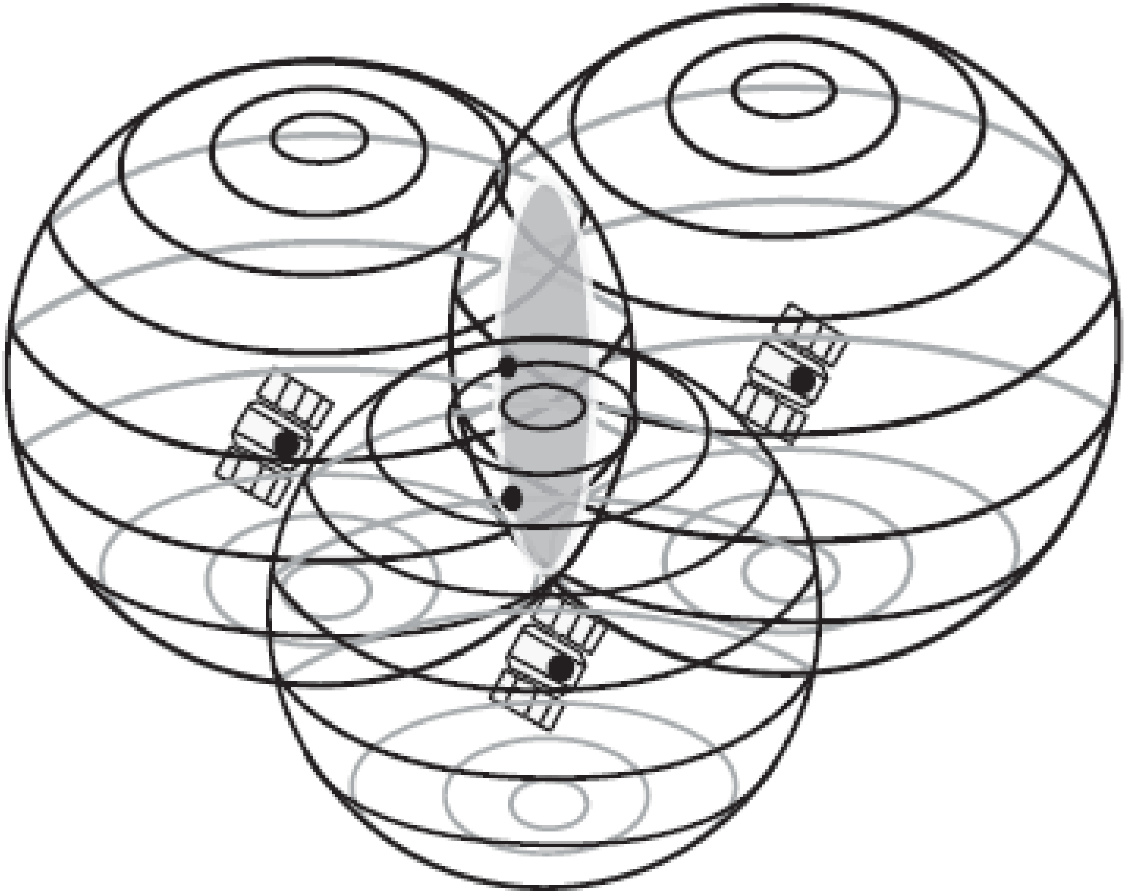
Three measurements

2D Example – Effect of Imperfect User Clock



Three measurements

Positioning in Three Dimensions



With three satellites and perfect user clock, one can estimate user location; with imperfect user clock, four satellites are needed to determine user location and clock error (4 equations, 4 unknowns).

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Time and Distance Conversions

- **Speed of light, c = 299792458 m/s**
- **Some conversions:**
  - **1 ns ~ 0.3 m**
  - **1 μs ~ 300 m**
  - **1 ms ~ 300 km**
  - **1 s ~ 300,000 km**

CENTER FOR ADVANCED AVIATION SYSTEM DEVELOPMENT (CAASD)

GPS Space and Control Segments

Chris Hegarty

April 2010

GPS Space and Control Segment

- GPS space segment
  - Block I
  - Block II/IIA
  - Block IIR/IIR-M
  - Block IIF
  - Block IIIA
  - Other systems (for comparison)
    - GLONASS
    - GALILEO
- Control segment
  - Legacy architecture
  - Architecture evolution plan (AEP)
  - Operations

Block I

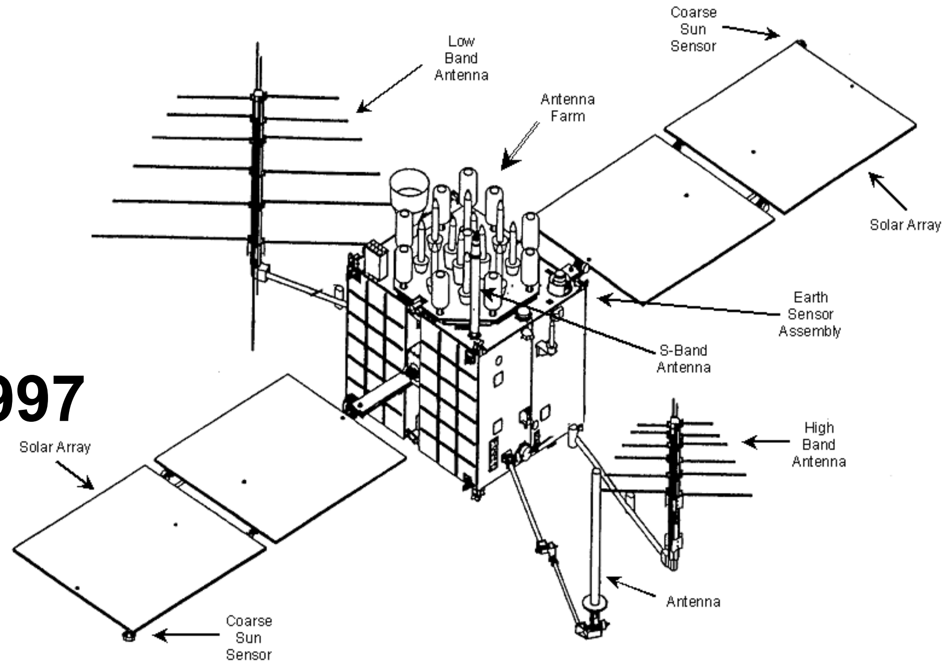
- **Developmental prototypes**
- **11 total vehicles, built by Rockwell International**
- **Launched between 1978 and 1985**
- **Two cesium and two rubidium clocks each**
- **5 year design life**
  - **None remain operational today**

Block II/IIA

- Initial and upgraded production satellites
- 28 total vehicles (9 Block II and 19 Block IIA), built by Rockwell International
- Launched between 1989 and 1997
- Two cesium and two rubidium clocks each
- Radiation hardened
- 7.5 year design life
  - 11 Block II/IIA are still in operation as of April 2010

Block IIR

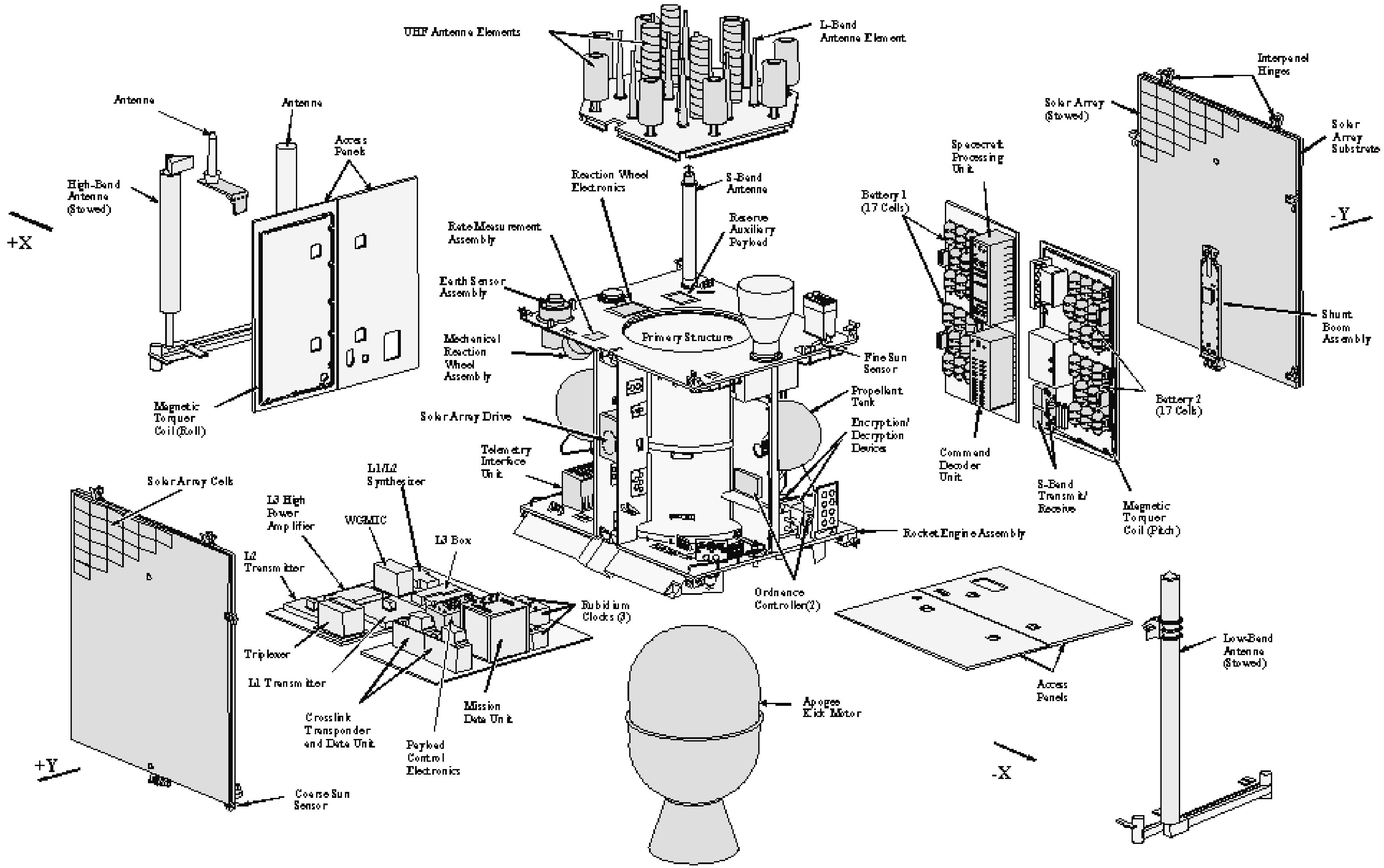
- **Replenishment satellites**
- **21 total vehicles, built by Lockheed-Martin**
  - **IIR-1 lost on launch**
  - **Last 8 satellites have been modernized (IIR-M)**
- **12 IIRs launched between 1997 and 2004**
- **Three rubidium clocks each**
- **7.5 year design life**
  - **All 12 successfully-launched IIRs are still in operation as of April 2010**



Block IIR-M

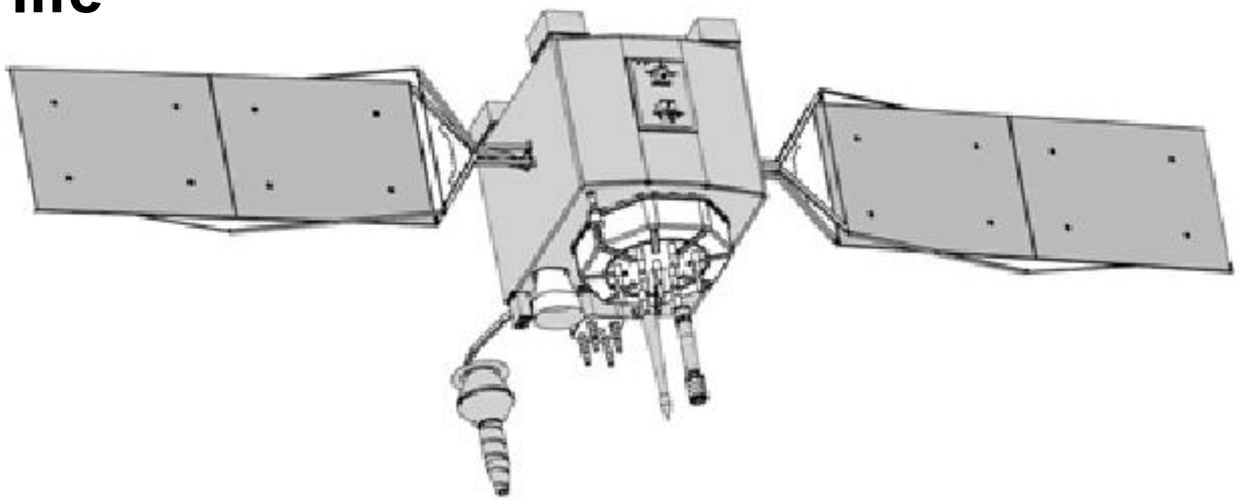
- Modernized replenishment satellites
- 8 total vehicles, built by Lockheed-Martin
  - Last 8 satellites of 21-satellite IIR procurement
- First IIR-M launched in September 2005
  - Now: all have been launched; seven in operation
- Adds new civil and military signal capability
  - L2 Civil (L2C) signal
  - Military (M)-code signals on L1 and L2
  - Demonstration L5 package on IIR-M(20)

Block IIR-M – Expanded View



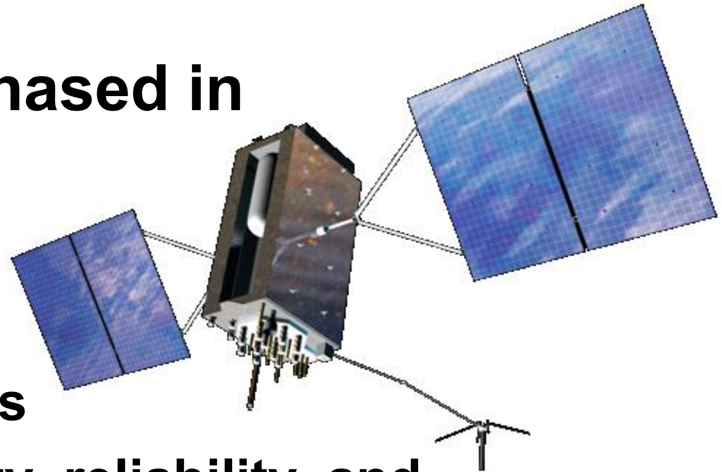
Block IIF

- “Follow-on” satellites
- 12 total vehicles, being built by Boeing
- First launch anticipated in 2010
- Adds new civil L5 signal at 1176.45 MHz
- Two rubidium and one cesium clock each
- 12 year design life

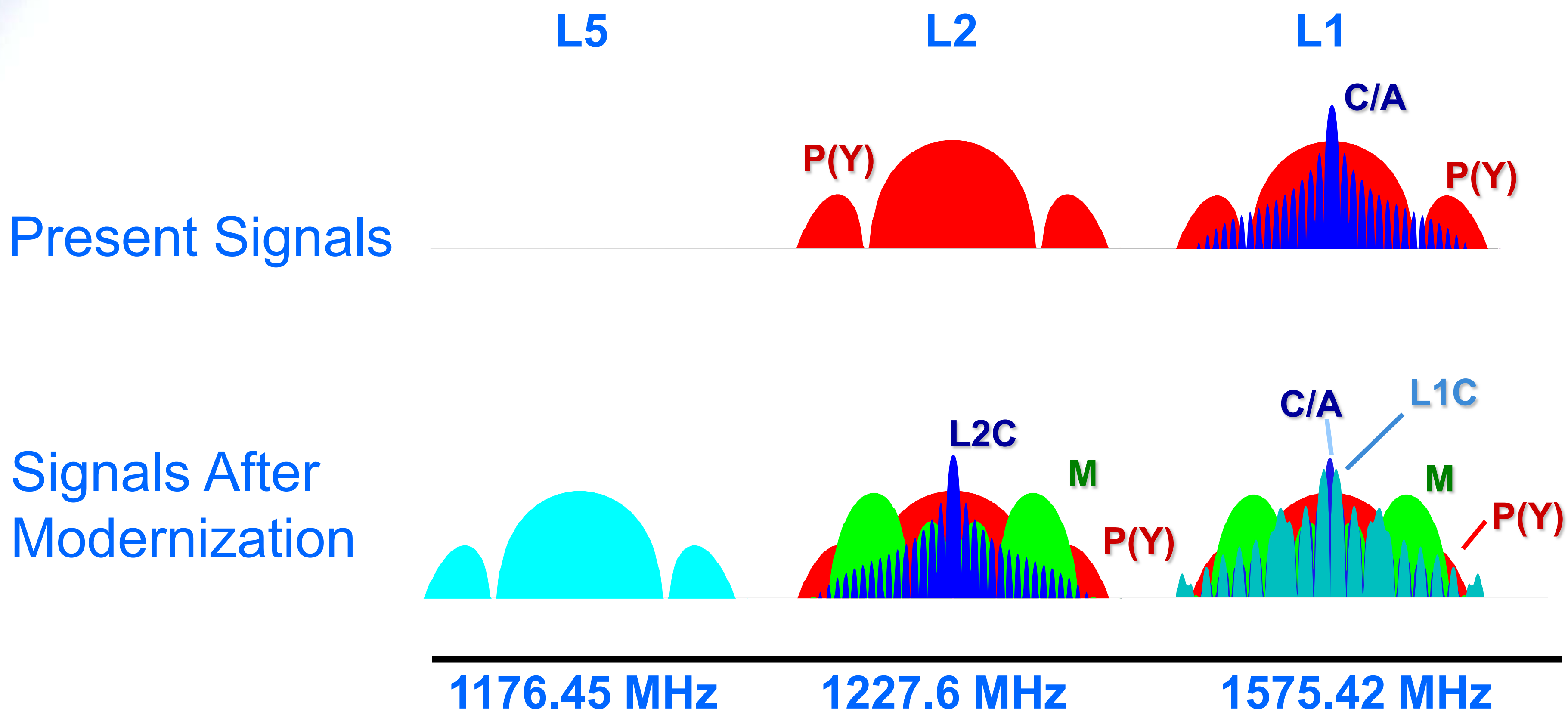


Block III

- **Next generation satellites**
- **Contract awarded to Lockheed Martin to build the first two Block III (IIIA) satellites in May 2008**
  - **With options for up to 10 additional IIIA vehicles**
- **First launch anticipated ~2014**
- **Potential Block III features (to be phased in incrementally):**
  - **New L1 civil (L1C) signal**
  - **"Spot beam" for military M-code signal**
  - **High-speed uplink/downlinks/crosslinks**
  - **Increased levels of accuracy, availability, reliability, and integrity**



GPS Signal Evolution



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Nominal GPS 24-Satellite Constellation

Slot	RAAN (deg)	Argument of Latitude (deg)	Slot	RAAN (deg)	Argument of Latitude (deg)
A1	272.847	268.126	D1	92.847	135.226
A2	272.847	161.786	D2	92.847	265.446
A3	272.847	11.676	D3	92.847	35.136
A4	272.847	41.806	D4	92.847	167.356
B1	332.847	80.956	E1	152.847	197.046
B2	332.847	173.336	E2	152.847	302.596
B3	332.847	309.976	E3	152.847	66.066
B4	332.847	204.376	E4	152.847	333.686
C1	32.847	111.876	F1	212.847	238.886
C2	32.847	11.796	F2	212.847	345.226
C3	32.847	339.666	F3	212.847	105.206
C4	32.847	241.556	F4	212.847	135.346

Defined Epoch: 0000Z, 1 July 1993; Greenwich Hour Angle: 18 h 36 min 14.4 s referenced to FK5/J2000.00 coordinates

Semimajor axis = 26559.7 km, inclination = 55 deg, eccentricity = 0

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Expandable 24-Slot Constellation Slot Assignments

Expandable Slot		RAAN	Argument of Latitude	GEC (GLAN)
B1 Expands To:	B1F	332.847°	94.916°	101.25°
	B1A	332.847°	66.356°	86.97°
D2 Expands To:	D2F	92.847°	282.676°	135.13°
	D2A	92.847°	257.976°	122.78°
F2 Expands To:	F2F	212.847°	0.456°	114.02°
	F2A	212.847°	334.016°	100.80°

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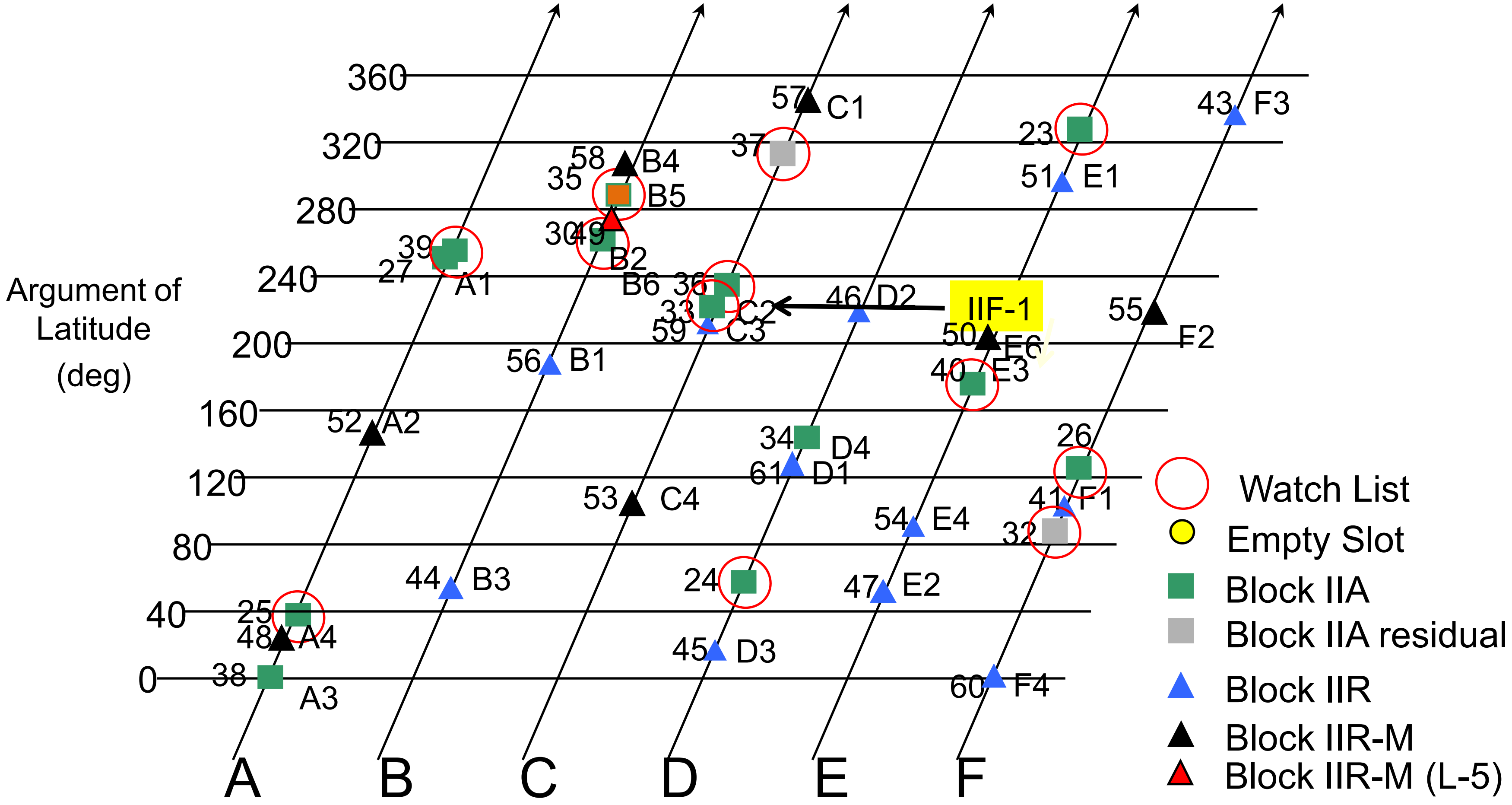
Constellation Status as of September 2009



SVN-26 Rephase Complete: Sept '09

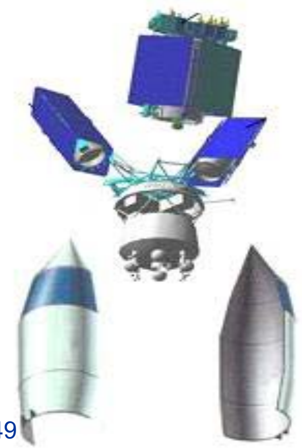
Source: United States Air Force, Civil GPS Services Interface Committee, September 2009.

GPS Constellation Design Features

- **Asymmetric spacing in each plane**
  - Selected based upon robustness considerations (e.g., performance of the constellation when satellite failures occur)
- **Orbital period is ~one-half a sidereal day**
  - Ground-tracks repeat ~once/day
- **Orbital parameters selected in part by launch vehicle**
  - Space shuttle was originally intended to be primary launch vehicle prior to the Challenger disaster

GLONASS Satellite

• Guaranteed Life-time	3 years
• Satellite mass	1 415 kg
• Power supply	1 000 W
• Navigation payload	
– Mass	180 kg
– Power consumption	600 W
• Clock stability (24 hours)	$5*10^{-13}$
• Attitude control accuracy	0.5 deg
• Solar panel pointing accuracy	5 deg



Group launch by PROTON

Total launched 81 satellites
Actual life-time 4.5 years

Source: Various public Russian Federation presentations.

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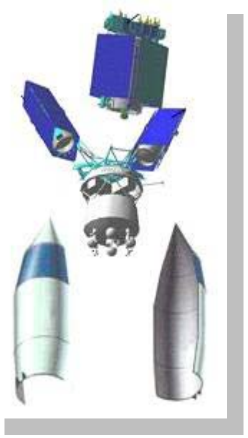
GLONASS-M Satellite

• Guaranteed Life-time	7 years
• Satellite mass	1 415 kg
• Power supply	1 450 W
• Navigation payload	
– Mass	250 kg
– Power consumption	580 W
• Clock stability (24 hours)	$1*10^{-13}$
• Attitude control accuracy	0.5 deg
• Solar panel pointing accuracy	2 deg

GLONASS-M new features:

- Extended life-time
- L2 civil signal
- Improved clock stability
- Improved solar panel pointing
- Improved dynamic model, less level of unpredicted accelerations



Group launch by PROTON

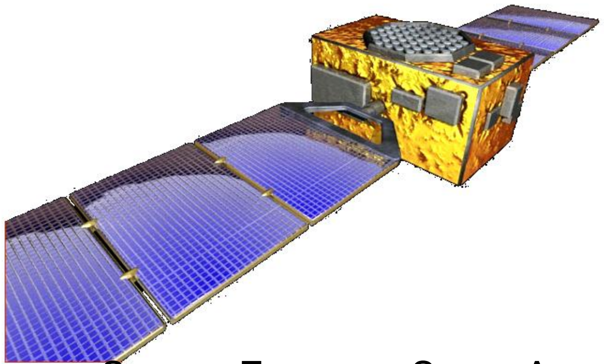
GLONASS Orbital Parameters (as of April 2010)

NS	TΩ	Trev	e	i	LΩ	ω	δt2	nl	ΔT
01	37202.53	40544.0	0.00046	64.73842	-31.469822	19.813843	1.7166138E-4	1	0.0014038086
02	1868.9375	40544.04	0.00037	64.50616	116.26007	14.655762	1.373291E-4	-4	0.0014038086
03	6900.7188	40543.984	0.00011	64.51371	95.249405	83.57849	3.8146973E-5	5	0.0013427734
04	11862.594	40544.0	0.00161	64.722275	74.38483	134.73633	7.6293945E-5	6	0.0013427734
05	16932.719	40543.973	0.00058	64.72571	53.1946	75.42114	1.411438E-4	1	0.0013427734
06									
07	27119.781	40544.14	0.0008	63.661926	7.6463127	120.64636	2.0980835E-4	5	0.0014648438
08	32189.75	40543.938	0.00019	64.50204	-10.437012	37.23816	6.866455E-5	6	0.0014648438
09	35295.53	40544.03	0.00047	65.18868	95.8909	55.464478	1.296997E-4	-2	4.8828125E-4
10	40233.75	40544.027	0.00166	65.5835	74.899124	161.81763	1.2207031E-4	-7	4.272461E-4
11	4841.6875	40544.0	0.00198	65.2007	-136.85463	0.14282227	1.2207031E-4	0	7.324219E-4
12									
13	15094.6875	40544.062	0.00053	65.18817	-179.70354	91.16455	2.632141E-4	-2	6.1035156E-4
14	19931.938	40544.027	0.00142	65.56891	159.69933	159.35669	1.411438E-4	-7	5.493164E-4
15	25212.531	40544.504	0.0025	65.567535	137.63054	-5.7403564	-1.1444092E-5	0	4.8828125E-4
16									
17	33829.906	40544.08	0.00176	64.898056	-137.53955	-170.09583	2.822876E-4	4	-0.0020141602
18	38907.53	40544.08	0.0028	64.79506	-159.16786	-22.000122	1.5258789E-5	-3	-0.0020141602
19	3387.6562	40544.04	0.00019	64.882095	-10.361309	-69.28528	1.2207031E-4	3	-0.0020751953
20	8544.875	40544.12	0.0015	64.89531	-31.882324	-16.078491	6.866455E-5	2	-0.0021362305
21	13520.406	40544.047	0.002	64.80313	-53.05195	-178.84644	1.8692017E-4	4	-0.0020751953
22	18626.25	40544.008	0.0034	64.7791	-74.5788	-6.234741	1.1444092E-5	-3	-0.0020751953
23	23682.812	40544.055	0.00004	64.761765	-95.70019	106.98486	1.449585E-4	3	-0.0020751953
24	28760.062	40543.992	0.00054	64.77189	-116.92097	92.27966	3.8146973E-6	2	-0.0020141602

See www.glonass-ianc.rsa.ru for updated data and definitions.

GALILEO

- European contribution to the GNSS
  - Jointly financed by European Commission (EC) and European Space Agency (ESA)
  - Program gained significant boost in March 2002 with release of ~$1.1B euro
- 27+ satellite constellation
  - 3-planes
  - 56 deg inclination
  - ~23,200 km altitude



Source: European Space Agency.

- Two test satellite launched in 2005, 2008
- 4 in-orbit validation (IOV) satellites anticipated to be launched ~ late 2010/early 2011

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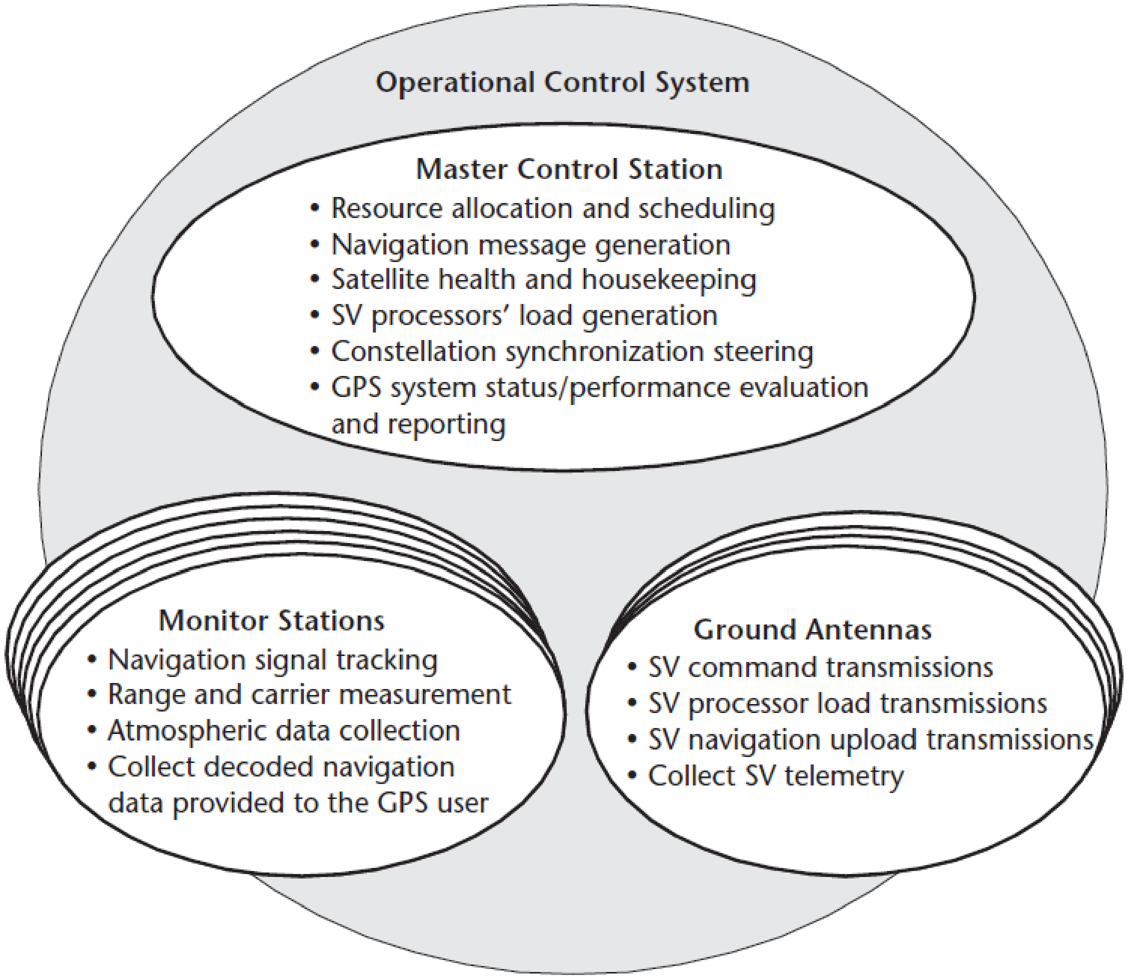
GALILEO Constellation Design

- **27-satellite nominal constellation + 3 spares**
  - **3-planes**
  - **56 deg inclination**
  - **23,616 km altitude (semimajor axis = 29600.318 km)**
- **Walker 27/3/1 configuration**
  - **Notation means 27 satellites, 3 planes, with 9 satellites equally spaced in each plane, and a relative phasing of satellites between planes of 13.3 deg**
  - **In general, Walker *T/P/F* means:**
    - ***T* total satellites**
    - ***P* orbital planes**
    - ***F* design factor, which provides the offset in mean anomaly between the first satellite in each adjacent orbital plane as (offset in degrees) = 360 × *F/T***

GPS Control Segment – Functional Overview



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Legacy Architecture and Architecture Evolution Plan (AEP)

- **Legacy architecture (prior to September 2007)**
  - MCS software was hosted on an IBM mainframe under Multiple Virtual Storage operating system
  - Since the 1980's the legacy architecture implemented a partitioned Kalman filter to estimate satellite positions and clock errors
    - Only up to six satellites and up to six monitor stations per partition
    - Limited accuracy
- **Architecture Evolution Plan (AEP) (now in use)**
  - Replaces mainframe with distributed Sun workstation configuration
  - Improved graphical user interface for GPS operators
  - Provides infrastructure for incremental control segment upgrades, e.g., to support new satellite blocks

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Legacy Control Segment

- Master Control Station (MCS): **Satellite control, System operations**
- Alternate Master Control Station: **Training, Back-up**
- Monitor Station (MS): **L-band; Collect range data, Monitor nav signals**
- Ground Antenna (GA): **S-band; Transmit data/commands, Collect telemetry**

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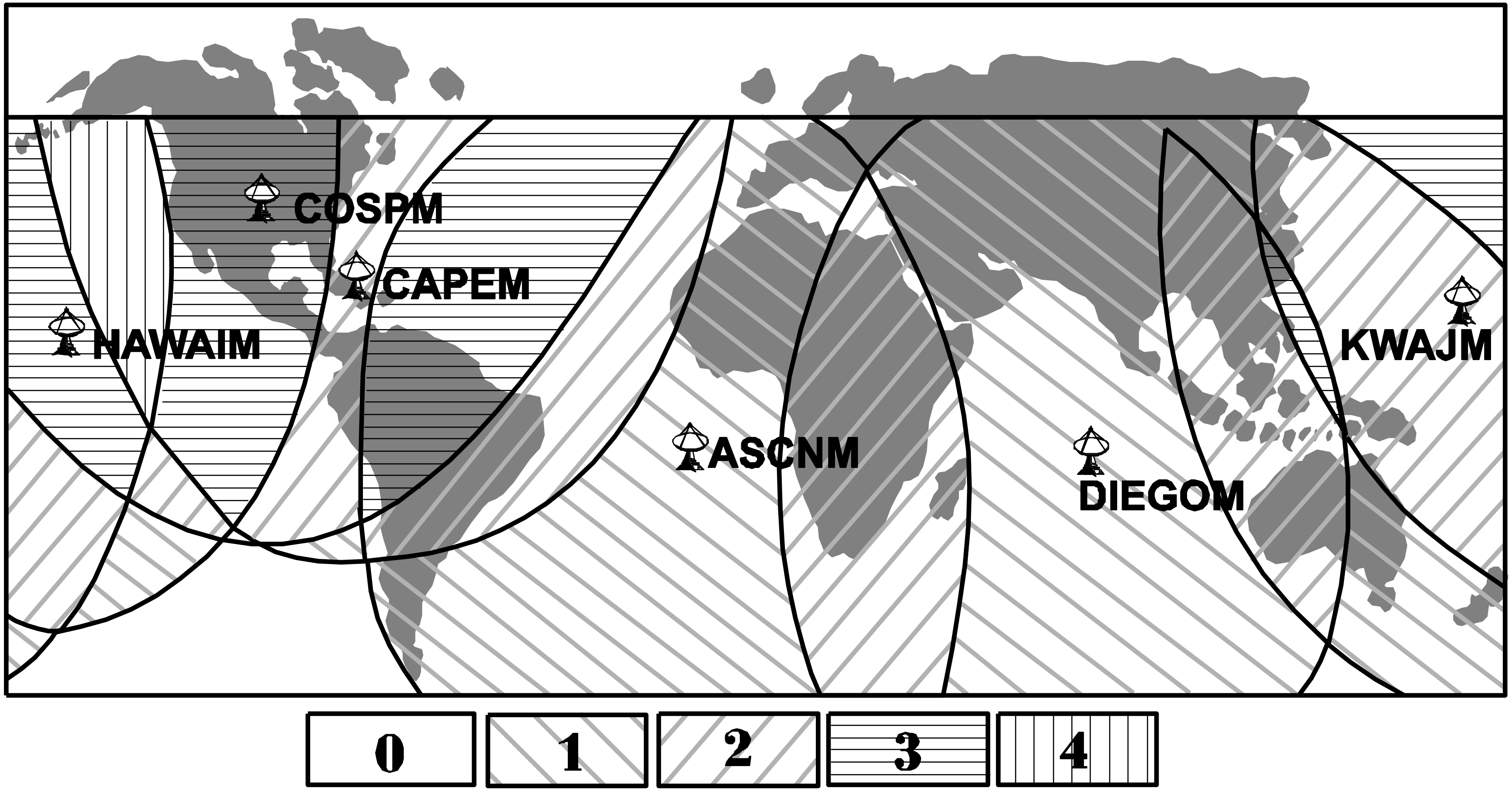
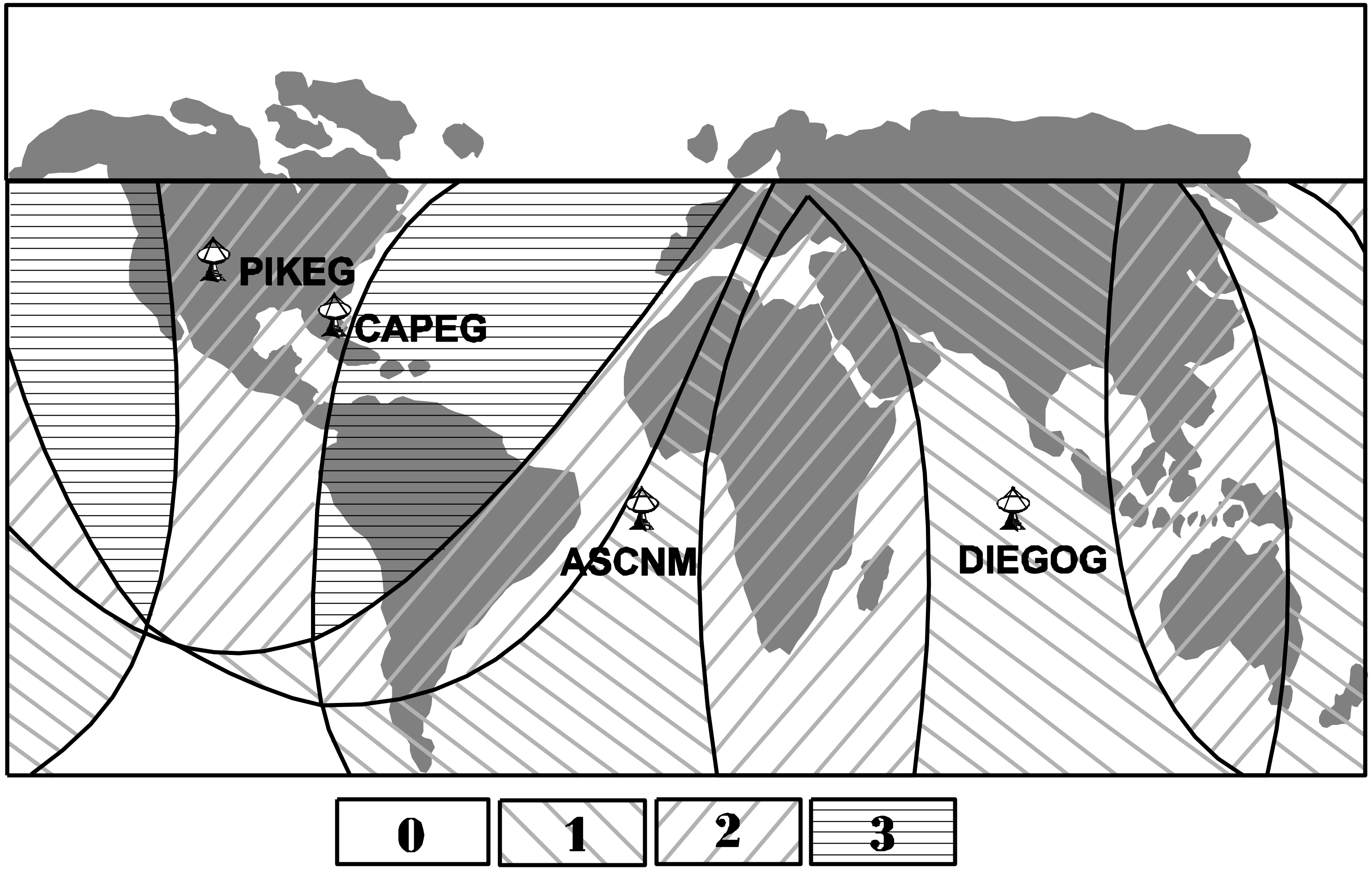
Legacy Monitor Station Coverage



Ground Antenna Coverage



Legacy Accuracy Improvement Initiative (L-AII)

- **Initiative began with legacy control segment**
- **Enabled up to 20 monitor stations and 32 satellites per Kalman filter partition**
- **National Geospatial-Intelligence Agency (NGA) monitor stations added to the existing AF monitor stations**
  - **Significantly improve system accuracy**

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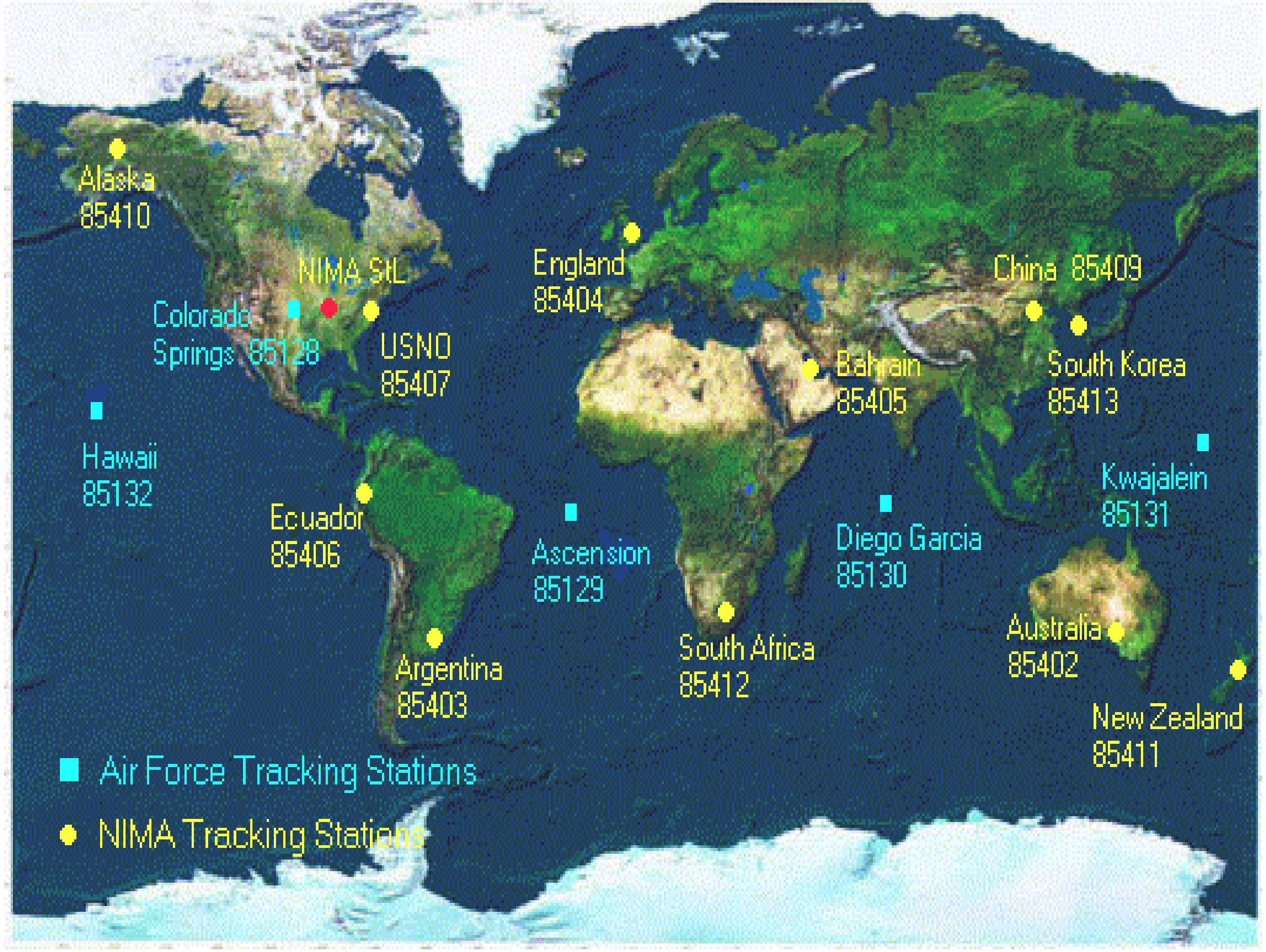
Air Force and NGA Monitor Stations



Current Control Segment

L-Band

S-Band

■ MCS at Schriever AFB, CO & Alternate MCS at VAFB

● 16 Monitor Stations 6 OCS + 10 NGA

▲ 12 Ground Antennas 4 GPS + 8 AFSCN

Not currently used operationally.

MCS Master Control Station Schriever AFB	AMCS Alternate MCS Vandenberg AFB

- GPS Ground Antennas
- AFSCN Remote Tracking Stations

Source: U.S. Air Force, Munich Satnav Summit, 2009.

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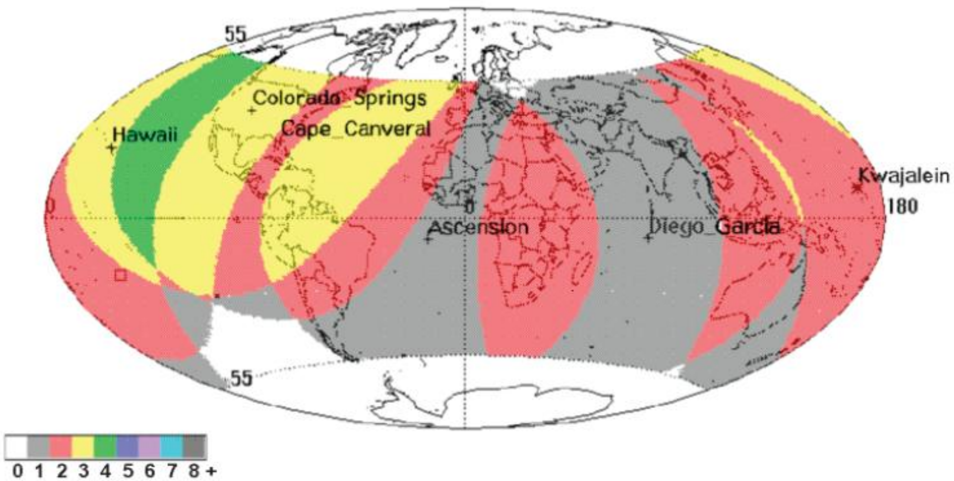
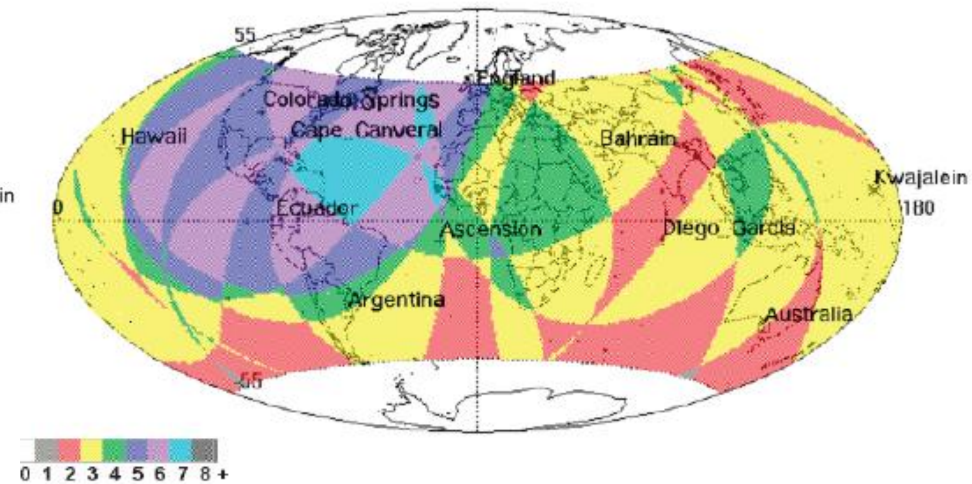
Coverage of AF and NGA Monitor Stations



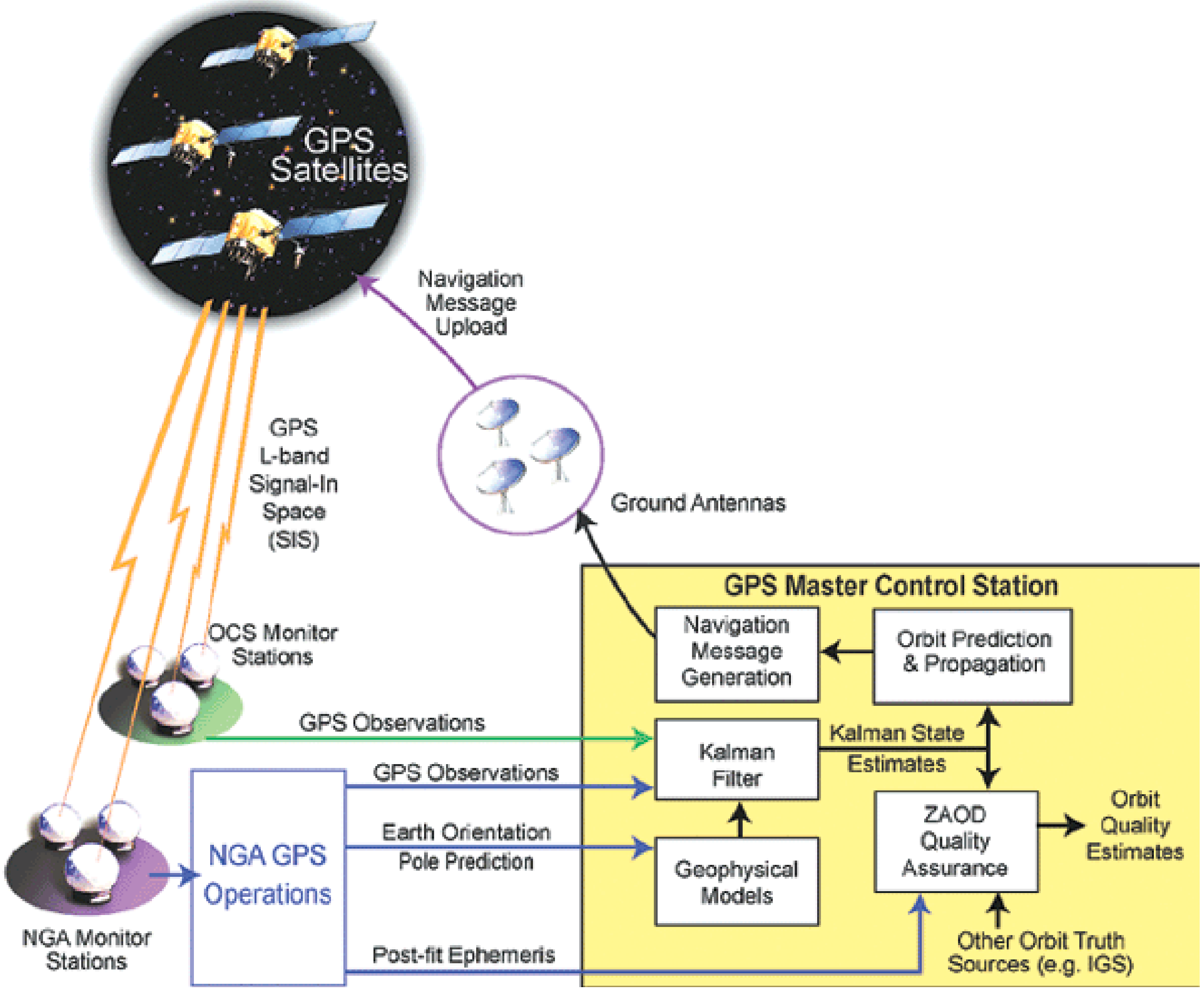
AF only



AF + NGA

Each plot shows the number of monitor stations that can see a satellite at a particular location (note that the white areas near the poles are locations where GPS satellites cannot go due to their 55 deg inclination orbits)

Control Segment – Data Flow



Source: Creel et al., "New, Improved GPS", *GPS World*, March 2006.

L-All Model Improvements

Model	Baseline GPS Model	L-AII GPS Update Model	Date Implemented
Geopotential model	WGS84 8x8 gravitational harmonics	EGM 96 (12 x 12) gravitational harmonics	Aug 31, 2005
Station Tide Displacement	Solid Tide displacement accounting for lunar and solar vertical component only	IERS 1996, including vertical and horizontal components	Jun 8, 2005
Earth Orientation Parameters	No zonal or diurnal/semi-diurnal tidal com-pensation	Restoration of zonal tides and application of diurnal/semi-diurnal tidal corrections	Jun 25, 2005
Solar Radiation Pressure model	Rockwell Rock42 model for Block II/IIA and Lockheed Martin Lookup Table for Block IIR	JPL empirically-derived solar pressure model	Sep 21 – Dec 12, 2005
Troposphere Model	Hopfield/Black model	Neill/Saastamoinen model	Jun 8, 2005

Source: Creel et al., “New, Improved GPS”, *GPS World*, March 2006.

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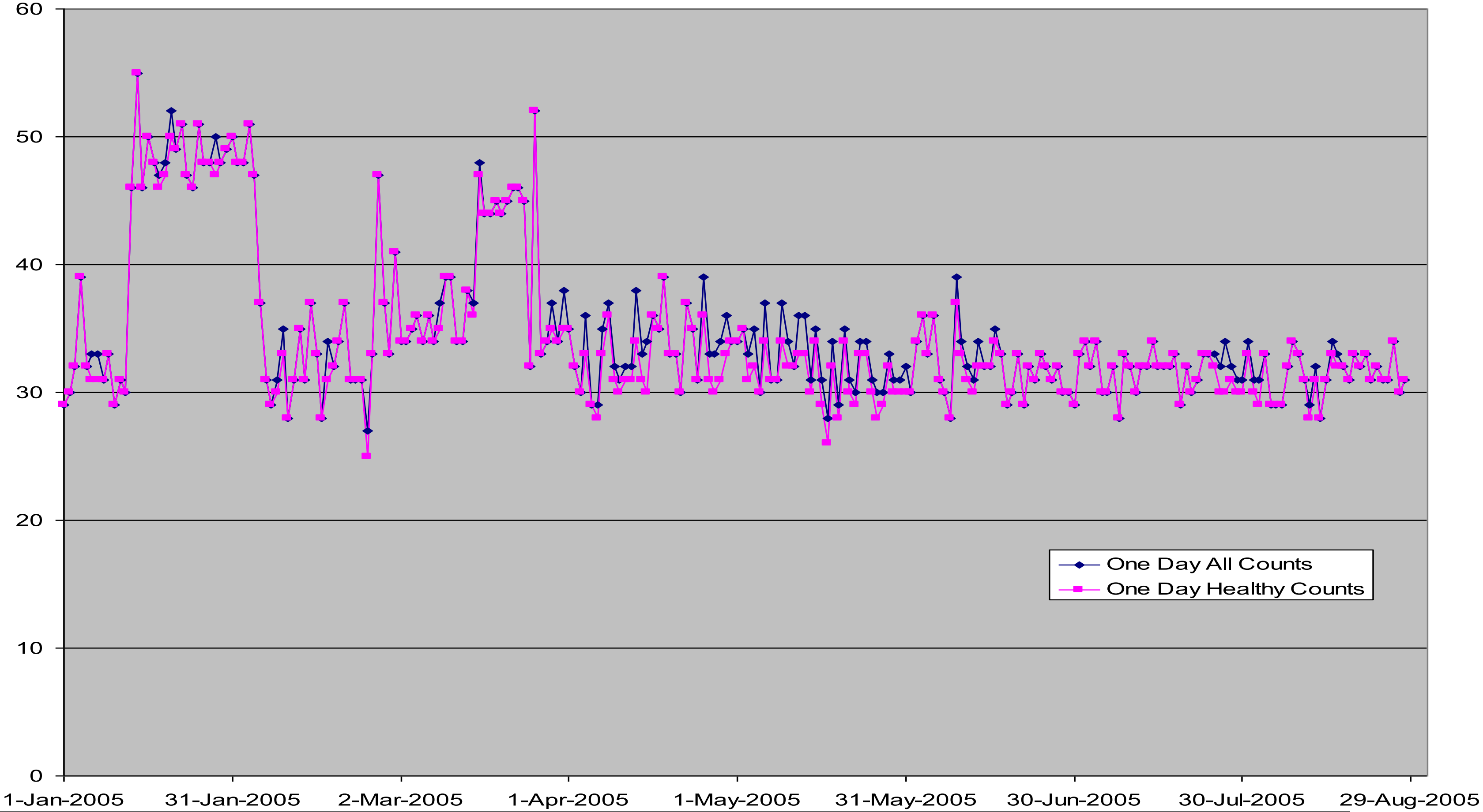
MCS Data Processing

- Monitor stations measure pseudorange and carrier phase on L1 and L2 P(Y)-code signals
  – New measurements every 1.5 s
  – Data is smoothed to mitigate noise
- Kalman filter in MCS operates upon data from all monitor stations
  – Continually (15 min epochs) estimates satellite clocks and positions (and derivatives, tropospheric delays, solar radiation pressure levels, monitor station clocks, etc)
- MCS next predicts positions and clock errors for the satellites over the maximum period between satellite uploads
  – Curve fits are performed to obtain broadcast navigation data for nominal 4 hour fit intervals
  – Nominal transmission interval/set is 2 hours

Frequency of Daily Uploads



Daily uploads for each satellite is typical. More frequent uploads scheduled when necessary for poor-performing satellites.

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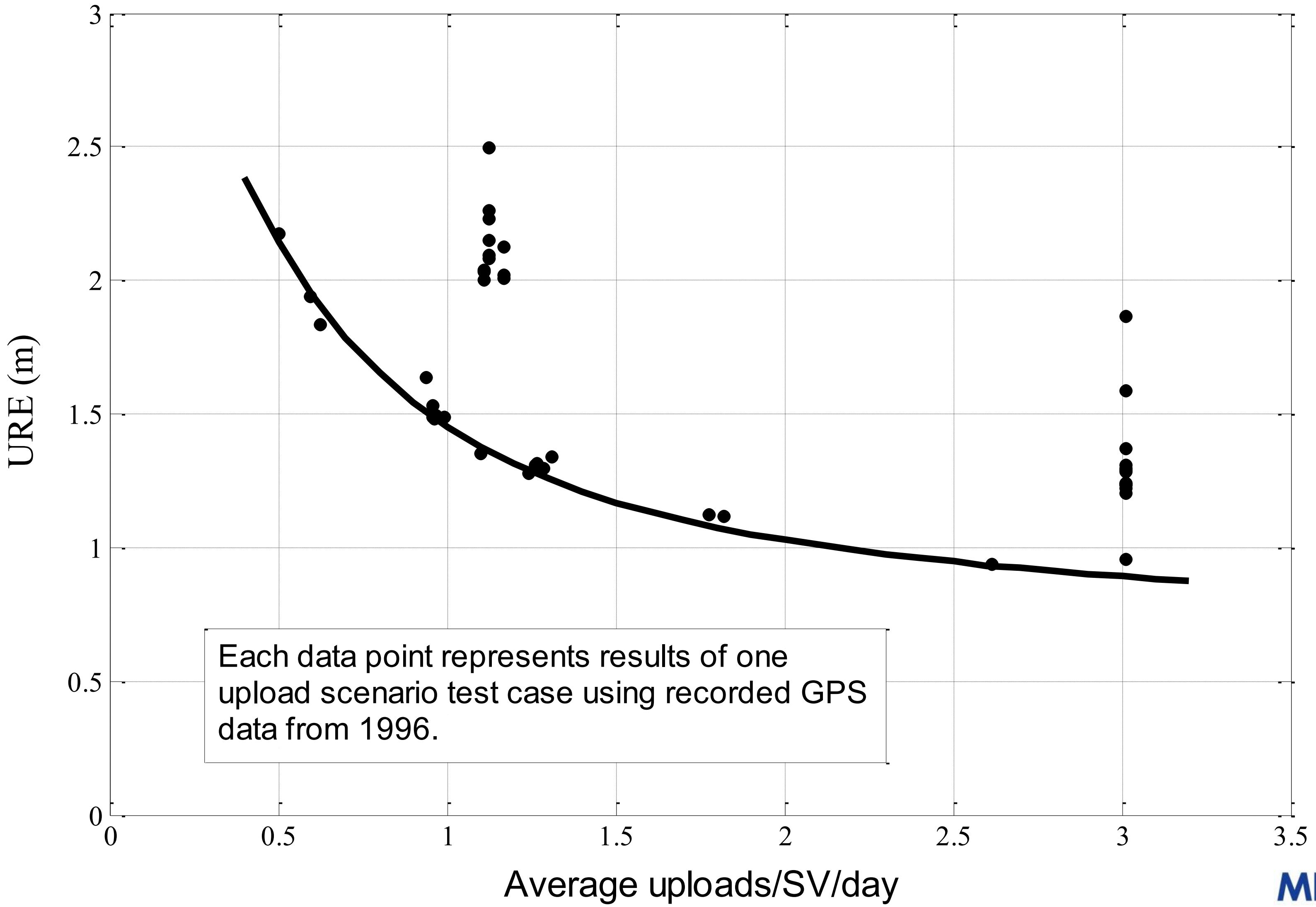
System Range Accuracy vs Upload Frequency



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CENTER FOR ADVANCED AVIATION SYSTEM DEVELOPMENT (CAASD)

GPS Measurements and Error Sources

Chris Hegarty

April 2010

CAASD

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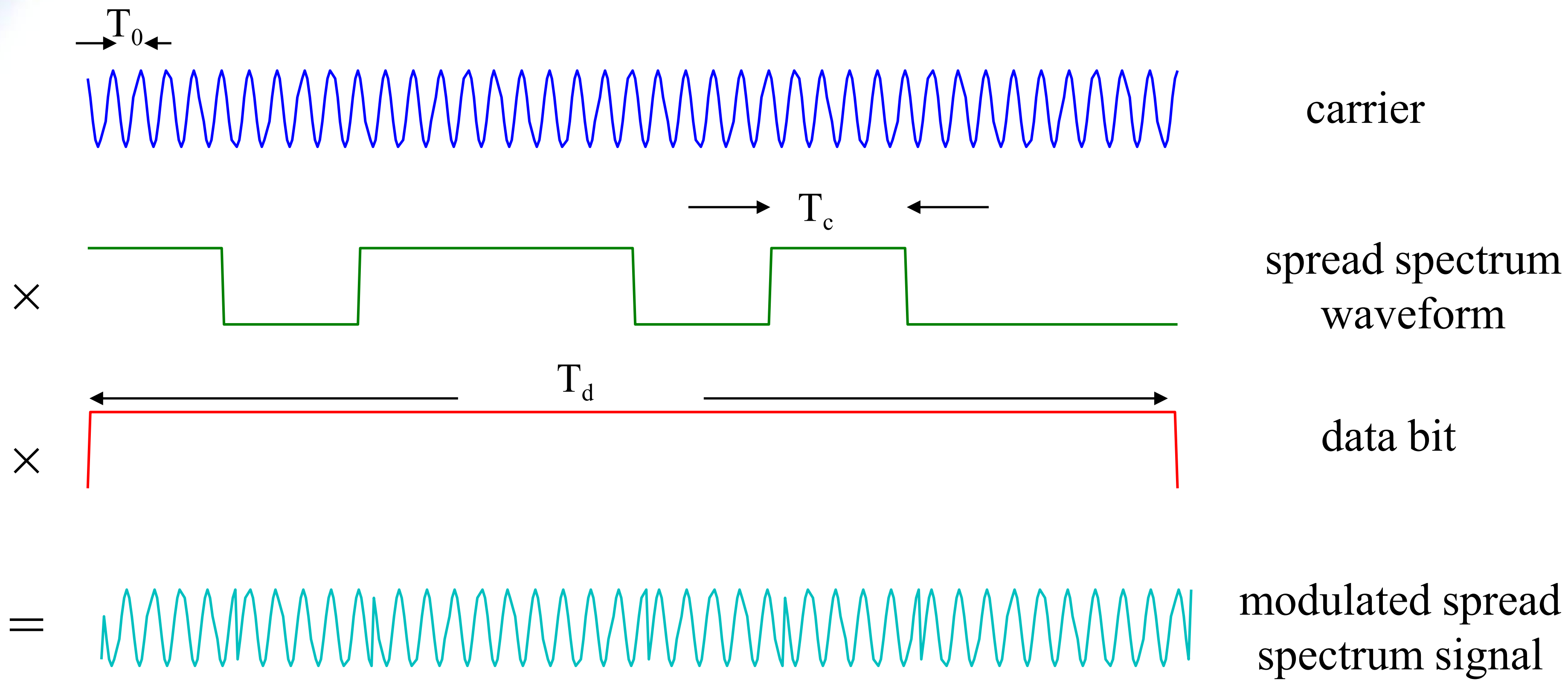
GPS Measurements and Error Sources

- **Measurements**
  - Pseudorange
  - Carrier phase
- **Error sources**
  - Satellite ephemeris errors
  - Satellite clock errors
  - Selective availability (discontinued in 2000)
  - Ionospheric and tropospheric delay
  - Multipath
  - Receiver noise, interference, and biases
- **Measurement combinations**
  - Carrier smoothing
  - Single and double differences
  - Dual-frequency

GPS C/A-code and P(Y)-code Signal Components (not to scale)

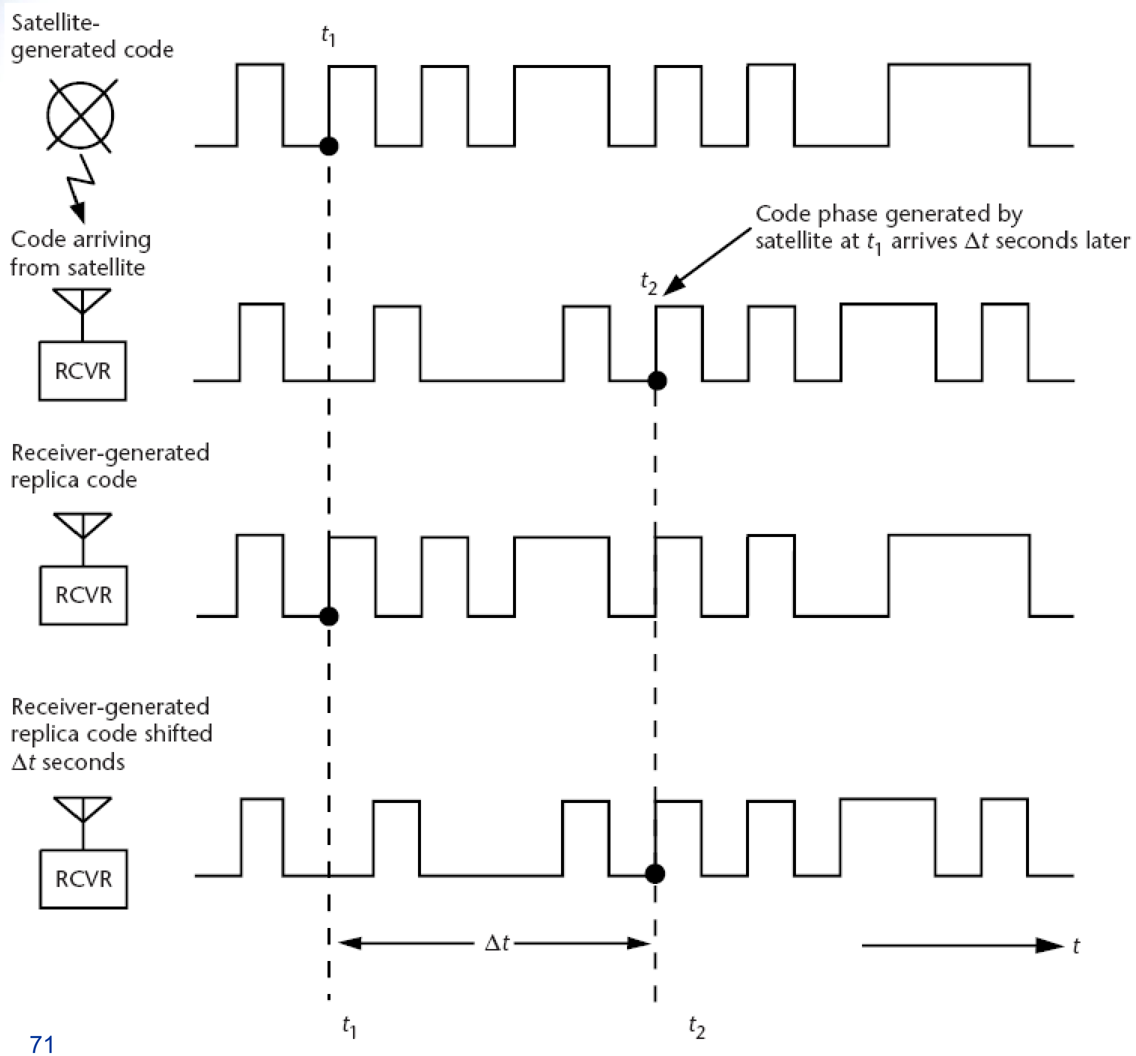


$R_c = 1/T_c$ = chipping rate (chips/s)
= 1.023 MHz (C/A), 10.23 MHz (P/Y)

$R_d = 1/T_d$ = data rate (bits/s)
= 50 bps

$f_0 = 1/T_0$ = carrier frequency (Hz)
= 1575.42 MHz (L1), 1227.6 MHz (L2)

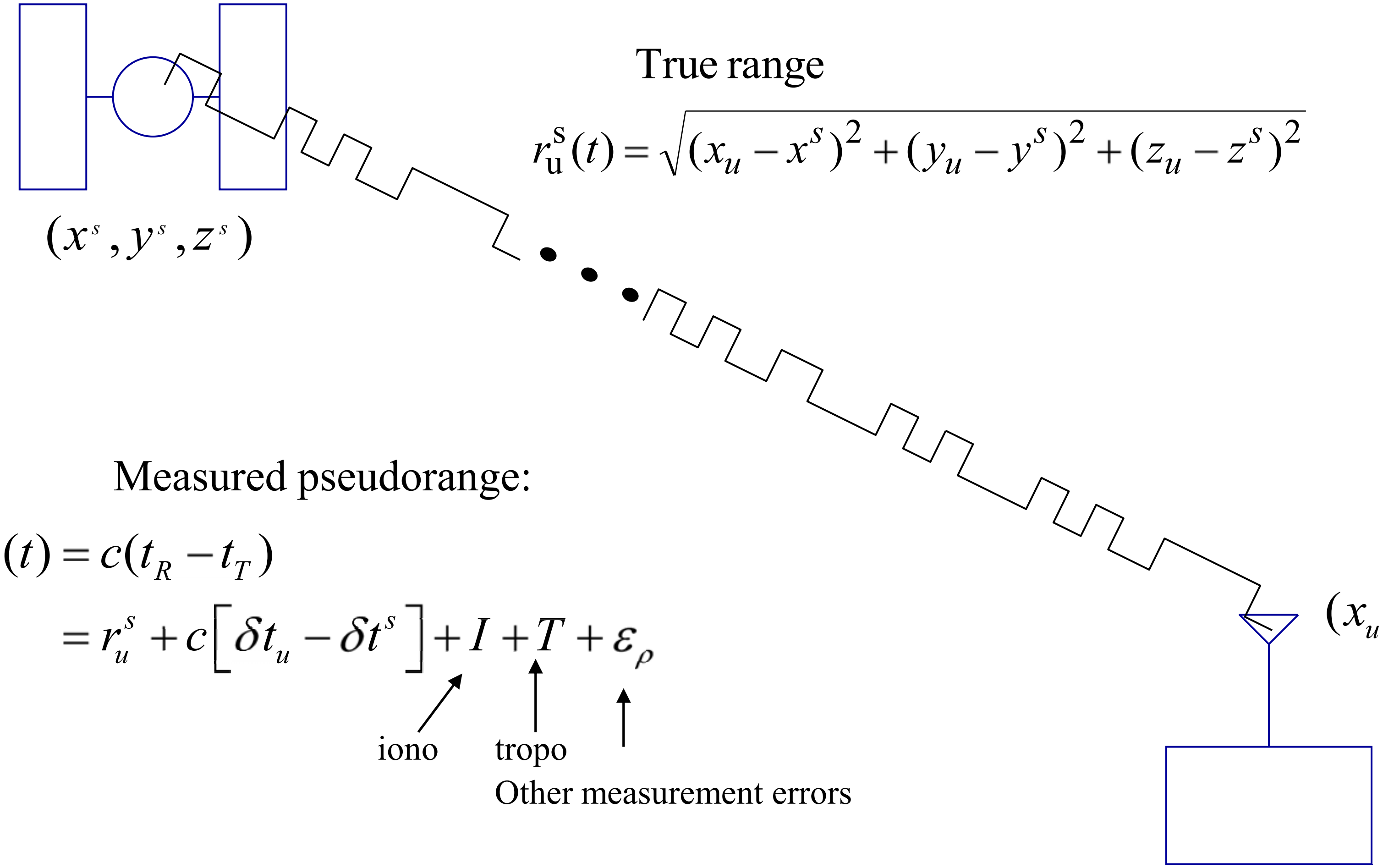
GPS Pseudorange Measurement



- **With a perfect user clock, signal transit time can be multiplied by the speed of light to yield a range measurement**
- **With an imperfect user clock, measured transit time for each satellite's signals are biased by a common user clock error – measurements are referred to as *pseudorange*.**

Pseudorange Model

Satellite clock: $t^{S}(t) = t + \delta t^{S}(t)$



(x^{s}, y^{s}, z^{s})

True range

$$r_{u}^{s}(t) = \sqrt{(x_u - x^s)^2 + (y_u - y^s)^2 + (z_u - z^s)^2}$$

Measured pseudorange:

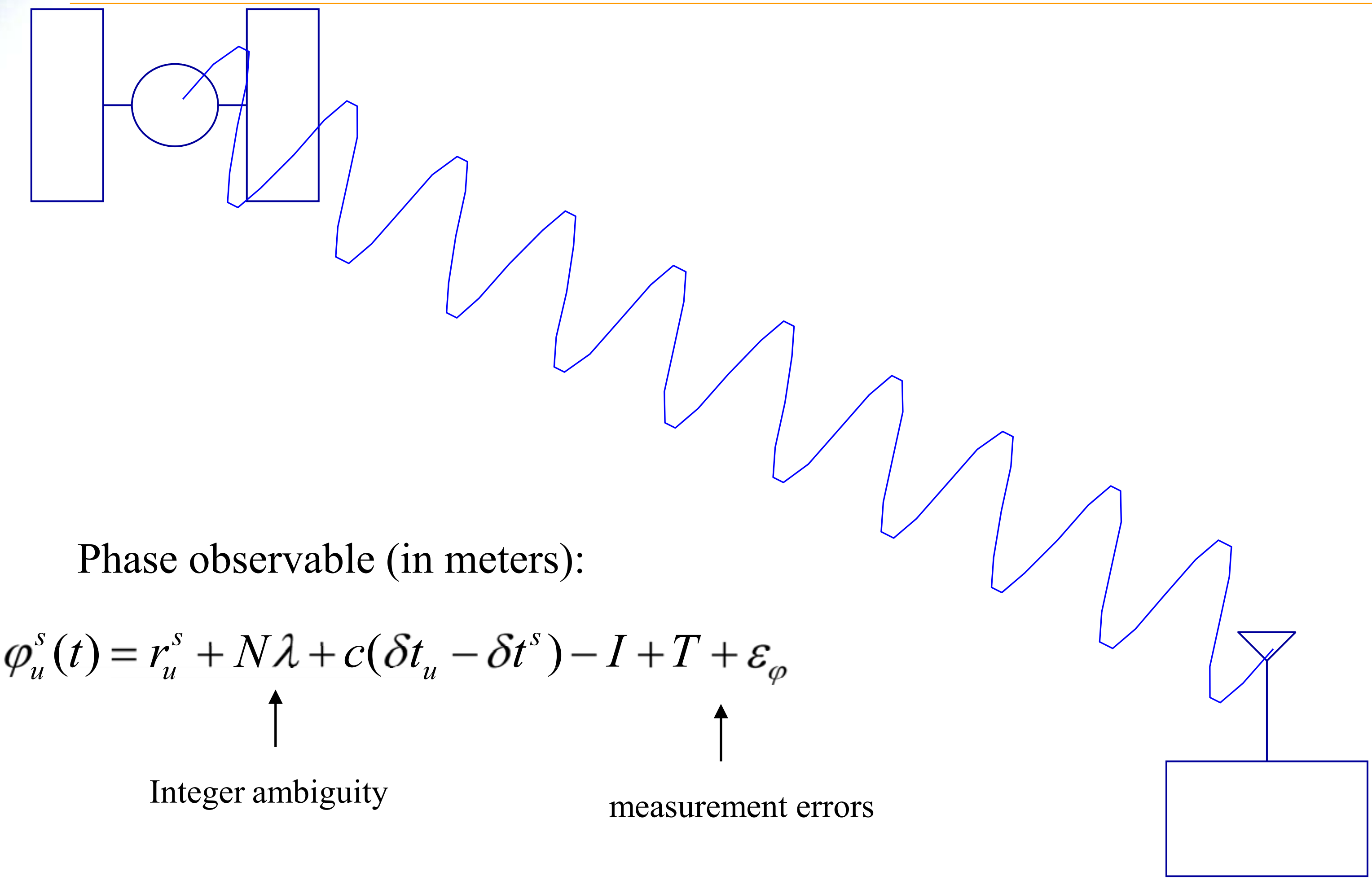
$$\rho_{u}^{s}(t) = c(t_R - t_T)$$
$$= r_{u}^{s} + c\left[\delta t_u - \delta t^{s}\right] + I + T + \varepsilon_{\rho}$$

iono — tropo — Other measurement errors

(x_u, y_u, z_u)

User clock: $t_u(t) = t + \delta t_u(t)$

Carrier Phase Model



Phase observable (in meters):

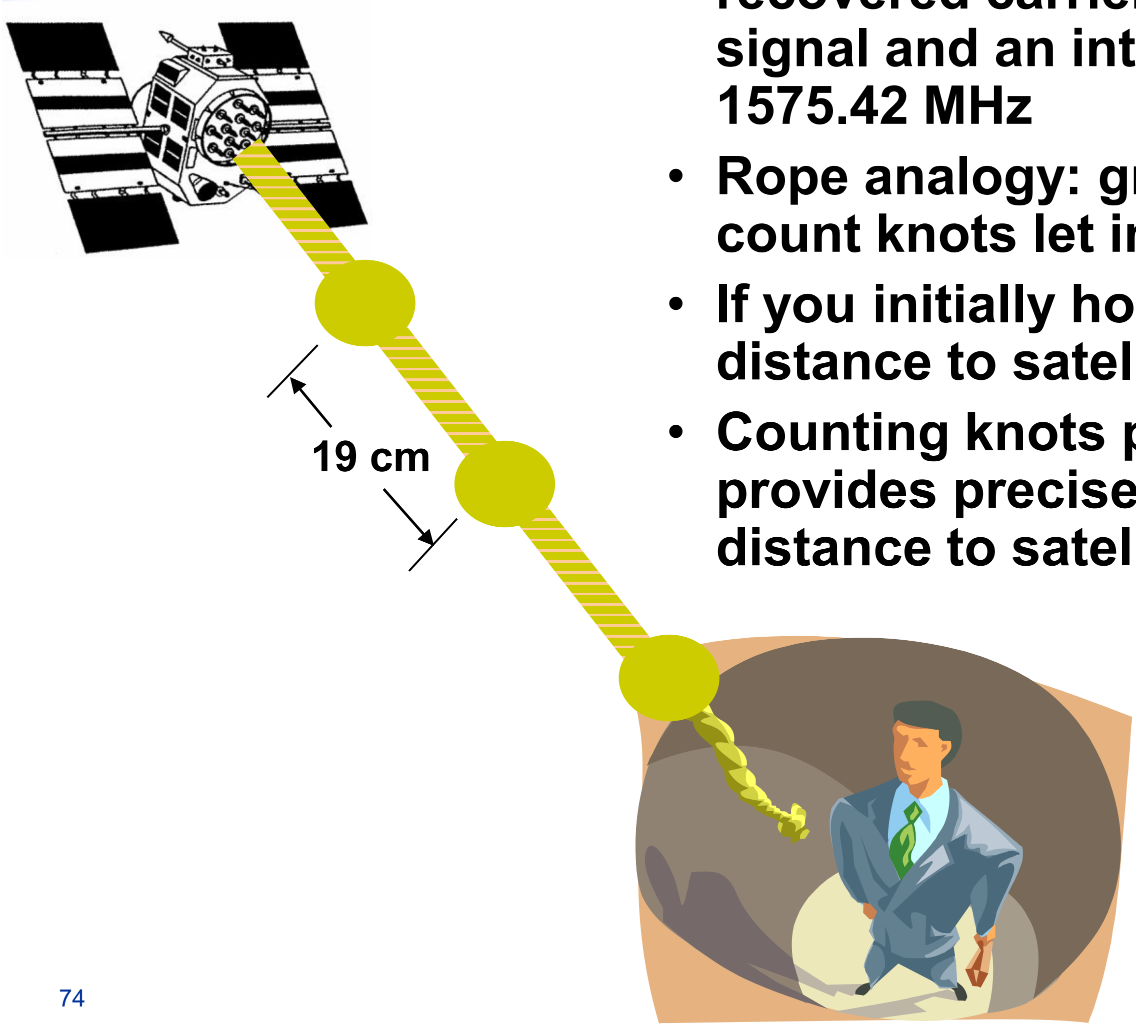
$$\varphi_u^s(t) = r_u^s + N\lambda + c(\delta t_u - \delta t^s) - I + T + \varepsilon_\varphi$$

Integer ambiguity

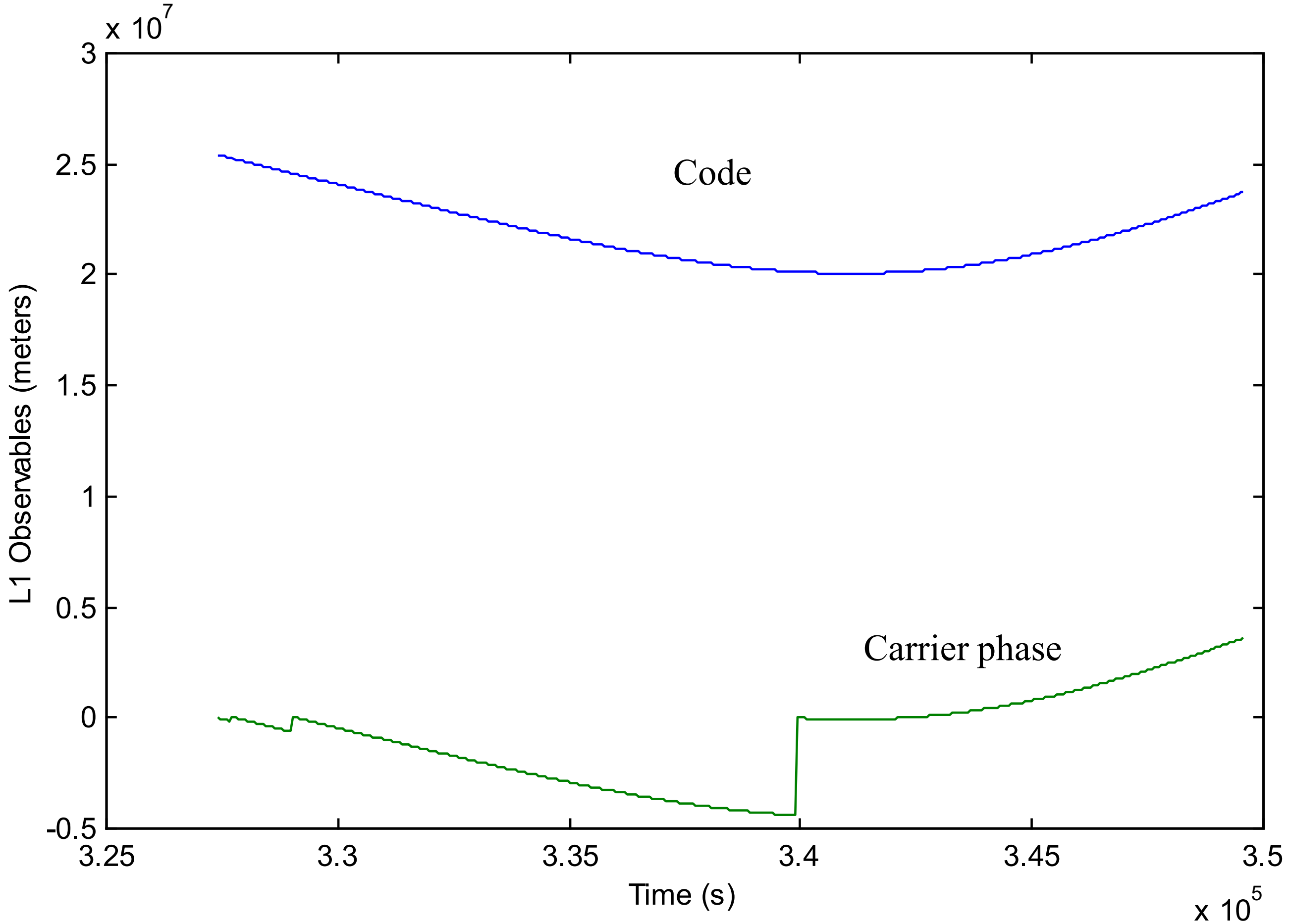
measurement errors

Understanding Carrier Phase Measurements

- Receiver tracks phase difference between recovered carrier component of received signal and an internally generated carrier at 1575.42 MHz
- Rope analogy: grab rope tied to satellite and count knots let in/out
- If you initially hold a knot, you know that distance to satellite is 19 cm *N*
- Counting knots passing through your hand provides precise measurement of change in distance to satellite with time (velocity)



SV2 Observables - One Pass



Key features: Code pseudorange provides unambiguous absolute range. Carrier phase has ambiguity, suffers cycle slips, but is much less noisy.

Getting Familiar with GPS Data

- Raw GPS measurement data available for free on the Internet from a wide variety of sources
- One example: International GNSS Service (IGS)
  - Logs raw measurement data from a global network with over 350 active stations
  - Wide variety of other products (GPS clock/orbit estimates, ionospheric/tropospheric delay data, etc)
- Common file format for data exchange is Receiver Independent Exchange Format (RINEX)
  - Simple ASCII file types
  - Description available at: ftp://igscb.jpl.nasa.gov/igscb/data/format/rinex300.pdf

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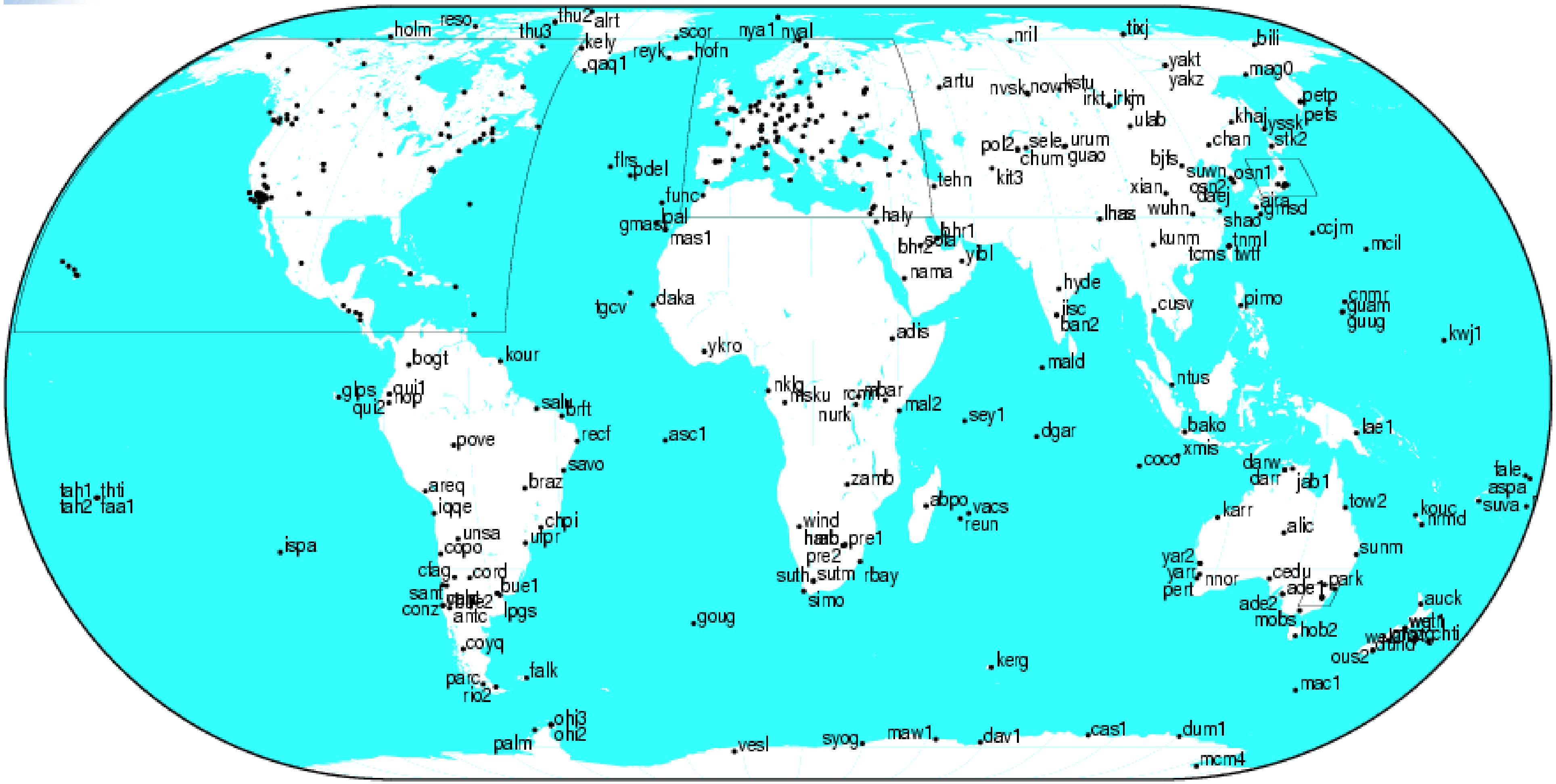
IGS Network



GMT 2009 Mar 15 16:48:34

Source: http://igscb.jpl.nasa.gov/network/complete.html

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Raw GNSS Data in RINEX Format from Ferrara, Italy (UNFE0010.09o)

RINEX header with:
- Station information
- Types of measurements
- Date/time of 1st measurement

First data record with:
- Date/time of measurements
- List of satellites
- Raw data (for this file, L1 phase, L2 phase, C/A pseudorange, L1 P pseudorange, L2 P pseudorange, L1 doppler, and L2 doppler)

```
     2.00           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
teqc  2000Feb29                         20090101 23:05:10UTCPGM / RUN BY / DATE
UNFE                                                        MARKER NAME
12756M001                                                   MARKER NUMBER
Marco Gatti         University of Ferrara - Italy           OBSERVER / AGENCY
12128               Ashtech Z18         0065      ZT17      REC # / TYPE / VERS
10401               ASH701941.B                             ANT # / TYPE
  4438219.7300   910967.3706  4474248.9608                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     7    L1    L2    C1    P1    P2    D1    D2            # / TYPES OF OBSERV
    30.0000                                                 INTERVAL
     0                                                      LEAP SECONDS
  2009     1     1     9     0    0.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER
 09  1  1  9  0  0.0000000  0 10G31G30G18G24G16G21G29R 6R 7R23
 -17155756.848 9 -13368122.035 7  22550586.505    22550586.505    22550587.553
     -2614.069       -2036.928
  -2498026.027 7  -1946514.208 4  23400100.190    23400100.190    23400103.027
     -3546.647       -2763.634
  -3136315.532 7  -2443877.227 2  24723584.578    24723584.578    24723587.121
      3131.241        2439.917
  -3919173.767 8  -3053900.030 6  21441677.839    21441677.839    21441680.733
     -2068.477       -1611.793
  -5872549.652 9  -4576012.646 7  21555128.309    21555128.309    21555129.449
      2264.219        1764.342
 -18530496.051 9 -14439344.591 7  20828216.730    20828216.730    20828217.680
      1086.477         846.615
 -17158968.306 8 -13370621.401 6  21820347.325    21820347.325    21820348.760
     -2145.233       -1671.595
 -12783419.939 7  -9942654.090 7  21684821.176    21684821.176    21684816.339
       899.959         699.957
  -9328997.785 7  -7255883.174 6  22691019.446    22691019.446    22691019.457
      3841.262        2987.630
 -11113637.034 8  -8643937.987 6  22585160.757    22585160.757    22585163.188
      2868.101        2230.743
```

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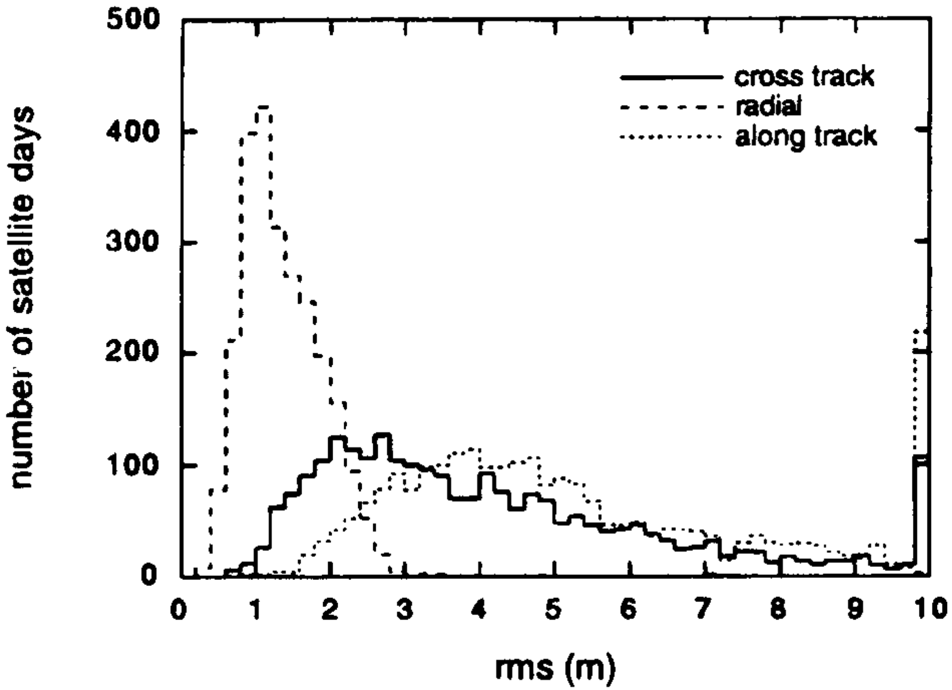
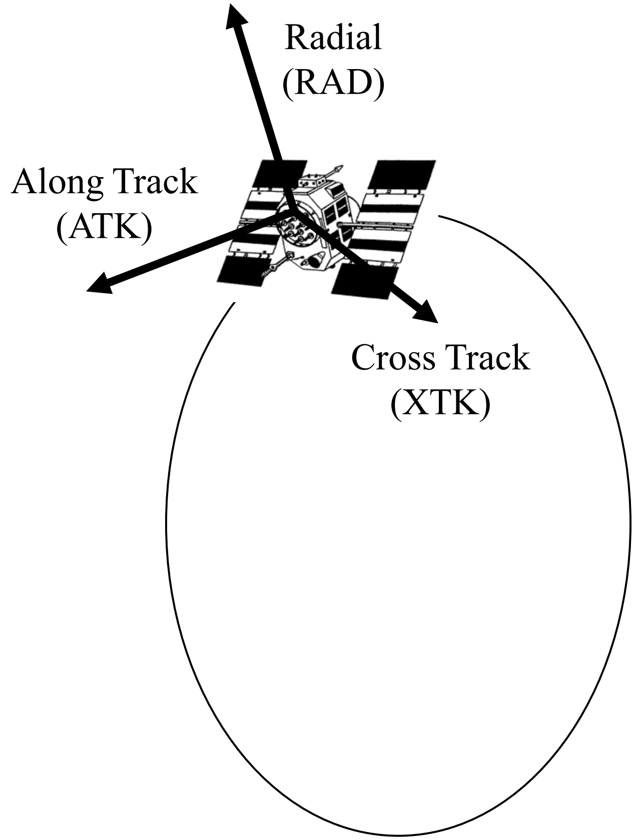
Satellite Ephemeris Errors

- **Ephemeris errors are differences between true satellite position and position computed using GPS navigation message**
- **Sources:**
  - **Selective Availability (SA) epsilon (was never observed - SA now discontinued)**
  - **GPS Control Segment estimation errors**
  - **Age of GPS navigation message data**

Typical Ephemeris Errors

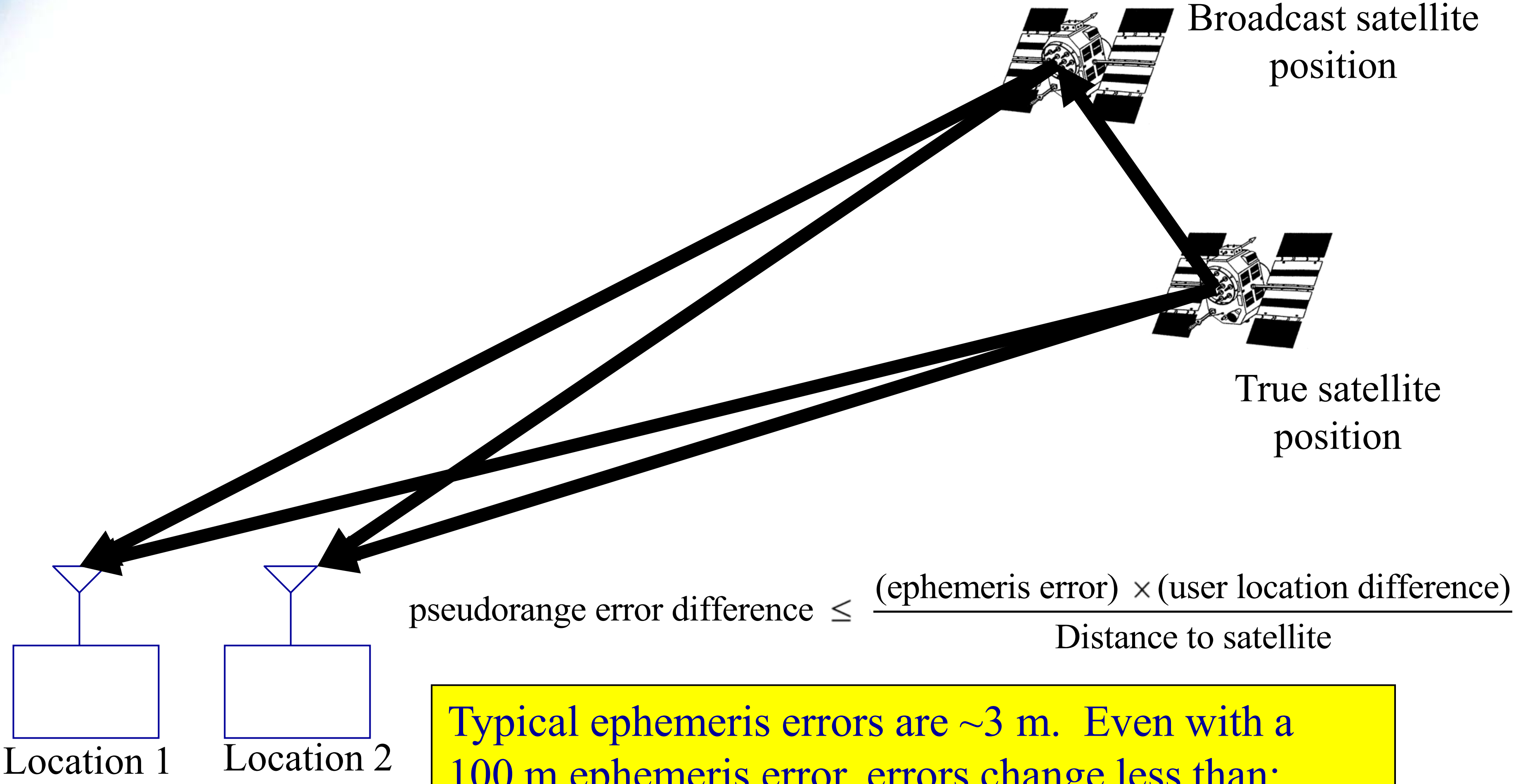


Distribution of Ephemeris Errors over ~4 months



Reference: Zumberge and Bertiger, "Ephemeris and Clock Navigation Message Accuracy," in Parkinson/Spilker, *GPS: Theory and Applications*, AIAA, 1996. More recent data suggests a mean 3D error of ~3 m, with a 90th percentile of ~6 m.

Variation of Ephemeris Errors with Location



$$\text{pseudorange error difference} \leq \frac{(\text{ephemeris error}) \times (\text{user location difference})}{\text{Distance to satellite}}$$

Typical ephemeris errors are ~3 m. Even with a 100 m ephemeris error, errors change less than: (5 mm/km) (difference in user position in km)

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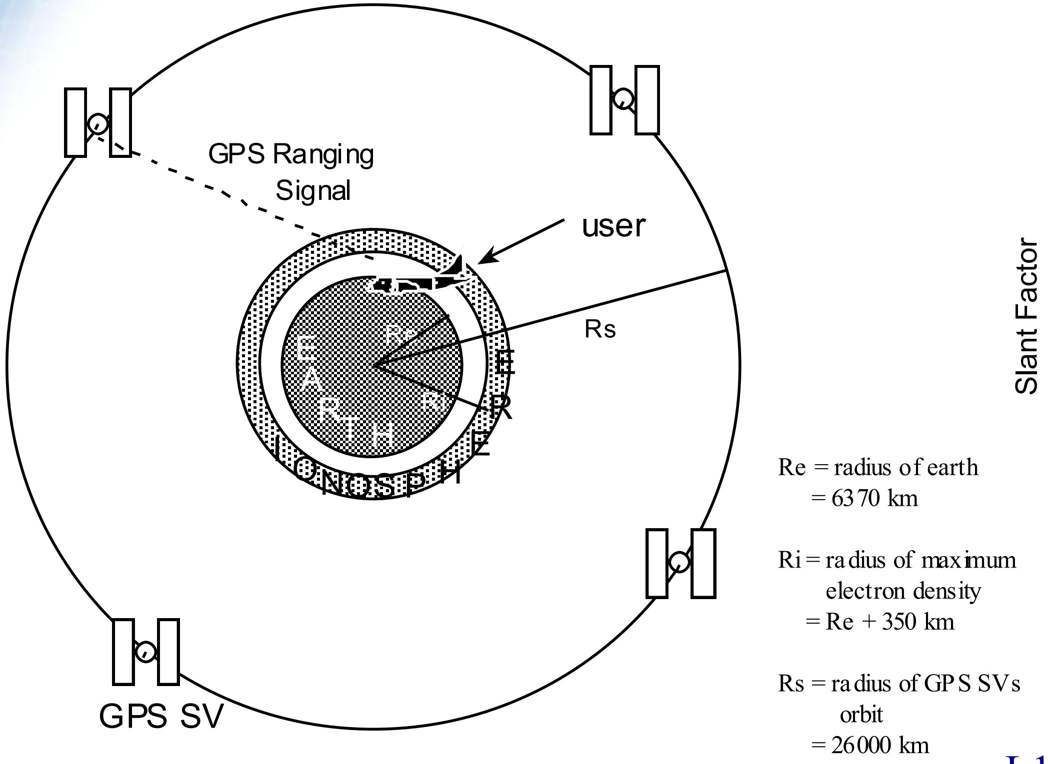
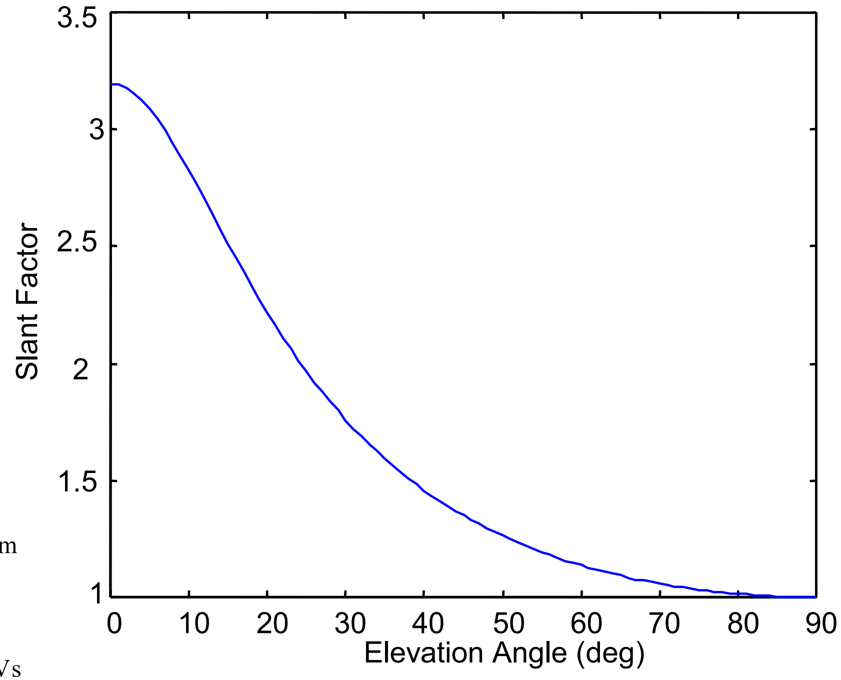
Ionospheric Delay Errors

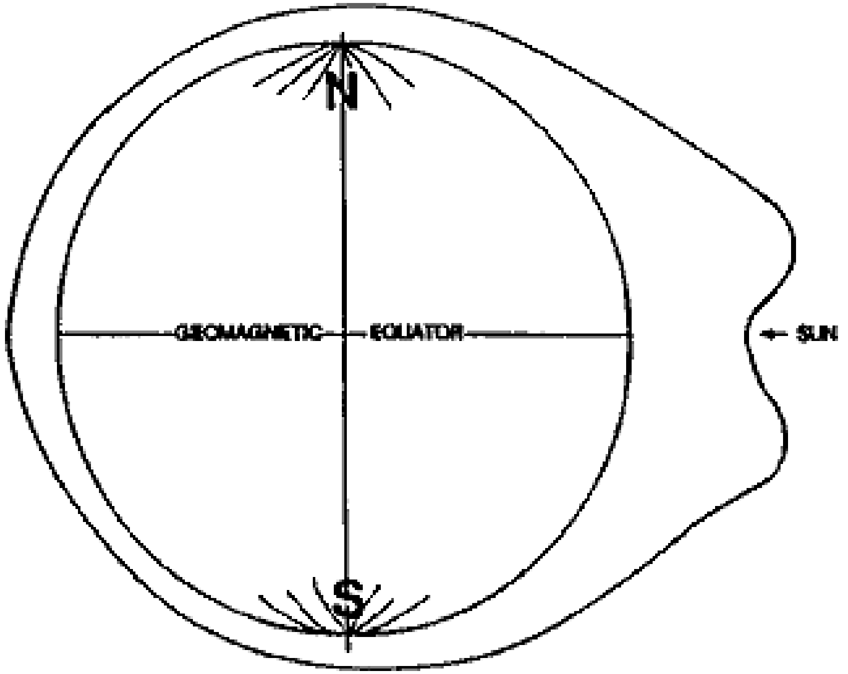




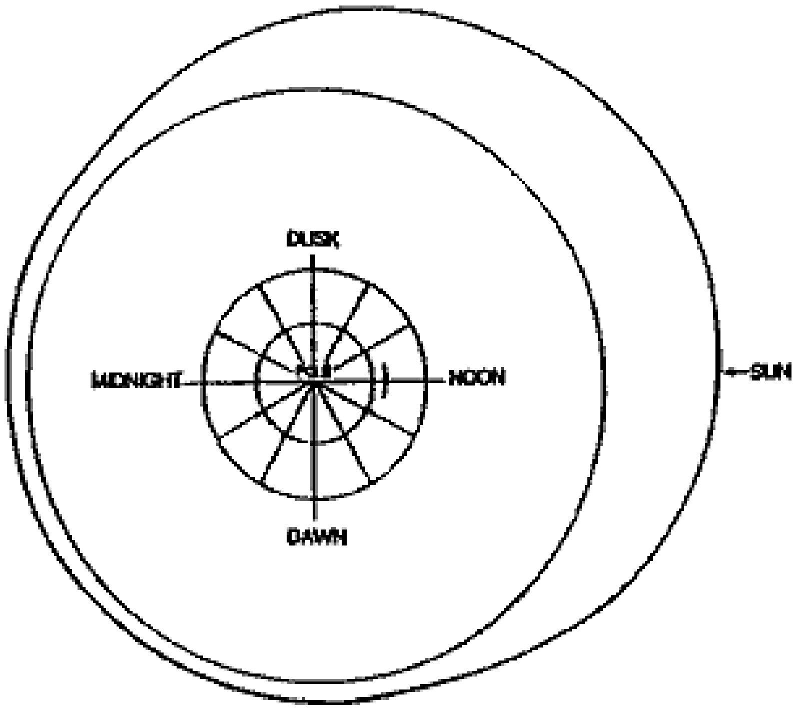
GPS signals are delayed as they traverse through the ionized layer of the Earth's atmosphere

L1 vertical delays range from 3 - 30 m (depending on time of day, phase of 11-year solar cycle, and solar activity), delays for signals along oblique paths may be greater by a factor of 3 for low elevation angles (L2 delays are ~1.6 times larger than L1 delays)

Variation of Ionosphere with Time of Day and Latitude

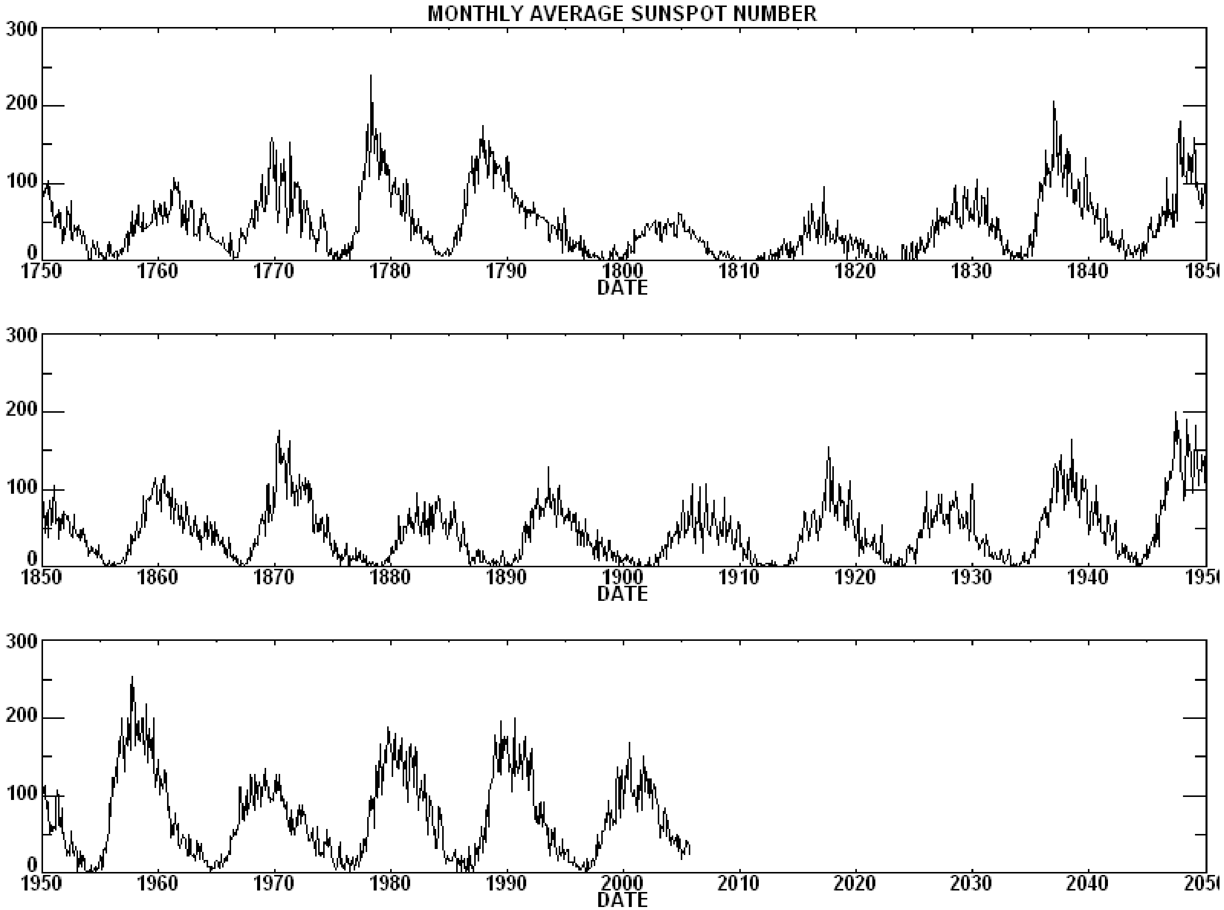


Latitude Profile



Longitude Profile

11 Year Solar Cycle



Next solar maximum expected ~2011-2012. Ionospheric delays, on average, are greatest during solar maximum (max/min average delay ratio ~2-3).

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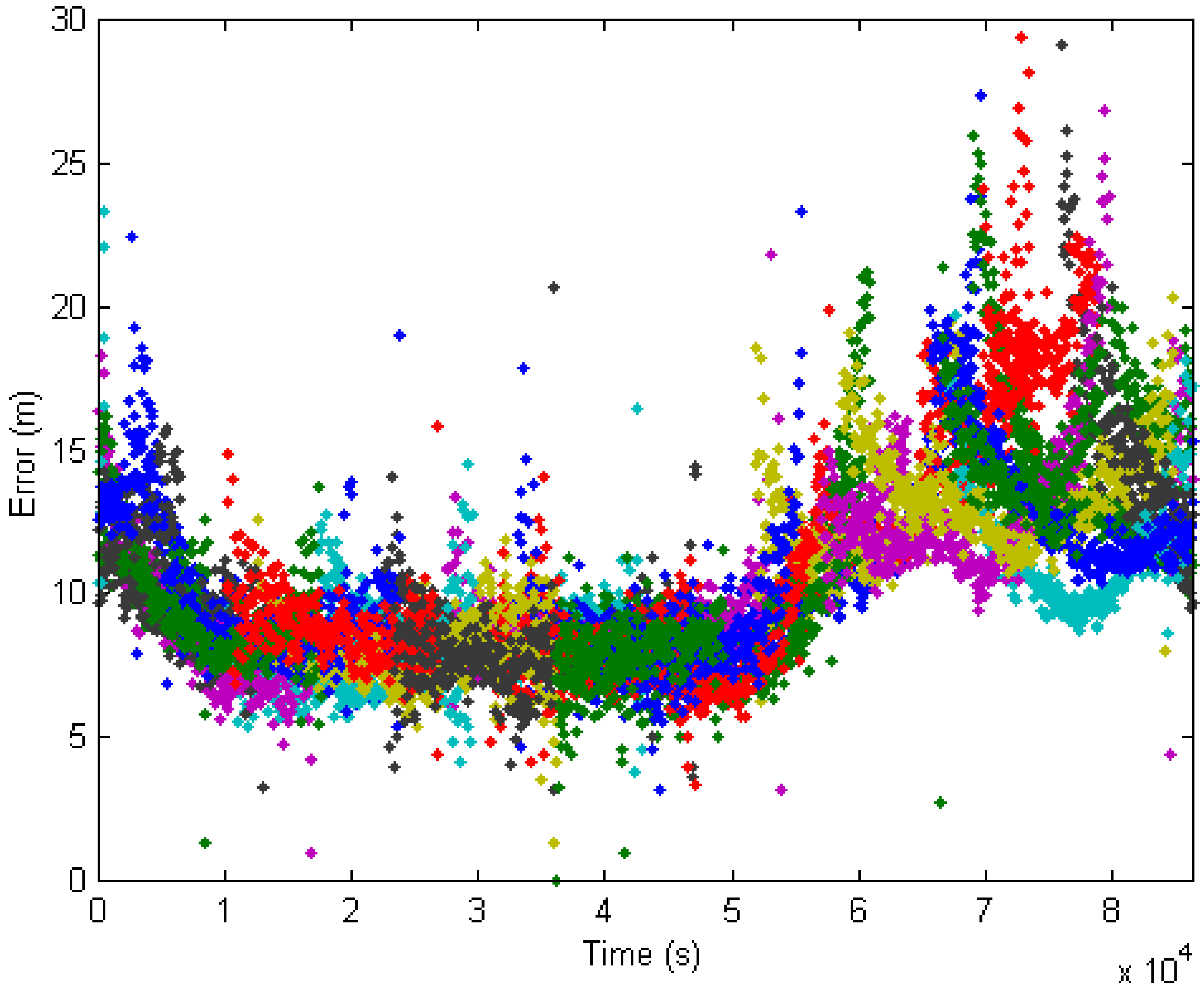
Typical Ionospheric Delay Errors



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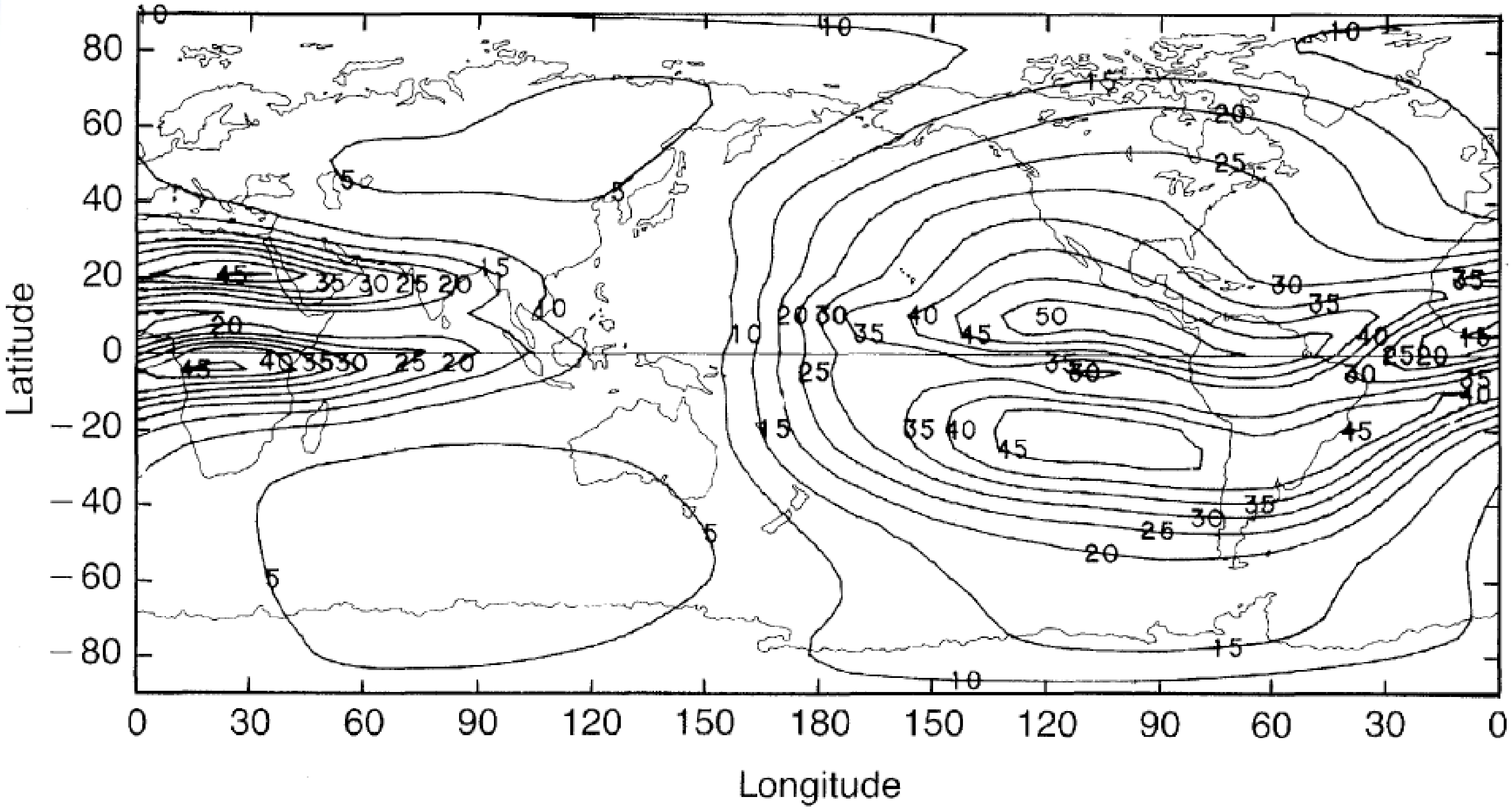
Vertical Ionospheric Delays at Solar Maximum



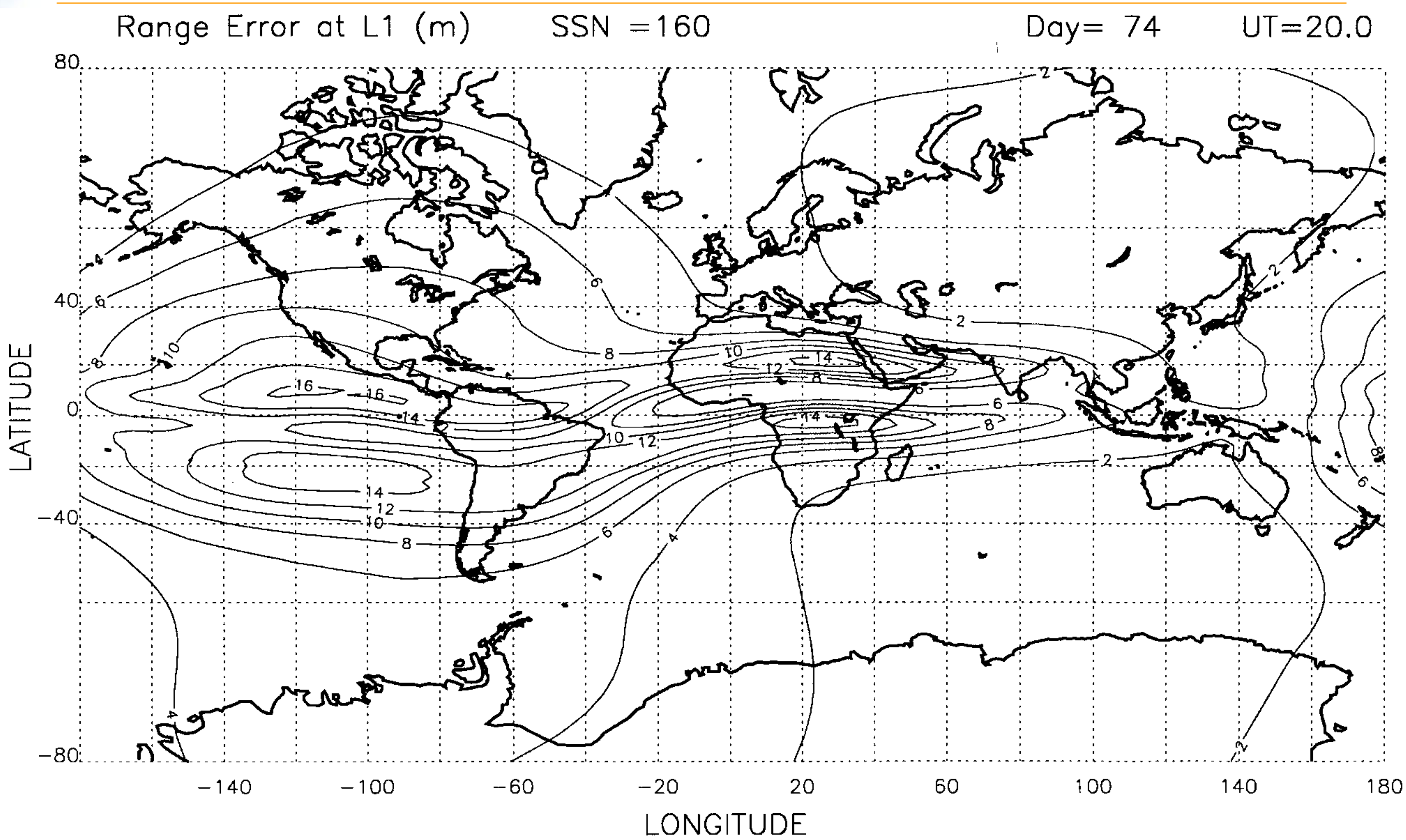
Maximum monthly average L1 vertical Ionospheric delays in nanoseconds (10 ns ~ 3 meters) for March 1990 at 2000 UTC.

Source: Klobuchar, *GPS World*, April 1991.

Correcting for Ionospheric Delay Errors

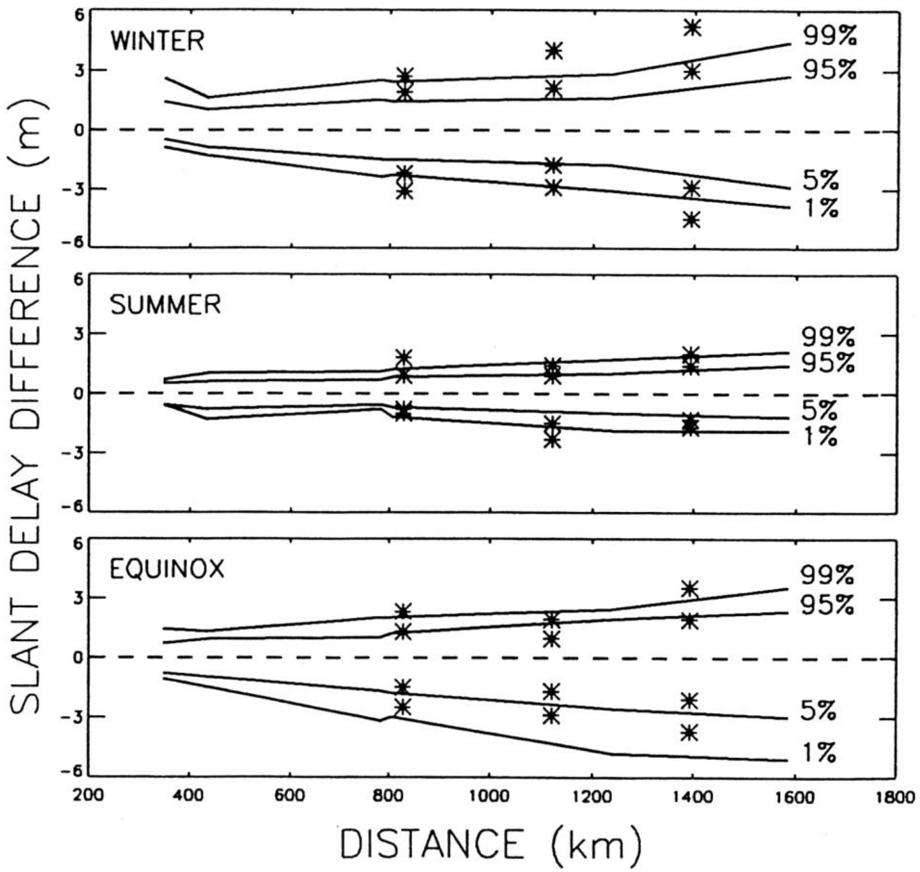
- Dual-frequency user
  - Ionospheric delay effects are dispersive – delays are inversely proportional to carrier frequency squared
  - With dual-frequency equipment, user equipment can nearly perfectly remove ionospheric delay errors
- Single-frequency user
  - Each GPS satellite broadcasts ionospheric delay model coefficients (Klobuchar model)
  - Model typically removes ~half of ionospheric delay range error
  - 90% of time, residual range error is < 10 m
  - Residual errors are highly correlated
    - Model tends to either overestimate or underestimate ionospheric delay on all satellites simultaneously
- DGPS

Spatial Variability at Solar Maximum



SSN = sunspot number

Spatial Variability of Ionospheric Delay



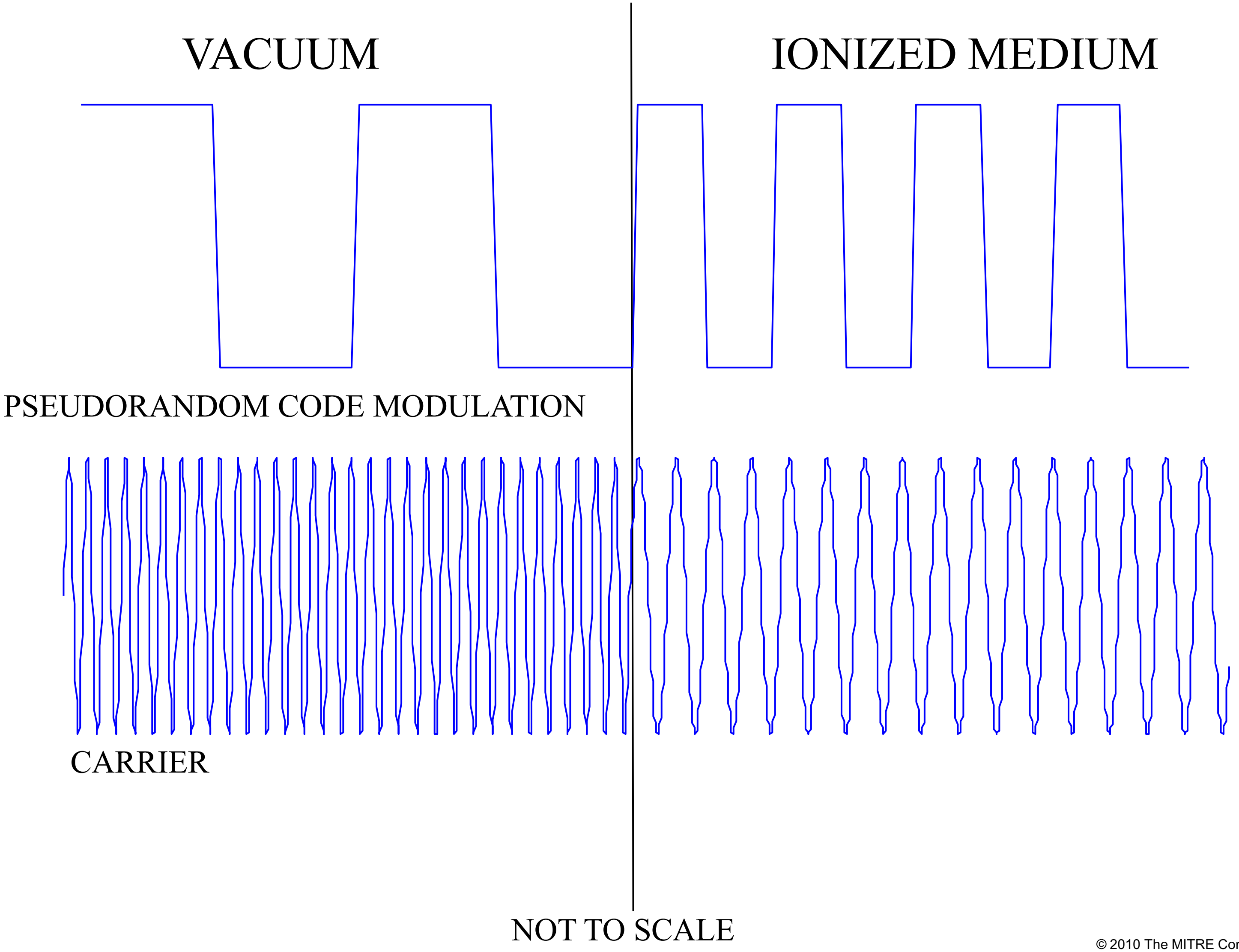
1992-1993 L1 data(daytime hours only), Solid lines = North-South station orientation, Asterisks = East-West station orientation, errors ~2× larger during solar maximum

Ref: Klobuchar, Doherty, El-Arini, "Potential Ionospheric Limitations to Wide-Area DGPS," ION GPS-93

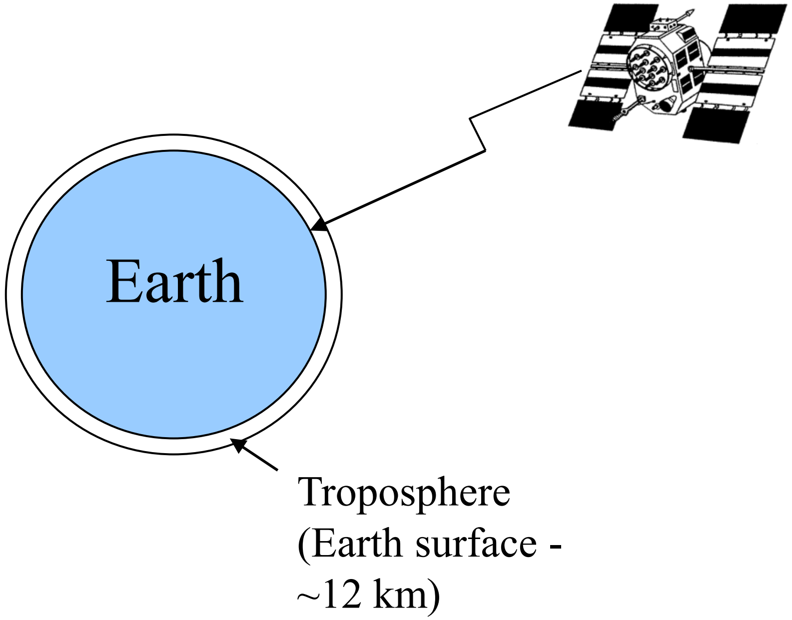
Ionospheric Divergence

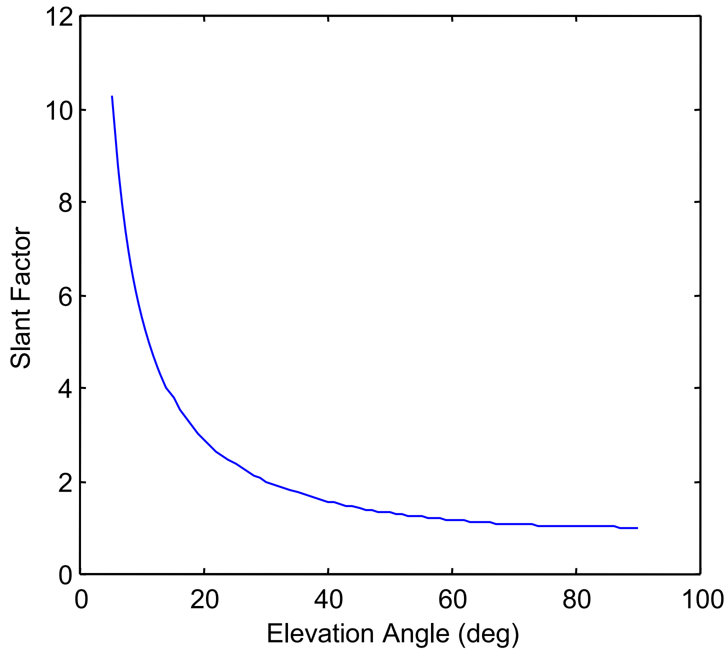
- As the distance between the user and a GPS satellite grows:
  - the number of carrier wavelengths in between grows
  - the number of code chips in between grows
- When the number of free electrons in the ionosphere grows (with distance between the user and GPS satellite fixed):
  - the number of carrier wavelengths between the user and the GPS satellite decreases (phase velocity increases)
  - the number of code chips between the user and the GPS satellite increases (group velocity decreases)
- Care must be taken when aiding code pseudorange measurements with carrier phase measurements

Ionospheric Divergence (cont'd)



Tropospheric Delay





GPS signals are delayed as they traverse through the dry gases and water vapor comprising the Earth's lower atmosphere

Vertical delays range from ~2 - 3 m (depending on altitude of user and local weather), delays for signals along oblique paths may be greater by a factor of 10 for low elevation angles

Correcting for Tropospheric Errors

- **Many correction models exist**
- **Simplest:**
  - Use global average weather conditions to determine vertical tropospheric error
  - Apply altitude correction and elevation angle scale factor
  - Residual errors ~25 cm for overhead satellite
- **More complicated:**
  - Use average local weather (e.g., pressure, temperature, humidity) determined from table lookup
  - Apply altitude correction and elevation angle scale factor
  - Residual errors ~5 cm for overhead satellite
- **Most complicated:**
  - Use meteorological sensors
  - Residual errors ~3 cm for overhead satellite

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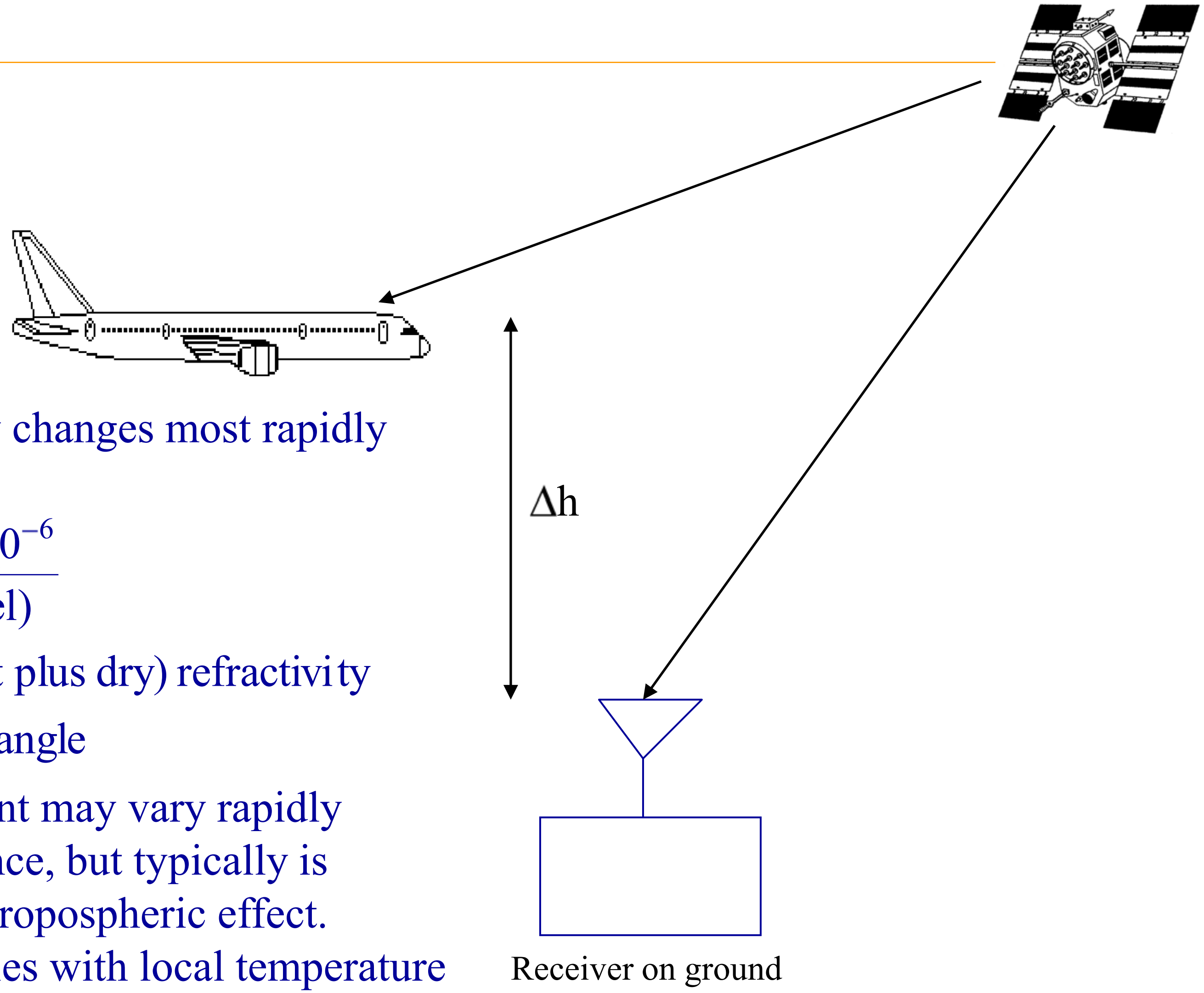
Variation of Tropospheric Delay with Altitude



Tropospheric delay changes most rapidly with altitude:

$$\Delta\tau \approx N_R \frac{\Delta h \times 10^{-6}}{\sin(el)}$$

N_R = Total (wet plus dry) refractivity

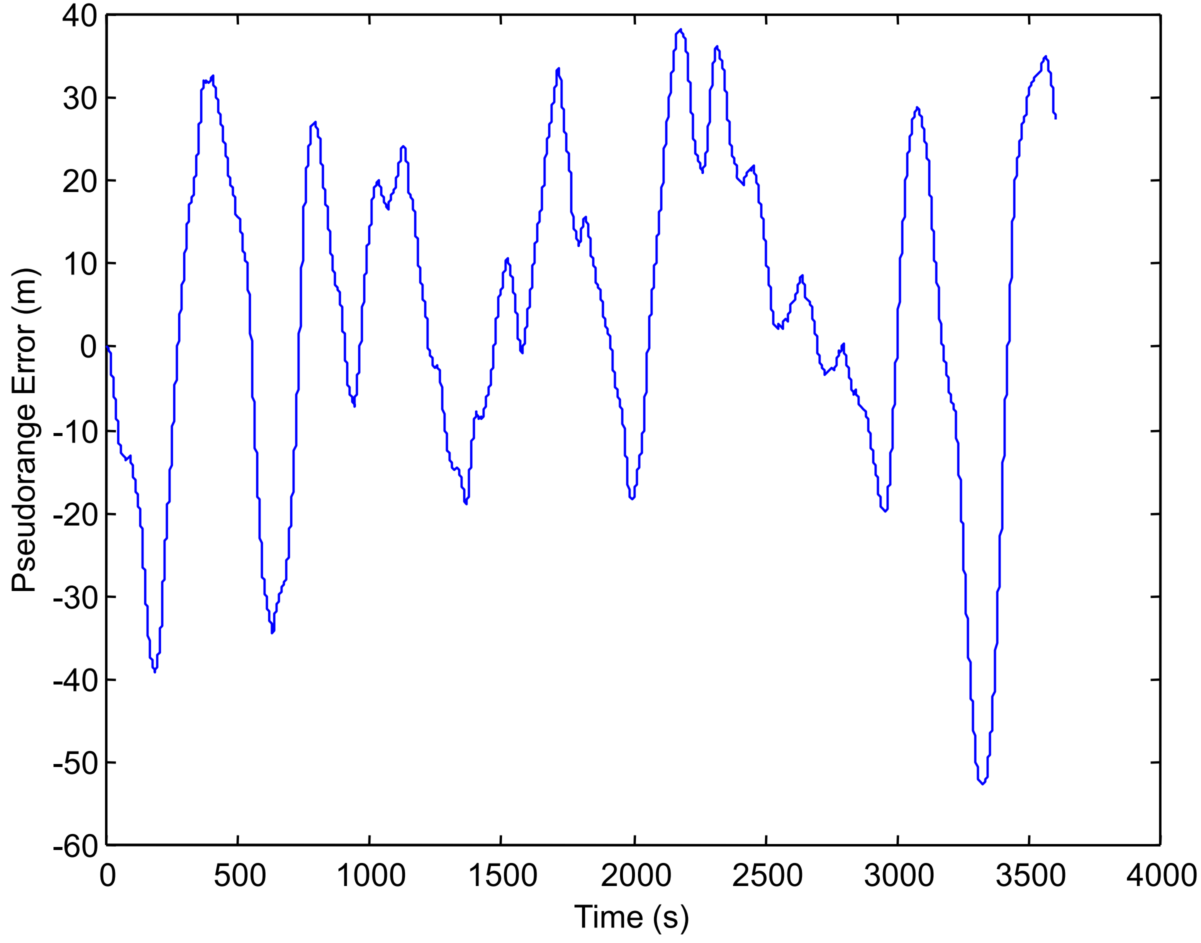
el = elevation angle

Wet delay component may vary rapidly with time and distance, but typically is only ~1/10 of total tropospheric effect.
Dry component varies with local temperature and pressure (correlated over 10's of km)

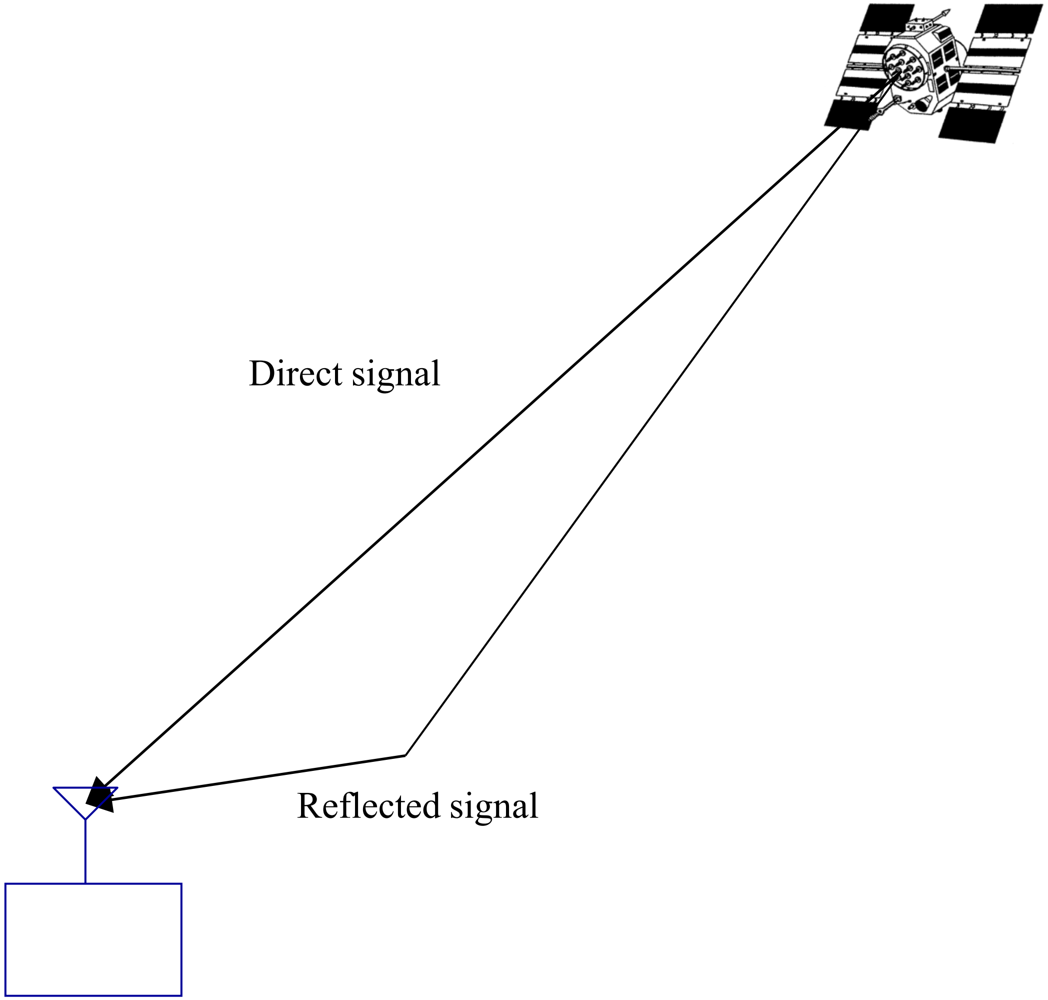
Satellite Clock Errors

- **Prior to discontinuance on May 1, 2000, Selective Availability (SA) was dominant error for civil users**
  - **Understanding SA still important - influenced design of many operational DGPS systems**
  - **20 - 30 m RMS ranging error, 2 - 5 min time constant**
  - **DGPS corrections effectively removed SA, small residual errors due to latency ~$1/2at^2$ ($a \approx$ 2-4 mm/s^2)**
- **Without SA, satellite clock errors are typically ~0.5 - 1 m (one upload/day)**
  - **Satellite atomic clock stability <1 part in 10^{13}**
  - **Virtually eliminated with DGPS**

Simulated SA Pseudorange Errors



Multipath



With care, multipath errors can be kept below ~1 meter. Multipath seen by two receivers is NOT the same (need to root sum square errors for DGPS)

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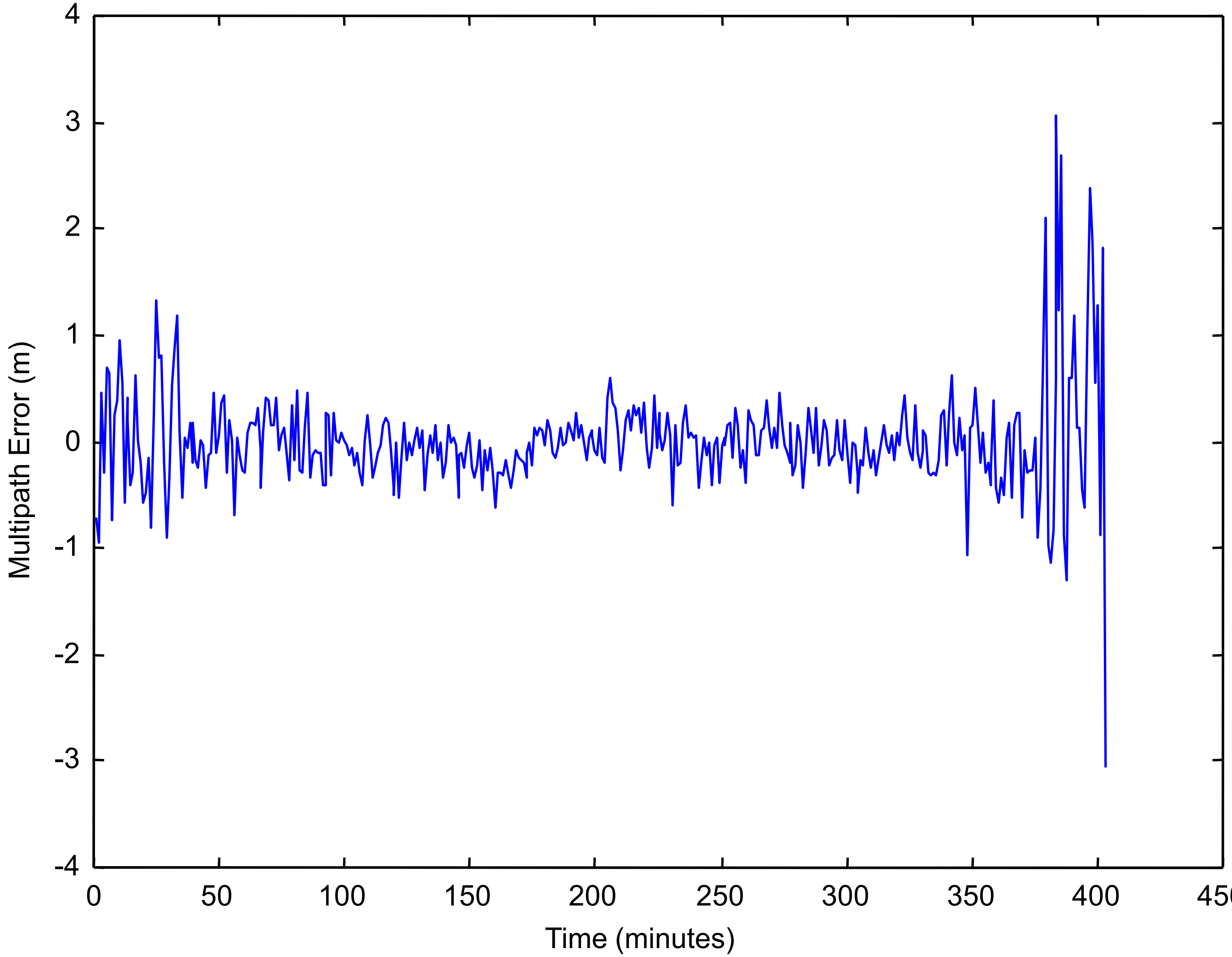
Typical Multipath - One Satellite Pass



Point Loma, CA
January 1, 1999
Ashtech XII3

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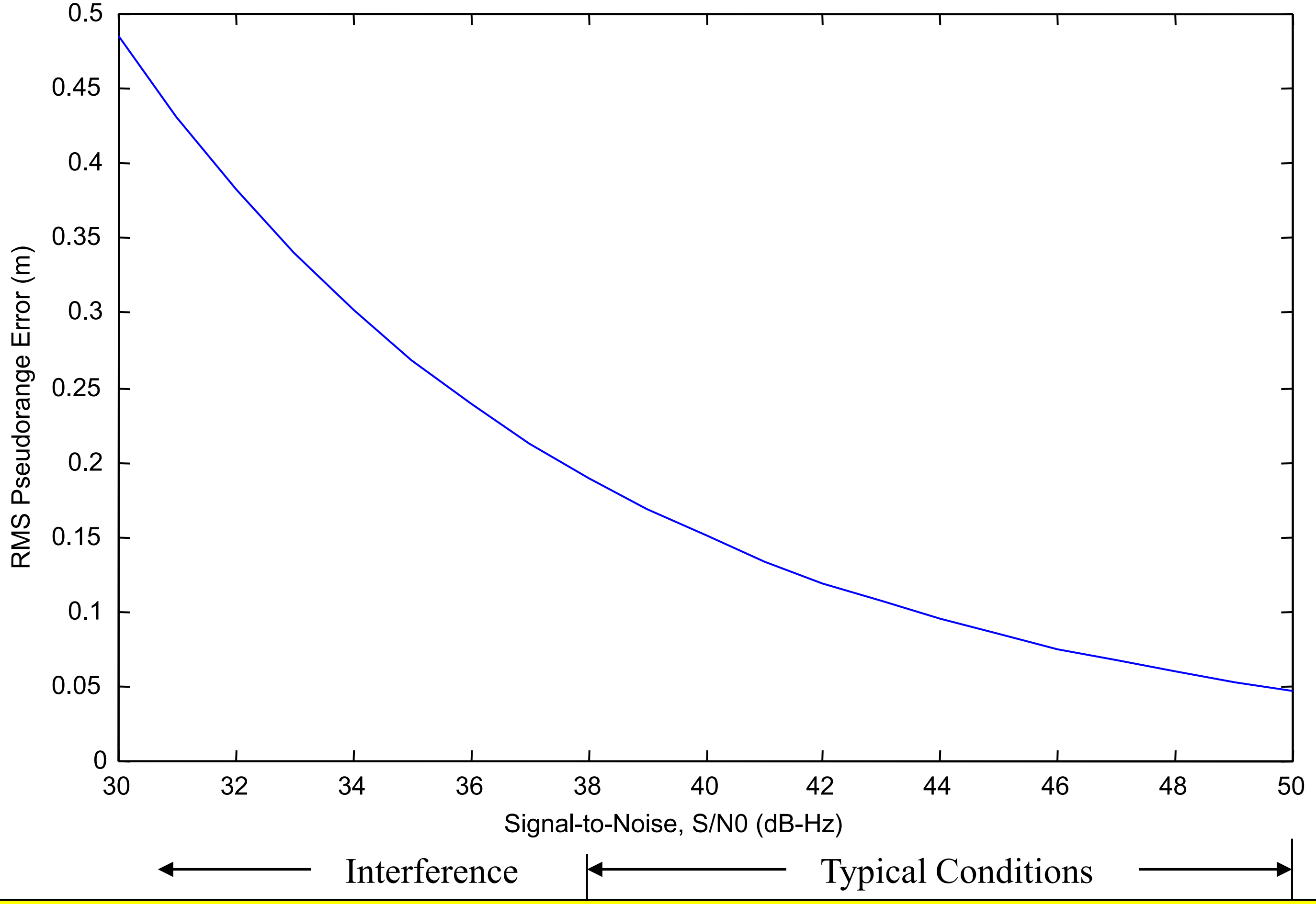
Receiver Noise and Interference



Receiver tracking errors due to thermal noise and interference varies greatly with implementation. Values from 0.1 - 0.7 meters RMS are typical.

Factors Influencing Noise and Interference Tracking Performance

Code tracking error in chips^2 where 1 C/A-code chip ~ 300 m, 1 P(Y)-code chip ~30 m:

$$\sigma_\tau^2 = \frac{B_L d}{2S/N_0}\left[1 + \frac{1}{S/N_0 T}\right]$$

B_L = loop bandwidth (Hz) (typically 1/400 - 1/20 Hz with carrier - aiding)

d = early - late correlator spacing (chips)

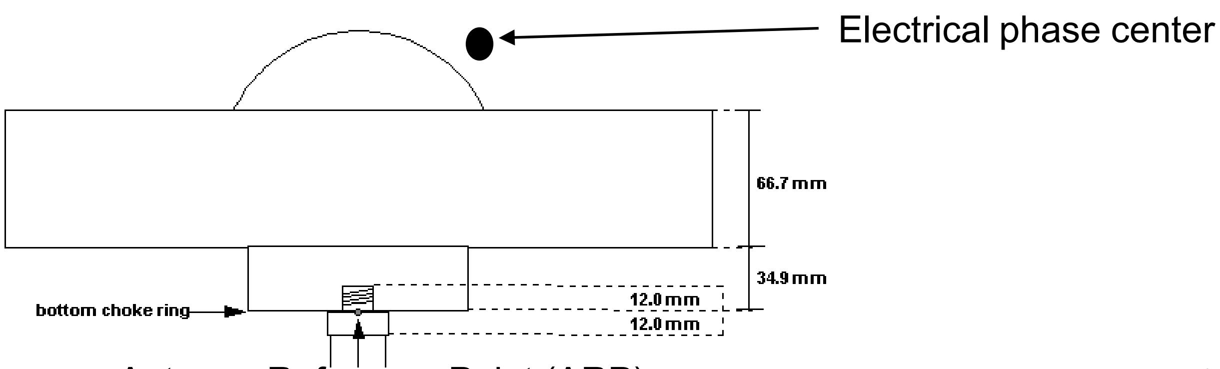
S = received signal power (W)

N_0 = thermal noise and interference level (W/Hz)

T = predetection integration interval (s) (typically 20 ms)

Antenna Biases

- **Antenna Reference Point (ARP) – physical point on antenna (usually center of bottom)**
- **Apparent antenna location from measurements will reside at different points**
  - **May be within or outside of physical antenna package**
  - **Varies with direction of signal arrival and between carrier/code – less so with well-designed antennas**
  - **Calibration employed for some applications**



Antenna Reference Point (ARP)

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Receiver Biases

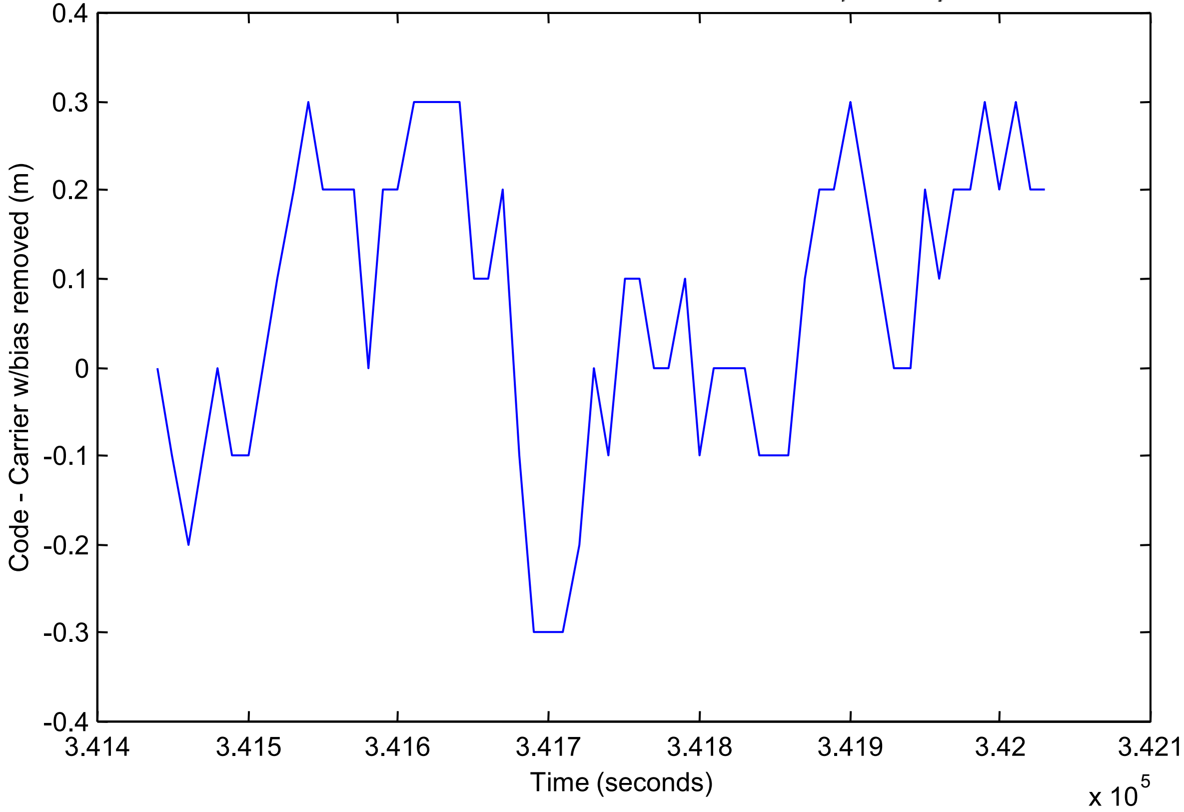
- **All signals experience group and phase delay as they travel from antenna through digital tracking loops**
  - **Common delays drop out in estimate of user receiver clock error in navigation solution**
  - **Not consequential for most navigation users, but of great importance to timing users**
- **Errors affecting positioning can arise when non-common biases occur**
  - **Due to e.g., processing of different signal types or signals on different frequencies**
  - **Can be significant even if common front-end is utilized, because of variations in group/phase delay with frequency across the passband**

Measurement Combinations

- Raw pseudorange and carrier phase measurements from one or more receivers are often combined
  - To reduce/eliminate errors, or
  - To observe errors
- Combinations include:
  - Code (pseudorange) minus carrier
  - Carrier-smoothed code
  - Single-, double-, and triple- differences
  - Ionospheric-free and geometry-free

Code Minus Carrier

$$\rho_u^s(t) - \phi_u^s(t) = 2I - N\lambda + \varepsilon_\rho - \varepsilon_\phi$$



Differencing code and carrier observables and removing the bias (for short data segments) is oft-used method for estimating code measurement noise.

Carrier Smoothed Code

- **Code pseudorange measurements are unambiguous, but noisy**
- **Carrier phase measurements are very precise, but include integer cycle ambiguity**
  - **Differenced phase measurements provide very precise delta-range, with no ambiguity**
- **Popular algorithm to reduce pseudorange noise:**

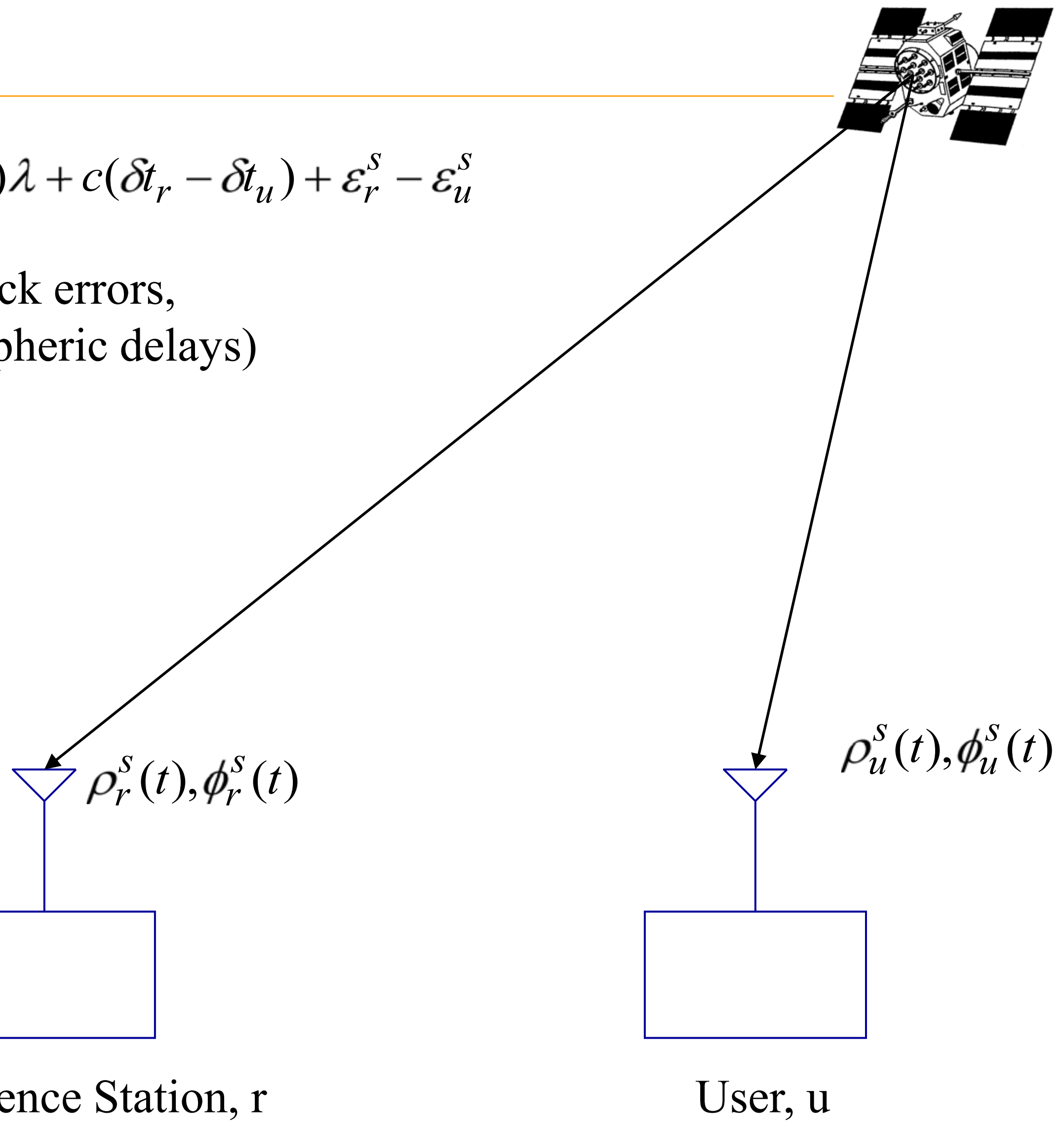
$$\hat{\rho}_u^s(t_k) = (1-\alpha)[\hat{\rho}_u^s(t_{k-1}) + \varphi(t_k) - \varphi(t_{k-1})] + \alpha\rho_u^s(t_k)$$

Smoothing constant (when setting, be mindful of ionospheric divergence!)

Single-Difference

$$\phi_{r,u}^s \equiv \phi_r^s - \phi_u^s = r_r^s - r_u^s + (N_r^s - N_u^s)\lambda + c(\delta t_r - \delta t_u) + \varepsilon_r^s - \varepsilon_u^s$$

(eliminates satellite clock errors, tropospheric and ionospheric delays)

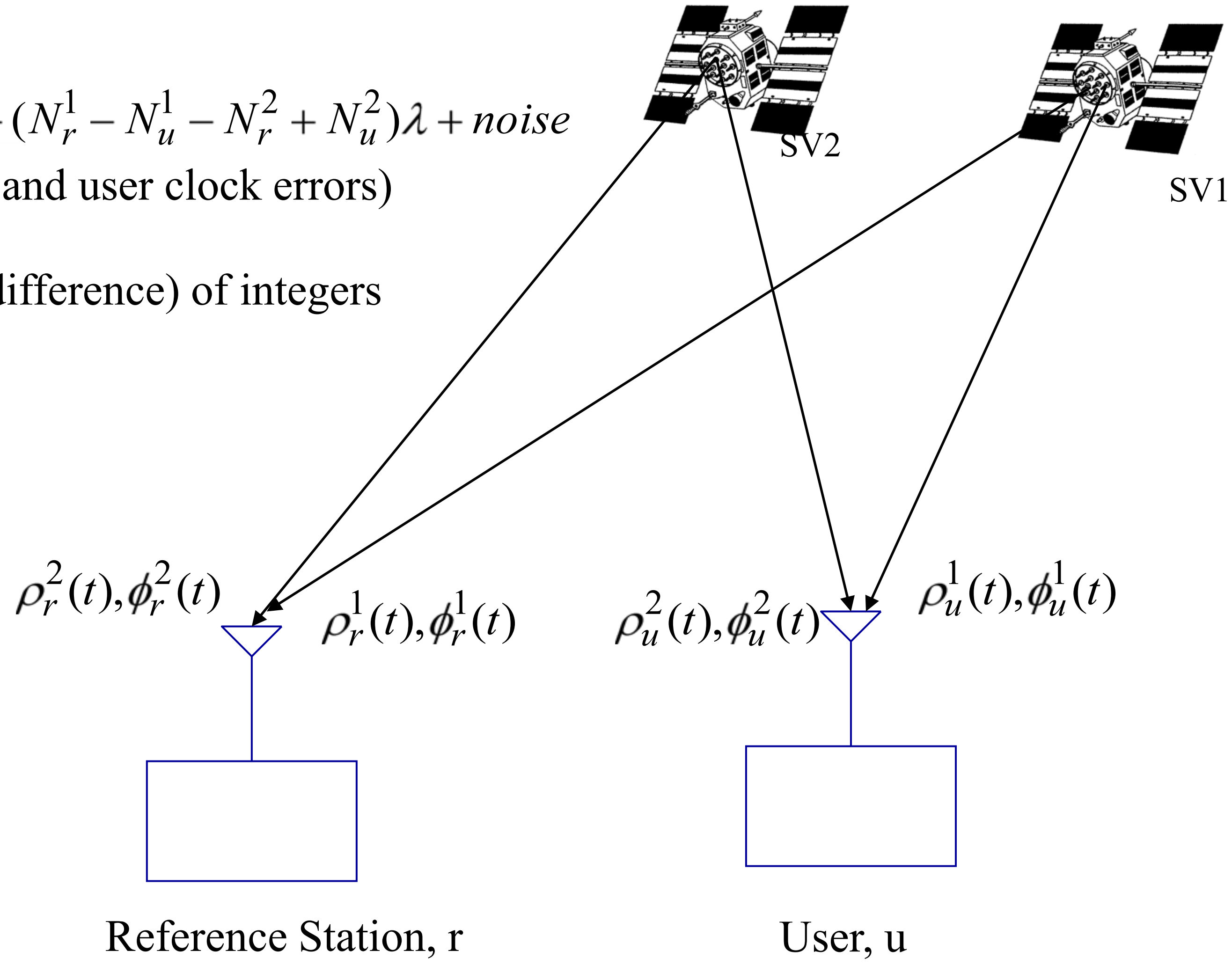


Double-Difference

$$\phi_{r,u}^{1,2} \equiv \phi_{r,u}^{1} - \phi_{r,u}^{2}$$
$$= r_r^1 - r_u^1 - r_r^2 + r_u^2 + (N_r^1 - N_u^1 - N_r^2 + N_u^2)\lambda + noise$$

(eliminates satellite and user clock errors)

Note that a sum (or difference) of integers is an integer.



Ionospheric-free and Ionospheric Combinations

- **Ionospheric delays are frequency dependent**

$$I = \frac{40.3 \cdot TEC}{f^2}$$

Total electron count (electons/m²) along line-of-sight from the user to the satellite

- **L1/L2 measurements can be combined to remove ionospheric delay or estimate it**

$$\rho_{ionofree} = \frac{\rho_{L2} - \gamma\rho_{L1}}{1-\gamma}$$

$$= r + c(\delta t^s - \delta t_u) + T + \frac{\varepsilon_{L2} - \gamma\varepsilon_{L1}}{1-\gamma}$$

Note: noise is enhanced

$$\gamma = \frac{{f_{L1}}^2}{{f_{L2}}^2} \approx 1.65$$

Ionospheric-free and Ionospheric Combinations (continued)

$$\hat{I}_1 = \frac{(1+\gamma)\rho_{L1} - \rho_{L2}}{1-\gamma}$$

$$= I_1 + \frac{\varepsilon_1 - \varepsilon_2}{1-\gamma}$$

Estimate of ionospheric delay on L1 pseudorange.

$$\hat{I}_2 = \gamma\left[\frac{(1+\gamma)\rho_{L1} - \rho_{L2}}{1-\gamma}\right]$$

$$= I_2 + \gamma\frac{\varepsilon_1 - \varepsilon_2}{1-\gamma}$$

Estimate of ionospheric delay on L2 pseudorange.

Ionospheric delay estimates are sometimes referred to as "geometry-free" since they are linear combinations of pseudoranges that no longer depend on the distance between the user and the satellite.

Widelane Observable

$$\phi_w \equiv \lambda_w \left(\frac{\phi_{L1}}{\lambda_1} - \frac{\phi_{L2}}{\lambda_2} \right)$$

(in meters)

$$= r_u^s + N_w \lambda_w + c\left[\delta t_u - \delta t^s \right] - I_w + T + \varepsilon_w$$

Where $\lambda_w = \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)^{-1} \approx 86 \text{ cm}$

is the wavelength of the beat frequency between L1 and L2.

This combination of L1 and L2 phase measurements appears as a carrier phase measurement made at the beat frequency of $f_{L1} - f_{L2}$. The much larger wavelength facilitates ambiguity resolution.

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Relationship Between Range and Position Errors

Basic Idea: **Position Error = "Dilution of Precision" × "Range Error"**

Derivation:

$$\rho_u^1 = \sqrt{(x_u - x^1)^2 + (y_u - y^1)^2 + (z_u - z^1)^2} + c\delta t_u + \varepsilon_u^1$$

$$\vdots$$

$$\rho_u^N = \sqrt{(x_u - x^N)^2 + (y_u - y^N)^2 + (z_u - z^N)^2} + c\delta t_u + \varepsilon_u^N$$

pseudoranges from user, u, to N satellites

$$\Delta\boldsymbol{\rho} = \mathbf{G}\Delta\mathbf{x} + \Delta\boldsymbol{\varepsilon}$$

matrix version, linearized around user's estimated position and time

$$\Delta\mathbf{x} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\Delta\boldsymbol{\rho}$$

least-squares solution for error in user's a priori position/time estimate

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Relationship Between Range and Position Errors (cont'd)

$$\mathrm{cov}(\Delta\mathbf{x}) = (\mathbf{G}^T\mathbf{G})^{-1}\sigma_\rho{}^2$$

Covariance of "state" (position plus user clock offset), assuming pseudoranges to each satellite are independent with standard deviation σ_ρ

The "DOPs":

Vertical (VDOP) - relates 1-σ vertical position error to 1-σ pseudorange error

Horizontal (HDOP) - relates 1-σ horizontal position error to 1-σ pseudorange error

Position (PDOP) - relates 1-σ position error (3D) to 1-σ pseudorange error

Time (TDOP) - relates 1-σ user clock error to 1-σ pseudorange error

Geometric (GDOP) - relates RSS of East, North, Up, and Time errors to 1- σ pseudorange error

The same mathematics applies for code-based DGPS, except that the solution provides the *difference between the reference station and user clock error*.

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Typical Pre-2000 SPS Error Budget (with Selective Availability)

	One-sigma Error (meters)		
Error Source	Bias	Random	Total
Ephemeris	1.0	0.0	1.0
Satellite clock	20.0	0.7	20.0
Ionosphere	4.0	0.5	4.0
Troposphere	0.5	0.0	0.5
Multipath	0.2	0.2	0.3
Receiver noise	0.0	0.1	0.1
User equivalent range error, rms	20.5	0.9	20.5
Filtered UERE, rms	20.5	0.4	20.5
Vertical one-sigma errors - VDOP = 1.7			**34.8**
Horizontal one-sigma errors - HDOP = 1.0			**20.5**

Note: Chart format from Parkinson and Enge, 1996. Values reflect typical 2010 performance (except SA).

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Typical Single-Frequency Error Budget (no SA)

	One-sigma Error (meters)		
Error Source	Bias	Random	Total
Ephemeris	0.8	0.0	0.8
Satellite clock	1.0	0.0	1.0
Ionosphere	7.0*	0.0	7.0
Troposphere	0.2	0.0	0.2
Multipath	0.2	0.2	0.3
Receiver noise	0.0	0.1	0.1
User equivalent range error, rms	7.1	0.2	7.1
Filtered UERE, rms	7.1	0.1	7.1
Vertical one-sigma errors - VDOP = 1.7			**12.1***
Horizontal one-sigma errors - HDOP = 1.0			**7.1***

Note: Chart format from Parkinson and Enge, 1996. Values reflect typical 2010 performance.
*Note that residual ionospheric delay errors tend to be highly correlated among satellites, and thus observed position-domain errors tend to be less than predicted by DOP·UERE.

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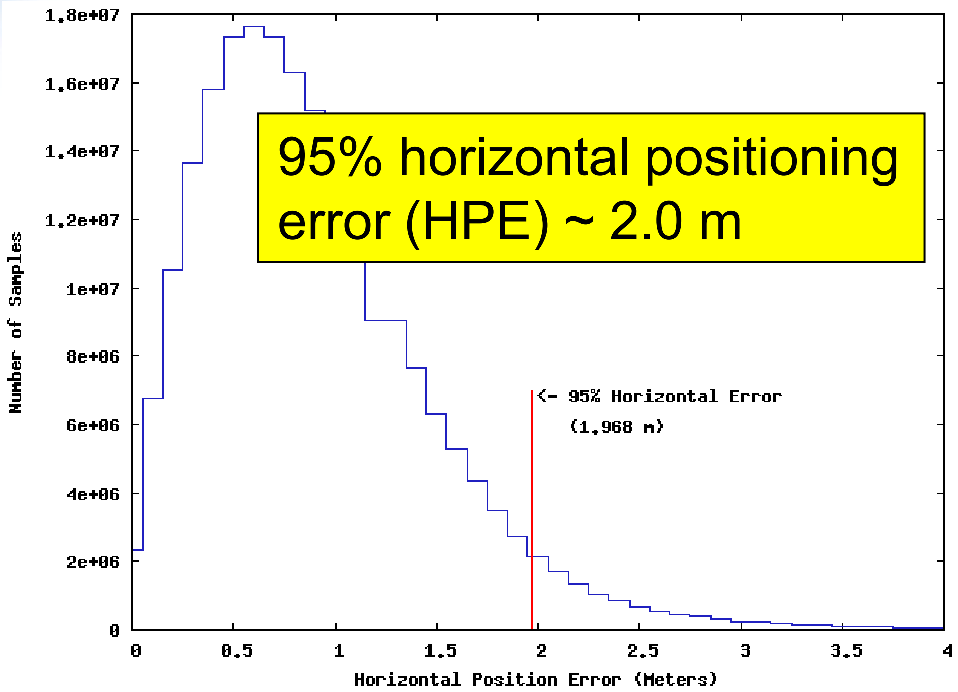
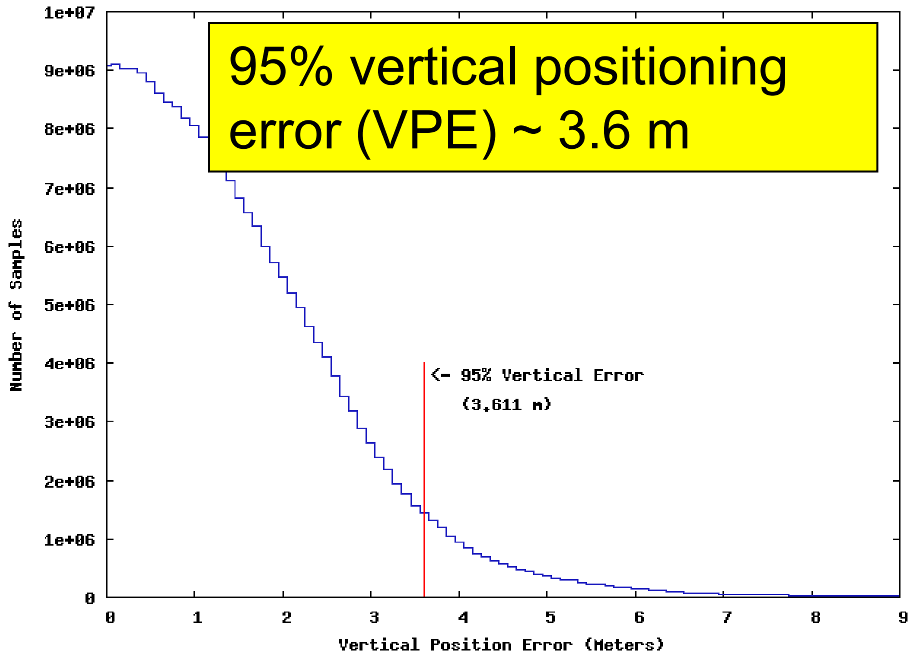
Typical Mid-Latitude SPS Positioning Accuracy





GPS position accuracy data from 28 sites distributed throughout North America for 3 month period (July 2009 – September 2009)

Source: October 2009 Federal Aviation Administration SPS Performance Analysis Report.

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Typical Dual-Frequency Error Budget (no SA)

	One-sigma Error (meters)		
Error Source	Bias	Random	Total
Ephemeris	0.8	0.0	0.8
Satellite clock	1.0	0.0	1.0
Ionosphere	0.1	0.0	0.1
Troposphere	0.2	0.0	0.2
Multipath	0.2	0.2	0.3
Receiver noise	0.0	0.1	0.1
User equivalent range error, rms	1.3	0.2	1.3
Filtered UERE, rms	1.3	0.1	1.3
Vertical one-sigma errors - VDOP = 1.7			**2.3**
Horizontal one-sigma errors - HDOP = 1.0			**1.3**

Note: Chart format from Parkinson and Enge, 1996. Values reflect typical 2010 performance.