

GPS TEC calibration: details and practical aspects

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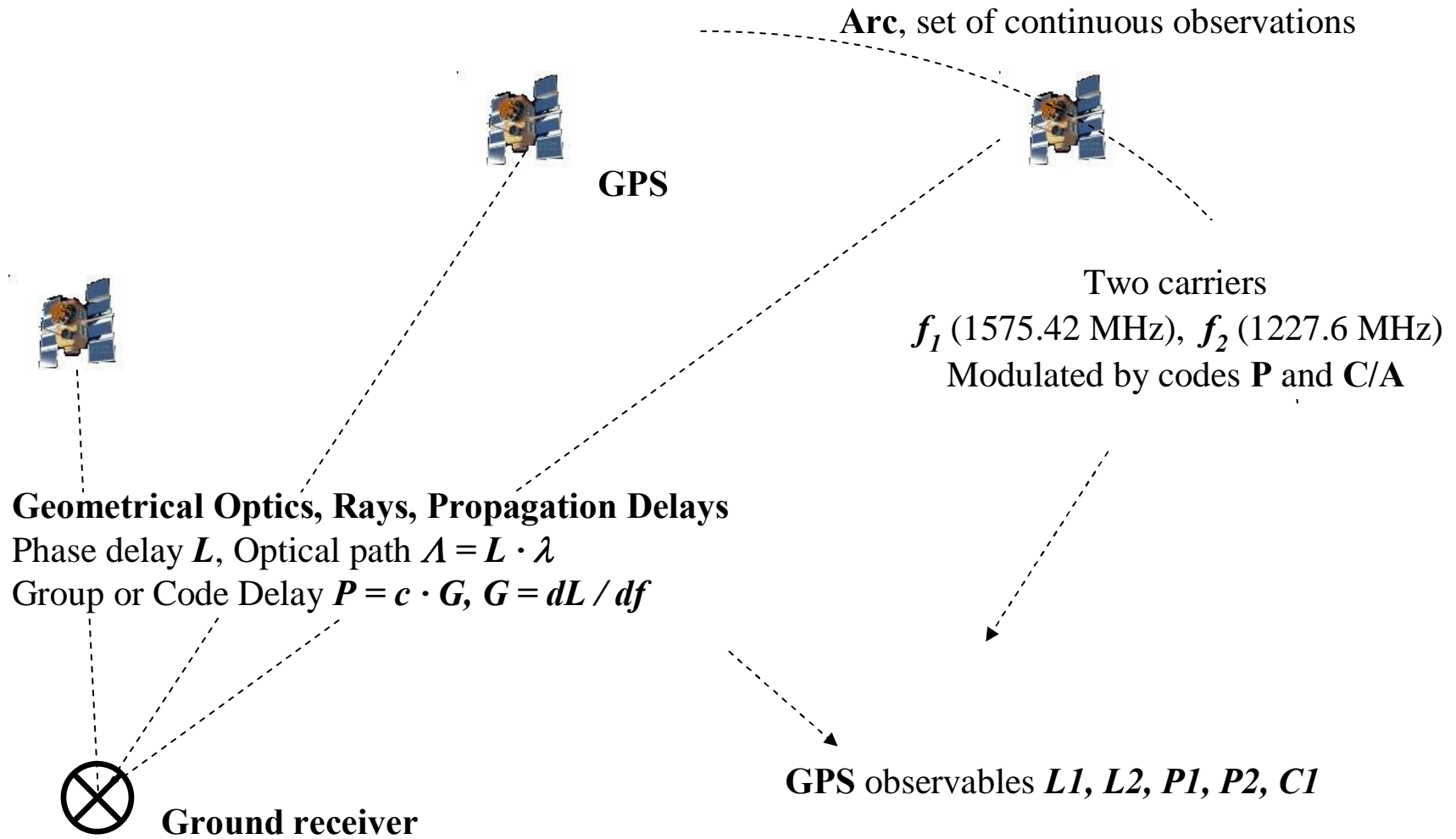
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**Second Workshop on Satellite Navigation Science and Technology for
Africa**

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the Abdus Salam ICTP, Trieste, Italy

GPS scenario



Geometrical Optics, Rays, Propagation Delays

Phase delay L , Optical path $\Lambda = L \cdot \lambda$

Group or Code Delay $P = c \cdot G, G = dL / df$

Arc, set of continuous observations

GPS

Two carriers

f_1 (1575.42 MHz), f_2 (1227.6 MHz)

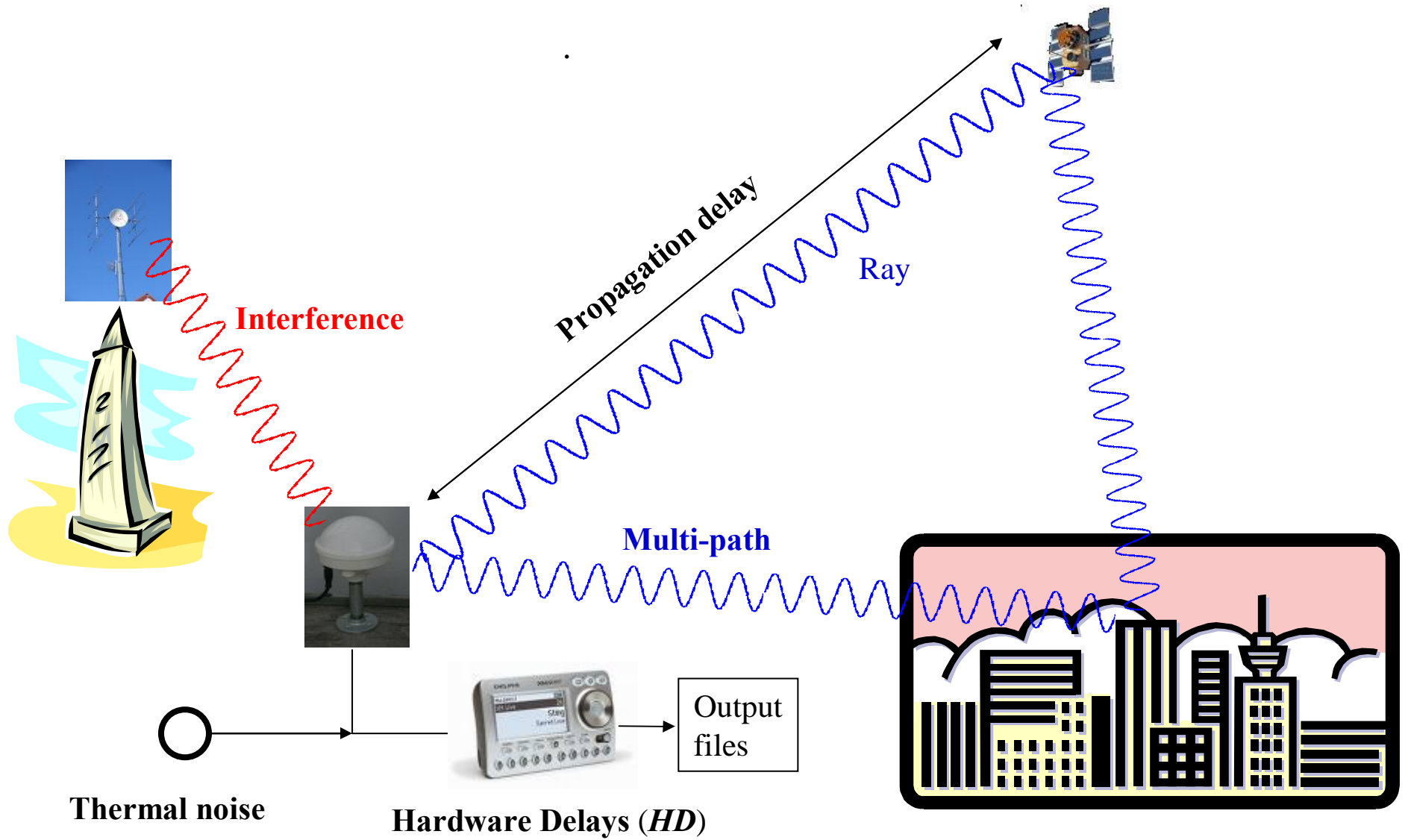
Modulated by codes P and C/A



Ground receiver

GPS observables $L1, L2, P1, P2, C1$

Propagation delays, Disturbances, Hardware Delays, Multi-Path



Propagation Delays

Propagation and Atmospheric contributions to optical path A :

Geometric (Distance), Tropospheric, Ionospheric

$$A = D + T + I$$

Equivalent Group Path $P =$ Group delay $G \times$ speed of light

$$P = G \cdot c = D + T - I$$

Refractivity $R = n - 1$, n Index of Refraction

$$T = \int R_{atm}(s) ds \quad I = \int R_{Iono}(s) ds \quad R_{Iono} = -\frac{40.3 \cdot N_e}{f^2},$$

$$TEC = \int N_e(s) ds, \quad I = -\frac{40.3 \cdot TEC}{f^2}$$

$$L = \frac{D + T + I}{\lambda} = \frac{f}{c}(D + T) - \frac{40.3 TEC}{cf}$$

$$G = \frac{dL}{df} = \frac{D + T}{c} + \frac{40.3 TEC}{cf^2}$$

Measurements introduce additional "delays"

Hardware electronic delays originating

in satellite and receiver, β, γ

Offset (delay, ambiguity) for phase Ω

Noise n

Multipath m

User clock offset τ

Code delay affected by user clock offset is *pseudorange*

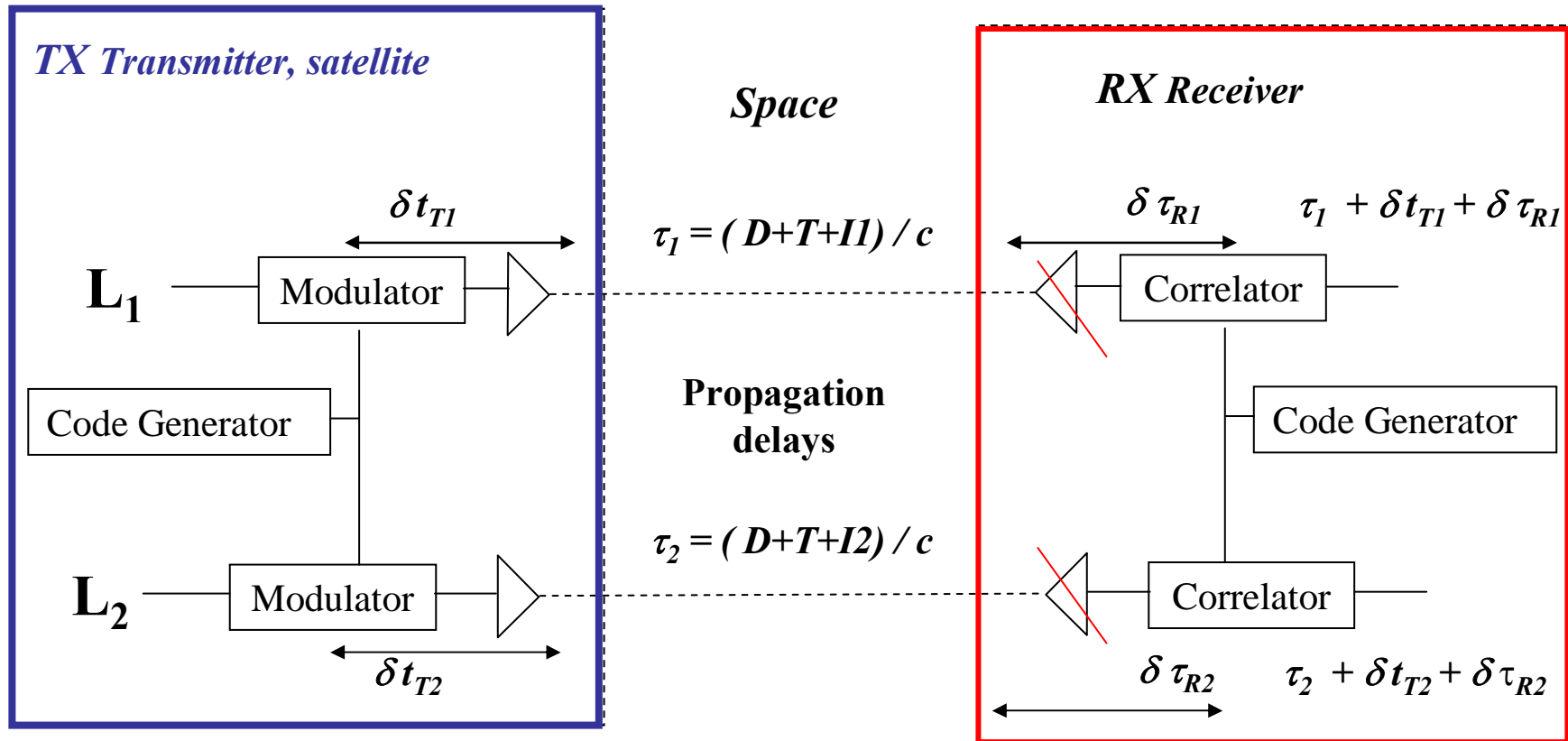
$$P = D + T - I + \beta + \gamma + n + m + \tau$$

For following discussion, noise and multipath can be neglected for phase delays.

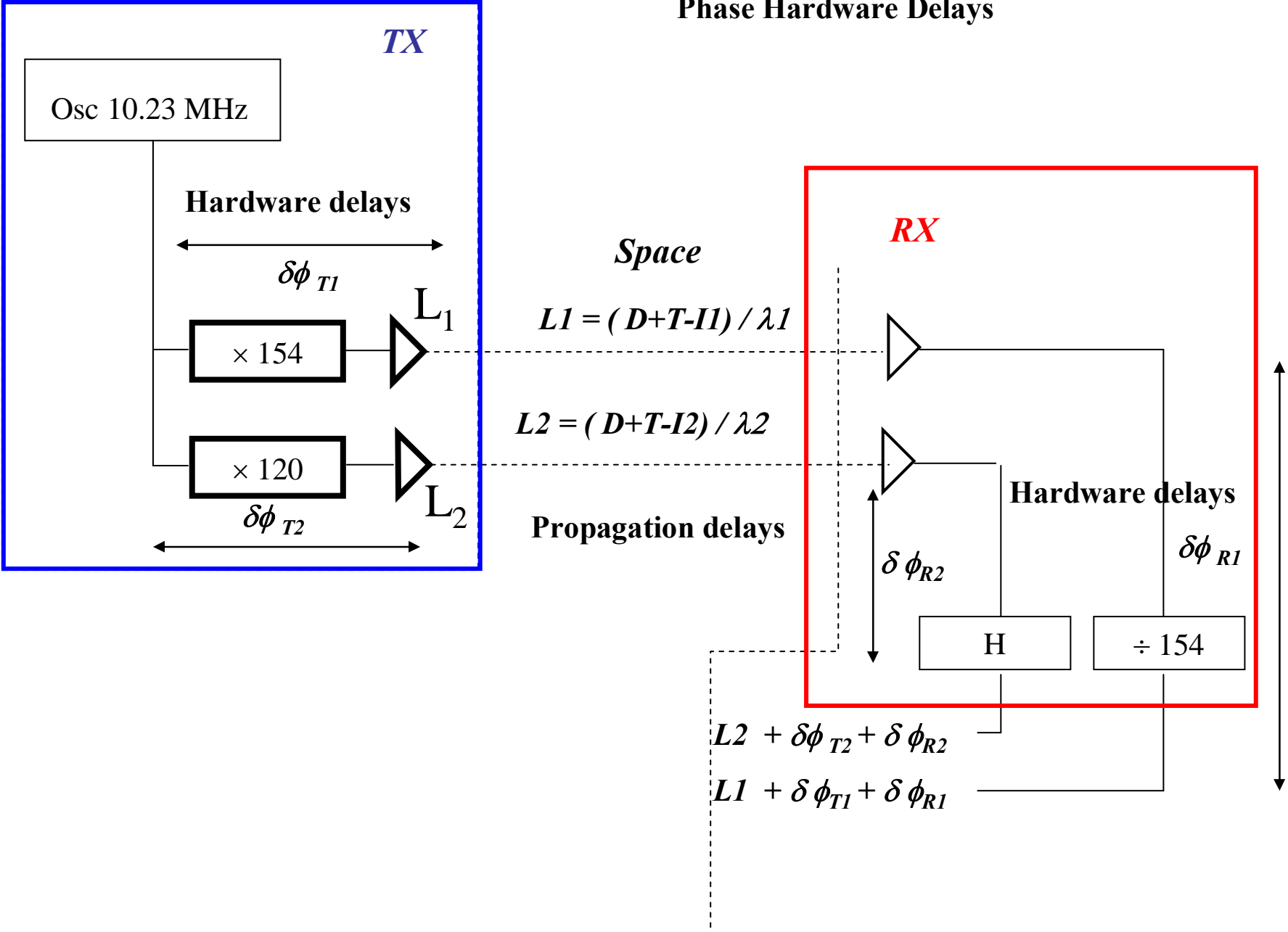
Hardware delays for phase are included in Ω

$$A = D + T + I + \Omega$$

Code hardware delays



Phase Hardware Delays



Availing GPS delays *P1, P2, L1, L2, C1*

Users aiming to determine their position, will get rid of ionospheric contribution taking proper combinations of them.

Users aiming to investigate ionosphere, will simply compute differential delays

Differential pseudorange

$$P2 - P1$$

Differential phase path

$$\Lambda1 - \Lambda2 = L1 \cdot \lambda1 - L2 \cdot \lambda2$$

Both differential delays are in meters.

Following steps:

Show dependence on *TEC*

Transform to *TEC units* (10^{16} electrons/m²), *TECu*

The differential Delays

For the carrier i ($i = 1,2$), contributions with no index do not depend on frequency and cancel out forming differential delays

$$P_i = G_i \cdot c = D + T - I_i + \beta_i + \gamma_i + n_i + m_i + \tau,$$

$$\underline{\Delta P = P_2 - P_1 = I_1 - I_2 + \Delta\beta + \Delta\gamma + \Delta n + \Delta m}$$

$$A_i = D + T + I_i + \Omega_i$$

$$\underline{\Delta A = A_1 - A_2 = I_1 - I_2 + \Delta\Omega}$$

$$I_2 - I_1 = k \cdot TEC \quad k = 40.3 TEC \left(\frac{1}{f_2^2} - \frac{1}{f_1^2} \right)$$

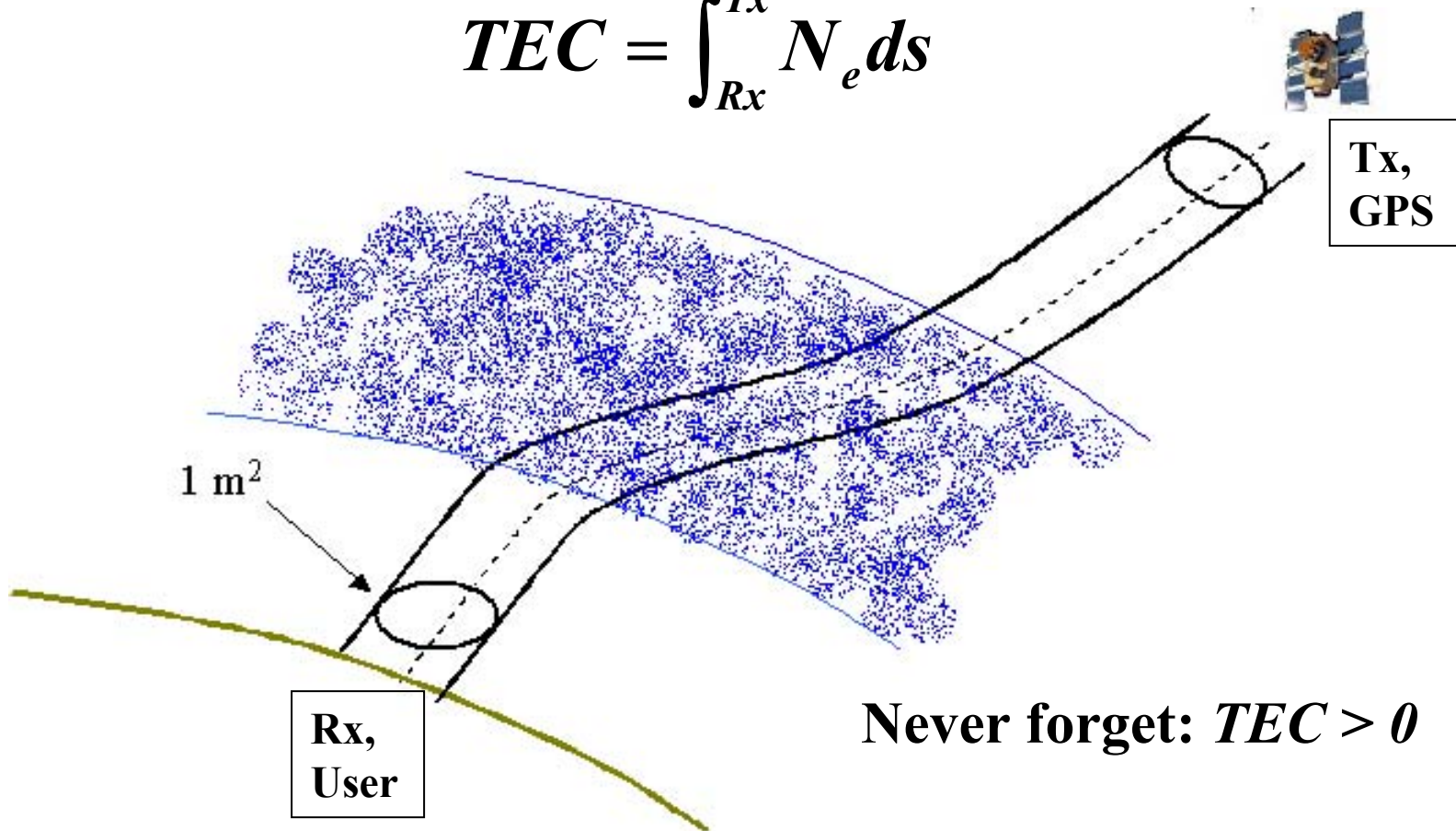
Divide by $k \cdot 10^{-16}$, drop out the Δ symbol to obtain the *phase slants* S_p and *group or code slants* S_c in *TECu*, 1 *TECu* = 10^{16} electrons/m², disregard radio noise n

$$S_p = \frac{1}{k} \cdot (A_1 - A_2) = TEC + \Omega$$

$$S_c = \frac{1}{k} \cdot (P_2 - P_1) = TEC + m + \beta + \gamma$$

The classical interpretation of *TEC* as the **numbers of electrons** contained in a column of unitary base along the ray

$$TEC = \int_{Rx}^{Tx} N_e ds$$



Note for the following: expressions for observations like

$$S = TEC + b$$

denote the set of all available observations used for performing some specific task.

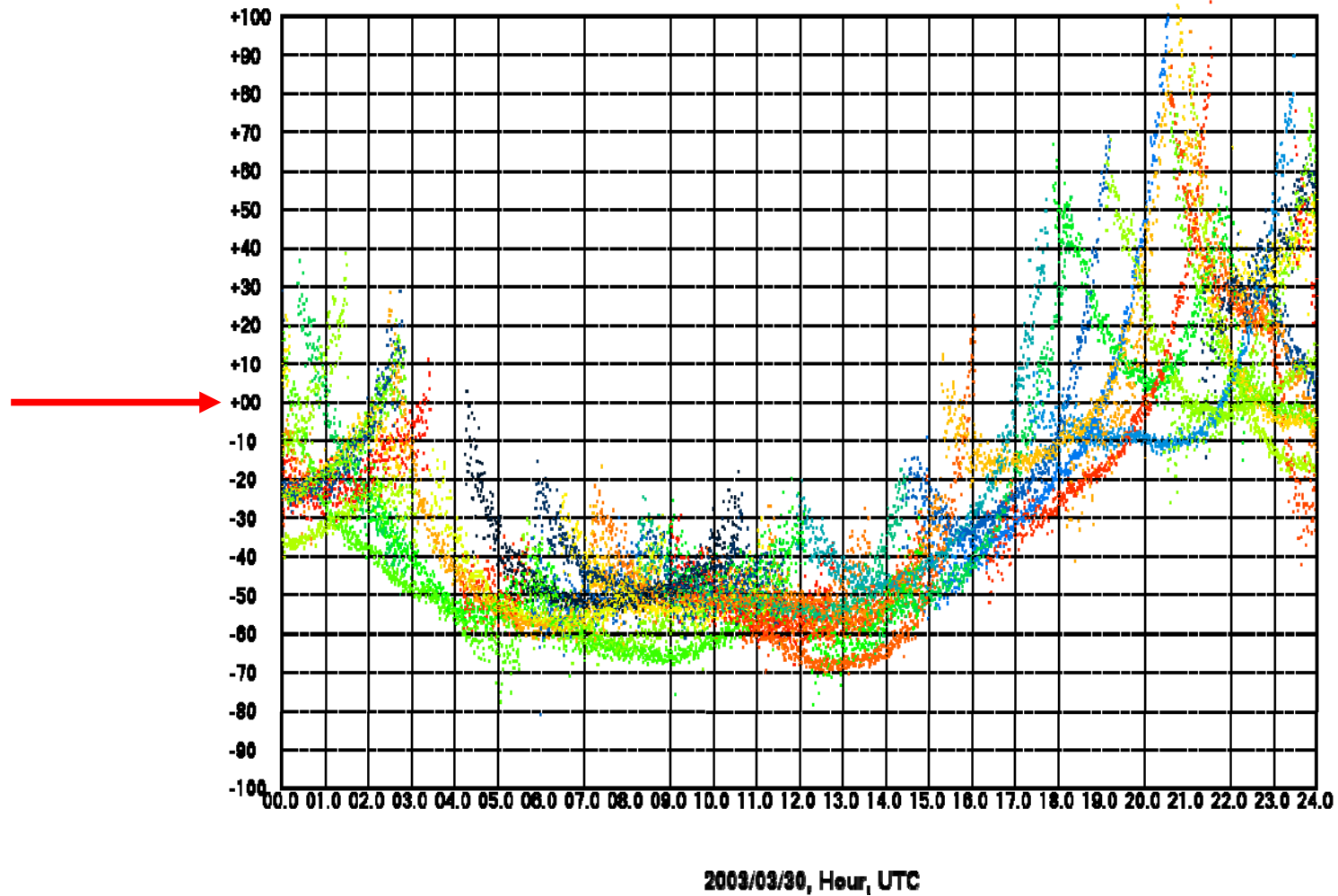
Actually observations should be indexed as S_{ijt} meaning that the individual observed quantity, the “slant”, refers to i^{th} satellite, j^{th} station, t^{th} time.

Biasing terms can still be indexed according to satellite and station (not time as assumed to be constant), but also according to the specific observed arc.

When needed for clarity, indexing will be explicitly adopted.

Plot of S_C arcs for one day

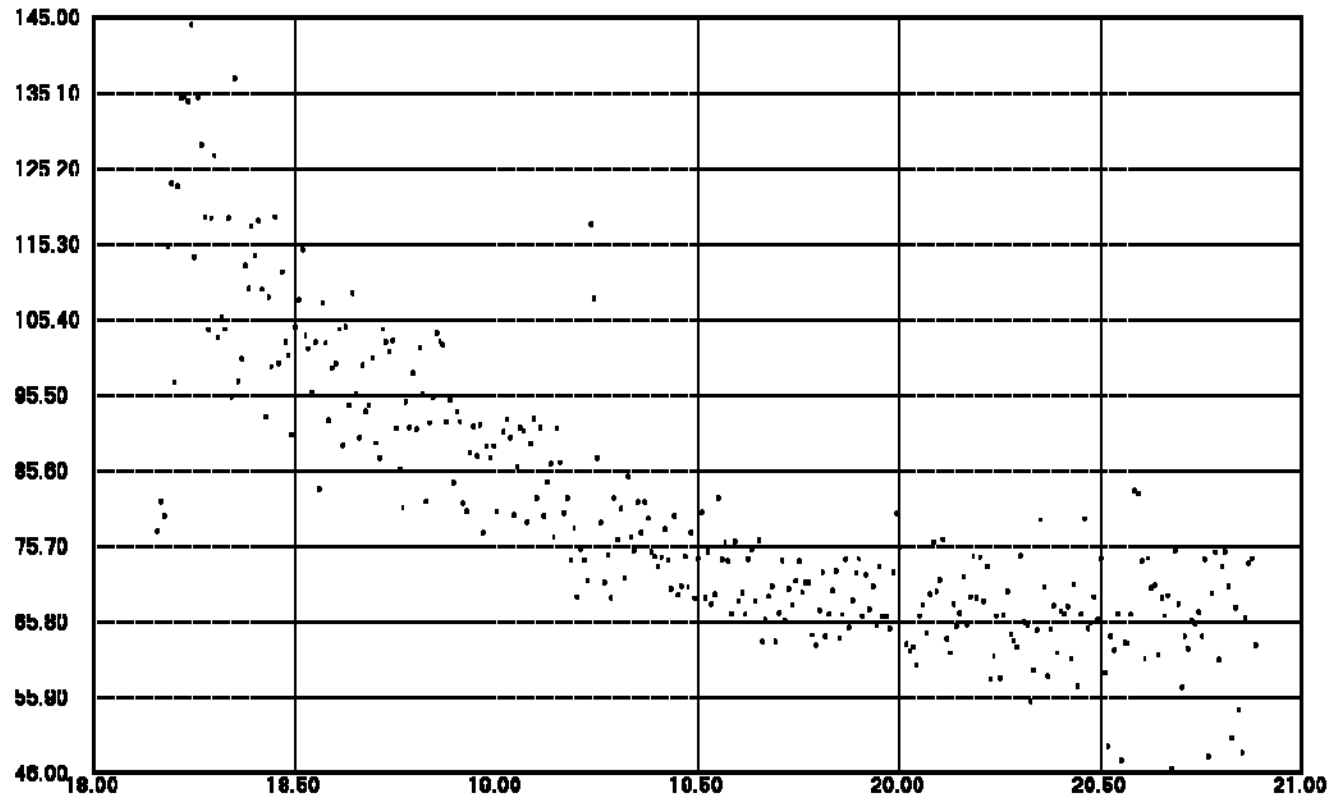
TEC(10^{16}) albh Lat=48.4N Lon=-123.6E
2025 AOA BENCHMARK ACT 3.3.32.2N lk 99/07/2



*** Evidence that calibration is needed: TEC is a positive quantity**

Sample S_C , one arc: the common situation

Code Slant (TECu), PRN# = 25 mats Lat = 40.6N Lon = 16.7E RecType = Var = 21580 TRIMBLE 4000SSI NAV 7.29 SIG 3.07

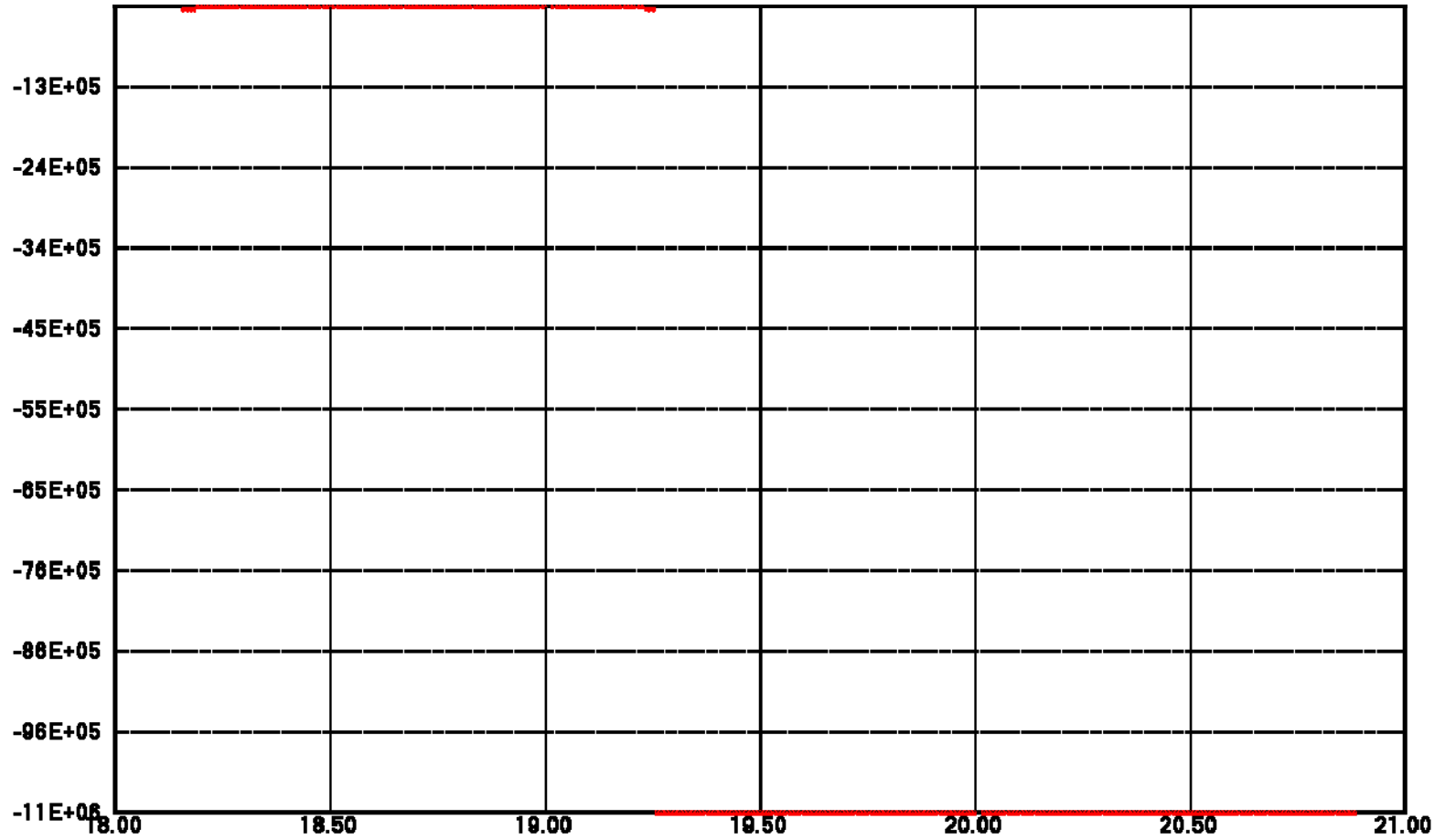


2000/03/30, Hour, UTC

Sample S_p , one arc: the common situation (phase jumps)

Phase Slant (TECu), PRN# = 25 meta Lat = 40.6N Lon = 16.7E RecTypeVer = 21580

TRIMBLE 4000SSI NAV 7.29 SIG 3.07

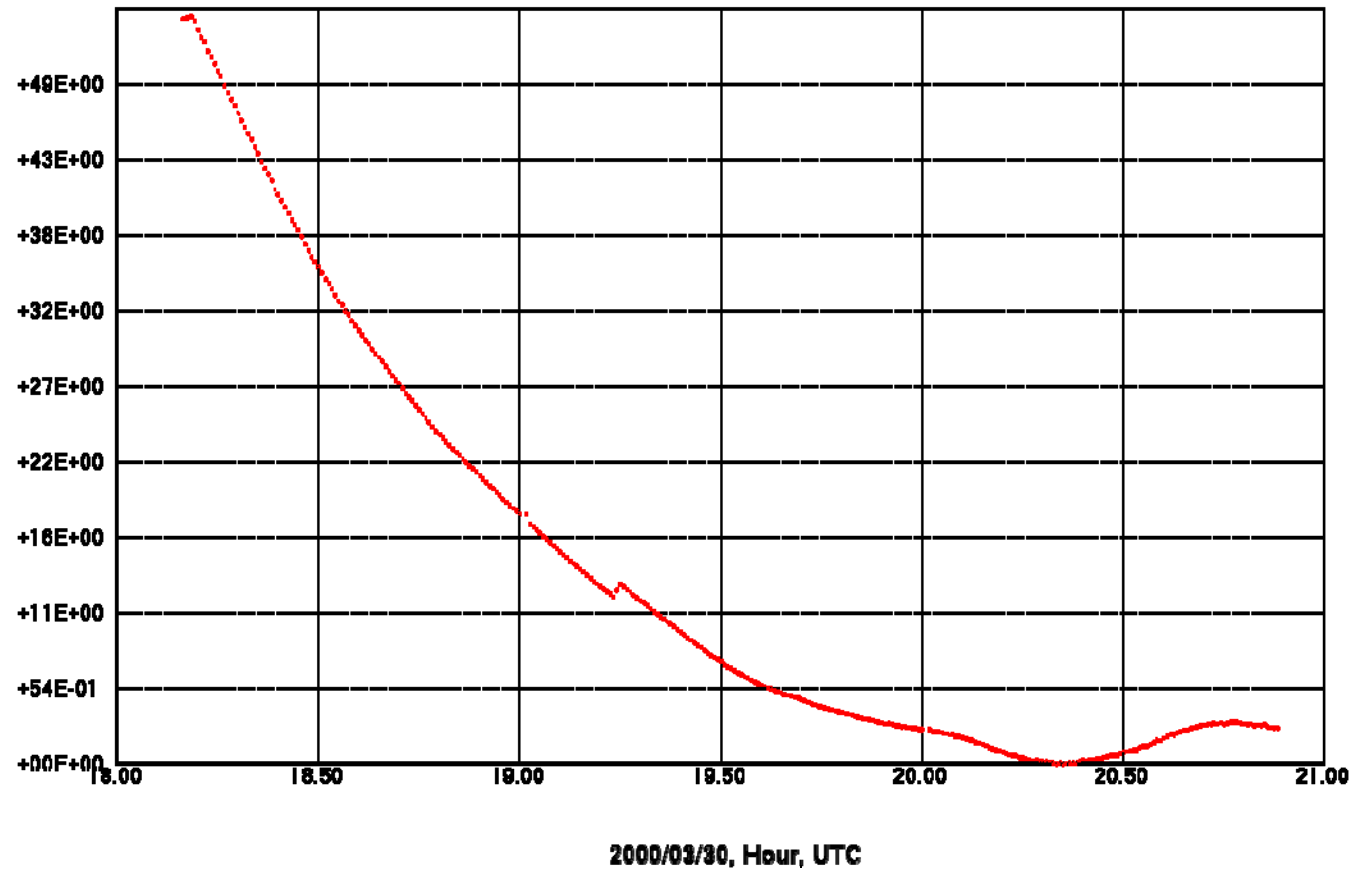


2000/03/30, Hour, UTC

Sample S_p , one arc „ : after removing jumps, fixing the minimum to zero

Phase Slant (TECu), PRNF=25 mase Lat=40.6N Lon=16.7E RecTypeVer = 21680

TRIMBLE 4000SSI NAV 7.29 SIG 3.07



Offset Ω is an arbitrary quantity: can we set it in some useful way?

A new set of observables: Phase slants leveled to Code

Operator $\langle \cdot \rangle$ is a properly selected weighted (possibly robust) average

Build, arc by arc, the leveled slants S_L

$$S_L = S_P - \langle S_P - S_C \rangle$$

$$\langle S_P - S_C \rangle = \Omega - \langle m \rangle - \beta - \gamma$$

$$S_L = TEC + \langle m \rangle + \beta + \gamma$$

Properties of S_L

Noise is the same (neglected) of phase slants

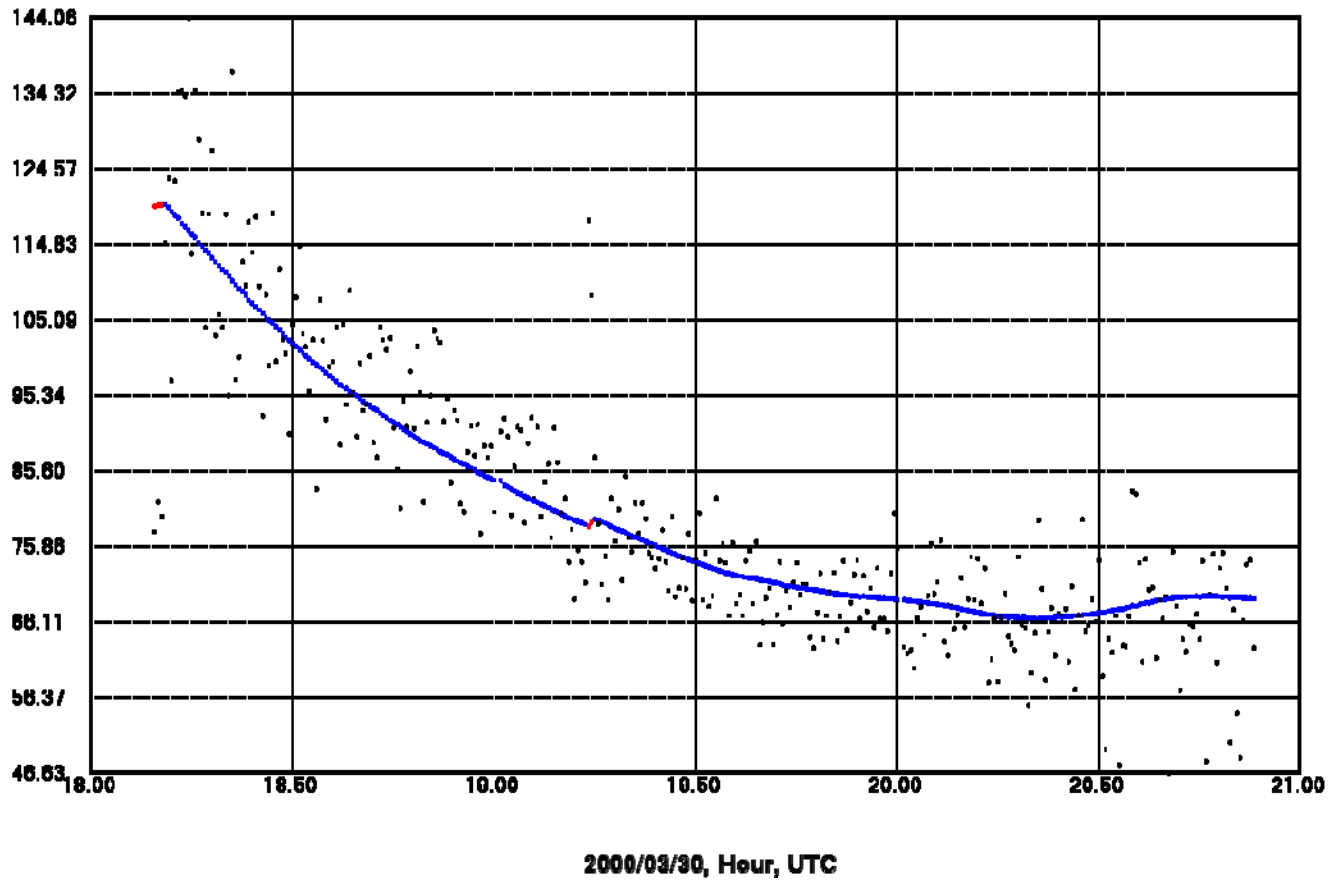
Biased exactly as code slants

But: an arc dependent constant leveling error $\lambda = \langle n \rangle + \langle m \rangle$ appears

Sample S_C and S_P with properly selected phase offset $\Omega = S_L$

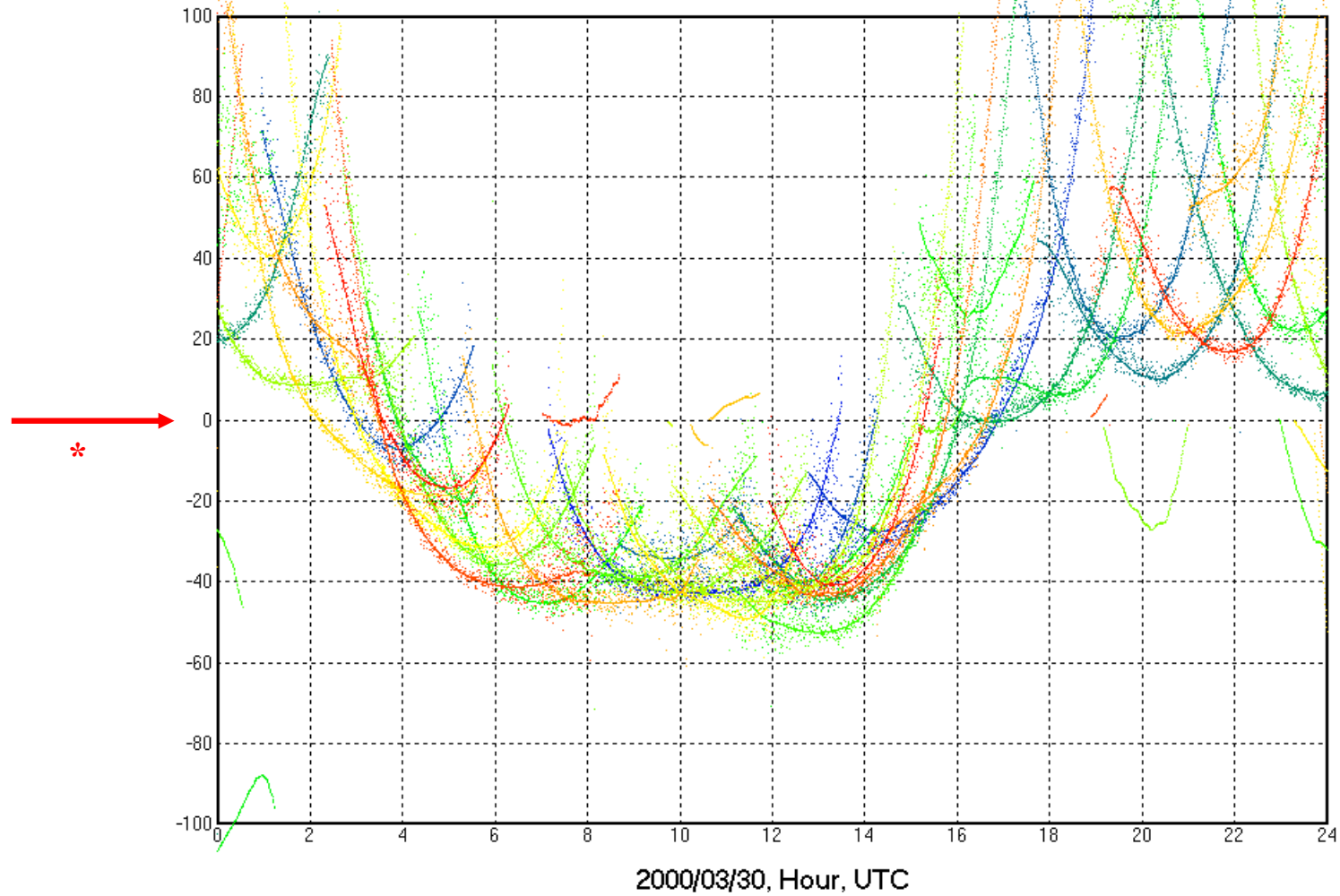
meta Lat=40.6N Lon=16.7E RecTypeVer = 21580

TRIMBLE 4000SSI NAV 7.29 SIG 3.07



One day, S_C and S_L arcs

TEC(10^{16}) albh Lat=48.4N Lon=123.5W



*** Evidence that calibration is needed: TEC is a positive quantity**

Summary of the observables

$$S_p = TEC + \Omega$$

$$S_C = TEC + m + \beta + \gamma$$

$$S_L = TEC + \lambda_{Arc} + \beta + \gamma$$

- Ω Offset, constant but arbitrarily changing from arc to arc
- β, γ Hardware biases: delays in electronics of transmitter and receiver.
One β for satellite, one γ per station.
- m Multi-path,
- λ Leveling error, $\langle m \rangle$, changing generally (but not arbitrarily) from arc to arc.
- TEC The quantity to estimate, variable from observation to observation

Following topics will be discussed in the following

GPS ionospheric observables

Reliability of leveled slants

Problems with multipath

Problems with receivers?

***TEC* expansion**

Reliability of the thin shell approximation

Calibration

The thin-shell, single-station, multi-day solution

of individual arc offsets

Validation

Use of ionospheric models to validate the calibration techniques

Features of observations, Code slants

$$S_C = TEC + n + m + \beta + \gamma$$

Advantages: the electronic delays are physical quantities, stable or undergoing slow aging in controlled environmental conditions: they are generally considered constants over long times (up to 1 month).

One β per satellite, one γ for station: a favorable unknowns/observations budget.

n : strong radio noise (non linear techniques used to evaluate pseudo-ranges), but still a stochastic variable with zero mean (resulting in consistent estimations)

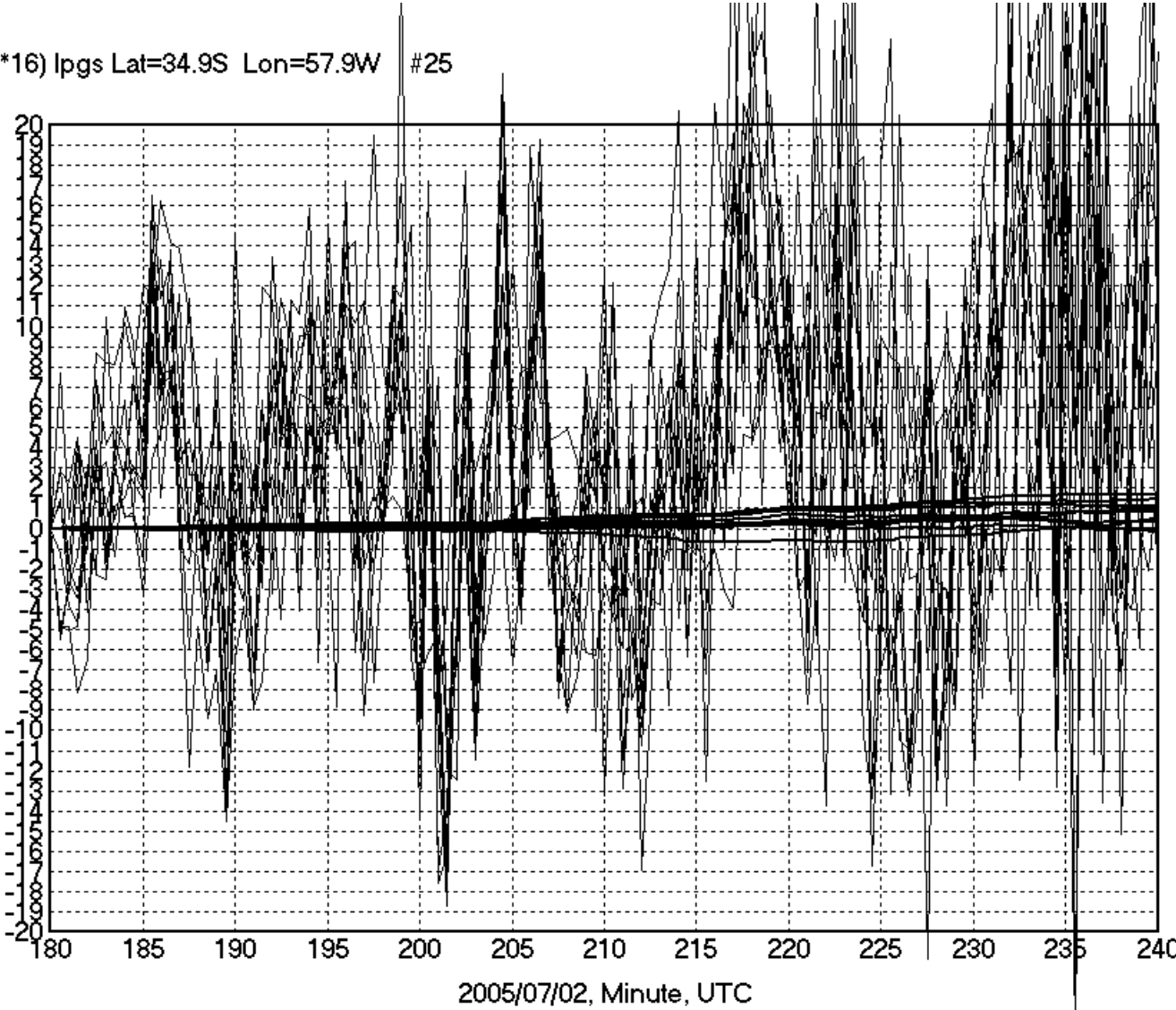
Can multipath m be considered a disturbance?

How to distinguish it from noise? Period of GPS orbits is 12 sidereal hours: day after day the same satellite will occupy the same position with an advance of ≈ 4 minutes: if same environment day after day, m will advance by the same amount.

Plot a fraction of arc of the same satellite day by day with an advance of ≈ 4 minutes

Note: to avoid TEC variability, what is plotted for each arc is $TEC(t) - TEC(t_0)$, t_0 being the beginning of each arc. Both S_G and S_ϕ relative to the same arc are plotted .

TEC(10**16) lpgs Lat=34.9S Lon=57.9W #25



Features of observations: Phase slants

$$S_p = TEC + \Omega$$

No significant noise and multipath (above slide)

Modest equations/unknown budget: one unknown per arc

Global single day solution, 200 stations

Unknowns: coefficients of TEC expansion plus around 1000 unknown offsets, compared to 200+30 hardware biases.

Possibility to use first differences (in time) of the observations of one arc. Only TEC coefficients remain: calibration relies entirely on the model used for the expansion.

Other possibility: solving by geodetic techniques for the ambiguities and therefore for the offsets.

Leveled slants: $S_L = TEC + \lambda + \beta + \gamma$

$$\lambda = \langle m \rangle$$

As for code slants, one unknown per satellite β and for station γ

Same observations/unknown budget of phase slants S_p , apart the leveling error, constant arc by arc

Commonly assumed: disregard leveling error $\lambda = \langle m \rangle$

In leveling error, the mean of a stochastic variable, $\langle n \rangle$ has been neglected as a quantity with (likely) zero mean: it can be considered a disturbance that will not significantly affect the ultimate accuracy of calibration.

Does the same holds for $\langle m \rangle$?

No: multi-path is **not** a stochastic variable and it has **no zero mean**

The **close stations experiment** can evidence this statement

Availability of close stations

Many co-located IGS stations are available:

darr/darw, dav1/davr, gode/godz, gol2/gold, kou1/kour, mad2/madr, mat1/mate
ohi2/ohi3, reyk/reyz, tcms/tnml, thu2/thu3, tid1/tid2, tid1/tidb, tid2/tidb, zimj/zimz
and the combinations of *wtza, wtzj, wtzr, wtzt*.

Besides IGS stations, a special set of observation has been set up by the group of La Plata University, Argentina (C.Brunini, F.Azpilicueta).

Close to the IGS station "*lpgs*", the additional stations "*blue*", "*red0*" and "*asht*" have been set up for present investigation, whose characteristics will be described in (*).

Duration: days 182/205 and 262/269 , 2005

(*) *Journal of Geodesy*

DOI 10.1007/s00190-006-0093-1

Calibration Errors on Experimental Slant Total Electron Content (TEC) Determined with GPS
L. Ciralo, F. Azpilicueta, C. Brunini, A. Meza, S. M. Radicella

Updated availability of close station (2008)

cagl/cagz; cont/conz; darr/darw; dav1/davr; gode/godz;
gol2/gold; harb/hrao; hers/hert; irkj/irkm; irkj/irkt;
irkm/irkt; joz2/joze; kir0/kiru; lhas/lhaz; mad2/madr;
mat1/mate; mdvj/mdvo; mets/metz; mobj/mobn; nya1/nyal;
ohi2/ohi3; suth/sutm; tcms/tnml; thu2/thu3; tid1/tid2;
tid1/tidb; tid2/tidb; tixi/tixj; tro1/trom; tsk2/tskb;
usn3/usno; wtza/wtzj; wtza/wtzt; wtza/wtzs; wtza/wtzz;
wtzj/wtzt; wtzj/wtzs; wtzj/wtzz; wtzt/wtzs; wtzt/wtzz;
wtzs/wtzz; yakt/yakz; yar2/yarr; zimj/zimm;

The close stations experiment

Station 1

$$S1_{PRN} = TEC + \lambda1 + \beta_{PRN} + \gamma1$$

TEC

Station 2

$$S2_{PRN} = TEC + \lambda2 + \beta_{PRN} + \gamma2$$



< 100 m



β

$$S1 - S2 = \gamma1 - \gamma2 + \lambda1 - \lambda2$$

Not dependent on PRN

The close stations experiment

In equations of observation

$$S = TEC + \beta + \gamma + \lambda$$

Consider observations to satellite i from stations j e k

$$S_{ij} = TEC_{ij} + \beta_i + \gamma_j + \lambda_{Arc_i}$$

$$S_{ik} = TEC_{ik} + \beta_i + \gamma_k + \lambda_{Arc_k}$$

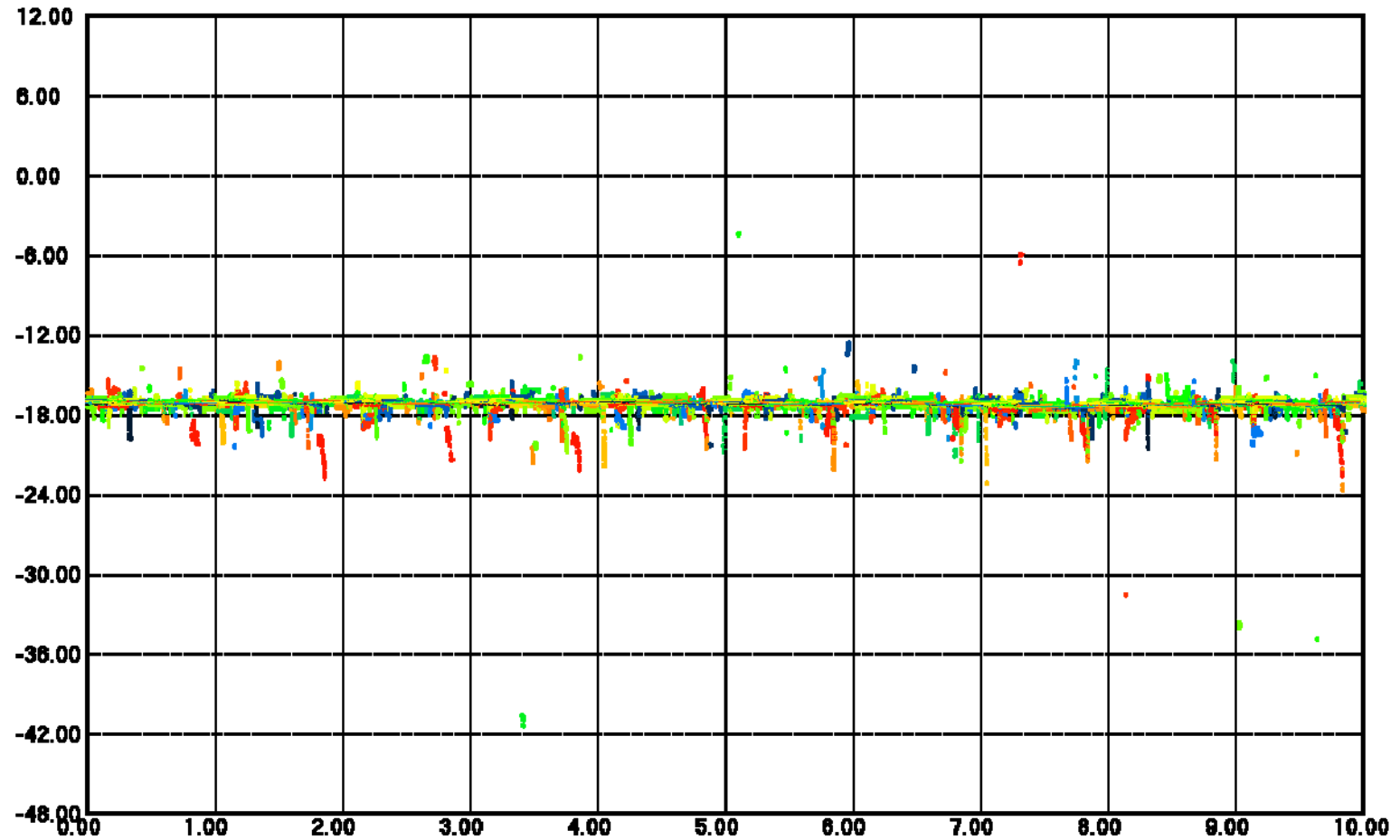
For close stations (up to few km) $TEC_{ij} = TEC_{ik}$ satellite bias contribution is canceled

$$S_{ij} - S_{ik} = \gamma_j - \gamma_k + \lambda_{Arc_i} - \lambda_{Arc_k}$$

If contribution of leveling error is not significant, plotting $S_{ij} - S_{ik}$ one gets points close to the difference $\gamma_j - \gamma_k$, **a constant quantity for the investigated pair of stations.**

$$S_{i1} - S_{i2}, i=1..all\ satellites, TECu$$

gol2 Lat=35.4N Lon=116.9E RecTypeVer = UC2200524002 ASHTECH UZ-12 CQ00
gold Lat=35.4N Lon=116.9E RecTypeVer = LP03572 ASHTECH Z-XII3 CC00 1s soc2mx



2008/01/10

The situation for *gol2/gold* is rather uncommon

Most of times the situation is quite different as

a significant spread among satellites appears

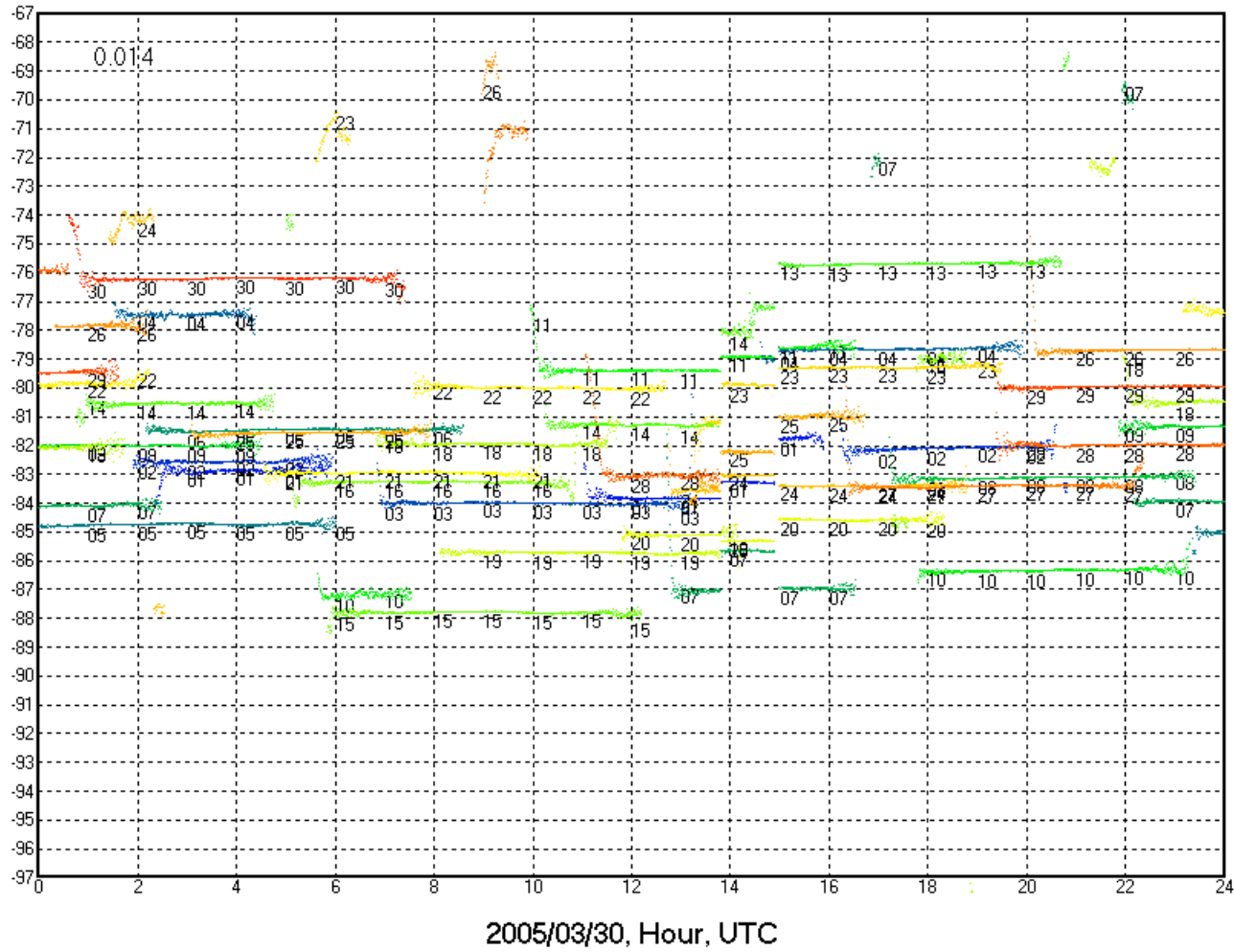
As shown in following slides

Possible cause

the leveling error $\lambda = \langle m \rangle ?$

$S_{L1} - S_{L2}$, all satellites

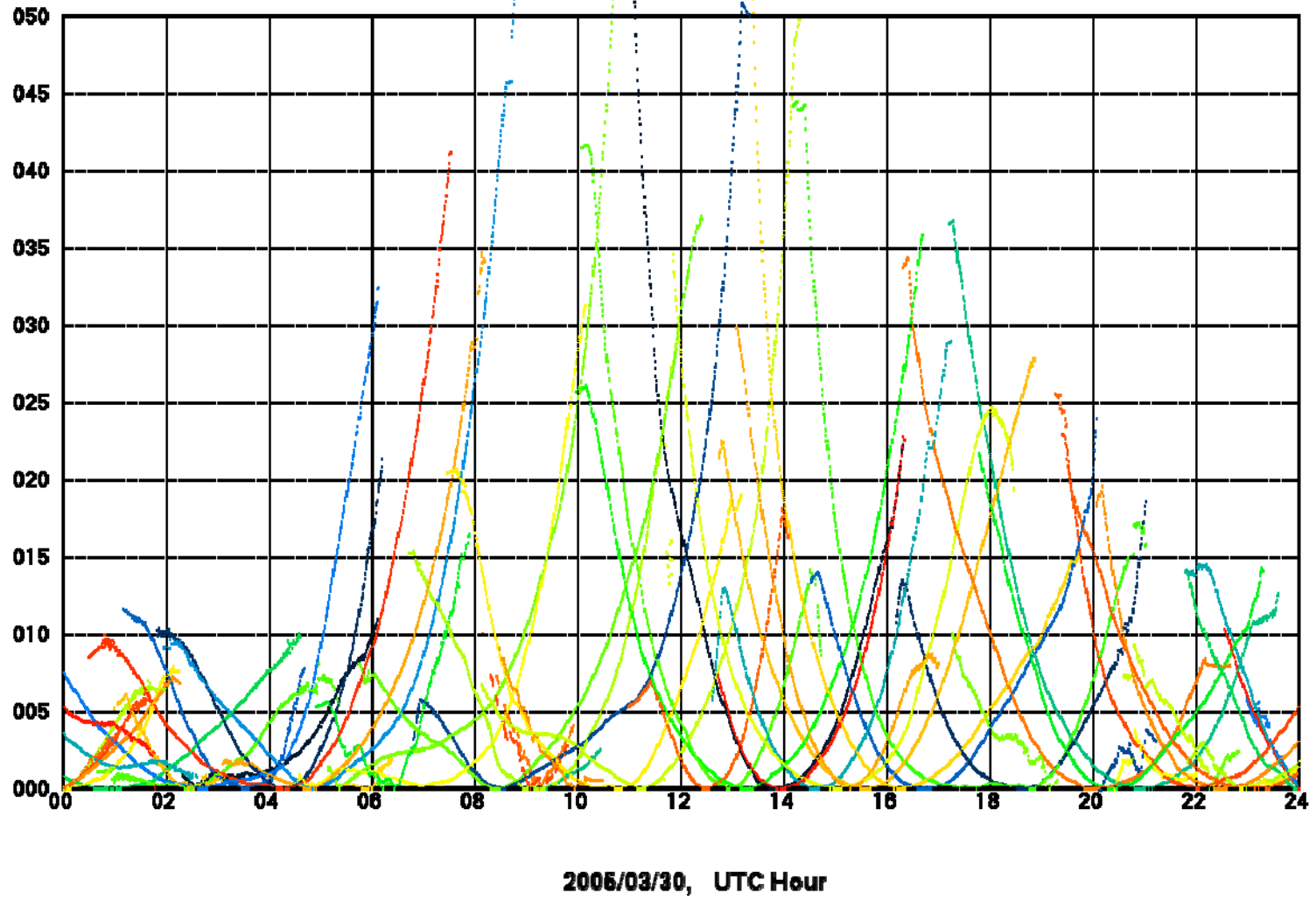
TEC(10**16) zimm - zimj Lat=46.9N Lon=7.5E



TEC(10**16) zimj Lat=46.9N Lon=7.6E
MT300342603 JPS LEGACY 2.3 APR,28,2004 P4

$S_p, zimj$

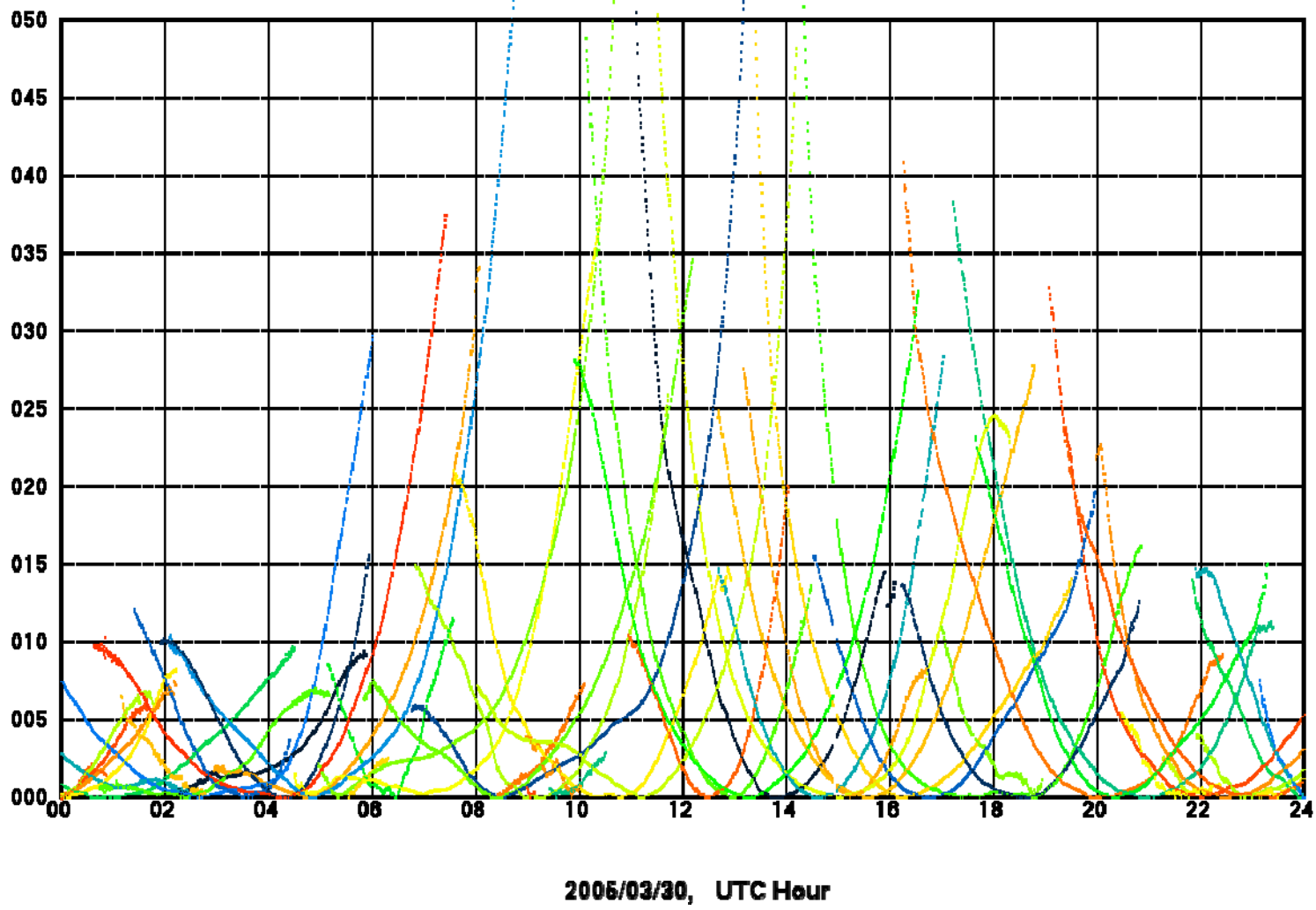
Arcs leveled to 0 minimum value



$S_p, zimm$

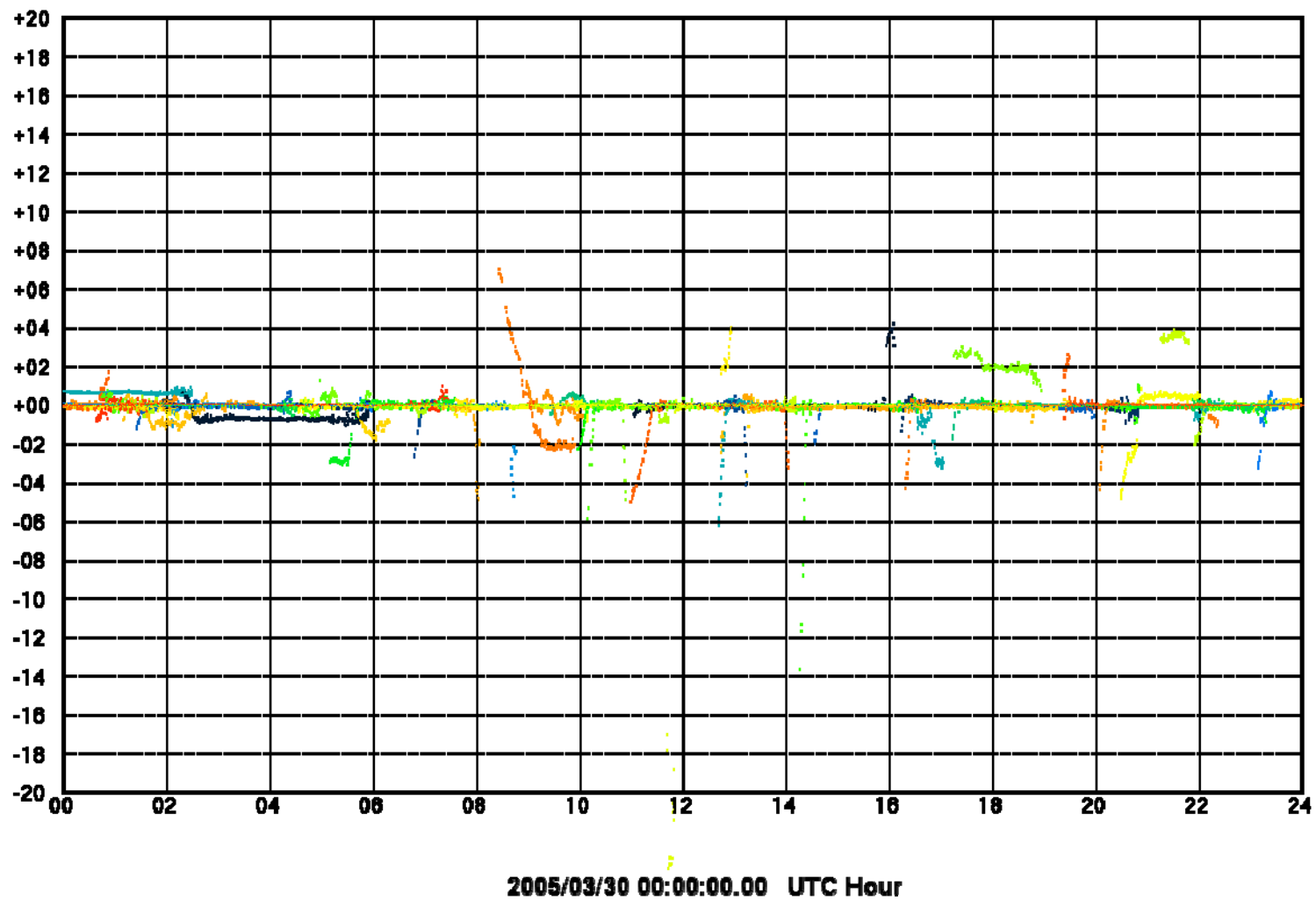
Arcs leveled to 0 minimum value

TEC(10**16) zimm Lat=46.9N Lon=7.5E
20224769 TRIMBLE 4700 Nav 1.30 / Boot 1



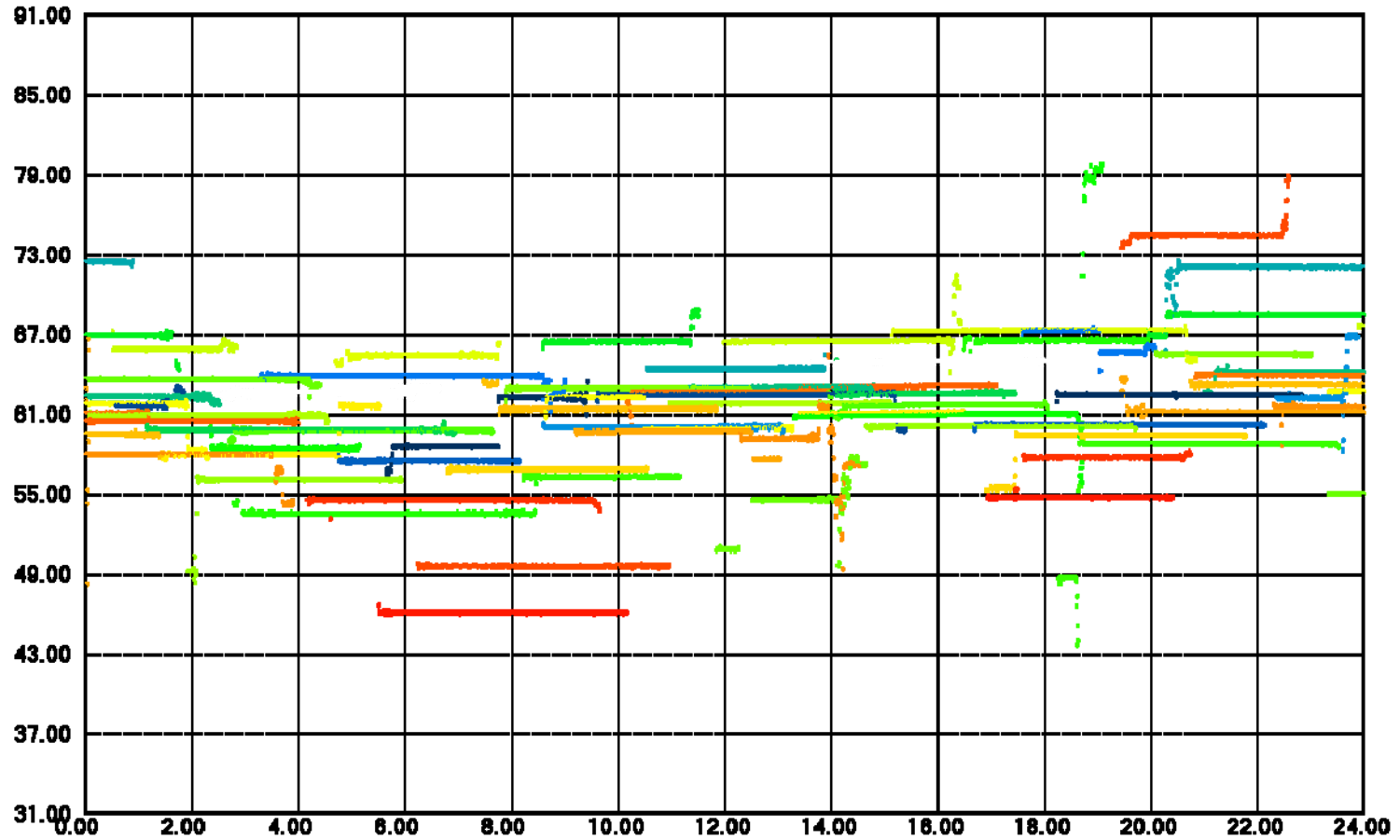
$$S_p(zimj) - S_p(zimm)$$

TEC(10**16) zimj Lat=46.9N Lon=7.5E
MT300342503 JPS LEGACY 2.3 APR,28,2004 P4



$S_{L1} - S_{L2}$, all satellites

dav1 Lat=68.6S Lon=78.0E RecTypeVer = 2006 ASHTECH UZ-12 CN00
davr Lat=68.6S Lon=78.0E RecTypeVer = 4614K01058 TRIMBLE NETR5 3.30

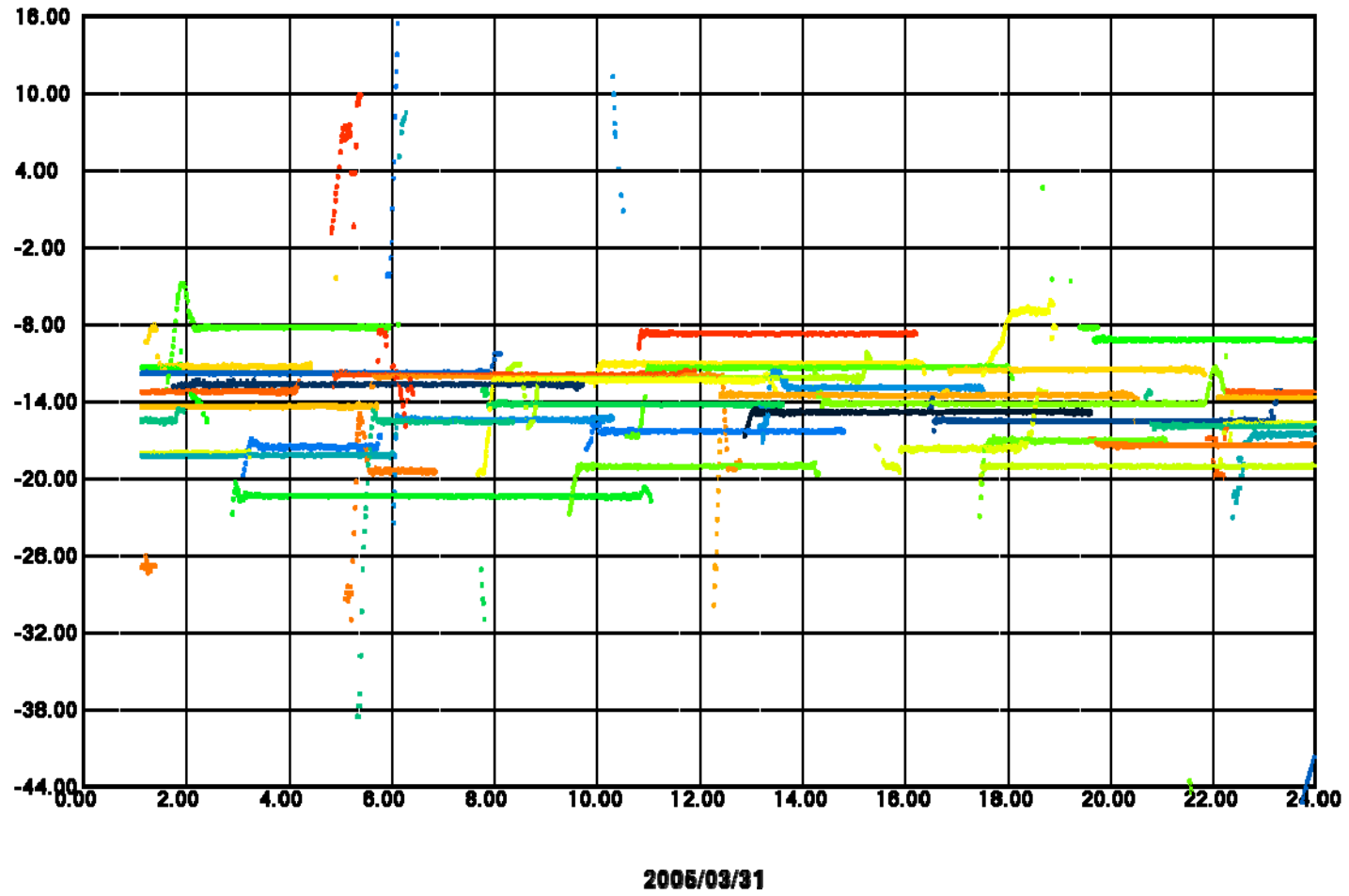


2008/03/30

$S_{L1} - S_{L2}$, all satellites

tcms Lat=24.8N Lon=121.0E RecTypeVer = 80057
trml Lat=24.8N Lon=121.0E RecTypeVer = 1132

LEICA RS500 4.00
AOA BENCHMARK ACT 1900.05.02



Is this spread due to multipath?

The spread among satellites, according to

$$S_{ij} - S_{ik} = \gamma_j - \gamma_k + \lambda_{Arc_i} - \lambda_{Arc_k}$$

provides with an estimation of the spread of $\lambda_{Arc_i} - \lambda_{Arc_k}$ around $\gamma_j - \gamma_k$

The split antenna experiment seems to confirm it.

The receivers of "*blue*" and "*red0*", of the same firm, have been fed from the same antenna.

Implications: "*blue*" and "*red0*" see exactly the same multipath.

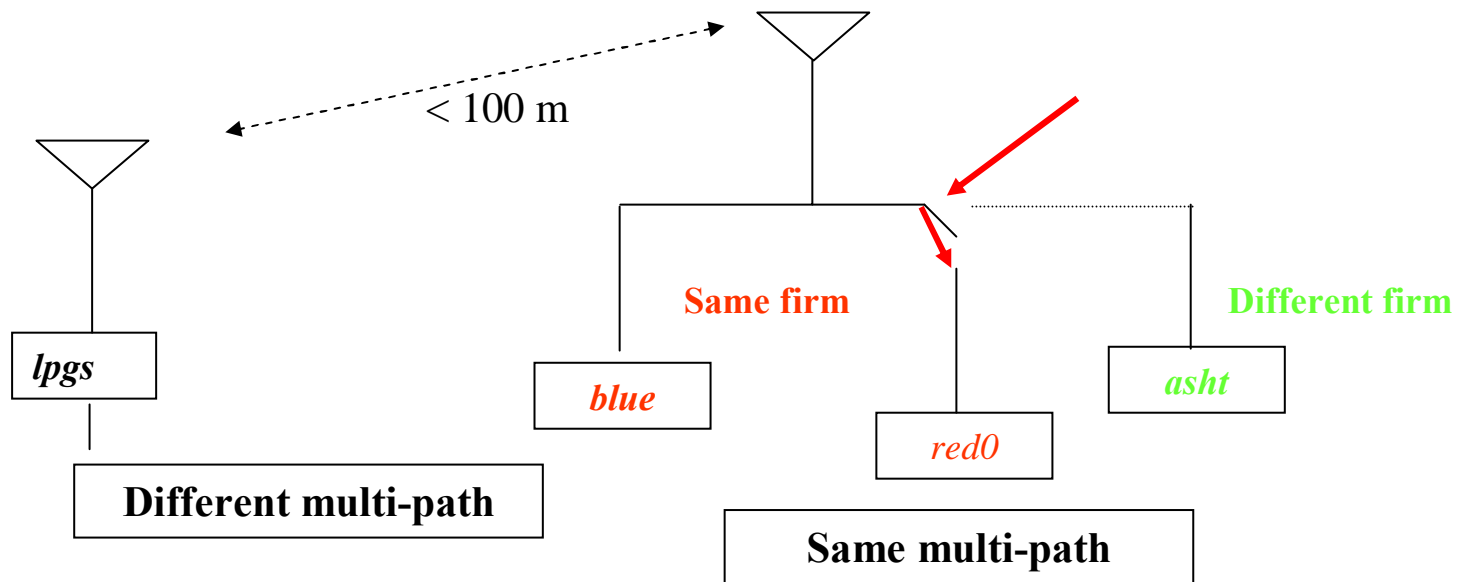
Besides IGS stations, a special set of observation has been set up by the group of La Plata University, Argentina (C.Brunini, F.Azpiliqueta).

Close to the IGS station “*lpgs*”, the additional stations “*blue*”, “*red0*” and “*asht*” have been set up to perform the following experiments

Close stations: different multipath; same or different way of processing multipath

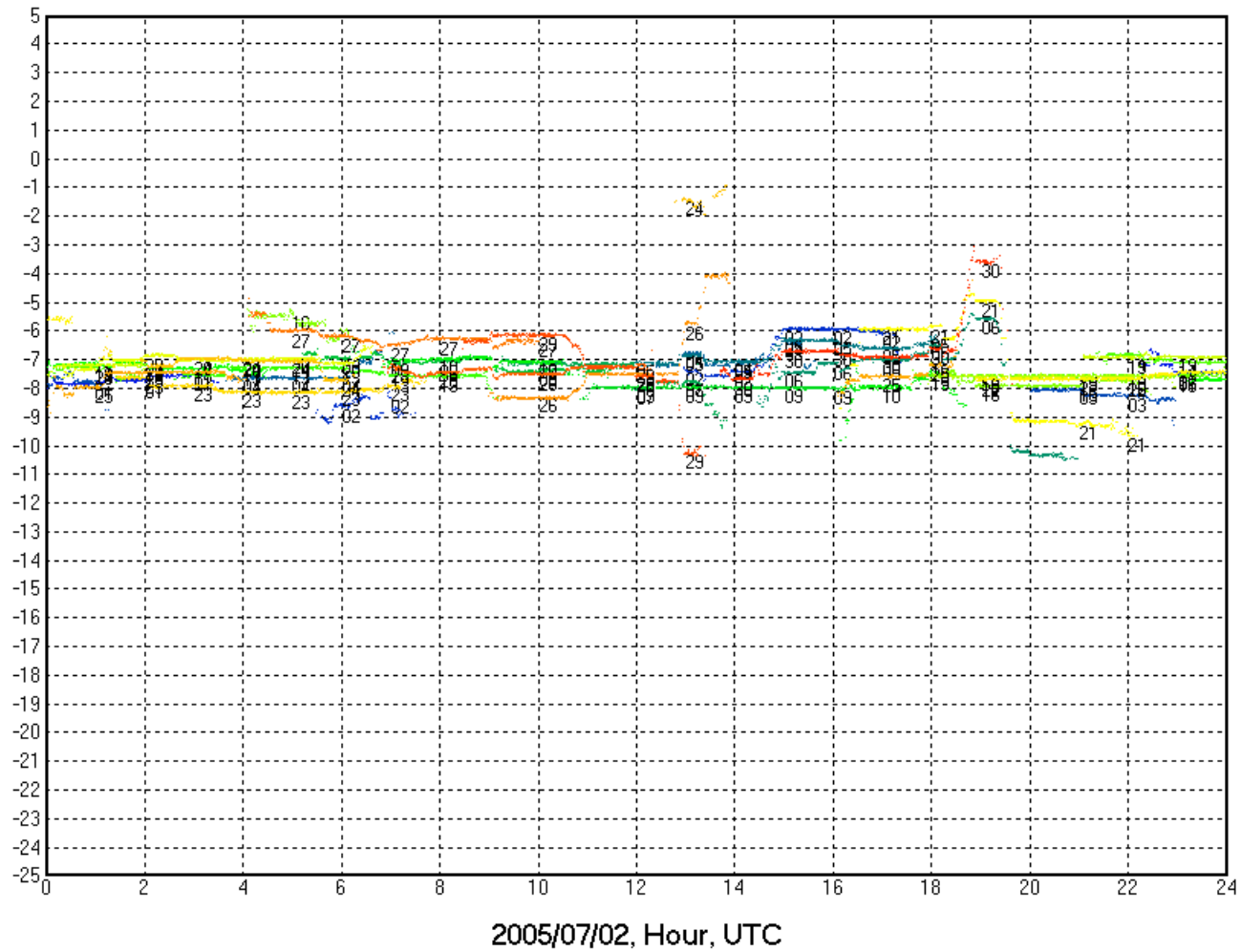
Split antenna, receivers of same firm: same multipath, same way of processing it

Split antenna, receivers of different firms: same multipath, different way of processing



Split antenna, same multipath, same type of receiver

TEC(10**16) blue - red0 Lat=34.9S Lon=57.9W



To reduce errors in observations, what is needed is

Recipes to reduce multipath effects

-care antenna environment and radio-technical coupling

In the normal situation, the observed discrepancies amount to several *TECu*.

If this is due to multi-path only, great care must be taken in selecting a weighted average $\langle \cdot \rangle$ using small weights when multi-path is expected to be large:

-avoid short arcs

-care the selection of weights

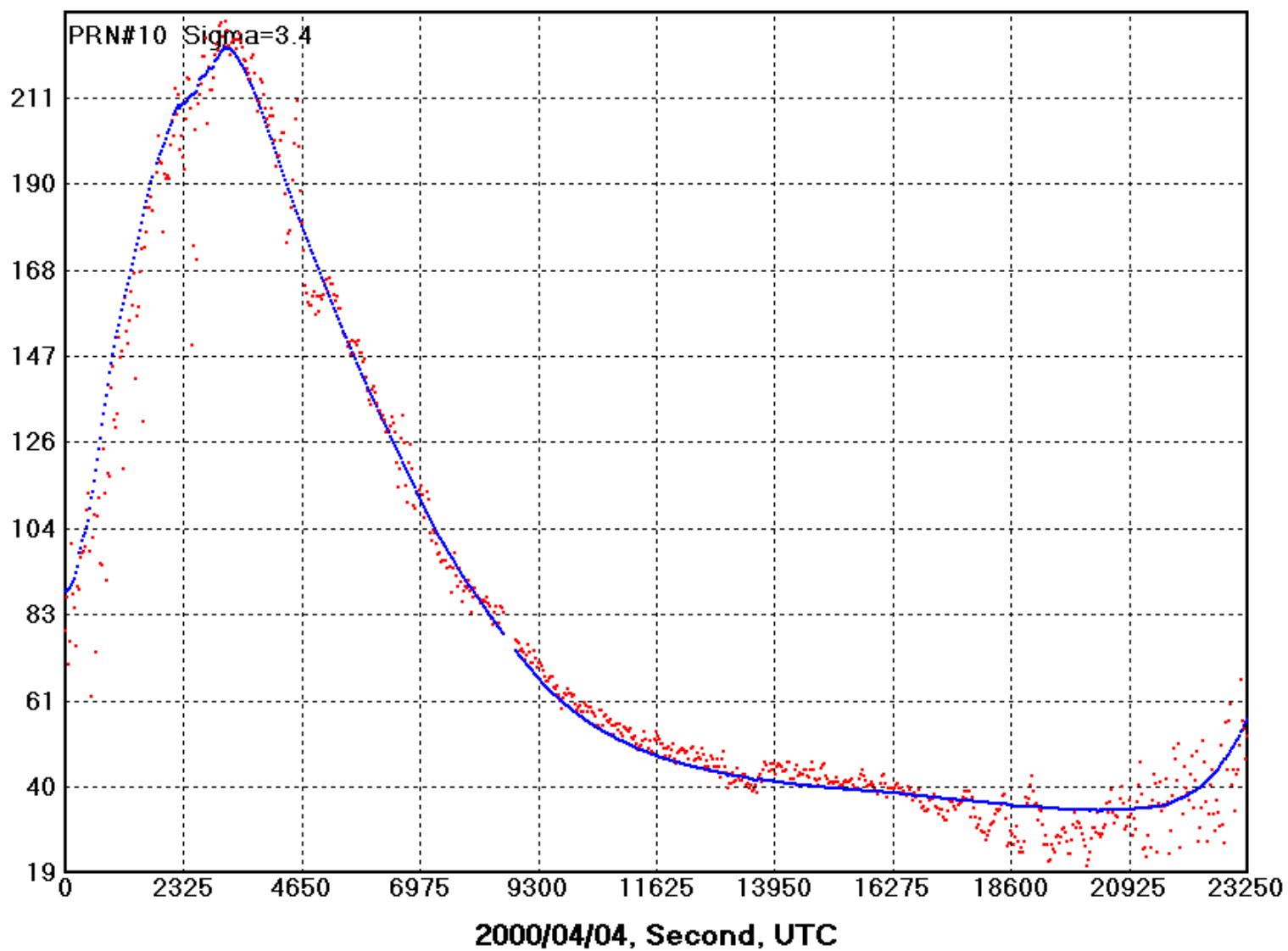
-use an elevation mask as higher as possible (where *m* is reasonably less strong)

empirically, using past experience

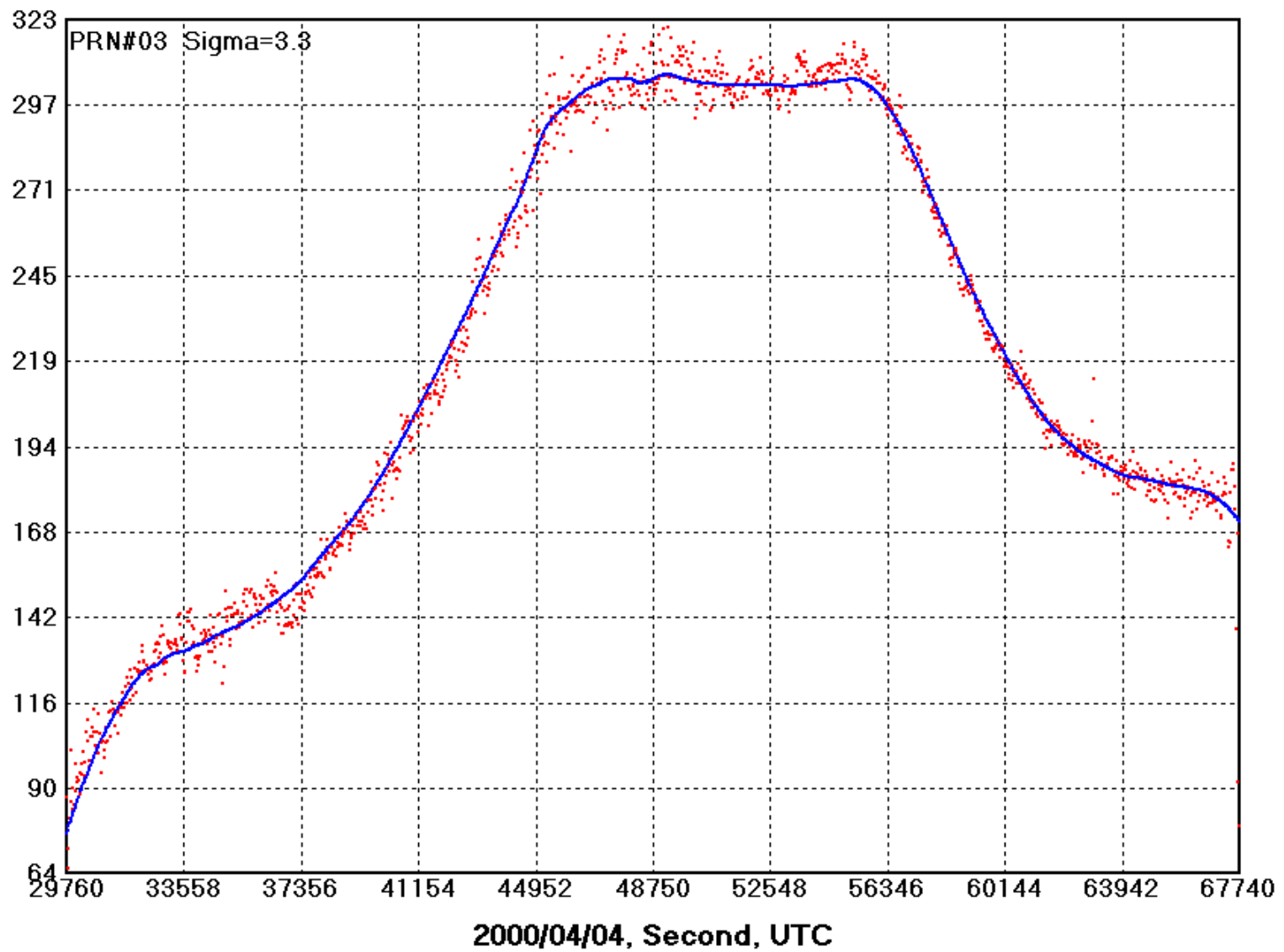
trying to estimate them from the plots of $S_G - S$, which according to the equations of the reported observables is $m + n - \langle m + n \rangle$

$$W = 1 \text{ if } \text{Abs}(S_G - S) < \text{Sigma}_{SG-S}$$
$$W = \left\{ \frac{\text{Sigma}_{SG-S}}{\text{Abs}(S_G - S)} \right\}^{2h}$$

TEC(10**16) asc1 Lat=08.0S Lon=14.4W



TEC(10^{16}) asc1 Lat=08.0S Lon=14.4W



But are we dealing with actual multipath only?

For some station pairs, strange patterns appear.

In the following, station "wtzj" compared to the colocated "wtza", "wtzr", "wtzt", "wtzz", exhibits a strange pattern.

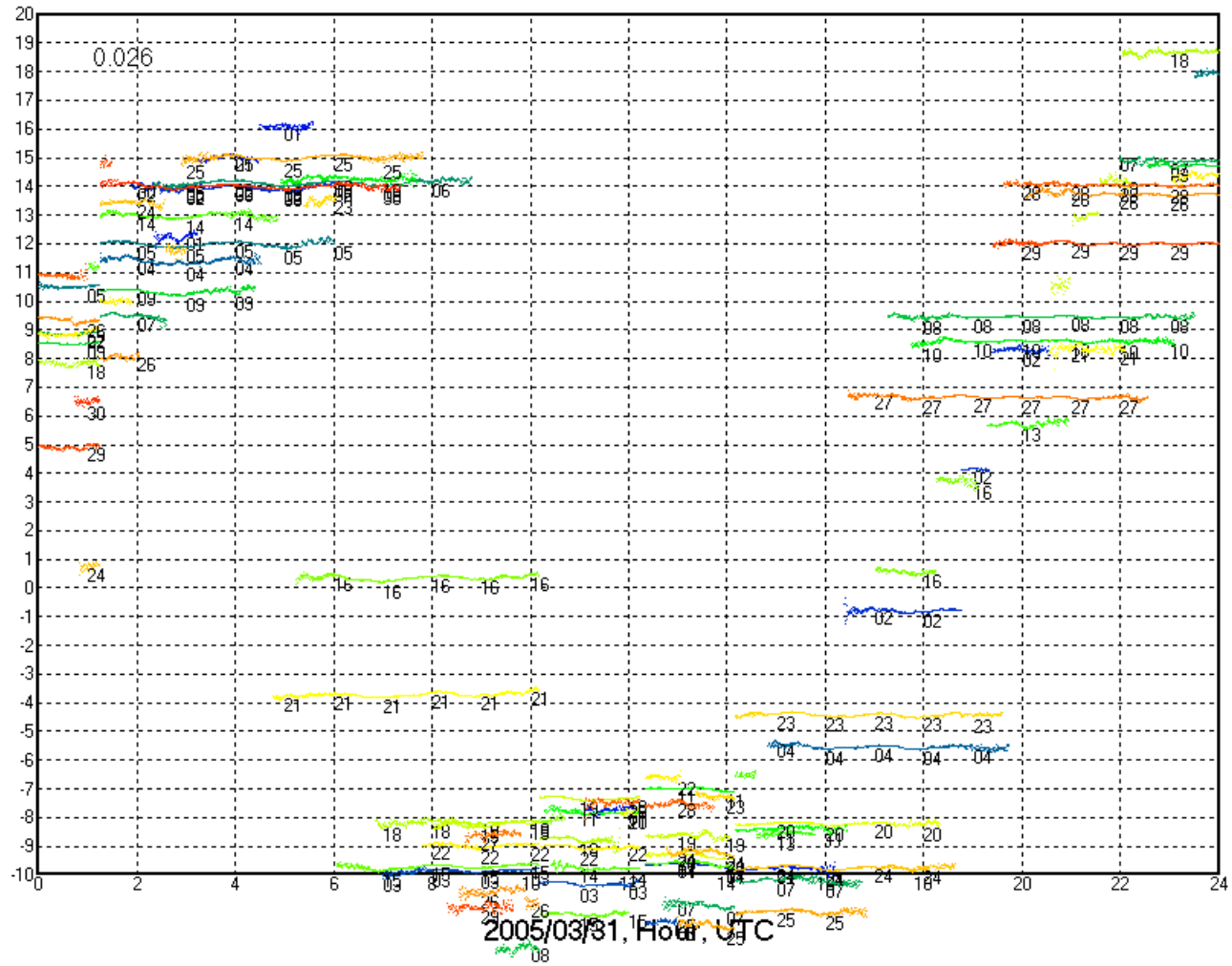
The problem is limited to "wtzj", as the plots for other pairs are "normal".

Is it a thermal drift of station bias?

What will it happen to the calibration with discrepancies amounting to almost 25 $TECu$, and having no knowledge of the behavior of the station (evidenced only by the availability of close stations) ?

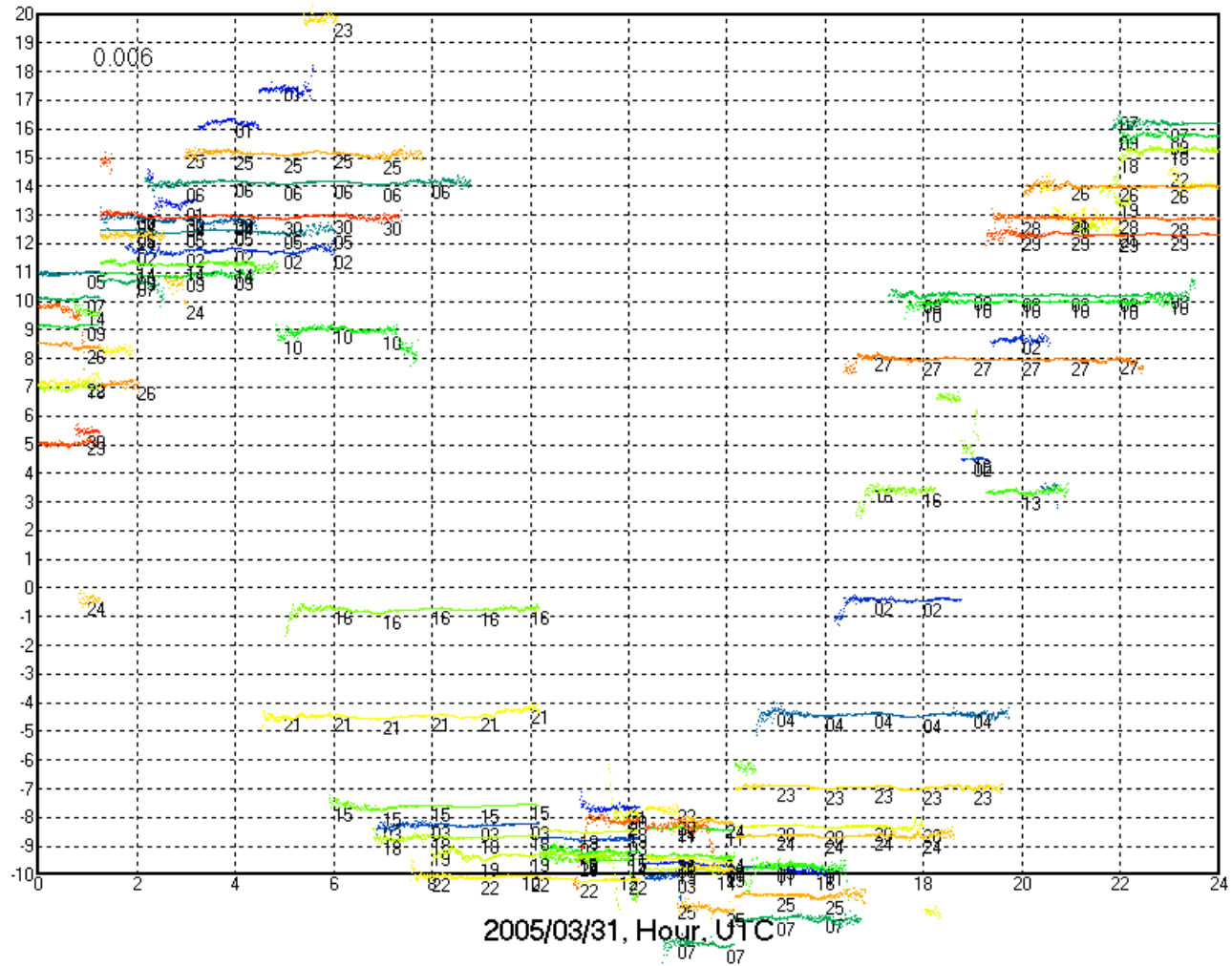
$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtza - wtzj Lat=49.1N Lon=12.9E



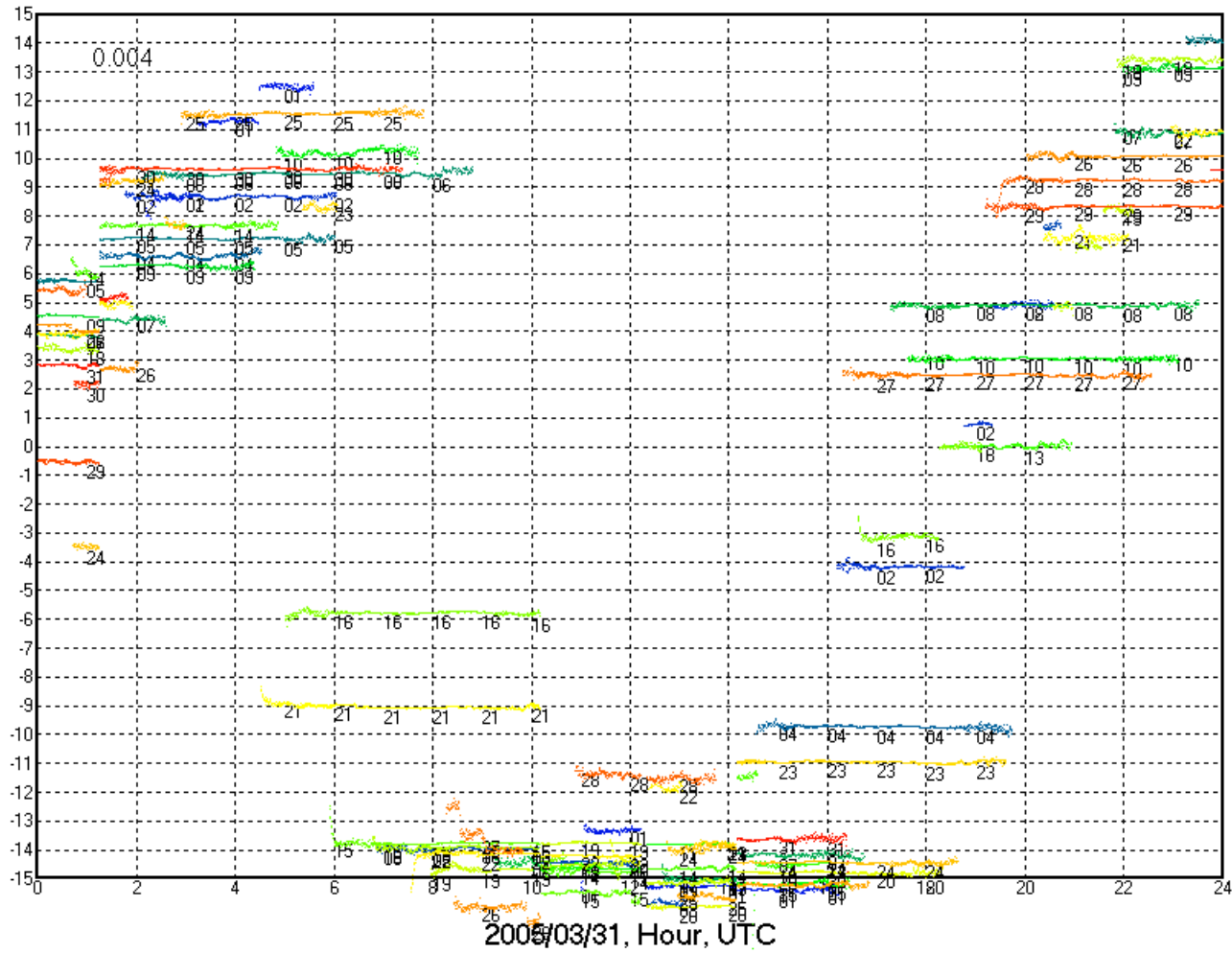
$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtzi - wtzj Lat=49.1N Lon=12.9E



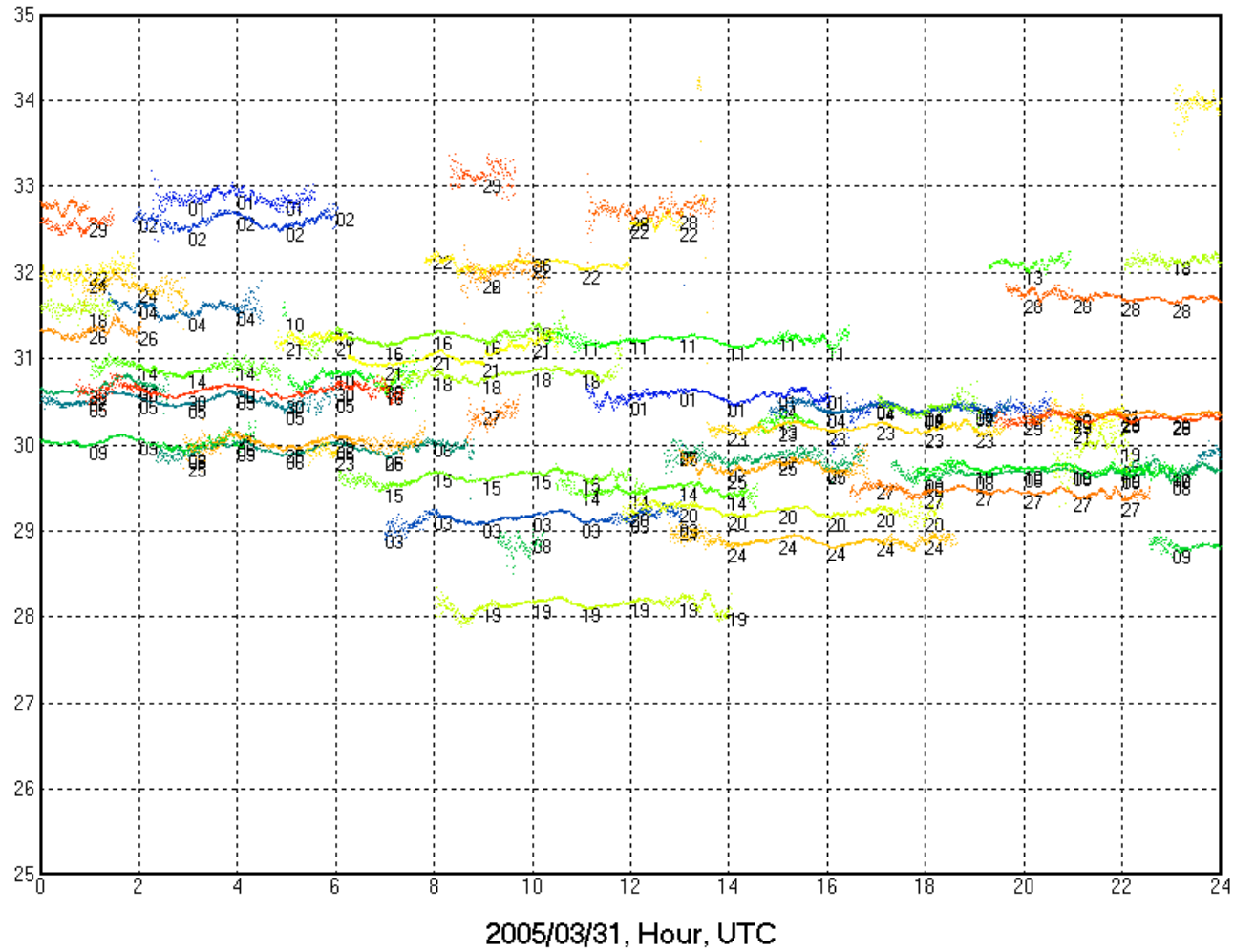
$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtzz - wtzj Lat=49.1N Lon=12.9E



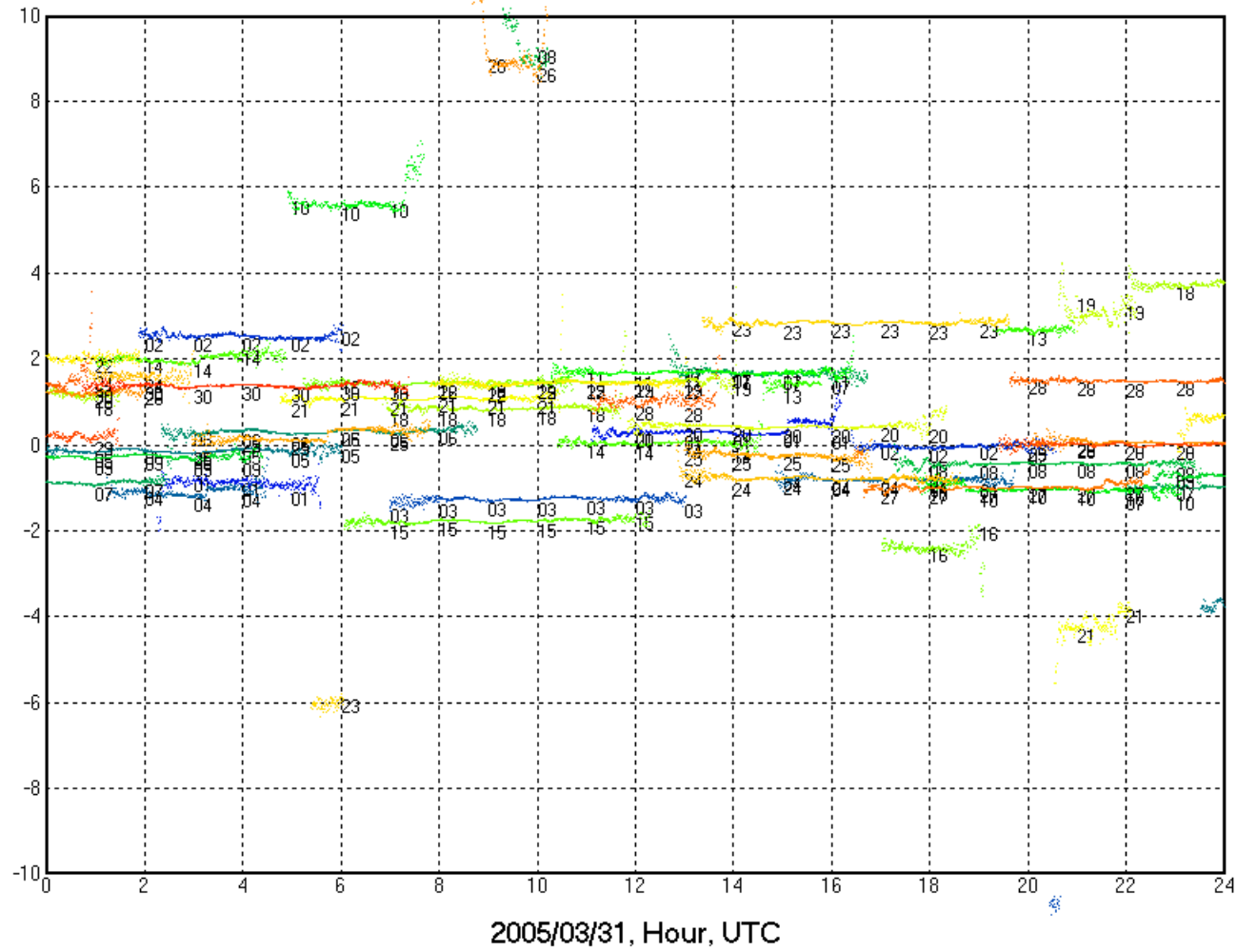
$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtza - wtzr Lat=49.1N Lon=12.9E



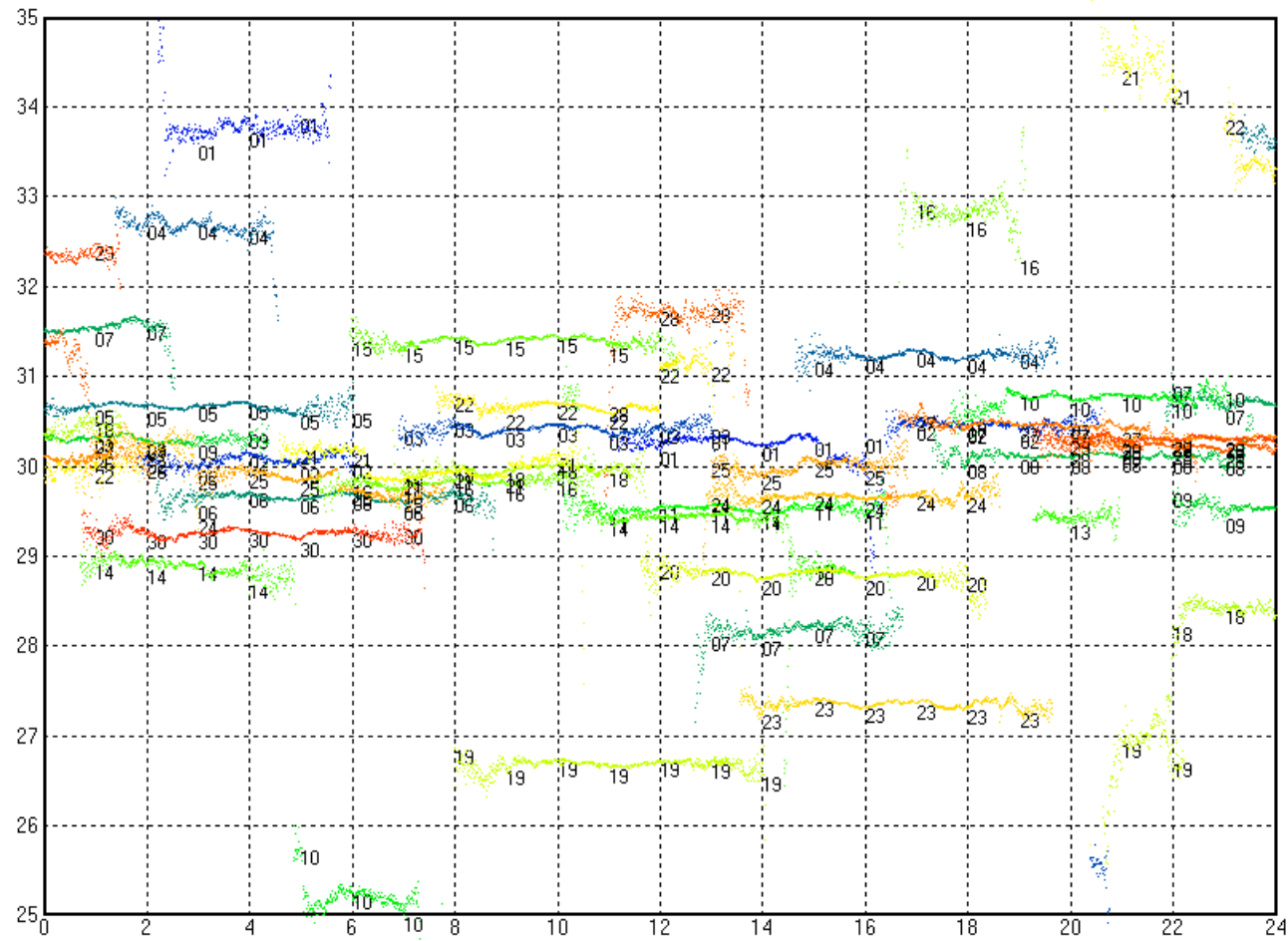
$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtza - wtzi Lat=49.1N Lon=12.9E



$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtzt - wtzr Lat=49.1N Lon=12.9E

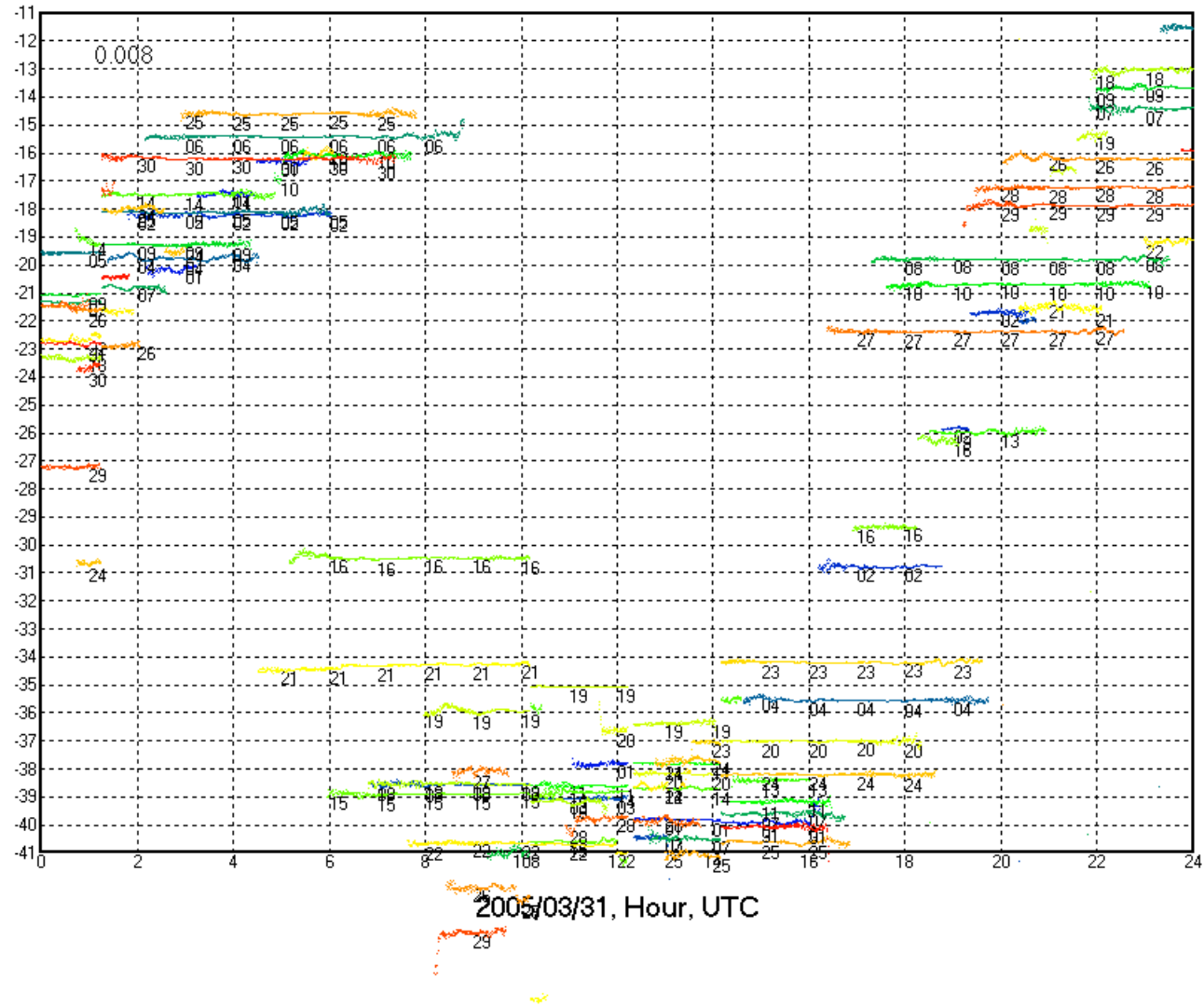


2005/03/31, Hour, UTC

26

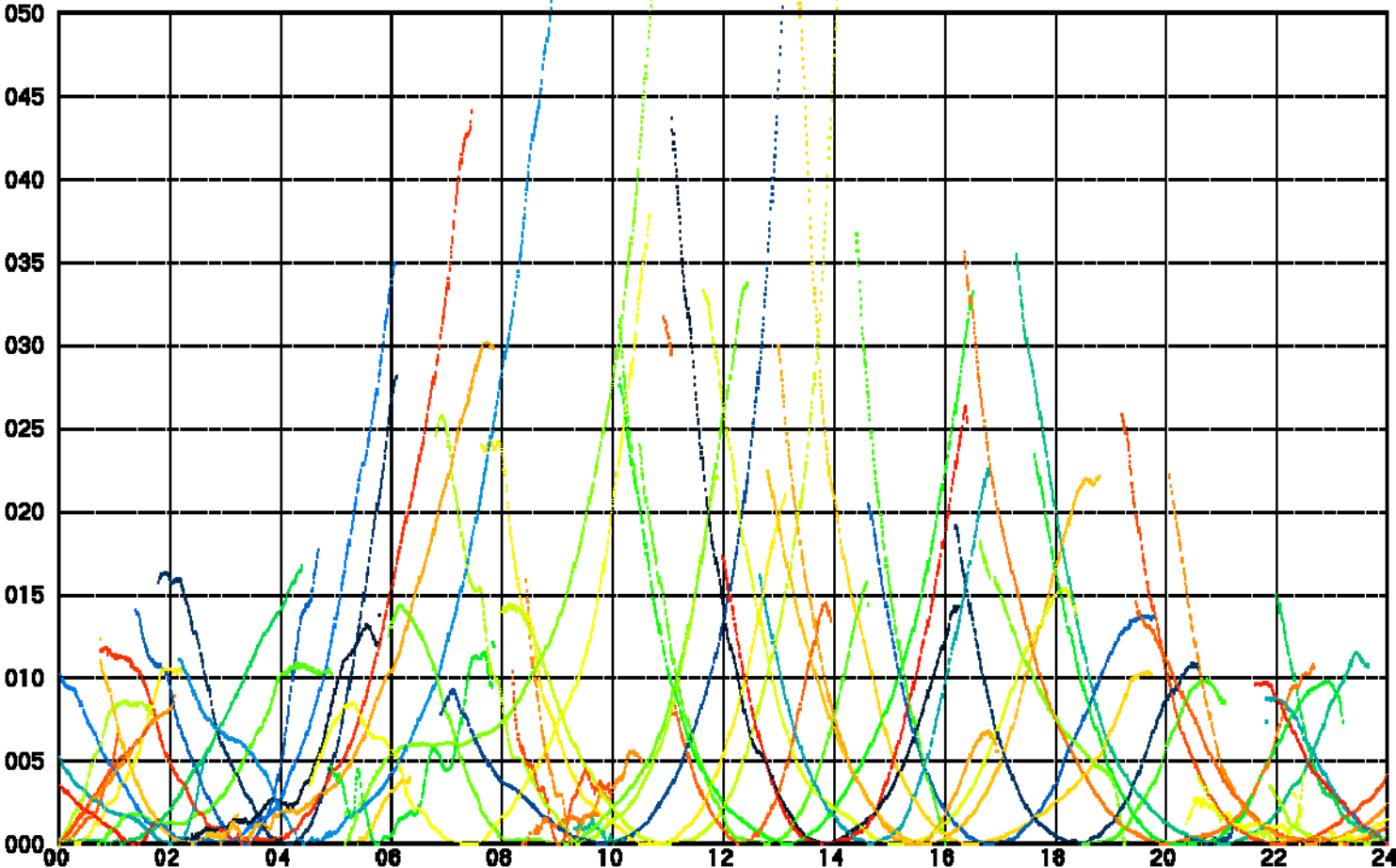
$S_{L1} - S_{L2}$, all satellites

TEC(10**16) wtzr - wtzj Lat=49.1N Lon=12.9E



Arcs leveled to 0 minimum value

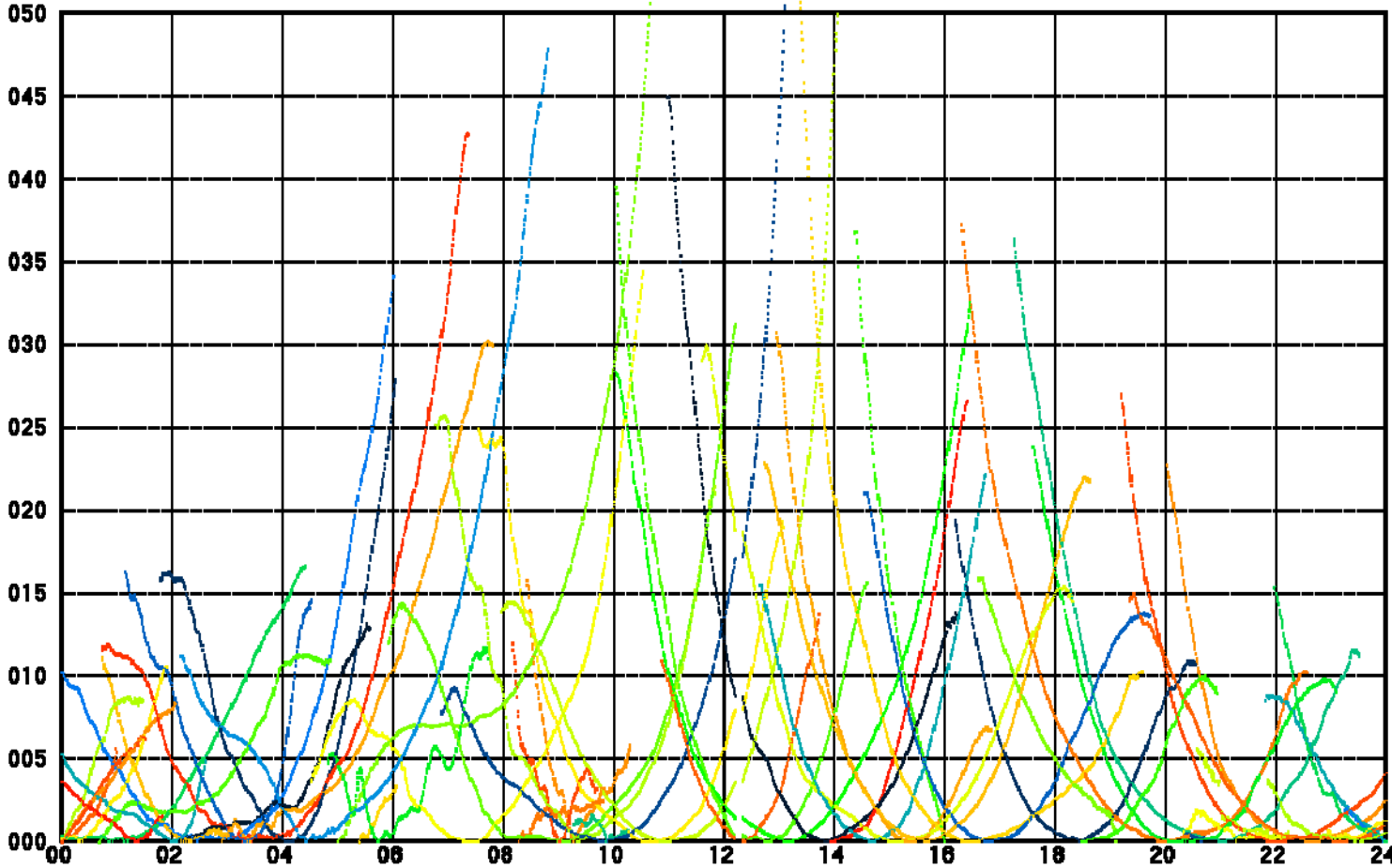
TEC(10**16) wtzr Lat=49.1N Lon=12.9E
T-317U AOA SNR-8000 ACT 3.3.32.5



2005/03/31, UTC Hour

Arcs leveled to 0 minimum value

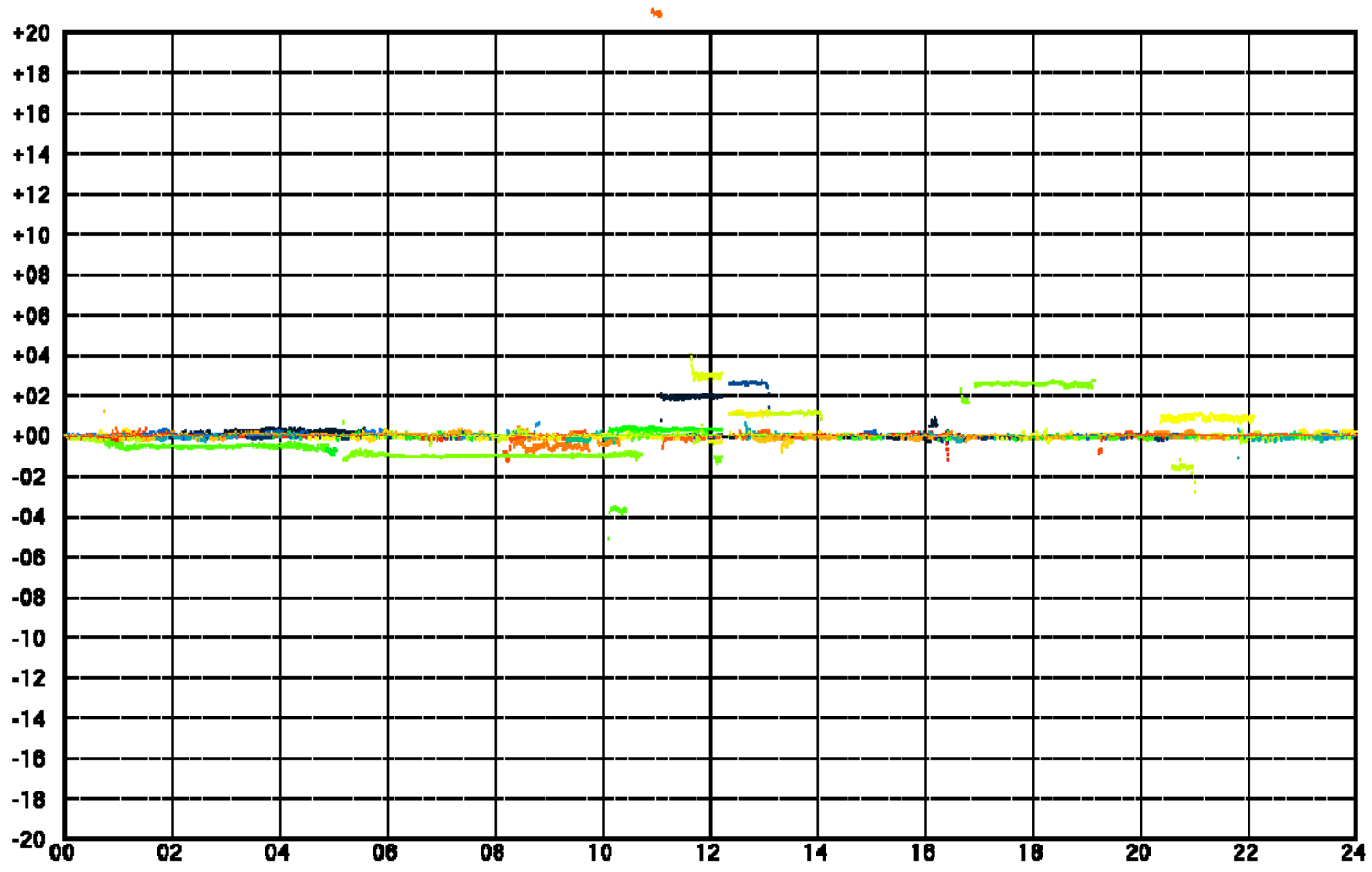
TEC(10**16) wtz| Lat=49.1N Lon=12.9E
MT312211422 JPS LEGACY EURO_3/2.3P4



2005/03/31, UTC Hour

$$S_P(wtzj) - S_P(wtzt)$$

TEC(10**16) wtzr Lat=49.1N Lon=12.9E
T-317U AOA SNR-8000 ACT 3.3.32.5



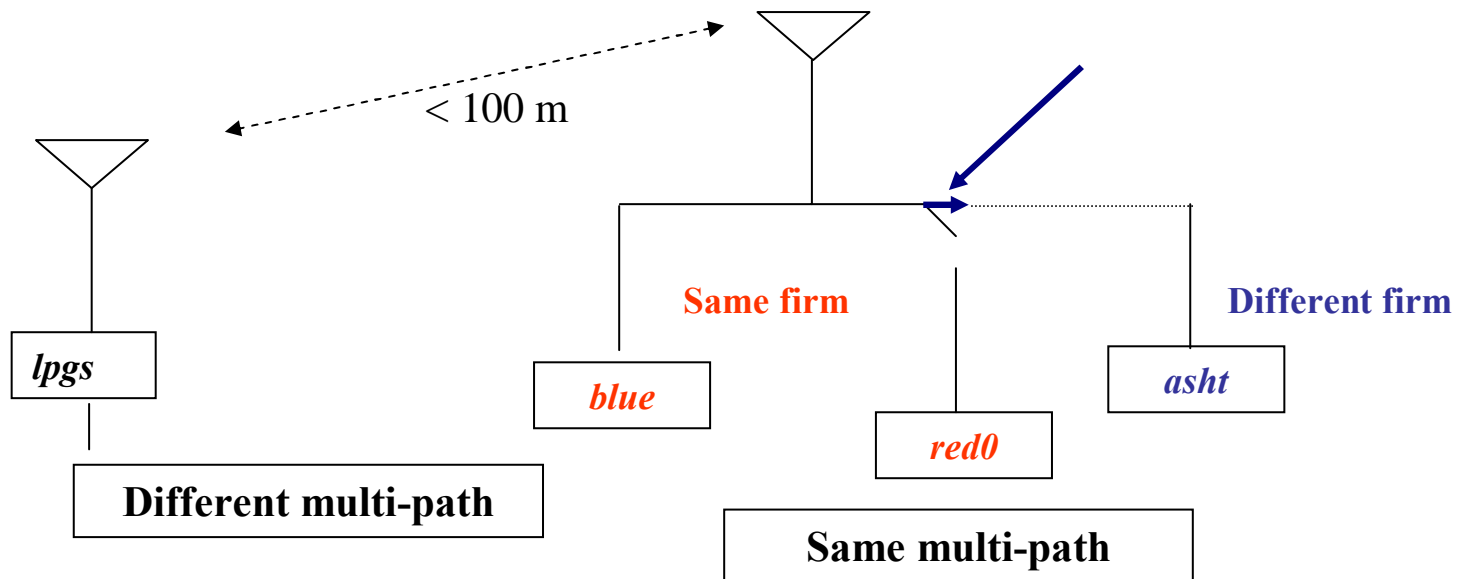
2005/03/31 00:00:00.00 UTC Hour

Still: **only multipath or some other problem?**

Back to the split antenna experiment,

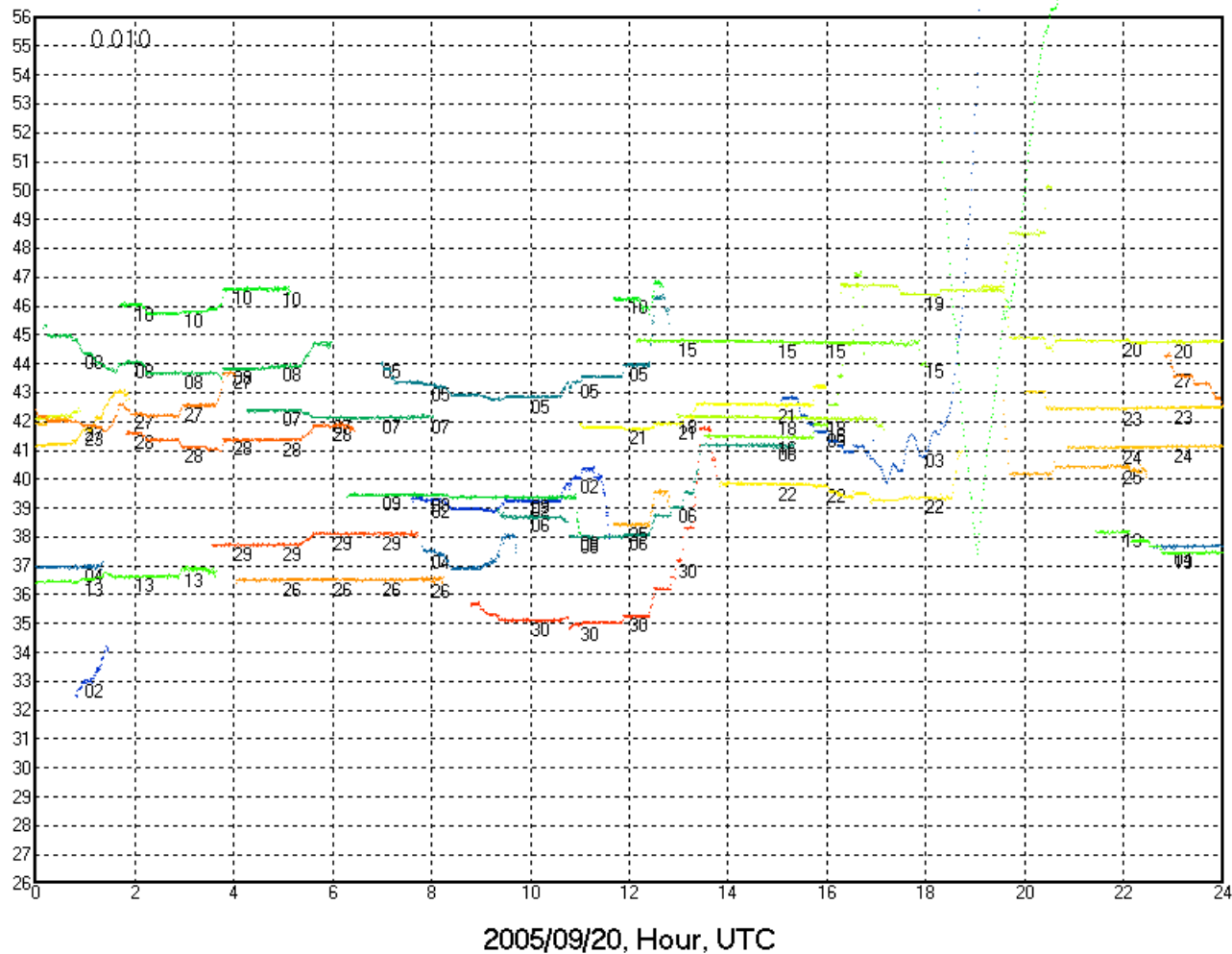
but using receivers of different firms.

Spread will appear again, suggesting that its cause is more the way by which multipath is processed rather than multipath itself.



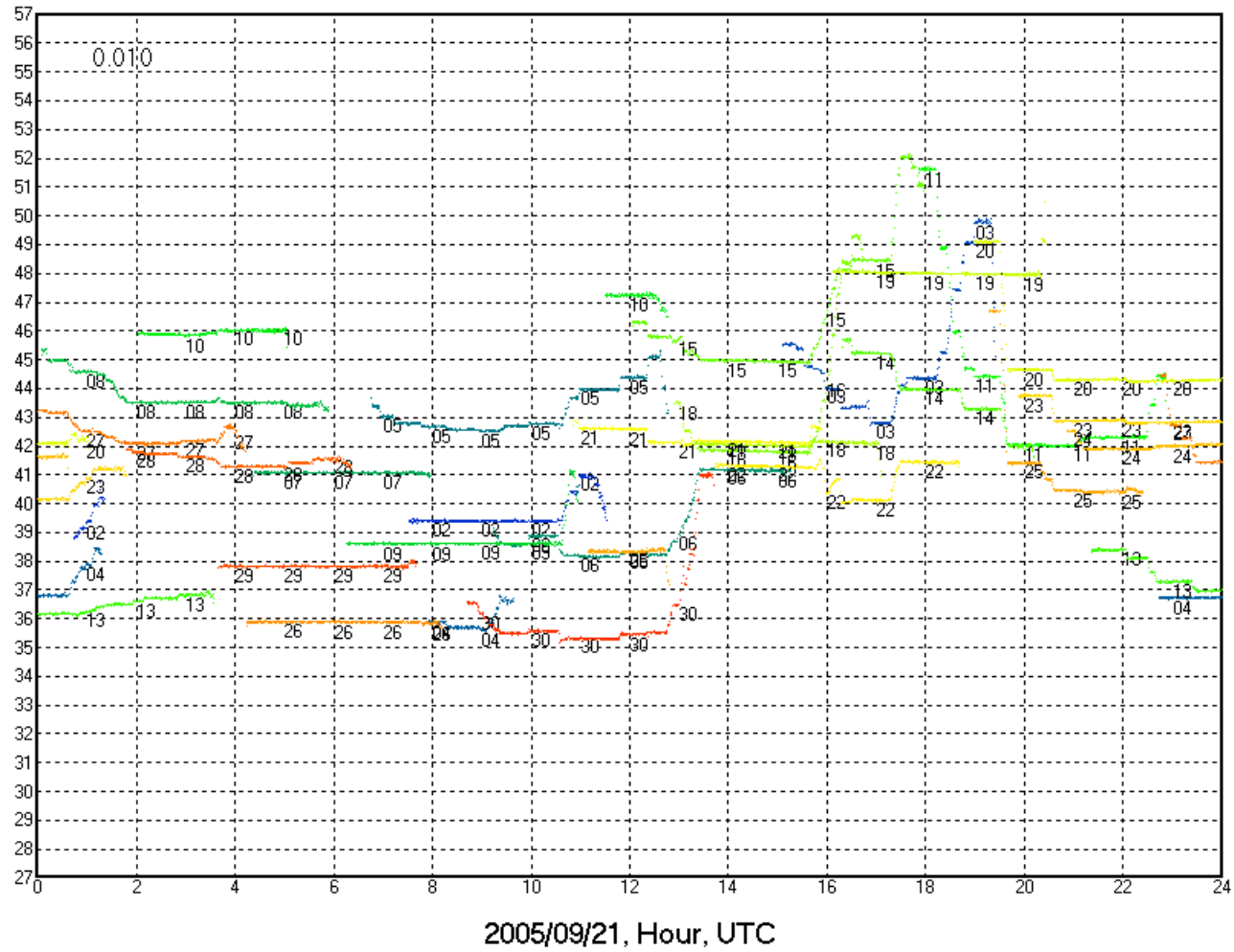
Split antenna, same multipath, different type of receiver

TEC(10**16) asht - blue Lat=34.9S Lon=57.9W

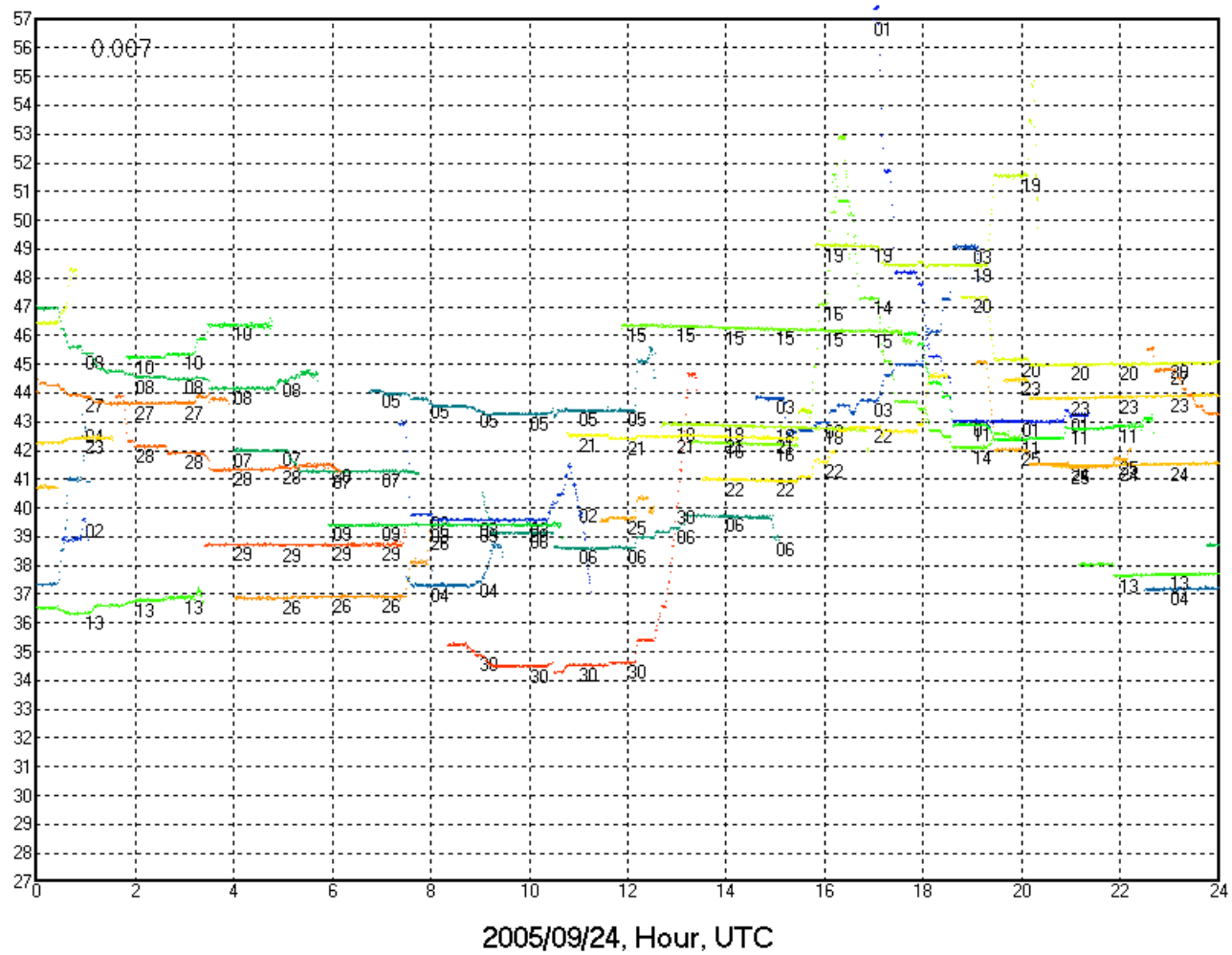


Split antenna, same multipath, different type of receiver

TEC(10**16) asht - blue Lat=34.9S Lon=57.9W



TEC(10**16) asht - blue Lat=34.9S Lon=57.9W



Conclusion of above experiments

Leveled to code slants are affected by the leveling error λ

The leveling error λ is most likely due to multipath (*)

Receivers of the same type produce similar λ 's, but there is no way to estimate their magnitude

Different types of receivers produce different λ 's observing the same ray

(*) other possible cause are possible, but not up to now investigated: studying scintillation it has been evidenced effect due to interference of other GPS satellites (still sidereal-time synchronous effects)

Is it correct modeling leveled slants S_L disregarding λ ?

For many station pairs, answer is negative

Still: no a priori method exists to notice that something is wrong unless availing two or more stations (see above plot of slants from close stations).

The results of the close stations experiment seem to evidence the need to introduce an additional satellite “bias”, the leveling error λ , dependent on the receiving station

(and the receiver type == way of extracting pseudorange).

Leveling error λ is an arc dependent unknown: this implies that

No advantage is taken using leveled slants S_L with respect to phase slants (but this will need introducing one unknown per arc).

The choice of the calibration method

Aiming to

a simple solution (thin shell)

avoiding the problems of slants leveled to code S_L
(when leveling error is disregarded)

mitigating the errors of mapping function

It is natural to select a **single station** solution using **phase slants S_P** or leveled slants S_L

Notes about V_{Eq} approach

It takes automatically into account of plasmaspheric contribution

It is easier to model at low latitudes than actual vertical **TEC**

It presents some more difficulty to model at low elevations

The single station solution: Calibration

Observations

Phase slants S_p

Assumptions

One thin shell at 400 km

Elevation mask: 10°

TEC expressed through V_{Eq} at the ionospheric point, by the mapping function $sec \chi$

V_{Eq} expressed as a proper expansion of horizontal coordinates l, f with one set of coefficients at each time $V_{Eq}(l, f) = \sum_n c_n p_n(l, f)$

$$S_{ijt} = \sum_n c_n^{(t)} p_n(l_{ijt}, f_{ijt}) sec \chi_{ijt} + \Omega_{Arc}$$

The unknowns are now the coefficients $c_n^{(t)}$ and the offsets Ω_{Arc}

To solve the system

$$S_{ijt} = \sum_n c_n^{(t)} p_n(l_{ijt}, f_{ijt}) \sec \chi_{ijt} + \Omega_{Arc}$$

extra assumptions are taken to reduce the number of coefficients $\sum_n c_n^{(t)}$

Using as horizontal coordinates *Modified Dip Angle* and *Local Time*, we can assume that for a set of adjacent epochs (up to ± 15 minutes), the coefficients $c_n^{(t)}$ keep constant.

This allows also reducing computing resources during solution using commonly used standard methods for sparse systems.

After the solution of the system, we avail with

Calibrated slants along the observed rays $TEC_{ijt} = S_{ijt} - \Omega_{Arc}$

“Mapped slants” at given coordinates l_{ijt}, f_{ijt}

Vertical *TEC* above the station (ionospheric point at the its zenith)

$$VTec(t) = \sum_n c_n^{(t)} p_n(l_{ijt}^{Zenith}, f_{ijt}^{Zenith}) \sec \chi_{ijt}$$

Performance of the proposed calibration method must be now investigated

- 1) **A first look:** will it provide same *TEC*'s from colocated stations?
- 2) **Internal consistency:** compute the residuals

$$R_{ijt} = S_{ijt} - \sum_n c^{(t)}_n p_n(l_{ijt}, f_{ijt}) \sec \chi_{ijt} - \Omega_{Arc}$$

Small residuals mean good internal consistency, but do not help in asserting the accuracy of the method.

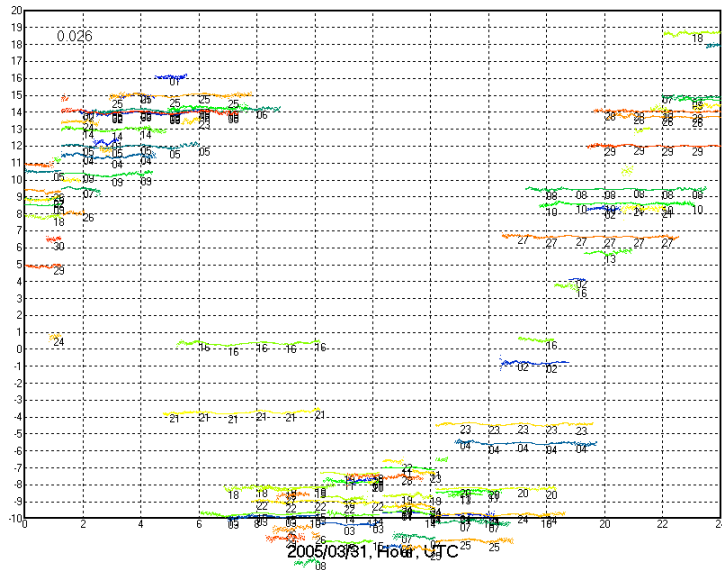
3) External consistency, namely the comparison with completely independent observations, should be the only way to assert the accuracy.

Possible observations: Incoherent Scatter Radar (**ISR**), Two-Frequency Radar Altimeter (**RA-2**). Problems: very few *ISR*'s, **RA-2** needs its own calibration.

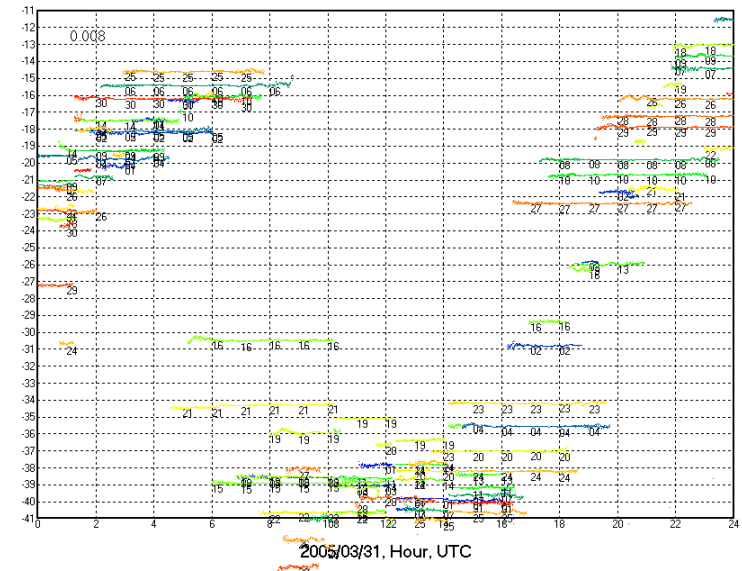
Only possibility: using artificial truth data obtained using ionospheric models

A first look: worth adopting the above procedure for calibration?

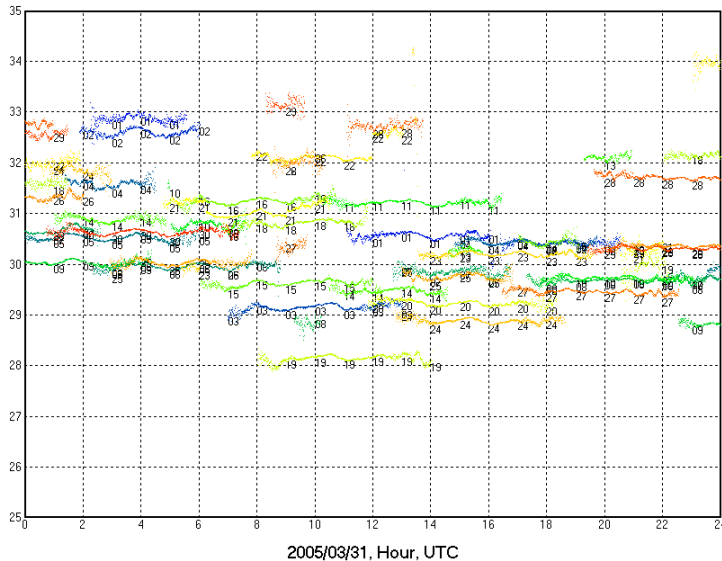
TEC(10**16) wtza - wtzj Lat=49.1N Lon=12.9E



TEC(10**16) wtzr - wtzj Lat=49.1N Lon=12.9E



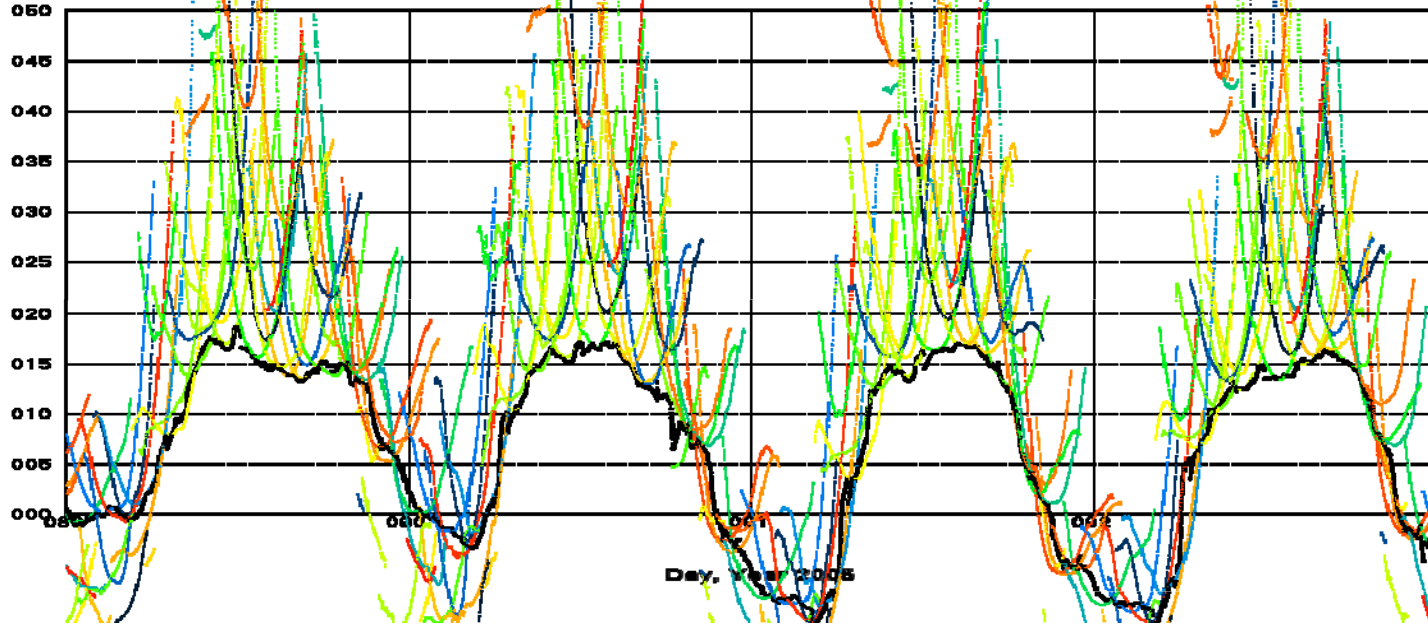
TEC(10**16) wtza - wtzr Lat=49.1N Lon=12.9E



Close station plots for *wtza*, *wtzj*, *wtzr* suggest that something is wrong with *wtzj*. Try arc offsets and standard biases calibration for the above stations

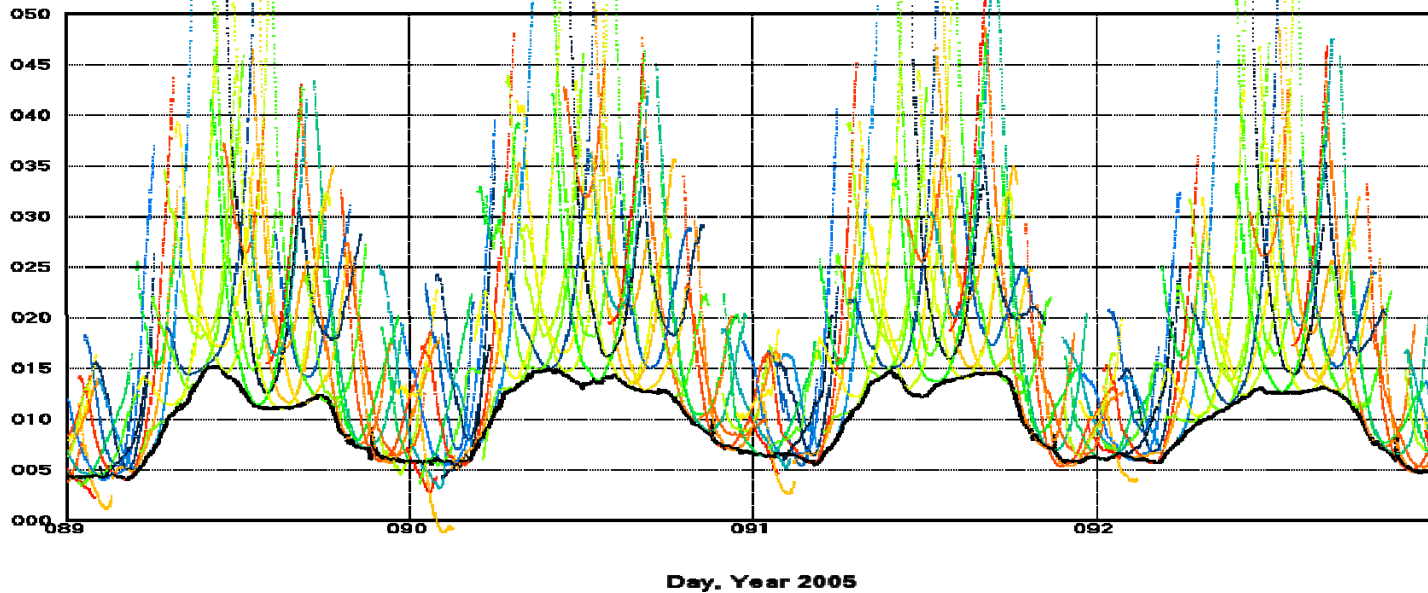
TEC(10**16) wtz| Lat=49.1N Lon=12.9E
MT312211422 JPS LEGACY EURO_3/2.3P4

Standard solution, $S_L, \beta+\gamma$



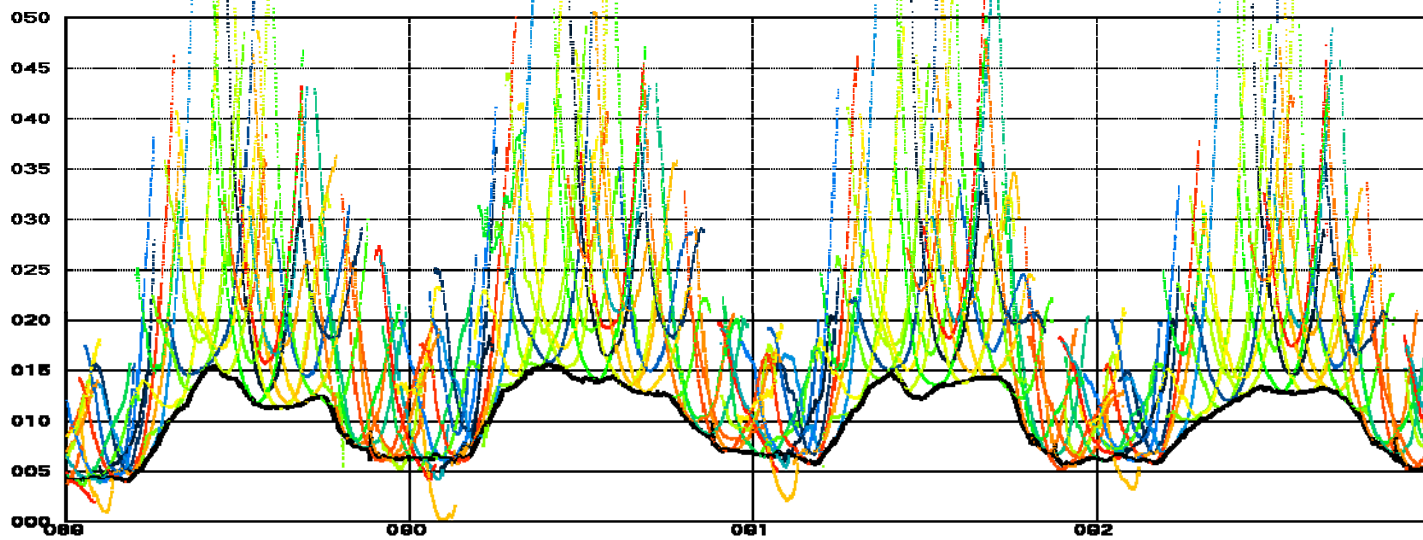
TEC(10**16) wtz| Lat=49.1N Lon=12.9E
MT312211422 JPS LEGACY EURO_3/2.3P4

Proposed solution, S_{pt}, Ω_{Arc}



TEC(10**16) wtzr Lat=49.1N Lon=12.9E
T-317U AOA SNR-8000 ACT 3.3.32.6

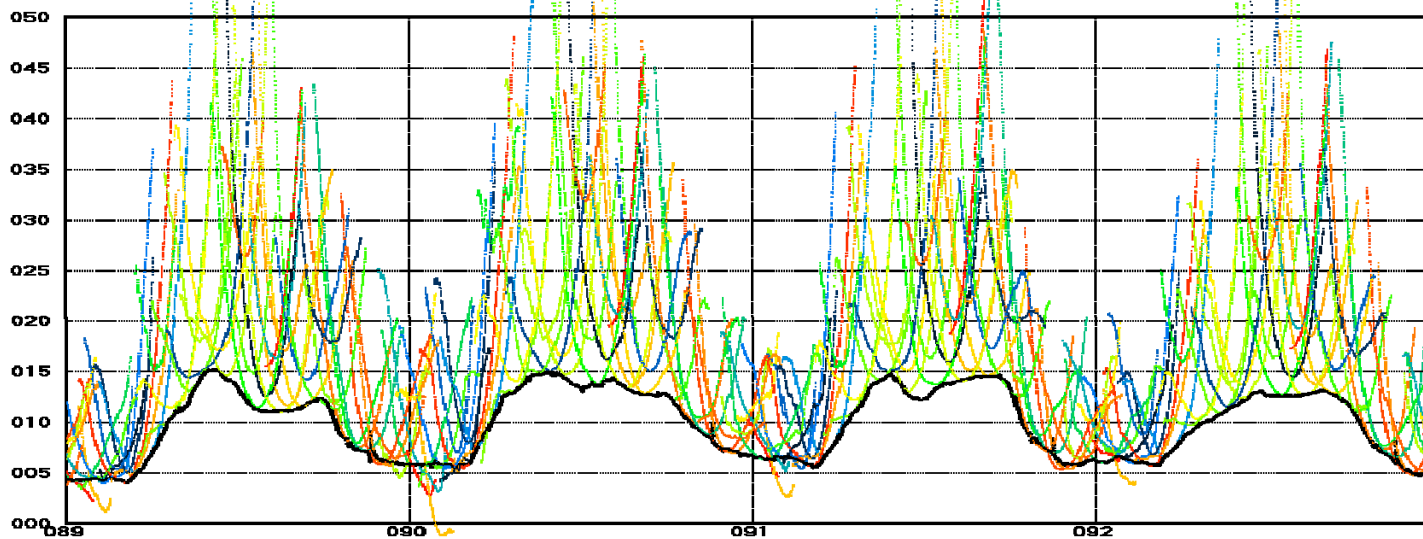
Proposed solution, S_p, Ω_{Arc}



Day, Year 2005

TEC(10**16) wtzr Lat=49.1N Lon=12.9E
MT312211422 JPS LEGACY EURO_3/2.3P4

Proposed solution, S_p, Ω_{Arc}



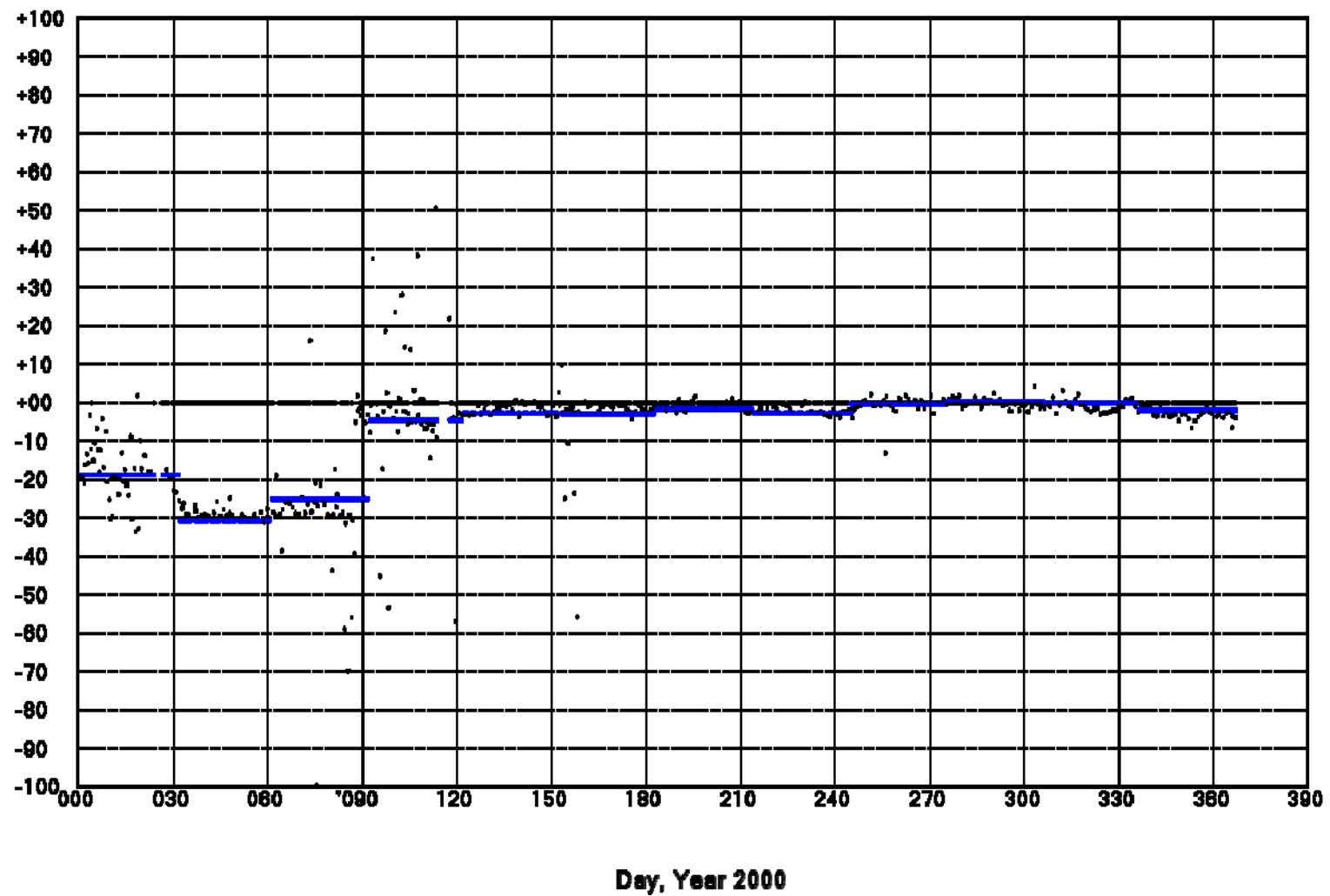
Day, Year 2005

How do traditional and proposed solution compare?

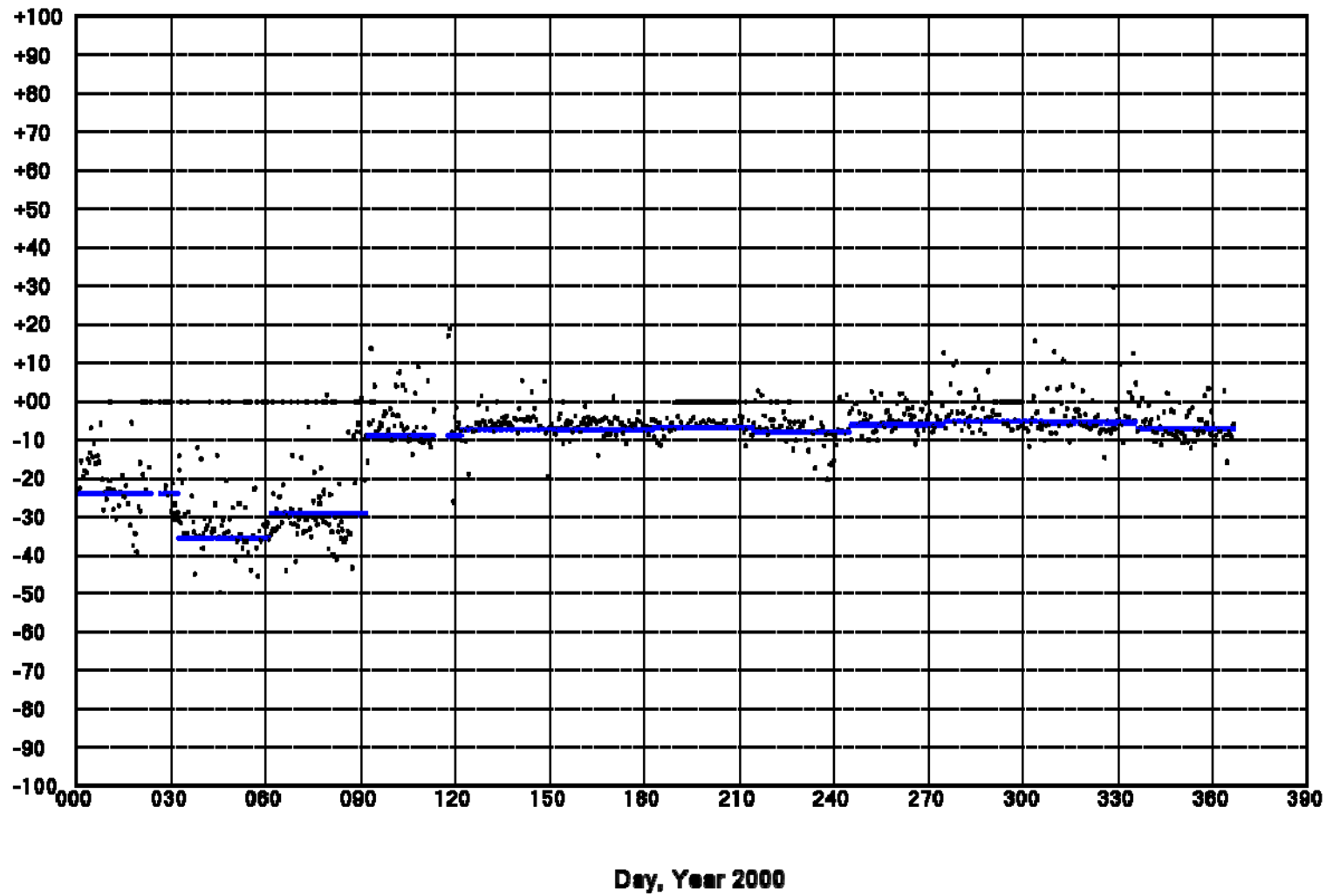
In the following slides it can be seen that the two solutions agree in the average, but the difference in bias can amount to 10 *TECu*

The pattern of the jumps, similar for different satellites, simply indicates that something has changed in the receiver

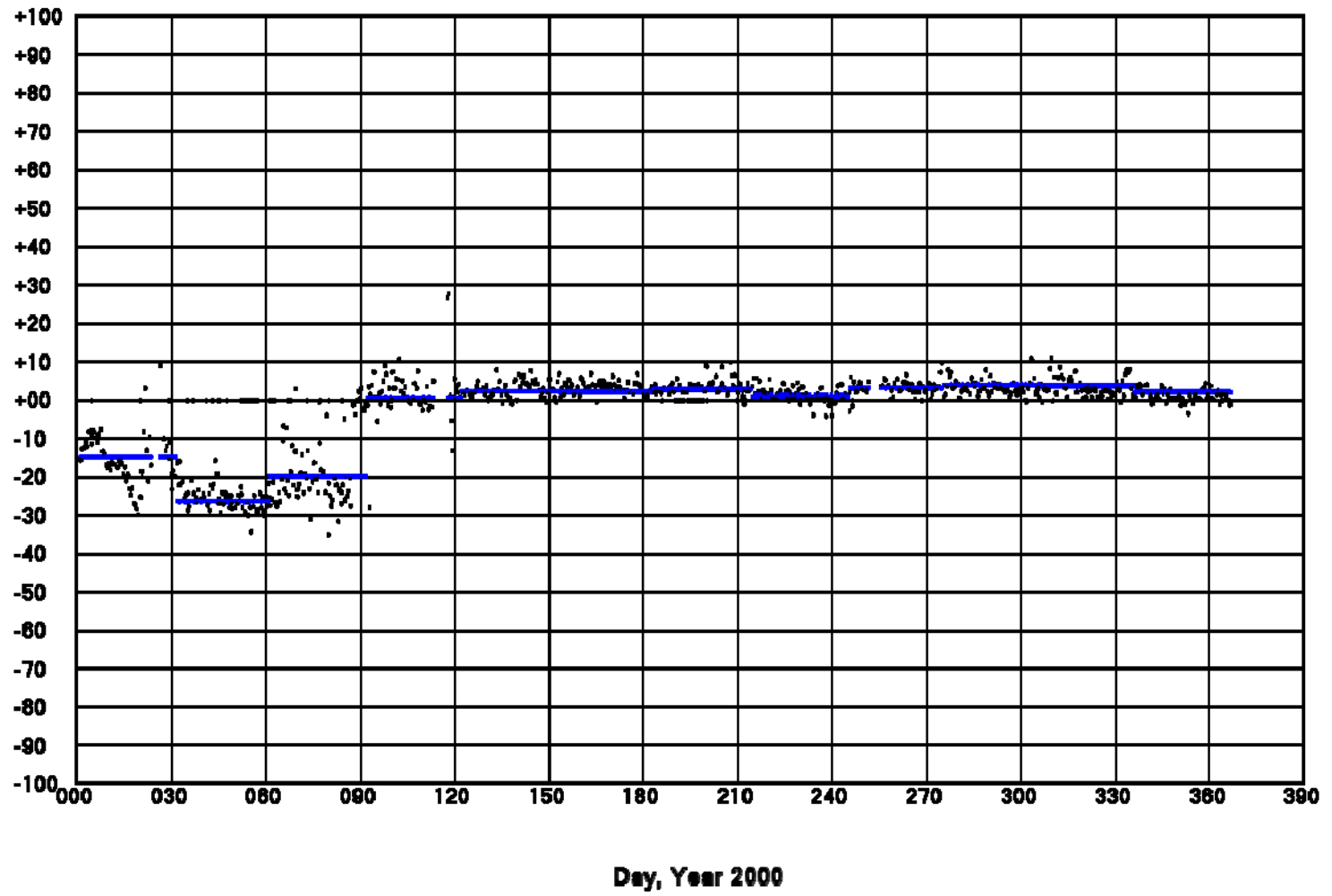
Station brus PRN #1



Station brus PRN #4



Station brue PRN #24



**Next topic: how can artificial data help in estimating
the reliability of calibration techniques?**

How accuracy of calibration techniques can be estimated

Examination of residuals

$$Res_{ijt} = S_{ijt} - \sum_n c_n^t p_n(l_{ijt}, f_{ijt}) \sec \chi_{ijt} - \Omega_{Arc}$$

After a calibration run will provide with useful information about the

Internal consistency of the solution

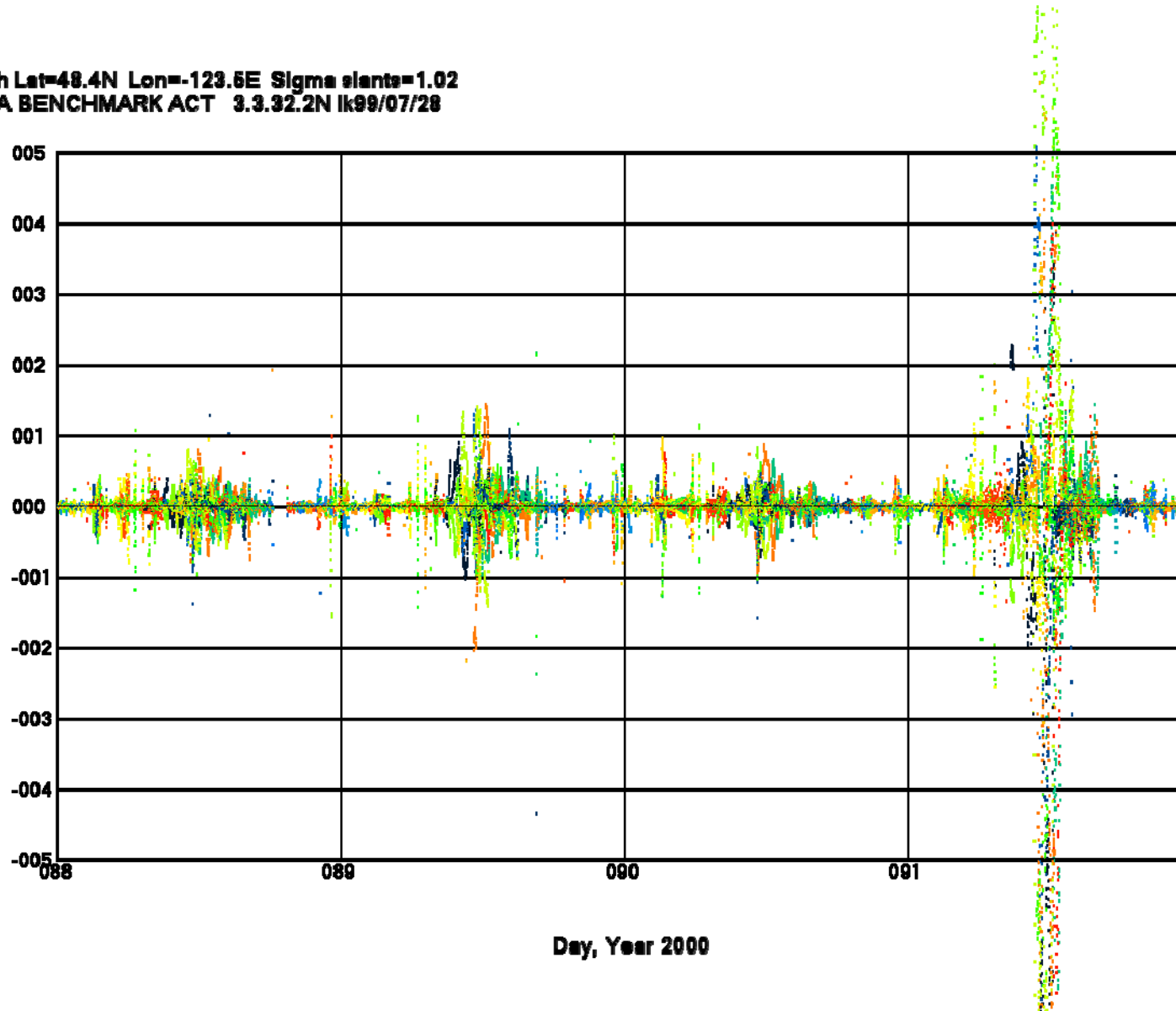
Residuals are plotted in the following examples for few sample stations.

Standard deviation of the individual samples is reported.

Internal consistency of the method is estimated from the residuals (actual data)

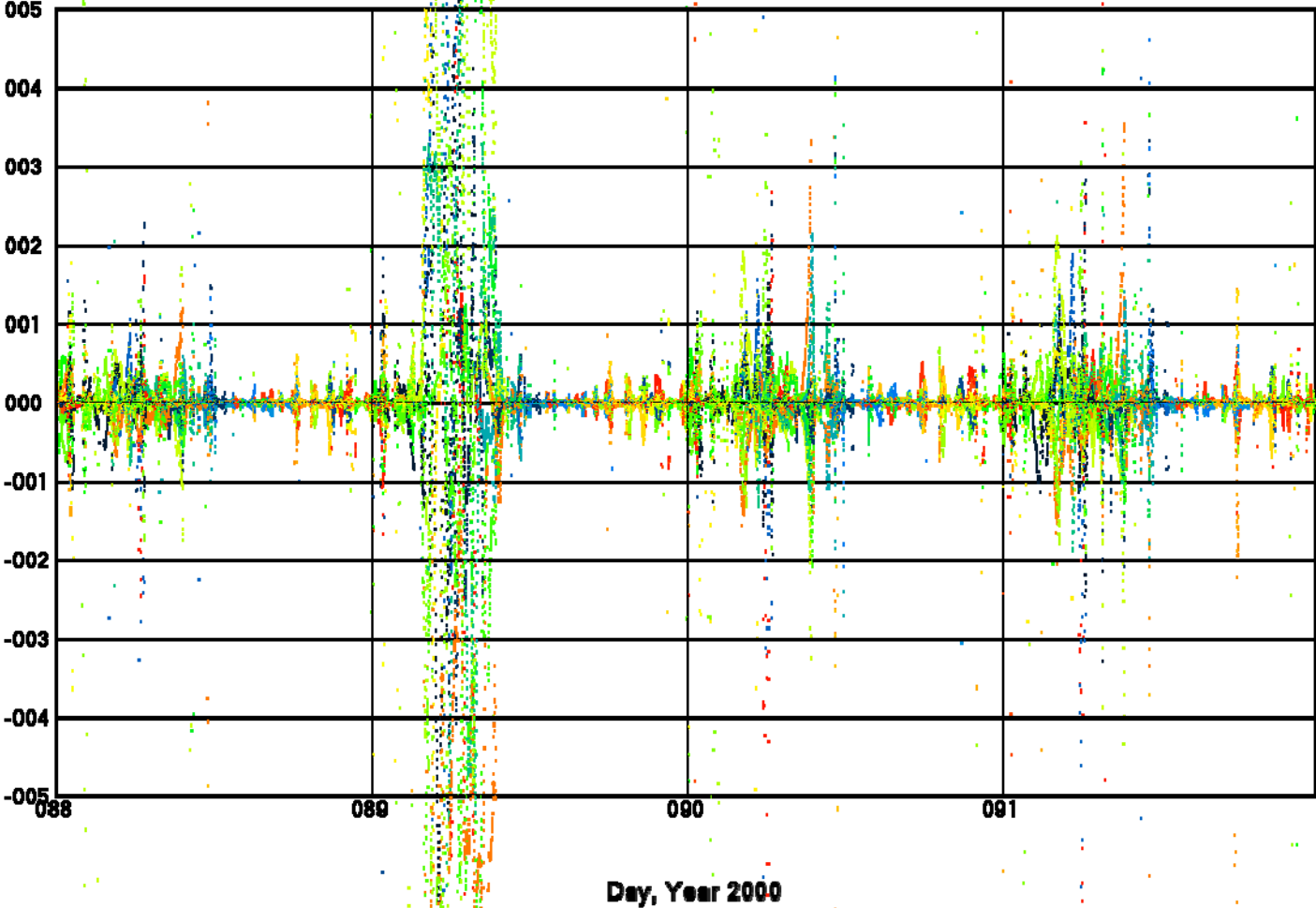
$$Res_{ijt} = S_{ijt} - \sum_n c_n^t p_n(l_{ijt}, f_{ijt}) \sec \chi_{ijt} - \Omega_{Arc}$$

TEC(10**16) slbh Lat=48.4N Lon=-123.6E Sigma slants=1.02
2026 AOA BENCHMARK ACT 3.3.32.2N Ik99/07/28



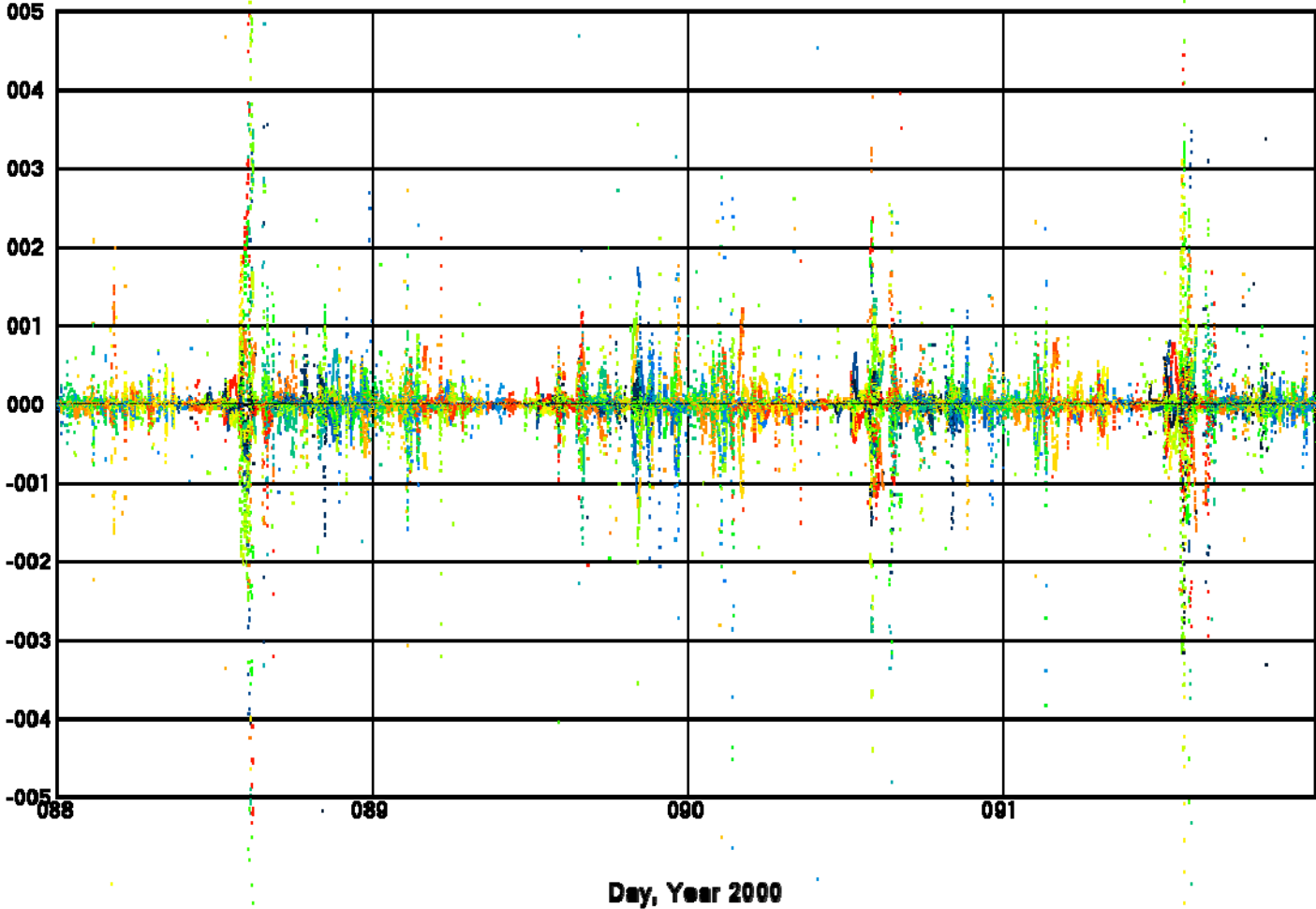
Residuals, actual data

TEC(10**16) slc Lat=23.7S Lon=133.9E Sigma slants=1.59
C126U AOA ICS-4000Z ACT 00.01.14 / 3.3.32.3



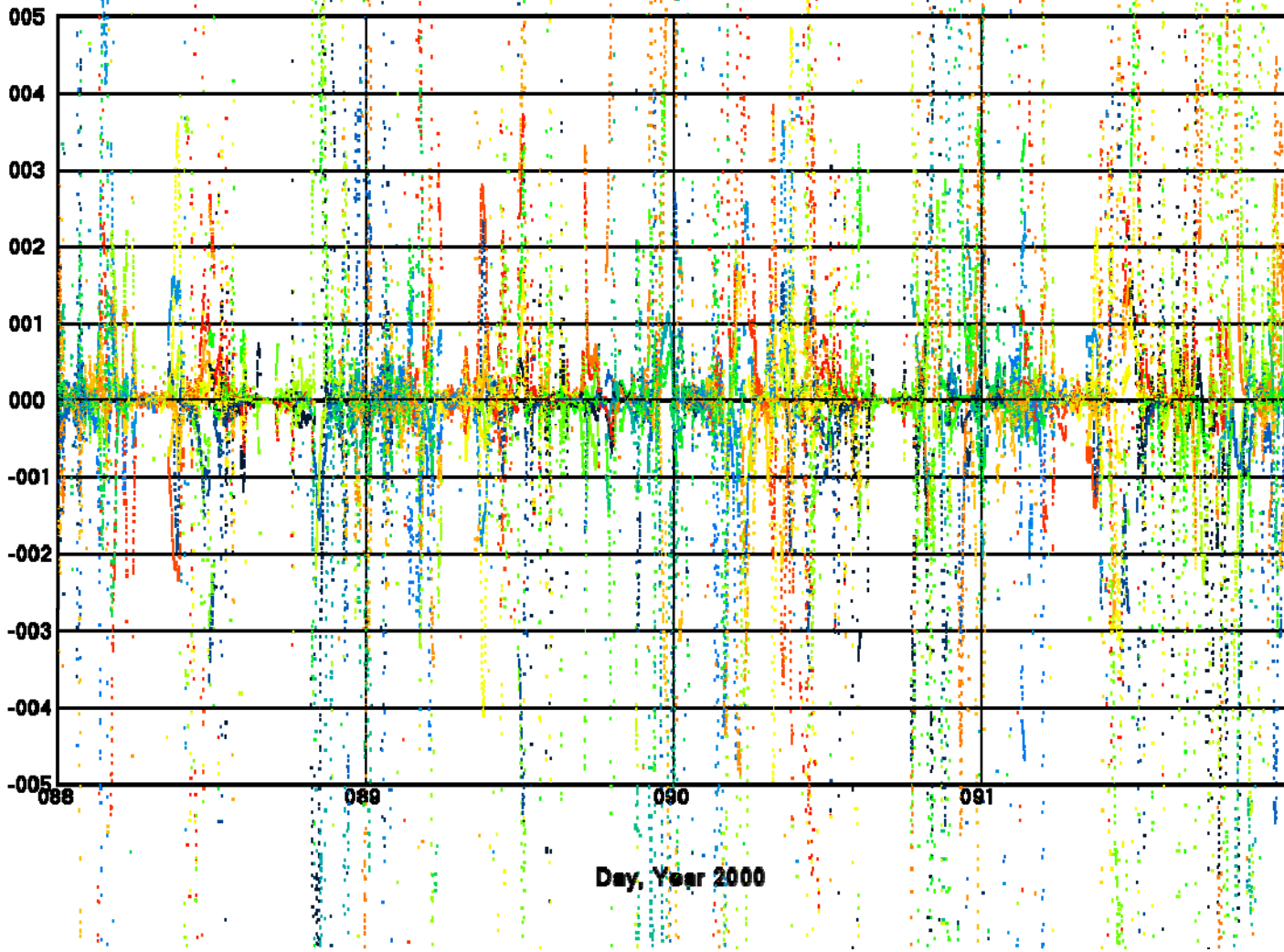
Residuals, actual data

TEC(10**16) cro1 Lat=17.8N Lon=-64.6E Sigma slants=0.43
R141 AOA SNR=8100 ACT 3.3.32.2



Residuals, actual data

TEC(10**16) for Lat=03.9S Lon=-38.4E Sigma slants=3.72
T149 ROGUE SNR=8000 3.2.32.8



Day, Year 2000

Sigma of the shown sample residuals ranges from $\approx .5$ to 4 *TECu* according to latitude.

Is this an estimation of the accuracy of the calibration?

No, as this requires a comparison with truth data, which are unavailable (Incoherent Scatter Radar, Radar Altimeter may help, but are not sufficient).

What can look more like truth data?

Artificial data produced by Ionospheric Models.

But keeping in mind that agreement with artificial data is a condition **necessary but not sufficient** to validate the method

The artificial data

Ionospheric models enable to estimate median electron density at some time at some geographic location, i.e. given date and time, latitude, longitude, height.

$$N_e = N_e(t, \phi, \lambda, h)$$

TEC is the integral of electron density along the ray-path from satellite to receiver,

$$TEC = \int N_e(P) ds$$

which will be numerically evaluated as the sum

$$TEC \approx \sum N_e(P_i) \delta s_i$$

or with any more effective numerical algorithm (Gauss, ...)

Model TEC computation

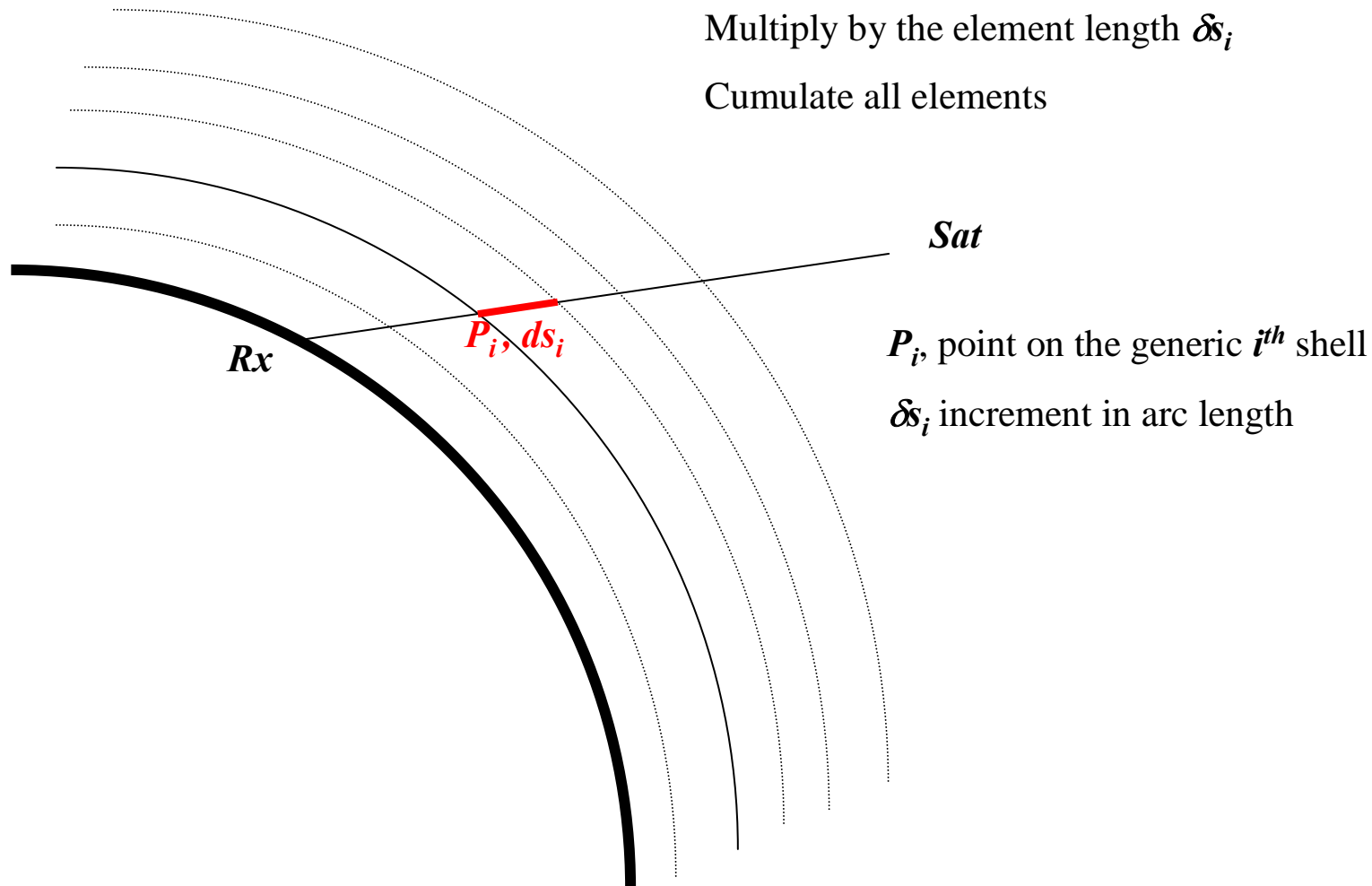
$$TEC = \int N_e(P) ds \approx \sum N_e(P_i) \delta s_i$$

Divide the path in elements δs_i

At each point P_i compute the electron density $N_e(P_i)$ provided by the model

Multiply by the element length δs_i

Cumulate all elements



Simple uses of artificial data: the mapping function

Which errors do affect the **standard approach** (actual vertical *TEC*) of mapping function?

Using an artificial ionosphere:

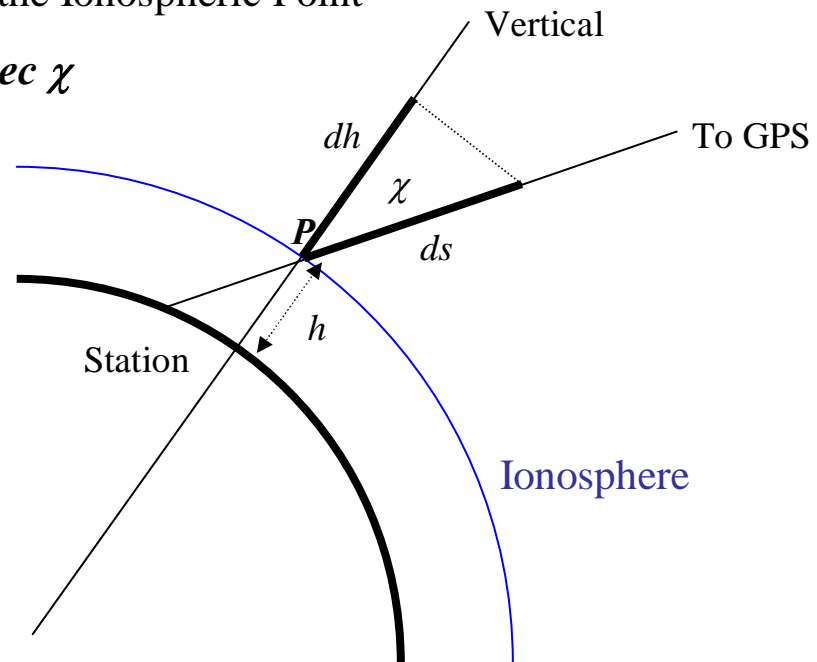
Compute χ

Compute Slant S

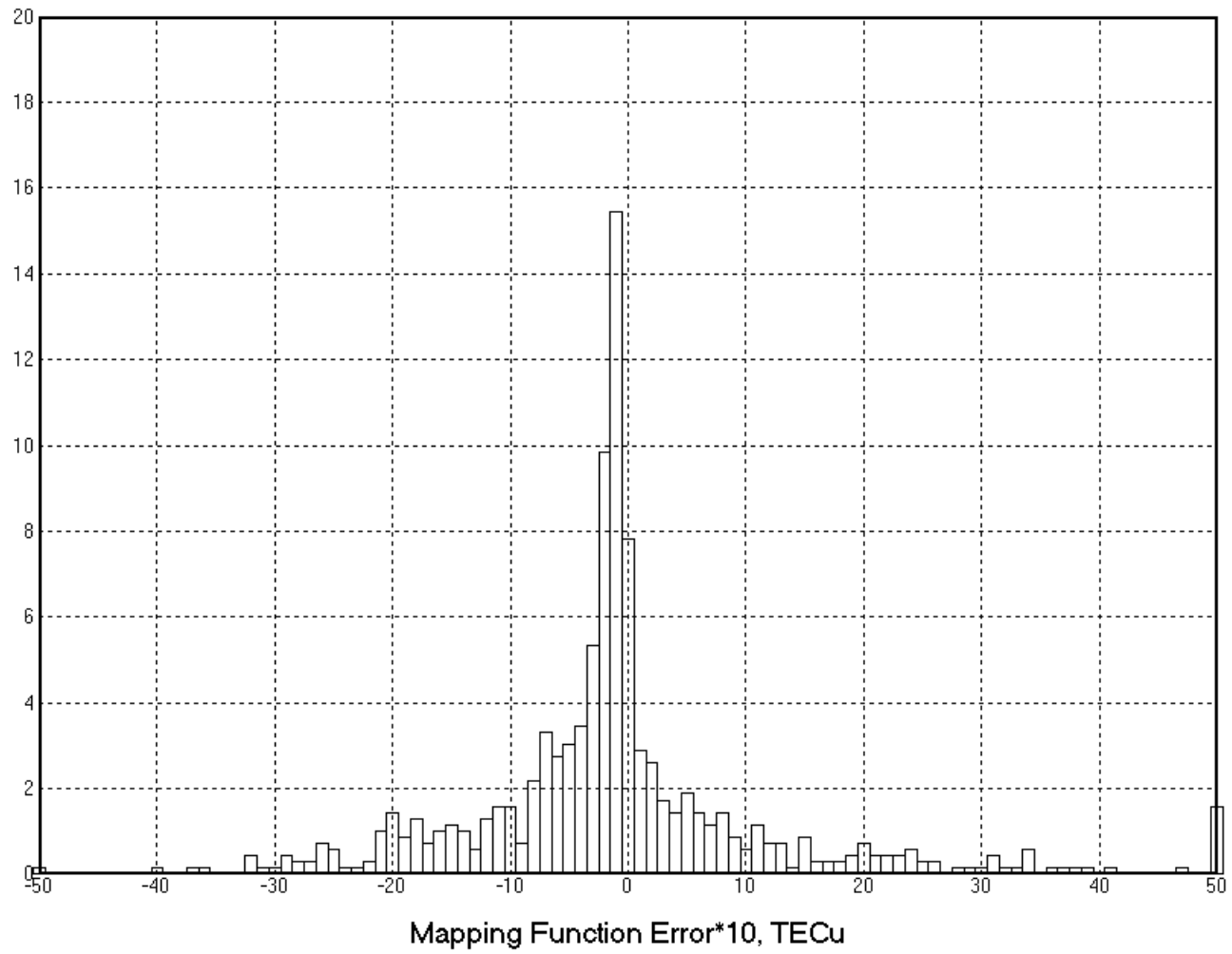
Compute Vertical TEC V at the Ionospheric Point

Error: $S - V \sec \chi$

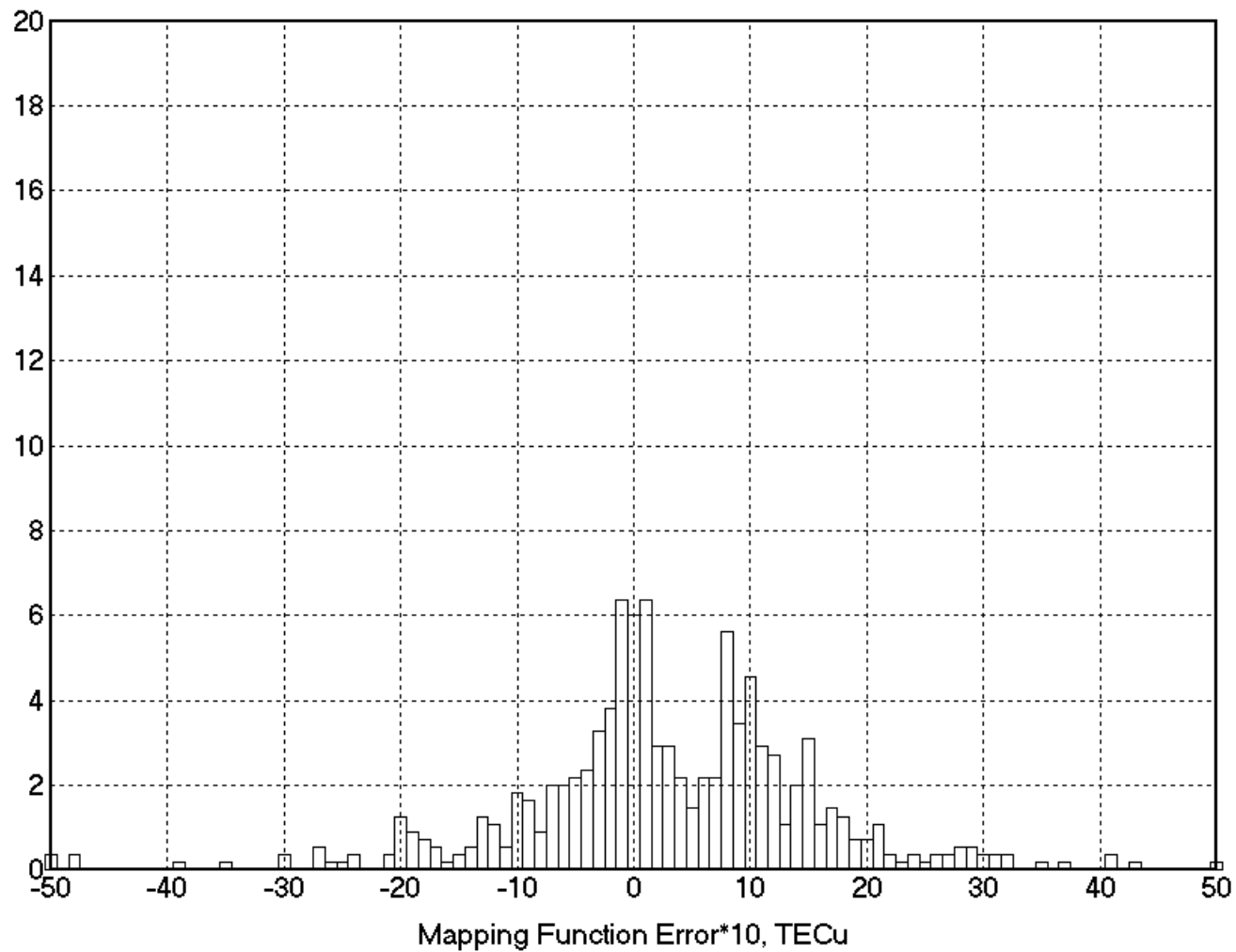
Plot Error distribution



Occurrence %, ajac Lat=41.9N Lon=8.8E



Occurrence %, areq Lat=16.5S Lon=71.5W



Simple uses of artificial data: *VEC and VEq*

In the Single-Station / Arc Offset calibration the *Vertical Equivalent TEC VEq* for which it is exactly $S = VEq \sec \chi$ is used.

How different is *VEq* from actual Vertical *TEC (VEC)* ?

Using an artificial ionosphere:

Compute χ

Compute Slant S

By definition $V Eq = S \cos \chi$

Compute Vertical TEC V at the Ionospheric Point *VEC*

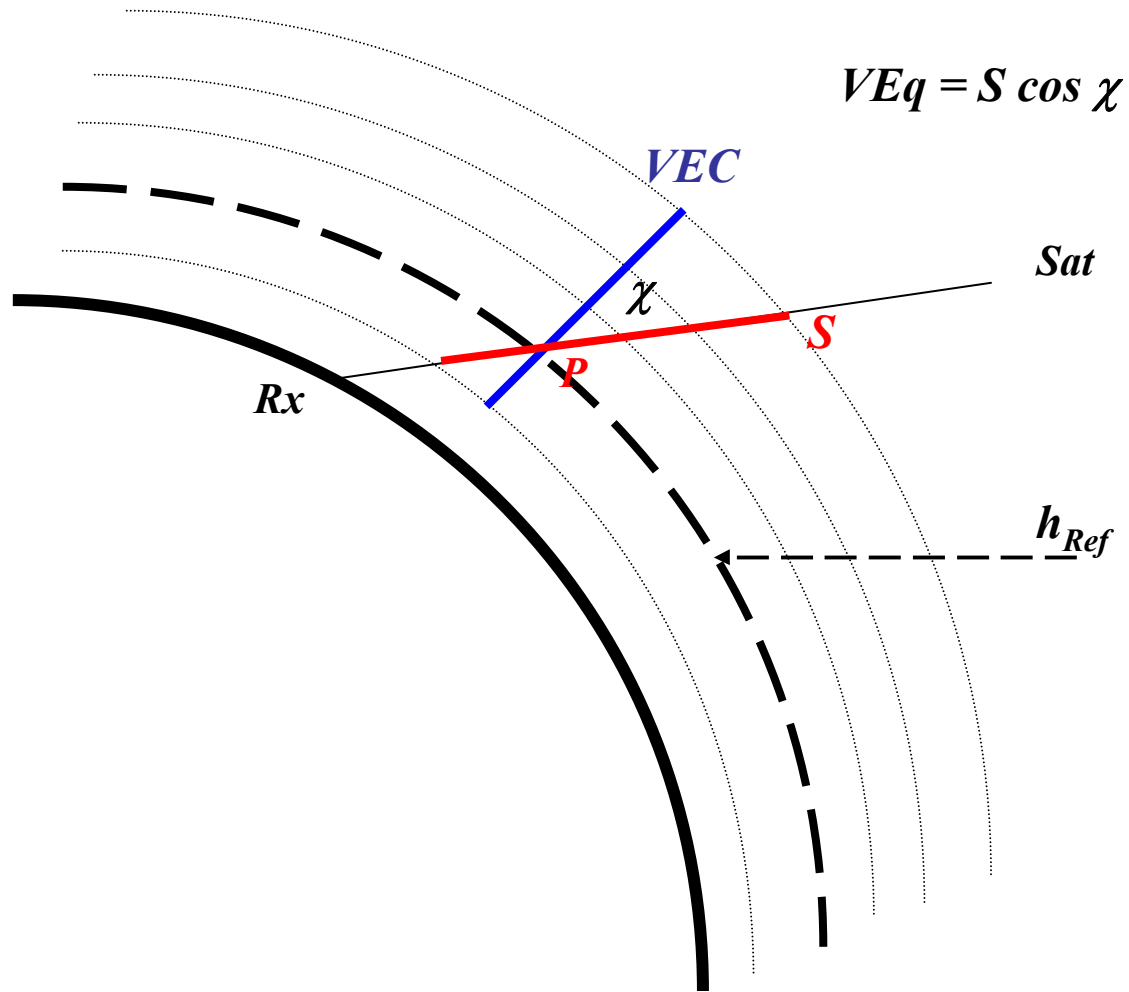
Plot *VEC, VEq*

Plasmasphere can be included too using a suitable model

Integration paths for

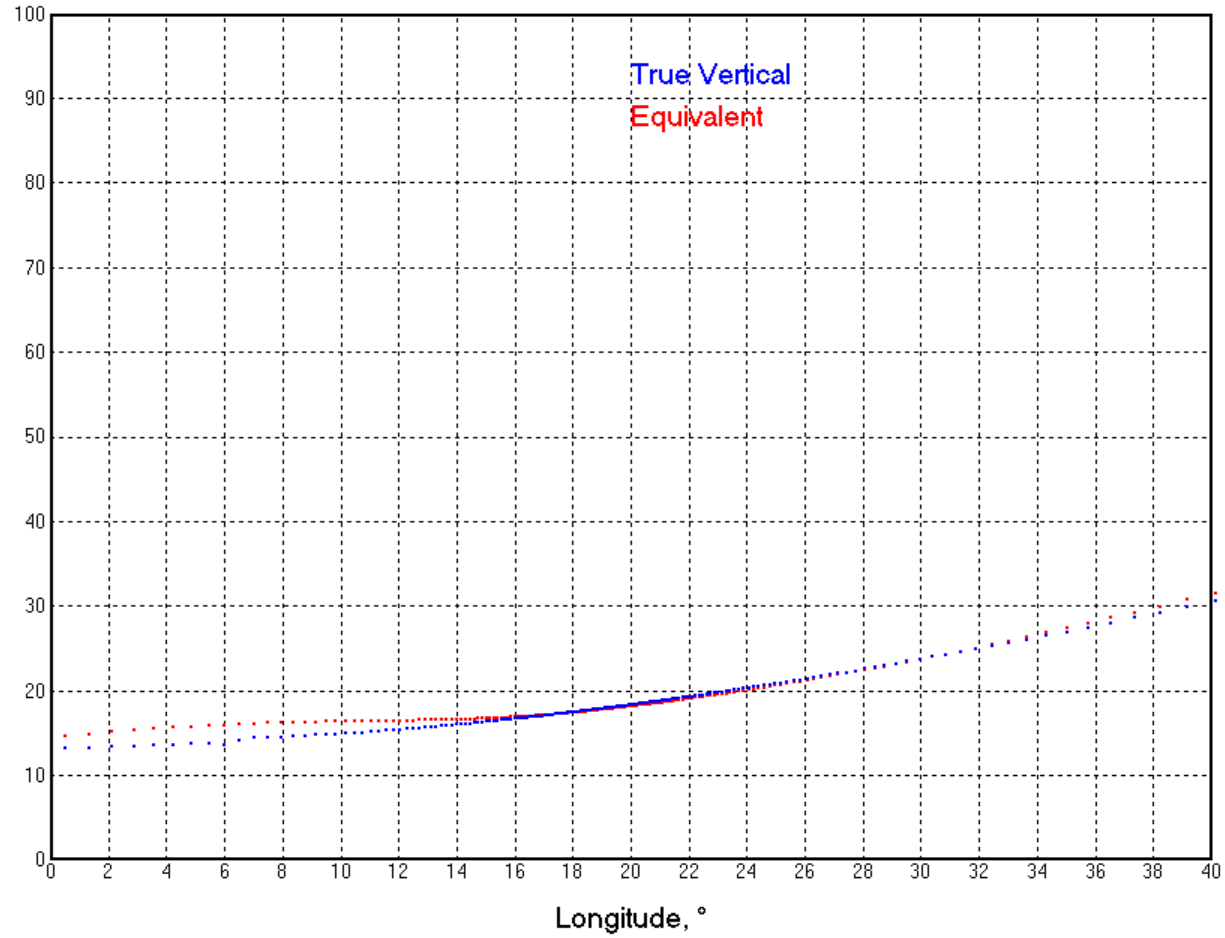
S 

VEC 

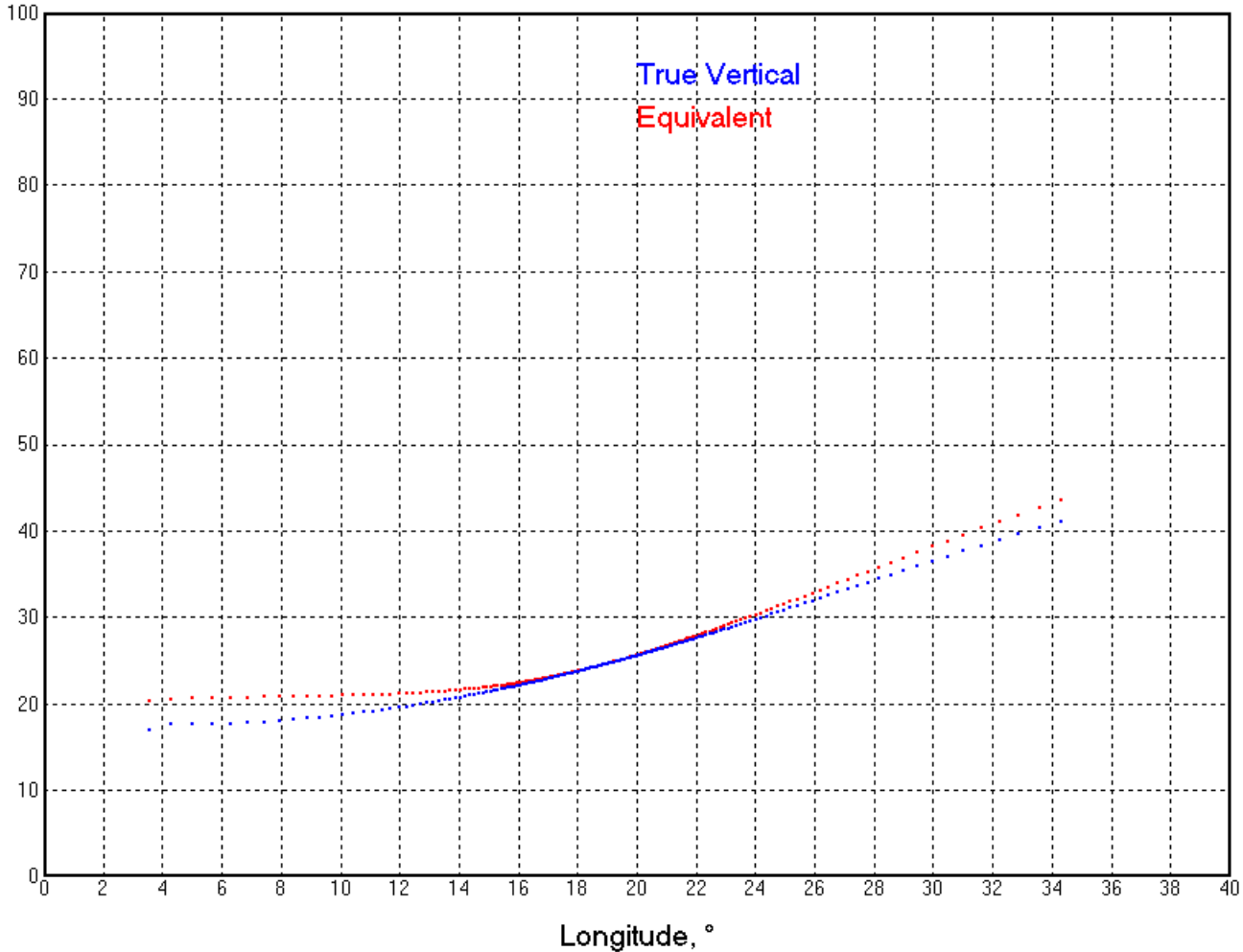


Simple uses of artificial data: How much VEC and VEq differ?

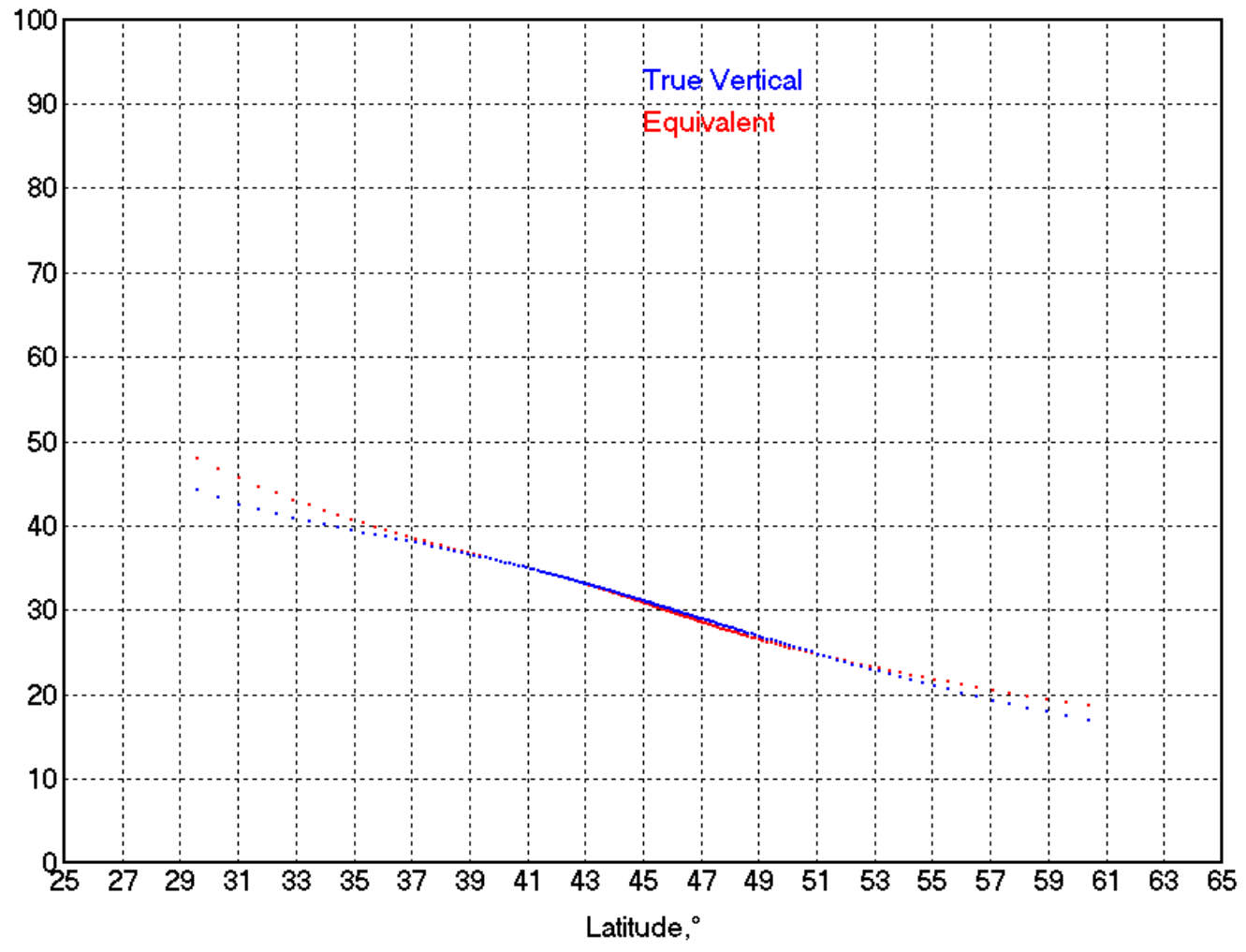
TEC, 10^{16} e/m², Station Lat=+45.0



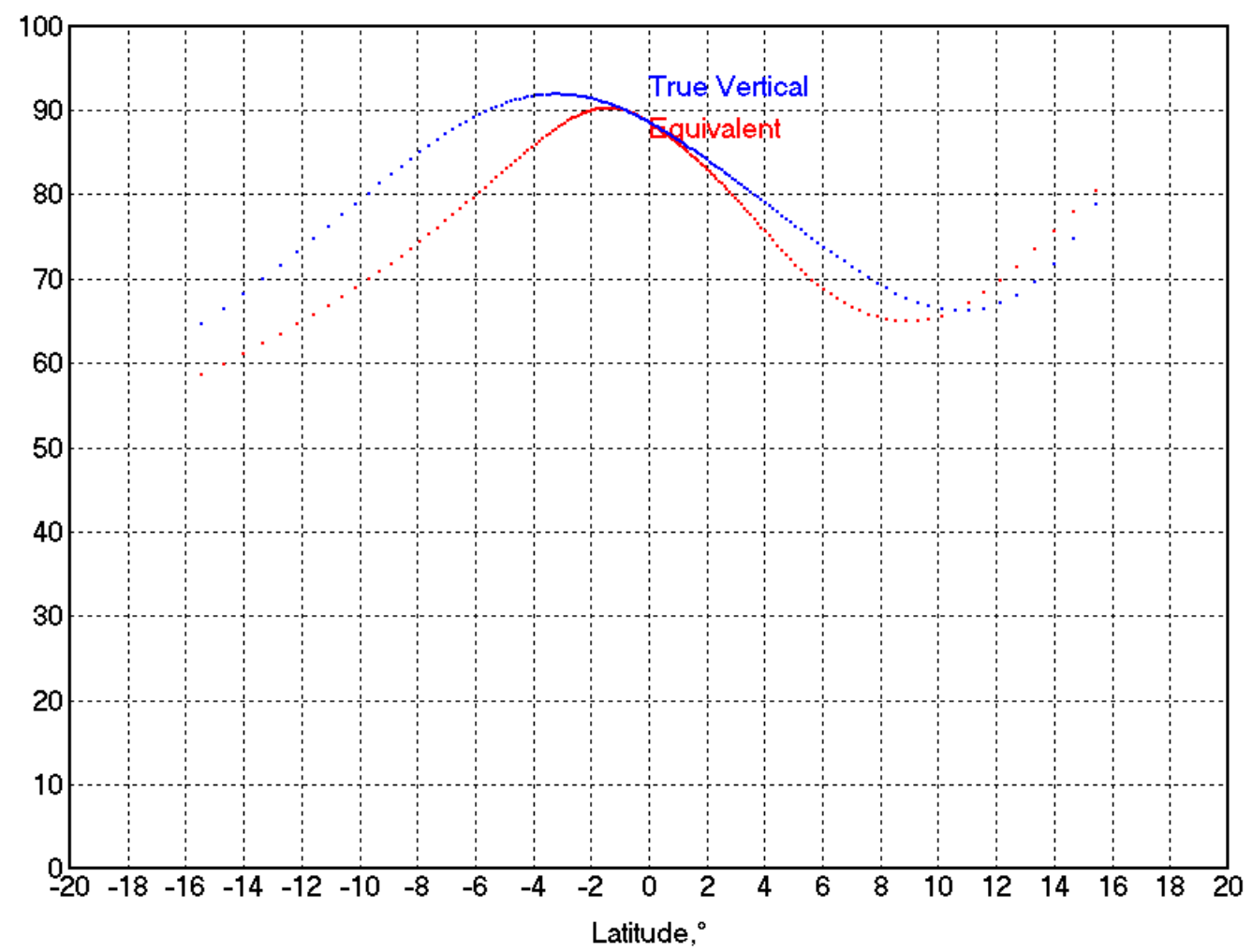
TEC, 10^{16} e/m², Station Lat=+00.0



TEC, 10^{16} el/m², Lon=+20.0



TEC, 10^{16} el/m², Lon=+20.0



Test of Single-Station, Arc-Offset solution

Generation of artificial truth data

Given all slants actually observed and archived

in a (quasi) complete set of IGS stations (≈ 200 per day)
for year 2000
for days 88-91 (March 28-31)

Re-compute them using

NeQuick ($Az = 150$), integrating up to 2000 km

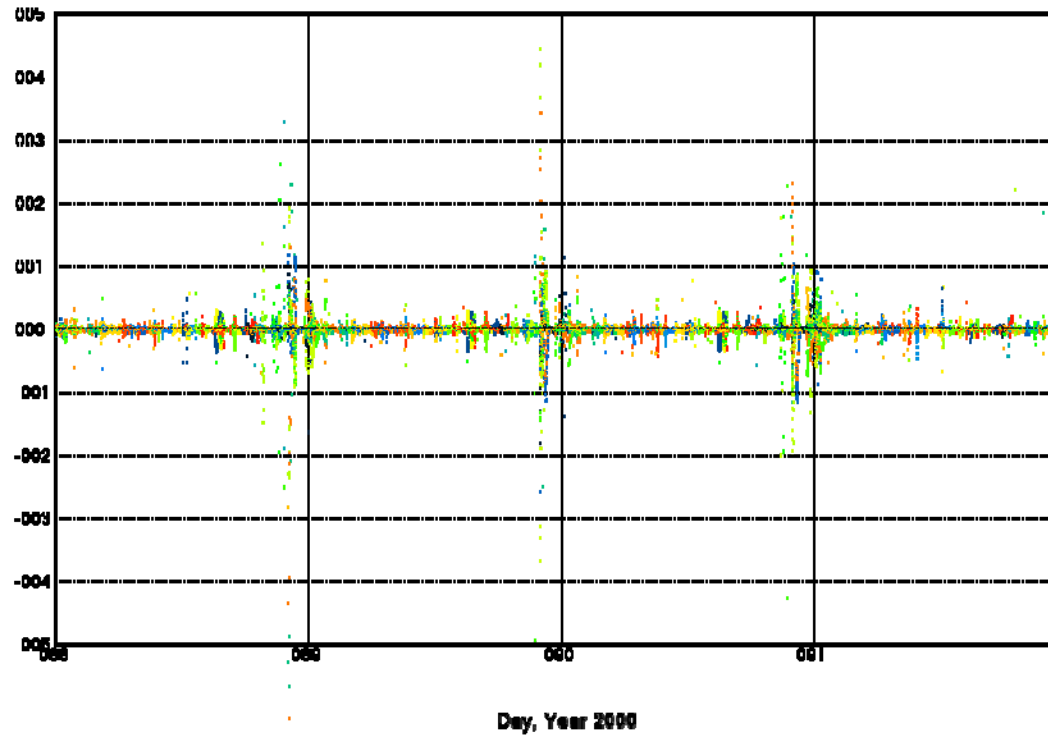
Therefore:

Not only the actual GPS constellation has been preserved for the reference period, but also the possible lack of observations (this will affect the solution)

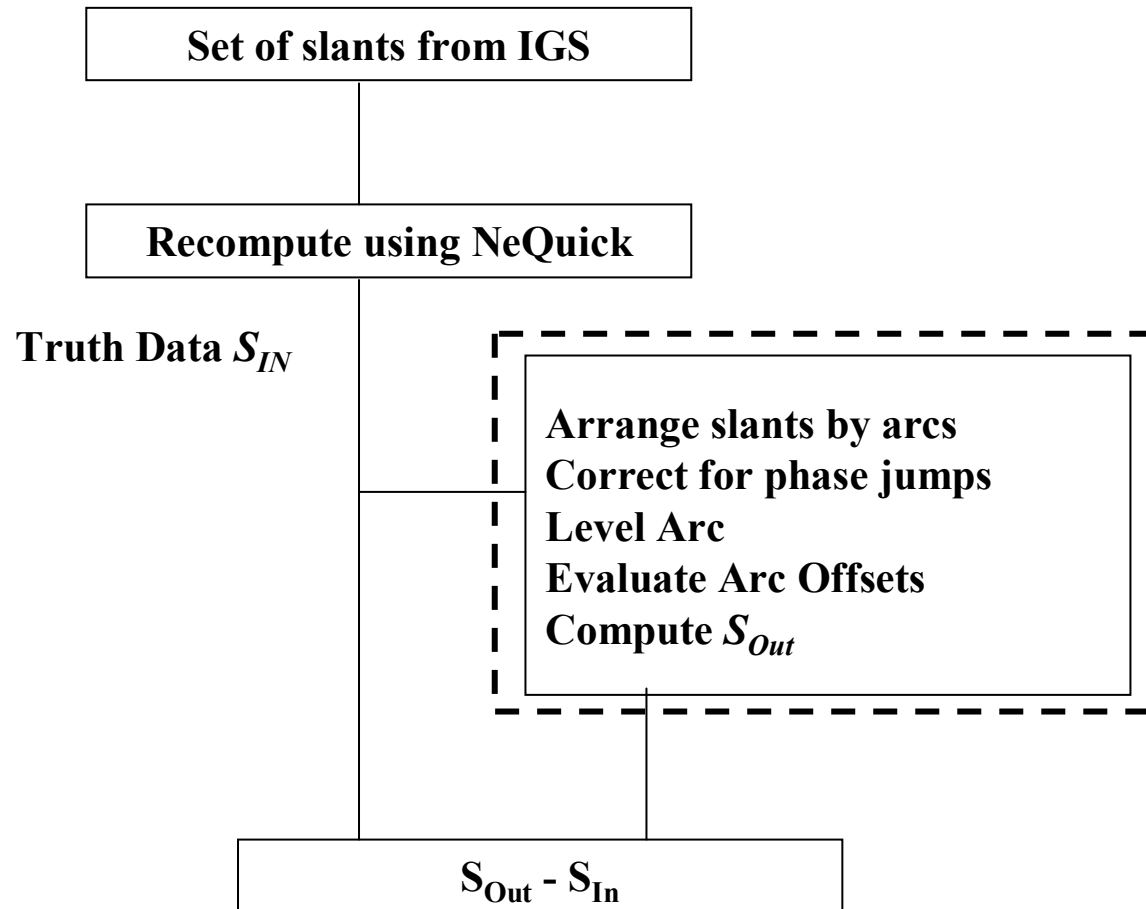
Internal consistency: Residuals, simulated data

$$Res_{ijt} = S_{ijt} - \sum_n c_n^t p_n(l_{ijt}, f_{ijt}) \sec \chi_{ijt} - \Omega_{Arc}$$

TEC(10¹⁶) lamp Lat=35.6N Lon=12.6E Sigma slants=0.22
21821 TRIMBLE 4000S81 Nov 7.28 Sig 3.07



Testing the calibration procedure



$S_{Out} - S_{In}$ are plotted vs time

Worth (but expected) noting that errors at low latitudes are larger

Remark about highlighted arc:

errors show a weakness of the solution.

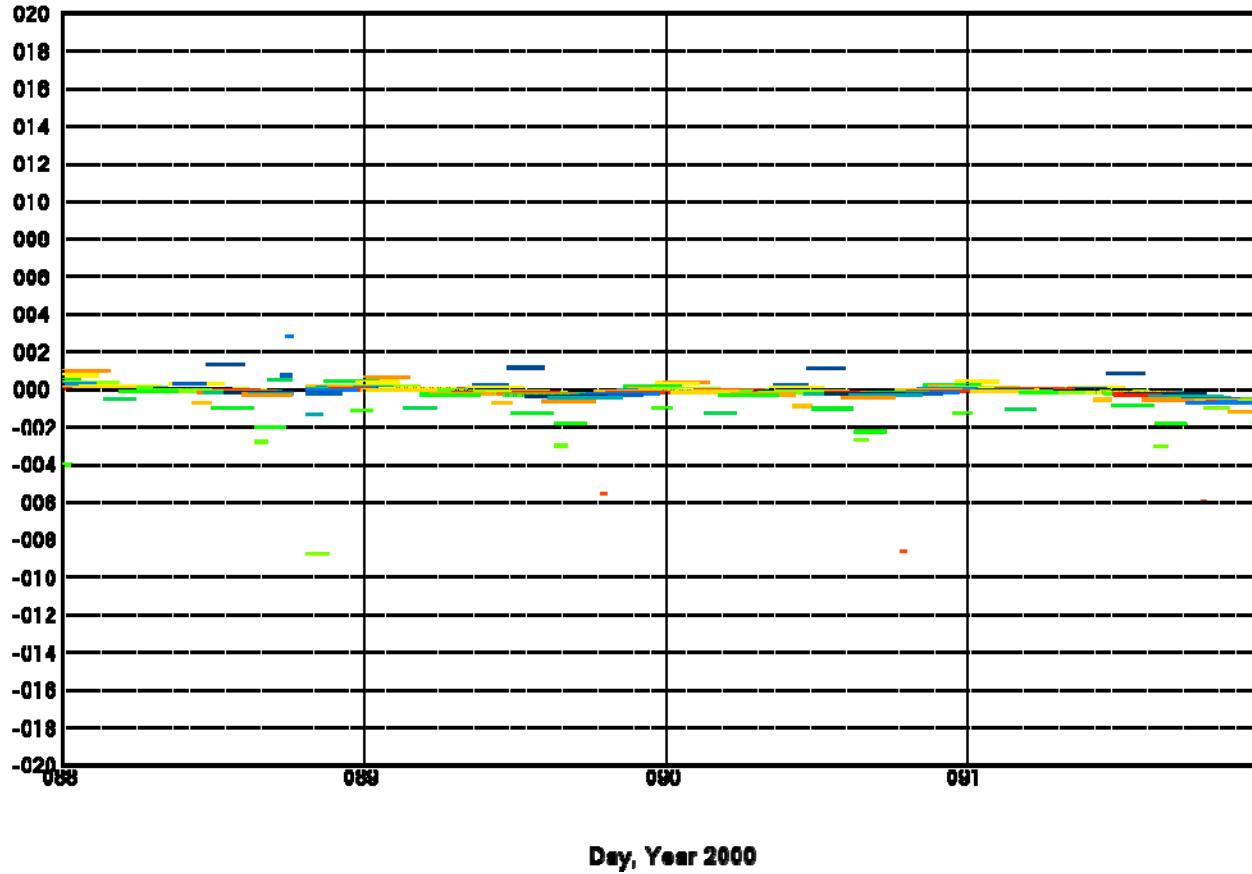
These errors occur for arcs of low elevation also if, in some case, of long duration.

Processing real data, there is no chance to know if the subject arc is ill-calibrated (unless in presence of very strong errors)

Testing the solution with simulated data will (likely) enable to find a more effective way of avoiding such errors, or in a last instance, rejecting them

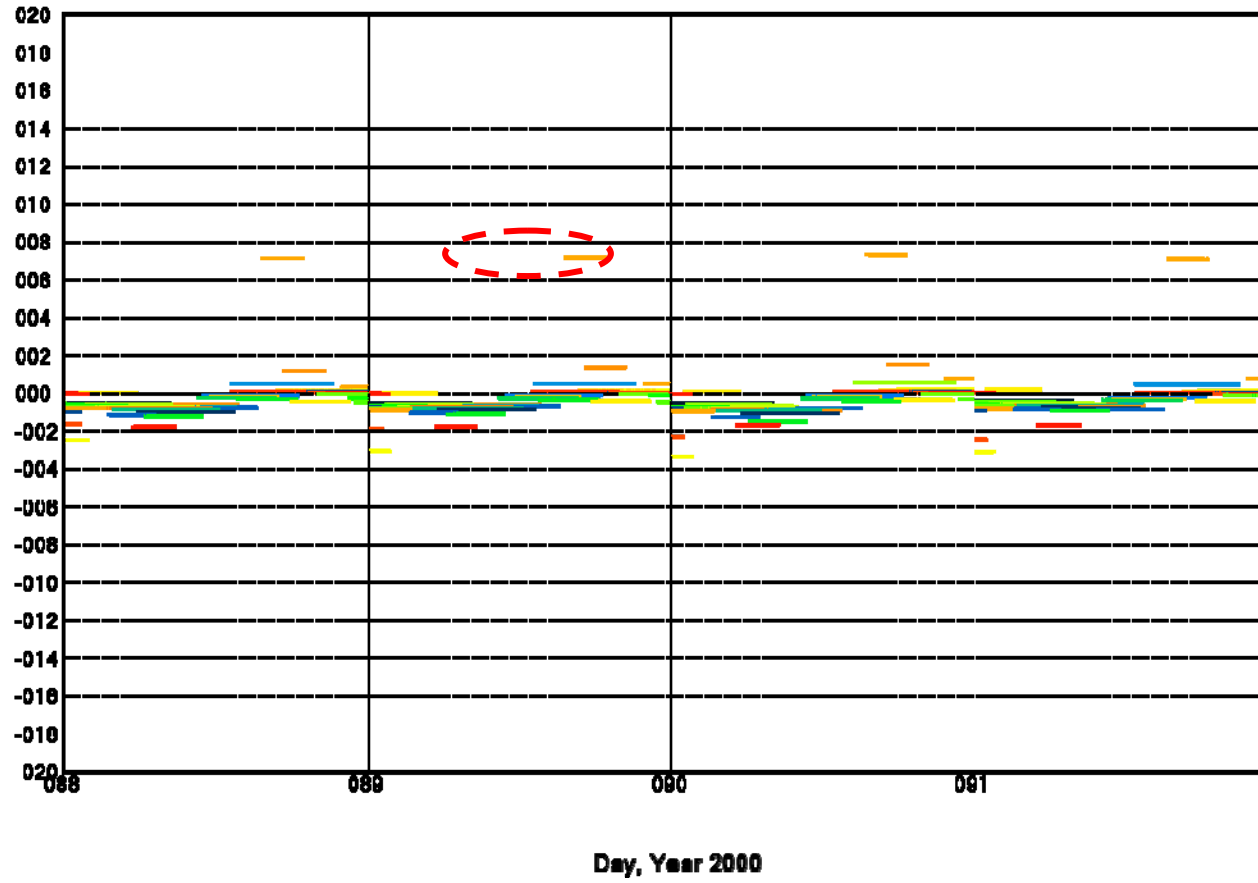
Slant_{Out}-Slant_{In}, TECu

TEC(10¹⁶) albh Lat=48.4N Lon=-129.5E
2025 AOA BENCHMARK ACT 3.3.32.2N Ik99/07/28



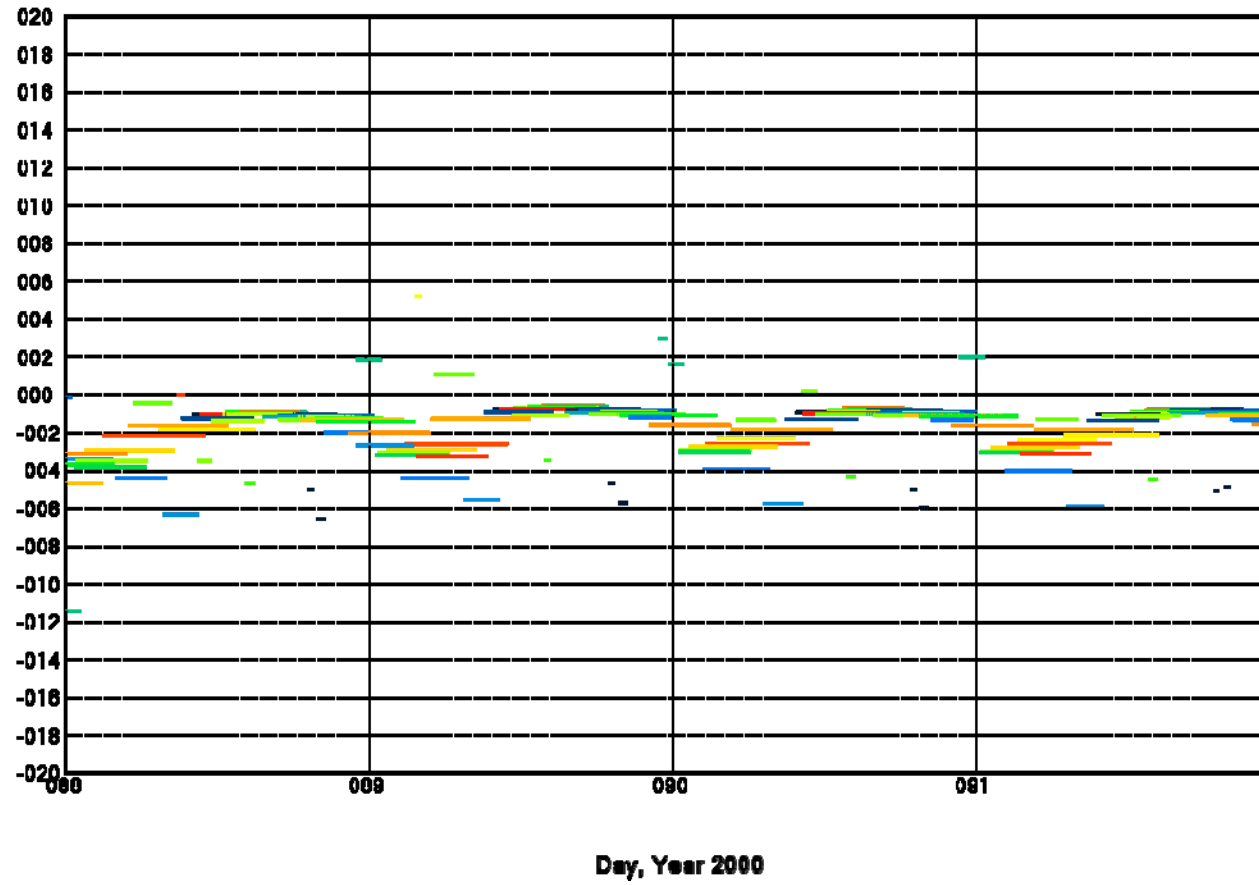
Slant_{Out}-Slant_{In}, TECu

TEC(10**16) alic Lat=23.7S Lon=133.9E
C126U AOA ICS-4000Z ACT 00.01.14 / 3.3.32.3



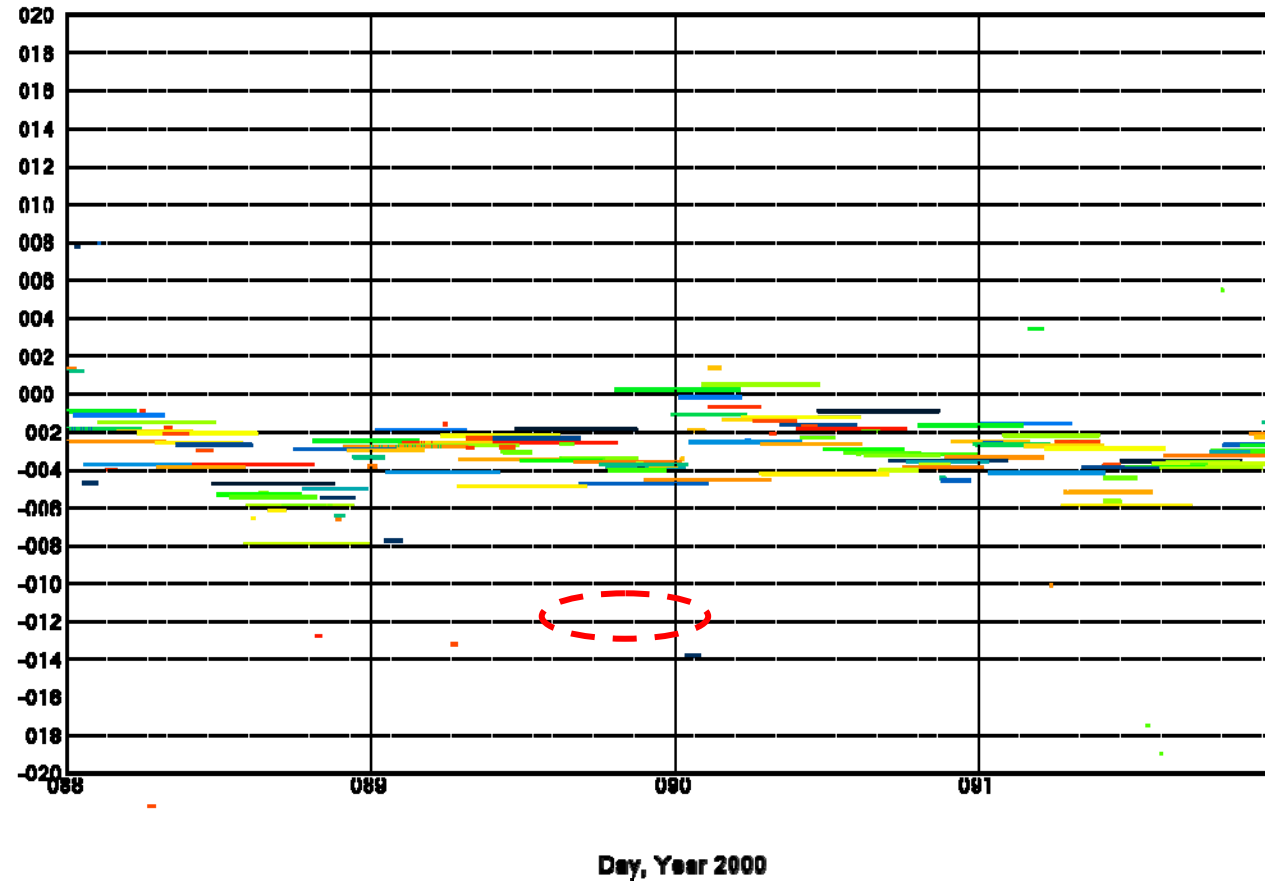
Slant_{Out}-Slant_{In}, TECu

TEC(10**16) cro1 Lat=17.8N Lon=-84.6E
R141 AOA SNR-8100 ACT 3.3.32.2



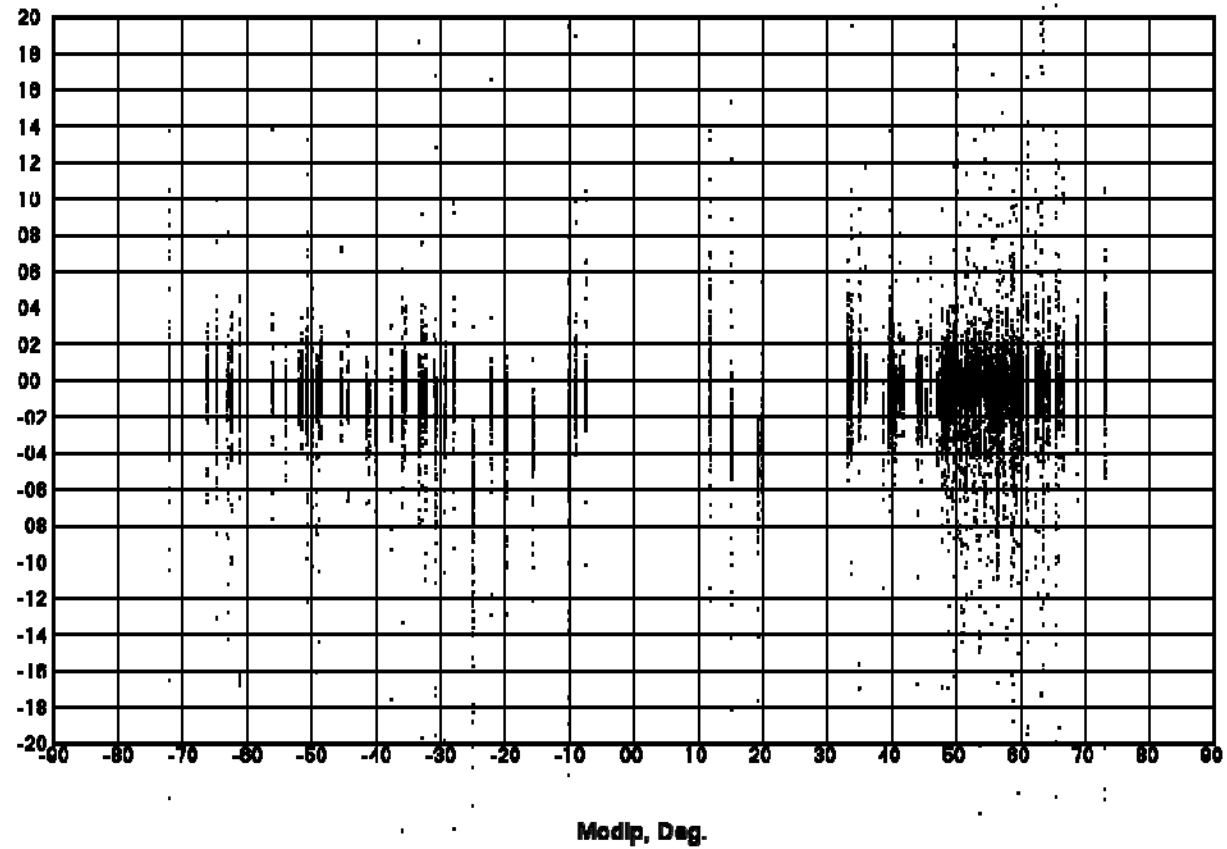
Slant_{Out}-Slant_{In}, TECu

TEC(10**16) fort Lat=03.9S Lon=-38.4E
T149 ROGUE SNR-8000 3.2.92.8



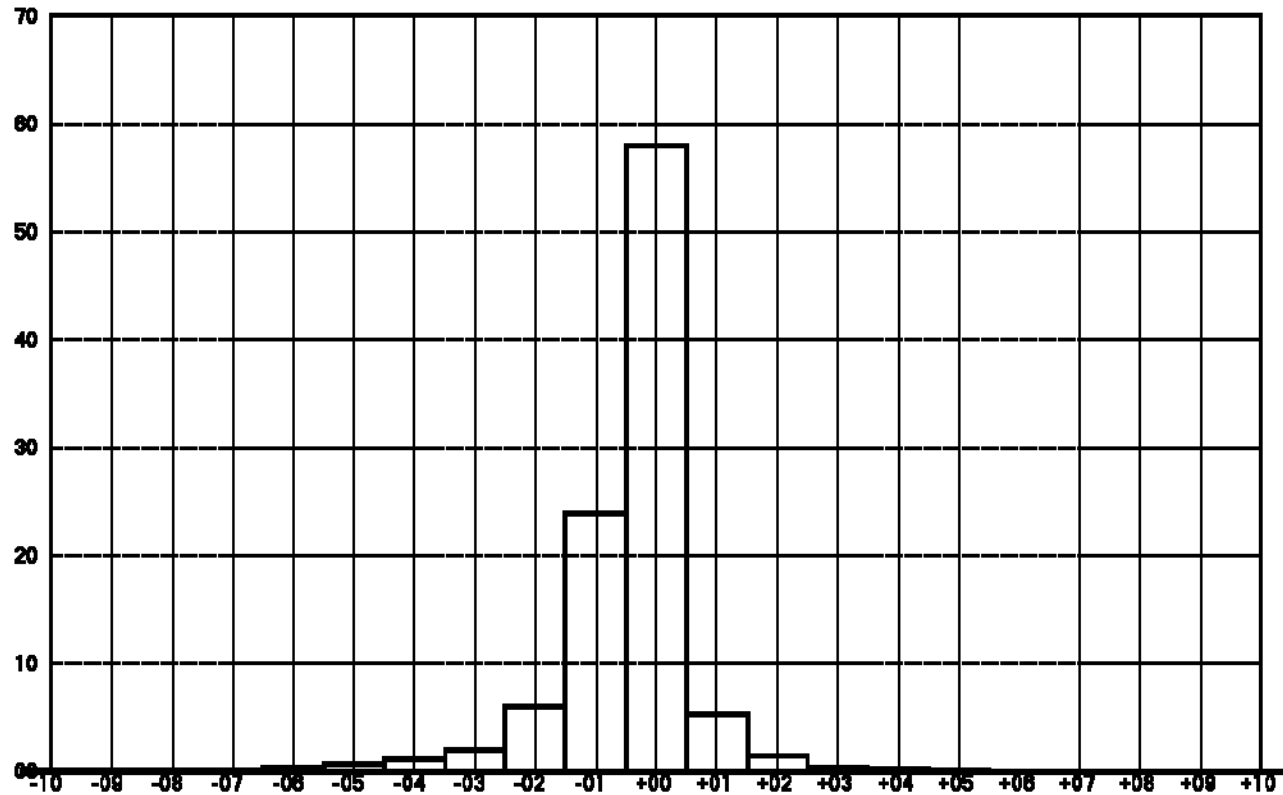
An overall look to the errors: $S_{Out} - S_{In}$, whole set

Slant out - Truth, TECu



An overall look to the errors: $S_{Out} - S_{In}$, probability density

Probability Density, % (Number of slants of sample=1.88E+07)



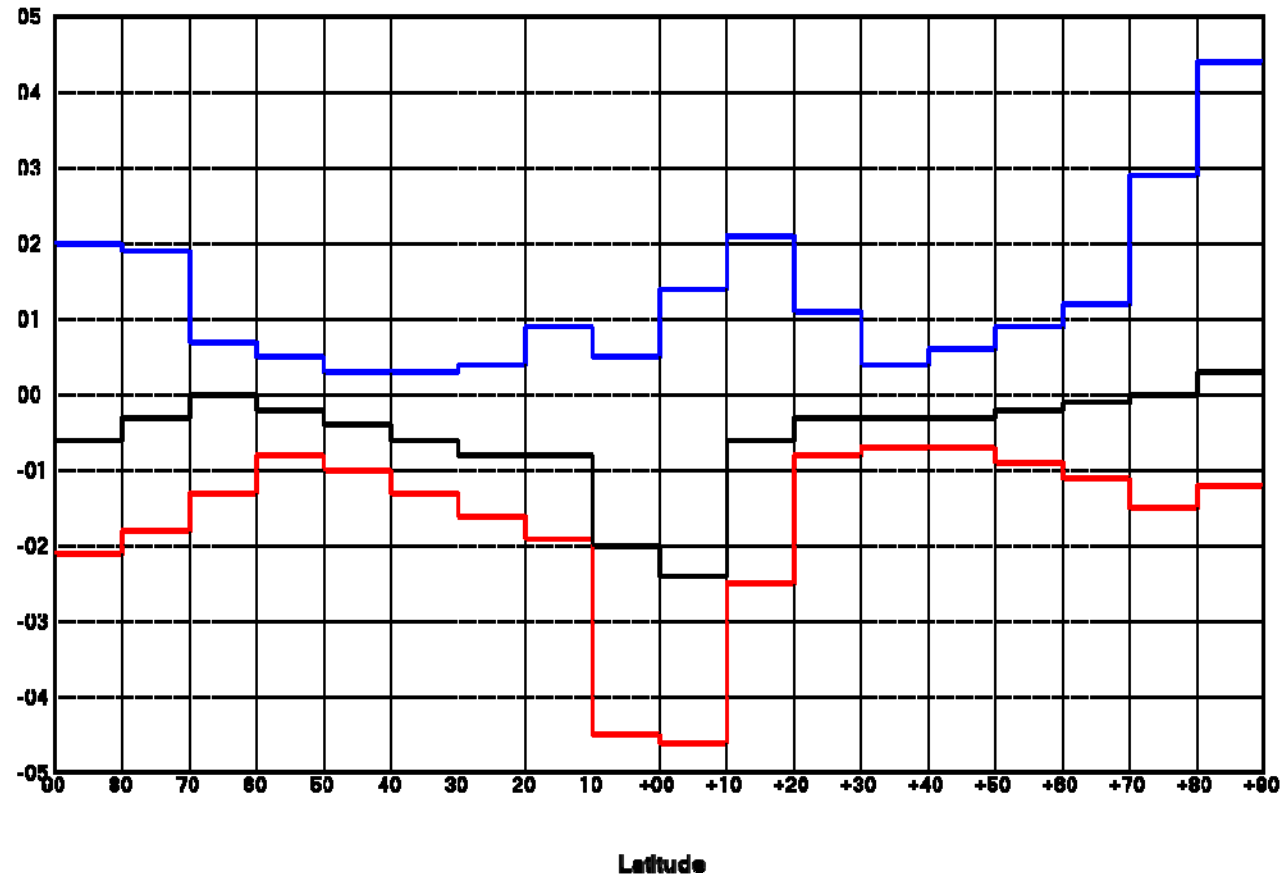
0.12% < -10

Error (SlantOut-SlantIn), TECu

0.067% > 10

Error's behavior vs latitude: percentiles, whole set

Error 5% (Red), 50% (Black), 95% (Blue) Percentiles, TECu



Simulation: role of multi-path contribution λ

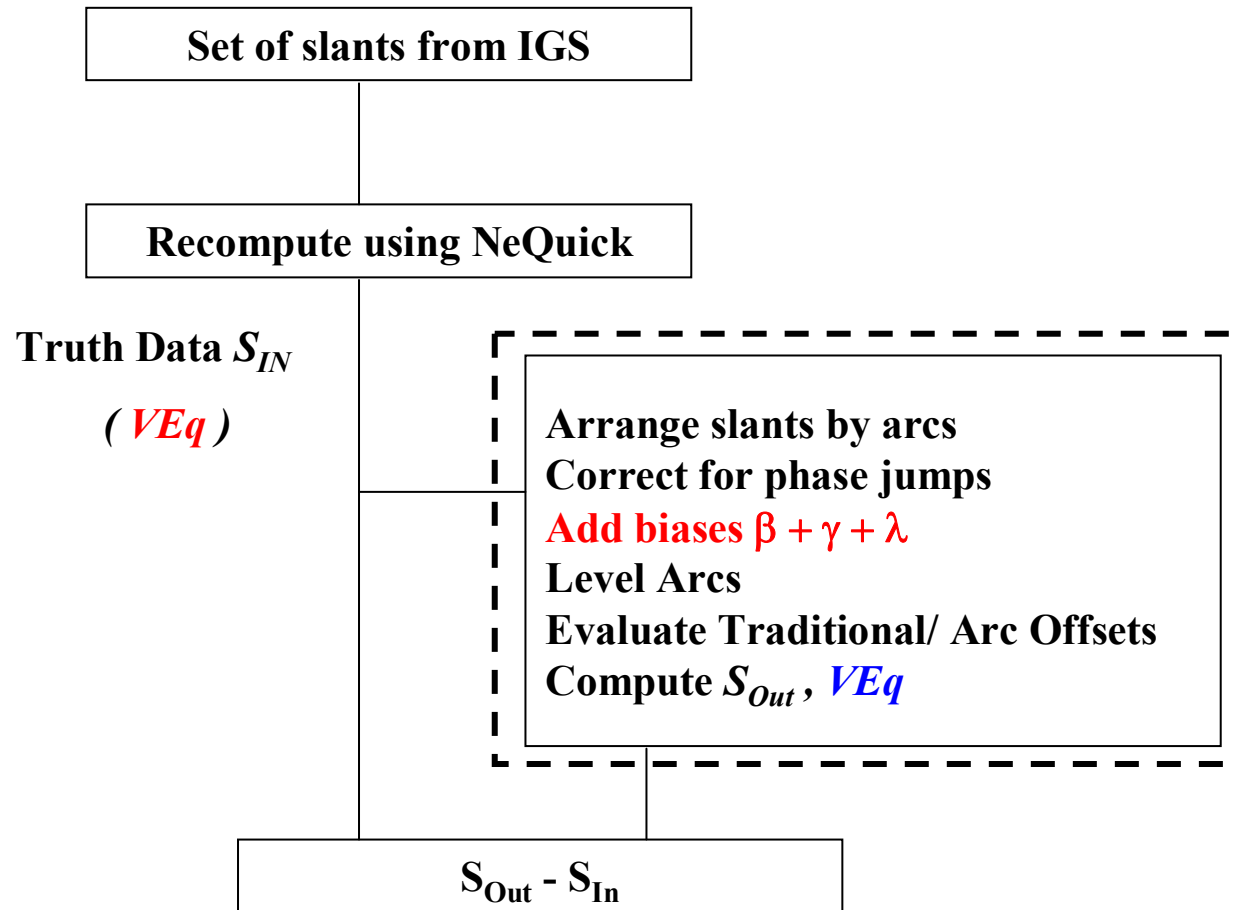
An arbitrary set of *satellite + receiver biases + multipath* errors is added to model slants

Station bias $\gamma = 25$

Satellite biases $\beta_i = 10 * (Rnd() - Rnd()) , i=1,..,32$

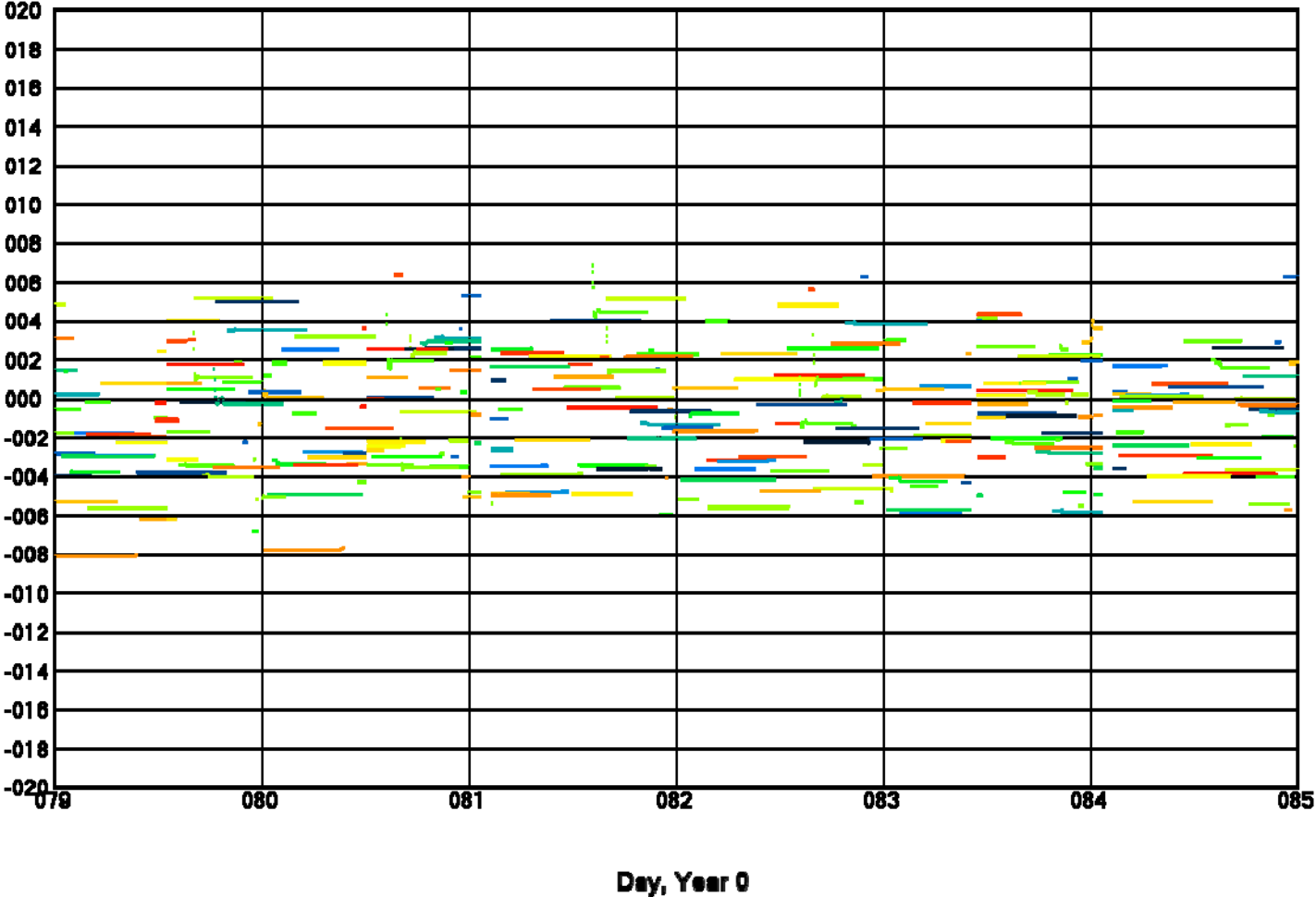
LevelingError $\lambda_{Arc} = 10 * Rnd()$
Arc Offset $\Omega_{Arc} = 1000 * Rnd()$ $\left\{ \begin{array}{l} \text{Arc} = 1.. \text{Number of Arcs} \end{array} \right.$

NextData are processed both by traditional and arc offset single-station calibration.



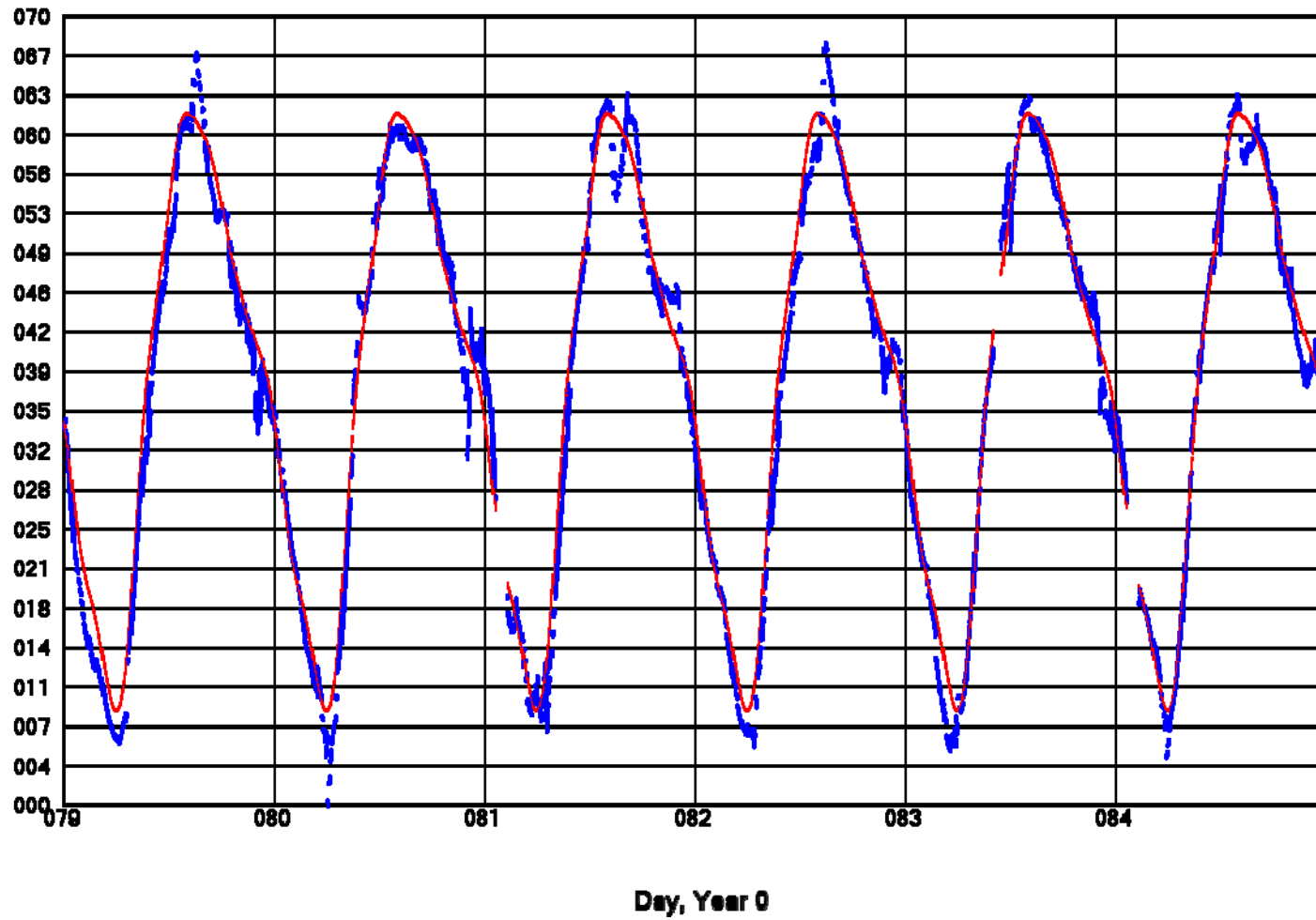
Traditional, SOut - SIn

TEC(10**16) asc1 Lat=08.0S Lon=14.4W RecTypeVer =



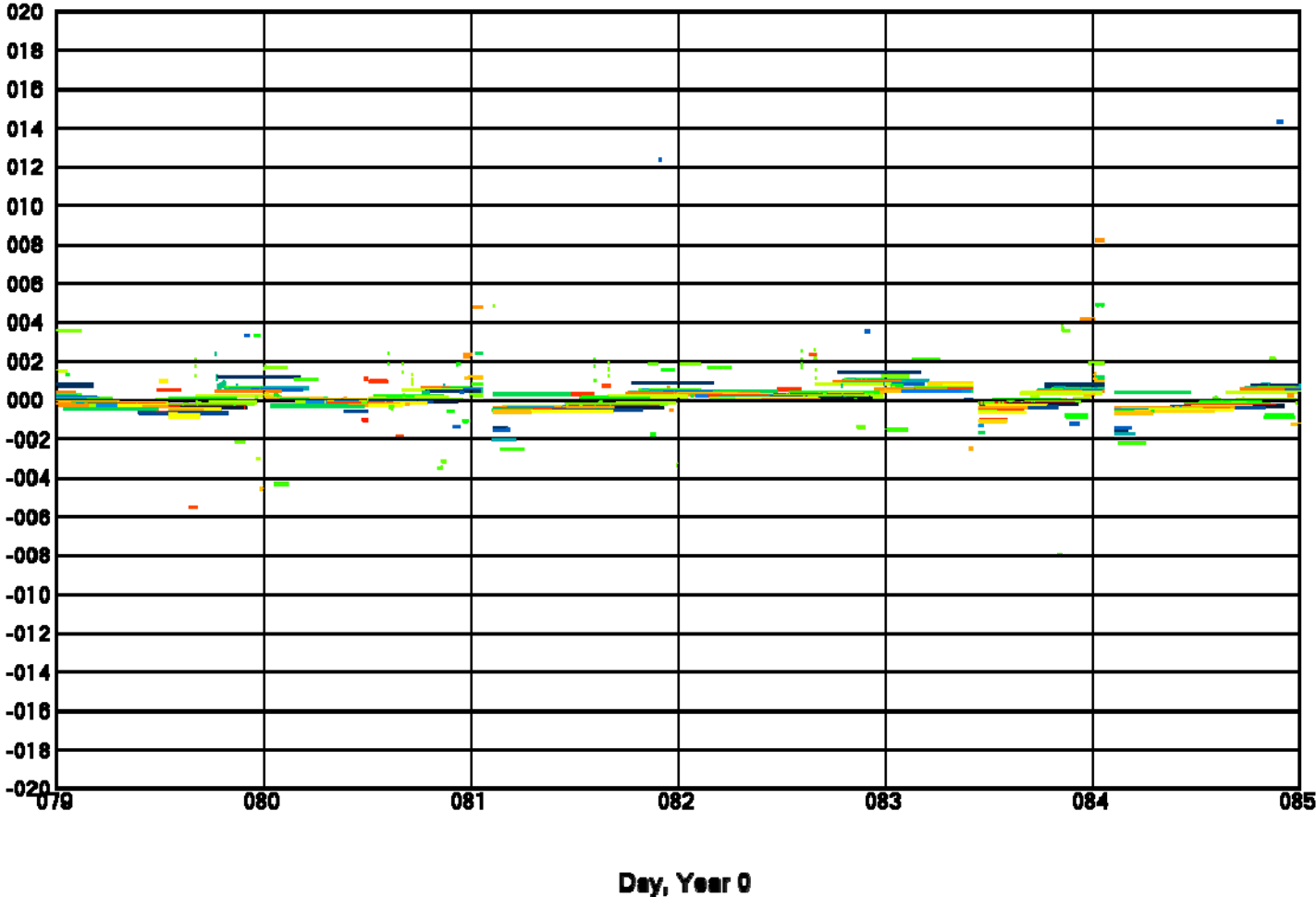
Traditional, VEq computed / VEq True

TEC(10^{16}) asc1 Lat=08.0S Lon=14.4W RecTypeVer =



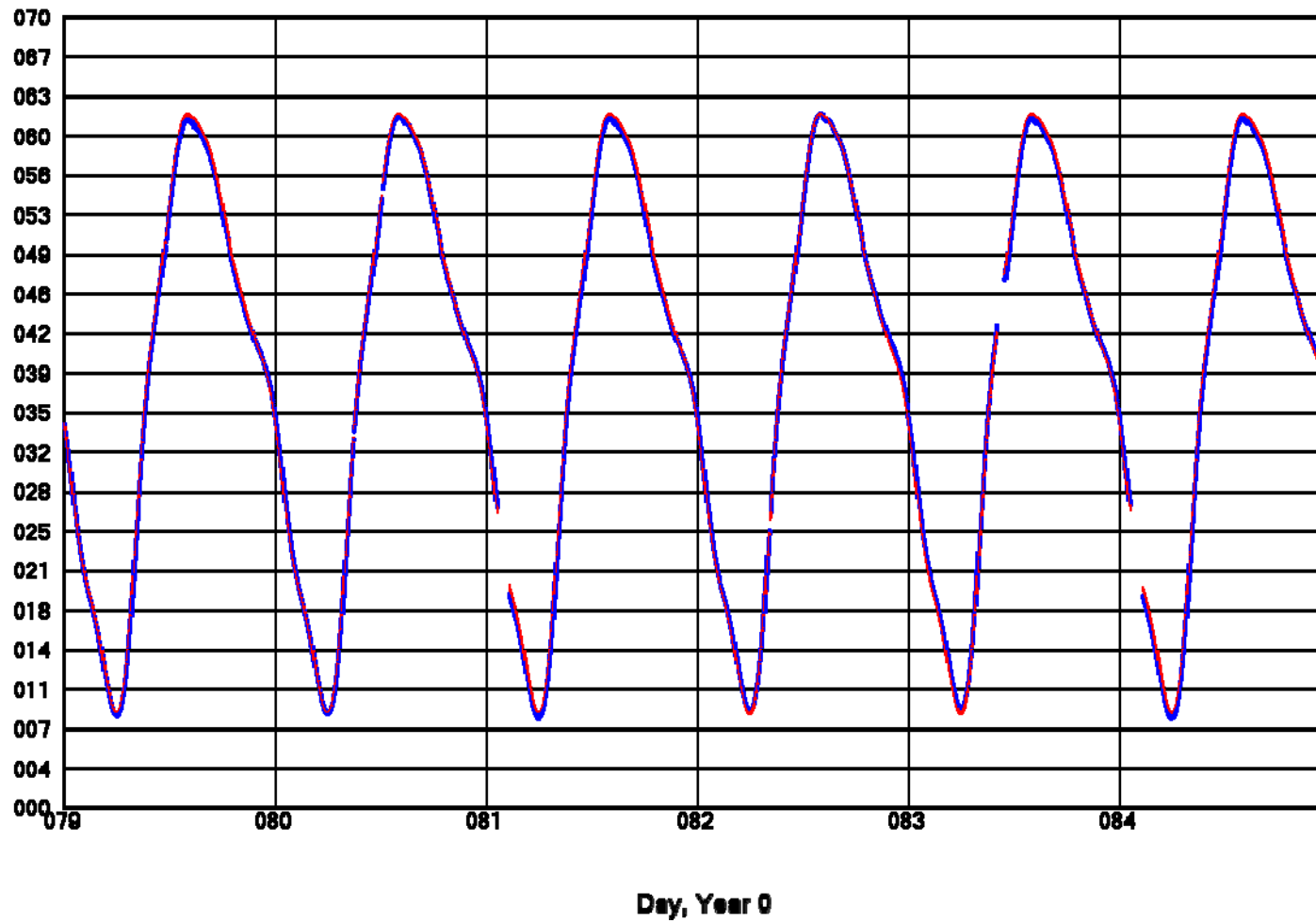
Arc Offset, SOUT - SIn

TEC(10**16) asc1 Lat=08.0S Lon=14.4W RecTypeVer =



Arc Offset, VEq computed / VEq True

TEC(10^{16}) asc1 Lat=08.0S Lon=14.4W RecTypeVer =



Thank you