TEC Calibration techniques:

Single-station estimation of arc offsets

L. Ciraolo

IFAC-CNR, Firenze / ICTP, Trieste

E-mail: l.ciraolo@ifac.cnr.it, lciraolo@ictp.it

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Objective of the presentation

What is calibration

How it is traditionally performed

Why proposing the

Single station, multi day, arc offset TEC calibration

Effort will be spent in avoiding details, that will be developed in the last section.

The observations

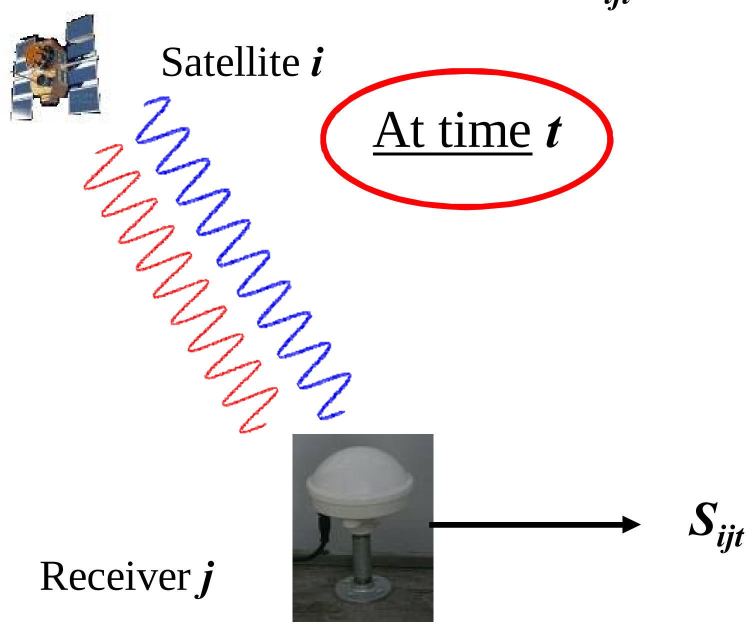
Properly processing GPS measurements

forming differential delays (dual frequency receiver)

combining them to obtain '**leveled slants**'

one gets slant Total Electron Content (***TEC)*** measurements affected by biasing terms β_i , γ_j , (λ_{Arc})

$$S_{ijt} = TEC_{ijt} + \beta_i + \gamma_j + (\lambda_{Arc})$$



i = 1, 2 , …,32 available GPS satellites

j = 1, .., available receivers

Arc = common to all continuous observations performed by receiver j on satellite i at times contiguos to t

Why bracketing λ_{Arc}? Because this term is disregarded in the traditional approach but basic for the proposed "arc offset" solution.

Arc:

a sub-set of continuous observations from one receiver to one satellite

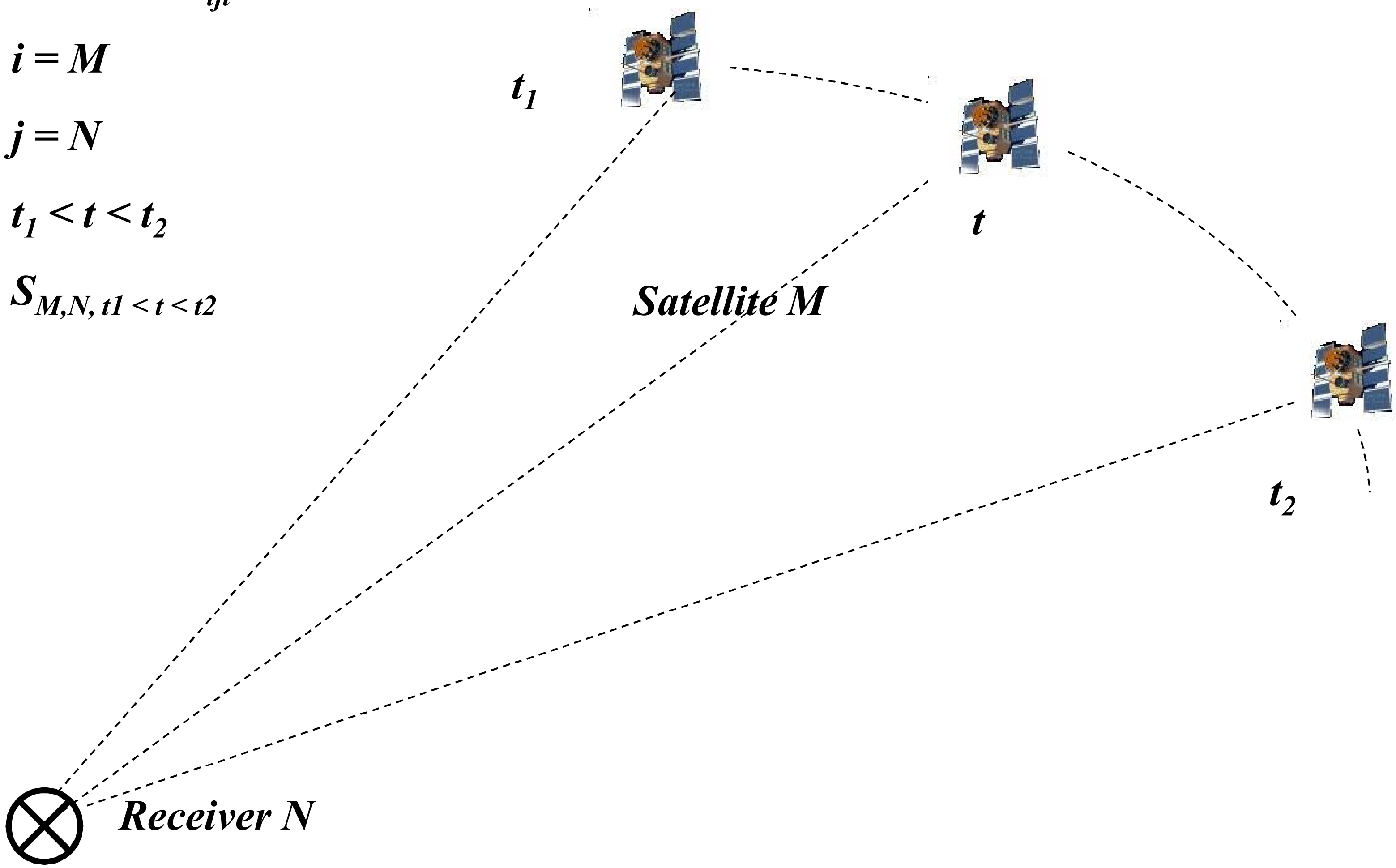
Subset of $\boldsymbol{S_{ijt}}$

$\boldsymbol{i = M}$

$\boldsymbol{j = N}$

$\boldsymbol{t_1 < t < t_2}$

$\boldsymbol{S_{M,N,\, t1 < t < t2}}$



Description of the biasing terms

$$S_{ijt} = TEC_{ijt} + \beta_\imath + \gamma_j + (\lambda_{Arc})$$

β differential hardware delays in satellite electronic circuitry

γ the same for receiver circuitry

λ the average contribution of differential multi-path along an arc

All biasing terms can be considered as constants

For ionospheric investigation and its applications (ionospheric corrections) an algorithm is needed able to estimate the biasing terms in order to have only ***TEC***

$$TEC_{ijt} = S_{ijt} - \beta_\imath - \gamma_j - (\lambda_{Arc})$$

This algorithm is known as

CALIBRATION or DE-BIASING

Red: unknowns

Blue: estimates

The calibration or de-biasing of GPS leveled slants

The system of the equations of observation is linear in all unknown terms

$$S_{ijt} = TEC_{ijt} + \beta_\iota + \gamma_j + (\lambda_{Arc})$$

but contains more unknowns than equations.

Number of unknowns = number of TEC**s *plus number of unknown (constant) terms***

$$\beta_\iota, \gamma_j, (\lambda_{Arc})$$

How is it possible performing the calibration?

TEC's are not actually uncorrelated: at some location, at some time they depend on the electron density distribution N_e.

Assume electron density N_e can be written as a function of position P, time t and a set of K parameters $Z_1, Z_2, \dots$

$$TEC = \int N_e(P, Q, t, Z_1, Z_{2,\dots})ds$$

Calibration is performed finding the values $Z_1, Z_2, \dots, \beta_\iota, \gamma_j, (\lambda_{Arc})$ which minimize the sum of the square of the residuals

$$\varepsilon_{ijt} = S_{ijt} - TEC(P, t, Z_1, Z_2, \dots) - \beta_\iota - \gamma_j - (\lambda_{Arc})$$

$$\Sigma\varepsilon^2_{ijt} \Rightarrow Minimum$$

Example: Ionospheric model ***NeQuick*** computes electron density N_e at given point P, at given time t, as a function of a Solar Flux equivalent parameter Az.

$$\varepsilon_{ijt} = S_{ijt} - TEC\,(P,\, t\,,\, Az) - \beta_\iota - \gamma_j - (\lambda_{Arc})$$

Find Az, β_ι, γ_j, (λ_{Arc}) *such that* $\Sigma\varepsilon^2_{ijt} \Rightarrow$ ***Minimum***

Observations/Unknowns budget: very favorable

Problems

Non linear minimization methods needed / Dependence on parameters is not analytical but numerical

Models provide with excellent median values whereas calibration requires that the model describes very precisely the actual N_e distribution

But:

Excellent perspectives for the future

Writing TEC

Better using formulations in which also actual gradients (not only median ones provided by the Ionospheric Models) can be taken into account, possibly linear in all unknowns.

$$S_{ijt} = TEC_{ijt} + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

Writing TEC ➔ Write the integral $$TEC = \int N_e(P, Q, t, Z_1, Z_{2,\ldots})ds$$

Possible (linear) expansions of ***TEC***

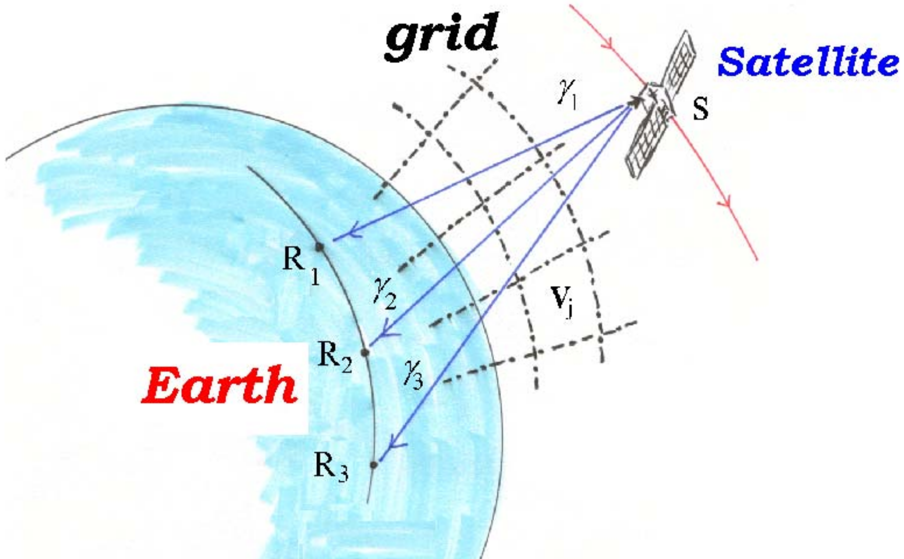
3D (Tomography)

Multi shell

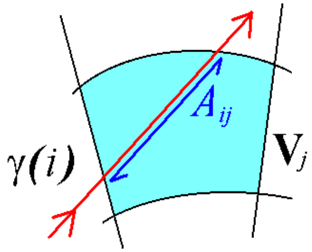
Thin shell

3D-4D approach (Tomography)

the ionosphere is divided in elements of volume (voxels) inside which $\boldsymbol{N_e}$ is constant. $\boldsymbol{N_e}$ of voxels are the unknowns. Evolution with time of $\boldsymbol{N_e}$ is considered to improve the budget unknowns/observations. Vertical behavour of $\boldsymbol{N_e}$ is expanded in Empirical Orthogonal Functions (EOF)



$$d_i = \int_{\gamma(i)} N(\mathbf{r},t)\,ds = \sum_j x_j \int_{\gamma(i)} b_j(\mathbf{r})\,ds \equiv \sum_j A_{ij} x_j$$

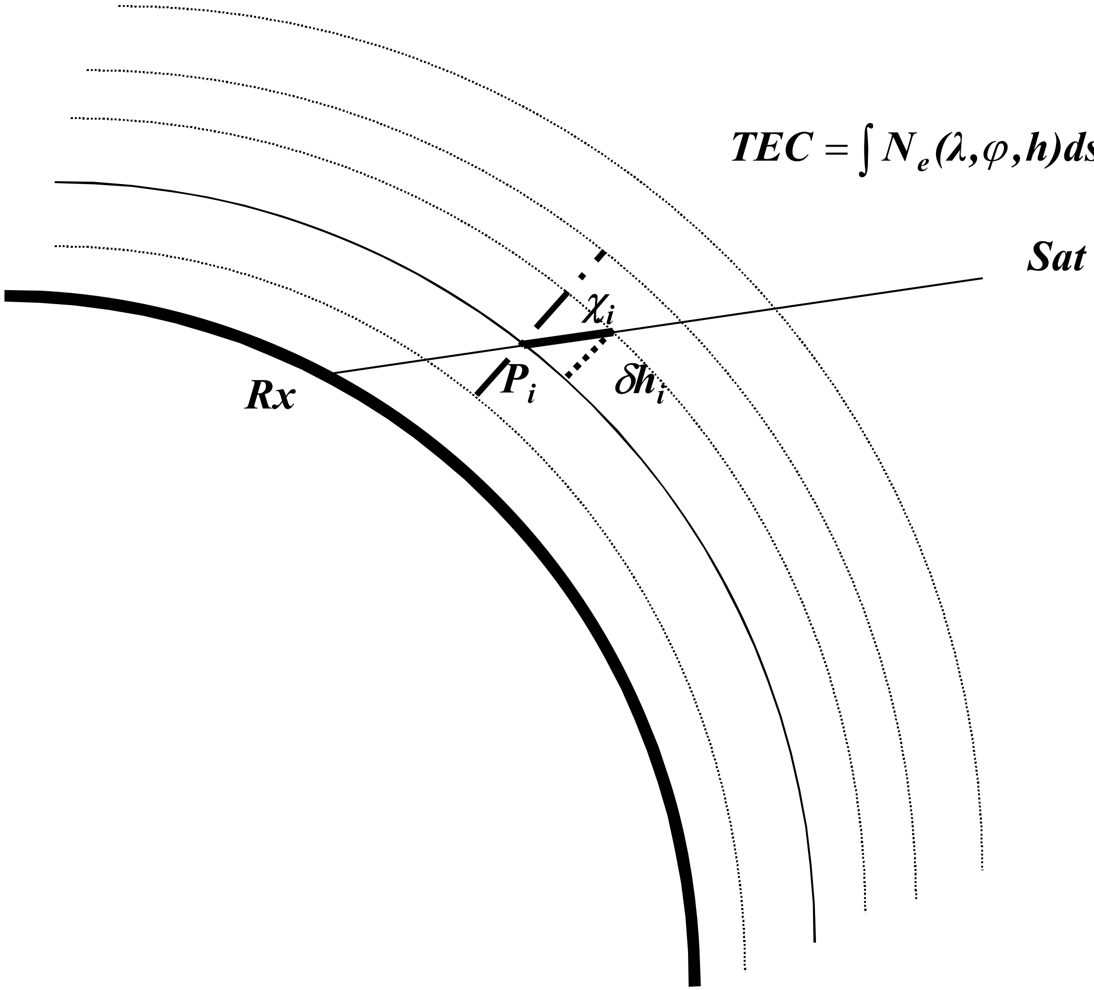


$$\underline{d} = A\underline{x} \quad \Rightarrow \quad \underline{x} = A^{-1}_{SVD}\underline{d}$$

3D: The multishell method

If many shells are used, this is exactly the method by which numerical integration is carried out. For each shell, a suitable 2D expansion in horizontal coordinates is assumed.

$$TEC = \int N_e(\lambda,\varphi,h)ds \approx \sum N_e(P_i)\delta s_i = \sum N_e(P_i)sec\chi_i\delta h_i$$



P_i, point on the generic i^{th} shell

δh_i increment in height

δs_i increment in arc length

$\delta s_i = \delta h_i \sec \chi_i$

The classical thin shell model

Reducing down the number of shells, ***and in principle the expected accuracy***,

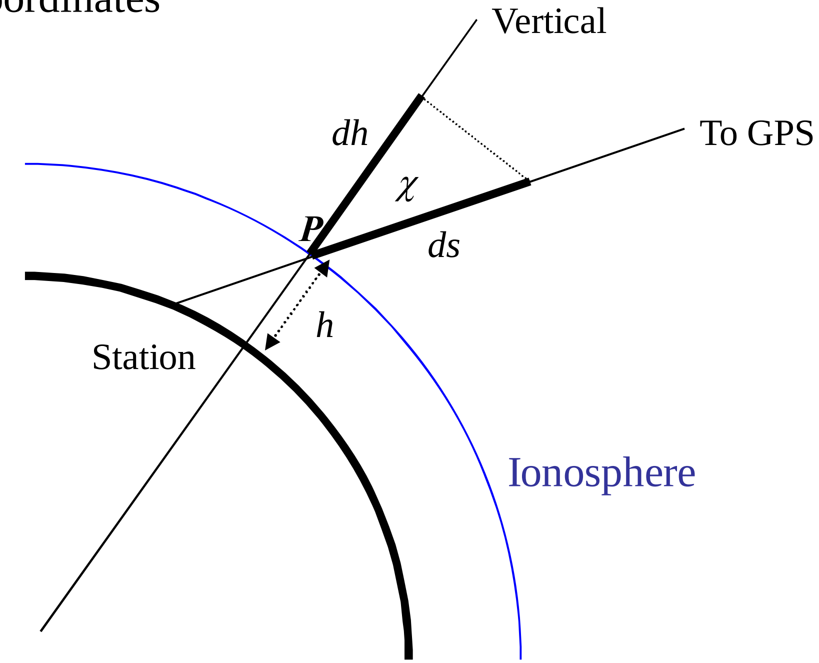
take only one **(thin) shell at some reference height *h***

$$TEC = V(P) \sec \chi$$

V(P) is the ***TEC*** along the vertical of the ionospheric point ***P***

(**V**ertical **E**lectron **C**ontent, ***VEC***)

V(P) is a **2D** function of horizontal coordinates



Note

Thanks to its simplicity and despite its known limitations, the thin shell approach has been and is very widely used also in application in which integrity is a basic requirement, such as

Satellite Based Augmentation Systems (SBAS)

In which ***VEC*** dependence on horizontal coordinates is implemented interpolating values of a grid of points covering the area of application

Klobuchar model uses this approach too

In the following, only the thin shell approach will be considered

Calibration using the thin shell approximation

Given

The observations $S_{ijt} = TEC_{ijt} + \beta_\imath + \gamma_j + (\lambda_{Arc})$

The thin shell assumption $TEC_{ijt} = V(P_{ijt})\ sec\ \chi_{ijt}$

Write Vertical $V(P_{ijt})$ as expansion in horizontal coordinates (geographic, geomagnetic or equivalent latitude Φ and longitude Λ)

$$V(P_{ijt}) = \Sigma_n c^{(t)}{}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt})$$

$$S_{ijt} = TEC_{ijt} + \beta_\imath + \gamma_j + \lambda_{Arc} = V(P_{ijt})\ sec\ \chi_{ijt} + \beta_\imath + \gamma_j + (\lambda_{Arc}) =$$

$$= sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt}) + \beta_\imath + \gamma_j + (\lambda_{Arc})$$

Representing the linear system of equations of observation to be solved in the unknowns

$$c^{(t)}{}_n, \beta_\imath, \gamma_j, (\lambda_{Arc})$$

Some simple example for ***VEC*** expansion

$$V(P_{ijt}) = \Sigma_n c^{(t)}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt})$$

Single-station: assume, at time t, that ***VEC*** is constantover the station horizon, $VEC = V_0^{(t)}$:

$$V(P_{ijt}) = V_0^{(t)}$$

Single-station : assume ***VEC*** varies linearly with latitude Φ and longitude Λ

$$V(P_{ijt}) = V^{(t)}_0 + a^{(t)} (\Phi - \Phi_0) + b^{(t)} (\Lambda - \Lambda_0)$$

Which can be improved up to bi-linear, bi-polynomial expansion and the full spherical harmonics expansion for global solutions

Rewrite equations of observation

$$S_{ijt} = TEC_{ijt} + \beta_{\iota} + \gamma_{j} + \lambda_{Arc} = V(P_{ijt})\ sec\ \chi_{ijt} + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt}) + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

Symbolically written as

$$S = Ax$$

Unknowns x will be solved using Least Squares or equivalent (and more sophisticated) methods

$$x = (A^T A)^{-1} A^T S$$

Going back to the equations of observations, knowing solution x means knowing

The coefficients of the expansion of vertical TEC $c^{(t)}{}_n$

The biasing terms β_{ι} , γ_j , (λ_{Arc})

After the numerical solution

Having solved for $c^{(t)}_n, \beta_\iota, \gamma_j, (\lambda_{Arc})$, available products are

The calibrated slants

Calibrated slants will be available as $TEC_{ijt} = S_{ijt} - \beta_\iota - \gamma_j - (\lambda_{Arc})$

The Vertical TEC

In addition, as a by-product of calibration, knowledge of the coefficients $c^{(t)}_n$ of ***TEC*** expansion will enable to estimate slants along directions different from the ones of the actual observations.

$$TEC_{ijt} = \sec \chi_{ijt} \Sigma_n c^{(t)}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt})$$

The most familiar is vertical ***TEC (VEC)***, the Total Electron Content relative to the zenith of the station of coordinates Φ^*_j, Λ^*_j

$$VEC(j,t) = TEC_{jt} = \Sigma_n c^{(t)}_n \Psi_n (\Phi^*_j, \Lambda^*_j)$$

Summary

All solutions for calibration follow the reported scheme

Extraction of un-calibrated slants from GPS observations

Solution of the system in unknown *VEC* coefficients and biasing terms

According to the geographical distribution of stations and the time span in which observations are available, several solutions are possible getting the possible combinations of one solution per line

Hourly / Single-day / Multi-day

Single-station / Regional /Global

Factors affecting the reliability of calibration

Modelling of observations

$$S = VEC \sec \chi + \beta + \gamma + (\lambda_{Arc})$$

Mapping function accuracy, constancy of biases, role of (λ_{Arc})

Adequacy of the model used for the expansion of *VEC*

$$VEC(P, t) = \Sigma c \, \Psi(P, t)$$

Conditioning of the resulting systems of equations

Still under investigation: biasing terms and VEC strongly correlated

The traditional method: assumptions

Accept the known limitations of the thin shell approach

Accept the constancy of biases

Disregard the multi-path contribution

Solve the system

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt}) + \beta_\iota + \gamma_j$$

In the unknowns $c^{(t)}{}_n$, β_ι, γ_j

The traditional method: Advantages

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt}) + \beta_\iota + \gamma_j$$

Excellent observations/unknowns budget

Coefficients of VEC expansion plus
one β per satellite, one γ per receiver, both constant

No need to perform calibration for every new set of data:

just compute the leveled slants and subtract an available set of pre-computed β_ι, γ_j

$$TEC_{ijt} = S_{ijt} - \beta_\iota - \gamma_j$$

Use pre-computed values during storm periods or at extreme latitudes (inadequacy of VEC expansion)

Use pre-computed values provided by others

Use of pre-computed values

Slants to calibrate

From a set of IGS stations (RINEX files)

Work has been already done by IGS: monthly values biases for satellites and IGS stations are available at

ftp://ftp.unibe.ch/aiub/CODE/

For user owning their own receiver

Use CODE for satellite biases, set up a calibration algorithm to estimate the bias of the receiver γ

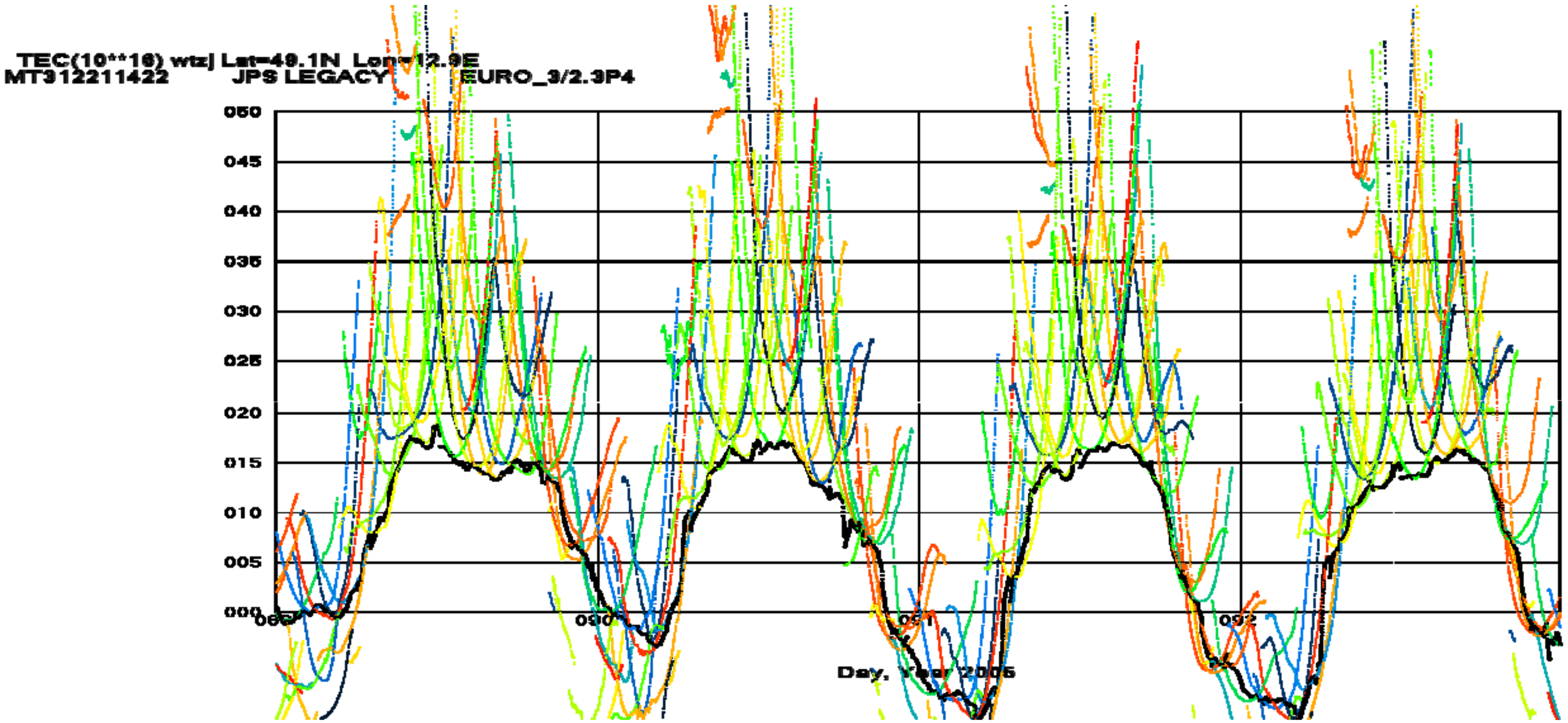
$$S_{ijt} - \beta_\iota = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt}) + \gamma$$

Why proposing a different solution?

Reported gossips on the traditional solution:

Slants (to the same satellite) of co-located receivers are not the same

Possible occurrence of negative TECs



Which of the reported limitations can produce this errors?

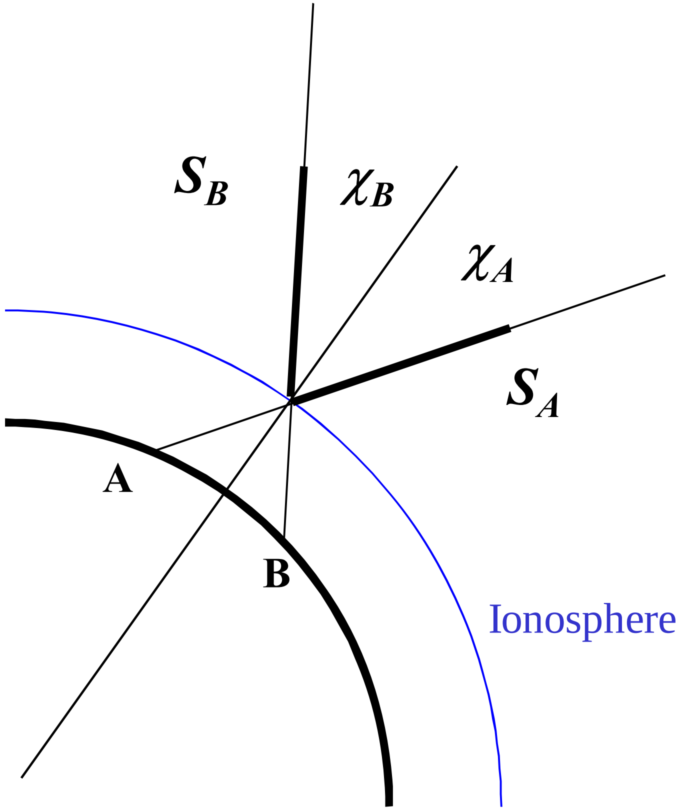
Limitations of the thin shell assumption?

The thin shell assumption is self-evidently poor:

TEC is the same for rays passing through the same ionospheric point,

disregarding at all gradients

If $\chi_A = \chi_B$ then $\boldsymbol{S}_A = \boldsymbol{S}_B$



But shall we discard the thin shell approach?

A new interpretation

For a given ray, rearrange ***TEC*** definition using **sec** χ_{REF} at a given reference height

$$TEC = \int N_e ds = \int N_e sec\chi dh = sec\chi_{REF} \int N_e \frac{sec\chi}{sec\chi_{REF}} dh = sec\chi_{REF} V_{eq}$$

$$V_{Eq} = \int N_e \frac{sec\chi}{sec\chi_{REF}} ds \qquad \boxed{TEC = sec\chi_{REF} V_{eq}}$$

The expression is formally identical to the mapping function approximation,

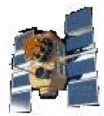
but it is **exact** provided V_{Eq}, a 2D Function (elevation/azimut or displacement of horizontal coordinates from the station) is not interpreted as the vertical ***TEC***.

V_{Eq} will change for stations in different locations, **<u>so its use is limited to the calibration performed by the single station solution</u>**.

Calibration requires a relationship correlating the various slants: for the single station solution the properly interpreted mapping function does not implies errors other than the capability to map V_{Eq} in satisfactory way.

Which of the reported limitations can produce this errors?

Disregarding the multi-path error λ_{Arc}



β

The close stations experiment

Station 1

$$S1_{PRN} = TEC + \lambda 1 + \beta_{PRN} + \gamma 1$$

TEC

Station 2

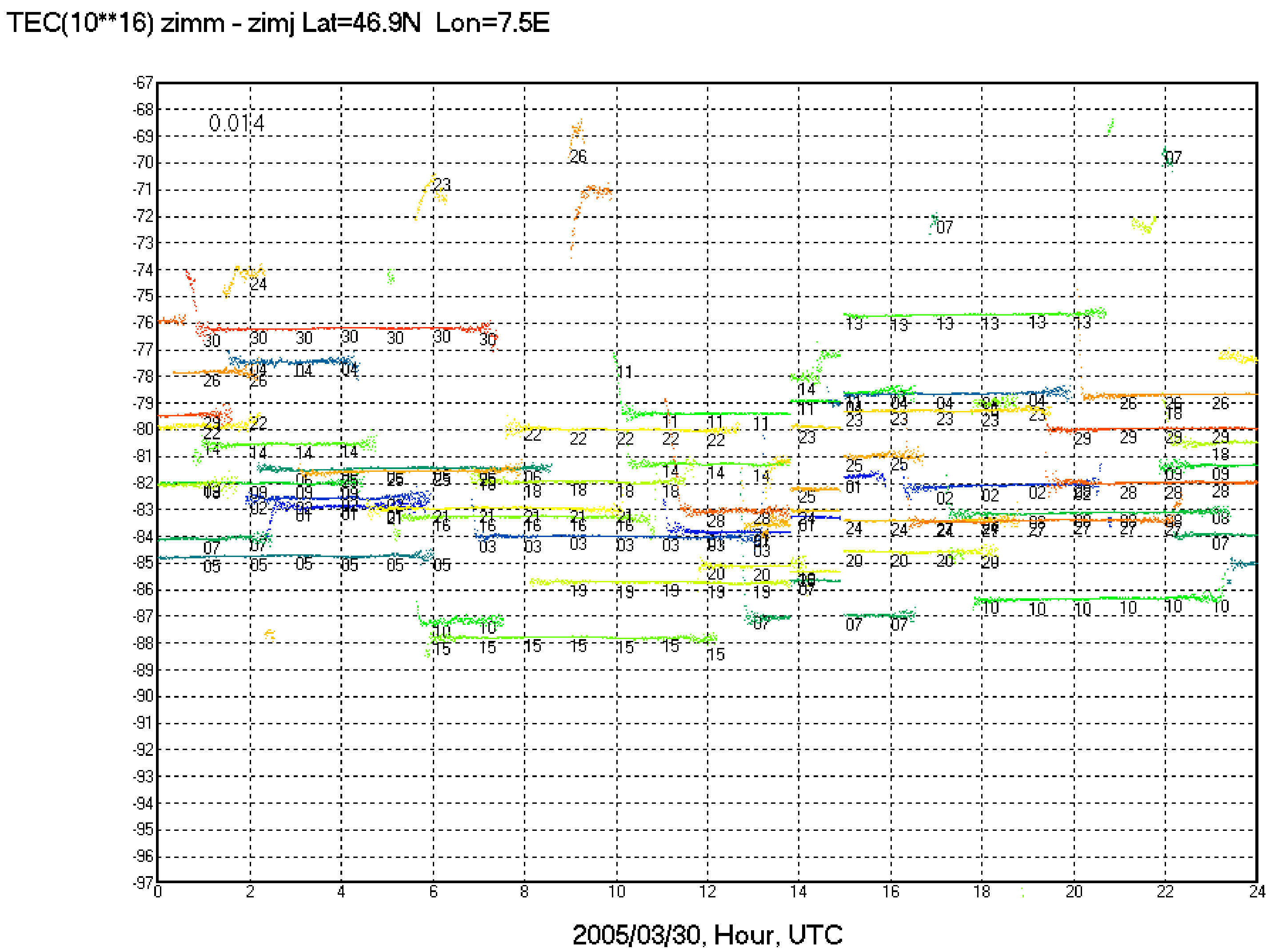
$$S2_{PRN} = TEC + \lambda 2 + \beta_{PRN} + \gamma 2$$

< 100 m

$$S1 - S2 = \gamma 1 - \gamma 2 + \lambda 1 - \lambda 2$$

Not dependent on PRN

$S_{L1} - S_{L2}$, all satellites



How the λ_{Arc} contribution affects the observations?

$$S_{ijt} = TEC_{ijt} + \beta_{\iota} + \gamma_{j} + \lambda_{Arc}$$

This term results from the contribution of multi-path (and the way it is processed by the receiver) along any individual arc. For the same satellite, the same receiver, the overall contribution $\beta_{\iota} + \gamma_{j} + \lambda_{Arc}$ will be different arc by arc.

Proposed solution

Consider the observations affected by an unknown overall arc dependent bias

$$\Omega_{Arc} = \beta_{\iota} + \gamma_{j} + \lambda_{Arc}$$

Implement only single-station, possibly multi-day, solutions for calibration (getting in part rid of problems with the mapping function)

The system of observation equations becomes

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n \Psi_n\ (\Phi_{ijt}, \Lambda_{ijt}) + \Omega_{Arc}$$

Expecting that from a numerical point of view the proposed solution will be less conditioned than the traditional one, but free from reported errors, Notes:

having assumed the validity of the thin shell approximation in the single-station solution, in the observations

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n\ \Psi_n\ (\Phi_{ijt}, \Lambda_{ijt}) + \Omega_{Arc}$$

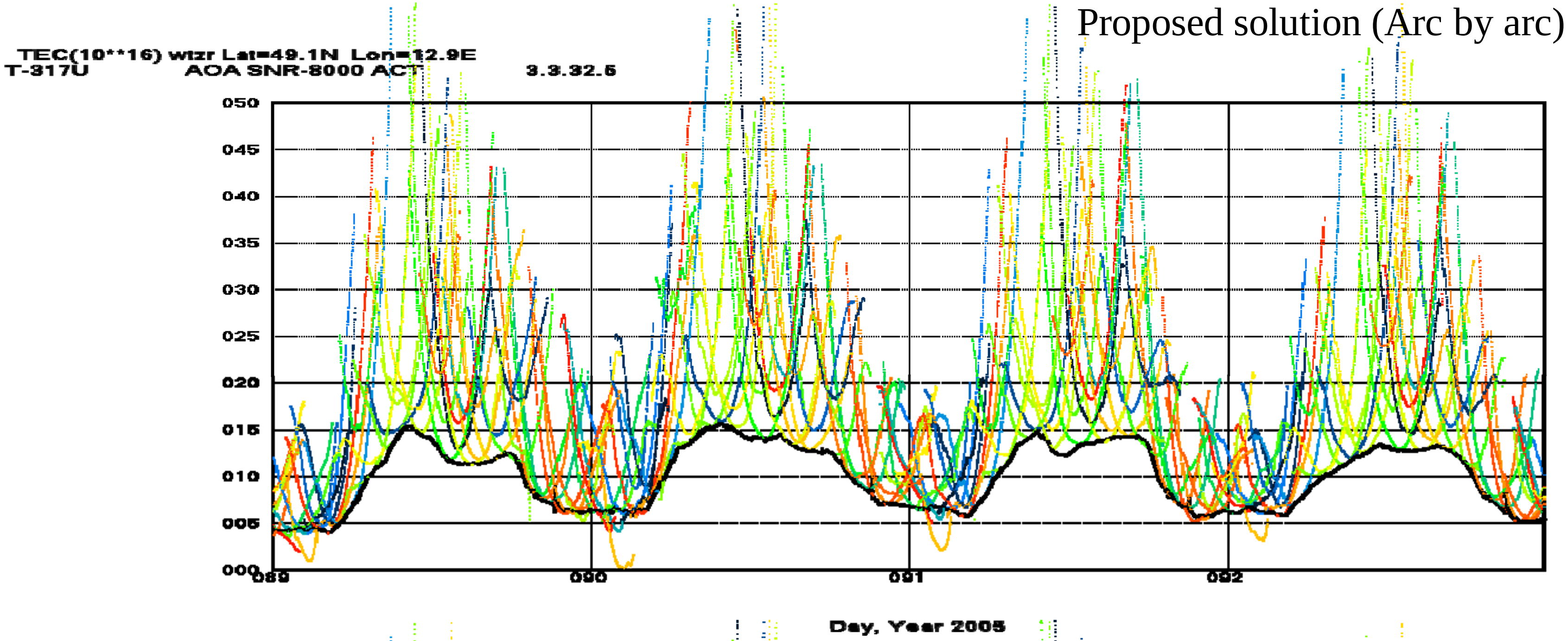
the expansion $\Sigma_n c^{(t)}{}_n\ \Psi_n\ (\Phi_{ijt}, \Lambda_{ijt})$ represents the **Vertical Equivalent Content** (***VEq***) and not the actual **Vertical Electron Content** (***VEC***)

VEq takes automatically into account of plasmaspheric contribution.

Considering Vertical ***TEC*** over the station, nothing will change as ***VEC*** and ***VEq*** coincide.

No possibility to use pre-computed biases

But the solution for co-located receiver will look much more reliable


Proposed solution (Arc by arc)
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T-317U AOA SNR-8000 ACT 3.3.32.5
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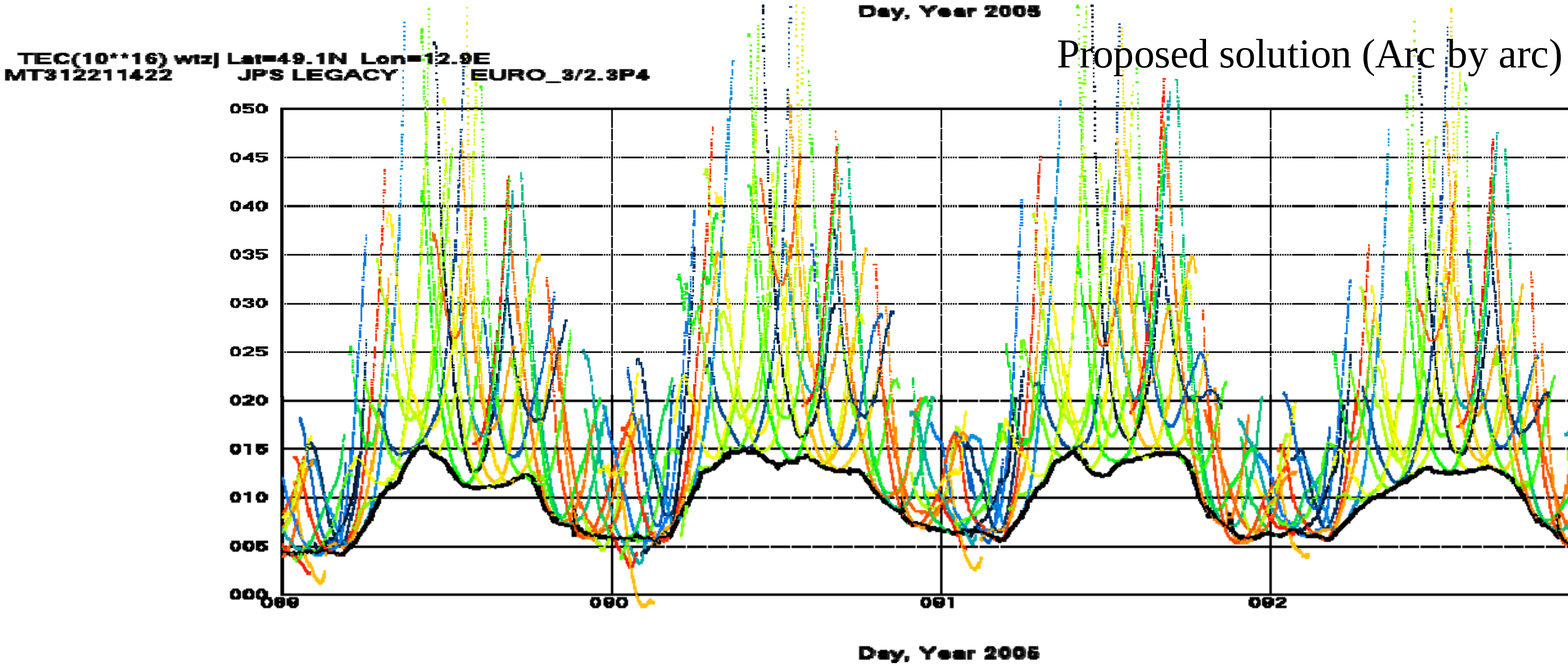
Proposed solution (Arc by arc)
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Summary of Proposed Solution characteristics

Observations

Leveled slants or directly phase slants

Assumptions

One thin shell at 400 km

Elevation mask: 10^{o}

TEC expressed through V_{Eq} at the ionospheric point, by the mapping function $\sec \chi$

V_{Eq} expressed as a proper expansion of horizontal coordinates l, f with one set of coefficients at each time $V_{Eq}(l, f) = \Sigma_n c_n p_n(l,f)$

$$S_{ijt} = \Sigma_n c^{(t)}{}_n p_n (l_{ijt} , f_{ijt}) \sec \chi_{ijt} + \Omega_{Arc}$$

The unknowns are now the coefficients $c_n^{(t)}$ and the offsets Ω_{Arc}

The adopted horizontal coordinates

Using as horizontal coordinates ***Modified Dip Angle*** and ***Local Time***, we can assume that for a set of adjacent epochs (up to ±15 minutes), the coefficients $c_n^{(t)}$ keep constant.

This allows also reducing computing resources during solution using commonly used standard methods for sparse systems.

After the solution of the system, we avail with :

Calibrated slants along the observed rays $TEC_{ijt} = S_{ijt} - \Omega_{Arc}$

"Mapped slants" at given coordinates l_{ijt}, f_{ijt}

Vertical ***TEC*** above the station (ionospheric point at the its zenith)

$$VTec(t) = \sum_n c_n^{(t)} p_n\left(l_{ijt}^{Zenith}, f_{ijt}^{Zenith}\right) \sec \chi_{ijt}$$

Why multi-day solution

A multi-day solution is performed, avoiding day to day discontinuities in calibrated slants, except that at the beginning and the end of the solution.

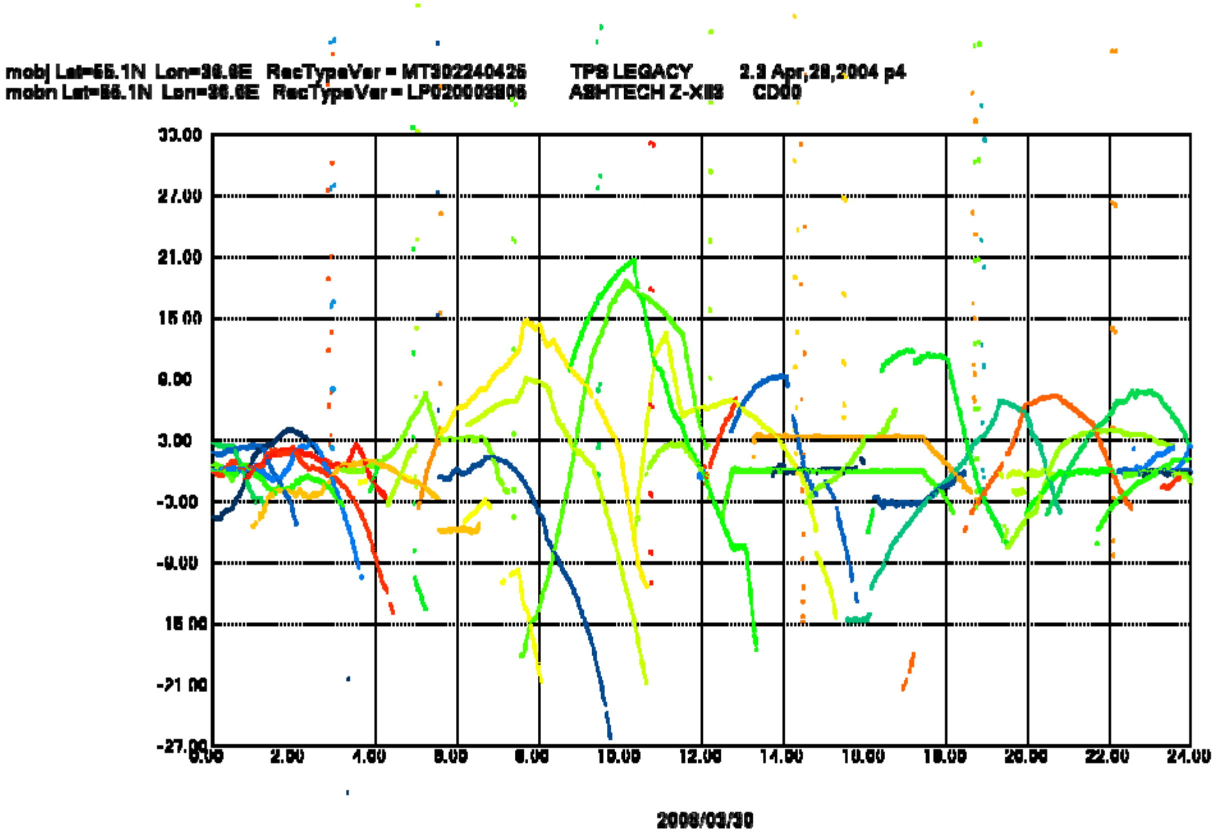
Still, at the beginning and the end of the set of data, broken arcs occur.

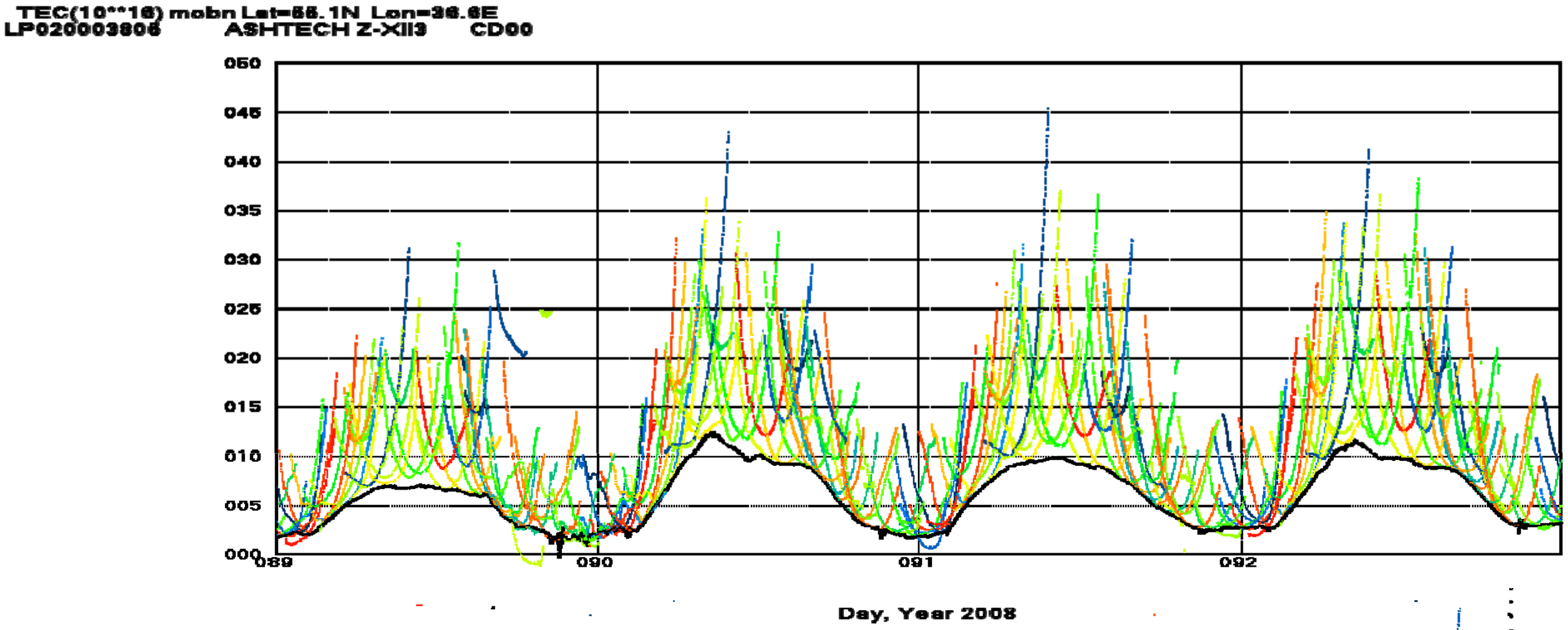
Broken arcs are generally shorter implying

1. worse results during leveling
2. worse numerical conditioning for the solution

To reduce these problems, in order to calibrate ***N*** days, ***N+2*** days are actually processed: first and last day of the ***N+2*** set are discarded.

Will it work everytime? Yes, provided phase slants $\boldsymbol{S_P}$ are reliable. For some pair of stations (namely $\boldsymbol{S_P}$ [***mobj***] - $\boldsymbol{S_P}$ [***mobn***]), the situation looks like here, showing that, for at least one of them, observations are not reliable. Still, no a priori way exists to know what is going wrong. For the present sample, the solutions of individual stations (next slide) show that the problem arises with "***mobj***".

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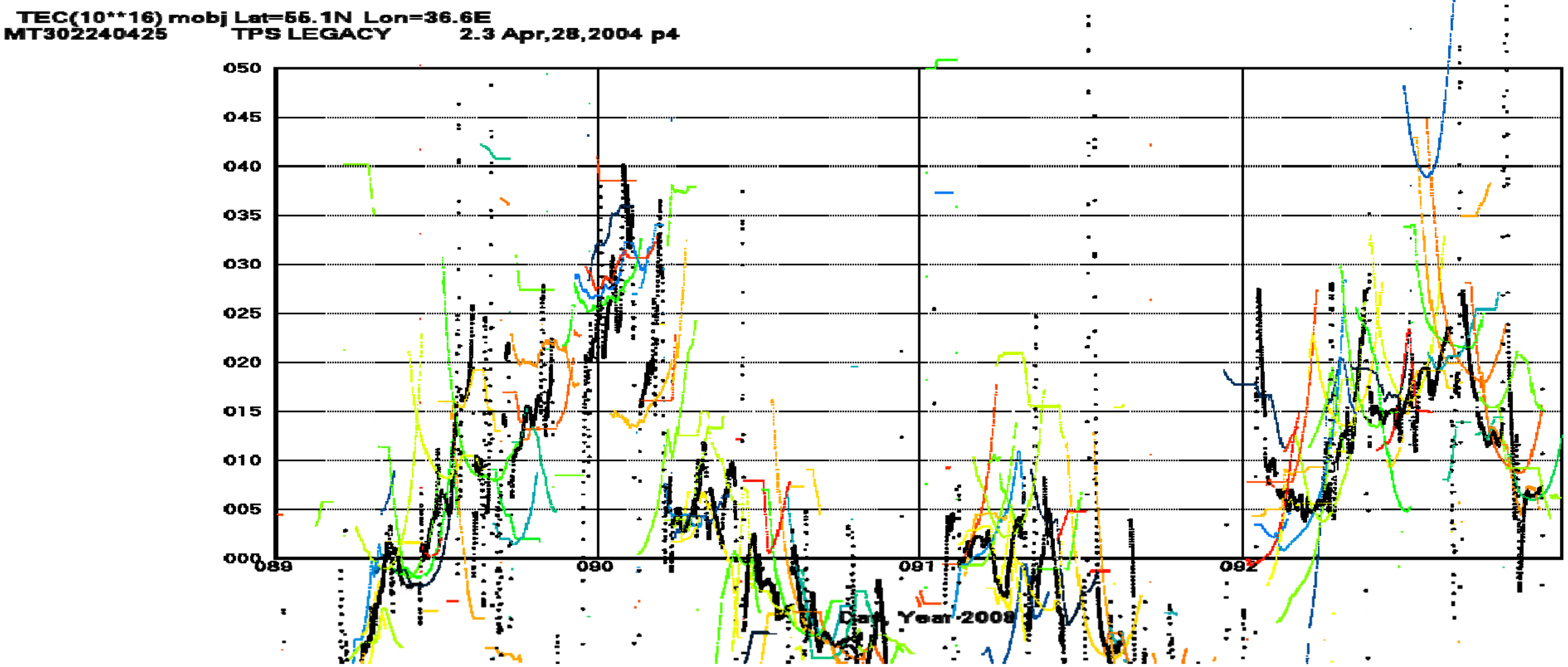
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Conclusions for the single-station, multi-day, arc-offset solution

Is it better than the traditional solutions?

A direct answer is ***not*** possible because reliable truth data to perform comparison are not available.

Models of the electron density can provide with "artificial data" to check the performance of the technique used for the calibration,

but they will not "simulate" the problems of the observations (multi-path)

Some positive aspects:

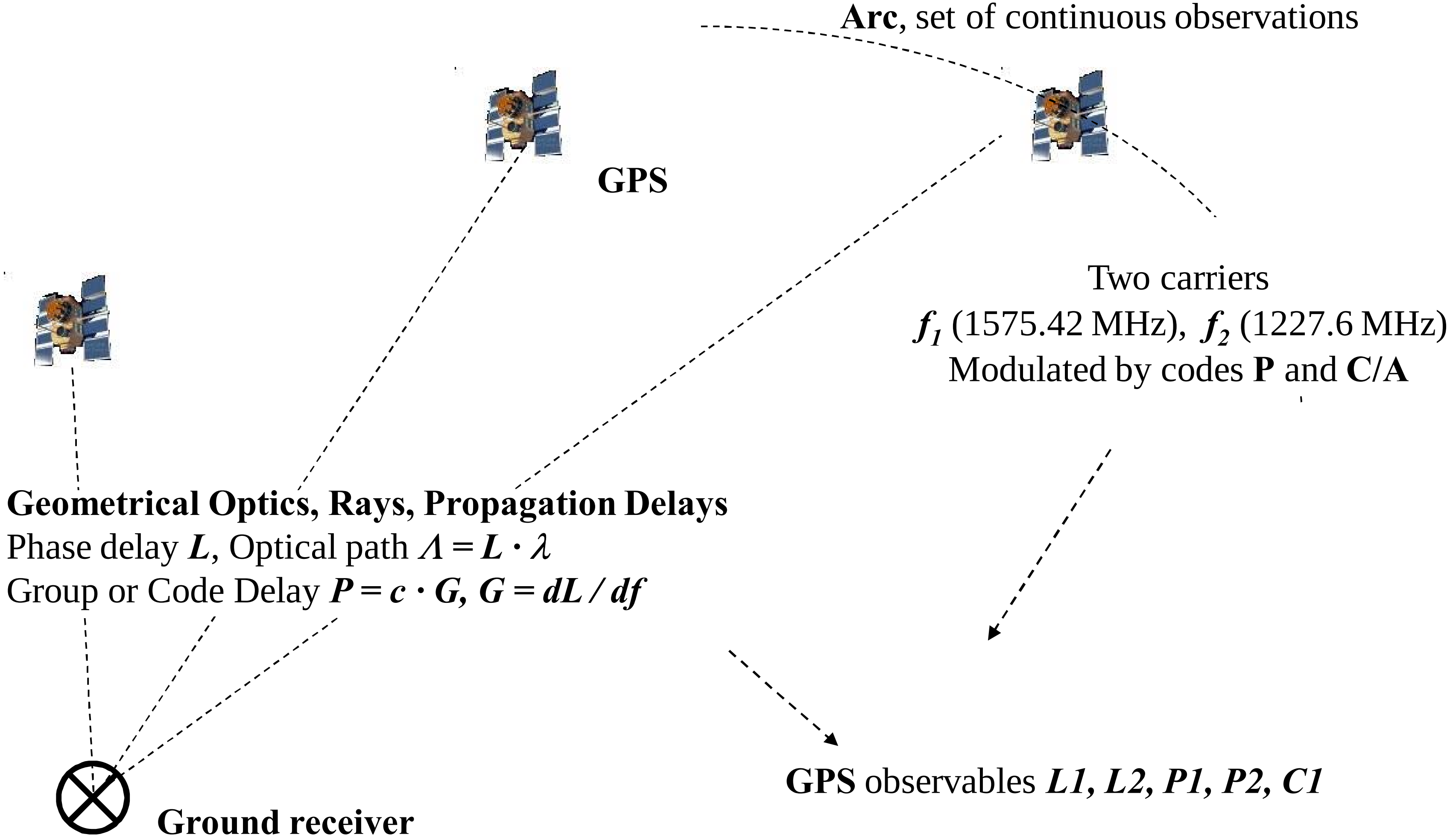
TEC's from two co-located receivers is the same

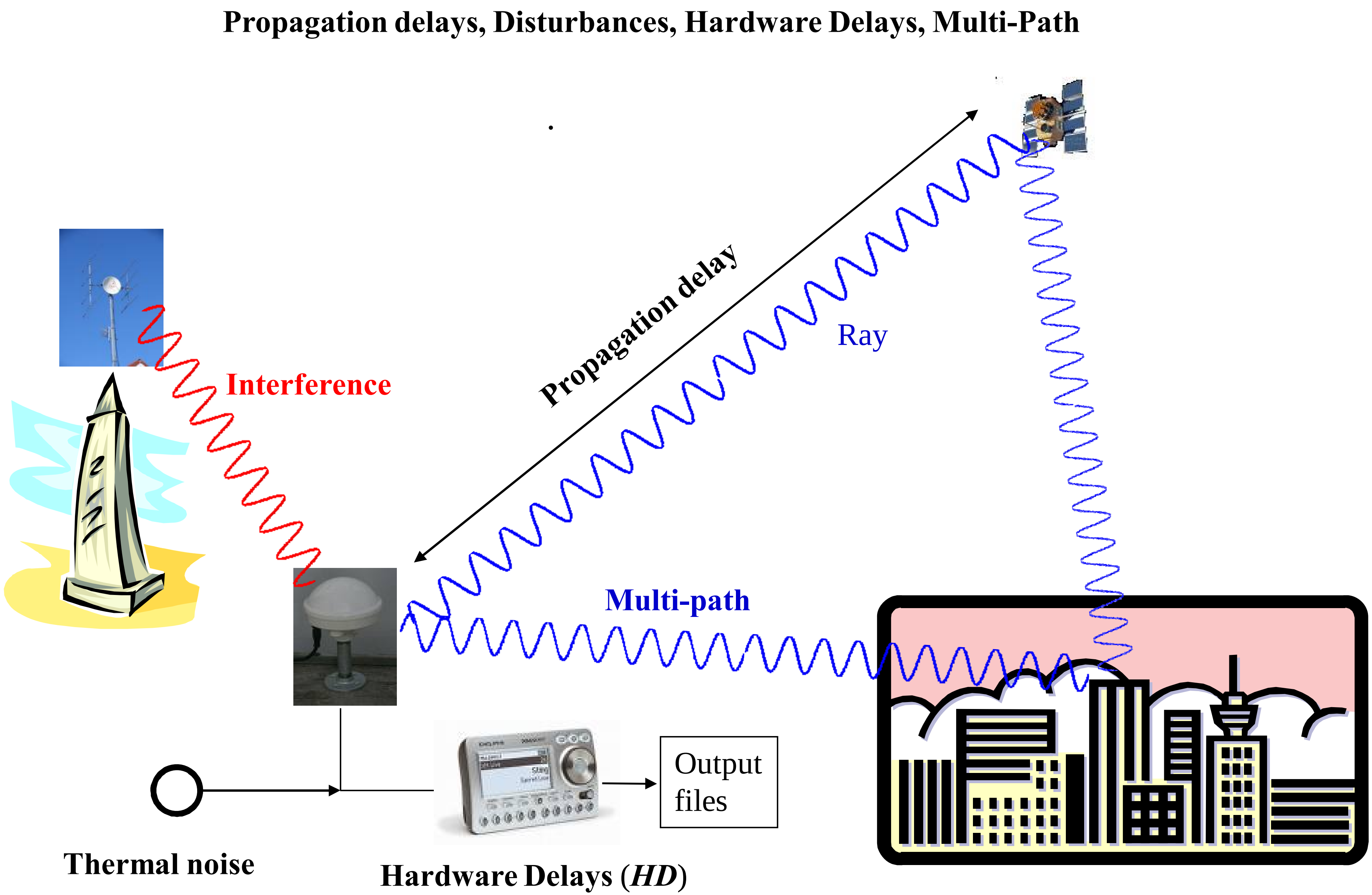
In the following, "details" and use of "artificial data" will be briefly discussed

End of the description of calibration:

Some detail

GPS scenario




Propagation delays, Disturbances, Hardware Delays, Multi-Path
Propagation delay
Ray
Interference
Multi-path
Output files
Thermal noise
Hardware Delays (*HD*)

Propagation Delays

Propagation and Atmospheric contributions to optical path Λ:

Geometric (**D**istance), Tropospheric, **I**onospheric

$$\Lambda \quad = \quad D + T + I$$

Equivalent Group Path P = Group delay $G \times$ speed of light

$$P = G \cdot c = D + T - I$$

Refractivity $R = n - 1$, n Index of Refraction

$$T = \int R_{atm}(s)ds \qquad I = \int R_{Iono}(s)ds \qquad R_{Iono} = -\frac{40.3 \cdot N_e}{f^2},$$

$$TEC = \int N_e(s)ds, \qquad I = -\frac{40.3 \cdot TEC}{f^2}$$

$$L = \frac{D + T + I}{\lambda} = \frac{f}{c}(D + T) - \frac{40.3TEC}{cf}$$

$$G = \frac{dL}{df} = \frac{D + T}{c} + \frac{40.3TEC}{cf^2}$$

Measurements introduce additional "delays"

Hardware electronic delays originating in satellite and receiver,	β, γ
Offset (delay, ambiguity) for phase	$\boldsymbol{\Omega}$
Noise	$\boldsymbol{n}$
Multipath	$\boldsymbol{m}$
User clock offset	τ

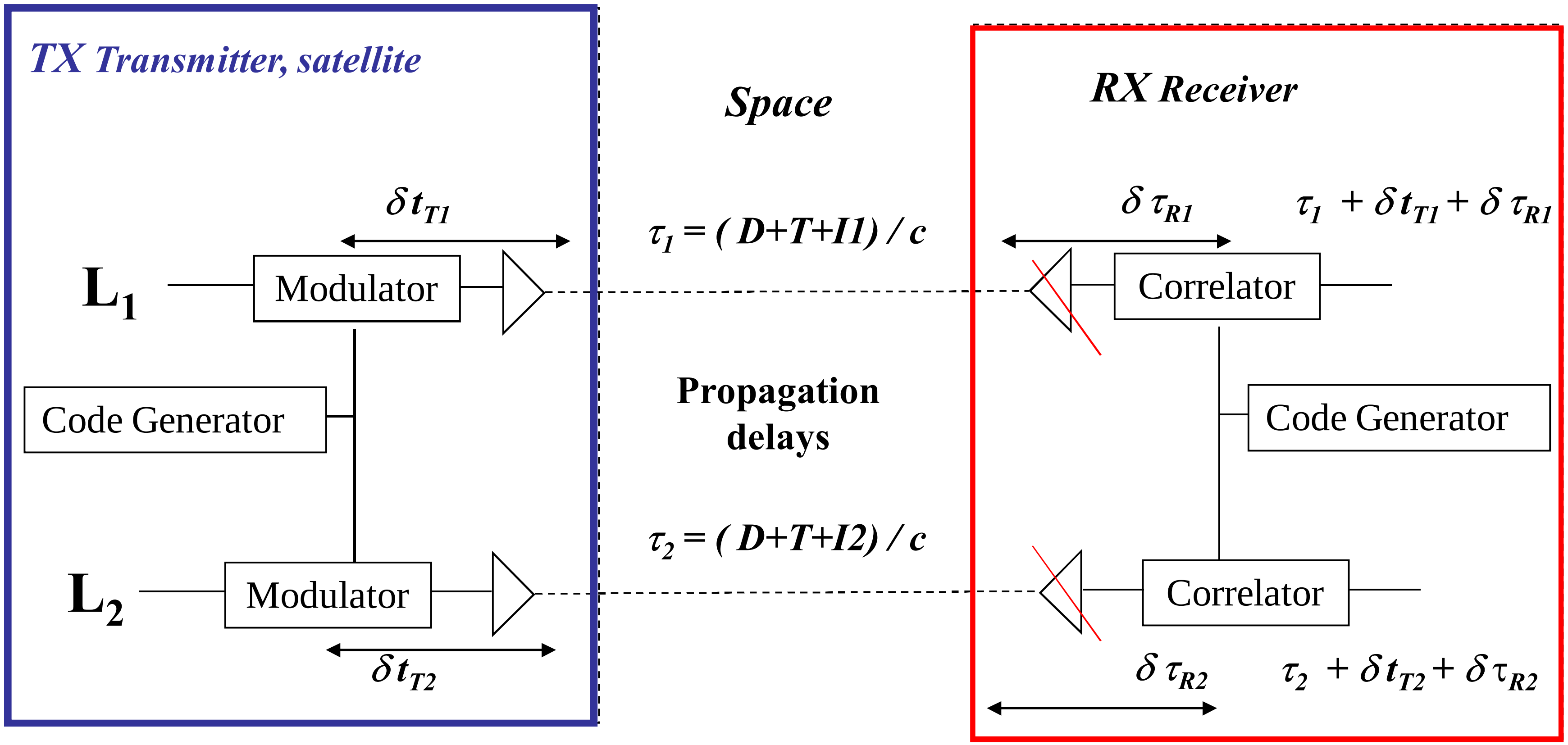
Code delay affected by user clock offset is ***pseudorange***

$$\boldsymbol{P} = \boldsymbol{D} + \boldsymbol{T} - \boldsymbol{I} + \beta + \gamma + \boldsymbol{n} + \boldsymbol{m} + \tau$$

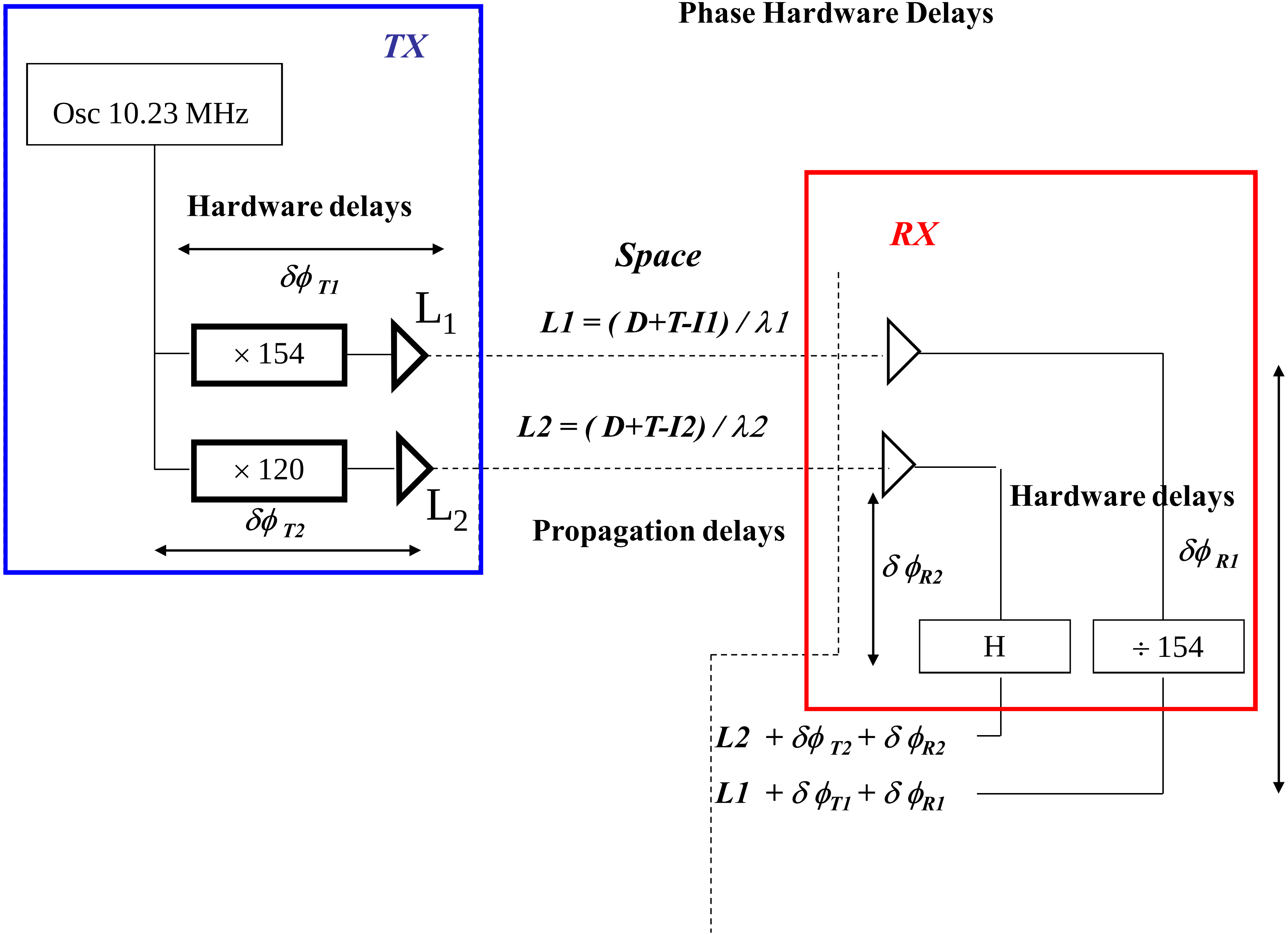
For following discussion, noise and multipath can be neglected for phase delays. Hardware delays for phase are included in $\boldsymbol{\Omega}$

$$\Lambda = \boldsymbol{D} + \boldsymbol{T} + \boldsymbol{I} + \boldsymbol{\Omega}$$

Code hardware delays



Phase Hardware Delays



Availing GPS delays ***P1, P2, L1, L2, C1***

Users aiming to determine their position, will get rid of ionospheric contribution taking proper combinations of them.

Users aiming to investigate ionosphere, will simply compute differential delays

Differential pseudorange

$$\boldsymbol{P2 - P1}$$

Differential phase path

$$\boldsymbol{\Lambda 1 - \Lambda 2 = L1 \cdot \lambda 1 - L2 \cdot \lambda 2}$$

Both differential delays are in meters.

Following steps:

Show dependence on ***TEC***

Transform to ***TEC units*** **(10^{16} electrons/m^2),** ***TECu***

The differential Delays

For the carrier i (i = 1,2), contributions with no index do not depend on frequency and cancel out forming differential delays

$$P_i = G_i \cdot c = D + T - I_i + \beta_i + \gamma_i + n_i + m_i + \tau,$$

$$\underline{\Delta P = P2 - P1 = I1 - I2 + \Delta\beta + \Delta\gamma + \Delta n + \Delta m}$$

$$\Lambda_i = D + T + I_i + \Omega_i$$

$$\underline{\Delta\Lambda = \Lambda 1 - \Lambda 2 = I1 - I2 + \Delta\Omega}$$

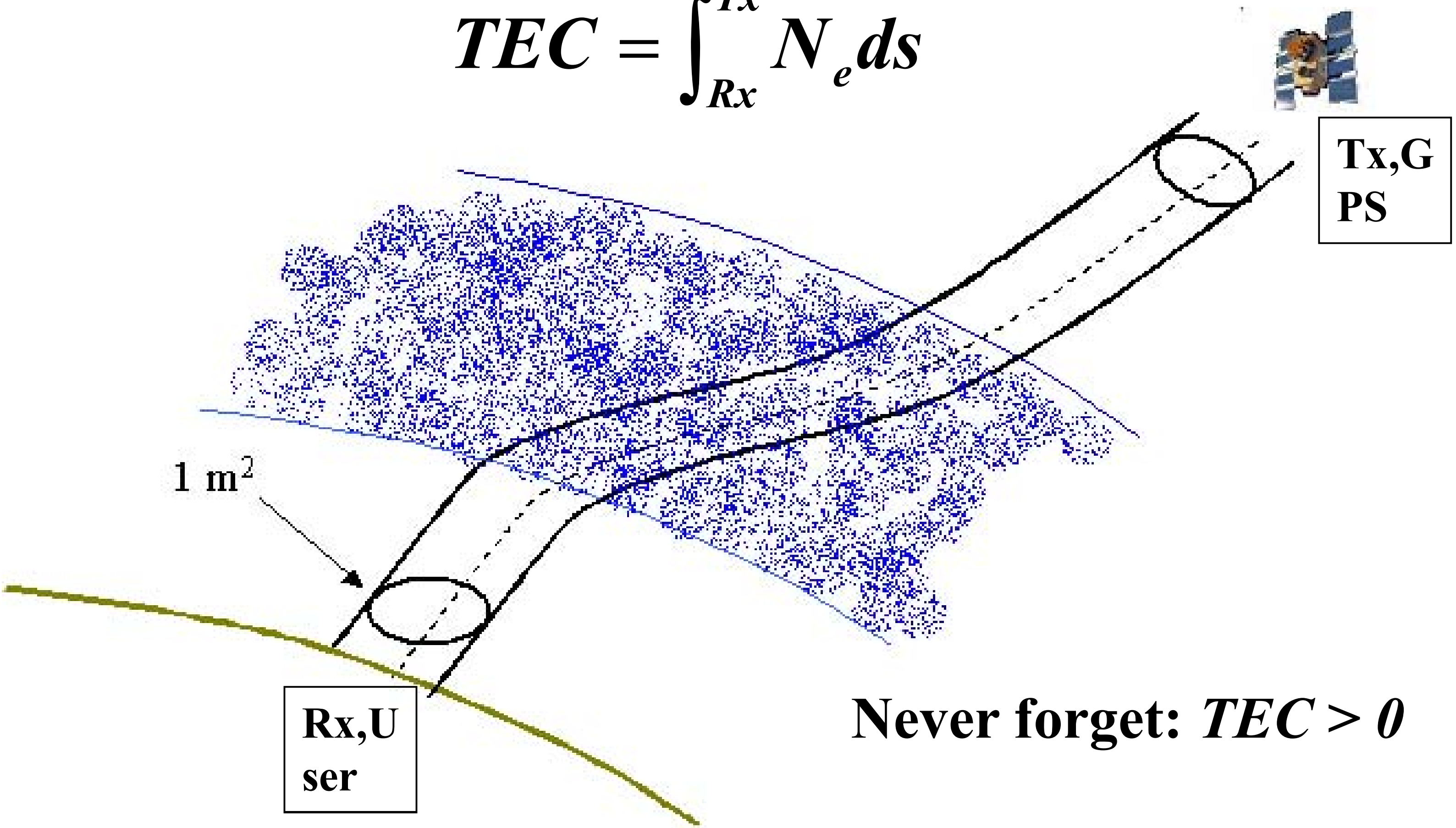
$$I2 - I1 = k \cdot TEC \quad k = 40.3TEC\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right)$$

Divide by $k \cdot 10^{-16}$, <u>drop out</u> the Δ symbol to obtain the ***phase slants*** S_P and ***group or code slants*** S_C in ***TECu***, 1 ***TECu*** = 10^{16} electrons/m^2, disregard radio noise n

$$\boxed{\begin{aligned} S_P &= \frac{1}{k} \cdot (\Lambda 1 - \Lambda 2) = TEC + \Omega \\ S_C &= \frac{1}{k} \cdot (P2 - P1) = TEC + m + \beta + \gamma \end{aligned}}$$

The classical interpretation of ***TEC*** as the **numbers of electrons** contained in a column of unitary base along the ray

$$\boldsymbol{TEC = \int_{Rx}^{Tx} N_e ds}$$



Never forget: *TEC > 0*

Note for the following: expressions for observations like

$$\boldsymbol{S = TEC + b}$$

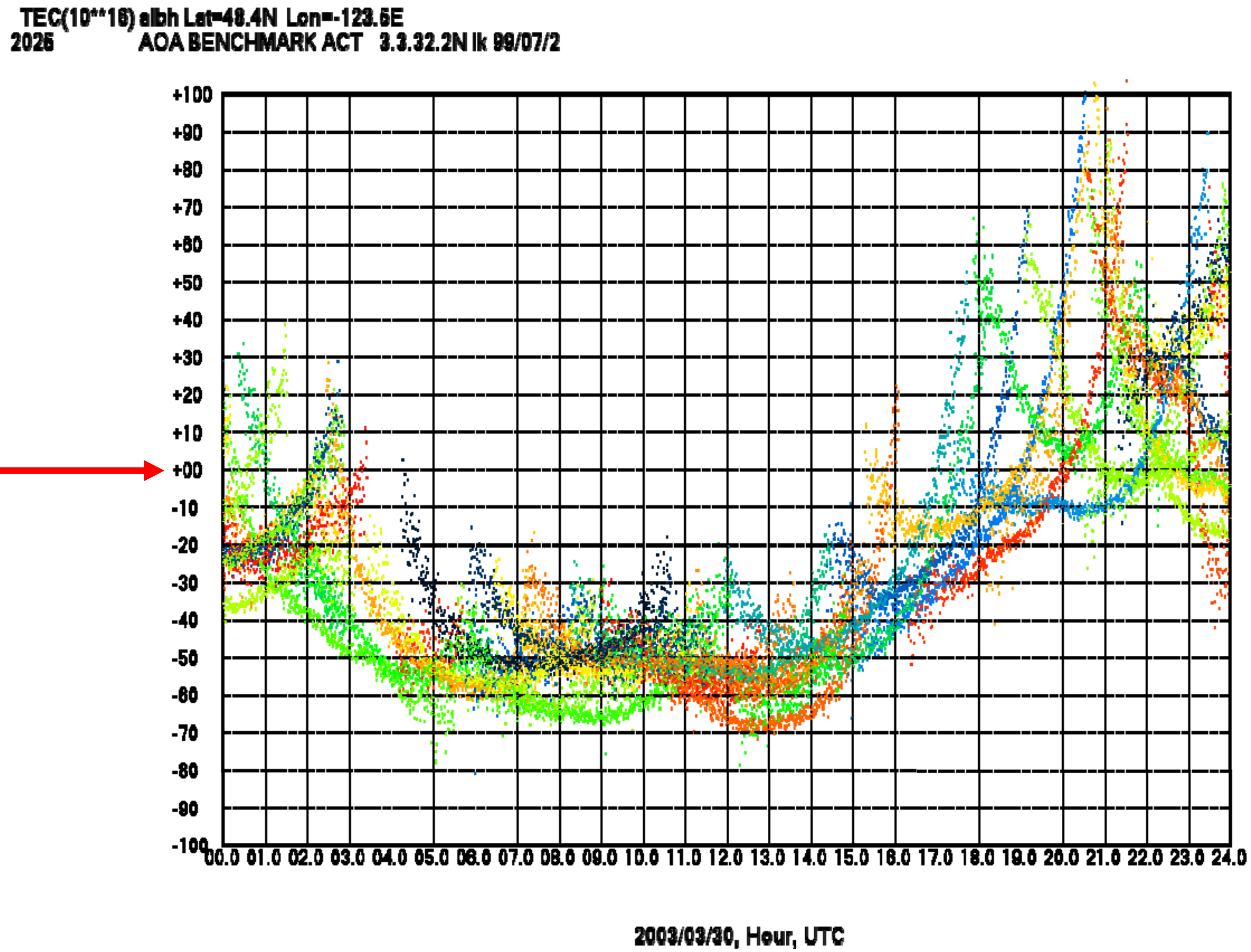
denote the set of all available observations used for performing some specific task.

Actually observations should be indexed as $\boldsymbol{S_{ijt}}$ meaning that the individual observed quantity , the ***“slant ”***, refers to $\boldsymbol{i^{th}}$ satellite, $\boldsymbol{j^{th}}$ station, $\boldsymbol{t^{th}}$ time.

Biasing terms can still be indexed according to satellite and station (not time as assumed to be constant), but also according to the specific observed arc.

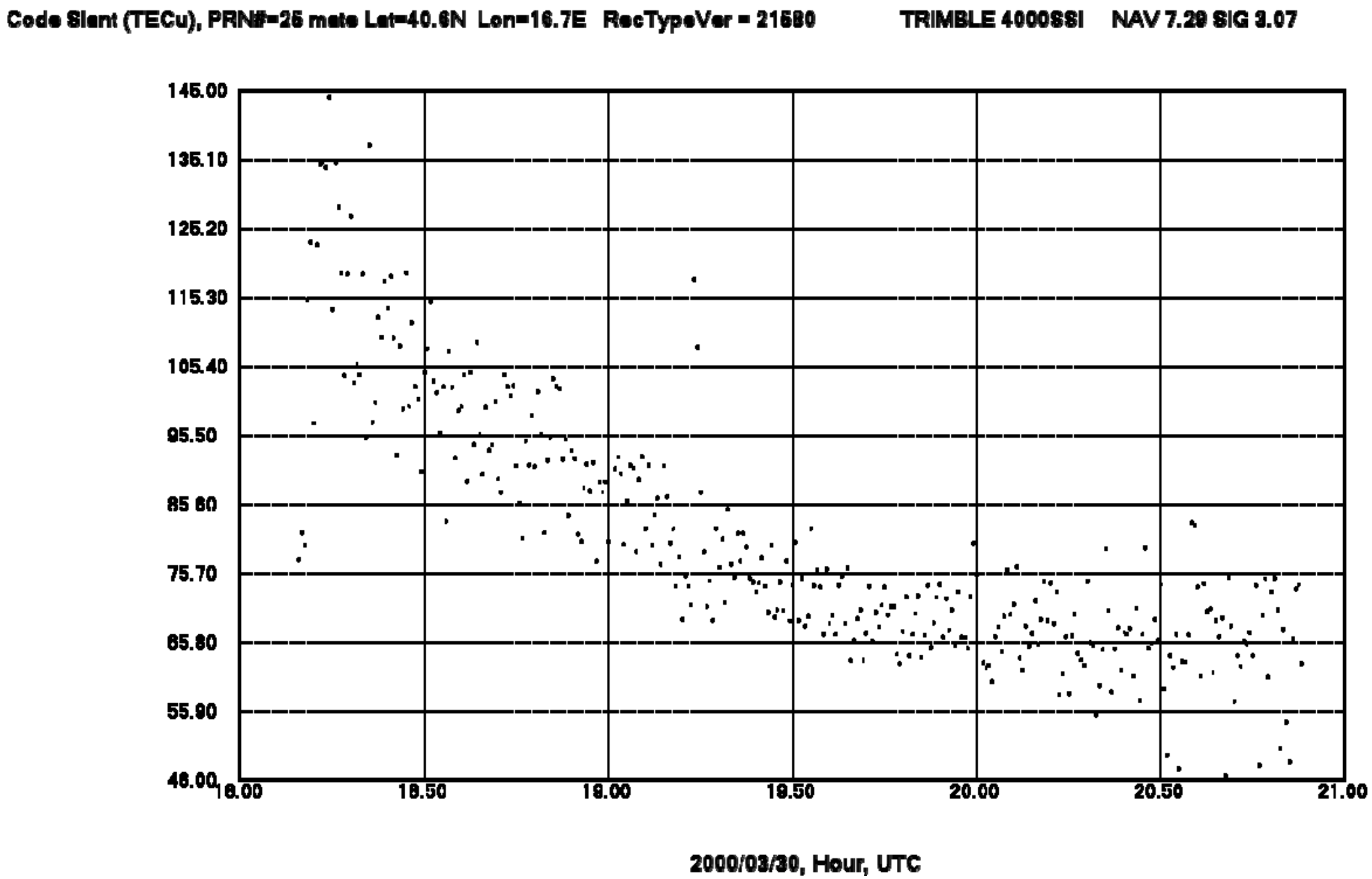
When needed for clarity, indexing will be explicitly adopted.

Plot of S_C arcs for one day

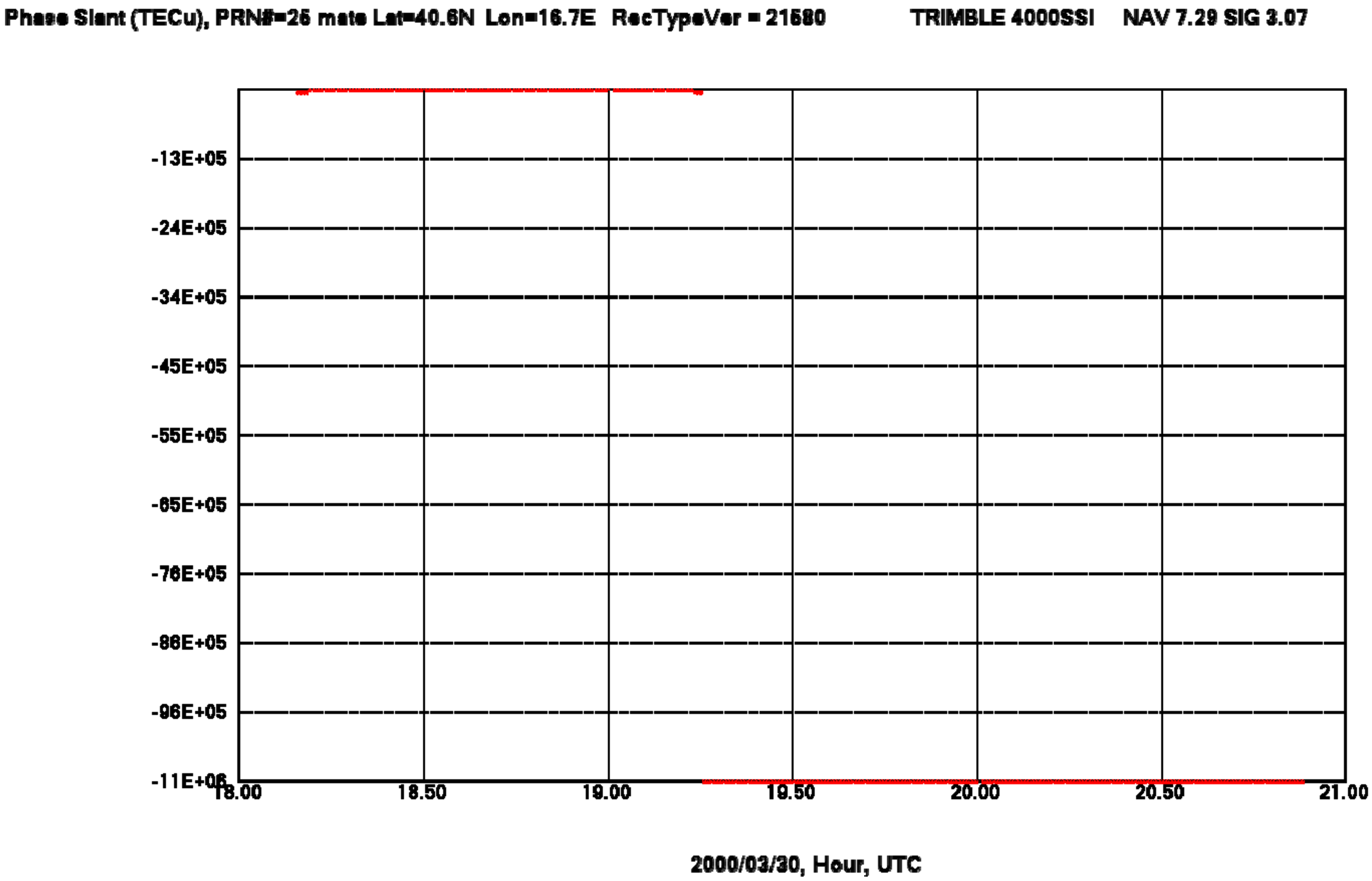


* Evidence that calibration is needed: TEC is a positive quantity

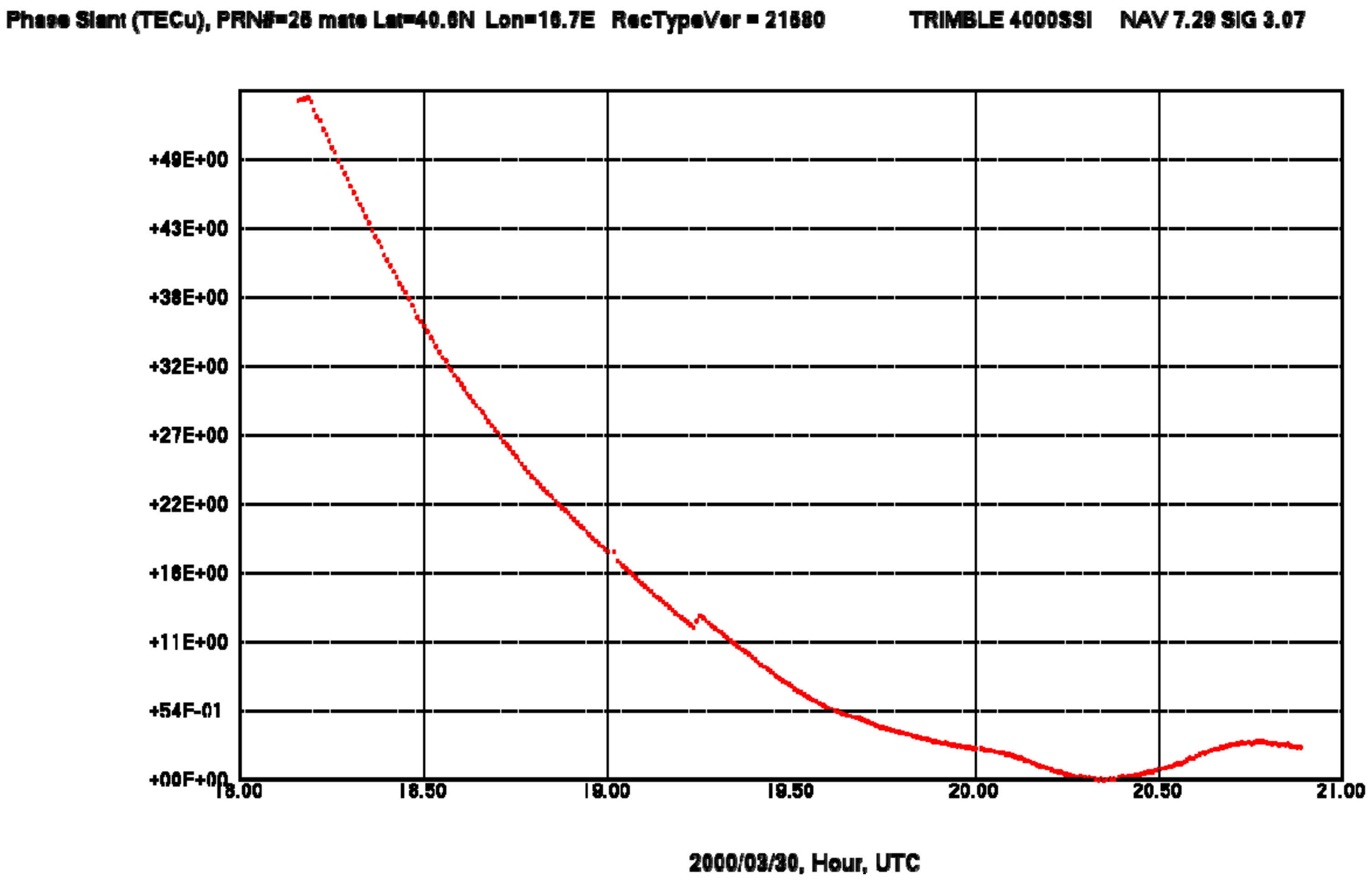
Sample S_C , one arc: the common situation



Sample $\boldsymbol{S_P}$, one arc: the common situation (phase jumps)



Sample $\boldsymbol{S_P}$, one arc$_{„}$: after removing jumps, fixing the minimum to zero



Offset Ω is an arbitrary quantity: can we set it in some useful way?

A new set of observables: Phase slants leveled to Code

Operator **<·>** is a properly selected weighted (possibly robust) average

Build, arc by arc, the <u>leveled</u> slants $\boldsymbol{S_L}$

$$\boldsymbol{S_L = S_P - < S_P - S_C >}$$

$$\boldsymbol{< S_P - S_C > = \Omega - < m > - \beta - \gamma}$$

$$\boldsymbol{S_L = TEC + < m > + \beta + \gamma}$$

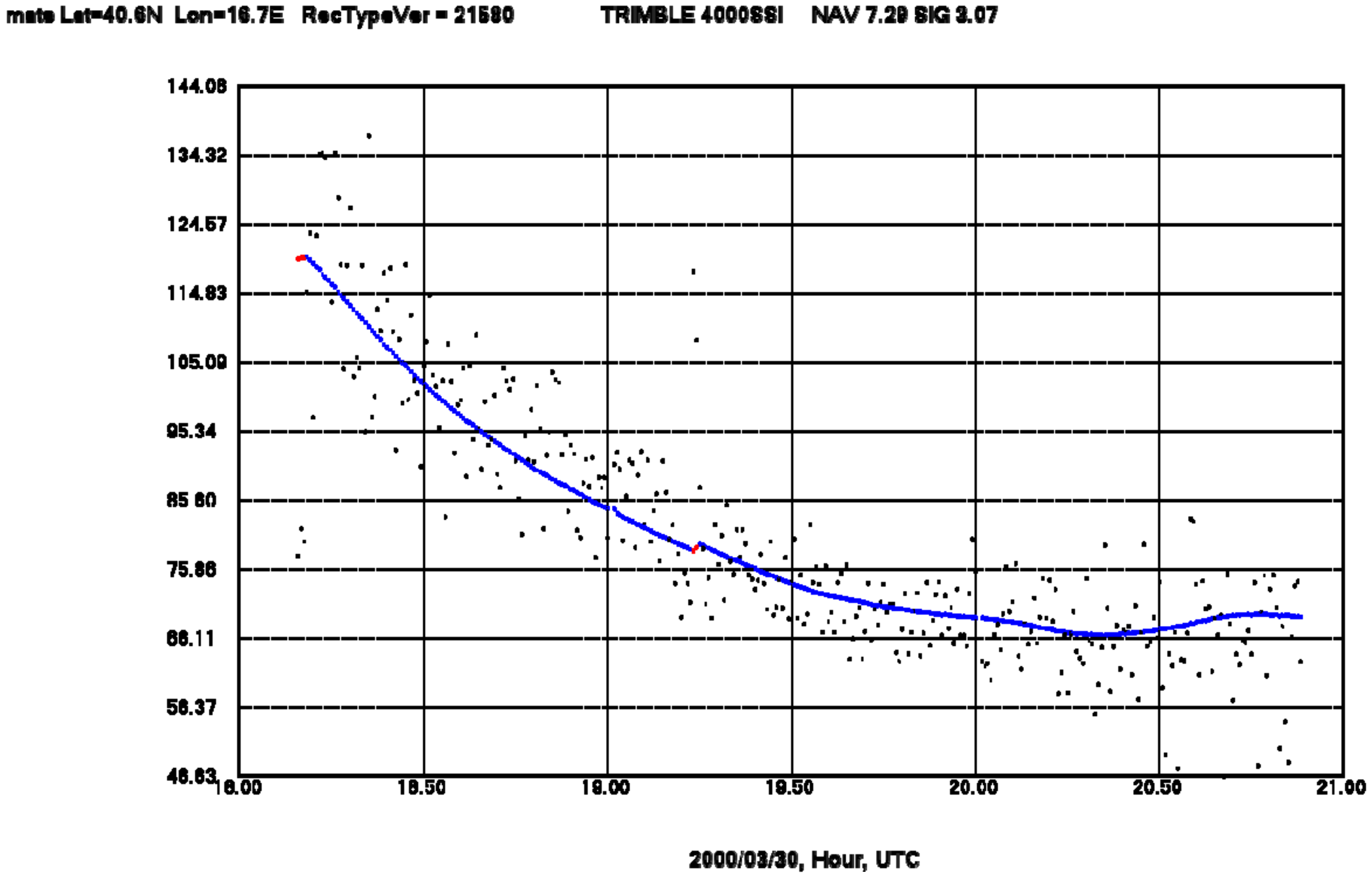
Properties of $\boldsymbol{S_L}$

Noise is the same (neglected) of phase slants

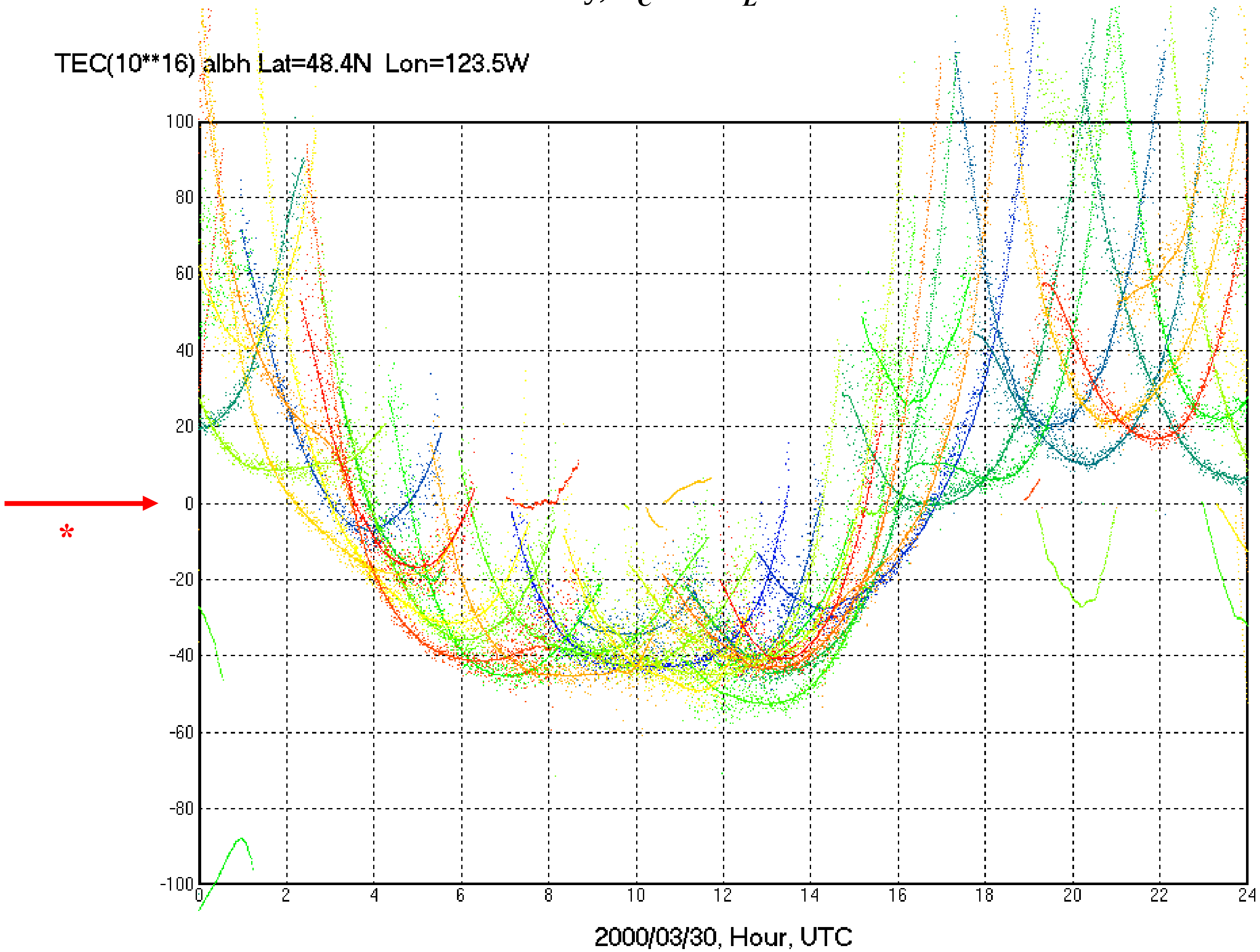
Biased exactly as code slants

But: an arc dependent constant leveling error $\lambda = \boldsymbol{< n > + < m >}$ appears

Sample $\boldsymbol{S_C}$ and $\boldsymbol{S_P}$ with properly selected phase offset $\boldsymbol{\Omega = S_L}$



One day, S_C and S_L arcs



*** Evidence that calibration is needed: TEC is a positive quantity**

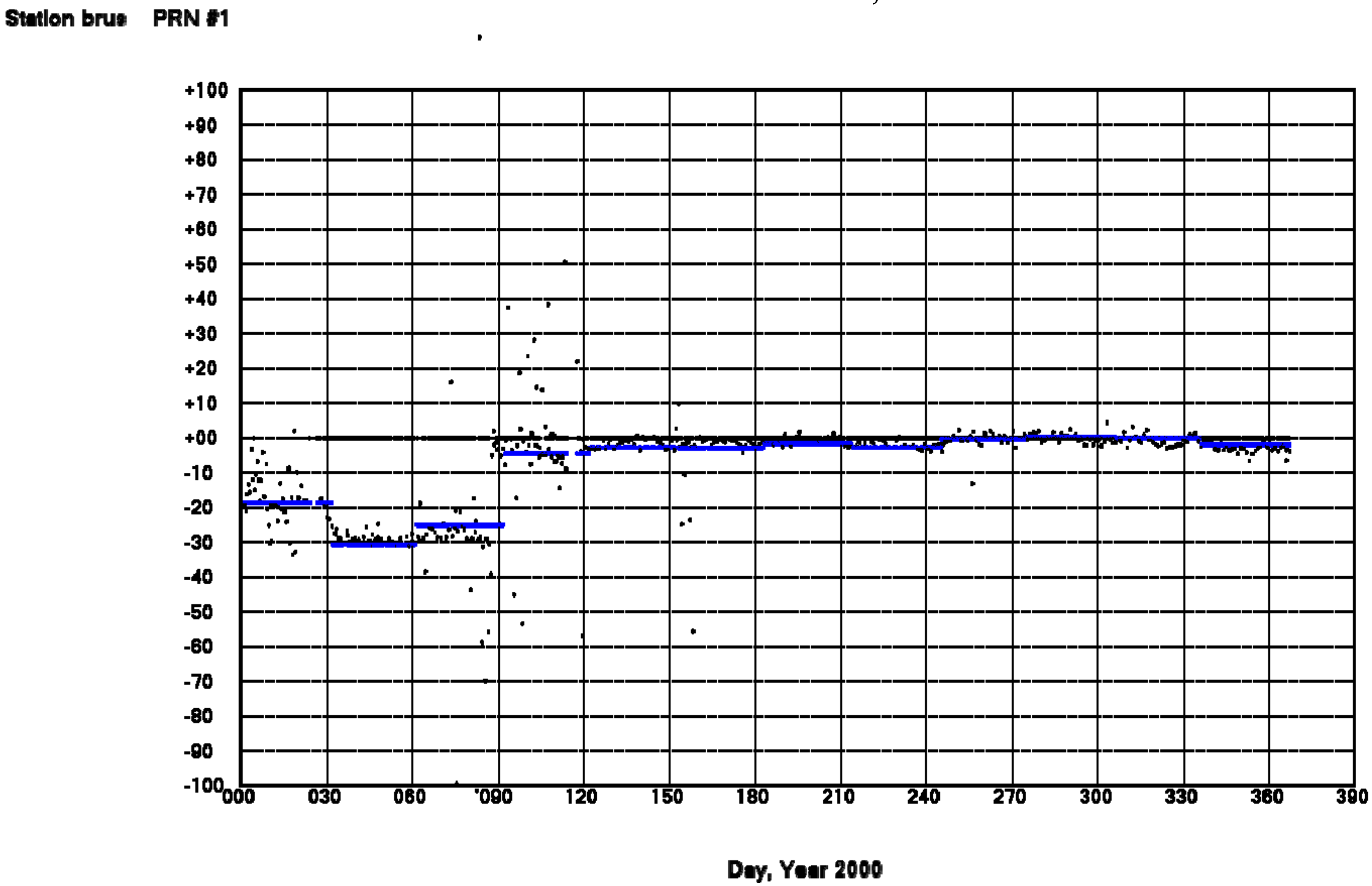
How do traditional and proposed solution compare?

In the following slides it can be seen that the two solutions agree in the average, but the difference in bias can amount to 10 ***TECu***

The pattern of the jumps, similar for different satellites, simply indicates that something has changed in the receiver

CODE Station + Satellite Biases

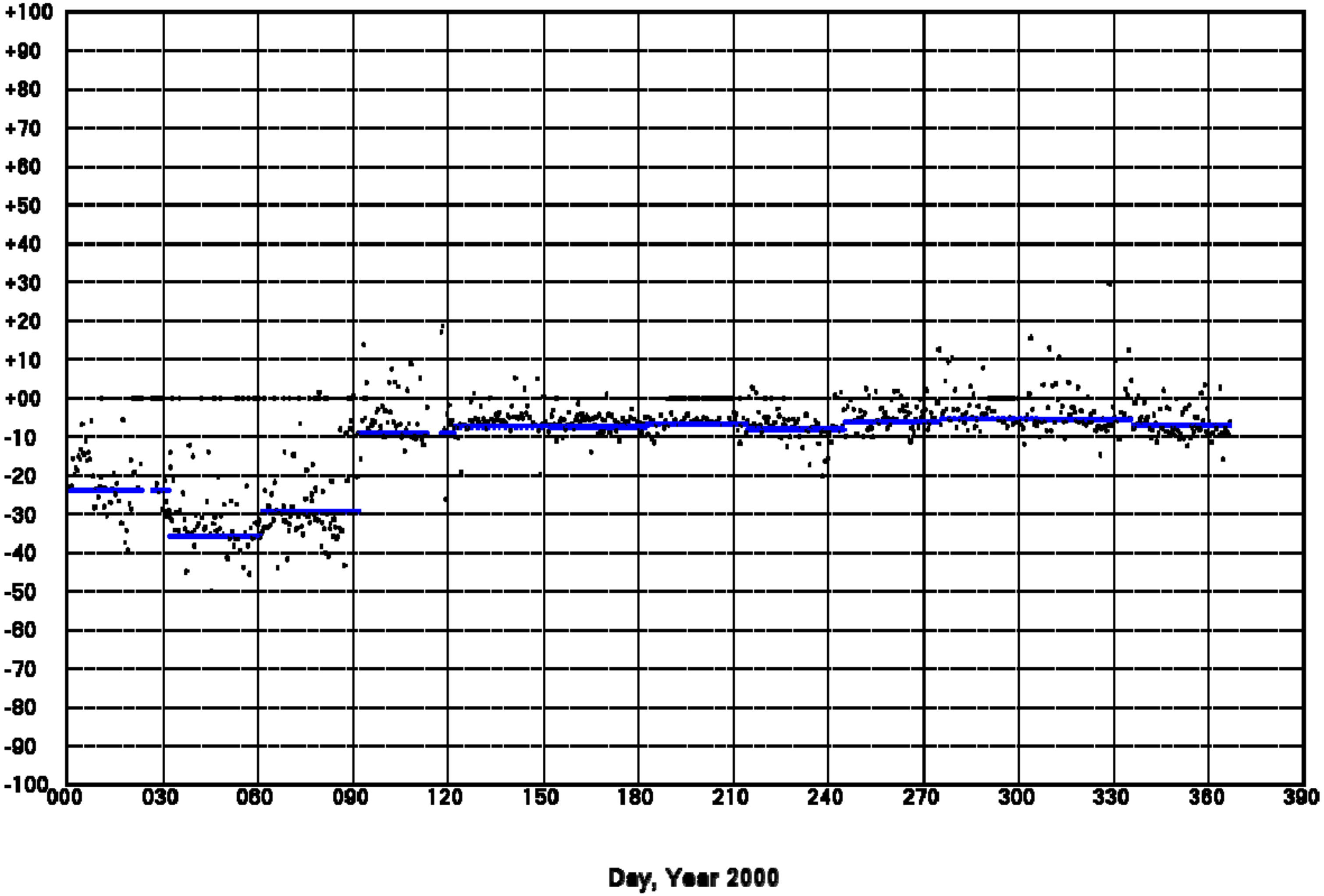
Arc offset solution, individual values



CODE Station + Satellite Biases

Station brus PRN #4

Arc offset solution, individual values



Thank you